

11.1

GIVEN:  $X = 4t^4 - 6t^3 + 2t - 1$   $x$ -m,  $t$ -sFIND:  $X$ ,  $V$ , AND  $A$  AT  $t = 2$  S

HAVE ..  $X = 4t^4 - 6t^3 + 2t - 1$   
 THEN  $V = \frac{dX}{dt} = 16t^3 - 18t^2 + 2$   
 AND  $A = \frac{dV}{dt} = 48t^2 - 36t$

AT  $t = 2$  S:  $X_2 = 4(2)^4 - 6(2)^3 + 2(2) - 1$  OR  $X_2 = 19$  m  
 $V_2 = 16(2)^3 - 18(2)^2 + 2$  OR  $V_2 = 58 \frac{\text{m}}{\text{s}}$   
 $A_2 = 48(2)^2 - 36(2)$  OR  $A_2 = 120 \frac{\text{m}}{\text{s}^2}$

11.2

GIVEN:  $X = 3t^4 + 4t^3 - 7t^2 - 5t + 8$   $x$ -mm,  $t$ -sFIND:  $X$ ,  $V$ , AND  $A$  AT  $t = 3$  S

HAVE ..  $X = 3t^4 + 4t^3 - 7t^2 - 5t + 8$   
 THEN  $V = \frac{dX}{dt} = 12t^3 + 12t^2 - 14t - 5$   
 AND  $A = \frac{dV}{dt} = 36t^2 + 24t - 14$

AT  $t = 3$  S:  $X_3 = 3(3)^4 + 4(3)^3 - 7(3)^2 - 5(3) + 8$  OR  $X_3 = 281$  mm  
 $V_3 = 12(3)^3 + 12(3)^2 - 14(3) - 5$  OR  $V_3 = 385 \frac{\text{mm}}{\text{s}}$   
 $A_3 = 36(3)^2 + 24(3) - 14$  OR  $A_3 = 382 \frac{\text{mm}}{\text{s}^2}$

11.3

GIVEN:  $X = 6t^2 - 8 + 40 \cos \pi t$   $x$ -in.,  $t$ -sFIND:  $X$ ,  $V$ , AND  $A$  AT  $t = 6$  S

HAVE ..  $X = 6t^2 - 8 + 40 \cos \pi t$   
 THEN  $V = \frac{dX}{dt} = 12t - 40\pi \sin \pi t$   
 AND  $A = \frac{dV}{dt} = 12 - 40\pi^2 \cos \pi t$

AT  $t = 6$  S:  $X_6 = 6(6)^2 - 8 + 40 \cos 6\pi$  OR  $X_6 = 248$  in.  
 $V_6 = 12(6) - 40\pi \sin 6\pi$  OR  $V_6 = 72 \frac{\text{in.}}{\text{s}}$   
 $A_6 = 12 - 40\pi^2 \cos 6\pi$  OR  $A_6 = -383 \frac{\text{in.}}{\text{s}^2}$

11.4

GIVEN:  $X = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$   $x$ -ft,  $t$ -sFIND:  $t$ ,  $X$ , AND  $A$  WHEN  $V = 0$ 

HAVE ..  $X = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$   
 THEN  $V = \frac{dX}{dt} = 5t^2 - 5t - 30$   
 AND  $A = \frac{dV}{dt} = 10t - 5$

WHEN  $V = 0$ :  $5t^2 - 5t - 30 = 5(t^2 - t - 6) = 0$   
 OR  $t = 3$  S AND  $t = -2$  S (REJECT)  $\therefore t = 3$  S

AT  $t = 3$  S:  $X_3 = \frac{5}{3}(3)^3 - \frac{5}{2}(3)^2 - 30(3) + 8$  OR  $X_3 = -59.5$  ft  
 $A_3 = 10(3) - 5$  OR  $A_3 = 25 \frac{\text{ft}}{\text{s}^2}$

11.5

GIVEN:  $X = 6t^4 - 2t^3 - 12t^2 + 3t + 3$   $x$ -m,  $t$ -sFIND:  $t$ ,  $X$ , AND  $V$  WHEN  $A = 0$ 

HAVE ..  $X = 6t^4 - 2t^3 - 12t^2 + 3t + 3$   
 THEN  $V = \frac{dX}{dt} = 24t^3 - 6t^2 - 24t + 3$   
 AND  $A = \frac{dV}{dt} = 72t^2 - 12t - 24$

WHEN  $A = 0$ :  $72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$   
 OR  $(3t - 2)(2t + 1) = 0$   
 OR  $t = \frac{2}{3}$  S AND  $t = -\frac{1}{2}$  S (REJECT)  $\therefore t = 0.667$  S

AT  $t = \frac{2}{3}$  S:  $X_{\frac{2}{3}} = 6(\frac{2}{3})^4 - 2(\frac{2}{3})^3 - 12(\frac{2}{3})^2 + 3(\frac{2}{3}) + 3$  OR  $X_{\frac{2}{3}} = 0.259$  m  
 $V_{\frac{2}{3}} = 24(\frac{2}{3})^3 - 6(\frac{2}{3})^2 - 24(\frac{2}{3}) + 3$  OR  $V_{\frac{2}{3}} = -8.56 \frac{\text{m}}{\text{s}}$

11.6

GIVEN:  $X = 3t^3 - 6t^2 - 12t + 5$   $x$ -m,  $t$ -sFIND: (a)  $t$  WHEN  $V = 0$ (b)  $X$ ,  $A$ , TOTAL DISTANCE TRAVELED WHEN  $t = 4$  S

HAVE ..  $X = 3t^3 - 6t^2 - 12t + 5$   
 THEN  $V = \frac{dX}{dt} = 9t^2 - 12t - 12$   
 AND  $A = \frac{dV}{dt} = 18t - 12$

(a) WHEN  $V = 0$ :  $9t^2 - 12t - 12 = 3(3t^2 - 4t - 4) = 0$

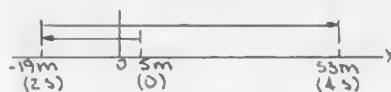
OR  $(3t + 2)(t - 2) = 0$

OR  $t = 2$  S AND  $t = -\frac{2}{3}$  S (REJECT)  $\therefore t = 2$  S

(b) AT  $t = 4$  S:  $X = 3(4)^3 - 6(4)^2 - 12(4) + 5$  OR  $X_4 = 53$  m  
 $A = 18(4) - 12$  OR  $A_4 = 60 \frac{\text{m}}{\text{s}^2}$

FIRST OBSERVE THAT ..  $0 \leq t < 2$  S:  $V < 0$   
 $t > 2$  S:  $V > 0$

NOW .. AT  $t = 0$ :  $X_0 = 5$  m  
 $t = 2$  S:  $X_2 = 3(2)^3 - 6(2)^2 - 12(2) + 5 = -19$  m



THEN  $|X_2 - X_0| = |-19 - 5| = 24$  m

$X_4 - X_2 = 53 - (-19) = 72$  m

$\therefore$  TOTAL DISTANCE TRAVELED =  $(24 + 72)$  m = 96 m

11.7

GIVEN:  $X = t^3 - 9t^2 + 24t - 8$   $x$ -in.,  $t$ -sFIND: (a)  $t$  WHEN  $V = 0$ (b)  $X$  AND TOTAL DISTANCE TRAVELED WHEN  $A = 0$ 

HAVE ..  $X = t^3 - 9t^2 + 24t - 8$   
 THEN  $V = \frac{dX}{dt} = 3t^2 - 18t + 24$   
 AND  $A = \frac{dV}{dt} = 6t - 18$

(a) WHEN  $V = 0$ :  $3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0$   
 OR  $(t - 2)(t - 4) = 0$

OR  $t = 2$  S AND  $t = 4$  S

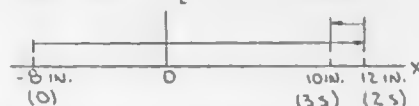
(b) WHEN  $A = 0$ :  $6t - 18 = 0$  OR  $t = 3$  S

AT  $t = 3$  S:  $X_3 = (3)^3 - 9(3)^2 + 24(3) - 8$  OR  $X_3 = 10$  in.

FIRST OBSERVE THAT ..  $0 \leq t < 2$  S:  $V > 0$   
 $2 < t \leq 3$  S:  $V < 0$

NOW .. AT  $t = 0$ :  $X_0 = -8$  in.

AT  $t = 2$  S:  $X_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12$  in.



THEN  $X_2 - X_0 = 12 - (-8) = 20$  in.

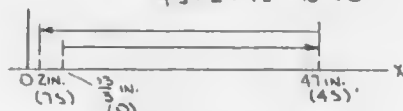
$|X_3 - X_2| = |10 - 12| = 2$  in.

$\therefore$  TOTAL DISTANCE TRAVELED =  $(20 + 2)$  in. = 22 in.

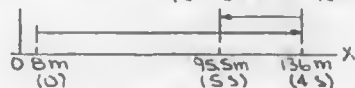
11.8

GIVEN:  $x = t^3 - 6t^2 - 36t - 40$   $x = 0$ ,  $t = 5$ FIND: (a)  $t$  WHEN  $v = 0$ (b)  $v$ ,  $a$ , AND TOTAL DISTANCE TRAVELED WHEN  $x = 0$ HAVE --  $x = t^3 - 6t^2 - 36t - 40$ THEN  $v = \frac{dx}{dt} = 3t^2 - 12t - 36$ AND  $a = \frac{dv}{dt} = 6t - 12$ (a) WHEN  $v = 0$ :  $3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$ OR  $(t+2)(t-6) = 0$ OR  $t = -2$  (REJECT) AND  $t = 6$   $\therefore t = 6$  s(b) WHEN  $x = 0$ :  $t^3 - 6t^2 - 36t - 40 = 0$ FACTORS --  $(t-10)(t+2)(t+2) = 0$  OR  $t = 10$  sNOW OBSERVE THAT..  $0 \leq t < 6$  s:  $v < 0$  $6 \leq t \leq 10$  s:  $v > 0$ AND AT  $t = 0$ :  $x_0 = -40$  ft $t = 6$  s:  $x_6 = (6)^3 - 6(6)^2 - 36(6) - 40 = -256$  ft $t = 10$  s:  $x_{10} = 3(10)^3 - 12(10) - 36 = 144$  ft $a_0 = 6(0) - 12$  OR  $a_0 = -12$  ft/s<sup>2</sup>THEN  $|x_6 - x_0| = |-256 - (-40)| = 216$  ft $x_{10} - x_6 = 144 - (-256) = 400$  ft $\therefore$  TOTAL DISTANCE TRAVELED =  $(216 + 400)$  ft = 616 ft

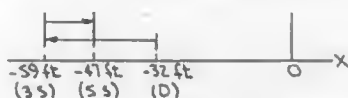
11.10

GIVEN:  $a = kt$ ; AT  $t = 0$ ,  $v = 16$  m/s; AT  $t = 1$  s,  $v = 15$  m/s,  $x = 20$  m.FIND:  $v$ ,  $x$ , AND TOTAL DISTANCE TRAVELED AT  $t = 7$  sHAVE --  $a = kt$   $k = \text{CONSTANT}$ NOW  $\frac{dv}{dt} = a = kt$ AT  $t = 0$ ,  $v = 16$  m/s:  $\int_{16}^v dv = \int_0^t kt dt$ OR  $v - 16 = \frac{1}{2}kt^2$ OR  $v = 16 + \frac{1}{2}kt^2$  (m/s)AT  $t = 1$  s,  $v = 15$  m/s:  $15 = 16 + \frac{1}{2}k(1)^2$ OR  $k = -2$  m/s<sup>3</sup> AND  $v = 16 - t^2$ ALSO  $\frac{dx}{dt} = v = 16 - t^2$ AT  $t = 1$  s,  $x = 20$  m:  $\int_{20}^x dx = \int_1^t (16 - t^2) dt$ OR  $x - 20 = [16t - \frac{1}{3}t^3]$ OR  $x = -\frac{1}{3}t^3 + 16t + \frac{17}{3}$  (m)THEN.. AT  $t = 7$  s:  $v_7 = 16 - (7)^2$  OR  $v_7 = -33$  m/s $x_7 = -\frac{1}{3}(7)^3 + 16(7) + \frac{17}{3}$  OR  $x_7 = 2.00$  mWHEN  $v = 0$ :  $16 - t^2 = 0$  OR  $t = 4$  sAT  $t = 0$ :  $x_0 = \frac{17}{3}$  $t = 4$  s:  $x_4 = -\frac{1}{3}(4)^3 + 16(4) + \frac{17}{3} = 47$  m.NOW OBSERVE THAT..  $0 \leq t < 4$  s:  $v > 0$  $4 \leq t \leq 7$  s:  $v < 0$ THEN  $x_4 - x_0 = 47 - \frac{17}{3} = 42.67$  m. $|x_7 - x_4| = |2 - 47| = 45$  m. $\therefore$  TOTAL DISTANCE TRAVELED =  $(42.67 + 45)$  m = 87.7 m.

11.11

GIVEN:  $a = A - 6t^2$ ; AT  $t = 0$ ,  $x = 8$  m,  $v = 0$ ;AT  $t = 1$  s,  $v = 30$  m/sFIND: (a)  $t$  WHEN  $v = 0$ (b) TOTAL DISTANCE TRAVELED WHEN  $t = 5$  sHAVE --  $a = A - 6t^2$   $A = \text{CONSTANT}$ NOW  $\frac{dv}{dt} = a = A - 6t^2$ AT  $t = 0$ ,  $v = 0$ :  $\int_0^v dv = \int_0^t (A - 6t^2) dt$ OR  $v = At - 2t^3$  (m/s)AT  $t = 1$  s,  $v = 30$  m/s:  $30 = A(1) - 2(1)^3$ OR  $A = 32$  m/s<sup>2</sup> AND  $v = 32t - 2t^3$ ALSO  $\frac{dx}{dt} = v = 32t - 2t^3$ AT  $t = 0$ ,  $x = 8$  m:  $\int_8^x dx = \int_0^t (32t - 2t^3) dt$ OR  $x = 8 + 16t^2 - \frac{1}{2}t^4$  (m)(a) WHEN  $v = 0$ :  $32t - 2t^3 = 2t(16 - t^2) = 0$ OR  $t = 0$  AND  $t = 4$  s(b) AT  $t = 4$  s:  $x_4 = 8 + 16(4)^2 - \frac{1}{2}(4)^4 = 136$  m $t = 5$  s:  $x_5 = 8 + 16(5)^2 - \frac{1}{2}(5)^4 = 95.5$  mNOW OBSERVE THAT  $0 < t < 4$  s:  $v > 0$  $4 \leq t \leq 5$  s:  $v < 0$ THEN  $x_4 - x_0 = 136 - 8 = 128$  m $|x_5 - x_4| = |95.5 - 136| = 40.5$  m $\therefore$  TOTAL DISTANCE TRAVELED =  $(128 + 40.5)$  m = 168.5 m

11.9

GIVEN:  $a = 6$  ft/s<sup>2</sup>; AT  $t = 0$ ,  $x = -32$  ft;AT  $t = 2$  s,  $v = -6$  ft/sFIND:  $v$ ,  $x$ , AND TOTAL DISTANCE TRAVELED AT  $t = 5$  sHAVE --  $\frac{dv}{dt} = a = 6$  ft/s<sup>2</sup>AT  $t = 2$  s,  $v = -6$  ft/s:  $\int_{-6}^v dv = \int_2^t 6 dt$ OR  $v - (-6) = 6(t - 2)$ OR  $v = 6t - 18$  (ft/s)ALSO..  $\frac{dx}{dt} = v = 6t - 18$ AT  $t = 0$ ,  $x = -32$  ft:  $\int_{-32}^x dx = \int_0^t (6t - 18) dt$ OR  $x - (-32) = 3t^2 - 18t$ OR  $x = 3t^2 - 18t - 32$  (ft)AT  $t = 5$  s:  $v_5 = 6(5) - 18$  OR  $v_5 = 12$  ft/s $x_5 = 3(5)^2 - 18(5) - 32$  OR  $x_5 = -47$  ftWHEN  $v = 0$ :  $6t - 18 = 0$  OR  $t = 3$  sAT  $t = 3$  s:  $x_3 = 3(3)^2 - 18(3) - 32 = -59$  ftNOW OBSERVE THAT  $0 \leq t < 3$  s:  $v < 0$  $3 \leq t \leq 5$  s:  $v > 0$ THEN  $|x_5 - x_0| = |-47 - (-32)| = 15$  ft $|x_3 - x_5| = |-59 - (-47)| = 12$  ft $\therefore$  TOTAL DISTANCE TRAVELED =  $(15 + 12)$  ft = 27 ft



11.12

GIVEN:  $a = kt^2$ ; AT  $t=0$ ,  $x=24$  m; AT  $t=6$  s,  
 $x=96$  m,  $v=18$  m/s  
 FIND:  $x(t)$  AND  $v(t)$

HAVE..  $a = kt^2$   $k = \text{CONSTANT}$

NOW  $\frac{dv}{dt} = a = kt^2$

$$\text{AT } t=6 \text{ s, } v=18 \frac{\text{m}}{\text{s}}: \int_0^6 dv = \int_0^6 kt^2 dt$$

$$\text{OR } v-0 = \frac{1}{3}k(t^3-0)$$

$$\text{OR } v = \frac{1}{3}k(t^3-0) \quad (\frac{\text{m}}{\text{s}})$$

ALSO  $\frac{dx}{dt} = v = \frac{1}{3}k(t^3-0)$

$$\text{AT } t=0, x=24 \text{ m: } \int_{24}^x dx = \int_0^t (18 + \frac{1}{3}k(t^3-0)) dt$$

$$\text{OR } x-24 = 18t + \frac{1}{12}k(t^4-0)$$

$$\text{NOW.. AT } t=6 \text{ s, } x=96 \text{ m: } 96-24 = 18(6) + \frac{1}{12}k(6^4-0)$$

$$\text{OR } k = \frac{1}{3} \frac{\text{m}}{\text{s}^2}$$

$$\text{THEN.. } x-24 = 18t + \frac{1}{12}(\frac{1}{3})(t^4-0)$$

$$\text{OR } x(t) = \frac{1}{108}t^4 + 18t + 24 \quad \blacktriangleleft$$

$$\text{AND } v = 18 + \frac{1}{3}(\frac{1}{3})(t^3-0)$$

$$\text{OR } v(t) = \frac{1}{27}t^3 + 18 \quad \blacktriangleleft$$

11.13

GIVEN: FOR  $2 \leq t \leq 10$  s,  $a = \frac{t^2}{2}$ ; AT  $t=2$  s,  
 $v = -15$  m/s; AT  $t=10$  s,  $v = 0.36$  m/s;  
 $|x_2| = 2|x_0|$

FIND: (a)  $x$  AT  $t=2$  s AND AT  $t=10$  s  
 (b) TOTAL DISTANCE TRAVELED FROM  
 $t=2$  s TO  $t=10$  s

HAVE..  $a = \frac{t^2}{2}$   $k = \text{CONSTANT}$

NOW  $\frac{dv}{dt} = a = \frac{t^2}{2}$

$$\text{AT } t=2 \text{ s, } v = -15 \frac{\text{m}}{\text{s}}: \int_{-15}^v dv = \int_2^t \frac{t^2}{2} dt$$

$$\text{OR } v - (-15) = \frac{1}{6}(t^3 - 2^3)$$

$$\text{OR } v = \frac{1}{6}(t^3 - 8) - 15 \quad (\frac{\text{m}}{\text{s}})$$

$$\text{AT } t=10 \text{ s, } v = 0.36 \frac{\text{m}}{\text{s}}: 0.36 = \frac{1}{6}(\frac{1}{6}(10^3 - 8) - 15)$$

$$\text{OR } k = 128 \frac{\text{m}}{\text{s}^2}$$

$$\text{AND } v = 1 - \frac{t^2}{2} \quad (\frac{\text{m}}{\text{s}})$$

(a) HAVE  $\frac{dx}{dt} = v = 1 - \frac{t^2}{2}$

THEN  $\int dx = \int (1 - \frac{t^2}{2}) dt = C$   $C = \text{CONSTANT}$

$$\text{OR } x = t - \frac{t^3}{6} + C \quad (\text{m})$$

$$\text{NOW } x_2 = 2x_0: 2 + \frac{1}{6}(10^3 - 8) + C = 2(2 + \frac{1}{6}(2^3 - 8) + C)$$

$$\text{OR } C = 1.2 \text{ m}$$

$$\text{AND } x = t - \frac{t^3}{6} + 1.2 \quad (\text{m})$$

$$\therefore \text{AT } t=2 \text{ s: } x_2 = 2 - \frac{8}{6} + 1.2 \quad \text{OR } x_2 = 35.2 \text{ m} \quad \blacktriangleleft$$

$$t=10 \text{ s: } x_{10} = 10 - \frac{1000}{6} + 1.2 \quad \text{OR } x_{10} = 17.6 \text{ m} \quad \blacktriangleleft$$

NOTE: A SECOND SOLUTION EXISTS FOR THE CASE

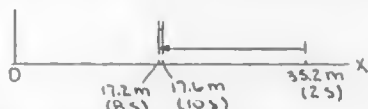
$$x_2 > 0, x_{10} < 0. \text{ FOR THIS CASE } C = -22 \frac{1}{3} \text{ m}$$

$$\text{AND } x_2 = 11 \frac{1}{3} \text{ m}, x_{10} = -5 \frac{13}{3} \text{ m}$$

(b) WHEN  $v=0$ :  $1 - \frac{t^2}{2} = 0$  OR  $t = 8$  s

$$\text{AT } t=8 \text{ s: } x_8 = 8 - \frac{512}{6} + 1.2 = 17.2 \text{ m}$$

NOW OBSERVE THAT  $2 \leq t < 8$  s:  $v < 0$   
 $8 \leq t \leq 10$  s:  $v > 0$



$$\text{THEN } |x_8 - x_2| = |17.2 - 35.2| = 18 \text{ m}$$

$$x_{10} - x_8 = 17.6 - 17.2 = 0.4 \text{ m}$$

$$\therefore \text{TOTAL DISTANCE TRAVELED} = (18 + 0.4) \text{ m} = 18.4 \text{ m} \quad \blacktriangleleft$$

NOTE: THE TOTAL DISTANCE TRAVELED IS THE SAME  
 FOR BOTH CASES.

11.14

GIVEN:  $a = -8 \frac{\text{m}}{\text{s}^2}$ ; AT  $t=4$  s,  $x=20$  m;  
 WHEN  $v=16$  m/s,  $x=4$  m

FIND: (a)  $t$  WHEN  $v=0$

(b)  $v$  AND TOTAL DISTANCE TRAVELED  
 AT  $t=11$  s

HAVE  $\frac{dv}{dt} = a = -8 \frac{\text{m}}{\text{s}^2}$

THEN  $\int dv = \int -8 dt + C$   $C = \text{CONSTANT}$

$$\text{OR } v = -8t + C \quad (\frac{\text{m}}{\text{s}})$$

ALSO  $\frac{dx}{dt} = v = -8t + C$

$$\text{AT } t=4 \text{ s, } x=20 \text{ m: } \int_{20}^x dx = \int_4^t (-8t + C) dt$$

$$\text{OR } x-20 = [-4t^2 + Ct]_4^t$$

$$\text{OR } x = -4t^2 + C(t-4) + 84 \quad (\text{m})$$

$$\text{WHEN } v=16 \frac{\text{m}}{\text{s}}, x=4 \text{ m: } 16 = -8t + C \Rightarrow C = 16 + 8t$$

$$4 = -4t^2 + C(t-4) + 84$$

$$\text{COMBINING.. } 0 = -4t^2 + (16+8t)(t-4) + 80$$

$$\text{SIMPLIFYING.. } t^2 - 4t + 4 = 0$$

$$\text{OR } t = 2 \text{ s}$$

$$\text{AND } C = 32 \frac{\text{m}}{\text{s}}$$

$$v = -8t + 32 \quad (\frac{\text{m}}{\text{s}})$$

$$x = -4t^2 + 32t - 44 \quad (\text{m})$$

$$(a) \text{ WHEN } v=0: -8t + 32 = 0 \quad \text{OR } t = 4 \text{ s} \quad \blacktriangleleft$$

$$(b) \text{ AT } t=0: x_0 = -44 \text{ m}$$

$$t=4 \text{ s: } x_4 = 20 \text{ m}$$

$$t=11 \text{ s: } x_{11} = -4(11)^2 + 32(11) - 44 = -176 \text{ m}$$

NOW OBSERVE THAT  $0 \leq t < 4$  s:  $v > 0$

$$4 \leq t \leq 11 \text{ s: } v < 0$$



$$\text{THEN } x_4 - x_0 = 20 - (-44) = 64 \text{ m}$$

$$|x_{11} - x_4| = |-176 - 20| = 196 \text{ m}$$

$$\therefore \text{TOTAL DISTANCE TRAVELED} = (64 + 196) \text{ m} = 260 \text{ m} \quad \blacktriangleleft$$

11.15

GIVEN:  $a = k(100-x)$ ,  $k = \text{CONSTANT}$ ;  $v=0$

AT  $x=40$  mm,  $x=160$  mm; WHEN

$x=100$  mm,  $v=18$  mm/s

FIND: (a)  $k$

(b)  $v$  WHEN  $x=120$  mm

(a) HAVE  $v \frac{dv}{dx} = a = k(100-x)$

$$\text{WHEN } x=40 \text{ mm, } v=0: \int_0^v v dv = \int_{40}^x k(100-x) dx$$

$$\text{OR } \frac{1}{2}v^2 = k[100x - \frac{1}{2}x^2]_{40}^x$$

$$\text{OR } \frac{1}{2}v^2 = k(100x - \frac{1}{2}x^2 - 3200)$$

$$\text{WHEN } x=100 \text{ mm, } v=18 \frac{\text{mm}}{\text{s}}:$$

$$\frac{1}{2}(18)^2 = k[100(100) - \frac{1}{2}(100)^2 - 3200]$$

$$\text{OR } k = 0.09 \frac{\text{s}^2}{\text{mm}}$$

$$(b) \text{ WHEN } x=120 \text{ mm: } \frac{1}{2}v^2 = 0.09[100(120) - \frac{1}{2}(120)^2 - 3200]$$

$$= 144$$

$$\text{OR } v = \pm 16.97 \frac{\text{mm}}{\text{s}} \quad \blacktriangleleft$$

11.16

GIVEN:  $a = k/(x+4)^2$ ,  $k$  - CONSTANT; WHEN  $x=0$ ,  $v=0$ ; WHEN  $x=8$  m,  $v=4$  m/s

FIND: (a)  $k$   
(b)  $x$  WHEN  $v=4.5$  m/s  
(c)  $v_{\text{MAX}}$

(a) HAVE  $v \frac{dv}{dx} = a = \frac{k}{(x+4)^2}$   
WHEN  $x=0$ ,  $v=0$ :  $\int_0^v v dv = \int_0^x \frac{k}{(x+4)^2} dx$   
OR  $\frac{1}{2} v^2 = -k \left( \frac{1}{x+4} - \frac{1}{4} \right)$

WHEN  $x=8$  m,  $v=4$  m/s:  $\frac{1}{2} (4)^2 = -k \left( \frac{1}{8+4} - \frac{1}{4} \right)$   
OR  $k = 48 \frac{\text{m}^3}{\text{s}^2}$

(b) WHEN  $v=4.5$  m/s:  $\frac{1}{2} (4.5)^2 = -48 \left( \frac{1}{x+4} - \frac{1}{4} \right)$   
OR  $x = 21.6$  m

(c) NOTE THAT WHEN  $v = v_{\text{MAX}}$ ,  $a=0$ . NOW..  
 $a \rightarrow 0$  AS  $x \rightarrow \infty$  SO THAT  
 $\frac{1}{2} v_{\text{MAX}}^2 = 48 \lim_{x \rightarrow \infty} \left( \frac{1}{4} - \frac{1}{x+4} \right) = 48 \left( \frac{1}{4} \right)$   
OR  $v_{\text{MAX}} = 4.90$  m/s

11.17

GIVEN:  $a = 6x - 14$ ,  $a$  - ft/s<sup>2</sup>;  $x$  - ft;  
WHEN  $x=0$ ,  $v=4$  ft/s

FIND: (a)  $x_{\text{MAX}}$   
(b)  $v$  WHEN TOTAL DISTANCE TRAVELED = 1 ft

HAVE  $v \frac{dv}{dx} = a = 6x - 14$   
WHEN  $x=0$ ,  $v=4$  ft/s:  $\int_4^v v dv = \int_0^x (6x - 14) dx$   
OR  $\left[ \frac{1}{2} v^2 \right]_4^v = \left[ 3x^2 - 14x \right]_0^x$   
OR  $\frac{1}{2} v^2 = 3x^2 - 14x + 8$

(a) FIRST DETERMINE WHERE  $v=0$ ..  
 $3x^2 - 14x + 8 = (3x-2)(x-4) = 0$   
OR  $x = \frac{2}{3}$  ft AND  $x = 4$  ft

NOW OBSERVE THAT AS THE PARTICLE PASSES THROUGH  $x=0$ ,  $v>0$  AND  $a<0$  AND THAT AT  $x=\frac{2}{3}$  ft,  $v=0$  AND  $a<0$ . THUS, THE PARTICLE WILL NEVER REACH  $x=4$  ft AND, THEREFORE,

$$x_{\text{MAX}} = 0.667 \text{ ft}$$

(b) THE PARTICLE WILL HAVE TRAVELED A TOTAL DISTANCE OF 1 ft WHEN IT PASSES THROUGH  $x = \frac{1}{3}$  ft FOR THE SECOND TIME AND IS MOVING TO THE LEFT. THEN..

AT  $x = \frac{1}{3}$  ft:  $\frac{1}{2} v^2 = 3 \left( \frac{1}{3} \right)^2 - 14 \left( \frac{1}{3} \right) + 8 = \frac{11}{3}$   
OR  $v = 2.71$  ft/s

11.18 CONTINUED

NOW..  $\frac{v_B}{v_A} = 2$ :  $\frac{\frac{1}{2} v_B^2}{\frac{1}{2} v_A^2} = (2)^2 = \frac{k(31.5 - A \ln 8)}{k \left( \frac{3}{2} - A \ln 2 \right)}$

OR  $6 - 4A \ln 2 = 31.5 - A \ln 8$   
OR  $25.5 = A(\ln 8 - 4 \ln 2) = A(\ln 8 - \ln 2^4) = A \ln \left( \frac{8}{16} \right)$   
OR  $A = -36.8$  ft<sup>2</sup>

WHEN  $x=16$  ft,  $v=29$  ft/s:  $\frac{1}{2} (29)^2 = k \left[ \frac{1}{2} (16)^2 - \frac{25.5}{\ln(16)} \ln(16) - \frac{1}{2} \right]$

NOTING THAT  $\ln(16) = 4 \ln 2$  AND  $\ln \left( \frac{1}{2} \right) = -\ln(2)$   
HAVE ..  $841 = k \left[ 256 - \frac{25.5}{-\ln(2)} \cdot 4 \ln(2) - 1 \right]$   
OR  $k = 1.832 \frac{\text{ft}^2}{\text{s}^2}$

11.19

GIVEN:  $a = k(1 - e^{-x})$ ,  $k$  - CONSTANT;  
WHEN  $x=2$  m,  $v=6$  m/s; WHEN  $x=0$ ,  $v=0$

FIND: (a)  $k$   
(b)  $v$  WHEN  $x=-1$  m

(a) HAVE  $v \frac{dv}{dx} = a = k(1 - e^{-x})$   
WHEN  $x=2$  m,  $v=6$  m/s:  $\int_0^v v dv = \int_0^x k(1 - e^{-x}) dx$   
OR  $\frac{1}{2} (v^2 - 36) = k \left[ x + e^{-x} \right]_0^x$   
OR  $\frac{1}{2} v^2 = k(x + e^{-x} + 2 - e^{-2}) + 18$

WHEN  $x=0$ ,  $v=0$ :  $0 = k(1 + 2 - e^{-2}) + 18$   
OR  $k = 4.1011 \frac{\text{m}^2}{\text{s}^2}$

(b) WHEN  $x=-1$  m:  $\frac{1}{2} v^2 = 4.1011(-1 + 2 - e^{-2}) + 18$   
OR  $v = 2.43$  m/s

11.20

GIVEN:  $a = -(0.1 + \sin \frac{x}{0.8})$ ,  $a$  - m/s<sup>2</sup>;  $x$  - m;  
 $b = 0.8$  m; WHEN  $x=0$ ,  $v=1$  m/s

FIND: (a)  $v$  WHEN  $x=-1$  m  
(b)  $x$  WHERE  $v = v_{\text{MAX}}$   
(c)  $v_{\text{MAX}}$

HAVE  $v \frac{dv}{dx} = a = -(0.1 + \sin \frac{x}{0.8})$   
WHEN  $x=0$ ,  $v=1$  m/s:  $\int_1^v v dv = \int_0^x -(0.1 + \sin \frac{x}{0.8}) dx$   
OR  $\frac{1}{2} (v^2 - 1) = - \left[ 0.1x - 0.8 \cos \frac{x}{0.8} \right]_0^x$   
OR  $\frac{1}{2} v^2 = -0.1x + 0.8 \cos \frac{x}{0.8} - 0.3$

(a) WHEN  $x=-1$  m:  $\frac{1}{2} v^2 = -0.1(-1) + 0.8 \cos \frac{-1}{0.8} - 0.3$   
OR  $v = 2.323$  m/s

(b) WHEN  $v = v_{\text{MAX}}$ ,  $a=0$ :  $-(0.1 + \sin \frac{x}{0.8}) = 0$   
OR  $x = -0.080134$  m  $x = -0.0801$  m

(c) WHEN  $x = -0.080134$  m:  
 $\frac{1}{2} v_{\text{MAX}}^2 = -0.1(-0.080134) + 0.8 \cos \frac{-0.080134}{0.8} - 0.3$   
OR  $v_{\text{MAX}} = 1.004$  m/s

11.18

GIVEN:  $a = k \left( x - \frac{A}{x} \right)$ ,  $k$  AND  $A$  ARE CONSTANTS; AT  $t=0$ ,  $x=1$  ft,  $v=0$ ;  
WHEN  $x=16$  ft,  $v=29$  ft/s;  
 $v(x=8 \text{ ft}) = 2 \left( v(x=2 \text{ ft}) \right)$

FIND:  $A$  AND  $k$

HAVE  $v \frac{dv}{dx} = a = k \left( x - \frac{A}{x} \right)$   
WHEN  $x=1$  ft,  $v=0$ :  $\int_0^v v dv = \int_1^x k \left( x - \frac{A}{x} \right) dx$   
OR  $\frac{1}{2} v^2 = k \left[ \frac{1}{2} x^2 - A \ln x \right]_1^x$   
 $= k \left( \frac{1}{2} x^2 - A \ln x - \frac{1}{2} \right)$

AT  $x=2$  ft:  $\frac{1}{2} v_1^2 = k \left[ \frac{1}{2} (2)^2 - A \ln 2 - \frac{1}{2} \right] = k \left( \frac{3}{2} - A \ln 2 \right)$   
 $x=8$  ft  $\frac{1}{2} v_2^2 = k \left[ \frac{1}{2} (8)^2 - A \ln 8 - \frac{1}{2} \right] = k(31.5 - A \ln 8)$   
(CONTINUED)

11.21

GIVEN:  $a = 0.8\sqrt{v^2 + 49}$ ,  $a \sim m/s^2$ ,  $v \sim m/s$ WHEN  $x=0$ ,  $v=0$ FIND: (a)  $x$  WHEN  $v=24 m/s$ (b)  $v$  WHEN  $x=40 m$ 

HAVE  $v \frac{dv}{dx} = a = 0.8\sqrt{v^2 + 49}$   
 WHEN  $x=0$ ,  $v=0$ :  $\int_0^v \frac{v dv}{\sqrt{v^2 + 49}} = \int_0^x 0.8 dx$

OR  $[\sqrt{v^2 + 49}]_0^v = 0.8x$

OR  $\sqrt{v^2 + 49} - 7 = 0.8x$

(a) WHEN  $v=24 m/s$ :  $\sqrt{24^2 + 49} - 7 = 0.8x$

OR  $x = 22.5 m$

(b) WHEN  $x=40 m$ :  $\sqrt{v^2 + 49} - 7 = 0.8(40)$

OR  $v = 38.4 m/s$

11.22

GIVEN:  $a = -k\sqrt{v}$ ,  $k = \text{CONSTANT}$ ; AT  $t=0$ ,  
 $x=0$ ,  $v=81 m/s$ ; WHEN  $x=18 m$ ,  
 $v=36 m/s$

FIND: (a)  $v$  WHEN  $x=20 m$ (b)  $t$  WHEN  $v=0$ 

(a) HAVE  $v \frac{dv}{dx} = a = -k\sqrt{v}$

SO THAT  $\sqrt{v} dv = -k dx$

WHEN  $x=0$ ,  $v=81 m/s$ :  $\int_{81}^v \sqrt{v} dv = \int_0^x -k dx$

OR  $\frac{2}{3} [v^{3/2}]_{81}^v = -kx$

OR  $\frac{2}{3} (v^{3/2} - 729) = -kx$

WHEN  $x=18 m$ ,  $v=36 m/s$ :  $\frac{2}{3} (36^{3/2} - 729) = -k(18)$

OR  $k = 19 \frac{m}{s^{3/2}}$

FINALLY.. WHEN  $x=20 m$ :  $\frac{2}{3} (v^{3/2} - 729) = -19(20)$

OR  $v^{3/2} = 159$

$v = 29.3 m/s$

(b) HAVE  $\frac{dv}{dt} = a = -19\sqrt{v}$

AT  $t=0$ ,  $v=81 m/s$ :  $\int_{81}^0 \frac{dv}{\sqrt{v}} = \int_0^t -19 dt$

OR  $2[\sqrt{v}]_{81}^0 = -19t$

OR  $2(\sqrt{0} - 9) = -19t$

WHEN  $v=0$ :  $2(-9) = -19t$

OR  $t = 0.947 s$

11.23

GIVEN:  $a = -kv^{1.5}$ ,  $k = \text{CONSTANT}$ ; AT  $t=0$ ,  
 $x=0$ ,  $v=16 in/s$ ; WHEN  $x=6 in$ ,  
 $v=4 in/s$

FIND: (a)  $v$  WHEN  $x=5 in$ (b)  $t$  WHEN  $v=9 in/s$ 

(a) HAVE  $v \frac{dv}{dx} = a = -kv^{1.5}$

SO THAT  $v^{1.5} dv = -k dx$

WHEN  $x=0$ ,  $v=16 in/s$ :  $\int_{16}^v v^{1.5} dv = \int_0^x -k dx$

OR  $-\frac{2}{3} [v^{-0.5}]_{16}^v = -kx$

OR  $2(\frac{1}{\sqrt{v}} - \frac{1}{4}) = kx$

WHEN  $x=6 in$ ,  $v=4 in/s$ :  $2(\frac{1}{\sqrt{4}} - \frac{1}{4}) = k(6)$

OR  $k = \frac{1}{12} \frac{1}{in/s}$

FINALLY.. WHEN  $x=5 in$ :  $2(\frac{1}{\sqrt{v}} - \frac{1}{4}) = \frac{1}{12}(5)$

OR  $\frac{1}{\sqrt{v}} = \frac{11}{24}$

$v = 4.76 in/s$

(CONTINUED)

11.23 CONTINUED

(b) HAVE  $\frac{dv}{dt} = a = -\frac{1}{12} v^{1.5}$

AT  $t=0$ ,  $v=16 in/s$ :  $\int_{16}^0 v^{-1.5} dv = \int_0^t -\frac{1}{12} dt$

OR  $-\frac{2}{3} [v^{-0.5}]_{16}^0 = -\frac{1}{12} t$

OR  $\frac{2}{3} (\frac{1}{\sqrt{v}} - \frac{1}{4}) = \frac{t}{12}$

WHEN  $v=9 in/s$ :  $\frac{2}{3} (\frac{1}{\sqrt{9}} - \frac{1}{4}) = \frac{t}{12}$

OR  $t = 0.1713 s$

11.24

GIVEN:  $a = -5/(2v_0 - v)$ ,  $a \sim ft/s^2$ ,  $v \sim ft/s$ AT  $t=0$ ,  $x=0$ ,  $v=v_0$ ; AT  $t=2 s$ , $v=0.5v_0$ FIND: (a)  $v_0$ (b)  $t$  WHEN  $v=0$ (c)  $x$  WHEN  $v=1 ft/s$ 

(a) HAVE  $\frac{dv}{dt} = a = -\frac{5}{2v_0 - v}$

AT  $t=0$ ,  $v=v_0$ :  $\int_{v_0}^v (2v_0 - v) dv = \int_0^t -5 dt$

OR  $-\frac{1}{2} [2v_0 v - v^2]_{v_0}^v = -5t$

OR  $(2v_0 - v)^2 - v_0^2 = 10t$

AT  $t=2 s$ ,  $v=0.5v_0$ :  $(2v_0 - 0.5v_0)^2 - v_0^2 = 10(2)$

OR  $\frac{9}{4} v_0^2 = 20$

$v_0 = 4 \frac{ft}{s}$

(b) HAVE  $(8 - v)^2 - 16 = 10t$

WHEN  $v=0$ :  $(8)^2 - 16 = 10t$

OR  $t = 4.8 s$

(c) HAVE  $v \frac{dv}{dx} = a = -\frac{5}{2v_0 - v}$

WHEN  $x=0$ ,  $v=v_0 = 4 \frac{ft}{s}$ :  $\int_{v_0}^v (2v_0 - v) dv = \int_0^x -5 dx$

OR  $[4v^2 - \frac{1}{2}v^3]_{v_0}^v = -5x$

OR  $(4v^2 - \frac{1}{2}v^3) - [4(4)^2 - \frac{1}{2}(4)^3] = -5x$

OR  $(4v^2 - \frac{1}{2}v^3) - \frac{128}{2} = -5x$

WHEN  $v=1 \frac{ft}{s}$ :  $[4(1)^2 - \frac{1}{2}(1)^3] - \frac{128}{2} = -5x$

OR  $x = 7.80 ft$

11.25

GIVEN:  $a = 0.4(1 - kv)$ ,  $k = \text{CONSTANT}$ AT  $t=0$ ,  $x=4 m$ ,  $v=0$ ; AT  $t=1 s$ , $v=4 m/s$ FIND: (a)  $k$ (b)  $x$  WHEN  $v=6 m/s$ (c)  $v_{max}$ 

(a) HAVE  $\frac{dv}{dt} = a = 0.4(1 - kv)$

AT  $t=0$ ,  $v=0$ :  $\int_0^v \frac{dv}{1 - kv} = \int_0^t 0.4 dt$

OR  $-\frac{1}{k} [\ln(1 - kv)]_0^v = 0.4t$

OR  $\ln(1 - kv) = -0.4kt$  (1)

AT  $t=1 s$ ,  $v=4 m/s$ :  $\ln(1 - 4k) = -0.4k(1s)$

$= -0.4k$

SOLVING YIELDS  $k = 0.145703 \frac{1}{m}$

OR  $k = 0.1457 \frac{1}{m}$

(b) HAVE  $v \frac{dv}{dx} = a = 0.4(1 - kv)$

WHEN  $x=4 m$ ,  $v=0$ :  $\int_0^v \frac{v dv}{1 - kv} = \int_4^x 0.4 dx$

NOW..  $\frac{v}{1 - kv} = -\frac{1}{k} + \frac{1/k}{1 - kv}$

THEN  $\int_0^v [-\frac{1}{k} + \frac{1/k}{1 - kv}] dv = \int_4^x 0.4 dx$

OR  $[-\frac{v}{k} - \frac{1}{k^2} \ln(1 - kv)]_0^v = 0.4(x)_4$

(CONTINUED)

# 11.25 CONTINUED

OR  $-\left[\frac{v}{k} + \frac{1}{k^2} \ln(1-kv)\right] = 0.4(x-4)$   
 WHEN  $v = 6 \frac{m}{s}$ :  
 $-\left[\frac{6}{0.145703} + \frac{1}{(0.145703)^2} \ln(1-0.145703 \cdot 6)\right] = 0.4(x-4)$   
 OR  $0.4(x-4) = 56.4778$

OR  $x = 145.2 \text{ m}$

(C) THE MAXIMUM VELOCITY OCCURS WHEN  $a = 0$ .

$\therefore a = 0: 0.4(1-kv_{max}) = 0$

OR  $v_{max} = \frac{1}{0.145703}$

OR  $v_{max} = 6.86 \frac{m}{s}$

AN ALTERNATIVE SOLUTION IS TO BEGIN WITH EQ.(1).

$\ln(1-kv) = -0.4kt$

THEN  $v = \frac{1}{k}(1 - e^{-0.4kt})$

THUS,  $v_{max}$  IS ATTAINED AS  $t \rightarrow \infty \dots$

$v_{max} = \frac{1}{k} \dots$  AS ABOVE

# 11.26

GIVEN:  $a = -0.6v^{3/2}$   $a = \frac{m}{s^2}$ ,  $v = \frac{m}{s}$ ;

AT  $t = 0$ ,  $x = 0$ ,  $v = 9 \frac{m}{s}$

FIND: (a)  $x$  WHEN  $v = 4 \frac{m}{s}$

(b)  $t$  WHEN  $v = 1 \frac{m}{s}$

(c)  $t$  WHEN  $x = 6 \text{ m}$

(a) HAVE  $\frac{dv}{dx} = a = -0.6v^{3/2}$   
 WHEN  $x = 0$ ,  $v = 9 \frac{m}{s}$ :  $\int_9^v v^{-3/2} dv = \int_0^x -0.6 dx$   
 OR  $2[v^{-1/2}]_9^v = -0.6x$

OR  $x = \frac{1}{0.3}(3 - v^{1/2})$  (1)

WHEN  $v = 4 \frac{m}{s}$ :  $x = \frac{1}{0.3}(3 - 4^{1/2})$

OR  $x = 3.33 \text{ m}$

(b) HAVE  $\frac{dv}{dt} = a = -0.6v^{3/2}$   
 WHEN  $t = 0$ ,  $v = 9 \frac{m}{s}$ :  $\int_9^v v^{-3/2} dv = \int_0^t -0.6 dt$

OR  $-2[v^{-1/2}]_9^v = -0.6t$

OR  $\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$

WHEN  $v = 1 \frac{m}{s}$ :  $\frac{1}{1} - \frac{1}{3} = 0.3t$

OR  $t = 2.22 \text{ s}$

(c) HAVE  $\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$   
 OR  $v = \left(\frac{3}{1+0.9t}\right)^2 = \frac{9}{(1+0.9t)^2}$

NOW..  $\frac{dx}{dt} = v = \frac{9}{(1+0.9t)^2}$

AT  $t = 0$ ,  $x = 0$ :  $\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2} dt$

OR  $x = 9\left[-\frac{1}{0.9} \frac{1}{1+0.9t}\right]_0^t$   
 $= 10\left(1 - \frac{1}{1+0.9t}\right)$

$= \frac{9t}{1+0.9t}$

WHEN  $x = 6 \text{ m}$ :  $6 = \frac{9t}{1+0.9t}$

OR  $t = 1.667 \text{ s}$

AN ALTERNATIVE SOLUTION IS TO BEGIN WITH EQ.(1).

$x = \frac{1}{0.3}(3 - v^{1/2})$

THEN  $\frac{dx}{dt} = v = (3 - 0.3x)^2$

NOW.. AT  $t = 0$ ,  $x = 0$ :  $\int_0^x \frac{dx}{(3 - 0.3x)^2} = \int_0^t dt$

OR  $t = \frac{1}{0.3} \left[ \frac{1}{3 - 0.3x} \right]_0^x = \frac{x}{9 - 0.9x}$

WHICH LEADS TO THE SAME EQUATION AS ABOVE.

# 11.27



GIVEN:  $v = 7.5(1 - 0.04x)^{0.3}$   $v = \frac{m}{h}$ ,

$x = \text{mi}$ ; AT  $t = 0$ ,  $x = 0$

FIND: (a)  $x$  AT  $t = 1 \text{ h}$

(b)  $a$  ( $\frac{m}{h^2}$ ) AT  $t = 0$

(c)  $t$  WHEN  $x = 6 \text{ mi}$

(a) HAVE  $\frac{dx}{dt} = v = 7.5(1 - 0.04x)^{0.3}$   
 AT  $t = 0$ ,  $x = 0$ :  $\int_0^x \frac{dx}{(1 - 0.04x)^{0.7}} = \int_0^t 7.5 dt$

OR  $\frac{1}{0.7}(-0.04)[(1 - 0.04x)^{0.7}]_0^x = 7.5t$

OR  $1 - (1 - 0.04x)^{0.7} = 0.21t$  (1)

OR  $x = \frac{1}{0.04}[1 - (1 - 0.21t)^{1/0.7}]$

AT  $t = 1 \text{ h}$ :  $x = \frac{1}{0.04}[1 - (1 - 0.21(1))^{1/0.7}]$

OR  $x = 7.15 \text{ mi}$

(b) HAVE  $a = v \frac{dv}{dx}$   
 $= 7.5(1 - 0.04x)^{0.3} \frac{d}{dx}[7.5(1 - 0.04x)^{0.3}]$   
 $= 7.5^2(1 - 0.04x)^{0.3}[(0.3)(-0.04)(1 - 0.04x)^{-0.7}]$   
 $= -0.675(1 - 0.04x)^{-0.4}$

AT  $t = 0$ ,  $x = 0$ :  $a_0 = -0.675 \frac{m}{h^2} = \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2$   
 OR  $a_0 = -275 \cdot 10^{-6} \frac{ft}{s^2}$

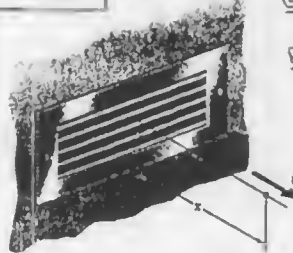
(c) FROM EQ.(1):  $t = \frac{1}{0.21}[1 - (1 - 0.04x)^{0.7}]$

WHEN  $x = 6 \text{ mi}$ :  $t = \frac{1}{0.21}[1 - (1 - 0.04(6))^{0.7}]$

$= 0.83229 \text{ h}$

OR  $t = 49.9 \text{ min}$

# 11.28



GIVEN:  $v = \frac{0.18v_0}{x}$   $v = \frac{m}{s}$ ,  $x = \text{m}$ ;

$v_0 = 3.6 \frac{m}{s}$

FIND: (a)  $a$  WHEN  $x = 2 \text{ m}$

(b) TIME FOR AIR TO FLOW FROM  $x = 1 \text{ m}$  TO  $x = 3 \text{ m}$

(a) HAVE  $a = v \frac{dv}{dx}$   
 $= \frac{0.18v_0}{x} \frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$   
 $= -\frac{0.0324 v_0^2}{x^2}$

WHEN  $x = 2 \text{ m}$ :  $a = -\frac{0.0324(3.6)^2}{(2)^2}$

OR  $a = -0.0525 \frac{m}{s^2}$

(b) HAVE  $\frac{dx}{dt} = v = \frac{0.18v_0}{x}$

FROM  $x = 1 \text{ m}$  TO  $x = 3 \text{ m}$ :  $\int_1^3 x dx = \int_{t_1}^{t_3} 0.18v_0 dt$

OR  $[\frac{1}{2}x^2]_1^3 = 0.18v_0(t_3 - t_1)$

OR  $(t_3 - t_1) = \frac{\frac{1}{2}(9 - 1)}{0.18(3.6)}$

OR  $t_3 - t_1 = 6.17 \text{ s}$

11.29



GIVEN:  $a = -32.2 / (1 + (y/20.9 \times 10^6)^2)$   
 $a = -y/s^2$ ,  $y = ft$

FIND:  $y_{max}$  WHEN

(a)  $v_0 = 1800$  ft/s

(b)  $v_0 = 3000$  ft/s

(c)  $v_0 = 36,700$  ft/s

HAVE  $v \frac{dv}{dy} = a = - \frac{32.2}{(1 + \frac{y}{20.9 \times 10^6})^2}$

WHEN  $y = 0$ ,  $v = v_0$

$y = y_{max}$ ,  $v = 0$

THEN..  $\int_{v_0}^0 v dv = \int_0^{y_{max}} \frac{-32.2}{(1 + \frac{y}{20.9 \times 10^6})^2} dy$

OR  $-\frac{1}{2} v_0^2 = -32.2 \left[ 20.9 \times 10^6 \frac{1}{1 + \frac{y}{20.9 \times 10^6}} \right]_0^{y_{max}}$

OR  $v_0^2 = 1345.96 \times 10^6 \left( 1 - \frac{1}{1 + \frac{y_{max}}{20.9 \times 10^6}} \right)$

OR  $y_{max} = \frac{v_0^2}{64.4 - \frac{v_0^2}{20.9 \times 10^6}}$

(a)  $v_0 = 1800$  ft/s:

$y_{max} = \frac{(1800)^2}{64.4 - \frac{(1800)^2}{20.9 \times 10^6}}$

OR  $y_{max} = 50.4110$  ft

(b)  $v_0 = 3000$  ft/s:

$y_{max} = \frac{(3000)^2}{64.4 - \frac{(3000)^2}{20.9 \times 10^6}}$

OR  $y_{max} = 140.7110$  ft

(c)  $v_0 = 36,700$  ft/s:

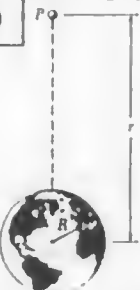
$y_{max} = \frac{(36,700)^2}{64.4 - \frac{(36,700)^2}{20.9 \times 10^6}}$

OR  $y_{max} = -3.0310$  ft

THE VELOCITY 36,700 ft/s IS APPROXIMATELY THE ESCAPE VELOCITY  $v_0$  FROM THE EARTH. FOR  $v_0$

$y_{max} \rightarrow \infty$

11.30



GIVEN:  $a = -\frac{gR^2}{r^2}$ ,  $R = 3960$  mi,  
 WHEN  $r = \infty$ ,  $v = 0$

FIND:  $v_0$ 

HAVE  $v \frac{dv}{dr} = a = -\frac{gR^2}{r^2}$

WHEN  $r = R$ ,  $v = v_0$

$r = \infty$ ,  $v = 0$

THEN  $\int_{v_0}^0 v dv = \int_R^{\infty} -\frac{gR^2}{r^2} dr$

(CONTINUED)

11.30 CONTINUED

OR  $-\frac{1}{2} v_0^2 = gR^2 \left[ \frac{1}{r} \right]_R^{\infty}$

OR  $v_0 = \sqrt{2gR}$

$= (2 \times 32.2 \frac{ft}{s^2} \times 3960 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}})^{1/2}$

OR  $v_0 = 36,710 \frac{ft}{s}$

11.31

GIVEN:  $y = v_0 [1 - \sin(\frac{\pi t}{T})]$ ; AT  $t = 0$ ,  $x = 0$ ,  
 $v = v_0$

FIND: (a)  $x$  AND  $a$  AT  $t = 3T$ (b)  $v_{AVE}$  DURING  $t = 0$  TO  $t = T$ 

(a) HAVE  $\frac{dx}{dt} = v = v_0 [1 - \sin(\frac{\pi t}{T})]$   
 AT  $t = 0$ ,  $x = 0$ :  $\int_0^x dx = \int_0^t v_0 [1 - \sin(\frac{\pi t}{T})] dt$

OR  $x = v_0 \left[ t + \frac{T}{\pi} \cos(\frac{\pi t}{T}) \right]_0^t$   
 $= v_0 \left[ t + \frac{T}{\pi} \cos(\frac{\pi t}{T}) - \frac{T}{\pi} \right]$  (1)

AT  $t = 3T$ :  $x_{3T} = v_0 \left[ 3T + \frac{T}{\pi} \cos(\frac{\pi \cdot 3T}{T}) - \frac{T}{\pi} \right]$   
 $= v_0 (3T - \frac{2T}{\pi})$

ALSO..  $a = \frac{dv}{dt} = \frac{d}{dt} \left\{ v_0 [1 - \sin(\frac{\pi t}{T})] \right\}$   
 $= -v_0 \frac{\pi}{T} \cos(\frac{\pi t}{T})$

AT  $t = 3T$ :  $a_{3T} = -v_0 \frac{\pi}{T} \cos(\frac{\pi \cdot 3T}{T})$

OR  $a_{3T} = \frac{\pi v_0}{T}$

(b) USING EQ. (1) ..

AT  $t = 0$ :  $x_0 = v_0 \left[ 0 + \frac{T}{\pi} \cos(0) - \frac{T}{\pi} \right] = 0$

AT  $t = T$ :  $x_T = v_0 \left[ T + \frac{T}{\pi} \cos(\frac{\pi T}{T}) - \frac{T}{\pi} \right]$   
 $= v_0 (T - \frac{2T}{\pi})$   
 $= 0.363 v_0 T$

NOW..  $v_{AVE} = \frac{x_T - x_0}{\Delta t}$   
 $= \frac{0.363 v_0 T - 0}{T - 0}$

OR  $v_{AVE} = 0.363 v_0$

11.32

GIVEN:  $y = v' \sin(\omega_N t + \phi)$ ; AT  $t = 0$ ,  $x = x_0$ ,

$v = v_0$ ;  $x_{max} = 2x_0$

SHOW: (a)  $v_0 = (\omega_N^2 + x_0^2 \omega_N^2) / 2x_0 \omega_N$

(b)  $v_{max}$  OCCURS WHEN

$x = x_0 [3 - (\omega_N / x_0 \omega_N)^2] / 2$

(a) AT  $t = 0$ ,  $y = v_0$ :  $v_0 = v' \sin(0 + \phi) = v' \sin \phi$

THEN  $\cos \phi = \sqrt{v'^2 - v_0^2} / v'$

NOW  $\frac{dx}{dt} = v = v' \sin(\omega_N t + \phi)$

AT  $t = 0$ ,  $x = x_0$ :  $\int_{x_0}^x dx = \int_0^t v' \sin(\omega_N t + \phi) dt$

OR  $x - x_0 = v' \left[ -\frac{1}{\omega_N} \cos(\omega_N t + \phi) \right]_0^t$

OR  $x = x_0 + \frac{v'}{\omega_N} [\cos \phi - \cos(\omega_N t + \phi)]$

NOW OBSERVE THAT  $x_{max}$  OCCURS WHEN  $\cos(\omega_N t + \phi) = -1$ . THEN..

$x_{max} = 2x_0 = x_0 + \frac{v'}{\omega_N} [\cos \phi - (-1)]$

SUBSTITUTING FOR  $\cos \phi$  --  $x_0 = \frac{v'}{\omega_N} \left( \frac{\sqrt{v'^2 - v_0^2}}{v'} + 1 \right)$

OR  $x_0 \omega_N - v' = \sqrt{v'^2 - v_0^2}$

SQUARING BOTH SIDES OF THIS EQUATION..

$x_0^2 \omega_N^2 - 2x_0 \omega_N v' + v'^2 = v'^2 - v_0^2$

OR  $v' = \frac{x_0^2 \omega_N^2 + v_0^2}{2x_0 \omega_N}$

Q.E.D.

(CONTINUED)

# 11.32 CONTINUED

(b) FIRST OBSERVE THAT  $N_{MAX}$  OCCURS WHEN  $\omega t + \phi = \frac{\pi}{2}$ . THE CORRESPONDING VALUE OF  $X$  IS

$$X_{N_{MAX}} = X_0 + \frac{N'}{\omega_N} [\cos \phi - \cos(\frac{\pi}{2})]$$

$$= X_0 + \frac{N'}{\omega_N} \cos \phi$$

SUBSTITUTING FIRST FOR  $\cos \phi$  AND THEN FOR  $N'$ :-

$$X_{N_{MAX}} = X_0 + \frac{N'}{\omega_N} \frac{\sqrt{N_0^2 - N_0^2}}{N_0}$$

$$= X_0 + \frac{1}{\omega_N} \left[ \left( \frac{N_0^2 + X_0^2 \omega_N^2}{2 X_0 \omega_N} \right)^2 - N_0^2 \right]^{1/2}$$

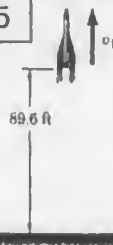
$$= X_0 + \frac{1}{2 X_0 \omega_N} (N_0^4 + 2 N_0^2 X_0^2 \omega_N^2 + X_0^4 \omega_N^4 - 4 X_0^2 \omega_N^2 N_0^2)^{1/2}$$

$$= X_0 + \frac{1}{2 X_0 \omega_N} [ (X_0^2 \omega_N^2 - N_0^2)^2 ]^{1/2}$$

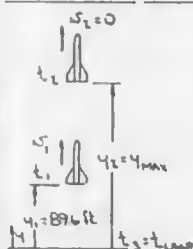
$$= X_0 + \frac{X_0^2 \omega_N^2 - N_0^2}{2 X_0 \omega_N}$$

$$= \frac{X_0}{2} \left[ 3 - \left( \frac{N_0}{X_0 \omega_N} \right)^2 \right] \quad Q.E.D.$$

# 11.35



GIVEN:  $a = -32.2 \text{ ft/s}^2$ ;  $t_{LAMB} - t_1 = 16 \text{ s}$   
 FIND: (a)  $N_1$   
 (b)  $y_{max}$



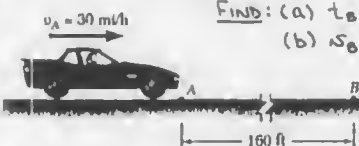
(a) HAVE..  $y = y_1 + N_1 t - \frac{1}{2} a t^2$   
 AT  $t_{LAMB}$ ,  $y = 0$   
 THEN..  $0 = 89.6 \text{ ft} + N_1 (16 \text{ s}) + \frac{1}{2} (-32.2 \text{ ft/s}^2) (16 \text{ s})^2$   
 OR  $N_1 = 252 \text{ ft/s}^2$

(b) HAVE..  $N^2 = N_1^2 + 2a(y - y_1)$   
 AT  $y = y_{max}$ ,  $N = 0$   
 THEN..  $0 = (252 \text{ ft/s}^2)^2 + 2(-32.2 \text{ ft/s}^2)(y_{max} - 89.6 \text{ ft})$   
 OR  $y_{max} = 1076 \text{ ft}$

# 11.36

GIVEN:  $a = 11 \text{ ft/s}^2$  - CONSTANT;  $N_A = 30 \text{ mi/h}$

FIND: (a)  $t_B$   
 (b)  $N_B$



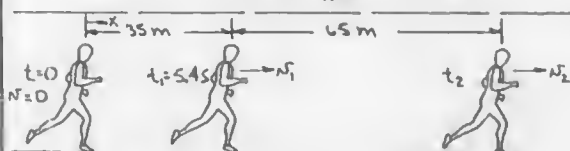
$t=0$   $N_A = 30 \text{ mi/h} = 44 \text{ ft/s}$   
 (a) HAVE..  $x = x_A^0 + N_A t + \frac{1}{2} a t^2$   
 WHEN  $x = x_B = 160 \text{ ft}$   
 $160 \text{ ft} = (44 \text{ ft/s}) t_B + \frac{1}{2} (11 \text{ ft/s}^2) t_B^2$   
 OR  $5.5 t_B^2 + 44 t_B - 160 = 0$  ( $t_B = -5$ )  
 SOLVING FOR THE POSITIVE ROOT..  $t_B = 2.7150 \text{ s}$   
 OR  $t_B = 2.72 \text{ s}$

(b) HAVE..  $N = N_A + at$   
 AT  $t = t_B$ ..  $N_B = 44 \text{ ft/s} + (11 \text{ ft/s}^2)(2.7150 \text{ s})$   
 $= 73.865 \text{ ft/s}$   
 OR  $N_B = 50.4 \text{ mi/h}$

# 11.37

GIVEN:  $0 \leq x \leq 35 \text{ m}$ ,  $a = \text{CONSTANT}$ ;  
 $35 \text{ m} < x \leq 100 \text{ m}$ ,  $N = \text{CONSTANT}$ ;  
 AT  $t=0$ ,  $N=0$ ; WHEN  $x=35 \text{ m}$ ,  $t=5.4 \text{ s}$

FIND: (a)  $a$   
 (b)  $N$  WHEN  $x=100 \text{ m}$   
 (c)  $t$  WHEN  $x=100 \text{ m}$



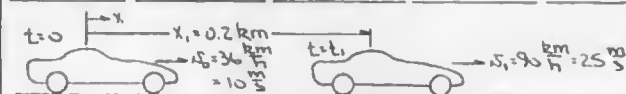
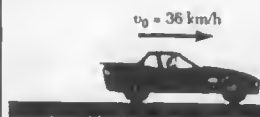
(a) HAVE..  $x = x_0^0 + N_0 t + \frac{1}{2} a t^2$  FOR  $0 \leq x \leq 35 \text{ m}$   
 AT  $t = 5.4 \text{ s}$ :  $35 \text{ m} = \frac{1}{2} a (5.4 \text{ s})^2$   
 OR  $a = 2.4005 \text{ m/s}^2$   $a = 2.40 \text{ m/s}^2$

(b) FIRST NOTE THAT  $N = N_{MAX}$  FOR  $35 \text{ m} \leq x \leq 100 \text{ m}$   
 NOW..  $N^2 = N_0^2 + 2a(x - x_0)$  FOR  $0 \leq x \leq 35 \text{ m}$   
 WHEN  $x = 35 \text{ m}$ :  $N_{MAX}^2 = 2(2.4005 \text{ m/s}^2)(35 \text{ m})$   
 (CONTINUED)

# 11.33

GIVEN:  $N_0 = 36 \text{ km/h}$ ,  $N_1 = 90 \text{ km/h}$ ;  
 UNIFORM ACCELERATION;  
 $\Delta x = 0.2 \text{ km}$

FIND: (a)  $a$   
 (b)  $t_1$



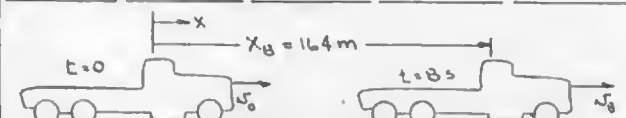
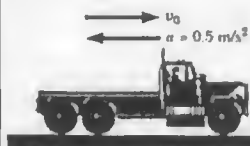
(a) HAVE..  $N_1^2 = N_0^2 + 2a(x_1 - x_0)$   
 OR  $(25 \text{ m/s})^2 = (10 \text{ m/s})^2 + 2a(200 \text{ m})$   
 OR  $a = 1.3125 \text{ m/s}^2$   $a = 1.313 \text{ m/s}^2$

(b) HAVE  $N_1 = N_0 + at_1$   
 OR  $25 \text{ m/s} = 10 \text{ m/s} + (1.3125 \text{ m/s}^2) t_1$   
 OR  $t_1 = 11.43 \text{ s}$

# 11.34

GIVEN:  $a = -0.5 \text{ m/s}^2$  - CONSTANT;  
 $\Delta x = 164 \text{ m}$ ,  $\Delta t = 8 \text{ s}$

FIND: (a)  $N_0$   
 (b)  $N$  AT  $8 \text{ s}$   
 (c)  $\Delta x$  AT  $0.6 \text{ s}$



(a) HAVE..  $x = x_0^0 + N_0 t + \frac{1}{2} a t^2$  (i)  
 AT  $t = 8 \text{ s}$ :  $164 \text{ m} = N_0 (8 \text{ s}) + \frac{1}{2} (-0.5 \text{ m/s}^2) (8 \text{ s})^2$   
 OR  $N_0 = 22.5 \text{ m/s}$

(b) HAVE..  $N = N_0 + at$   
 AT  $t = 8 \text{ s}$ :  $N_8 = 22.5 \text{ m/s} + (-0.5 \text{ m/s}^2) (8 \text{ s})$   
 OR  $N_8 = 18.5 \text{ m/s}$

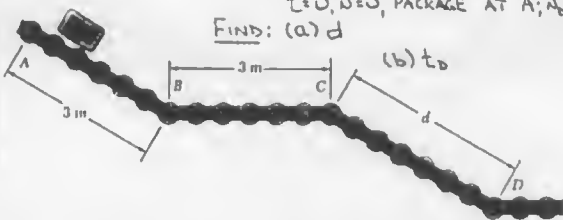
(c) USING EQ. (i)..  $x_{0.6} = (22.5 \text{ m/s})(0.6 \text{ s}) + \frac{1}{2} (-0.5 \text{ m/s}^2) (0.6 \text{ s})^2$   
 OR  $x_{0.6} = 13.41 \text{ m}$

# 11.37 CONTINUED

OR  $v_{max} = 12.9628 \frac{m}{s}$   $v_{max} = 12.96 \frac{m}{s}$   $\leftarrow$   
 (C) HAVE...  $x = x_1 + v_0(t - t_1)$  FOR  $35m < x \leq 100m$   
 WHEN  $x = 100m$ :  $100m = 35m + (12.9628 \frac{m}{s})(t_2 - 5.4)s$   
 OR  $t_2 = 10.41s$   $\leftarrow$

# 11.38

GIVEN:  $a_{AB} = a_{CD} = 4.8 \frac{m}{s^2}$ ;  $v_{BC} = \text{CONSTANT}$ ;  
 $t = 0, v = 0$ , PACKAGE AT A;  $v_0 = 7.2 \frac{m}{s}$   
 FIND: (a) d (b)  $t_D$



(a) FOR  $A \rightarrow B$  AND  $C \rightarrow D$  HAVE  $v^2 = v_0^2 + 2a(x - x_0)$   
 THEN... AT B..  $v_B^2 = v_A^2 + 2(4.8 \frac{m}{s^2})(3 - 0)m$   
 $= 28.8 \frac{m^2}{s^2}$  ( $v_{BC} = 5.3666 \frac{m}{s}$ )  
 AND AT D..  $v_D^2 = v_C^2 + 2a_0(x_D - x_C)$   $d = x_D - x_C$   
 OR  $(7.2 \frac{m}{s})^2 = (28.8 \frac{m^2}{s^2}) + 2(4.8 \frac{m}{s^2})d$   
 OR  $d = 2.40m$

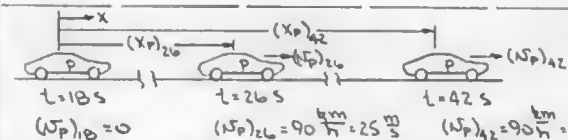
(b) FOR  $A \rightarrow B$  AND  $C \rightarrow D$  HAVE  $v = v_0 + at$   
 THEN...  $A \rightarrow B$ ..  $5.3666 \frac{m}{s} = 0 + (4.8 \frac{m}{s^2})t_{AB}$   
 OR  $t_{AB} = 1.11804s$   
 AND  $C \rightarrow D$ ..  $7.2 \frac{m}{s} = 5.3666 \frac{m}{s} + (4.8 \frac{m}{s^2})t_{CD}$   
 OR  $t_{CD} = 0.38196s$

NOW.. FOR  $B \rightarrow C$  HAVE  $x_C = x_B + v_{BC}t_{BC}$   
 OR  $3m = (5.3666 \frac{m}{s})t_{BC}$   
 OR  $t_{BC} = 0.55901s$

FINALLY,  $t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196)s$   
 OR  $t_D = 2.06s$   $\leftarrow$

# 11.39

GIVEN: AT  $t = 0$ ,  $x_M = x_P = 0$ ; AT  $t = 42s$ ,  $x_M = x_P$ ;  
 $v_M = \text{CONSTANT}$ ; FOR  $0 \leq t \leq 18s$ ,  $v_P = 0$ ;  
 FOR  $18s < t \leq 26s$ ,  $a_P = \text{CONSTANT}$ ;  
 AT  $t = 26s$ ,  $v_P = 90 \frac{km}{h}$ ;  
 FOR  $26s < t \leq 42s$ ,  $v_P = 90 \frac{km}{h}$   
 FIND: (a)  $x_P$  AT  $t = 42s$   
 (b)  $v_M$

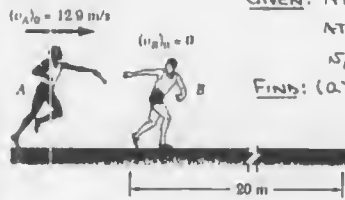


(a) PATROL CAR: FOR  $18s < t \leq 26s$ :  $v_P = (v_P)_{18} + a_P(t - 18)$   
 AT  $t = 26s$ :  $25 \frac{m}{s} = a_P(26 - 18)s$   
 OR  $a_P = 3.125 \frac{m}{s^2}$   
 ALSO,  $x_P = (x_P)_{18} + (v_P)_{18}(t - 18) + \frac{1}{2}a_P(t - 18)^2$   
 AT  $t = 26s$ :  $(x_P)_{26} = \frac{1}{2}(3.125 \frac{m}{s^2})(26 - 18)^2 = 100m$   
 FOR  $26s < t \leq 42s$ :  $x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$   
 AT  $t = 42s$ :  $(x_P)_{42} = 100m + (25 \frac{m}{s})(42 - 26)s$   
 $= 500m$  ( $(x_P)_{42} = 0.5km$ )  $\leftarrow$

(b) FOR THE MOTORIST'S CAR..  $x_M = (x_M)_0 + v_M t$   
 AT  $t = 42s$ ,  $x_M = x_P$ :  $500m = v_M(42s)$   
 OR  $v_M = 11.9048 \frac{m}{s}$   
 OR  $v_M = 42.9 \frac{km}{h}$   $\leftarrow$

# 11.40

GIVEN: AT  $t = 0$ ,  $x_A = x_B = 0$ ;  
 AT  $t = 1.82s$ ,  $x_A = x_B = 20m$ ,  
 $v_A = v_B$   
 FIND: (a)  $a_A$  AND  $a_B$  KNOWING  
 THAT BOTH ARE UNIFORM  
 (b)  $t_B$  WHEN RUNNER B  
 STARTS TO RUN



(a) FOR RUNNER A:  $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2$   
 AT  $t = 1.82s$ :  $20m = (12.9 \frac{m}{s})(1.82s) + \frac{1}{2}a_A(1.82s)^2$   
 OR  $a_A = -2.10 \frac{m}{s^2}$   $\leftarrow$

ALSO..  $v_A = (v_A)_0 + a_A t$   
 AT  $t = 1.82s$ :  $(v_A)_{1.82} = (12.9 \frac{m}{s}) + (-2.10 \frac{m}{s^2})(1.82s)$   
 $= 9.078 \frac{m}{s}$

FOR RUNNER B:  $v_B^2 = (v_B)_0^2 + 2a_B(x_B - (x_B)_0)$   
 WHEN  $x_B = 20m$ ,  $v_B = v_A$ :  $(9.078 \frac{m}{s})^2 = 2a_B(20m)$   
 OR  $a_B = 2.0603 \frac{m}{s^2}$   
 $a_B = 2.06 \frac{m}{s^2}$   $\leftarrow$

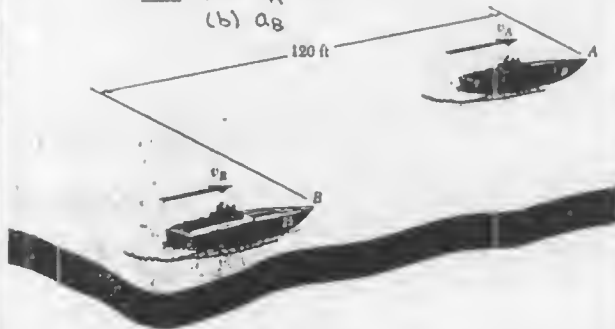
(b) FOR RUNNER B:  $v_B = (v_B)_0 + a_B(t - t_0)$   
 WHERE  $t_0$  IS THE TIME AT WHICH HE BEGINS  
 TO RUN.

AT  $t = 1.82s$ :  $9.078 \frac{m}{s} = (2.0603 \frac{m}{s^2})(1.82 - t_0)$   
 OR  $t_0 = -2.59s$

$\therefore$  RUNNER B SHOULD START TO RUN  
 2.59s BEFORE A REACHES THE EXCHANGE ZONE.  $\leftarrow$

# 11.41

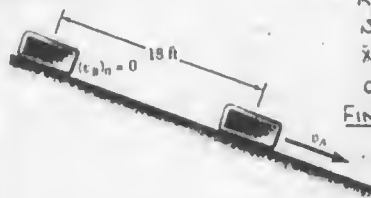
GIVEN: AT  $t = 0$ ,  $x_{AB} = 120ft$ ,  $v_A = v_B = 105 \frac{mi}{h}$ ;  
 $a_A, a_B = \text{CONSTANTS}$ ; AT  $t = 8s$ ,  $x_A = x_B$ ,  
 $v_A = 135 \frac{mi}{h}$   
 FIND: (a)  $a_A$   
 (b)  $a_B$



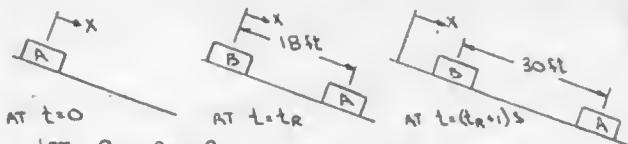
(a) HAVE..  $v_A = (v_A)_0 + a_A t$  ( $(v_A)_0 = 105 \frac{mi}{h} = 154 \frac{ft}{s}$ )  
 AT  $t = 8s$ :  $v_A = 135 \frac{mi}{h} = 198 \frac{ft}{s}$   
 THEN  $198 \frac{ft}{s} = 154 \frac{ft}{s} + a_A(8s)$  OR  $a_A = 5.50 \frac{ft}{s^2}$   $\leftarrow$

(b) HAVE..  $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2$  ( $(x_A)_0 = 120ft$ )  
 AND  $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2$  ( $(v_B)_0 = 154 \frac{ft}{s}$ )  
 AT  $t = 8s$ :  $x_A = x_B$   
 $\therefore 120ft + (154 \frac{ft}{s})(8s) + \frac{1}{2}(5.50 \frac{ft}{s^2})(8s)^2 = (154 \frac{ft}{s})(8s) + \frac{1}{2}a_B(8s)^2$   
 OR  $a_B = 9.25 \frac{ft}{s^2}$   $\leftarrow$

11.42

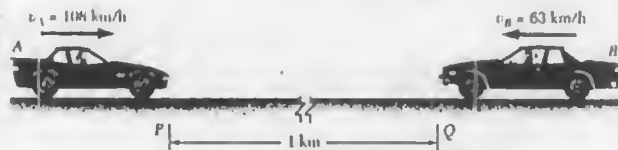


GIVEN: AT  $t=0$ ,  $x_A=0$ ,  $v_{A0}=0$ ;  
AT  $t=t_R$ ,  $x_A=18$  ft,  $x_B=0$ ;  
AT  $t=(t_R+1)$ ,  
 $x_{AB}=30$  ft;  $a_A=a_B$   
CONSTANT  
FIND: (a)  $t_R$   
(b)  $a_A$  AND  $a_B$



LET  $a_A = a_B = a$   
(a) FOR  $t \geq 0$ :  $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a t^2$   
 $t \geq t_R$ :  $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a (t - t_R)^2$   
AT  $t=t_R$ ,  $x_A=18$  ft:  $18 = \frac{1}{2} a t_R^2$  (1)  
 $t=(t_R+1)$  s,  $x_A - x_B = 30$  ft:  
 $30 = \frac{1}{2} a (t_R+1)^2 - \frac{1}{2} a [(t_R+1) - t_R]^2$   
 $= \frac{1}{2} a (t_R^2 + 2t_R)$  (2)  
EQ. (1)  $\Rightarrow \frac{1}{2} a = \frac{18}{t_R^2}$  SO THAT  $30 = \frac{18}{t_R^2} (t_R^2 + 2t_R)$   
OR  $t_R = 3$  s  
(b) SUBSTITUTING INTO EQ. (1) ..  $18 = \frac{1}{2} a (3)^2$   
OR  $a = a_A = a_B = 4 \frac{\text{ft}}{\text{s}^2}$

11.44

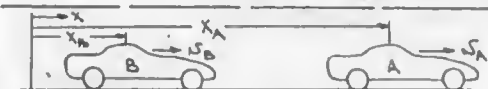


GIVEN: AT  $t=0$ , CAR A IS AT P, CAR B IS AT Q; AT  $t=40$  s, CAR A IS AT Q; AT  $t=42$  s, CAR B IS AT P  
FIND: (a)  $a_A$  AND  $a_B$  ( $a_A$  AND  $a_B$  ARE CONSTANT)  
(b)  $t$  WHEN CARS MEET  
(c)  $v_B$  WHEN CARS MEET

LET  $a_A = a_B = a$   
(a) HAVE..  $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a t^2$  ( $v_A)_0 = 108 \frac{\text{km}}{\text{h}} = 30 \frac{\text{m}}{\text{s}}$   
AT  $t=40$  s:  $1000 \text{ m} = (30 \frac{\text{m}}{\text{s}})(40 \text{ s}) + \frac{1}{2} a (40 \text{ s})^2$   
OR  $a_A = -0.250 \frac{\text{m}}{\text{s}^2}$   
ALSO..  $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a t^2$  ( $v_B)_0 = 63 \frac{\text{km}}{\text{h}} = 17.5 \frac{\text{m}}{\text{s}}$   
AT  $t=42$  s:  $1000 \text{ m} = (17.5 \frac{\text{m}}{\text{s}})(42 \text{ s}) + \frac{1}{2} a (42 \text{ s})^2$   
OR  $a_B = 0.30045 \frac{\text{m}}{\text{s}^2}$   $a_B = 0.300 \frac{\text{m}}{\text{s}^2}$   
(b) WHEN THE CARS PASS EACH OTHER  
 $x_A + x_B = 1000 \text{ m}$   
THEN..  $(30 \frac{\text{m}}{\text{s}})t_{AB} + \frac{1}{2}(-0.250 \frac{\text{m}}{\text{s}^2})t_{AB}^2 + (17.5 \frac{\text{m}}{\text{s}})t_{AB} + \frac{1}{2}(0.30045 \frac{\text{m}}{\text{s}^2})t_{AB}^2 = 1000 \text{ m}$   
OR  $0.05045 t_{AB}^2 + 95 t_{AB} - 2000 = 0$   
SOLVING..  $t = 20.822 \text{ s}$  AND  $t = -1904 \text{ s}$   
 $t > 0 \Rightarrow t_{AB} = 20.8 \text{ s}$   
(c) HAVE..  $v_B = (v_B)_0 + a_B t$   
AT  $t=t_{AB}$ :  $v_B = 17.5 \frac{\text{m}}{\text{s}} + (0.30045 \frac{\text{m}}{\text{s}^2})(20.822 \text{ s})$   
 $= 23.756 \frac{\text{m}}{\text{s}}$   
OR  $v_B = 85.5 \frac{\text{km}}{\text{h}}$

11.43

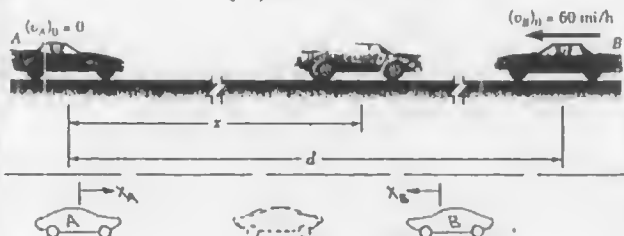
GIVEN: FOR  $t \geq 0$ ,  $a_A = 2 \frac{\text{m}}{\text{s}^2}$ ; FOR  $t \geq 2$  s,  
 $a_B = 3.6 \frac{\text{m}}{\text{s}^2}$ ; CARS START FROM REST  
FIND: (a)  $t$  AND  $x$  WHEN  $x_A = x_B$   
(b)  $v_A$  AND  $v_B$  WHEN  $x_A = x_B$



(a) FOR  $t \geq 0$ :  $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$   
 $t \geq 2$  s:  $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B (t - 2)^2$   
WHEN  $x_A = x_B$ ..  $\frac{1}{2}(2 \frac{\text{m}}{\text{s}^2})t^2 = \frac{1}{2}(3.6 \frac{\text{m}}{\text{s}^2})(t - 2)^2$   
EXPANDING AND SIMPLIFYING..  $t^2 - 9t + 9 = 0$   
SOLVING..  $t = 1.1459 \text{ s}$  AND  $t = 7.8541 \text{ s}$   
MUST REQUIRE  $t > 2$  s.  $\therefore t = 7.85 \text{ s}$   
AT  $t = 7.8541 \text{ s}$ :  $x_A = \frac{1}{2}(2 \frac{\text{m}}{\text{s}^2})(7.8541 \text{ s})^2$   
OR  $x_A = x_B = 61.7 \text{ m}$   
(b) FOR  $t \geq 0$ :  $v_A = (v_A)_0 + a_A t$   
AT  $t = 7.8541 \text{ s}$ :  $v_A = (2 \frac{\text{m}}{\text{s}^2})(7.8541 \text{ s})$   
OR  $v_A = 15.71 \frac{\text{m}}{\text{s}}$   
FOR  $t \geq 2$  s:  $v_B = (v_B)_0 + a_B (t - 2)$   
AT  $t = 7.8541 \text{ s}$ :  $v_B = (3.6 \frac{\text{m}}{\text{s}^2})(7.8541 - 2) \text{ s}$   
OR  $v_B = 21.1 \frac{\text{m}}{\text{s}}$

11.45

GIVEN: AT  $t=0$ ,  $v_A=0$ ,  $v_B=60 \frac{\text{mi}}{\text{h}}$ ; FOR  $t \geq 0$ ,  
 $a_A$  CONSTANT; FOR  $t \geq 5$  s,  $a_B = -\frac{a_A}{6}$ ;  
WHEN CARS MEET,  $x = 294$  ft,  $v_A = v_B$   
FIND: (a)  $a_A$   
(b)  $t$  WHEN  $x = 294$  ft  
(c)  $d$



FOR  $t \geq 0$ :  $v_A = (v_A)_0 + a_A t$   
 $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$   
 $0 \leq t < 5$  s:  $x_B = (x_B)_0 + (v_B)_0 t$  ( $v_B)_0 = 60 \frac{\text{mi}}{\text{h}} = 88 \frac{\text{ft}}{\text{s}}$   
AT  $t = 5$  s:  $x_B = (88 \frac{\text{ft}}{\text{s}})(5 \text{ s}) = 440 \text{ ft}$   
FOR  $t \geq 5$  s:  $v_B = (v_B)_0 + a_B (t - 5)$   $a_B = -\frac{1}{6} a_A$   
 $x_B = (x_B)_5 + (v_B)_5 (t - 5) + \frac{1}{2} a_B (t - 5)^2$   
ASSUME  $t > 5$  s WHEN THE CARS PASS EACH OTHER.  
AT THAT TIME ( $t_{AB}$ )  
 $v_A = v_B$ :  $a_A t_{AB} = (88 \frac{\text{ft}}{\text{s}}) - \frac{a_A}{6} (t_{AB} - 5)$   
 $x_A = 294 \text{ ft}$ :  $294 \text{ ft} = \frac{1}{2} a_A t_{AB}^2$  (CONTINUED)



# 11.45 CONTINUED

$$\text{THEN } \frac{a_A (\frac{1}{2} t_{AB} - \frac{5}{6})}{\frac{1}{2} a_A t_{AB}^2} = \frac{88}{294}$$

$$\text{OR } 44 t_{AB}^2 - 343 t_{AB} + 245 = 0$$

$$\text{SOLVING... } t_{AB} = 0.795 \text{ s AND } t_{AB} = 7.00 \text{ s}$$

$$(a) \text{ WITH } t_{AB} > 5 \text{ s, } 294 \text{ ft} = \frac{1}{2} a_A (7.00 \text{ s})^2$$

$$\text{OR } a_A = 12.00 \frac{\text{ft}}{\text{s}^2}$$

$$(b) \text{ FROM ABOVE } t_{AB} = 7.00 \text{ s}$$

NOTE: AN ACCEPTABLE SOLUTION CANNOT BE FOUND IF IT IS ASSUMED THAT  $t_{AB} \leq 5 \text{ s}$ .

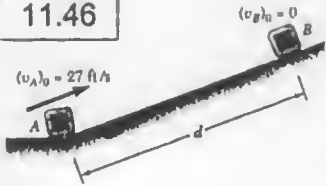
$$(c) \text{ HAVE... } d = x + (x_B)_{t_{AB}}$$

$$= 294 \text{ ft} + [440 \text{ ft} + (88 \frac{\text{ft}}{\text{s}})(7.00 \text{ s})$$

$$+ \frac{1}{2} (-\frac{1}{6} \times 12.00 \frac{\text{ft}}{\text{s}^2} \times (7.00 \text{ s})^2)$$

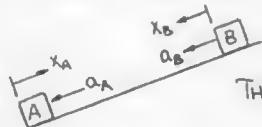
$$\text{OR } d = 906 \text{ ft}$$

# 11.46



GIVEN:  $(v_A)_0$  AND  $(v_B)_0$ ; AT  $t = 1 \text{ s}$ , BLOCKS PASS EACH OTHER, AT  $t = 3.4 \text{ s}$ ,  $x_B = d$ ;  $(x_A)_{\text{MAX}} = 21 \text{ ft}$ ;  $a_A, a_B$  ARE CONSTANT AND

FIND: (a)  $a_A$  AND  $a_B$   
(b)  $d$   
(c)  $v_A$  AT  $t = 1 \text{ s}$



(a) HAVE...  $v_A^2 = (v_A)_0^2 + 2a_A(x_A - (x_A)_0)$   
WHEN  $x_A = (x_A)_{\text{MAX}}$ ,  $v_A = 0$   
THEN...  $0 = (27 \frac{\text{ft}}{\text{s}})^2 + 2a_A(21 \text{ ft})$   
OR  $a_A = -17.3571 \frac{\text{ft}}{\text{s}^2}$   
OR  $a_A = 17.36 \frac{\text{ft}}{\text{s}^2}$

$$\text{NOW... } x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$\text{AND } x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

AT  $t = 1 \text{ s}$ , THE BLOCKS PASS EACH OTHER

$$\therefore (x_A)_1 + (x_B)_1 = d$$

AT  $t = 3.4 \text{ s}$ ,  $x_B = d$

$$\text{THUS... } (x_A)_1 + (x_B)_1 = (x_B)_{3.4}$$

$$\text{OR } [(27 \frac{\text{ft}}{\text{s}})(1 \text{ s}) + \frac{1}{2} (-17.3571 \frac{\text{ft}}{\text{s}^2})(1 \text{ s})^2] = [\frac{1}{2} a_B (3.4 \text{ s})^2]$$

$$\text{OR } a_B = 3.4700 \frac{\text{ft}}{\text{s}^2}$$

$$(b) \text{ AT } t = 3.4 \text{ s, } x_B = d: d = \frac{1}{2} (3.4700 \frac{\text{ft}}{\text{s}^2})(3.4 \text{ s})^2$$

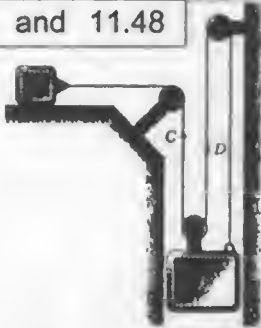
$$\text{OR } d = 20.1 \text{ ft}$$

$$(c) \text{ HAVE... } v_A = (v_A)_0 + a_A t$$

$$\text{AT } t = 1 \text{ s: } v_A = 27 \frac{\text{ft}}{\text{s}} + (-17.3571 \frac{\text{ft}}{\text{s}^2})(1 \text{ s})$$

$$\text{OR } v_A = 9.64 \frac{\text{ft}}{\text{s}}$$

# 11.47 and 11.48



GIVEN: BLOCKS A AND B AND THE PULLEY/CABLE SYSTEM SHOWN

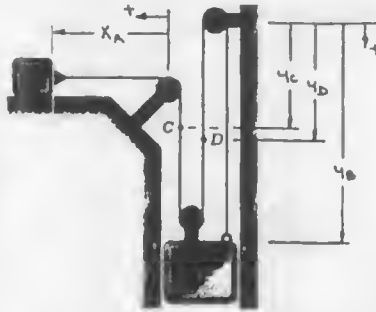
FROM THE DIAGRAM (NEXT COLUMN) HAVE..

$$x_A + 3y_B = \text{CONSTANT}$$

$$\text{THEN } v_A + 3v_B = 0 \quad (1)$$

$$\text{AND } a_A + 3a_B = 0 \quad (2) \quad (\text{CONTINUED})$$

# 11.47 and 11.48 CONTINUED



11.47 GIVEN:  $v_A = 6 \frac{\text{m}}{\text{s}}$

FIND: (a)  $v_B$

(b)  $v_D$

(c)  $v_C$

$$(a) \text{ SUBSTITUTING INTO EQ. (1)... } 6 \frac{\text{m}}{\text{s}} + 3v_B = 0$$

$$\text{OR } v_B = -2 \frac{\text{m}}{\text{s}}$$

$$(b) \text{ FROM THE DIAGRAM... } y_B + y_D = \text{CONSTANT}$$

$$\text{THEN } v_B + v_D = 0$$

$$\therefore v_D = 2 \frac{\text{m}}{\text{s}}$$

$$(c) \text{ FROM THE DIAGRAM... } x_A + y_C = \text{CONSTANT}$$

$$\text{THEN } v_A + v_C = 0 \therefore v_C = -6 \frac{\text{m}}{\text{s}}$$

$$\text{NOW... } v_C = v_D - v_B$$

$$= (-6 \frac{\text{m}}{\text{s}}) - (-2 \frac{\text{m}}{\text{s}}) = -8 \frac{\text{m}}{\text{s}}$$

$$\therefore v_C = 8 \frac{\text{m}}{\text{s}}$$

11.48 GIVEN: AT  $t = 0$ ,  $v_B = 0$ ;  $a_B = \text{CONSTANT}$ !

WHEN  $| \Delta x_A | = 0.4 \text{ m}$ ,  $v_A = 4 \frac{\text{m}}{\text{s}}$

FIND: (a)  $a_A$  AND  $a_B$

(b)  $v_B$  AND  $[y_B - (y_B)_0]$  AT  $t = 2 \text{ s}$

$$(a) \text{ EQ. (2): } a_A + 3a_B = 0 \text{ AND } a_B \text{ IS CONSTANT AND POSITIVE} \Rightarrow a_A \text{ IS CONSTANT AND NEGATIVE}$$

$$\text{ALSO, EQ. (1) AND } (v_B)_0 = 0 \Rightarrow (v_A)_0 = 0$$

$$\text{THEN } v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$$

$$\text{WHEN } | \Delta x_A | = 0.4 \text{ m: } (4 \frac{\text{m}}{\text{s}})^2 = 2a_A(0.4 \text{ m})$$

$$\text{OR } a_A = 20 \frac{\text{m}}{\text{s}^2}$$

$$\text{THEN... SUBSTITUTING INTO EQ. (2)... } -20 \frac{\text{m}}{\text{s}^2} + 3a_B = 0$$

$$\text{OR } a_B = \frac{20}{3} \frac{\text{m}}{\text{s}^2}$$

$$a_B = 6.67 \frac{\text{m}}{\text{s}^2}$$

$$(b) \text{ HAVE... } v_B = (v_B)_0 + a_B t$$

$$\text{AT } t = 2 \text{ s: } v_B = (\frac{20}{3} \frac{\text{m}}{\text{s}^2})(2 \text{ s})$$

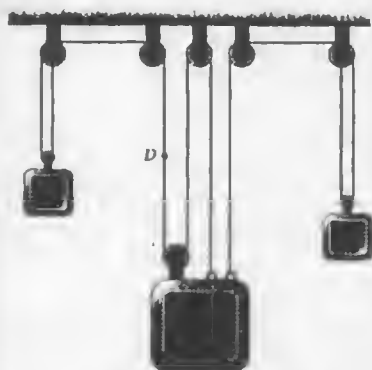
$$\text{OR } v_B = 13.33 \frac{\text{m}}{\text{s}}$$

$$\text{ALSO... } y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$\text{AT } t = 2 \text{ s: } y_B - (y_B)_0 = \frac{1}{2} (\frac{20}{3} \frac{\text{m}}{\text{s}^2})(2 \text{ s})^2$$

$$\text{OR } y_B - (y_B)_0 = 13.33 \text{ m}$$

# 11.49 and 11.50



GIVEN: BLOCKS A, B, AND C AND THE PULLEY/CABLE SYSTEM SHOWN

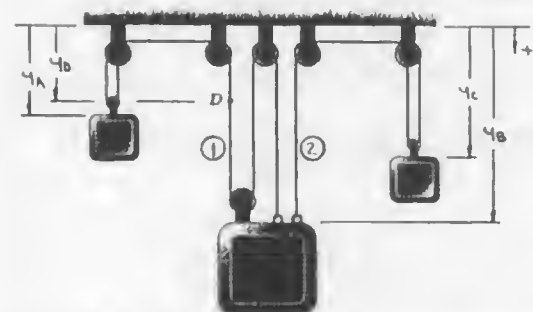
# 11.50 CONTINUED

SUBSTITUTING INTO EQ. (2)..  
 $2(1.5 \frac{\text{in}}{\text{s}}) + 3a_B = 0$   
 OR  $a_B = -1.0 \frac{\text{in}}{\text{s}^2}$   $\blacktriangleleft$   
 SUBSTITUTING INTO EQ. (4)..  
 $(-1.0 \frac{\text{in}}{\text{s}^2}) + 2a_C = 0$   
 OR  $a_C = 0.5 \frac{\text{in}}{\text{s}^2}$   $\blacktriangleleft$   
 (b) HAVE..  
 $v_B = (v_B)_0 + a_B t$   
 AT  $t = 8 \text{ s}$ :  $v_B = (-1.0 \frac{\text{in}}{\text{s}^2})(8 \text{ s})$   
 OR  $v_B = -8.0 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$   
 ALSO..  
 $y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$   
 AT  $t = 8 \text{ s}$ :  $y_B - (y_B)_0 = \frac{1}{2} (-1.0 \frac{\text{in}}{\text{s}^2})(8 \text{ s})^2$   
 OR  $y_B - (y_B)_0 = -32.0 \text{ in.}$   $\blacktriangleleft$

# 11.51 and 11.52



GIVEN: COLLARS A AND B AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM..

CABLE 1:  $2y_A + 3y_B = \text{CONSTANT}$   
 THEN..  $2v_A + 3v_B = 0$  (1)  
 AND  $2a_A + 3a_B = 0$  (2)

CABLE 2:  $y_B + 2y_C = \text{CONSTANT}$   
 THEN..  $v_B + 2v_C = 0$  (3)  
 AND  $a_B + 2a_C = 0$  (4)

11.49 GIVEN:  $v_B = 24 \frac{\text{in}}{\text{s}}$   $\downarrow$

FIND: (a)  $v_A$

(b)  $v_C$

(c)  $v_B$

(d)  $v_B/B$

(a) SUBSTITUTING INTO EQ. (1)..  
 $2v_A + 3(24 \frac{\text{in}}{\text{s}}) = 0$   
 OR  $v_A = -36 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$

(b) SUBSTITUTING INTO EQ. (3)..  
 $(24 \frac{\text{in}}{\text{s}}) + 2v_C = 0$   
 OR  $v_C = -12 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$

(c) FROM THE DIAGRAM..  $2y_A + y_B = \text{CONSTANT}$

THEN..  $2v_A + v_B = 0$

SUBSTITUTING FOR  $v_A$ ..  
 $2(-36 \frac{\text{in}}{\text{s}}) + v_B = 0$   
 OR  $v_B = 72 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$

(d) HAVE..  
 $v_{B/B} = v_B - v_B$   
 $= 72 \frac{\text{in}}{\text{s}} - 24 \frac{\text{in}}{\text{s}}$   
 OR  $v_{B/B} = 48 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$

11.50 GIVEN:  $(v_C)_0 = 0$ ;  $a_C = \text{CONSTANT}$   $\uparrow$ ; AT  $t = 12 \text{ s}$ ,  
 $v_A = 18 \frac{\text{in}}{\text{s}}$

FIND: (a)  $a_A$ ,  $a_B$ , AND  $a_C$

(b)  $v_B$  AND  $[y_B - (y_B)_0]$  AT  $t = 8 \text{ s}$

(a) EQS. (3) AND (1) AND  $(v_C)_0 = 0 \Rightarrow (v_A)_0 = (v_B)_0 = 0$

ALSO, EQS. (4) AND (2) AND  $a_C$  IS CONSTANT AND POSITIVE  $\Rightarrow a_B$  IS CONSTANT AND NEGATIVE

$a_A$  IS CONSTANT AND POSITIVE

THEN..  $v_A = (v_A)_0 + a_A t$

AT  $t = 12 \text{ s}$ :  $18 \frac{\text{in}}{\text{s}} = a_A (12 \text{ s})$  OR  $a_A = 1.5 \frac{\text{in}}{\text{s}^2}$   $\blacktriangleleft$

(CONTINUED)

11.51 GIVEN:  $(v_A)_0 = 0$ ;  $a_A = \text{CONSTANT}$   $\uparrow$ ; AT  $t = 8 \text{ s}$ ,  
 $|v_{B/A}| = 24 \frac{\text{in}}{\text{s}}$

FIND: (a)  $a_A$  AND  $a_B$

(b)  $v_B$  AND  $y_B - (y_B)_0$  AT  $t = 6 \text{ s}$

(a) EQ. (1) AND  $(v_A)_0 = 0 \Rightarrow (v_B)_0$

ALSO, EQ. (2) AND  $a_A$  IS CONSTANT AND NEGATIVE

$\Rightarrow a_B$  IS CONSTANT AND POSITIVE

THEN..  $v_A = (v_A)_0 + a_A t$   $v_B = (v_B)_0 + a_B t$

NOW..  $v_{B/A} = v_B - v_A = (a_B - a_A)t$

FROM EQ. (2)..  
 $a_B = -\frac{1}{2} a_A$  SO THAT  $v_{B/A} = -\frac{3}{2} a_A t$

AT  $t = 8 \text{ s}$ :  $24 \frac{\text{in}}{\text{s}} = -\frac{3}{2} a_A (8 \text{ s})$   
 OR  $a_A = 2 \frac{\text{in}}{\text{s}^2}$   $\blacktriangleleft$

AND THEN..  
 $a_B = -\frac{1}{2} (2 \frac{\text{in}}{\text{s}^2})$  OR  $a_B = -1 \frac{\text{in}}{\text{s}^2}$   $\blacktriangleleft$

(b) AT  $t = 6 \text{ s}$ :  $v_B = (1 \frac{\text{in}}{\text{s}})(6 \text{ s})$

OR  $v_B = 6 \frac{\text{in}}{\text{s}}$   $\blacktriangleleft$

NOW..  
 $y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

AT  $t = 6 \text{ s}$ :  $y_B - (y_B)_0 = \frac{1}{2} (1 \frac{\text{in}}{\text{s}^2})(6 \text{ s})^2$

OR  $y_B - (y_B)_0 = 18 \text{ in.}$   $\blacktriangleleft$

(CONTINUED)

## 11.52 CONTINUED

11.52 GIVEN:  $v_B = 12 \frac{\text{in}}{\text{s}}$

FIND: (a)  $v_A$

(b)  $v_C$

(c)  $v_{C/A}$

(a) SUBSTITUTING INTO EQ. (1)..  
 $v_A + 2(12 \frac{\text{in}}{\text{s}}) = 0$   
 OR  $v_A = -24 \frac{\text{in}}{\text{s}}$

(b) FROM THE DIAGRAM ..  $2v_A + v_C = \text{CONSTANT}$

THEN ..  $2v_A + v_C = 0$

SUBSTITUTING ..  $2(-24 \frac{\text{in}}{\text{s}}) + v_C = 0$

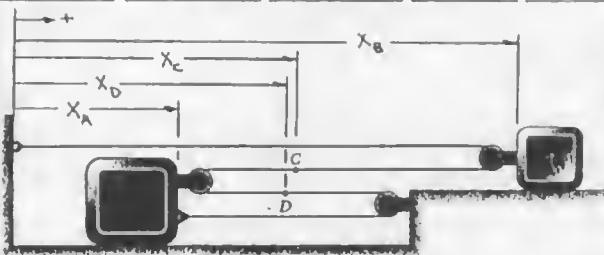
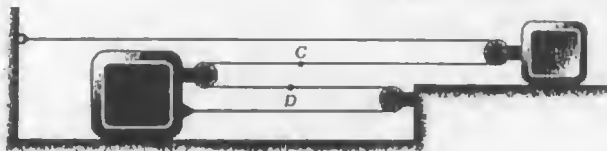
OR  $v_C = 48 \frac{\text{in}}{\text{s}}$

(c) HAVE ..  $v_{C/A} = v_C - v_A$   
 $= (48 \frac{\text{in}}{\text{s}}) - (-24 \frac{\text{in}}{\text{s}})$

OR  $v_{C/A} = 72 \frac{\text{in}}{\text{s}}$

## 11.53 and 11.54

GIVEN: BLOCKS A AND B AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM ..  $x_B + (x_B - x_A) - 2x_A = \text{CONSTANT}$

THEN ..  $2v_B - 3v_A = 0$  (1)

AND  $2v_B - 3v_A = 0$  (2)

11.53 GIVEN:  $v_B = 300 \frac{\text{mm}}{\text{s}}$

FIND: (a)  $v_A$

(b)  $v_C$

(c)  $v_D$

(d)  $v_{C/A}$

(a) SUBSTITUTING INTO EQ. (1)..  
 $2(300 \frac{\text{mm}}{\text{s}}) - 3v_A = 0$   
 OR  $v_A = 200 \frac{\text{mm}}{\text{s}}$

(b) FROM THE DIAGRAM ..  $x_B + (x_B - x_C) = \text{CONSTANT}$

THEN ..  $2v_B - v_C = 0$

SUBSTITUTING ..  $2(300 \frac{\text{mm}}{\text{s}}) - v_C = 0$

OR  $v_C = 600 \frac{\text{mm}}{\text{s}}$

(c) FROM THE DIAGRAM ..  $(x_C - x_A) + (x_D - x_A) = \text{CONSTANT}$

THEN ..  $v_C - 2v_A + v_D = 0$

SUBSTITUTING ..  $600 \frac{\text{mm}}{\text{s}} - 2(200 \frac{\text{mm}}{\text{s}}) + v_D = 0$

OR  $v_D = 200 \frac{\text{mm}}{\text{s}}$

(d) HAVE ..  $v_{C/A} = v_C - v_A$   
 $= 600 \frac{\text{mm}}{\text{s}} - 200 \frac{\text{mm}}{\text{s}}$

OR  $v_{C/A} = 400 \frac{\text{mm}}{\text{s}}$

\* ALSO HAVE ..  $-x_B - x_A = \text{CONSTANT}$

THEN ..  $v_B + v_A = 0$  (3)

(CONTINUED)

## 11.54 CONTINUED

11.54 GIVEN:  $(v_B)_0 = 150 \frac{\text{mm}}{\text{s}}$ ;  $a_B = \text{CONSTANT}$ ; WHEN  
 $x_A - (x_A)_0 = 240 \text{ mm}$ ,  $v_A = 60 \frac{\text{mm}}{\text{s}}$

FIND: (a)  $a_A$  AND  $a_B$

(b)  $a_B$

(c)  $v_B$  AND  $x_B - (x_B)_0$  AT  $t = 4 \text{ s}$

(a) FIRST OBSERVE THAT IF BLOCK A MOVES TO THE RIGHT,  $v_A \rightarrow$  AND EQ. (1)  $\Rightarrow v_B \rightarrow$ . THEN, USING EQ. (1) AT  $t = 0$  ..

$$2(150 \frac{\text{mm}}{\text{s}}) - 3(v_A)_0 = 0$$

$$\text{OR } (v_A)_0 = 100 \frac{\text{mm}}{\text{s}}$$

ALSO, EQ. (2) AND  $a_B = \text{CONSTANT} \Rightarrow a_A = \text{CONSTANT}$

THEN ..  $v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$

WHEN  $x_A - (x_A)_0 = 240 \text{ mm}$ :  $(60 \frac{\text{mm}}{\text{s}})^2 = (100 \frac{\text{mm}}{\text{s}})^2 + 2a_A(240 \text{ mm})$

$$\text{OR } a_A = -\frac{40}{3} \frac{\text{mm}}{\text{s}^2}$$

$$\text{OR } a_A = -13.33 \frac{\text{mm}}{\text{s}^2}$$

THEN, SUBSTITUTING INTO EQ. (2) ..

$$2a_B - 3(-\frac{40}{3} \frac{\text{mm}}{\text{s}^2}) = 0$$

$$\text{OR } a_B = -20 \frac{\text{mm}}{\text{s}^2}$$

$$a_B = 20.0 \frac{\text{mm}}{\text{s}^2}$$

(b) FROM THE SOLUTION TO PROBLEM 11.53 ..

$$v_B + v_A = 0$$

$$\text{THEN .. } a_B + a_A = 0$$

$$\text{SUBSTITUTING .. } a_B + (-\frac{40}{3} \frac{\text{mm}}{\text{s}^2}) = 0$$

$$\text{OR } a_B = 13.33 \frac{\text{mm}}{\text{s}^2}$$

(c) HAVE ..  $v_B = (v_B)_0 + a_B t$

$$\text{AT } t = 4 \text{ s: } v_B = 150 \frac{\text{mm}}{\text{s}} + (-20.0 \frac{\text{mm}}{\text{s}^2})(4 \text{ s})$$

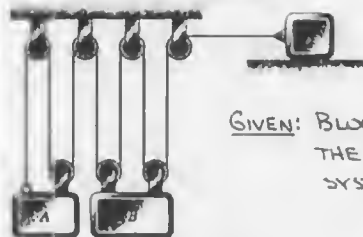
$$\text{OR } v_B = 70.0 \frac{\text{mm}}{\text{s}}$$

$$\text{ALSO .. } v_B = (v_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

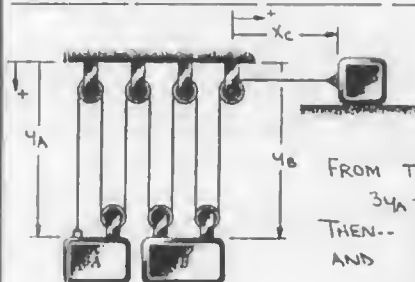
$$\text{AT } t = 4 \text{ s: } v_B - (v_B)_0 = (150 \frac{\text{mm}}{\text{s}})(4 \text{ s}) + \frac{1}{2}(-20.0 \frac{\text{mm}}{\text{s}^2})(4 \text{ s})^2$$

$$\text{OR } v_B - (v_B)_0 = 440 \text{ mm}$$

## 11.55 and 11.56



GIVEN: BLOCKS A, B, AND C AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM ..

$$3x_A + 4x_B + x_C = \text{CONSTANT}$$

$$\text{THEN .. } 3v_A + 4v_B + v_C = 0$$
 (1)

$$\text{AND } 3a_A + 4a_B + a_C = 0$$
 (2)

(CONTINUED)

# 11.55 and 11.56 CONTINUED

11.55 GIVEN:  $\dot{x}_B = 20 \frac{\text{mm}}{\text{s}} \uparrow$ ;  $(x_A)_0 = 30 \frac{\text{mm}}{\text{s}} \uparrow$ ;  
 $a_A = \text{CONSTANT}$ ; AT  $t = 3 \text{ s}$ ,  
 $x_C - (x_C)_0 = 57 \text{ mm} \rightarrow$

FIND: (a)  $(\dot{x}_C)_0$   
 (b)  $Q_A$  AND  $Q_C$   
 (c)  $y_A - (y_A)_0$  AT  $t = 5 \text{ s}$

(a) SUBSTITUTING INTO EQ. (1) AT  $t = 0..$

$$3(-30 \frac{\text{mm}}{\text{s}}) + 4(20 \frac{\text{mm}}{\text{s}}) + (\dot{x}_C)_0 = 0$$

$$\text{OR } (\dot{x}_C)_0 = 10 \frac{\text{mm}}{\text{s}} \rightarrow$$

(b) HAVE..  $x_C = (x_C)_0 + (\dot{x}_C)_0 t + \frac{1}{2} a_C t^2$

$$\text{AT } t = 3 \text{ s: } 57 \text{ mm} = (10 \frac{\text{mm}}{\text{s}})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$$

$$\text{OR } a_C = 6 \frac{\text{mm}}{\text{s}^2} \rightarrow$$

NOW..  $\dot{x}_B = \text{CONSTANT} \Rightarrow a_B = 0$

THEN.. SUBSTITUTING INTO EQ. (2)..

$$3Q_A + 4(0) + (6 \frac{\text{mm}}{\text{s}^2}) = 0$$

$$\text{OR } Q_A = 2 \frac{\text{mm}}{\text{s}^2} \uparrow$$

(c) HAVE..  $y_A = (y_A)_0 + (\dot{y}_A)_0 t + \frac{1}{2} a_A t^2$

$$\text{AT } t = 5 \text{ s: } y_A - (y_A)_0 = (-30 \frac{\text{mm}}{\text{s}})(5 \text{ s}) + \frac{1}{2} (-2 \frac{\text{mm}}{\text{s}^2})(5 \text{ s})^2$$

$$\text{OR } y_A - (y_A)_0 = -175 \text{ mm} \uparrow$$

11.56 GIVEN:  $(\dot{x}_B)_0 = 0$ ,  $Q_A = \text{CONSTANT}$ ,  
 $(Q_C) = 75 \frac{\text{mm}}{\text{s}^2} \rightarrow$ ; AT  $t = 2 \text{ s}$ ,  
 $\dot{x}_B = 480 \frac{\text{mm}}{\text{s}} \uparrow$ ,  $\dot{x}_C = 280 \frac{\text{mm}}{\text{s}} \rightarrow$

FIND: (a)  $Q_A$  AND  $Q_B$   
 (b)  $(\dot{x}_A)_0$  AND  $(\dot{x}_C)_0$   
 (c)  $x_C - (x_C)_0$  AT  $t = 3 \text{ s}$

(a) EQ. (2) AND  $Q_A = \text{CONSTANT}$  AND  
 $Q_C = \text{CONSTANT} \Rightarrow a_B = \text{CONSTANT}$

THEN..  $\dot{x}_B = (\dot{x}_B)_0 + a_B t$

$$\text{AT } t = 2 \text{ s: } 480 \frac{\text{mm}}{\text{s}} = a_B (2 \text{ s})$$

$$\text{OR } a_B = 240 \frac{\text{mm}}{\text{s}^2} \downarrow$$

SUBSTITUTING INTO EQ. (2)..

$$3Q_A + 4(240 \frac{\text{mm}}{\text{s}^2}) + (75 \frac{\text{mm}}{\text{s}^2}) = 0$$

$$\text{OR } Q_A = 345 \frac{\text{mm}}{\text{s}^2} \uparrow$$

(b) HAVE..  $\dot{x}_C = (\dot{x}_C)_0 + a_C t$

$$\text{AT } t = 2 \text{ s: } 280 \frac{\text{mm}}{\text{s}} = (\dot{x}_C)_0 + (75 \frac{\text{mm}}{\text{s}^2})(2 \text{ s})$$

$$\text{OR } (\dot{x}_C)_0 = 130 \frac{\text{mm}}{\text{s}} \rightarrow$$

THEN, SUBSTITUTING INTO EQ. (1) AT  $t = 0..$

$$3(\dot{x}_A)_0 + 4(0) + (130 \frac{\text{mm}}{\text{s}}) = 0$$

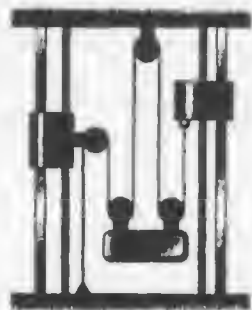
$$\text{OR } (\dot{x}_A)_0 = 43.3 \frac{\text{mm}}{\text{s}} \uparrow$$

(c) HAVE..  $x_C = (x_C)_0 + (\dot{x}_C)_0 t + \frac{1}{2} a_C t^2$

$$\text{AT } t = 3 \text{ s: } x_C - (x_C)_0 = (130 \frac{\text{mm}}{\text{s}})(3 \text{ s}) + \frac{1}{2} (75 \frac{\text{mm}}{\text{s}^2})(3 \text{ s})^2$$

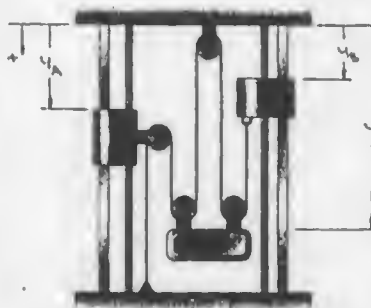
$$\text{OR } x_C - (x_C)_0 = 728 \text{ mm} \rightarrow$$

# 11.57 and 11.58



GIVEN: COLLARS A AND B,  
 BLOCK C, AND THE  
 PULLEY/CABLE  
 SYSTEM SHOWN

# 11.57 and 11.58 CONTINUED



FROM THE DIAGRAM..

$$-y_A + (y_C - y_A) + 2y_C$$

$$+ (y_C - y_B) = \text{CONSTANT}$$

THEN..

$$-2\dot{y}_A - \dot{y}_B + 4\dot{y}_C = 0 \quad (1)$$

AND

$$-2Q_A - Q_B + 4Q_C = 0 \quad (2)$$

11.57 GIVEN:  $(\dot{x}_A)_0 = 0$ ,  $(Q_A) = 7 \frac{\text{in.}}{\text{s}^2} \uparrow$ ;  $(\dot{x}_B)_0 = 8 \frac{\text{in.}}{\text{s}} \uparrow$ ;  
 $Q_B = \text{CONSTANT}$ ; AT  $t = 2 \text{ s}$ ,  $y_B - (y_B)_0 = 20 \text{ in.} \uparrow$

FIND: (a)  $Q_B$  AND  $Q_C$

(b)  $t$  WHEN  $\dot{x}_C = 0$

(c)  $y_C - (y_C)_0$  WHEN  $\dot{x}_C = 0$

(a) HAVE..  $y_B = (y_B)_0 + (\dot{y}_B)_0 t + \frac{1}{2} a_B t^2$

$$\text{AT } t = 2 \text{ s: } -20 \text{ in.} = (-8 \frac{\text{in.}}{\text{s}})(2 \text{ s}) + \frac{1}{2} a_B (2 \text{ s})^2$$

$$\text{OR } a_B = 2 \frac{\text{in.}}{\text{s}^2} \uparrow$$

THEN.. SUBSTITUTING INTO EQ. (2)..

$$-2(7 \frac{\text{in.}}{\text{s}^2}) - (-2 \frac{\text{in.}}{\text{s}^2}) + 4Q_C = 0$$

$$\text{OR } Q_C = 3 \frac{\text{in.}}{\text{s}^2} \downarrow$$

(b) SUBSTITUTING INTO EQ. (1) AT  $t = 0..$

$$-2(0) - (-8 \frac{\text{in.}}{\text{s}}) + 4(\dot{x}_C)_0 = 0 \quad \text{OR } (\dot{x}_C)_0 = -2 \frac{\text{in.}}{\text{s}}$$

NOW..  $\dot{x}_C = (\dot{x}_C)_0 + a_C t$

$$\text{WHEN } \dot{x}_C = 0: 0 = (-2 \frac{\text{in.}}{\text{s}}) + (3 \frac{\text{in.}}{\text{s}^2}) t$$

$$\text{OR } t = \frac{2}{3} \text{ s} \quad t = 0.667 \text{ s}$$

(c) HAVE..  $y_C = (y_C)_0 + (\dot{y}_C)_0 t + \frac{1}{2} a_C t^2$

$$\text{AT } t = \frac{2}{3} \text{ s: } y_C - (y_C)_0 = (-2 \frac{\text{in.}}{\text{s}})(\frac{2}{3} \text{ s}) + \frac{1}{2} (3 \frac{\text{in.}}{\text{s}^2})(\frac{2}{3} \text{ s})^2$$

$$\text{OR } y_C - (y_C)_0 = 0.667 \text{ in.} \uparrow$$

11.58 GIVEN:  $(\dot{x}_A)_0 = 0$ ,  $(\dot{x}_B)_0 = 0$ ;  $Q_A = 3 \frac{\text{ft}^2}{\text{s}^2} \uparrow$ ;  
 $Q_B = \text{CONSTANT}$ ; WHEN  $y_B - (y_B)_0 = 32 \text{ in.} \uparrow$ ,  
 $\dot{x}_B = 8 \frac{\text{ft}}{\text{s}}$

FIND: (a)  $a_C$

(b) DISTANCE TRAVELED BY C AT  $t = 3 \text{ s}$

(a) HAVE..  $\dot{x}_B = (\dot{x}_B)_0 + a_B t$

$$\text{WHEN } y_B - (y_B)_0 = 32 \text{ in.}: (8 \frac{\text{ft}}{\text{s}})^2 = 2Q_B (32 \text{ in.})$$

$$\text{OR } Q_B = 1 \frac{\text{ft}}{\text{s}^2}$$

THEN, SUBSTITUTING INTO EQ. (2)..

$$-2(-3 \frac{\text{ft}^2}{\text{s}^2}) - (1 \frac{\text{ft}^2}{\text{s}^2}) + 4Q_C = 0$$

$$\text{OR } Q_C = \frac{1}{4} (1 - 6 \frac{\text{ft}^2}{\text{s}^2}) \frac{\text{ft}}{\text{s}^2}$$

(b) SUBSTITUTING INTO EQ. (1) AT  $t = 0..$

$$-2(0) - (0) + 4(\dot{x}_C)_0 = 0 \quad \text{OR } (\dot{x}_C)_0 = 0$$

NOW..  $\frac{dy_C}{dt} = a_C = \frac{1}{4} (1 - 6t^2)$

$$\text{AT } t = 0, \dot{x}_C = 0: \int_0^{\dot{x}_C} d\dot{x}_C = \int_0^t \frac{1}{4} (1 - 6t^2) dt$$

$$\text{OR } \dot{x}_C = \frac{1}{4} (t - 2t^3)$$

THUS,  $\dot{x}_C = 0$  AT  $\frac{1}{4} (t - 2t^3) = 0$  OR  $t = 0, t = \frac{1}{\sqrt{2}} \text{ s}$

THEREFORE, BLOCK C INITIALLY MOVES DOWNWARDS

( $\dot{x}_C > 0$ ) AND THEN MOVES UPWARDS ( $\dot{x}_C < 0$ ).

NOW..  $\frac{dy_C}{dt} = \dot{x}_C = \frac{1}{4} (t - 2t^3)$

$$\text{AT } t = 0, y_C - (y_C)_0 = \int_0^t \frac{1}{4} (t - 2t^3) dt$$

$$\text{OR } y_C - (y_C)_0 = \frac{1}{8} (t^2 - \frac{1}{2} t^4)$$

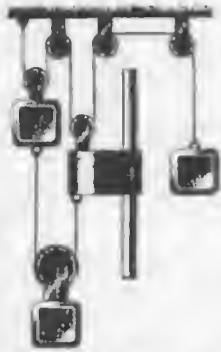
$$\text{AT } t = \frac{1}{\sqrt{2}} \text{ s: } y_C - (y_C)_0 = \frac{1}{8} [(\frac{1}{\sqrt{2}})^2 - (\frac{1}{\sqrt{2}})^4] = \frac{1}{32} \text{ in.}$$

$$\text{AT } t = 3 \text{ s: } y_C - (y_C)_0 = \frac{1}{8} [(3)^2 - (3)^4] = -9 \text{ in.}$$

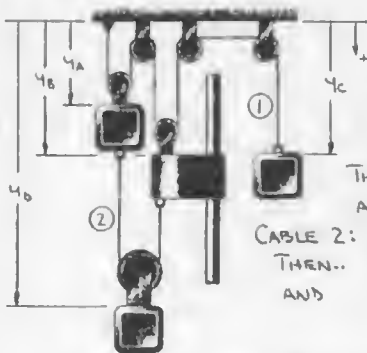
$$\therefore \text{TOTAL DISTANCE TRAVELED} = (\frac{1}{32}) + |-9 - \frac{1}{32}| = 9 \frac{1}{32} \text{ in.}$$

$$= 9.06 \text{ in.}$$

# 11.59 and \* 11.60



GIVEN: BLOCKS A, C, AND D, COLLAR B, AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM..  
CABLE 1:

$$2y_A + 2y_B + y_C = \text{CONSTANT}$$

$$\text{THEN.. } 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C = 0 \quad (1)$$

$$\text{AND } 2a_A + 2a_B + a_C = 0 \quad (2)$$

$$\text{CABLE 2: } (y_B - y_A) + (y_B - y_C) = \text{CONSTANT}$$

$$\text{THEN.. } -\dot{y}_A - \dot{y}_B + 2\dot{y}_C = 0 \quad (3)$$

$$\text{AND } -a_A - a_B + 2a_C = 0 \quad (4)$$

11.59 GIVEN: AT  $t=0$ ,  $\dot{y}_C=0$ ; ALL ACCELERATIONS CONSTANT;  $a_{C/B} = 60 \frac{\text{mm}}{\text{s}^2}$  ↑

$$a_{B/A} = 110 \frac{\text{mm}}{\text{s}^2} \uparrow$$

FIND: (a)  $\dot{y}_C$  AT  $t=3$  s

$$(b) y_B - (y_B)_0 \text{ AT } t=5$$

$$(a) \text{ HAVE.. } a_{C/B} = a_C - a_B = -60 \text{ OR } a_B = a_C + 60$$

$$\text{AND } a_{B/A} = a_B - a_A = 110 \text{ OR } a_A = a_B - 110$$

SUBSTITUTING INTO EQS (2) AND (4)...

$$\text{EQ (2): } 2(a_B - 110) + 2(a_C + 60) + a_C = 0$$

$$\text{OR } 3a_C + 2a_B = 100 \quad (5)$$

$$\text{EQ (4): } -(a_B - 110) - (a_C + 60) + 2a_C = 0$$

$$\text{OR } -a_C + a_B = -50 \quad (6)$$

SOLVING EQS (5) AND (6) FOR  $a_C$  AND  $a_B$ ...

$$a_C = 40 \frac{\text{mm}}{\text{s}^2} \quad a_B = -10 \frac{\text{mm}}{\text{s}^2}$$

$$\text{NOW.. } \dot{y}_C = (\dot{y}_C)_0 + a_C t$$

$$\text{AT } t=3 \text{ s: } \dot{y}_C = (40 \frac{\text{mm}}{\text{s}^2})(3 \text{ s})$$

$$\text{OR } \dot{y}_C = 120 \frac{\text{mm}}{\text{s}} \uparrow$$

$$(b) \text{ HAVE.. } y_B = (y_B)_0 + (\dot{y}_B)_0 t + \frac{1}{2} a_B t^2$$

$$\text{AT } t=5 \text{ s: } y_B - (y_B)_0 = \frac{1}{2} (-10 \frac{\text{mm}}{\text{s}^2})(5 \text{ s})^2$$

$$\text{OR } y_B - (y_B)_0 = -125 \text{ mm} \uparrow$$

11.60 GIVEN: AT  $t=0$ ,  $\dot{y}_C=0$ ,  $(y_A)_0 = (y_B)_0 = (y_C)_0$ ; ALL ACCELERATIONS CONSTANT; AT  $t=2$  s,  $y_{C/A} = 280 \text{ mm} \uparrow$ ; WHEN  $\dot{y}_{B/A} = 80 \frac{\text{mm}}{\text{s}} \uparrow$ ,  $y_A - (y_A)_0 = 160 \text{ mm} \uparrow$ ,  $y_B - (y_B)_0 = 320 \text{ mm} \uparrow$ ;  $a_B > 10 \frac{\text{mm}}{\text{s}^2}$

FIND: (a)  $a_A$  AND  $a_B$

$$(b) y_B - (y_B)_0 \text{ WHEN } \dot{y}_C = 600 \frac{\text{mm}}{\text{s}} \uparrow$$

$$(a) \text{ HAVE.. } y_A = (y_A)_0 + (\dot{y}_A)_0 t + \frac{1}{2} a_A t^2$$

$$\text{AND } y_C = (y_C)_0 + (\dot{y}_C)_0 t + \frac{1}{2} a_C t^2$$

(CONTINUED)

# 11.60 CONTINUED

$$\text{THEN.. } y_{C/A} = y_C - y_A = \frac{1}{2} (a_C - a_A) t^2$$

$$\text{AT } t=2 \text{ s, } y_{C/A} = -280 \text{ mm: } -280 \text{ mm} = \frac{1}{2} (a_C - a_A) (2 \text{ s})^2$$

$$\text{OR } a_C = a_A - 140 \quad (5)$$

$$\text{SUBSTITUTING INTO EQ (2).. } 2a_A + 2a_B + (a_A - 140) = 0$$

$$\text{OR } a_A = \frac{1}{3} (140 - 2a_B) \quad (6)$$

$$\text{NOW.. } \dot{y}_B = (\dot{y}_B)_0 + a_B t \quad \dot{y}_A = (\dot{y}_A)_0 + a_A t$$

$$\therefore \dot{y}_{B/A} = \dot{y}_B - \dot{y}_A = (a_B - a_A) t$$

$$\text{ALSO.. } y_B = (y_B)_0 + (\dot{y}_B)_0 t + \frac{1}{2} a_B t^2$$

$$\text{WHEN } \dot{y}_{B/A} = 80 \frac{\text{mm}}{\text{s}} \uparrow: 80 = (a_B - a_A) t \quad (7)$$

$$\Delta y_A = 160 \text{ mm: } 160 = \frac{1}{2} a_A t^2$$

$$\Delta y_B = 320 \text{ mm: } 320 = \frac{1}{2} a_B t^2$$

$$\text{THEN } 160 = \frac{1}{2} (a_B - a_A) t^2$$

$$\text{USING EQ (7).. } 320 = (80) t \quad \text{OR } t = 4 \text{ s}$$

$$\text{THEN } 160 = \frac{1}{2} a_A (4)^2 \quad \text{OR } a_A = 20 \frac{\text{mm}}{\text{s}^2} \uparrow$$

$$\text{AND } 320 = \frac{1}{2} a_B (4)^2 \quad \text{OR } a_B = 40 \frac{\text{mm}}{\text{s}^2} \uparrow$$

NOTE THAT EQ (6) IS NOT USED; THUS, THE PROBLEM IS OVER DETERMINED.

ALTERNATIVE SOLUTION:

$$\text{HAVE.. } \dot{y}_A^2 = (\dot{y}_A)_0^2 + 2a_A[y_A - (y_A)_0] \quad \dot{y}_B^2 = (\dot{y}_B)_0^2 + 2a_B[y_B - (y_B)_0]$$

$$\text{THEN.. } \dot{y}_{B/A} = \dot{y}_B - \dot{y}_A = \sqrt{2a_B[y_B - (y_B)_0]} - \sqrt{2a_A[y_A - (y_A)_0]}$$

$$\text{WHEN } \dot{y}_{B/A} = 80 \frac{\text{mm}}{\text{s}} \uparrow: 80 \frac{\text{mm}}{\text{s}} = \sqrt{2a_B(320 \text{ mm})} - \sqrt{2a_A(160 \text{ mm})}$$

$$\text{OR } 20 = \sqrt{2(20a_B - 10a_A)} \quad (8)$$

SOLVING EQS (6) AND (8) YIELDS  $a_A$  AND  $a_B$ .

(b) SUBSTITUTING INTO EQ (5)...

$$a_C = 20 - 140 = -120 \frac{\text{mm}}{\text{s}^2}$$

$$\text{AND INTO EQ (4).. } -(20 \frac{\text{mm}}{\text{s}^2}) - (40 \frac{\text{mm}}{\text{s}^2}) + 2a_B = 0$$

$$\text{OR } a_B = 30 \frac{\text{mm}}{\text{s}^2}$$

$$\text{NOW.. } \dot{y}_C = (\dot{y}_C)_0 + a_C t$$

$$\text{WHEN } \dot{y}_C = 600 \frac{\text{mm}}{\text{s}}: 600 \frac{\text{mm}}{\text{s}} = (-120 \frac{\text{mm}}{\text{s}^2}) t$$

$$\text{OR } t = 5 \text{ s}$$

$$\text{ALSO.. } y_B = (y_B)_0 + (\dot{y}_B)_0 t + \frac{1}{2} a_B t^2$$

$$\text{AT } t=5 \text{ s: } y_B - (y_B)_0 = \frac{1}{2} (30 \frac{\text{mm}}{\text{s}^2})(5 \text{ s})^2$$

$$\text{OR } y_B - (y_B)_0 = 375 \text{ mm} \uparrow$$

# 11.61 and 11.62

$a$  (m/s<sup>2</sup>)

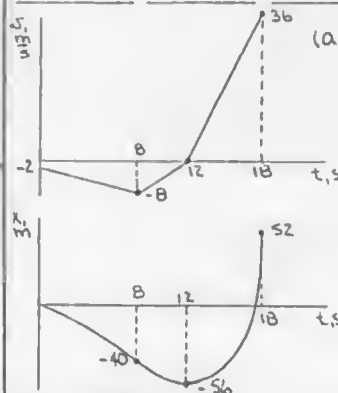


GIVEN:  $a-t$  CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT  $t=0$ ,

$x=0$ ,  $\dot{x}=-2 \frac{\text{m}}{\text{s}}$

CONSTRUCT: (a)  $\dot{x}-t$  AND

$x-t$  CURVES FOR  $0 \leq t \leq 18$  s



$$(a) \dot{x}_2 = \dot{x}_1 + (\text{AREA UNDER } a-t \text{ CURVE FROM } t_1 \text{ TO } t_2)$$

$$\text{AT } t=8 \text{ s: } \dot{x}_8 = -2 - (8)(0.75)$$

$$= -8.0 \frac{\text{m}}{\text{s}}$$

$$t=12 \text{ s: } \dot{x}_{12} = -8.0 + 4(2) = 0$$

$$t=18 \text{ s: } \dot{x}_{18} = 0 + 6(6)$$

$$= 36 \frac{\text{m}}{\text{s}}$$

$$x_2 = x_1 + (\text{AREA UNDER } \dot{x}-t \text{ CURVE FROM } t_1 \text{ TO } t_2)$$

$$\text{AT } t=8 \text{ s: } x_8 = 0 - 8(\frac{2}{2})$$

$$= -40 \text{ m}$$

(CONTINUED)

# 11.61 and 11.62 CONTINUED

At  $t = 12$  s:  $x_{12} = -40 - \frac{1}{2}(4)(8) = -56$  m  
 $t = 18$  s:  $x_{18} = -56 + \frac{1}{2}(6)(36) = 52$  m

11.61 FIND: (b)  $x$ ,  $v$ , AND TOTAL DISTANCE TRAVELED AT  $t = 18$  s

(b) READING FROM THE CURVES..  $x_{18} = 52$  m  
 $v_{18} = 36 \frac{\text{m}}{\text{s}}$

FROM  $t = 0$  TO  $t = 12$  s: DISTANCE TRAVELED = 56 m  
 $t = 12$  s TO  $t = 18$  s: DISTANCE TRAVELED = 52 - (-56) = 108 m

$\therefore$  TOTAL DISTANCE TRAVELED = (56 + 108) m = 164 m

11.62 FIND: (b)  $v_{\min}$   
 (c)  $x_{\min}$

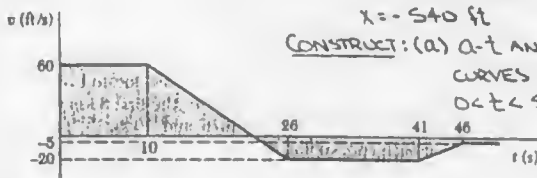
(b) READING FROM THE  $v$ - $t$  CURVE..  $v_{\min} = -8 \frac{\text{m}}{\text{s}}$

(c) READING FROM THE  $x$ - $t$  CURVE..  $x_{\min} = -56$  m

# 11.63 and 11.64

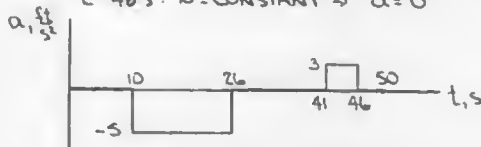
GIVEN:  $v$ - $t$  CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT  $t = 0$ ,  $x = -540$  ft

CONSTRUCT: (a)  $a$ - $t$  AND  $x$ - $t$  CURVES FOR  $0 \leq t \leq 50$  s



(a)  $a_t$  = SLOPE OF  $v$ - $t$  CURVE AT TIME  $t$

FROM  $t = 0$  TO  $t = 10$  s:  $v$  = CONSTANT  $\Rightarrow a = 0$   
 $t = 10$  s TO  $t = 26$  s:  $a = \frac{0 - 60}{26 - 10} = -5 \frac{\text{ft}}{\text{s}^2}$   
 $t = 26$  s TO  $t = 41$  s:  $v$  = CONSTANT  $\Rightarrow a = 0$   
 $t = 41$  s TO  $t = 46$  s:  $a = \frac{30 - 0}{46 - 41} = 3 \frac{\text{ft}}{\text{s}^2}$   
 $t > 46$  s:  $v$  = CONSTANT  $\Rightarrow a = 0$

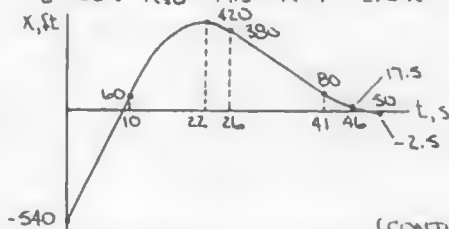


$x_2 = x_1 + (\text{AREA UNDER } v\text{-}t \text{ CURVE FROM } t_1 \text{ TO } t_2)$

AT  $t = 10$  s:  $x_{10} = -540 + 10(60) = 60$  ft

NEXT FIND TIME AT WHICH  $v = 0$ . USING SIMILAR TRIANGLES..  $\frac{t_{v=0} - 10}{60} = \frac{26 - 10}{80}$  OR  $t_{v=0} = 22$  s

AT  $t = 22$  s:  $x_{22} = 60 + \frac{1}{2}(12)(60) = 420$  ft  
 $t = 26$  s:  $x_{26} = 420 - \frac{1}{2}(4)(20) = 380$  ft  
 $t = 41$  s:  $x_{41} = 380 - 15(20) = 80$  ft  
 $t = 46$  s:  $x_{46} = 80 - 5(\frac{30 \times 5}{2}) = 17.5$  ft  
 $t = 50$  s:  $x_{50} = 17.5 - 4(5) = -2.5$  ft



(CONTINUED)

# 11.63 and 11.64 CONTINUED

11.63 FIND: (b) TOTAL DISTANCE TRAVELED AT  $t = 50$  s  
 (c)  $t$  WHEN  $x = 0$

(b) FROM  $t = 0$  TO  $t = 22$  s: DISTANCE TRAVELED = 420 - (-540) = 960 ft

$t = 22$  s TO  $t = 50$  s: DISTANCE TRAVELED = 12.5 - 420 = 422.5 ft

$\therefore$  TOTAL DISTANCE TRAVELED = (960 + 422.5) ft = 1382.5 ft

(c) USING SIMILAR TRIANGLES..

BETWEEN 0 AND 10 s:  $\frac{(t_{x=0})_1 - 0}{540} = \frac{10}{600}$

OR  $(t_{x=0})_1 = 9$  s

BETWEEN 46 s AND 50 s:  $\frac{(t_{x=0})_2 - 46}{17.5} = \frac{4}{20}$

OR  $(t_{x=0})_2 = 49.5$  s

11.64 FIND: (b)  $x_{\max}$

(c)  $t$  WHEN  $x = 100$  ft

(b) READING FROM THE  $x$ - $t$  CURVE..  $x_{\max} = 420$  ft

(c) BETWEEN 10 s AND 22 s..

100 ft = 420 ft - (AREA UNDER  $v$ - $t$  CURVE FROM  $t_1$  TO 22 s) ft

OR  $100 = 420 - \frac{1}{2}(22 - t_1)(v_1)$

OR  $(22 - t_1)(v_1) = 640$

USING SIMILAR TRIANGLES..

$\frac{v_1}{22 - t_1} = \frac{60}{12}$  OR  $v_1 = 5(22 - t_1)$

THEN..  $(22 - t_1)[5(22 - t_1)] = 640$

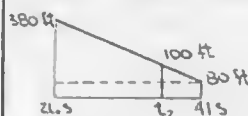
OR  $t_1 = 10.69$  s AND  $t_1 = 33.3$  s

HAVE  $10 \text{ s} < t_1 < 22 \text{ s}$

$t_1 = 10.69$  s

BETWEEN 26 s AND 41 s

USING SIMILAR TRIANGLES..



$\frac{41 - t_2}{20} = \frac{15}{300}$

OR  $t_2 = 40$  s

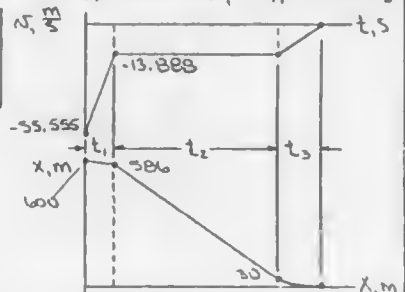
# 11.65



GIVEN: AT  $t = 0$ ,  $v = 200 \frac{\text{km}}{\text{h}}$ ,  $x = 600$  m;  
 FOR  $600 \text{ m} \leq x \leq 586 \text{ m}$ ,  
 $a$  = CONSTANT; FOR  
 $586 \text{ m} < x \leq 30 \text{ m}$ ,  $v = 50 \frac{\text{km}}{\text{h}}$ ;  
 WHEN  $x = 0$ ,  $v = 0$

FIND: (a)  $t_{\text{TOTAL}}$   
 (b)  $Q_{\text{INITIAL}}$

ASSUME SECOND DECELERATION IS CONSTANT. ALSO, NOTE THAT  
 $200 \frac{\text{km}}{\text{h}} = 55.555 \frac{\text{m}}{\text{s}}$ ,  $50 \frac{\text{km}}{\text{h}} = 13.888 \frac{\text{m}}{\text{s}}$



(CONTINUED)

# 11.65 CONTINUED

(a) NOW  $-\Delta x = \text{AREA UNDER } v-t \text{ CURVE FOR GIVEN TIME INTERVAL}$

$$\text{THEN.. } (586-600)\text{m} = -t_1 \left( \frac{55.555 + 13.888}{2} \right) \frac{\text{m}}{\text{s}}$$

$$\text{OR } t_1 = 0.4032 \text{ s}$$

$$(30-586)\text{m} = -t_2 (13.888 \frac{\text{m}}{\text{s}})$$

$$\text{OR } t_2 = 40.0346 \text{ s}$$

$$(0-30)\text{m} = -\frac{1}{2}(t_3)(13.888 \frac{\text{m}}{\text{s}})$$

$$\text{OR } t_3 = 4.3203 \text{ s}$$

$$\therefore t_{\text{TOTAL}} = (0.4032 + 40.0346 + 4.3203)\text{s}$$

$$\text{OR } t_{\text{TOTAL}} = 44.8 \text{ s}$$

(b) HAVE..  $Q_{\text{INITIAL}} = \frac{A v_{\text{INITIAL}}}{L_1}$   
 $= \frac{[-13.888 - (55.555)] \frac{\text{m}}{\text{s}}}{0.4032 \text{ s}}$

$$= 103.3 \frac{\text{m}}{\text{s}^2}$$

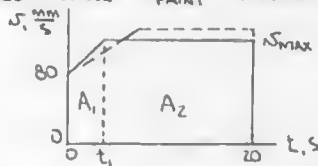
$$\text{OR } Q_{\text{INITIAL}} = 103.3 \frac{\text{m}}{\text{s}^2}$$

# 11.66

GIVEN: AT  $t = 20 \text{ s}$ ,  $x = 4 \text{ m}$ ;  $v_0 = 80 \frac{\text{mm}}{\text{s}}$ ,  
 $a_{\text{MAX}} = 60 \frac{\text{mm}}{\text{s}^2}$ ;  $t_{\text{PAINT}} = 15 \text{ s}$ ,  
 $v_{\text{PAINT}} = \text{CONSTANT}$

FIND:  $(L_{\text{MAX}})_{\text{MIN}}$

FIRST NOTE THAT  $(80 \frac{\text{mm}}{\text{s}})(20 \text{ s}) < 4000 \text{ mm}$ , SO THAT THE SPEED OF THE PALLET MUST BE INCREASED. SINCE  $v_{\text{PAINT}} = \text{CONSTANT}$ , IT FOLLOWS



THAT  $v_{\text{PAINT}} = v_{\text{MAX}}$  AND THEN  $t_1 \leq 5 \text{ s}$ . FROM THE  $v-t$  CURVE,  $A_1 + A_2 = 4000 \text{ mm}$  AND IT IS SEEN THAT  $(v_{\text{MAX}})_{\text{MIN}}$  OCCURS WHEN  $A_1 = (\frac{v_{\text{MAX}} - 80}{t_1})$  IS MAXIMUM. THUS

$$\frac{(v_{\text{MAX}} - 80) \frac{\text{mm}}{\text{s}}}{t_1 (\text{s})} = 60 \frac{\text{mm}}{\text{s}^2}$$

$$\text{OR } t_1 = (v_{\text{MAX}} - 80) / 60$$

$$\text{AND } t_1 (\frac{80 + v_{\text{MAX}}}{2}) + (20 - t_1)(v_{\text{MAX}}) = 4000$$

SUBSTITUTING FOR  $t_1$ ..

$$\frac{(v_{\text{MAX}} - 80)}{60} \left( \frac{80 + v_{\text{MAX}}}{2} \right) + (20 - \frac{v_{\text{MAX}} - 80}{60}) v_{\text{MAX}} = 4000$$

$$\text{SIMPLIFYING.. } v_{\text{MAX}}^2 - 2560 v_{\text{MAX}} + 486400 = 0$$

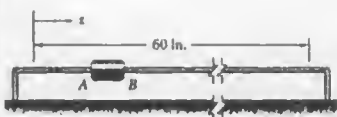
$$\text{SOLVING.. } v_{\text{MAX}} = 207 \frac{\text{mm}}{\text{s}} \text{ AND } v_{\text{MAX}} = 2353 \frac{\text{mm}}{\text{s}}$$

$$\text{FOR } v_{\text{MAX}} = 207 \frac{\text{mm}}{\text{s}}, t_1 < 5 \text{ s}$$

$$v_{\text{MAX}} = 2353 \frac{\text{mm}}{\text{s}}, t_1 > 5 \text{ s}$$

$$\therefore (v_{\text{MAX}})_{\text{MIN}} = 207 \frac{\text{mm}}{\text{s}}$$

# 11.67

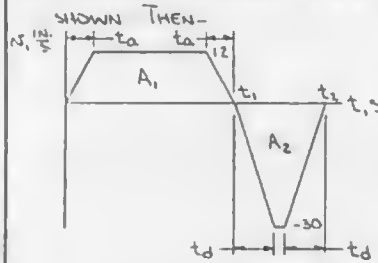


GIVEN:  $(v_{\text{MAX}})_{\text{RIGHT}} = 12 \frac{\text{in}}{\text{s}}$ ,  
 $(v_{\text{MAX}})_{\text{LEFT}} = 30 \frac{\text{in}}{\text{s}}$ ;  
 $a_{\text{RIGHT}} = \pm 6 \frac{\text{in}}{\text{s}^2}$   
 $a_{\text{LEFT}} = \pm 20 \frac{\text{in}}{\text{s}^2}$

FIND: (a)  $t_{\text{CYCLE}}$   
 CONSTRUCT (b)  $v-t$  AND  
 $x-t$  CURVES

# 11.67 CONTINUED

(a) AND (b) THE  $v-t$  CURVE IS FIRST DRAWN AS



$$t_a = \frac{v_{\text{RIGHT}}}{a_{\text{RIGHT}}} = \frac{12 \frac{\text{in}}{\text{s}}}{6 \frac{\text{in}}{\text{s}^2}} = 2 \text{ s}$$

$$t_d = \frac{v_{\text{LEFT}}}{a_{\text{LEFT}}} = \frac{30 \frac{\text{in}}{\text{s}}}{20 \frac{\text{in}}{\text{s}^2}} = 1.5 \text{ s}$$

$$\text{NOW.. } A_1 = 60 \text{ in.}$$

$$\text{OR } [(t_1 - 2)\text{s}](12 \frac{\text{in}}{\text{s}}) = 60 \text{ in.}$$

$$\text{OR } t_1 = 7 \text{ s}$$

$$\text{AND } A_2 = 60 \text{ in.}$$

$$\text{OR } [(t_2 - 7) - 1.5]\text{s} (30 \frac{\text{in}}{\text{s}}) = 60 \text{ in.}$$

$$\text{OR } t_2 = 10.5 \text{ s}$$

$$\therefore t_{\text{CYCLE}} = 10.5 \text{ s}$$

NOW..  $t_{\text{CYCLE}} = t_2$

HAVE..  $x_{i+1} = x_i + (\text{AREA UNDER } v-t \text{ CURVE FROM } t_i \text{ TO } t_{i+1})$

$$x_1 \text{ IN. AT } t = 2 \text{ s: } x_2 = \frac{1}{2}(2)(12) = 12 \text{ in.}$$

$$t = 5 \text{ s: } x_3 = 12 + (5-2)(12) = 48 \text{ in.}$$

$$t = 7 \text{ s: } x_4 = 60 \text{ in.}$$

$$t = 8.5 \text{ s: } x_5 = 60 - \frac{1}{2}(1.5)(30) = 37.5 \text{ in.}$$

$$t = 9 \text{ s: } x_6 = 37.5 - (0.5)(30) = 22.5 \text{ in.}$$

$$t = 10.5 \text{ s: } x_{10.5} = 0$$

# 11.68



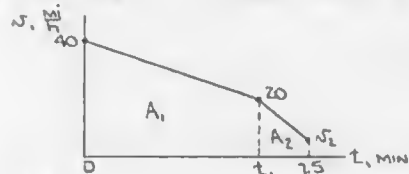
GIVEN: AT  $t = 0$ ,  $v = 40 \frac{\text{mi}}{\text{h}}$ ,  $x = 0$ ; WHEN  $x = 2.5 \text{ mi}$ ,  
 $v = 20 \frac{\text{mi}}{\text{h}}$ ; AT  $t = 7.5 \text{ min}$ ,  $x = 3 \text{ mi}$ ; CONSTANT  
 DECELERATIONS

FIND: (a)  $t$  WHEN  $x = 2.5 \text{ mi}$

(b)  $v$  WHEN  $x = 3 \text{ mi}$

(c)  $a_{\text{FINAL}}$

THE  $v-t$  CURVE IS FIRST DRAWN AS SHOWN.



(a) HAVE..  $A_1 = 2.5 \text{ mi}$   
 OR  $(t_1 \text{ MIN}) \left( \frac{40 + 20}{2} \right) \frac{\text{mi}}{\text{h}} = \frac{1 \text{ h}}{60 \text{ MIN}} = 2.5 \text{ mi}$   
 OR  $t_1 = 5 \text{ MIN}$

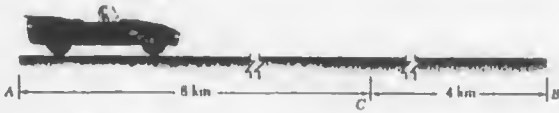
(b) HAVE..  $A_2 = 0.5 \text{ mi}$   
 OR  $(7.5 - 5) \text{ MIN} = \left( \frac{20 + v_2}{2} \right) \frac{\text{mi}}{\text{h}} = \frac{1 \text{ h}}{60 \text{ MIN}} = 0.5 \text{ mi}$   
 OR  $v_2 = 4 \frac{\text{mi}}{\text{h}}$

(c) HAVE  $a_{\text{FINAL}} = a_{12}$   
 $= \frac{(4 - 20) \frac{\text{mi}}{\text{h}}}{(7.5 - 5) \text{ MIN}} = \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{1 \text{ MIN}}{60 \text{ S}} \cdot \frac{1 \text{ h}}{3600 \text{ S}} = -0.156 \frac{\text{ft}}{\text{s}^2}$   
 OR  $a_{\text{FINAL}} = -0.156 \frac{\text{ft}}{\text{s}^2}$

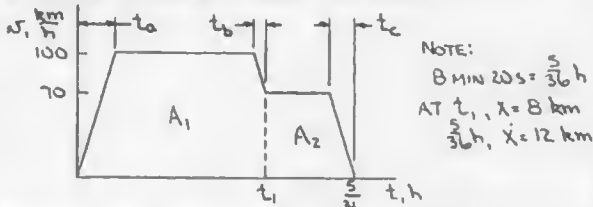
# 11.69

GIVEN:  $(N_{\max})_{AC} = 100 \frac{\text{km}}{\text{h}}$ ;  $(N_{\max})_{CB} = 70 \frac{\text{km}}{\text{h}}$ ;  
 $N_A = N_B = 0$ ;  $t_{AB} = 8 \text{ MIN}, 20 \text{ S}$ ;  
 $|a| = \text{CONSTANT}$ ;  $N = N_{\max}$  AS MUCH  
 AS POSSIBLE

FIND:  $a$



THE  $v-t$  CURVE IS FIRST DRAWN AS SHOWN, WHERE THE MAGNITUDES OF THE SLOPES (ACCELERATIONS) OF THE THREE INCLINED LINES ARE EQUAL.



DENOTING THE MAGNITUDE OF THE ACCELERATIONS BY  $a$  HAVE..

$a = \frac{100}{t_1}$   $a = \frac{30}{t_2}$   $a = \frac{70}{t_3}$   
 WHERE  $a$  IS IN  $\text{km/h}^2$  AND THE TIMES ARE IN  $\text{h}$ .

NOW..  $A_1 = 8 \text{ km}$ :  $(t_1)(100) - \frac{1}{2}(t_1)(100) = 8$   
 SUBSTITUTING..  $100t_1 - \frac{1}{2}(\frac{100}{a})(100) - \frac{1}{2}(\frac{30}{a})(30) = 8$

OR  $t_1 = 0.08 + \frac{54.5}{a}$   
 ALSO..  $A_2 = 4 \text{ km}$ :  $(\frac{5}{36} - t_1)(100) - \frac{1}{2}(t_2)(70) = 4$   
 SUBSTITUTING..  $(\frac{5}{36} - t_1)(100) - \frac{1}{2}(\frac{70}{a})(70) = 4$

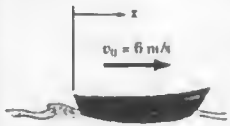
OR  $t_1 = \frac{1260}{a} - \frac{35}{a}$   
 THEN  $0.08 + \frac{54.5}{a} = \frac{1260}{a} - \frac{35}{a}$   
 OR  $a = 51.259 \frac{\text{km}}{\text{h}^2} = \frac{1000 \text{ m}}{\text{km}} \cdot (\frac{1 \text{ h}}{3600 \text{ s}})^2$   
 OR  $a = 3.96 \frac{\text{m}}{\text{s}^2}$

# 11.70

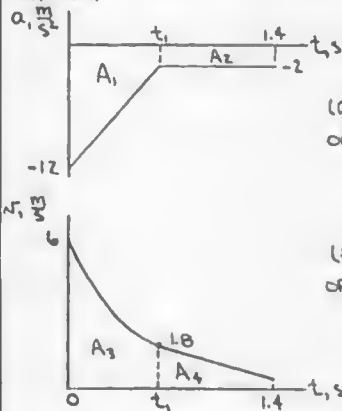
GIVEN:  $v_0 = 6 \frac{\text{m}}{\text{s}}$ ; FOR  $0 \leq t \leq t_1$ ,  $a = 2 \frac{\text{m}}{\text{s}^2}$ ;  
 FOR  $t_1 \leq t \leq 1.4 \text{ s}$ ,  $a = -2 \frac{\text{m}}{\text{s}^2}$ ;  
 AT  $t = 0$ ,  $a = -12 \frac{\text{m}}{\text{s}^2}$ ; AT  $t = t_1$ ,  
 $a = -2 \frac{\text{m}}{\text{s}^2}$ ,  $v = 1.8 \frac{\text{m}}{\text{s}}$

FIND: (a)  $t_1$

(b)  $v$  AND  $x$  AT  $t = 1.4 \text{ s}$



THE  $a-t$  AND  $v-t$  CURVES ARE FIRST DRAWN AS SHOWN.



(a) HAVE..  $v_{t_1} = v_0 + A_1$   
 OR  $1.8 \frac{\text{m}}{\text{s}} = 6 \frac{\text{m}}{\text{s}} - (t_1 \text{ s})(\frac{12+2}{2}) \frac{\text{m}}{\text{s}^2}$   
 OR  $t_1 = 0.6 \text{ s}$

(b) HAVE..  $v_{1.4} = v_{t_1} + A_2$   
 OR  $v_{1.4} = 1.8 \frac{\text{m}}{\text{s}} - (1.4 - 0.6 \text{ s})(2 \frac{\text{m}}{\text{s}^2})$   
 OR  $v_{1.4} = 0.20 \frac{\text{m}}{\text{s}}$

(CONTINUED)

# 11.70 CONTINUED

NOW..  $x_{1.4} = A_3 + A_4$ , WHERE  $A_3$  IS MOST EASILY DETERMINED USING INTEGRATING. THUS..

FOR  $0 \leq t \leq t_1$ :  $a = \frac{-2-12}{0.6}t - 12 = -\frac{20}{3}t - 12$   
 NOW..  $\frac{dv}{dt} = a = -\frac{20}{3}t - 12$

AT  $t=0$ ,  $v=6 \frac{\text{m}}{\text{s}}$ :  $\int_6^v dv = \int_0^{t_1} (-\frac{20}{3}t - 12) dt$   
 OR  $v = 6 - \frac{10}{3}t^2 - 12t$

HAVE..  $\frac{dx}{dt} = v = 6 - 12t - \frac{10}{3}t^2$   
 THEN..  $A_3 = \int_0^{t_1} dx = \int_0^{0.6} (6 - 12t - \frac{10}{3}t^2) dt$   
 $= [6t - 6t^2 - \frac{10}{9}t^3]_0^{0.6} = 2.04 \text{ m}$

ALSO..  $A_4 = (1.4 - 0.6)(\frac{1.8+0.2}{2}) = 0.8 \text{ m}$

THEN..  $x_{1.4} = (2.04 + 0.8) \text{ m}$  OR  $x_{1.4} = 2.84 \text{ m}$

# 11.71

GIVEN:  $N_T = 45 \frac{\text{mi}}{\text{h}}$ ; AT  $t=0$ ,  $x_T = 0$ ,  $x_B = 0$ ;

FOR  $0 \leq t \leq 2 \text{ MIN}$ ,  $N_B = 0$ ;

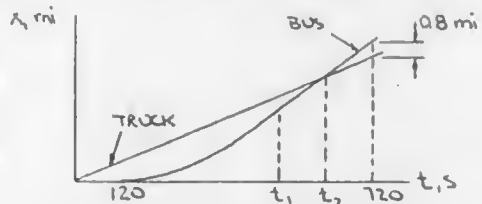
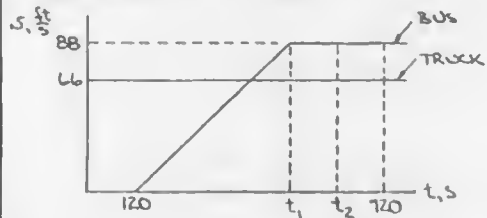
FOR  $t > 2 \text{ MIN}$ ,  $a_B = \text{CONST}$  UNTIL

$N_B = 60 \frac{\text{mi}}{\text{h}}$ , THEN  $a_B = 0$ ; AT  $t = 12 \text{ MIN}$ ,  
 $x_B - x_T = 0.8 \text{ mi}$

FIND: (a)  $t$  AND  $x$  WHEN  $x_B = x_T$   
 (b)  $a_B$

FIRST NOTE..  $45 \frac{\text{mi}}{\text{h}} = 66 \frac{\text{ft}}{\text{s}}$   $60 \frac{\text{mi}}{\text{h}} = 88 \frac{\text{ft}}{\text{s}}$

(a) ASSUMING THAT THE BUS REACHES  $60 \frac{\text{mi}}{\text{h}}$  (AT TIME  $t_1$ ) BEFORE IT PASSES THE TRUCK (AT TIME  $t_2$ ), THE  $v-t$  AND  $x-t$  CURVES CAN THEN BE DRAWN AS SHOWN.



AT  $t = 720 \text{ s}$  (12 MIN):  $x_B - x_T = 0.8 \text{ mi}$   
 OR  $[\frac{1}{2}(t_1 - 120) \text{ s} \cdot (88 \frac{\text{ft}}{\text{s}}) + (720 - t_1) \text{ s} \cdot (88 \frac{\text{ft}}{\text{s}})] - [720 \text{ s} \cdot (66 \frac{\text{ft}}{\text{s}})]$   
 $= 0.8 \text{ mi} = \frac{5280 \text{ ft}}{\text{mi}}$

OR  $t_1 = 144 \text{ s}$

AT  $t = t_2$ :  $x_B = x_T$   
 OR  $\frac{1}{2}(144 - 120) \text{ s} \cdot (88 \frac{\text{ft}}{\text{s}}) + (t_2 - 144) \text{ s} \cdot (88 \frac{\text{ft}}{\text{s}}) = (t_2 \text{ s}) \cdot (66 \frac{\text{ft}}{\text{s}})$   
 OR  $t_2 = 528 \text{ s}$

THEN  $t_2 > t_1$ , ASSUMPTION CORRECT

$\therefore t_2 = 528 \text{ s}$  OR  $t_2 = 8 \text{ MIN } 48 \text{ S}$

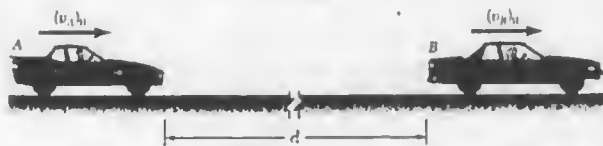
AT  $t = t_2$ :  $x_B = x_T = (528 \text{ s}) \cdot (66 \frac{\text{ft}}{\text{s}}) = 34848 \text{ ft}$   
 OR  $x_2 = 6.60 \text{ mi}$

(b) HAVE..  $a_B = \frac{(v_B)_{t_1} - 0}{t_1 - 120}$   
 $= \frac{88 \frac{\text{ft}}{\text{s}}}{(144 - 120) \text{ s}}$   
 OR  $a_B = 3.67 \frac{\text{ft}}{\text{s}^2}$

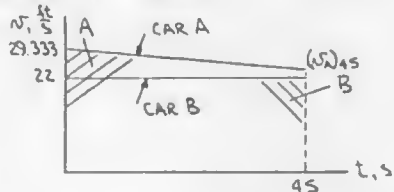


11.72

GIVEN: At  $t=0$ ,  $d=200$  ft;  $(v_A)_0 = 20$   $\frac{\text{mi}}{\text{h}}$ ,  $(v_B)_0 = 15$   $\frac{\text{mi}}{\text{h}}$ ; AT  $t=45$  s,  $x_A = x_B$ ; FOR  $t > 0$ ,  $a_A = \text{CONSTANT}$   
 FIND: (a)  $a_A$   
 (b)  $v_{A/B}$  AT  $t=45$  s



(a) FIRST NOTE..  $20 \frac{\text{mi}}{\text{h}} = 29.333 \frac{\text{ft}}{\text{s}}$   $15 \frac{\text{mi}}{\text{h}} = 22 \frac{\text{ft}}{\text{s}}$   
 THE  $v-t$  CURVES FOR THE TWO CARS ARE THEN DRAWN AS SHOWN.



AT  $t=45$  s,  $x_A = x_B$ :  $(\text{AREA})_A = (\text{AREA})_B + 200$  ft  
 OR  $(45 \text{ s}) \left( \frac{29.333 + v_A}{2} \right) = (45 \text{ s}) \left( \frac{22 + v_A}{2} \right) + 200$  ft  
 OR  $(v_A)_{45} = 23.555 \frac{\text{ft}}{\text{s}}$   
 THEN  $a_A = \frac{(v_A)_{45} - (v_A)_0}{t_{45}} = \frac{(23.555 - 29.333) \frac{\text{ft}}{\text{s}}}{45 \text{ s}}$

OR  $a_A = -0.1284 \frac{\text{ft}}{\text{s}^2}$

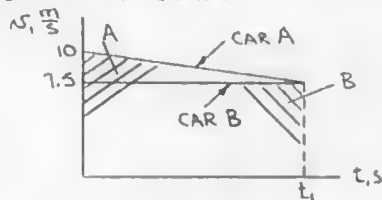
(b) HAVE --  $v_{A/B} = v_A - v_B = (23.555 - 22) \frac{\text{ft}}{\text{s}}$   
 $= 1.555 \frac{\text{ft}}{\text{s}}$   
 OR  $v_{A/B} = 1.060 \frac{\text{mi}}{\text{h}}$

11.73

GIVEN:  $(v_A)_0 = 36 \frac{\text{km}}{\text{h}}$ ,  $(v_B)_0 = 27 \frac{\text{km}}{\text{h}}$ ;  
 $a_A = -0.042 \frac{\text{m}}{\text{s}^2}$ ; CAR A JUST AVOIDS COLLIDING WITH CAR B  
 FIND:  $d$



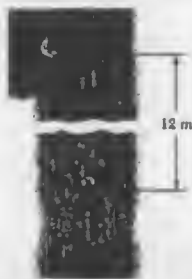
FIRST NOTE..  $36 \frac{\text{km}}{\text{h}} = 10 \frac{\text{m}}{\text{s}}$   $27 \frac{\text{km}}{\text{h}} = 7.5 \frac{\text{m}}{\text{s}}$   
 NOW ASSUME THAT  $v_A = v_B$  WHEN  $x_A = x_B$ ; THE  $v-t$  CURVES FOR THE TWO CARS ARE THEN DRAWN AS SHOWN.



NOW..  $a_A = \frac{(v_A)_{t_1} - (v_A)_0}{t_1}$   
 OR  $-0.042 \frac{\text{m}}{\text{s}^2} = \frac{(7.5 - 10) \frac{\text{m}}{\text{s}}}{t_1}$   
 OR  $t_1 = 59.524$  s

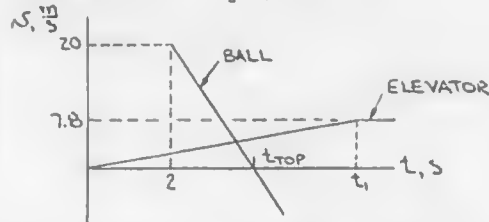
AT  $t=t_1$ ,  $x_A = x_B$ :  $(\text{AREA})_A = (\text{AREA})_B + d$   
 OR  $(59.524 \text{ s}) \left( \frac{10 + 7.5}{2} \right) \frac{\text{m}}{\text{s}} = (59.524 \text{ s}) \left( \frac{7.5}{2} \right) \frac{\text{m}}{\text{s}} + d$   
 OR  $d = 74.4$  m

11.74



GIVEN: AT  $t=0$ ,  $v_E=0$ ; FOR  $0 \leq t \leq 7.8$  s,  $a_E = 1.2 \frac{\text{m}}{\text{s}^2}$ ; FOR  $v_E = 7.8 \frac{\text{m}}{\text{s}}$ ,  $a_E = 0$ ;  
 AT  $t=2$  s,  $v_B = 20 \frac{\text{m}}{\text{s}}$ !  
 FIND:  $t$  WHEN THE BALL HITS THE ELEVATOR

THE  $v-t$  CURVES OF THE BALL AND THE ELEVATOR ARE FIRST DRAWN AS SHOWN. NOTE THAT THE INITIAL SLOPE OF THE CURVE FOR THE ELEVATOR IS  $1.2 \frac{\text{m}}{\text{s}^2}$ , WHILE THE SLOPE OF THE CURVE FOR THE BALL IS  $-9$  ( $-9.81 \frac{\text{m}}{\text{s}^2}$ ).



THE TIME  $t_1$  IS THE TIME WHEN  $v_E$  REACHES  $7.8 \frac{\text{m}}{\text{s}}$ . THUS..  $v_E = (v_E)_0 + a_E t$

OR  $7.8 \frac{\text{m}}{\text{s}} = (1.2 \frac{\text{m}}{\text{s}^2}) t_1$  OR  $t_1 = 6.5$  s

THE TIME  $t_{\text{TOP}}$  IS THE TIME AT WHICH THE BALL REACHES THE TOP OF ITS TRAJECTORY. THUS..  $v_B = (v_B)_0 - g(t - 2)$

OR  $0 = 20 \frac{\text{m}}{\text{s}} - (9.81 \frac{\text{m}}{\text{s}^2})(t_{\text{TOP}} - 2)$   
 OR  $t_{\text{TOP}} = 4.0387$  s

USING THE COORDINATE SYSTEM SHOWN, HAVE..

$0 \leq t \leq t_1$ :  $y_E = -12 \text{ m} + \frac{1}{2} a_E t^2$  m  
 AT  $t = t_{\text{TOP}}$ :  $y_B = \frac{1}{2} (4.0387 - 2) \text{ s} \cdot (20 \frac{\text{m}}{\text{s}})$   
 $= 20.387$  m  
 AND  $y_E = -12 \text{ m} + \frac{1}{2} (1.2 \frac{\text{m}}{\text{s}^2}) (4.0387 \text{ s})^2$   
 $= -2.213$  m

AT  $t = [2 + 2(4.0387 - 2)] \text{ s} = 6.0774$  s,  $y_B = 0$   
 AND AT  $t = t_1$ ,  $y_E = -12 \text{ m} + \frac{1}{2} (6.5 \text{ s}) (7.8 \frac{\text{m}}{\text{s}}) = 13.35$  m  
 $\therefore$  THE BALL HITS THE ELEVATOR ( $y_B = y_E$ ) WHEN  $t_{\text{TOP}} < t < t_1$ .

FOR  $t \geq t_{\text{TOP}}$ :  $y_B = 20.387 \text{ m} - [\frac{1}{2} g (t - t_{\text{TOP}})^2]$  m

THEN.. WHEN  $y_B = y_E$  -

$20.387 \text{ m} = -\frac{1}{2} (9.81 \frac{\text{m}}{\text{s}^2}) (t - 4.0387)^2$   
 $= -12 \text{ m} + \frac{1}{2} (1.2 \frac{\text{m}}{\text{s}^2}) (t \text{ s})^2$

OR  $5.505 t^2 - 39.6196 t + 47.619 = 0$

SOLVING..  $t = 1.525$  s AND  $t = 5.675$

NOW..  $t_{\text{TOP}} < t < t_1 \Rightarrow t = 5.675$

# 11.75

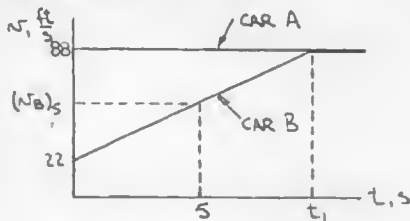
GIVEN:  $(v_A)_0 = 60 \frac{\text{mi}}{\text{h}}$ ,  $(v_B)_0 = 15 \frac{\text{mi}}{\text{h}}$ ; AT  $t=0$ ,  
 $(x_A)_0 = -380 \text{ ft}$ ,  $(x_B)_0 = 0$ ; AT  $t=5 \text{ s}$ ,  
 $x_B = 200 \text{ ft}$ ; FOR  $15 \frac{\text{mi}}{\text{h}} \leq v_B < 60 \frac{\text{mi}}{\text{h}}$ ,  
 $a_B = \text{CONSTANT}$ ; FOR  $v_B = 60 \frac{\text{mi}}{\text{h}}$ ,  
 $a_B = 0$

FIND:  $x_{B/A}$  WHEN  $v_B = 60 \frac{\text{mi}}{\text{h}}$

380 ft



FIRST NOTE..  $60 \frac{\text{mi}}{\text{h}} = 88 \frac{\text{ft}}{\text{s}}$   $15 \frac{\text{mi}}{\text{h}} = 22 \frac{\text{ft}}{\text{s}}$   
 THE  $v-t$  CURVES OF THE TWO CARS ARE THEN  
 DRAWN AS SHOWN.



USING THE COORDINATE SYSTEM SHOWN, HAVE..

$$\text{AT } t=5 \text{ s, } x_B = 200 \text{ ft:} \\ (5 \text{ s}) \left[ \frac{22 + (v_B)_5}{2} \right] \frac{\text{ft}}{\text{s}} = 200 \text{ ft} \\ \text{OR } (v_B)_5 = 58 \frac{\text{ft}}{\text{s}}$$

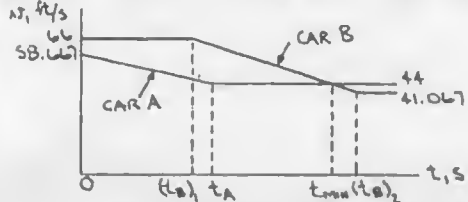
$$\text{THEN, USING SIMILAR TRIANGLES, HAVE..} \\ \frac{(88 - 22) \frac{\text{ft}}{\text{s}}}{t_1} = \frac{(58 - 22) \frac{\text{ft}}{\text{s}}}{5 \text{ s}} \quad (= a_B)$$

$$\text{OR } t_1 = 9.1667 \text{ s}$$

$$\text{FINALLY, AT } t=t_1 \\ x_{B/A} = x_B - x_A = \left[ (9.1667 \text{ s}) \left( \frac{22 + 58}{2} \right) \frac{\text{ft}}{\text{s}} \right] \\ - [-380 \text{ ft} + (9.1667 \text{ s}) (88 \frac{\text{ft}}{\text{s}})] \\ \text{OR } x_{B/A} = 77.5 \text{ ft}$$

# 11.76 CONTINUED

THE  $v-t$  CURVES OF THE TWO CARS ARE AS SHOWN.



AT  $t=0$ : CAR A ENTERS THE SPEED ZONE

$t=(t_B)_1$ : CAR B ENTERS THE SPEED ZONE

$t=t_A$ : CAR A REACHES ITS FINAL SPEED

$t=t_{\min}$ :  $v_A = v_B$

$t=(t_B)_2$ : CAR B REACHES ITS FINAL SPEED

$$(a) \text{ HAVE.. } a_A = \frac{(v_A)_{\text{FINAL}} - (v_A)_0}{t_A} \\ \text{OR } -16 \frac{\text{ft}}{\text{s}^2} = \frac{(44 - 58.667) \frac{\text{ft}}{\text{s}}}{t_A}$$

$$\text{OR } t_A = 0.91667 \text{ s}$$

$$\text{ALSO.. } 60 \text{ ft} = (t_B)_1 (v_B)_0$$

$$\text{OR } 60 \text{ ft} = (t_B)_1 (66 \frac{\text{ft}}{\text{s}}) \quad \text{OR } (t_B)_1 = 0.90909 \text{ s}$$

$$\text{AND } a_B = \frac{(v_B)_{\text{FINAL}} - (v_B)_0}{(t_B)_2 - (t_B)_1}$$

$$\text{OR } -20 \frac{\text{ft}}{\text{s}^2} = \frac{(41.067 - 66) \frac{\text{ft}}{\text{s}}}{[(t_B)_2 - 0.90909] \text{ s}}$$

$$\text{OR } (t_B)_2 = 2.15574 \text{ s}$$

(CAR B WILL CONTINUE TO OVERTAKE CAR A WHILE  $v_B > v_A$ . THEREFORE,  $(x_{A/B})_{\min}$  WILL OCCUR WHEN  $v_A = v_B$ , WHICH OCCURS FOR  $(t_B)_1 < t_{\min} < (t_B)_2$

FOR THIS TIME INTERVAL..

$$v_A = 44 \frac{\text{ft}}{\text{s}} \quad v_B = (v_B)_0 + a_B [t - (t_B)_1]$$

THEN.. AT  $t=t_{\min}$ :

$$44 \frac{\text{ft}}{\text{s}} = 66 \frac{\text{ft}}{\text{s}} + (-20 \frac{\text{ft}}{\text{s}^2}) (t_{\min} - 0.90909 \text{ s})$$

$$\text{OR } t_{\min} = 2.00909 \text{ s}$$

FINALLY..

$$(x_{A/B})_{\min} = (x_A)_{t_{\min}} - (x_B)_{t_{\min}} \\ = \left\{ t_A \left[ \frac{(v_A)_0 + (v_A)_{\text{FINAL}}}{2} \right] + (t_{\min} - t_A) (v_A)_{\text{FINAL}} \right\} \\ - \left\{ (v_B)_0 (t_B)_1 + [(v_B)_0 + (v_B)_{\text{FINAL}}] \left[ \frac{(t_B)_2 - (t_B)_1}{2} \right] \right\} \\ = \left\{ (0.91667 \text{ s}) \left( \frac{58.667 + 44}{2} \right) \frac{\text{ft}}{\text{s}} \right. \\ \left. + (2.00909 - 0.91667 \text{ s}) (44 \frac{\text{ft}}{\text{s}}) \right\} \\ - \left\{ -60 \text{ ft} + (0.90909 \text{ s}) (66 \frac{\text{ft}}{\text{s}}) \right. \\ \left. + (2.00909 - 0.90909 \text{ s}) \left( \frac{66 + 41.067}{2} \right) \frac{\text{ft}}{\text{s}} \right\} \\ = (47.057 + 48.066) \text{ ft} - (60 + 60.000 + 60.500) \text{ ft} \\ = 34.623 \text{ ft} \quad \text{OR } (x_{A/B})_{\min} = 34.6 \text{ ft}$$

(b) SINCE  $(x_{A/B}) \leq 60 \text{ ft}$  FOR  $t \leq t_{\min}$ , IT FOLLOWS

THAT  $x_{A/B} = 70 \text{ ft}$  FOR  $t > (t_B)_2$  [NOTE..

$(t_B)_2 \approx t_{\min}$ ]. THEN, FOR  $t > (t_B)_2$ ..

$$x_{A/B} = (x_{A/B})_{\min} + [(t - t_{\min}) (v_A)_{\text{FINAL}}] \\ - \left\{ [(t_B)_2 - t_{\min}] \left[ \frac{(v_A)_{\text{FINAL}} + (v_B)_{\text{FINAL}}}{2} \right] \right. \\ \left. + [t - (t_B)_2] (v_B)_{\text{FINAL}} \right\}$$

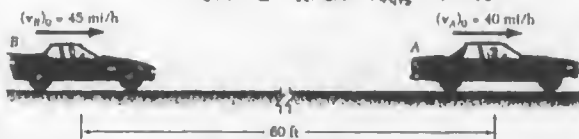
$$\text{OR } 70 \text{ ft} = 34.623 \text{ ft} + [(t - 2.00909 \text{ s}) (44 \frac{\text{ft}}{\text{s}})] \\ - \left\{ (2.15574 - 2.00909 \text{ s}) \left( \frac{44 + 41.067}{2} \right) \frac{\text{ft}}{\text{s}} \right. \\ \left. + (t - 2.15574 \text{ s}) (41.067 \frac{\text{ft}}{\text{s}}) \right\} \\ \text{OR } t = 14.14 \text{ s}$$

# 11.76

GIVEN:  $(v_A)_0 = 40 \frac{\text{mi}}{\text{h}}$ ; FOR  $30 \frac{\text{mi}}{\text{h}} \leq v_A \leq 40 \frac{\text{mi}}{\text{h}}$ ,  
 $a_A = -16 \frac{\text{ft}}{\text{s}^2}$ ; FOR  $v_A = 30 \frac{\text{mi}}{\text{h}}$ ,  $a_A = 0$ ;  
 $(x_{A/B})_0 = 60 \text{ ft}$ ;  $(v_A)_0 = 45 \frac{\text{mi}}{\text{h}}$ ; WHEN  
 $x_B = 0$ ,  $a_B = -20 \frac{\text{ft}}{\text{s}^2}$ ; FOR  $v_B = 28 \frac{\text{mi}}{\text{h}}$ ,  
 $a_B = 0$

FIND: (a)  $(x_{A/B})_{\min}$

(b)  $t$  WHEN  $x_{A/B} = 70 \text{ ft}$



$$\text{FIRST NOTE.. } 40 \frac{\text{mi}}{\text{h}} = 58.667 \frac{\text{ft}}{\text{s}} \quad 30 \frac{\text{mi}}{\text{h}} = 44 \frac{\text{ft}}{\text{s}} \\ 45 \frac{\text{mi}}{\text{h}} = 66 \frac{\text{ft}}{\text{s}} \quad 28 \frac{\text{mi}}{\text{h}} = 41.067 \frac{\text{ft}}{\text{s}}$$

AT  $t=0$ ...



(CONTINUED)

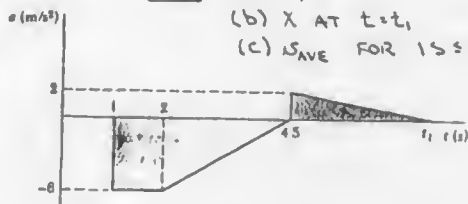
## 11.77

GIVEN: At  $t=0$ ,  $x=0$ ,  $v=54 \frac{\text{km}}{\text{h}}$ ; FOR  $t > 1$ ,  
 $v = 54 \frac{\text{km}}{\text{h}}$

FIND: (a)  $t_1$

(b)  $x$  AT  $t=t_1$

(c)  $v_{\text{AVE}}$  FOR  $1 \leq t \leq t_1$



FIRST NOTE..  $54 \frac{\text{km}}{\text{h}} = 15 \frac{\text{m}}{\text{s}}$

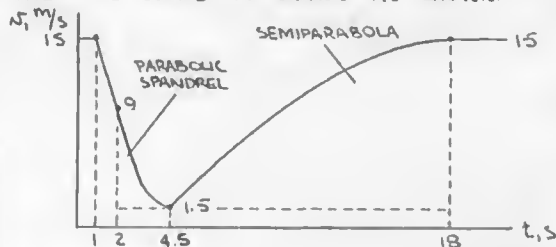
(a) HAVE..  $v_b = v_a + (\text{AREA UNDER } a-t \text{ CURVE FROM } t_a \text{ TO } t_b)$

THEN.. AT  $t = 2 \text{ s}$ :  $v = 15 - (1)(6) = 9 \frac{\text{m}}{\text{s}}$

$t = 4.5 \text{ s}$ :  $v = 9 - \frac{1}{2}(2.5)(6) = 1.5 \frac{\text{m}}{\text{s}}$

$t = t_1$ :  $15 = 1.5 + \frac{1}{2}(t_1 - 4.5)(2)$   
 OR  $t_1 = 18 \text{ s}$  ◀

(b) USING THE ABOVE VALUES OF THE VELOCITIES, THE  $v-t$  CURVE IS DRAWN AS SHOWN.



NOW..  $x$  AT  $t = 18 \text{ s}$ ..

$x_{18} = x_0^0 + \sum (\text{AREA UNDER THE } v-t \text{ CURVE FROM } t=0 \text{ TO } t=18 \text{ s})$

$$= (15)(15 \frac{\text{m}}{\text{s}}) + (15)(\frac{15-9}{2} \frac{\text{m}}{\text{s}}) + [(2.5)(1.5 \frac{\text{m}}{\text{s}}) + \frac{1}{2}(2.5)(7.5 \frac{\text{m}}{\text{s}})] + [(13.5)(1.5 \frac{\text{m}}{\text{s}}) + \frac{1}{2}(13.5)(13.5 \frac{\text{m}}{\text{s}})]$$

$$= [15 + 12 + (3.75 + 6.25) + (20.25 + 121.50)] \text{ m}$$

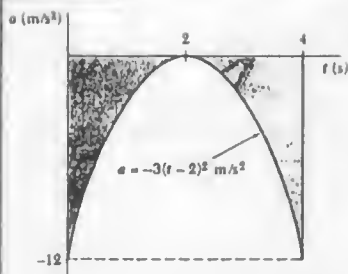
$$= 178.75 \text{ m} \quad \text{OR} \quad x_{18} = 178.8 \text{ m} \quad \blacktriangleleft$$

(c) FIRST NOTE..  $x_1 = 15 \text{ m}$   $x_{18} = 178.75 \text{ m}$

NOW..  $v_{\text{AVE}} = \frac{\Delta x}{\Delta t} = \frac{(178.75 - 15) \text{ m}}{(18 - 1) \text{ s}} = 9.6324 \frac{\text{m}}{\text{s}}$

OR  $v_{\text{AVE}} = 34.7 \frac{\text{km}}{\text{h}}$  ◀

## 11.78



GIVEN: At  $t=0$ ,  $x=0$ ,  
 $v=8 \frac{\text{m}}{\text{s}}$

CONSTRUCT: (a)  $v-t$  AND  
 $x-t$  CURVES  
 FOR  
 $0 \leq t \leq 4 \text{ s}$

FIND: (b)  $x$  AT  $t=3 \text{ s}$

(a) HAVE..  $v_2 = v_1 + (\text{AREA UNDER } a-t \text{ CURVE FROM } t_1 \text{ TO } t_2)$

AND..  $x_2 = x_1 + (\text{AREA UNDER } v-t \text{ CURVE FROM } t_1 \text{ TO } t_2)$

(CONTINUED)

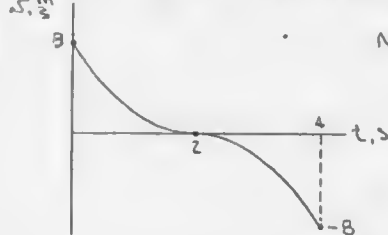
## 11.78 CONTINUED

THEN, USING THE FORMULA FOR THE AREA OF A PARABOLIC SPANDREL, HAVE..

AT  $t = 2 \text{ s}$ :  $v = 8 - \frac{1}{2}(2)(12) = 0$

$t = 4 \text{ s}$ :  $v = 0 - \frac{1}{2}(2)(12) = -8 \frac{\text{m}}{\text{s}}$

THE  $v-t$  CURVE IS THEN DRAWN AS SHOWN.



NOTE: THE AREA UNDER EACH PORTION OF THE CURVE IS A SPANDREL OF ORDER  $N=3$ .

NOW.. AT  $t = 2 \text{ s}$ :  $x = 0 + \frac{(2)(8)}{3+1} = 4 \text{ m}$

$t = 4 \text{ s}$ :  $x = 4 - \frac{(2)(8)}{3+1} = 0$

THE  $x-t$  CURVE IS THEN DRAWN AS SHOWN.



(b) HAVE.. AT  $t = 3 \text{ s}$ :  $a = -3(3-2)^2 = -3 \frac{\text{m}}{\text{s}^2}$

$v = 0 - \frac{1}{2}(1)(3) = -1 \frac{\text{m}}{\text{s}}$

$x = 4 - \frac{(1)(1)}{3+1}$

OR  $x_3 = 3.75 \text{ m}$  ◀

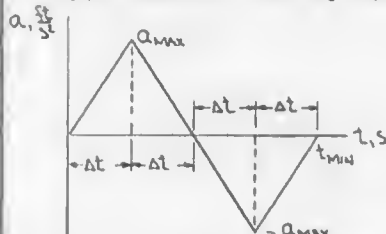
## 11.79

GIVEN: At  $t=0$ ,  $x=0$ ,  $v=0$ ;  $x_{\text{MAX}} = 1.2 \text{ ft}$ ;  
 WHEN  $x = x_{\text{MAX}}$ ,  $v=0$ ;  $(\frac{dv}{dt})_{\text{MAX}} = 4.8 \frac{\text{ft}}{\text{s}^2}$

FIND: (a)  $t_{\text{MIN}}$  FOR  $x_{\text{MAX}} = 1.2 \text{ ft}$

(b)  $v_{\text{MAX}}$  AND  $v_{\text{AVE}}$  FOR  $0 \leq t \leq t_{\text{MIN}}$

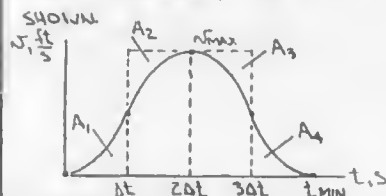
(a) OBSERVING THAT  $v_{\text{MAX}}$  MUST OCCUR AT  $t = \frac{1}{2} t_{\text{MIN}}$ , THE  $a-t$  CURVE MUST HAVE THE SHAPE SHOWN. NOTE THAT THE MAGNITUDE OF THE SLOPE OF EACH PORTION OF THE CURVE IS  $4.8 \frac{\text{ft}}{\text{s}^2}$ .



HAVE.. AT  $t = \Delta t$ :  $v = 0 + \frac{1}{2}(\Delta t)(a_{\text{MAX}}) = \frac{1}{2} a_{\text{MAX}} \Delta t$

$t = 2\Delta t$ :  $v_{\text{MAX}} = \frac{1}{2} a_{\text{MAX}} \Delta t + \frac{1}{2}(\Delta t)(a_{\text{MAX}})$   
 $= a_{\text{MAX}} \Delta t$

USING SYMMETRY, THE  $v-t$  IS THEN DRAWN AS SHOWN



(CONTINUED)

## 11.79 CONTINUED

NOTING THAT  $A_1 = A_2 = A_3 = A_4$  AND THAT THE AREA UNDER THE  $v-t$  CURVE IS EQUAL TO  $x_{\max}$ , HAVE..

$$(2\Delta t)(v_{\max}) = x_{\max}$$

$$v_{\max} = a_{\max} \Delta t \Rightarrow 2a_{\max} \Delta t^2 = x_{\max}$$

$$\text{NOW.. } \frac{a_{\max}}{\Delta t} = 4.8 \text{ ft/s}^2 \text{ SO THAT}$$

$$2(4.8 \Delta t \frac{\text{ft}}{\text{s}^2}) \Delta t^2 = 1.2 \text{ ft}$$

$$\text{OR } \Delta t = 0.5 \text{ s}$$

$$\text{THEN } t_{\min} = 4\Delta t \quad \text{OR } t_{\min} = 2.00 \text{ s}$$

$$(b) \text{ HAVE.. } v_{\max} = a_{\max} \Delta t = (4.8 \frac{\text{ft}}{\text{s}^2})(0.5 \text{ s})$$

$$= 4.8 \frac{\text{ft}}{\text{s}^2} \times (0.5 \text{ s})$$

$$\text{OR } v_{\max} = 1.2 \frac{\text{ft}}{\text{s}}$$

$$\text{ALSO.. } v_{\text{AVE}} = \frac{\Delta x}{\Delta t_{\text{TOTAL}}} = \frac{1.2 \text{ ft}}{2.00 \text{ s}}$$

$$\text{OR } v_{\text{AVE}} = 0.6 \frac{\text{ft}}{\text{s}}$$

## 11.80

GIVEN:  $x_{\max} = 1.6 \text{ mi}$ ;  $|a_{\max}| = 4 \frac{\text{ft}}{\text{s}^2}$ ,  
 $|(\frac{dv}{dt})_{\max}| = 0.8 \frac{\text{ft}}{\text{s}^2}$ ;  $v_{\max} = 20 \frac{\text{mi}}{\text{h}}$

FIND: (a)  $t_{\min}$  FOR  $x_{\max} = 1.6 \text{ mi}$

(b)  $v_{\text{AVE}}$

FIRST NOTE..  $20 \frac{\text{mi}}{\text{h}} = 29.333 \frac{\text{ft}}{\text{s}}$   $1.6 \text{ mi} = 8448 \text{ ft}$

(a) TO OBTAIN  $t_{\min}$ , THE TRAIN MUST ACCELERATE AND DECELERATE AT THE MAXIMUM RATE TO MAXIMIZE THE TIME FOR WHICH  $v = v_{\max}$ .

THE TIME  $\Delta t$  REQUIRED FOR THE TRAIN TO HAVE AN ACCELERATION OF  $4 \frac{\text{ft}}{\text{s}^2}$  IS FOUND FROM..

$$(\frac{dv}{dt})_{\max} = \frac{a_{\max}}{\Delta t}$$

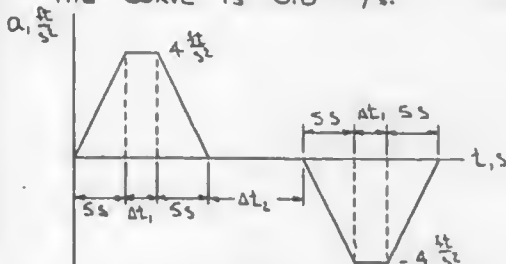
$$\text{OR } \Delta t = \frac{4 \frac{\text{ft}}{\text{s}^2}}{0.8 \frac{\text{ft}}{\text{s}^2}} \quad \text{OR } \Delta t = 5 \text{ s}$$

NOW.. AFTER 5 s THE SPEED OF THE TRAIN IS..

$$v_s = \frac{1}{2}(\Delta t)(a_{\max}) \quad (\text{SINCE } \frac{dv}{dt} = \text{CONSTANT})$$

$$\text{OR } v_s = \frac{1}{2}(5 \text{ s})(4 \frac{\text{ft}}{\text{s}^2}) = 10 \frac{\text{ft}}{\text{s}} = \text{CONSTANT}$$

THEN, SINCE  $v_s < v_{\max}$ , THE TRAIN WILL CONTINUE TO ACCELERATE AT  $4 \frac{\text{ft}}{\text{s}^2}$  UNTIL  $v = v_{\max}$ . THE  $a-t$  CURVE MUST THEN HAVE THE SHAPE SHOWN. NOTE THAT THE MAGNITUDE OF THE SLOPE OF EACH INCLINED PORTION OF THE CURVE IS  $0.8 \frac{\text{ft}}{\text{s}^2}$ .



NOW.. AT  $t = (10 + \Delta t) \text{ s}$ ,  $v = v_{\max}$ :

$$\therefore 2[\frac{1}{2}(5 \text{ s})(4 \frac{\text{ft}}{\text{s}^2}) + (\Delta t)(4 \frac{\text{ft}}{\text{s}^2})] = 29.333 \frac{\text{ft}}{\text{s}}$$

$$\text{OR } \Delta t = 2.3333 \text{ s}$$

THEN.. AT  $t = 5 \text{ s}$ :  $v = 0 + \frac{1}{2}(5)(4) = 10 \frac{\text{ft}}{\text{s}}$

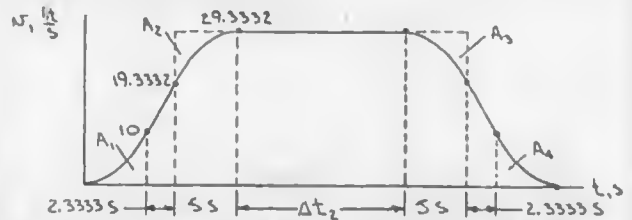
$$t = 7.3333 \text{ s}: v = 10 + (2.3333)(4) = 19.3332 \frac{\text{ft}}{\text{s}}$$

$$t = 12.3333 \text{ s}: v = 19.3332 + \frac{1}{2}(5)(4) = 29.3332 \frac{\text{ft}}{\text{s}}$$

USING SYMMETRY, THE  $v-t$  CURVE IS THEN DRAWN AS SHOWN.

(CONTINUED)

## 11.80 CONTINUED



NOTING THAT  $A_1 = A_2 = A_3 = A_4$  AND THAT THE AREA UNDER THE  $v-t$  CURVE IS EQUAL TO  $x_{\max}$ , HAVE..

$$2[(2.3333 \text{ s})(\frac{10 + 19.3332}{2} \frac{\text{ft}}{\text{s}}) + (10 + \Delta t)(5 \text{ s})(29.3332 \frac{\text{ft}}{\text{s}})] = 8448 \text{ ft}$$

$$\text{OR } \Delta t = 275.67 \text{ s}$$

THEN..  $t_{\min} = 4(5 \text{ s}) + 2(2.3333 \text{ s}) + 275.67 \text{ s}$

$$= 300.345 \text{ s}$$

$$\text{OR } t_{\min} = 5.01 \text{ MIN}$$

$$(b) \text{ HAVE.. } v_{\text{AVE}} = \frac{\Delta x}{\Delta t} = \frac{1.6 \text{ mi}}{300.345 \text{ s}} = \frac{3600 \text{ s}}{1 \text{ h}}$$

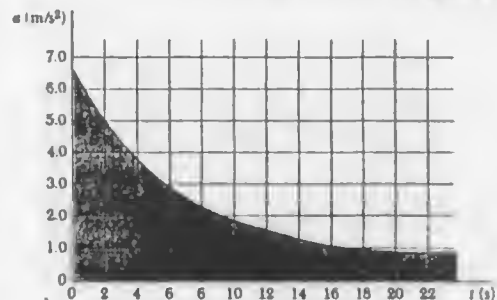
$$\text{OR } v_{\text{AVE}} = 19.18 \frac{\text{mi}}{\text{h}}$$

## 11.81

GIVEN:  $a-t$  CURVE; AT  $t=0$ ,  $x=0$ ,  $v=0$

FIND: (a)  $v$  AT  $t=8 \text{ s}$  BY APPROXIMATE MEANS

(b)  $x$  AT  $t=20 \text{ s}$  BY APPROXIMATE MEANS



### SOLUTION PROCEDURE

1. THE  $a-t$  CURVE IS FIRST APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH  $\Delta t = 2 \text{ s}$ . THE AREA  $(\Delta t)(a_{\text{AVE}})$  OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN VELOCITY  $\Delta v$  FOR THE SPECIFIED INTERVAL OF TIME. THUS,

$$\Delta v \approx a_{\text{AVE}} \Delta t$$

WHERE THE VALUES OF  $a_{\text{AVE}}$  AND  $\Delta v$  ARE GIVEN IN COLUMNS 1 AND 2, RESPECTIVELY, OF THE FOLLOWING TABLE.

2. NOTING THAT  $v_0 = 0$  AND THAT

$$v_L = v_1 + \Delta v_{12}$$

WHERE  $\Delta v_{12}$  IS THE CHANGE IN VELOCITY BETWEEN TIMES  $t_1$  AND  $t_2$ , THE VELOCITY AT THE END OF EACH 2 s INTERVAL CAN BE COMPUTED; SEE COLUMN 3 OF THE TABLE AND THE  $v-t$  CURVE.

3. THE  $v-t$  CURVE IS NEXT APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH  $\Delta t = 2 \text{ s}$ . THE AREA  $(\Delta t)(v_{\text{AVE}})$  OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN POSITION  $\Delta x$  FOR THE SPECIFIED INTERVAL OF TIME.

$$\text{THUS, } \Delta x \approx v_{\text{AVE}} \Delta t$$

(CONTINUED)

# 11.81 CONTINUED

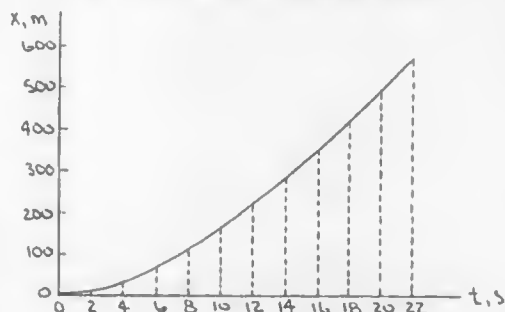
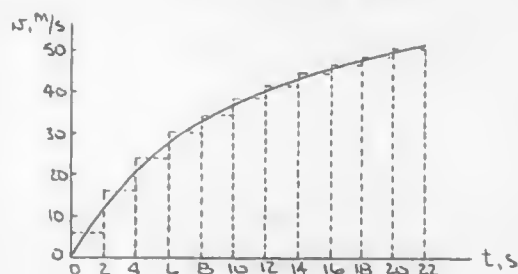
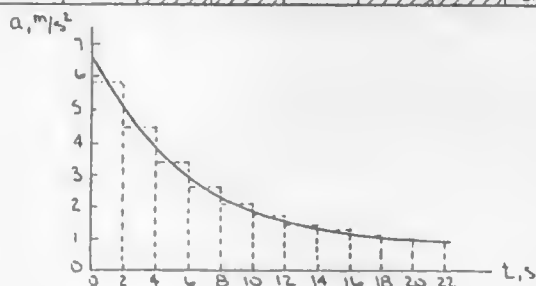
WHERE  $v_{ave}$  AND  $\Delta x$  ARE GIVEN IN COLUMNS 4 AND 5, RESPECTIVELY, OF THE TABLE.

4. WITH  $x_0 = 0$  AND NOTING THAT

$$x_2 = x_1 + \Delta x_{12}$$

WHERE  $\Delta x_{12}$  IS THE CHANGE IN POSITION BETWEEN TIMES  $t_1$  AND  $t_2$ , THE POSITION AT THE END OF EACH 2 S INTERVAL CAN BE COMPUTED; SEE COLUMN 6 OF THE TABLE AND THE  $x-t$  CURVE.

$t, s$	$a, m/s^2$	$a_{ave}, m/s^2$	$\Delta v, m/s$	$v, m/s$	$v_{ave}, m/s$	$\Delta x, m$	$x, m$
0	6.63	5.86	11.72	0	5.86	11.72	0
2	5.08	4.47	8.94	11.72	16.19	32.38	11.72
4	3.86	3.38	6.76	20.66	24.04	48.08	44.10
6	2.90	2.58	5.16	27.42	30.00	60.00	92.18
8	2.25	2.06	4.12	32.58	34.64	69.28	152.18
10	1.87	1.71	3.42	36.70	38.41	76.82	221.46
12	1.54	1.42	2.84	40.12	41.54	83.08	298.28
14	1.29	1.23	2.46	42.96	44.19	88.38	381.36
16	1.16	1.10	2.20	45.42	46.52	93.04	469.74
18	1.03	1.00	2.00	47.62	48.62	97.24	562.78
20	0.97	0.94	1.88	49.62	50.56	101.12	660.02
22	0.90			51.50			761.14



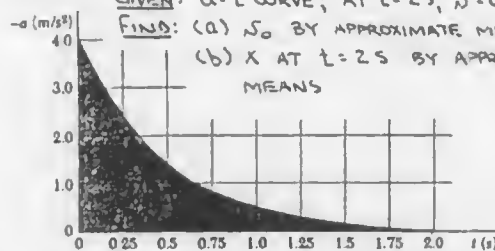
- (a) At  $t = 8 s$ ,  $v = 32.58 \frac{m}{s}$  or  $v = 117.3 \frac{km}{h}$   
 (b) At  $t = 20 s$ ,  $x = 660 m$

# 11.82

GIVEN:  $a-t$  CURVE; AT  $t = 2 s$ ,  $v = 0$

FIND: (a)  $v_0$  BY APPROXIMATE MEANS

(b)  $x$  AT  $t = 2 s$  BY APPROXIMATE MEANS



## SOLUTION PROCEDURE

1. THE  $a-t$  CURVE IS FIRST APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH  $\Delta t = 0.25 s$ . THE AREA  $(\Delta t)(a_{ave})$  OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN VELOCITY  $\Delta v$  FOR THE SPECIFIED INTERVAL OF TIME. THUS,

$$\Delta v \approx a_{ave} \Delta t$$

WHERE THE VALUES OF  $a_{ave}$  AND  $\Delta v$  ARE GIVEN IN COLUMNS 1 AND 2, RESPECTIVELY, OF THE FOLLOWING TABLE.

2. NOW --  $v(2) = v_0 + \int_0^2 a dt = 0$   
 AND APPROXIMATING THE AREA  $\int_0^2 a dt$  UNDER THE  $a-t$  CURVE BY  $\sum a_{ave} \Delta t = \sum \Delta v$ , THE INITIAL VELOCITY IS THEN EQUAL TO

$$v_0 = -\sum \Delta v$$

FINALLY, USING

$$v_2 = v_1 + \Delta v_{12}$$

WHERE  $\Delta v_{12}$  IS THE CHANGE IN VELOCITY BETWEEN TIMES  $t_1$  AND  $t_2$ , THE VELOCITY AT THE END OF EACH 0.25 INTERVAL CAN BE COMPUTED; SEE COLUMN 3 OF THE TABLE AND THE  $v-t$  CURVE.

3. THE  $v-t$  CURVE IS THEN APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH 0.25 s. THE AREA  $(\Delta t)(v_{ave})$  OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN POSITION  $\Delta x$  FOR THE SPECIFIED INTERVAL OF TIME. THUS --  $\Delta x \approx v_{ave} \Delta t$

WHERE  $v_{ave}$  AND  $\Delta x$  ARE GIVEN IN COLUMNS 4 AND 5, RESPECTIVELY, OF THE TABLE.

4. WITH  $x_0 = 0$  AND NOTING THAT

$$x_2 = x_1 + \Delta x_{12}$$

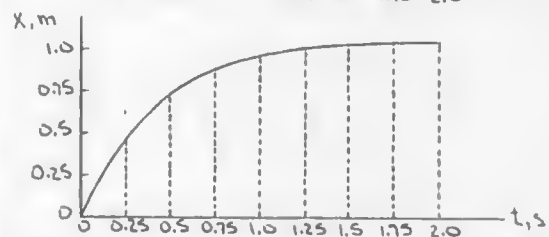
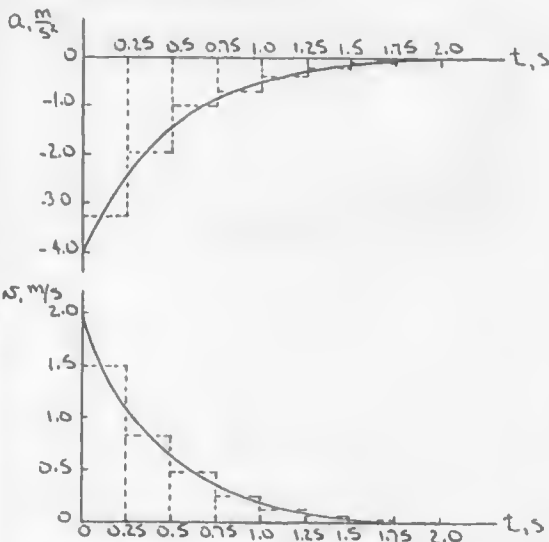
WHERE  $\Delta x_{12}$  IS THE CHANGE IN POSITION BETWEEN TIMES  $t_1$  AND  $t_2$ , THE POSITION AT THE END OF EACH 0.25 s INTERVAL CAN BE COMPUTED; SEE COLUMN 6 OF THE TABLE AND THE  $x-t$  CURVE.

$t, s$	$a, m/s^2$	$a_{ave}, m/s^2$	$\Delta v, m/s$	$v, m/s$	$v_{ave}, m/s$	$\Delta x, m$	$x, m$
0	-4.00	-3.215	-0.804	1.914	1.512	0.378	0
0.25	-2.43	-1.915	-0.479	1.110	0.871	0.218	0.378
0.50	-1.40	-1.125	-0.281	0.631	0.491	0.123	0.596
0.75	-0.85	-0.675	-0.169	0.350	0.266	0.067	0.719
1.00	-0.50	-0.390	-0.098	0.181	0.132	0.033	0.786
1.25	-0.28	-0.205	-0.051	0.083	0.058	0.015	0.819
1.50	-0.13	-0.095	-0.024	0.032	0.020	0.005	0.834
1.75	-0.06	-0.030	-0.008	0.008	0.004	0.001	0.839
2.00	0			0			0.840

$$\sum \Delta v = -1.914 m/s$$

(CONTINUED)

## 11.82 CONTINUED



- (a) HAS FOUND  
(b) AT  $t = 2$  s

$$v_5 = 1.914 \frac{m}{s}$$

$$x = 0.840 m$$

## 11.83 CONTINUED

FOR UNIFORMLY ACCELERATED MOTION..

$$v_2^2 = v_1^2 + 2a(x_2 - x_1) \quad v_2 = v_1 + a(t_2 - t_1)$$

$$\text{OR } \Delta x = \frac{v_2^2 - v_1^2}{2a} \quad \Delta t = \frac{v_2 - v_1}{a}$$

FOR THE FIVE REGIONS SHOWN ABOVE, HAVE..

REGION	$v_1$ , ft/s	$v_2$ , ft/s	$a$ , ft/s <sup>2</sup>	$\Delta x$ , ft	$\Delta t$ , s
1	126	120	-12.5	59.0	0.480
2	120	100	-33	66.7	0.606
3	100	80	-45.5	39.6	0.440
4	80	40	-54	44.4	0.741
5	40	0	-58	13.8	0.690
$\Sigma$				223.5	2.957

(a) FROM THE TABLE, WHEN  $v = 0$   $t = 2.96$  s

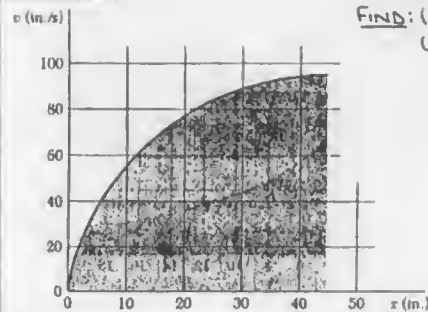
(b) FROM THE TABLE AND ASSUMING  $x_0 = 0$ , WHEN  $v = 0$   $x = 224$  ft

## 11.84

GIVEN:  $v$ - $x$  CURVE

FIND: (a)  $a$  WHEN  $x = 10$  IN.  
(b)  $a$  WHEN  $v = 80 \frac{in}{s}$

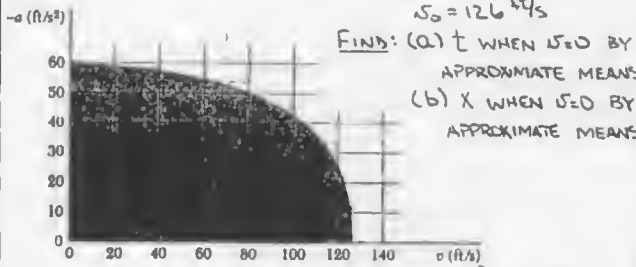
USE APPROXIMATE MEANS



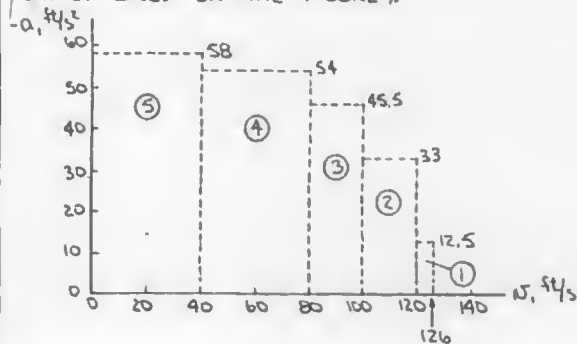
## 11.83

GIVEN:  $a$ - $v$  CURVE;  
 $v_0 = 126 \frac{ft}{s}$

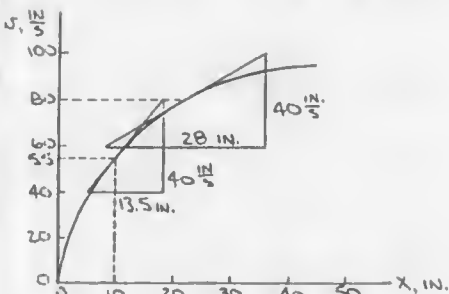
FIND: (a)  $t$  WHEN  $v = 0$  BY APPROXIMATE MEANS  
(b)  $x$  WHEN  $v = 0$  BY APPROXIMATE MEANS



THE GIVEN CURVE IS APPROXIMATED BY A SERIES OF UNIFORMLY ACCELERATED MOTIONS (THE HORIZONTAL DASHED LINES ON THE FIGURE).



(CONTINUED)



FIRST NOTE THAT THE SLOPE OF THE ABOVE CURVE IS  $\frac{dv}{dx}$ . NOW...

$$a = v \frac{dv}{dx}$$

(a) WHEN  $x = 10$  IN.,  $v = 55 \frac{in}{s}$

$$\text{THEN.. } a = 55 \frac{in}{s} \left( \frac{40 \frac{in/s}{13.5 \text{ IN.}}}{13.5 \text{ IN.}} \right)$$

$$\text{OR } a = 163.0 \frac{in}{s^2}$$

(b) WHEN  $v = 80 \frac{in}{s}$ , HAVE

$$a = 80 \frac{in}{s} \left( \frac{40 \frac{in/s}{28 \text{ IN.}}}{28 \text{ IN.}} \right)$$

$$\text{OR } a = 114.3 \frac{in}{s^2}$$

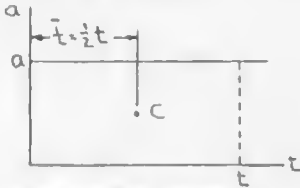
NOTE: TO USE THE METHOD OF MEASURING THE SUBNORMAL OUTLINED AT THE END OF SECTION 11.8, IT IS NECESSARY THAT THE SAME SCALE BE USED FOR THE  $x$  AND  $v$  AXES (e.g., 1 IN. = 50 IN., 1 IN. = 50 in/s). IN THE ABOVE SOLUTION,  $\Delta v$  AND  $\Delta x$  WERE MEASURED DIRECTLY, SO DIFFERENT SCALES COULD BE USED.

11.85

GIVEN: MOMENT-AREA METHOD OF SECTION 11.8

DERIVE:  $x = x_0 + v_0 t + \frac{1}{2} a t^2$  FOR A PARTICLE IN UNIFORMLY ACCELERATED RECTILINEAR MOTION

THE  $a$ - $t$  CURVE FOR UNIFORMLY ACCELERATED MOTION IS AS SHOWN.



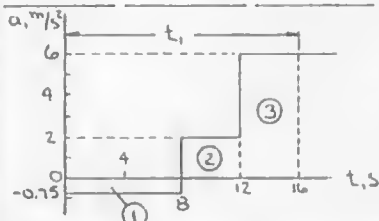
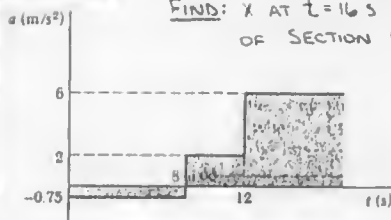
USING EQ. (11.13), HAVE..

$$\begin{aligned} x &= x_0 + v_0 t + (\text{AREA UNDER } a-t \text{ CURVE})(t - \bar{t}) \\ &= x_0 + v_0 t + (t \cdot a)(t - \frac{1}{2}t) \\ &= x_0 + v_0 t + \frac{1}{2} a t^2 \quad \text{Q.E.D.} \end{aligned}$$

11.86

GIVEN:  $a$ - $t$  CURVE;  $v_0 = -2 \frac{m}{s}$

FIND:  $x$  AT  $t = 16$  S USING THE METHOD OF SECTION 11.86



THE AREA UNDER THE CURVE IS FIRST DIVIDED INTO THREE REGIONS AS SHOWN.

FROM THE DISCUSSION FOLLOWING EQ. (11.13) AND ASSUMING  $x_0 = 0$ , HAVE..

$$x = x_0 + v_0 t + \sum A(t, \bar{t})$$

WHERE  $A$  IS THE AREA OF A REGION AND  $\bar{t}$  IS THE DISTANCE TO ITS CENTROID. THEN FOR  $t_1 = 16$  S...

$$\begin{aligned} x &= (-2 \frac{m}{s})(16 \text{ s}) + \left[ \left( (8 \text{ s})(-0.75 \frac{m}{s^2}) \right)(16 - 4) \right. \\ &\quad \left. + \left( (4 \text{ s})(2 \frac{m}{s^2}) \right)(16 - 10) \right] + \left[ (4 \text{ s})(6 \frac{m}{s^2}) \right](16 - 14) \text{ s} \\ &= [-32 + (-72 + 48 + 48)] \text{ m} \end{aligned}$$

$$\text{OR } x = -8.00 \text{ m}$$

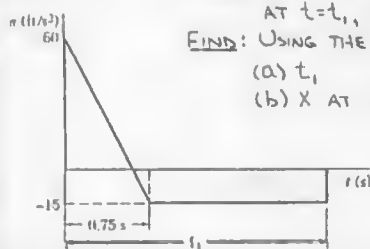
11.87

GIVEN:  $a$ - $t$  CURVE; AT  $t=0$ ,  $v = 7.5 \frac{ft}{s}$ ; AT  $t=t_1$ ,  $v=0$

FIND: USING THE METHOD OF SECTION 11.8

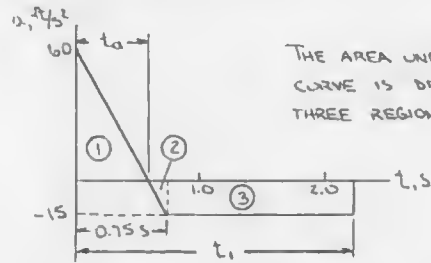
(a)  $t_1$

(b)  $x$  AT  $t=t_1$



(CONTINUED)

11.87 CONTINUED



THE AREA UNDER THE CURVE IS DIVIDED INTO THREE REGIONS AS SHOWN

(a) FIRST NOTE..  $\frac{t_0}{60} = \frac{0.75}{7.5}$  OR  $t_0 = 0.60 \text{ s}$

NOW..  $v = v_0 + \int_0^t a dt$

WHERE THE INTEGRAL IS EQUAL TO THE AREA UNDER THE  $a$ - $t$  CURVE. THEN, WITH  $v_0 = 7.5 \frac{ft}{s}$ ,  $v_{t_1} = 0$  HAVE..

$$\text{OR } t_1 = 2.375 \text{ s}$$

$$t_1 = 2.38 \text{ s}$$

(b) FROM THE DISCUSSION FOLLOWING EQ. (11.13) AND ASSUMING  $x_0 = 0$ , HAVE

$$x = x_0 + v_0 t + \sum A(t, \bar{t})$$

WHERE  $A$  IS THE AREA OF A REGION AND  $\bar{t}$  IS THE DISTANCE TO ITS CENTROID. THEN FOR  $t_1 = 2.375 \text{ s}$ ..

$$\begin{aligned} x &= (7.5 \frac{ft}{s})(2.375 \text{ s}) + \left[ \left( \frac{1}{2} (0.6 \text{ s})(7.5 \frac{ft}{s}) \right)(2.375 - 0.2) \right] \\ &\quad - \left[ \left( \frac{1}{2} (0.15 \text{ s})(15 \frac{ft}{s^2}) \right)(2.375 - 0.70) \right] \\ &\quad - \left[ \left( \frac{1}{2} (1.625 \text{ s})(15 \frac{ft}{s^2}) \right)(2.375 - (0.75 + \frac{1}{2} \cdot 1.625)) \right] \\ &= [17.8125 + (39.1500 - 1.8844 - 19.8047)] \text{ ft} \\ &\text{OR } x = 35.3 \text{ ft} \end{aligned}$$

11.88

GIVEN:  $v$ - $t$  CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT  $t=0$ ,

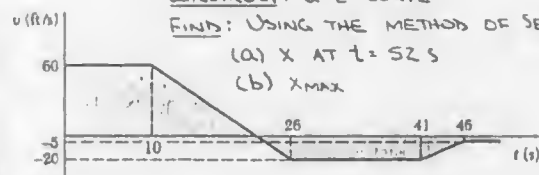
$$x = -540 \text{ ft}$$

CONSTRUCT:  $a$ - $t$  CURVE

FIND: USING THE METHOD OF SECTION 11.8

(a)  $x$  AT  $t = 52 \text{ s}$

(b)  $x_{\text{max}}$



HAVE..  $a = \frac{dv}{dt}$  WHERE  $\frac{dv}{dt}$  IS THE SLOPE OF THE  $v$ - $t$  CURVE. THEN..

FROM  $t=0$  TO  $t=10$  S:  $v = \text{CONSTANT} \Rightarrow a = 0$

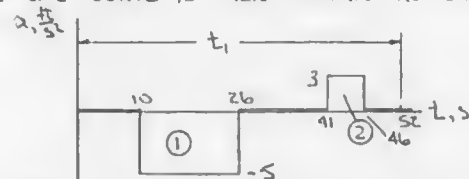
$$t=10 \text{ s TO } t=26 \text{ s: } a = \frac{-20 - 60}{26 - 10} = -5 \frac{ft}{s^2}$$

$$t=26 \text{ s TO } t=41 \text{ s: } v = \text{CONSTANT} \Rightarrow a = 0$$

$$t=41 \text{ s TO } t=46 \text{ s: } a = \frac{0 - (-20)}{46 - 41} = 4 \frac{ft}{s^2}$$

$$t > 46 \text{ s: } v = \text{CONSTANT} \Rightarrow a = 0$$

THE  $a$ - $t$  CURVE IS THEN DRAWN AS SHOWN.



(a) FROM THE DISCUSSION FOLLOWING EQ. (11.13), HAVE..

$$x = x_0 + v_0 t + \sum A(t, \bar{t})$$

(CONTINUED)

# 11.88 CONTINUED

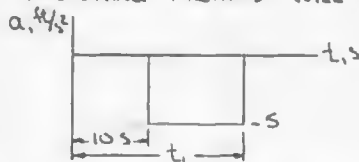
WHERE A IS THE AREA OF A REGION AND  $\bar{x}$  IS THE DISTANCE TO ITS CENTROID. THEN, FOR  $\bar{x} = 52.5$ ...

$$X = -540 \text{ ft} + (60 \frac{\text{ft}}{\text{s}})(52.5) + \left\{ \frac{1}{2} (16.5)(5 \frac{\text{ft}}{\text{s}})(52-18.5) + \frac{1}{2} (5)(3 \frac{\text{ft}}{\text{s}})(52-43.5) \right\}$$

$$= [-540 + (3120) + (-2720 + 127.5)] \text{ ft}$$

$$\text{OR } X = -12.50 \text{ ft}$$

(b) NOTING THAT  $X_{\text{MAX}}$  OCCURS WHEN  $\dot{X} = 0$  ( $\frac{dx}{dt} = 0$ ), IT IS SEEN FROM THE  $\dot{X}$ - $t$  CURVE THAT  $X_{\text{MAX}}$  OCCURS FOR  $10 \text{ s} < t < 26 \text{ s}$ . ALTHOUGH "SIMILAR TRIANGLES" COULD BE USED TO DETERMINE THE TIME AT WHICH  $X = X_{\text{MAX}}$  (SEE THE SOLUTION TO PROBLEM 11.63), THE FOLLOWING METHOD WILL BE USED.



FOR  $10 \text{ s} < t < 26 \text{ s}$ , HAVE

$$X = -540 + 60t - \left[ \frac{1}{2} (t-10)(5) \right] \left[ \frac{1}{2} (t-10) \right] \quad (\text{ft})$$

$$= -540 + 60t - \frac{5}{8} (t-10)^2$$

WHEN  $X = X_{\text{MAX}}$ :  $\frac{dX}{dt} = 60 - 5(t-10) = 0$   
 OR  $(t_1)_{X_{\text{MAX}}} = 22 \text{ s}$

THEN..  $X_{\text{MAX}} = -540 + 60(22) - \frac{5}{8} (22-10)^2$   
 OR  $X_{\text{MAX}} = 420 \text{ ft}$

# 11.89

GIVEN:  $X = 4t^4 - 6t$ ,  $Y = 6t^3 - 2t^2$   $X, Y$  - mm,  $t$  - s

FIND:  $\dot{X}$  AND  $\dot{Y}$  AT

- (a)  $t = 1 \text{ s}$   
 (b)  $t = 2 \text{ s}$   
 (c)  $t = 4 \text{ s}$

HAVE..  $X = 4t^4 - 6t$   $Y = 6t^3 - 2t^2$   
 THEN  $\dot{X} = \frac{dX}{dt} = 16t^3 - 6$   $\dot{Y} = \frac{dY}{dt} = 18t^2 - 4t$   
 AND  $a_x = \frac{d\dot{X}}{dt} = 48t^2$   $a_y = \frac{d\dot{Y}}{dt} = 36t - 4$

(a) AT  $t = 1 \text{ s}$ :  $\dot{X} = 16(1)^3 - 6 = 10 \frac{\text{mm}}{\text{s}}$   $\dot{Y} = 18(1)^2 - 4(1) = 14 \frac{\text{mm}}{\text{s}}$   
 OR  $\dot{X} = 17.20 \frac{\text{mm}}{\text{s}}$   $\angle 54.5^\circ$

$a_x = 48(1)^2 = 48 \frac{\text{mm}}{\text{s}^2}$   $a_y = 36(1) - 4 = 32 \frac{\text{mm}}{\text{s}^2}$   
 OR  $a = 57.7 \frac{\text{mm}}{\text{s}^2}$   $\angle 33.7^\circ$

(b) AT  $t = 2 \text{ s}$ :  $\dot{X} = 16(2)^3 - 6 = 122 \frac{\text{mm}}{\text{s}}$   $\dot{Y} = 18(2)^2 - 4(2) = 64 \frac{\text{mm}}{\text{s}}$   
 OR  $\dot{X} = 137.8 \frac{\text{mm}}{\text{s}}$   $\angle 27.7^\circ$

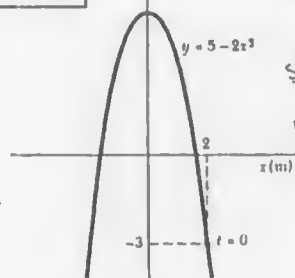
$a_x = 48(2)^2 = 192 \frac{\text{mm}}{\text{s}^2}$   $a_y = 36(2) - 4 = 68 \frac{\text{mm}}{\text{s}^2}$   
 OR  $a = 204 \frac{\text{mm}}{\text{s}^2}$   $\angle 19.5^\circ$

(c) AT  $t = 4 \text{ s}$ :  $\dot{X} = 16(4)^3 - 6 = 1018 \frac{\text{mm}}{\text{s}}$   $\dot{Y} = 18(4)^2 - 4(4) = 272 \frac{\text{mm}}{\text{s}}$   
 OR  $\dot{X} = 1054 \frac{\text{mm}}{\text{s}}$   $\angle 14.9^\circ$

$a_x = 48(4)^2 = 768 \frac{\text{mm}}{\text{s}^2}$   $a_y = 36(4) - 4 = 140 \frac{\text{mm}}{\text{s}^2}$   
 OR  $a = 781 \frac{\text{mm}}{\text{s}^2}$   $\angle 10.3^\circ$

# 11.90

$y(m)$



GIVEN:  $X = 2 \cos \pi t$ ,  $Y = 1 - 4 \cos 2\pi t$   $X, Y$  - m,  $t$  - s

SHOW: PATH IS THE PARABOLA  $Y = 5 - 2X^2$

FIND:  $\dot{X}$  AND  $\dot{Y}$  AT

- (a)  $t = 0$   
 (b)  $t = 1.5 \text{ s}$

HAVE..  $X = 2 \cos \pi t$   $Y = 1 - 4 \cos 2\pi t$

THEN..  $Y = 1 - 4(2 \cos^2 \pi t - 1)$   
 $= 5 - 8(\frac{X}{2})^2$

OR  $Y = 5 - 2X^2$  Q.E.D.

NOW..  $\dot{X} = \frac{dX}{dt} = -2\pi \sin \pi t$   $\dot{Y} = \frac{dY}{dt} = 8\pi \sin 2\pi t$

AND  $a_x = \frac{d\dot{X}}{dt} = -2\pi^2 \cos \pi t$   $a_y = \frac{d\dot{Y}}{dt} = 16\pi^2 \cos 2\pi t$

(a) AT  $t = 0$ :  $\dot{X} = 0$   $\dot{Y} = 0$   $\therefore \dot{X} = 0$   
 $a_x = -2\pi^2 \cos \pi t$   $a_y = 16\pi^2 \cos 2\pi t$

OR  $a = 159.1 \frac{\text{m}}{\text{s}^2}$   $\angle 82.9^\circ$

(b) AT  $t = 1.5 \text{ s}$ :  $\dot{X} = -2\pi \sin(1.5\pi)$   $\dot{Y} = 8\pi \sin(2\pi \cdot 1.5)$   
 $= 2\pi \frac{\text{m}}{\text{s}}$   $= 0$

OR  $\dot{X} = 6.28 \frac{\text{m}}{\text{s}}$

$a_x = -2\pi^2 \cos(1.5\pi)$   $a_y = 16\pi^2 \cos(2\pi \cdot 1.5)$   
 $= 0$   $= -16\pi^2$

OR  $a = 157.9 \frac{\text{m}}{\text{s}^2}$

# 11.91

GIVEN:  $X = \frac{1}{12}(t-2)^3 + t^2$ ,  $Y = \frac{1}{12} - \frac{1}{2}(t-1)^2$   $X, Y$  - ft,  $t$  - s

FIND: (a)  $\dot{X}_{\text{MIN}}$

(b)  $t$ ,  $X$ ,  $Y$ , AND DIRECTION OF  $\dot{X}$  WHEN  $\dot{X} = \dot{X}_{\text{MIN}}$

(a) HAVE..  $X = \frac{1}{12}(t-2)^3 + t^2$   $Y = \frac{1}{12} - \frac{1}{2}(t-1)^2$   
 THEN..  $\dot{X} = \frac{dX}{dt} = \frac{1}{4}(t-2)^2 + 2t$   $\dot{Y} = \frac{dY}{dt} = \frac{1}{4}t^2 - (t-1)$   
 $= \frac{1}{4}t^2 + t + 1$   $= \frac{1}{4}t^2 - t + 1$   
 $= \frac{1}{4}(t+2)^2$   $= \frac{1}{4}(t-2)^2$

NOW..  $\dot{X}^2 = \dot{X}_x^2 + \dot{X}_y^2 = \frac{1}{16}[(t+2)^4 + (t-2)^4]$

NOTING THAT  $\dot{X}$  IS MINIMUM WHEN  $\dot{X}^2$  IS MINIMUM,

HAVE..  $\frac{d\dot{X}^2}{dt} = \frac{1}{4}[(t+2)^3 + (t-2)^3] = 0$

EXPANDING..  $(t^3 + 6t^2 + 12t + 8) + (t^3 - 6t^2 + 12t - 8) = 0$   
 OR  $2(t^3 + 12t) = 0$

THE ONLY REAL ROOT OF THIS EQUATION IS  $t = 0$ .

$\therefore \dot{X}_{\text{MIN}} = \frac{1}{16}[(0+2)^4 + (0-2)^4] = 2$

OR  $\dot{X}_{\text{MIN}} = 1.414 \frac{\text{ft}}{\text{s}}$

(b) WHEN  $\dot{X} = \dot{X}_{\text{MIN}}$

$X = \frac{1}{12}(0-2)^3 + (0)^2$  OR  $X = -0.667 \text{ ft}$

$Y = \frac{1}{12} - \frac{1}{2}(0-1)^2$  OR  $Y = -0.500 \text{ ft}$

AND  $\dot{X}_x = \frac{1}{4}(0+2)^2 = 1 \frac{\text{ft}}{\text{s}}$   $\dot{X}_y = \frac{1}{4}(0-2)^2 = 1 \frac{\text{ft}}{\text{s}}$

THEN  $\tan \theta = \frac{\dot{X}_y}{\dot{X}_x} = 1$

OR  $\theta_{\text{MIN}} = 45^\circ$



# 11.92

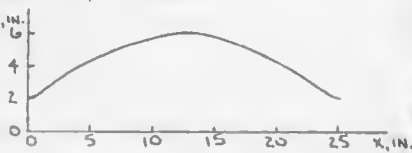
GIVEN:  $x = 4t - 2 \sin t$ ,  $y = 4 - 2 \cos t$   
 $x, y$  in in.,  $t$  in s

SKETCH: PATH OF THE PARTICLE

FIND: (a)  $v_{\min}$  AND  $v_{\max}$   
 (b)  $t$ ,  $x$ ,  $y$ , AND DIRECTION OF  $v$  WHEN  $v = v_{\min}$  AND  $v = v_{\max}$

HAVE...  $x = 4t - 2 \sin t$   $y = 4 - 2 \cos t$

$t, s$	$x, \text{in.}$	$y, \text{in.}$
0	0	2.0
$\frac{\pi}{2}$	4.28	4.0
$\pi$	12.57	6.0
$\frac{3\pi}{2}$	20.8	4.0
$2\pi$	25.1	2.0



(a) HAVE...  $x = 4t - 2 \sin t$   $y = 4 - 2 \cos t$   
 THEN...  $v_x = \frac{dx}{dt} = 4 - 2 \cos t$   $v_y = \frac{dy}{dt} = 2 \sin t$

$$\text{NOW... } v^2 = v_x^2 + v_y^2 = (4 - 2 \cos t)^2 + (2 \sin t)^2 = 20 - 16 \cos t$$

BY OBSERVATION... FOR  $v_{\min}$ ,  $\cos t = 1$  SO THAT  
 $v_{\min} = 4$  OR  $v_{\min} = 2 \frac{\text{in.}}{\text{s}}$

FOR  $v_{\max}$ ,  $\cos t = -1$  SO THAT  
 $v_{\max} = 6$  OR  $v_{\max} = 6 \frac{\text{in.}}{\text{s}}$

(b) WHEN  $v = v_{\min}$ :  $\cos t = 1$  OR  $t = 2N\pi$   
 WHERE  $N = 0, 1, 2, \dots$

THEN...  $x = 4(2N\pi) - 2 \sin(2N\pi)$  OR  $x = 8N\pi$  in.  
 $y = 4 - 2(-1)$  OR  $y = 2$  in.

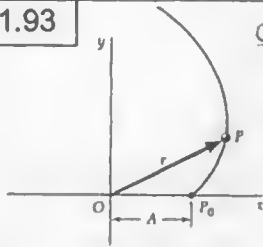
ALSO...  $v_x = 4 - 2(1) = 2 \frac{\text{in.}}{\text{s}}$   $v_y = 2 \sin(2N\pi) = 0$   
 $\therefore \theta_{v_{\min}} = 0$

WHEN  $v = v_{\max}$ :  $\cos t = -1$  OR  $t = (2N+1)\pi$   
 WHERE  $N = 0, 1, 2, \dots$

THEN  $x = 4(2N+1)\pi - 2 \sin(2N+1)\pi$  OR  $x = 4(2N+1)\pi$  in.  
 $y = 4 - 2(-1)$  OR  $y = 6$  in.

ALSO...  $v_x = 4 - 2(-1) = 6 \frac{\text{in.}}{\text{s}}$   $v_y = 2 \sin(2N+1)\pi = 0$   
 $\therefore \theta_{v_{\max}} = 0$

# 11.93



GIVEN:  $r = A(\cos t + t \sin t)\mathbf{i} + A(\sin t - t \cos t)\mathbf{j}$   
 $t$  in s

FIND: (a)  $t$  SO THAT  $r$  AND  $Q$  ARE PERPENDICULAR  
 (b)  $t$  SO THAT  $r$  AND  $Q$  ARE PARALLEL

HAVE...  $r = A(\cos t + t \sin t)\mathbf{i} + A(\sin t - t \cos t)\mathbf{j}$

THEN  $v = \frac{dr}{dt} = A(-\sin t + \sin t + t \cos t)\mathbf{i} + A(\cos t - \cos t - t \sin t)\mathbf{j}$   
 $= A(t \cos t)\mathbf{i} + A(-t \sin t)\mathbf{j}$

AND  $Q = \frac{dv}{dt} = A(\cos t - t \sin t)\mathbf{i} + A(\sin t + t \cos t)\mathbf{j}$

(a) WHEN  $r$  AND  $Q$  ARE PERPENDICULAR,  $r \cdot Q = 0$

$$\therefore A[(\cos t + t \sin t)(\sin t - t \cos t) + (\sin t - t \cos t)(\cos t - t \sin t)] = 0$$

$$\text{OR } (\cos t + t \sin t)(\cos t - t \sin t) + (\sin t - t \cos t)(\sin t + t \cos t) = 0$$

(CONTINUED)

# 11.93 CONTINUED

$$\text{OR } (\cos^2 t - t^2 \sin^2 t) + (\sin^2 t - t^2 \cos^2 t) = 0$$

$$\text{OR } 1 - t^2 = 0 \quad \text{OR } t = 1 \text{ s}$$

(b) WHEN  $r$  AND  $Q$  ARE PARALLEL,  $r \times Q = 0$

$$\therefore A[(\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}] \times A[(\cos t - t \sin t)\mathbf{i} + (\sin t + t \cos t)\mathbf{j}] = 0$$

$$\text{OR } [(\cos t + t \sin t)(\sin t + t \cos t) - (\sin t - t \cos t)(\cos t - t \sin t)]\mathbf{k} = 0$$

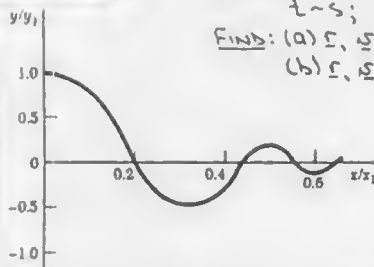
$$\text{EXPANDING... } (\sin t \cos t + t + t^2 \sin t \cos t) - (\sin t \cos t - t + t^2 \sin t \cos t) = 0$$

$$\text{OR } 2t = 0 \quad \text{OR } t = 0$$

# 11.94

GIVEN:  $r = x_1(1 - \frac{1}{t+1})\mathbf{i} + (y_1 e^{-\frac{\pi t}{2}} \cos 2\pi t)\mathbf{j}$   
 $t$  in s;  $x_1 = 30 \text{ mm}$ ,  $y_1 = 20 \text{ mm}$

FIND: (a)  $r$ ,  $v$ , AND  $Q$  AT  $t = 0$   
 (b)  $r$ ,  $v$ , AND  $Q$  AT  $t = 1.5 \text{ s}$



HAVE...  $r = 30(1 - \frac{1}{t+1})\mathbf{i} + 20(e^{-\frac{\pi t}{2}} \cos 2\pi t)\mathbf{j}$

THEN...  $v = \frac{dr}{dt} = 30 \frac{1}{(t+1)^2}\mathbf{i} + 20(-\frac{\pi}{2}e^{-\frac{\pi t}{2}} \cos 2\pi t - 2\pi e^{-\frac{\pi t}{2}} \sin 2\pi t)\mathbf{j}$   
 $= 30 \frac{1}{(t+1)^2}\mathbf{i} - 20\pi e^{-\frac{\pi t}{2}}(\frac{1}{2} \cos 2\pi t + 2 \sin 2\pi t)\mathbf{j}$

AND  $Q = \frac{dv}{dt} = -30 \frac{2}{(t+1)^3}\mathbf{i} - 20\pi[-\frac{\pi}{2}e^{-\frac{\pi t}{2}}(\frac{1}{2} \cos 2\pi t + 2 \sin 2\pi t) + e^{-\frac{\pi t}{2}}(-\pi \sin 2\pi t + 4 \cos 2\pi t)]\mathbf{j}$   
 $= -\frac{60}{(t+1)^3}\mathbf{i} + 10\pi^2 e^{-\frac{\pi t}{2}}(\frac{1}{4} \sin 2\pi t - 7.5 \cos 2\pi t)\mathbf{j}$

(a) AT  $t = 0$ :  $r = 30(1 - \frac{1}{1})\mathbf{i} + 20(1)\mathbf{j}$   
 OR  $r = 20 \text{ mm} \uparrow$

$v = 30(\frac{1}{1})\mathbf{i} - 20\pi[(\frac{1}{2})(\frac{1}{2} + 0)]\mathbf{j}$   
 OR  $v = 43.4 \frac{\text{mm}}{\text{s}} \angle 46.3^\circ$

$Q = -\frac{60}{(1)^3}\mathbf{i} + 10\pi^2(1)(\frac{1}{4} - 7.5)\mathbf{j}$   
 OR  $Q = 743 \frac{\text{mm}}{\text{s}^2} \angle 85.4^\circ$

(b) AT  $t = 1.5 \text{ s}$ :  $r = 30(1 - \frac{1}{2.5})\mathbf{i} + 20e^{-0.75\pi}(\cos 3\pi)\mathbf{j}$   
 $= (18 \text{ mm})\mathbf{i} + (-1.8956 \text{ mm})\mathbf{j}$

OR  $r = 18.10 \text{ mm} \angle 6.01^\circ$

$v = \frac{30}{(2.5)^2}\mathbf{i} - 20\pi e^{-0.75\pi}(\frac{1}{2} \cos 3\pi + 0)\mathbf{j}$   
 $= (4.80 \frac{\text{mm}}{\text{s}})\mathbf{i} + (2.9778 \frac{\text{mm}}{\text{s}})\mathbf{j}$

OR  $v = 5.65 \frac{\text{mm}}{\text{s}} \angle 31.8^\circ$

$Q = -\frac{60}{(2.5)^3}\mathbf{i} + 10\pi^2 e^{-0.75\pi}(\frac{1}{4} - 7.5 \cos 3\pi)\mathbf{j}$   
 $= (-3.84 \frac{\text{mm}}{\text{s}^2})\mathbf{i} + (70.1582 \frac{\text{mm}}{\text{s}^2})\mathbf{j}$

OR  $Q = 70.3 \frac{\text{mm}}{\text{s}^2} \angle 86.9^\circ$

11.95

GIVEN:  $\vec{r} = (Rt \cos \omega_n t) \hat{i} + ct \hat{j} + (Rt \sin \omega_n t) \hat{k}$   
 FIND:  $\vec{v}$  AND  $a$

HAVE..  $\vec{r} = (Rt \cos \omega_n t) \hat{i} + ct \hat{j} + (Rt \sin \omega_n t) \hat{k}$

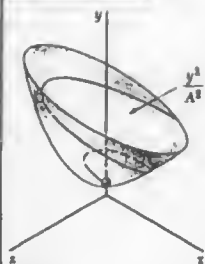
THEN..  $\vec{v} = \frac{d\vec{r}}{dt} = R(\cos \omega_n t - \omega_n t \sin \omega_n t) \hat{i} + c \hat{j} + R(\sin \omega_n t + \omega_n t \cos \omega_n t) \hat{k}$

AND  $a = \frac{d\vec{v}}{dt} = R(-\omega_n \sin \omega_n t - \omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t) \hat{i} + R(\omega_n \cos \omega_n t + \omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t) \hat{k}$   
 $= R(-2\omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t) \hat{i} + R(2\omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t) \hat{k}$

NOW..  $v^2 = v_x^2 + v_y^2 + v_z^2$   
 $= [R(\cos \omega_n t - \omega_n t \sin \omega_n t)]^2 + (c)^2 + [R(\sin \omega_n t + \omega_n t \cos \omega_n t)]^2$   
 $= R^2[\cos^2 \omega_n t - 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \sin^2 \omega_n t + (\sin^2 \omega_n t + 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \cos^2 \omega_n t)]$   
 $= R^2(1 + \omega_n^2 t^2) + c^2$   
 OR  $v = \sqrt{R^2(1 + \omega_n^2 t^2) + c^2}$

ALSO..  $a^2 = a_x^2 + a_y^2 + a_z^2$   
 $= [R(-2\omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t)]^2 + (0)^2 + [R(2\omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t)]^2$   
 $= R^2[4\omega_n^2 \sin^2 \omega_n t + 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + 4\omega_n^4 t^2 \cos^2 \omega_n t + 4\omega_n^2 \cos^2 \omega_n t - 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + 4\omega_n^4 t^2 \sin^2 \omega_n t]$   
 $= R^2(4\omega_n^2 + \omega_n^4 t^2)$   
 OR  $a = R\omega_n \sqrt{4 + \omega_n^2 t^2}$

\* 11.96



GIVEN:  $\vec{r} = (At \cos t) \hat{i} + (A\sqrt{t^2+1}) \hat{j} + (Bt \sin t) \hat{k}$   
 $r \sim \sqrt{t^2+1}$ ,  $t \sim s$ ;  $A=3$ ,  $B=1$

SHOW:  $(\frac{x}{A})^2 + (\frac{z}{B})^2 = 1$

FIND: (a)  $\vec{v}$  AND  $a$  AT  $t=0$

(b)  $t_{min}$  ( $t=0$ ) SO THAT  $\vec{r}$  AND  $\vec{v}$  ARE PERPENDICULAR

HAVE  $\vec{r} = (At \cos t) \hat{i} + (A\sqrt{t^2+1}) \hat{j} + (Bt \sin t) \hat{k}$

OR  $x = At \cos t$ ,  $y = A\sqrt{t^2+1}$ ,  $z = Bt \sin t$

THEN  $\cos t = \frac{x}{At}$ ,  $\sin t = \frac{z}{Bt}$ ,  $t^2 = (\frac{y}{A})^2 - 1$

NOW..  $\cos^2 t + \sin^2 t = 1 \Rightarrow (\frac{x}{At})^2 + (\frac{z}{Bt})^2 = 1$

THEN..  $(\frac{y}{A})^2 - 1 = (\frac{x}{A})^2 + (\frac{z}{B})^2$   
 OR  $(\frac{y}{A})^2 - (\frac{x}{A})^2 - (\frac{z}{B})^2 = 1$  Q.E.D.

(a) WITH  $A=3$  AND  $B=1$ , HAVE..

$\vec{v} = \frac{d\vec{r}}{dt} = 3(\cos t - t \sin t) \hat{i} + 3\frac{t}{\sqrt{t^2+1}} \hat{j} + (\sin t + t \cos t) \hat{k}$

AND  $a = \frac{d\vec{v}}{dt} = 3(-\sin t - \sin t - t \cos t) \hat{i} + 3\frac{1-t^2}{(t^2+1)^{3/2}} \hat{j} + (\cos t + \cos t - t \sin t) \hat{k}$   
 $= -3(2\sin t + t \cos t) \hat{i} + 3\frac{1-t^2}{(t^2+1)^{3/2}} \hat{j} + (2\cos t - t \sin t) \hat{k}$

AT  $t=0$ :  $\vec{v} = 3(1-0) \hat{i} + (0) \hat{j} + (0) \hat{k}$

OR  $v = 3 \frac{ft}{s}$

(CONTINUED)

11.96 CONTINUED

AND  $a = -3(0) \hat{i} + 3(1) \hat{j} + (2-0) \hat{k}$

THEN  $a^2 = (0)^2 + (3)^2 + (2)^2 = 13$  OR  $a = 3.61 \frac{ft}{s^2}$

(b) IF  $\vec{r}$  AND  $\vec{v}$  ARE PERPENDICULAR,  $\vec{r} \cdot \vec{v} = 0$

$\therefore [3t \cos t] \hat{i} + [3\sqrt{t^2+1}] \hat{j} + [t \sin t] \hat{k} \cdot [3(\cos t - t \sin t) \hat{i} + 3\frac{t}{\sqrt{t^2+1}} \hat{j} + (\sin t + t \cos t) \hat{k}] = 0$

OR  $(3t \cos t)[3(\cos t - t \sin t)] + (3\sqrt{t^2+1})(3\frac{t}{\sqrt{t^2+1}}) + (t \sin t)(\sin t + t \cos t) = 0$

EXPANDING..  $(9t \cos^2 t - 9t^2 \sin t \cos t) + (9t) + (t \sin^2 t + t^2 \sin t \cos t) = 0$

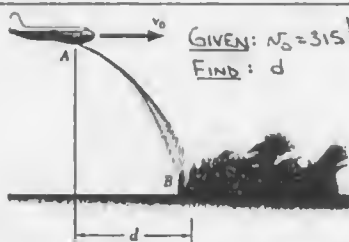
OR (WITH  $t \neq 0$ )  $10 + 8 \cos^2 t - 8t \sin t \cos t = 0$

OR  $7 + 2 \cos 2t - 2t \sin 2t = 0$

USING "TRIAL AND ERROR" OR NUMERICAL METHODS, THE SMALLEST ROOT IS

NOTE: THE NEXT ROOT IS  $t = 4.38$  s.

11.97



GIVEN:  $v_0 = 315 \frac{km}{h}$ ;  $h = 80$  m  
 FIND:  $d$

FIRST NOTE..  $v_0 = 315 \frac{km}{h} = 87.5 \frac{m}{s}$

VERTICAL MOTION

(UNIFORMLY ACCEL. MOTION)

$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

AT B..  $-80 \text{ m} = -\frac{1}{2}(9.81 \frac{m}{s^2}) t^2$

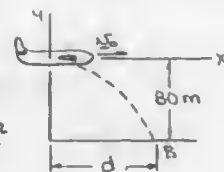
OR  $t_0 = 4.038$  s

HORIZONTAL MOTION (UNIFORM)

$x = x_0 + (v_x)_0 t$

AT B..  $d = (87.5 \frac{m}{s})(4.038 \text{ s})$

OR  $d = 353$  m



11.98

GIVEN:  $v_0$  IS HORIZONTAL; PATH OF SNOWBALL

FIND: (a)  $v_0$

(b)  $d$



(a) VERTICAL MOTION

(UNIFORMLY ACCEL. MOTION)

$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

AT B..  $-1 \text{ m} = -\frac{1}{2}(9.81 \frac{m}{s^2}) t^2$  OR  $t_0 = 0.451$  s

HORIZONTAL MOTION (UNIFORM)

$x = x_0 + (v_x)_0 t$

AT B..  $7 \text{ m} = v_0 (0.451 \text{ s})$

OR  $v_0 = 15.503 \frac{m}{s}$

$v_0 = 15.50 \frac{m}{s}$

(b) VERTICAL MOTION: AT C..  $-3 \text{ m} = -\frac{1}{2}(9.81 \frac{m}{s^2}) t^2$

OR  $t_c = 0.782$  s

(CONTINUED)

# 11.98 CONTINUED

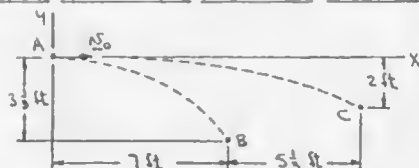
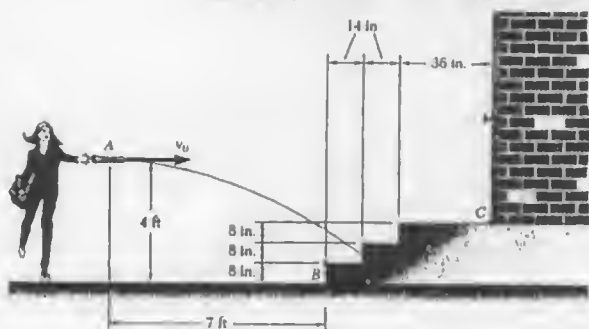
## HORIZONTAL MOTION

AT C:  $(7+d)m = (15.5031 \frac{m}{s})(0.782062 s)$   
OR  $d = 5.12 m$

# 11.99

GIVEN:  $N_0$  IS HORIZONTAL

FIND: RANGE OF VALUES OF  $N_0$  IF NEWSPAPER LANDS BETWEEN B AND C



VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = v_0 t$$

AT B:  $y: -3\frac{1}{3} ft = -\frac{1}{2} (32.2 \frac{ft}{s^2}) t^2$  OR  $t_B = 0.455016 s$

THEN..  $x: 7 ft = (v_0)_x (0.455016 s)$   
OR  $(v_0)_x = 15.38 \frac{ft}{s}$

AT C:  $y: -2 ft = -\frac{1}{2} (32.2 \frac{ft}{s^2}) t^2$  OR  $t_C = 0.352454 s$

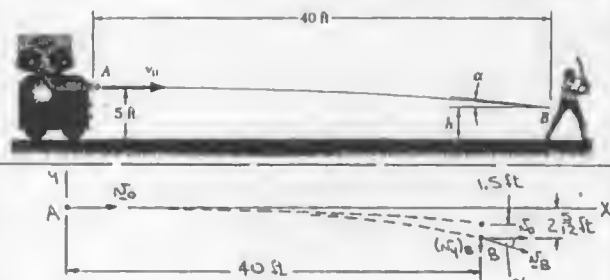
THEN..  $x: 12\frac{1}{3} ft = (v_0)_x (0.352454 s)$   
OR  $(v_0)_x = 35.0 \frac{ft}{s}$   
 $\therefore 15.38 \frac{ft}{s} \leq v_0 \leq 35.0 \frac{ft}{s}$

# 11.100

GIVEN:  $N_0$  IS HORIZONTAL; 31 IN.  $\leq h \leq 42$  IN.

FIND: (a) RANGE OF VALUES OF  $N_0$

(b)  $\alpha$  WHEN  $h = 31$  IN. AND  $h = 42$  IN.



(a) VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = v_0 t$$

WHEN  $h = 31$  IN.,  $y = -2\frac{1}{12} ft: -2\frac{1}{12} ft = -\frac{1}{2} (32.2 \frac{ft}{s^2}) t^2$   
OR  $t_{31} = 0.387432 s$

(CONTINUED)

# 11.100 CONTINUED

THEN..  $40 ft = (v_0)_{31} (0.387432 s)$

OR  $(v_0)_{31} = 103.244 \frac{ft}{s} = 70.4 \frac{mi}{h}$

WHEN  $h = 42$  IN.,  $y = -1.5 ft: -1.5 ft = -\frac{1}{2} (32.2 \frac{ft}{s^2}) t^2$

OR  $t_{42} = 0.305234 s$

THEN..  $40 ft = (v_0)_{42} (0.305234 s)$

OR  $(v_0)_{42} = 131.047 \frac{ft}{s} = 89.4 \frac{mi}{h}$

$\therefore 70.4 \frac{mi}{h} \leq v_0 \leq 89.4 \frac{mi}{h}$

(b) FOR THE VERTICAL MOTION

$$v_y = (v_y)_0 - g t$$

NOW  $TAN \alpha = \frac{(v_y)_0}{(v_x)_0} = \frac{g t}{v_0}$

WHEN  $h = 31$  IN.:  $TAN \alpha = \frac{(32.2 \frac{ft}{s^2})(0.387432 s)}{103.244 \frac{ft}{s}} = 0.120833$

OR  $\alpha_{31} = 6.89^\circ$

WHEN  $h = 42$  IN.:  $TAN \alpha = \frac{(32.2 \frac{ft}{s^2})(0.305234 s)}{131.047 \frac{ft}{s}} = 0.075000$

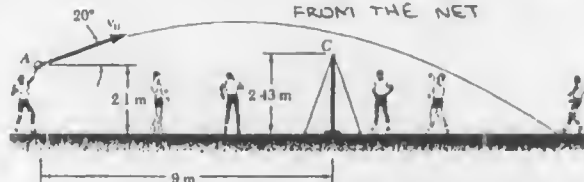
OR  $\alpha_{42} = 4.29^\circ$

# 11.101

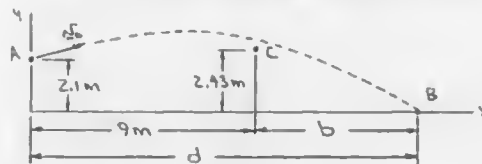
GIVEN:  $N_0 = 13.40 \frac{m}{s}$

FIND: (a) IF BALL CLEARS THE NET

(1.) DISTANCE THE BALL LANDS FROM THE NET



FIRST NOTE..  $(v_x)_0 = (13.40 \frac{m}{s}) \cos 20^\circ = 12.5919 \frac{m}{s}$   
 $(v_y)_0 = (13.40 \frac{m}{s}) \sin 20^\circ = 4.5831 \frac{m}{s}$



(a) HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

AT C..  $9 m = (12.5919 \frac{m}{s}) t$  OR  $t_C = 0.71475 s$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

AT C..  $y_C = 2.1 m + (4.5831 \frac{m}{s})(0.71475 s) - \frac{1}{2} (9.81 \frac{m}{s^2})(0.71475 s)^2$   
 $= 2.87 m$

$\therefore y_C > 2.43 m$  (HEIGHT OF NET)  $\Rightarrow$  BALL CLEARS NET

(b) AT B,  $y = 0: 0 = 2.1 m + (4.5831 \frac{m}{s}) t - \frac{1}{2} (9.81 \frac{m}{s^2}) t^2$

SOLVING..  $t_B = 1.271175 s$  (THE OTHER ROOT IS NEGATIVE)

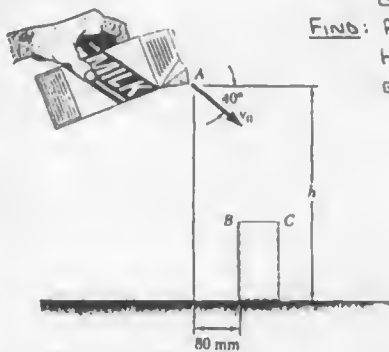
THEN..  $d = (v_x)_0 t_B = (12.5919 \frac{m}{s})(1.271175 s)$   
 $= 16.01 m$

$\therefore$  THE BALL LANDS  $b = (16.01 - 9.00) m = 7.01 m$  FROM THE NET

11.102

GIVEN:  $v_0 = 1.2 \frac{m}{s}$ ;  $h_c = 140 \text{ mm}$ ,  
 $d_{BC} = 66 \text{ mm}$

FIND: RANGE OF VALUES OF  
 $h$  SO THAT MILK  
 ENTERS THE GLASS



FIRST NOTE..

$$(v_x)_0 = (1.2 \frac{m}{s}) \cos 40^\circ = 0.91925 \frac{m}{s}$$

$$(v_y)_0 = -(1.2 \frac{m}{s}) \sin 40^\circ = -0.77135 \frac{m}{s}$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

MILK ENTERS GLASS AT B

$$x: 0.08 \text{ m} = (0.91925 \frac{m}{s}) t \quad \text{OR } t_B = 0.087028 \text{ s}$$

$$y: 0.140 \text{ m} = h_0 + (-0.77135 \frac{m}{s})(0.087028 \text{ s}) - \frac{1}{2}(9.81 \frac{m}{s^2})(0.087028 \text{ s})^2$$

$$\text{OR } h_0 = 0.244 \text{ m}$$

MILK ENTERS GLASS AT C

$$x: 0.146 \text{ m} = (0.91925 \frac{m}{s}) t \quad \text{OR } t_c = 0.158825 \text{ s}$$

$$y: 0.140 \text{ m} = h_c + (-0.77135 \frac{m}{s})(0.158825 \text{ s}) - \frac{1}{2}(9.81 \frac{m}{s^2})(0.158825 \text{ s})^2$$

$$\text{OR } h_c = 0.386 \text{ m}$$

$$\therefore 0.244 \text{ m} \leq h \leq 0.386 \text{ m}$$

11.103

GIVEN:  $v_0 = 160 \frac{ft}{s}$

FIND:  $d$



FIRST NOTE..  $(v_x)_0 = (160 \frac{ft}{s}) \cos 25^\circ$

$$(v_y)_0 = (160 \frac{ft}{s}) \sin 25^\circ$$

AND AT B..  $x_B = d \cos 5^\circ$   $y_B = -d \sin 5^\circ$

NOW.. HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

$$\text{AT B.. } d \cos 5^\circ = (160 \cos 25^\circ) t \quad \text{OR } t_B = \frac{\cos 5^\circ}{160 \cos 25^\circ} d$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{ft}{s^2})$$

$$\text{AT B.. } -d \sin 5^\circ = (160 \sin 25^\circ) t_B - \frac{1}{2} g t_B^2$$

$$\text{SUBSTITUTING FOR } t_B \dots -d \sin 5^\circ = (160 \sin 25^\circ) \left( \frac{\cos 5^\circ}{160 \cos 25^\circ} d \right) - \frac{1}{2} g \left( \frac{\cos 5^\circ}{160 \cos 25^\circ} d \right)^2$$

$$\text{OR } d = \frac{2}{32.2 \cos 5^\circ} (160 \cos 25^\circ)^2 (\tan 5^\circ + \tan 25^\circ)$$

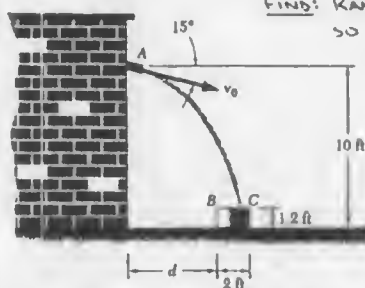
$$= 726.06 \text{ ft}$$

$$\text{OR } d = 242 \text{ yd}$$

11.104

GIVEN:  $v_0 = 2.5 \frac{m}{s}$

FIND: RANGE OF VALUES OF  $d$   
 SO THAT WATER ENTERS  
 THE TROUGH



FIRST NOTE..  $(v_x)_0 = (2.5 \frac{m}{s}) \cos 15^\circ = 2.4148 \frac{m}{s}$

$$(v_y)_0 = -(2.5 \frac{m}{s}) \sin 15^\circ = -0.64705 \frac{m}{s}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

AT THE TOP OF THE TROUGH..

$$-0.8 \text{ ft} = (-0.64705 \frac{m}{s}) t - \frac{1}{2} (32.2 \frac{ft}{s^2}) t^2$$

$$\text{OR } t_{tr} = 0.719491 \text{ s} \quad (\text{THE OTHER ROOT IS NEGATIVE})$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

$$\text{IN TIME } t_{tr} \dots x_{tr} = (2.4148 \frac{m}{s})(0.719491 \text{ s}) = 1.737 \text{ ft}$$

THUS, THE TROUGH MUST BE PLACED SO THAT

$$x_B \leq 1.737 \text{ ft} \quad x_C \geq 1.737 \text{ ft}$$

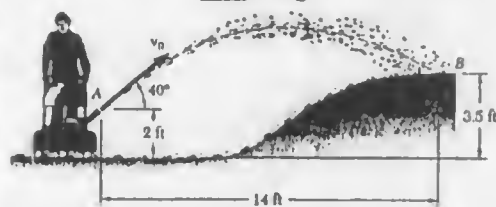
SINCE THE TROUGH IS 2 ft WIDE, IT THEN FOLLOWS THAT

$$0 \leq d \leq 1.737 \text{ ft}$$

11.105

GIVEN: SNOW DISCHARGED AS SHOWN

FIND:  $v_0$



FIRST NOTE..

$$(v_x)_0 = v_0 \cos 40^\circ$$

$$(v_y)_0 = v_0 \sin 40^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

$$\text{AT B.. } 14 = (v_0 \cos 40^\circ) t \quad \text{OR } t_B = \frac{14}{v_0 \cos 40^\circ}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{ft}{s^2})$$

$$\text{AT B.. } 1.5 = (v_0 \sin 40^\circ) t_B - \frac{1}{2} g t_B^2$$

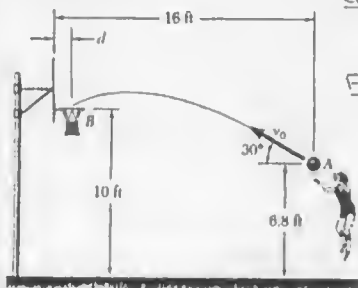
SUBSTITUTING FOR  $t_B$ ..

$$1.5 = (v_0 \sin 40^\circ) \left( \frac{14}{v_0 \cos 40^\circ} \right) - \frac{1}{2} g \left( \frac{14}{v_0 \cos 40^\circ} \right)^2$$

$$\text{OR } v_0^2 = \frac{\frac{1}{2} (32.2)(196) / \cos^2 40^\circ}{-1.5 + 14 \tan 40^\circ}$$

$$\text{OR } v_0 = 22.9 \frac{ft}{s}$$

11.106



GIVEN: TRAJECTORY OF A BASKETBALL AS SHOWN

FIND: (a)  $v_0$  WHEN  $d = 9$  IN.  
(b)  $v_0$  WHEN  $d = 17$  IN.

FIRST NOTE..

$$(v_x)_0 = v_0 \cos 30^\circ$$

$$(v_y)_0 = v_0 \sin 30^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

At B..  $(16 - d) = (v_0 \cos 30^\circ) t$  OR  $t_B = \frac{16 - d}{v_0 \cos 30^\circ}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{ft}{s^2})$$

At B..  $3.2 = (v_0 \sin 30^\circ) t_B - \frac{1}{2} g t_B^2$

SUBSTITUTING FOR  $t_B$ ..

$$3.2 = (v_0 \sin 30^\circ) \left( \frac{16 - d}{v_0 \cos 30^\circ} \right) - \frac{1}{2} g \left( \frac{16 - d}{v_0 \cos 30^\circ} \right)^2$$

$$\text{OR } v_0^2 = \frac{2g(16 - d)^2}{3 \left[ \frac{1}{\sin 30^\circ} (16 - d) - 3.2 \right]} \quad d \sim ft$$

(a)  $d = 9$  IN.:  $v_0^2 = \frac{2(32.2)(16 - \frac{9}{12})^2}{3 \left[ \frac{1}{\sin 30^\circ} (16 - \frac{9}{12}) - 3.2 \right]}$

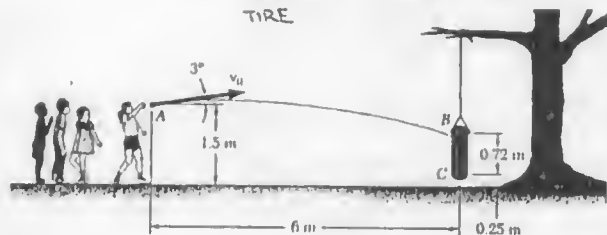
$$\text{OR } v_0 = 29.8 \frac{ft}{s}$$

(b)  $d = 17$  IN.:  $v_0^2 = \frac{2(32.2)(16 - \frac{17}{12})^2}{3 \left[ \frac{1}{\sin 30^\circ} (16 - \frac{17}{12}) - 3.2 \right]}$

$$\text{OR } v_0 = 29.6 \frac{ft}{s}$$

11.107

GIVEN: TRAJECTORY OF A BALL AS SHOWN  
FIND: RANGE OF VALUES OF  $v_0$  SO THAT BALL GOES THROUGH THE TIRE



FIRST NOTE..

$$(v_x)_0 = v_0 \cos 3^\circ$$

$$(v_y)_0 = v_0 \sin 3^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

When  $x = 6$  m:  $6 = (v_0 \cos 3^\circ) t$  OR  $t_B = \frac{6}{v_0 \cos 3^\circ}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{m}{s^2})$$

WHEN THE BALL REACHES THE TIRE,  $t = t_B$ .

$$\therefore y_{B,C} = (v_0 \sin 3^\circ) \left( \frac{6}{v_0 \cos 3^\circ} \right) - \frac{1}{2} g \left( \frac{6}{v_0 \cos 3^\circ} \right)^2$$

$$\text{OR } v_0^2 = \frac{18(9.81)}{\cos^2 3^\circ (6 \tan 3^\circ - y_{B,C})}$$

(CONTINUED)

11.107 CONTINUED

$$\text{OR } v_0^2 = \frac{177.065}{0.314447 - y_{B,C}}$$

At B,  $y = -0.53$  m:  $v_0^2 = \frac{177.065}{0.314447 - (-0.53)}$

$$\text{OR } (v_0)_B = 14.48 \frac{m}{s}$$

At C,  $y = -1.25$  m:  $v_0^2 = \frac{177.065}{0.314447 - (-1.25)}$

$$\text{OR } (v_0)_C = 10.64 \frac{m}{s}$$

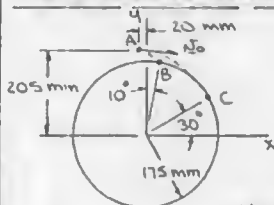
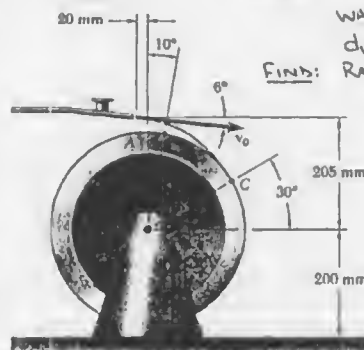
$$\therefore 10.64 \frac{m}{s} \leq v_0 \leq 14.48 \frac{m}{s}$$

11.108

GIVEN: TRAJECTORY OF COOLING WATER AS SHOWN;

$d_{\text{WHEEL}} = 350$  mm

FIND: RANGE OF VALUES OF  $v_0$  SO THAT THE WATER LANDS BETWEEN POINTS B AND C



FIRST NOTE..

$$(v_x)_0 = v_0 \cos 6^\circ$$

$$(v_y)_0 = -v_0 \sin 6^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{m}{s^2})$$

At POINT B:  $x = (0.175 \text{ m}) \sin 10^\circ$ ,  $y = (0.175 \text{ m}) \cos 10^\circ$

$$x: 0.175 \sin 10^\circ = -0.020 + (v_0 \cos 6^\circ) t$$

$$\text{OR } t_B = \frac{0.050388}{v_0 \cos 6^\circ}$$

$$y: 0.175 \cos 10^\circ = 0.205 + (-v_0 \sin 6^\circ) t_B - \frac{1}{2} g t_B^2$$

$$\text{SUBSTITUTING FOR } t_B \dots$$

$$-0.032659 = (-v_0 \sin 6^\circ) \left( \frac{0.050388}{v_0 \cos 6^\circ} \right) - \frac{1}{2} (9.81) \left( \frac{0.050388}{v_0 \cos 6^\circ} \right)^2$$

$$\text{OR } v_0^2 = \frac{\frac{1}{2} (9.81) (0.050388)^2}{\cos^2 6^\circ (0.032659 - 0.050388 \tan 6^\circ)}$$

$$\text{OR } (v_0)_B = 0.678 \frac{m}{s}$$

At POINT C:  $x = (0.175 \text{ m}) \cos 30^\circ$ ,  $y = (0.175 \text{ m}) \sin 30^\circ$

$$x: 0.175 \cos 30^\circ = -0.020 + (v_0 \cos 6^\circ) t$$

$$\text{OR } t_C = \frac{0.171554}{v_0 \cos 6^\circ}$$

$$y: 0.175 \sin 30^\circ = 0.205 + (-v_0 \sin 6^\circ) t_C - \frac{1}{2} g t_C^2$$

$$\text{SUBSTITUTING FOR } t_C \dots$$

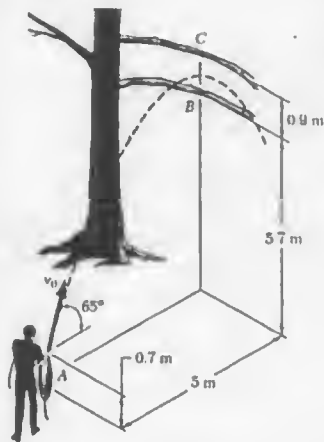
$$-0.117500 = (-v_0 \sin 6^\circ) \left( \frac{0.171554}{v_0 \cos 6^\circ} \right) - \frac{1}{2} (9.81) \left( \frac{0.171554}{v_0 \cos 6^\circ} \right)^2$$

$$\text{OR } v_0^2 = \frac{\frac{1}{2} (9.81) (0.171554)^2}{\cos^2 6^\circ (0.117500 - 0.171554 \tan 6^\circ)}$$

$$\text{OR } (v_0)_C = 1.211 \frac{m}{s}$$

$$\therefore 0.678 \frac{m}{s} \leq v_0 \leq 1.211 \frac{m}{s}$$

11.109



GIVEN: TRAJECTORY OF A ROPE AS SHOWN.

FIND: RANGE OF VALUES OF  $v_0$  SO THAT THE ROPE GOES OVER ONLY THE LOWEST LIMB.

FIRST NOTE..

$$(v_x)_0 = v_0 \cos 65^\circ \quad (v_y)_0 = v_0 \sin 65^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t$$

AT EITHER B OR C,  $x = 5\text{ m}$

$$5 = (v_0 \cos 65^\circ) t_{BC}$$

$$\text{OR } t_{BC} = \frac{5}{(v_0 \cos 65^\circ)}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{\text{m}}{\text{s}^2})$$

AT THE TREE LIMBS,  $t = t_{BC}$

$$y_{BC} = (v_0 \sin 65^\circ) \left( \frac{5}{v_0 \cos 65^\circ} \right) - \frac{1}{2} g \left( \frac{5}{v_0 \cos 65^\circ} \right)^2$$

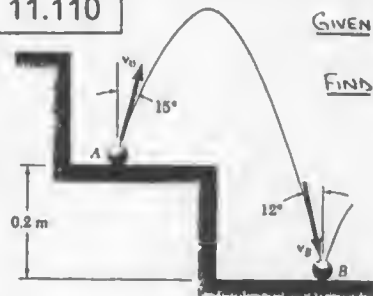
$$\text{OR } v_0^2 = \frac{\frac{1}{2} (9.81) (25)}{\cos^2 65^\circ (5 \tan 65^\circ - y_{BC})}$$

$$\text{AT POINT B: } v_0^2 = \frac{686.566}{5 \tan 65^\circ - 5} \quad \text{OR } (v_0)_B = 10.95 \frac{\text{m}}{\text{s}}$$

$$\text{AT POINT C: } v_0^2 = \frac{686.566}{5 \tan 65^\circ - 5.9} \quad \text{OR } (v_0)_C = 11.93 \frac{\text{m}}{\text{s}}$$

$$\therefore 10.95 \frac{\text{m}}{\text{s}} \leq v_0 \leq 11.93 \frac{\text{m}}{\text{s}}$$

11.110



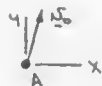
GIVEN: TRAJECTORY OF A BALL AS SHOWN

FIND:  $v_0$

FIRST NOTE..  $(v_x)_0 = v_0 \sin 15^\circ$   
 $(v_y)_0 = v_0 \cos 15^\circ$

HORIZONTAL MOTION (UNIFORM)

$$v_x = (v_x)_0 = v_0 \sin 15^\circ$$



(CONTINUED)

11.110 CONTINUED

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$v_y = (v_y)_0 - g t \quad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$= v_0 \cos 15^\circ - g t$$

AT POINT B,  $v_y = 0$

$$\text{THEN.. } \tan 12^\circ = \frac{(v_x)_B}{|(v_y)_B|} = \frac{v_0 \sin 15^\circ}{g t_B - v_0 \cos 15^\circ}$$

$$\text{OR } t_B = \frac{v_0 \left( \frac{\sin 15^\circ}{\tan 12^\circ} + \cos 15^\circ \right)}{g} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$$

$$= 0.22259 v_0$$

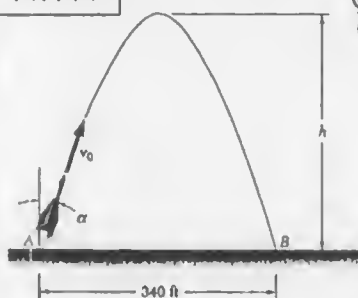
NOTING THAT  $y_B = -0.2\text{ m}$ , HAVE..

$$-0.2 = (v_0 \cos 15^\circ) (0.22259 v_0)$$

$$- \frac{1}{2} (9.81) (0.22259 v_0)^2$$

$$\text{OR } v_0 = 2.67 \frac{\text{m}}{\text{s}}$$

11.111



GIVEN:  $v_0 = 280 \frac{\text{ft}}{\text{s}}$

FIND: (a)  $\alpha$

(b)  $h$

(c)  $t_B$

FIRST NOTE..  $(v_x)_0 = v_0 \sin \alpha = (280 \frac{\text{ft}}{\text{s}}) \sin \alpha$

$$(v_y)_0 = v_0 \cos \alpha = (280 \frac{\text{ft}}{\text{s}}) \cos \alpha$$

(a) HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (280 \sin \alpha) t$$

AT POINT B:  $340 = (280 \sin \alpha) t$

$$\text{OR } t_B = \frac{17}{14 \sin \alpha}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 = (280 \cos \alpha) t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{\text{ft}}{\text{s}^2})$$

AT POINT B,  $t = t_B$ ,  $y = 0$ :

$$0 = (280 \cos \alpha) \left( \frac{17}{14 \sin \alpha} \right) - \frac{1}{2} g \left( \frac{17}{14 \sin \alpha} \right)^2$$

$$\text{OR } 280 \sin \alpha \cos \alpha - \frac{1}{2} g \left( \frac{17}{14} \right)^2 = 0$$

$$\text{OR } \sin 2\alpha = \frac{1}{2} \left( \frac{17}{14} \right) \left( \frac{32.2}{280} \right)$$

$$\text{OR } \alpha = 4.01359^\circ$$

$$\alpha = 4.01^\circ$$

(b) HAVE..  $v_y = (v_y)_0 - g t = 280 \cos \alpha - g t$

WHEN  $y = y_{\max} = h$ ,  $v_y = 0$ :  $0 = 280 \cos \alpha - g t$

$$\text{OR } t_h = \frac{280 \cos 4.01359^\circ}{32.2} = 8.67433 \text{ s}$$

THEN..  $h = (280 \cos \alpha) t_h - \frac{1}{2} g t_h^2$

$$= (280 \cos 4.01359^\circ) (8.67433) - \frac{1}{2} (32.2) (8.67433)^2$$

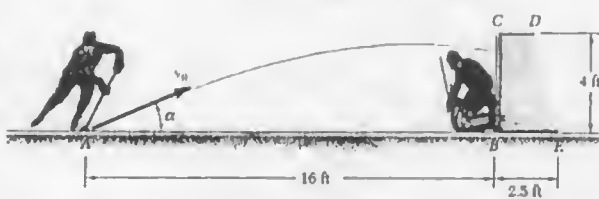
$$\text{OR } h = 1211 \text{ ft}$$

(c) HAS FOUND..  $t_B = \frac{17}{14 \sin \alpha}$

$$= \frac{17}{14 \sin 4.01359^\circ}$$

$$\text{OR } t_B = 17.35 \text{ s}$$

11.112

GIVEN:  $v_0 = 105 \frac{\text{mi}}{\text{h}}$ FIND: (a)  $\alpha_{\text{max}} (< 45^\circ)$  FOR WHICH THE  
PICK ENTERS THE NET(b)  $t$  WHEN  $\alpha = \alpha_{\text{max}}$ FIRST NOTE..  $v_0 = 105 \frac{\text{mi}}{\text{h}} = 154 \frac{\text{ft}}{\text{s}}$ AND  $(v_x)_0 = v_0 \cos \alpha = (154 \frac{\text{ft}}{\text{s}}) \cos \alpha$  $(v_y)_0 = v_0 \sin \alpha = (154 \frac{\text{ft}}{\text{s}}) \sin \alpha$ 

(a) HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (154 \cos \alpha) t$$

AT THE FRONT OF THE NET,  $x = 16 \text{ ft}$ 

$$\text{THEN.. } 16 = (154 \cos \alpha) t$$

$$\text{OR } t_{\text{ENTER}} = \frac{16}{154 \cos \alpha}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 = (154 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{\text{ft}}{\text{s}^2})$$

AT THE FRONT OF THE NET..

$$y_{\text{FRONT}} = (154 \sin \alpha) t_{\text{ENTER}} - \frac{1}{2} g t_{\text{ENTER}}^2$$

$$= (154 \sin \alpha) \left( \frac{16}{154 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{16}{154 \cos \alpha} \right)^2$$

$$= 16 \tan \alpha - \frac{32g}{5929 \cos^2 \alpha}$$

$$\text{NOW.. } \frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$$

$$\text{THEN } y_{\text{FRONT}} = 16 \tan \alpha - \frac{32g}{5929} (1 + \tan^2 \alpha)$$

$$\text{OR } \tan^2 \alpha - \frac{5929}{2g} \tan \alpha + \left( 1 + \frac{5929}{32g} y_{\text{FRONT}} \right) = 0$$

$$\text{THEN } \tan \alpha = \frac{\frac{5929}{2g} \pm \left[ \left( \frac{5929}{2g} \right)^2 - 4 \left( 1 + \frac{5929}{32g} y_{\text{FRONT}} \right) \right]^{1/2}}{2}$$

$$\text{OR } \tan \alpha = \frac{5929}{4 \times 32.2} \pm \left[ \left( \frac{5929}{4 \times 32.2} \right)^2 - \left( 1 + \frac{5929}{32 \times 32.2} y_{\text{FRONT}} \right) \right]^{1/2}$$

$$\text{OR } \tan \alpha = 46.0326 \pm \left[ (46.0326)^2 - (1 + 5.7541 y_{\text{FRONT}}) \right]^{1/2}$$

NOW..  $0 \leq y_{\text{FRONT}} \leq 4 \text{ ft}$  SO THAT THE POSITIVE  
ROOT WILL YIELD VALUES OF  $\alpha > 45^\circ$  FOR  
ALL VALUES OF  $y_{\text{FRONT}}$ . WHEN THE NEGATIVE ROOT  
IS SELECTED,  $\alpha$  INCREASES AS  $y_{\text{FRONT}}$  IS  
INCREASED. THEREFORE, FOR  $\alpha_{\text{MAX}}$  SET

$$y_{\text{FRONT}} = y_c = 4 \text{ ft}$$

$$\text{THEN.. } \tan \alpha = 46.0326 - \left[ (46.0326)^2 - (1 + 5.7541 \times 4) \right]^{1/2}$$

$$\text{OR } \alpha_{\text{MAX}} = 14.6604^\circ \quad \alpha_{\text{MAX}} = 14.66^\circ$$

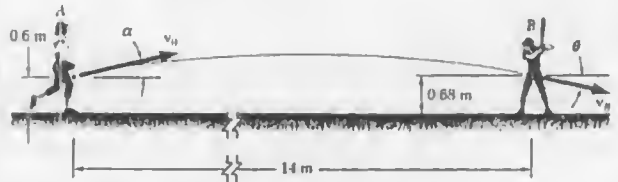
(b) HAD FOUND

$$t_{\text{ENTER}} = \frac{16}{154 \cos \alpha}$$

$$= \frac{16}{154 \cos 14.6604^\circ}$$

$$\text{OR } t_{\text{ENTER}} = 0.1074 \text{ s}$$

11.113

GIVEN:  $v_0 = 72 \frac{\text{km}}{\text{h}}$ FIND: (a)  $\alpha$ (b)  $\theta$ 

FIRST NOTE..

$$v_0 = 72 \frac{\text{km}}{\text{h}} = 20 \frac{\text{m}}{\text{s}}$$

$$\text{AND } (v_x)_0 = v_0 \cos \alpha = (20 \frac{\text{m}}{\text{s}}) \cos \alpha$$

$$(v_y)_0 = v_0 \sin \alpha = (20 \frac{\text{m}}{\text{s}}) \sin \alpha$$

(a) HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (20 \cos \alpha) t$$

$$\text{AT POINT B: } 14 = (20 \cos \alpha) t \quad \text{OR } t_B = \frac{14}{10 \cos \alpha}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 = (20 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{\text{m}}{\text{s}^2})$$

$$\text{AT POINT B: } 0.08 = (20 \sin \alpha) t_B - \frac{1}{2} g t_B^2$$

SUBSTITUTING FOR  $t_B$ ..

$$0.08 = (20 \sin \alpha) \left( \frac{14}{10 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{14}{10 \cos \alpha} \right)^2$$

$$\text{OR } 8 = 1400 \tan \alpha - \frac{1}{2} g \frac{49}{\cos^2 \alpha}$$

$$\text{NOW.. } \frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$$

$$\text{THEN.. } 8 = 1400 \tan \alpha - 24.5g (1 + \tan^2 \alpha)$$

$$\text{OR } 240.345 \tan^2 \alpha - 1400 \tan \alpha + 248.345 = 0$$

$$\text{SOLVING.. } \alpha = 10.3786^\circ \quad \text{AND } \alpha = 79.949^\circ$$

REJECTING THE SECOND ROOT BECAUSE IT IS NOT  
PHYSICALLY REASONABLE, HAVE

$$\alpha = 10.38^\circ$$

$$(b) \text{ HAVE } v_x = (v_x)_0 = 20 \cos \alpha$$

$$\text{AND } v_y = (v_y)_0 - g t = 20 \sin \alpha - g t$$

$$\text{AT POINT B: } (v_y)_B = 20 \sin \alpha - g t_B$$

$$= 20 \sin \alpha - \frac{19}{10 \cos \alpha}$$

NOTING THAT AT POINT B,  $v_y < 0$ , HAVE

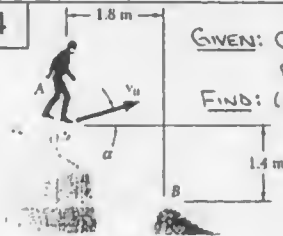
$$\tan \theta = \frac{1(v_y)_B}{v_x}$$

$$= \frac{\frac{19}{10 \cos \alpha} - 20 \sin \alpha}{20 \cos \alpha}$$

$$= \frac{\frac{1}{200} \cos 10.3786^\circ - \sin 10.3786^\circ}{\cos 10.3786^\circ}$$

$$\text{OR } \theta = 9.74^\circ$$

\* 11.114

GIVEN: CLIMBER JUMPS  
FROM A TO BFIND:  $(v_0)_{\text{MIN}}$  AND  $\alpha$ FIRST NOTE..  $(v_x)_0 = v_0 \cos \alpha$ 

$$(v_y)_0 = v_0 \sin \alpha$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (v_0 \cos \alpha) t$$

$$\text{AT POINT B: } 1.8 = (v_0 \cos \alpha) t$$

$$\text{OR } t_B = \frac{1.8}{v_0 \cos \alpha}$$

(CONTINUED)

# 11.114 CONTINUED

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 = (v_0 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{m}{s^2})$$

At Point B:  $-1.4 = (v_0 \sin \alpha) t_B - \frac{1}{2} g t_B^2$

SUBSTITUTING FOR  $t_B$ ..

$$-1.4 = (v_0 \sin \alpha) \left( \frac{1.8}{v_0 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{1.8}{v_0 \cos \alpha} \right)^2$$

$$\text{OR } v_0^2 = \frac{1.62g}{\cos^2 \alpha (1.8 \tan \alpha + 1.4)}$$

$$= \frac{1.62g}{0.9 \sin 2\alpha - 1.4 \cos^2 \alpha}$$

NOW MINIMIZE  $v_0^2$  WITH RESPECT TO  $\alpha$ . HAVE..

$$\frac{dv_0^2}{d\alpha} = 1.62g \frac{-(1.8 \cos 2\alpha - 2.8 \cos \alpha \sin \alpha)}{(0.9 \sin 2\alpha - 1.4 \cos^2 \alpha)^2} = 0$$

$$\text{OR } 1.8 \cos 2\alpha - 1.4 \sin 2\alpha = 0$$

$$\text{OR } \tan 2\alpha = \frac{1.8}{1.4}$$

$$\text{OR } \alpha = 26.0625^\circ \text{ AND } \alpha = 206.06^\circ$$

REJECTING THE SECOND VALUE BECAUSE IT IS NOT PHYSICALLY POSSIBLE, HAVE..

$$\alpha = 26.1^\circ$$

$$\text{FINALLY, } v_0^2 = \frac{1.62 \times 9.81}{\cos^2 26.0625^\circ (1.8 \tan 26.0625^\circ + 1.4)}$$

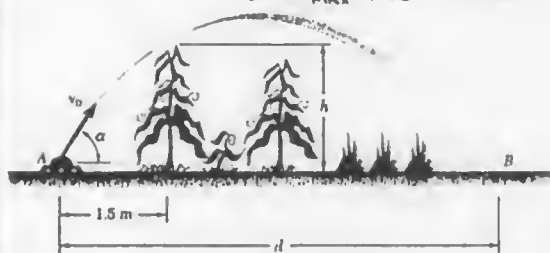
$$\text{OR } (v_0)_{\min} = 2.94 \frac{m}{s}$$

# 11.115

GIVEN:  $v_0 = 8 \frac{m}{s}$

FIND: (a)  $d_{\max}$  AND  $\alpha$  WHEN  $h=0$

(b)  $d_{\max}$  AND  $\alpha$  WHEN  $h=1.8 \text{ m}$



FIRST NOTE..  $(v_x)_0 = v_0 \cos \alpha = (8 \frac{m}{s}) \cos \alpha$

$$(v_y)_0 = v_0 \sin \alpha = (8 \frac{m}{s}) \sin \alpha$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (8 \cos \alpha) t$$

At Point B,  $x=d$ :  $d = (8 \cos \alpha) t$  OR  $t_B = \frac{d}{8 \cos \alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 = (8 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \frac{m}{s^2})$$

At Point B:  $0 = (8 \sin \alpha) t_B - \frac{1}{2} g t_B^2$

SIMPLIFYING AND SUBSTITUTING FOR  $t_B$ ..

$$0 = 8 \sin \alpha - \frac{1}{2} g \left( \frac{d}{8 \cos \alpha} \right)$$

$$\text{OR } d = \frac{64}{g} \sin 2\alpha \quad (1)$$

(a) WHEN  $h=0$ , THE WATER CAN FOLLOW ANY PHYSICALLY POSSIBLE TRAJECTORY. IT THEN FOLLOWS FROM EQ. (1) THAT  $d$  IS MAXIMUM WHEN  $2\alpha = 90^\circ$

$$\text{THEN } d = \frac{64}{9.81} \sin(2 \times 45^\circ)$$

$$\text{OR } d_{\max} = 6.52 \text{ m}$$

(b) BASED ON EQ. (1) AND THE RESULTS OF PART a, IT CAN BE CONCLUDED THAT  $d$  INCREASES IN VALUE AS  $\alpha$  INCREASES IN VALUE FROM (CONTINUED)

# 11.115 CONTINUED

0 TO  $45^\circ$  AND THEN  $d$  DECREASES AS  $\alpha$  IS FURTHER INCREASED. THUS,  $d_{\max}$  OCCURS FOR THE VALUE OF  $\alpha$  CLOSEST TO  $45^\circ$  AND FOR WHICH THE WATER JUST PASSES OVER THE FIRST ROW OF CORN PLANTS. AT THIS ROW  $x_{\text{corn}} = 1.5 \text{ m}$

$$\text{SO THAT } t_{\text{corn}} = \frac{1.5}{8 \cos \alpha}$$

ALSO, WITH  $y_{\text{corn}} = h$ , HAVE

$$h = (8 \sin \alpha) t_{\text{corn}} - \frac{1}{2} g t_{\text{corn}}^2$$

SUBSTITUTING FOR  $t_{\text{corn}}$  AND NOTING  $h=1.8 \text{ m}$ ,

$$1.8 = (8 \sin \alpha) \left( \frac{1.5}{8 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{1.5}{8 \cos \alpha} \right)^2$$

$$\text{OR } 1.8 = 1.5 \tan \alpha - \frac{2.25g}{128 \cos^2 \alpha}$$

$$\text{NOW.. } \frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$$

$$\text{THEN } 1.8 = 1.5 \tan \alpha - \frac{2.25(9.81)}{128} (1 + \tan^2 \alpha)$$

$$\text{OR } 0.172441 \tan^2 \alpha - 1.5 \tan \alpha + 1.972441 = 0$$

$$\text{SOLVING.. } \alpha = 58.229^\circ \text{ AND } \alpha = 81.965^\circ$$

FROM THE ABOVE DISCUSSION, IT FOLLOWS THAT  $d = d_{\max}$  WHEN

$$\alpha = 58.2^\circ$$

FINALLY, USING EQ (1)

$$d = \frac{64}{9.81} \sin(2 \times 58.229^\circ)$$

$$\text{OR } d_{\max} = 5.84 \text{ m}$$

# 11.116

GIVEN:  $v_0 = 11.5 \frac{m}{s}$

FIND: (a)  $d_{\max}$

(b)  $\alpha$  WHEN  $d = d_{\max}$



FIRST NOTE..  $(v_x)_0 = v_0 \cos \alpha = (11.5 \frac{m}{s}) \cos \alpha$

$$(v_y)_0 = v_0 \sin \alpha = (11.5 \frac{m}{s}) \sin \alpha$$

BY OBSERVATION,  $d_{\max}$  OCCURS

WHEN  $y_{\max} = 1.1 \text{ m}$ .

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$v_y = (v_y)_0 - g t \quad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$= (11.5 \sin \alpha) - g t \quad = (11.5 \sin \alpha) t - \frac{1}{2} g t^2$$

WHEN  $y = y_{\max}$  AT B,  $(v_y)_B = 0$

$$\text{THEN } (v_y)_B = 0 = (11.5 \sin \alpha) - g t$$

$$\text{OR } t_B = \frac{11.5 \sin \alpha}{g} \quad (g = 9.81 \frac{m}{s^2})$$

$$\text{AND } y_B = (11.5 \sin \alpha) t_B - \frac{1}{2} g t_B^2$$

SUBSTITUTING FOR  $t_B$  AND NOTING  $y_B = 1.1 \text{ m}$ ..

$$1.1 = (11.5 \sin \alpha) \left( \frac{11.5 \sin \alpha}{g} \right) - \frac{1}{2} g \left( \frac{11.5 \sin \alpha}{g} \right)^2$$

$$= \frac{1}{2g} (11.5)^2 \sin^2 \alpha$$

$$\text{OR } \sin^2 \alpha = \frac{2.2 \times 9.81}{11.5^2} \quad \alpha = 23.8265^\circ$$

(a) HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (v_x)_0 t = (11.5 \cos \alpha) t$$

At Point B,  $x = d_{\max}$  AND  $t = t_B$

$$\text{WHERE } t_B = \frac{11.5}{9.81} \sin 23.8265^\circ = 0.47356 \text{ s}$$

$$\text{THEN.. } d_{\max} = (11.5)(\cos 23.8265^\circ)(0.47356)$$

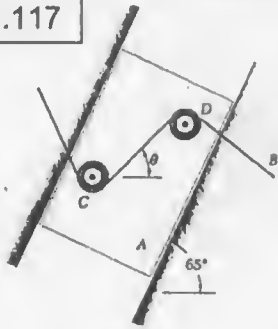
$$\text{OR } d_{\max} = 4.98 \text{ m}$$

(b) FROM ABOVE

$$\alpha = 23.8^\circ$$



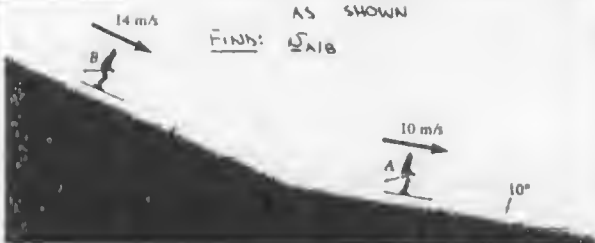
11.117



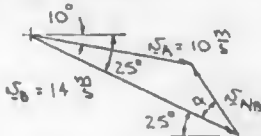
GIVEN:  $\vec{V}_A = 0.5 \frac{m}{s} \nearrow 65^\circ$   
 $\vec{V}_{C/A} = 2 \frac{m}{s} \searrow \theta$   
 FIND: (a)  $\vec{V}_{C/B}$  WHEN  $\theta = 45^\circ$   
 (b)  $\vec{V}_{C/B}$  WHEN  $\theta = 60^\circ$

HAVE ..  $\vec{V}_{C/B} = \vec{V}_A + \vec{V}_{C/A}$   
 WHERE  $\vec{V}_A = (0.5 \frac{m}{s})(-\cos 65^\circ \hat{j} - \sin 65^\circ \hat{i})$   
 $= (-0.21131 \frac{m}{s})\hat{i} - (0.45315 \frac{m}{s})\hat{j}$   
 AND  $\vec{V}_{C/A} = (2 \frac{m}{s})(\cos \theta \hat{i} + \sin \theta \hat{j})$   
 THEN ..  $\vec{V}_{C/B} = [(-0.21131 + 2 \cos \theta) \frac{m}{s}]\hat{i}$   
 $+ [(-0.45315 + 2 \sin \theta) \frac{m}{s}]\hat{j}$   
 (a) HAVE ..  $\vec{V}_{C/B} = (-0.21131 + 2 \cos 45^\circ)\hat{i}$   
 $+ (-0.45315 + 2 \sin 45^\circ)\hat{j}$   
 $= (1.20290 \frac{m}{s})\hat{i} + (0.96106 \frac{m}{s})\hat{j}$   
 OR  $\vec{V}_{C/B} = 1.540 \frac{m}{s} \nearrow 38.6^\circ$   
 (b) HAVE ..  $\vec{V}_{C/B} = (-0.21131 + 2 \cos 60^\circ)\hat{i}$   
 $+ (-0.45315 + 2 \sin 60^\circ)\hat{j}$   
 $= (0.78869 \frac{m}{s})\hat{i} + (1.27896 \frac{m}{s})\hat{j}$   
 OR  $\vec{V}_{C/B} = 1.503 \frac{m}{s} \nearrow 58.3^\circ$

11.118

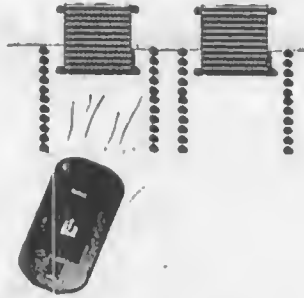


HAVE ..  $\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$   
 THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



THEN ..  $V_{A/B}^2 = 10^2 + 14^2 - 2(10)(14)\cos 15^\circ$   
 OR  $V_{A/B} = 5.05379 \frac{m}{s}$   
 AND  $\frac{10}{\sin \alpha} = \frac{5.05379}{\sin 15^\circ}$   
 OR  $\alpha = 30.8^\circ$   
 $\therefore \vec{V}_{A/B} = 5.05 \frac{m}{s} \nearrow 30.8^\circ$

11.119



GIVEN:  $\vec{V}_F = 9.8 \text{ KNOTS} \nearrow 70^\circ$   
 $\vec{V}_{FR} = 10 \text{ KNOTS} \searrow 30^\circ$   
 FIND:  $\vec{V}_R$

NOTE: "F" DENOTES THE FERRY AND "R" DENOTES THE RIVER..

HAVE ..  $\vec{V}_F = \vec{V}_R + \vec{V}_{FR}$  OR  $\vec{V}_F = \vec{V}_{FR} + \vec{V}_R$   
 THE GRAPHICAL REPRESENTATION OF THE SECOND EQUATION IS THEN AS SHOWN.

HAVE ..  $V_R^2 = 9.8^2 + 10^2 - 2(9.8)(10)\cos 10^\circ$   
 OR  $V_R = 1.737197 \text{ KNOTS}$   
 AND  $\frac{9.8}{\sin \alpha} = \frac{1.737197}{\sin 10^\circ}$   
 OR  $\alpha = 78.41^\circ$   
 NOTING THAT  
 $\therefore \vec{V}_R = 1.737 \text{ KNOTS} \nearrow 18.41^\circ$

11.120



GIVEN:  $\vec{V}_{C/A} = 235 \frac{mi}{h} \nearrow 75^\circ$   
 $\vec{V}_{C/B} = 260 \frac{mi}{h} \nearrow 40^\circ$   
 $\vec{V}_C = 24 \frac{mi}{h} \uparrow$   
 FIND: (a)  $\vec{V}_{B/A}$   
 (b)  $\vec{V}_A$   
 (c)  $\Delta t_{C/B}$  FOR  $\Delta t = 15 \text{ MIN}$

(a) HAVE ..  $\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$  AND  $\vec{V}_C = \vec{V}_B + \vec{V}_{C/B}$   
 THEN ..  $\vec{V}_A + \vec{V}_{C/A} = \vec{V}_B + \vec{V}_{C/B}$   
 OR  $\vec{V}_B - \vec{V}_A = \vec{V}_{C/A} - \vec{V}_{C/B}$   
 NOW ..  $\vec{V}_B - \vec{V}_A = \vec{V}_{B/A}$  SO THAT  
 $\vec{V}_{B/A} = \vec{V}_{C/A} - \vec{V}_{C/B}$  OR  $\vec{V}_{B/A} = \vec{V}_{C/B} - \vec{V}_{C/A}$   
 THE GRAPHICAL REPRESENTATION OF THE LAST EQUATION IS THEN AS SHOWN.

HAVE ..  $V_{B/A}^2 = 235^2 + 260^2 - 2(235)(260)\cos 65^\circ$   
 OR  $V_{B/A} = 266.798 \frac{mi}{h}$   
 AND  $\frac{260}{\sin \alpha} = \frac{266.798}{\sin 65^\circ}$   
 OR  $\alpha = 62.03^\circ$   
 $\therefore \vec{V}_{B/A} = 267 \frac{mi}{h} \nearrow 12.97^\circ$

(b) HAVE ..  $\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$   
 OR  $\vec{V}_A = (24 \frac{mi}{h})\hat{j} - (235 \frac{mi}{h})(-\cos 75^\circ \hat{i} - \sin 75^\circ \hat{j})$   
 (CONTINUED)

# 11.120 CONTINUED

$$\vec{v}_A = (60.822 \frac{\text{mi}}{\text{h}})\hat{i} + (250.99 \frac{\text{mi}}{\text{h}})\hat{j}$$

$$\text{OR } v_A = 258 \frac{\text{mi}}{\text{h}} \angle 76.4^\circ$$

(C) NOTING THAT THE VELOCITIES OF B AND C ARE CONSTANT, HAVE..

$$\vec{v}_B = (\vec{v}_B)_0 + \vec{v}_B t \quad \vec{v}_C = (\vec{v}_C)_0 + \vec{v}_C t$$

$$\text{NOW.. } \vec{v}_{C/B} = \vec{v}_C - \vec{v}_B = [(\vec{v}_C)_0 - (\vec{v}_B)_0] + (\vec{v}_C - \vec{v}_B)t$$

$$= [(\vec{v}_C)_0 - (\vec{v}_B)_0] + \vec{v}_{C/B} t$$

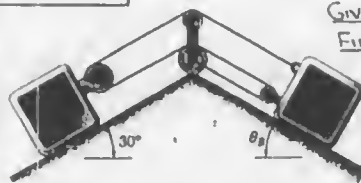
$$\text{THEN.. } \Delta \vec{v}_{C/B} = (\vec{v}_{C/B})_{t_2} - (\vec{v}_{C/B})_{t_1} = \vec{v}_{C/B} (t_2 - t_1)$$

$$= \vec{v}_{C/B} \Delta t$$

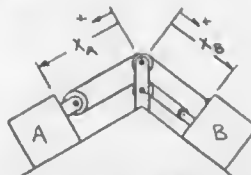
$$\text{FOR } \Delta t = 15 \text{ MIN: } \Delta \vec{v}_{C/B} = (260 \frac{\text{mi}}{\text{h}})(\frac{1}{4} \text{ h}) = 65 \text{ mi}$$

$$\therefore \Delta \vec{v}_{C/B} = 65 \text{ mi} \angle 40^\circ$$

# 11.122



GIVEN:  $v_{B/A} = 5.6 \frac{\text{m}}{\text{s}} \angle 70^\circ$   
FIND:  $v_A$  AND  $v_B$



FROM THE DIAGRAM..

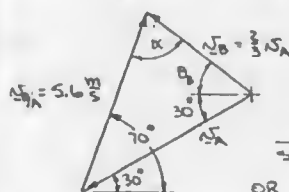
$$2x_A + 3x_B = \text{CONSTANT}$$

$$\text{THEN.. } 2v_A + 3v_B = 0$$

$$\text{OR } |v_B| = \frac{2}{3}v_A$$

$$\text{NOW.. } v_B = v_A + v_{B/A}$$

AND NOTING THAT  $v_A$  AND  $v_B$  MUST BE PARALLEL TO SURFACES A AND B, RESPECTIVELY, THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN. NOTE: ASSUMING THAT  $v_A$  IS DIRECTED UP THE INCLINE LEADS TO A VELOCITY DIAGRAM THAT DOES NOT "CLOSE."



FIRST NOTE..

$$\alpha = 180^\circ - (40^\circ + 30^\circ + \theta_B)$$

$$= 110^\circ - \theta_B$$

THEN

$$\frac{v_A}{\sin(110^\circ - \theta_B)} = \frac{\frac{2}{3}v_A}{\sin 40^\circ} = \frac{5.6}{\sin(30^\circ + \theta_B)}$$

$$\text{OR } v_A \sin 40^\circ = \frac{2}{3}v_A \sin(110^\circ - \theta_B)$$

$$\text{OR } \sin(110^\circ - \theta_B) = 0.96418$$

$$\text{OR } \theta_B = 35.3817^\circ \quad \text{AND } \theta_B = 4.6183^\circ$$

FOR  $\theta_B = 35.3817^\circ$ :

$$v_B = \frac{2}{3}v_A = \frac{5.6 \sin 40^\circ}{\sin(30^\circ + 35.3817^\circ)}$$

$$\text{OR } v_A = 5.94 \frac{\text{m}}{\text{s}}$$

$$\therefore v_A = 5.94 \frac{\text{m}}{\text{s}} \angle 30^\circ$$

$$v_B = 3.96 \frac{\text{m}}{\text{s}} \angle 35.4^\circ$$

FOR  $\theta_B = 4.6183^\circ$ :

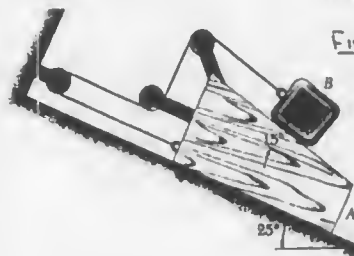
$$v_B = \frac{2}{3}v_A = \frac{5.6 \sin 40^\circ}{\sin(30^\circ + 4.6183^\circ)}$$

$$\text{OR } v_A = 9.50 \frac{\text{m}}{\text{s}}$$

$$\therefore v_A = 9.50 \frac{\text{m}}{\text{s}} \angle 30^\circ$$

$$v_B = 6.34 \frac{\text{m}}{\text{s}} \angle 4.62^\circ$$

# 11.123



GIVEN:  $v_A = 8 \frac{\text{in}}{\text{s}} \angle 25^\circ$   
 $Q_A = 6 \frac{\text{in}}{\text{s}} \angle 25^\circ$

FIND: (a)  $v_B$

(b)  $Q_B$

FROM THE DIAGRAM..

$$2x_A + x_{B/A} = \text{CONSTANT}$$

$$\text{THEN.. } 2v_A + v_{B/A} = 0$$

$$\text{OR } |v_{B/A}| = 16 \frac{\text{in}}{\text{s}}$$

$$\text{AND } 2Q_A + Q_{B/A} = 0$$

$$\text{OR } |Q_{B/A}| = 12 \frac{\text{in}}{\text{s}}$$

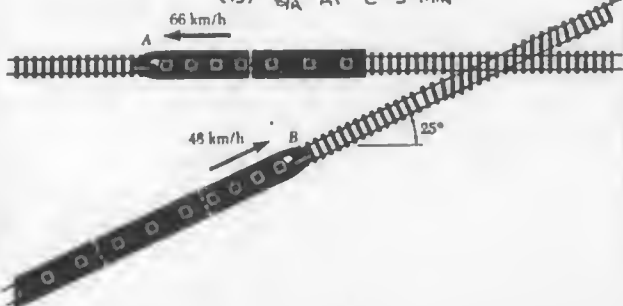
(CONTINUED)

# 11.121

GIVEN: CONSTANT VELOCITIES OF TRAINS A AND B; AT  $t=0$ , A IS AT THE CROSSING;  
AT  $t=10 \text{ MIN}$ , B IS AT THE CROSSING

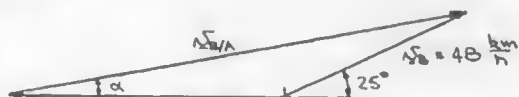
FIND: (a)  $v_{B/A}$

(b)  $r_{B/A}$  AT  $t=3 \text{ MIN}$



(a) HAVE..  $v_B = v_A + v_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



$$\text{THEN.. } v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155^\circ$$

$$\text{OR } v_{B/A} = 111.366 \text{ km/h}$$

$$\text{AND } \frac{48}{\sin \alpha} = \frac{111.366}{\sin 155^\circ}$$

$$\text{OR } \alpha = 10.50^\circ$$

$$\therefore v_{B/A} = 111.4 \frac{\text{km}}{\text{h}} \angle 10.50^\circ$$

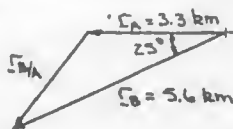
(b) FIRST NOTE THAT

AT  $t=3 \text{ MIN}$ , A IS  $(66 \frac{\text{km}}{\text{h}})(\frac{3}{60}) = 3.3 \text{ km}$  WEST OF THE CROSSING.

AT  $t=3 \text{ MIN}$ , B IS  $(48 \frac{\text{km}}{\text{h}})(\frac{3}{60}) = 2.4 \text{ km}$  SOUTHWEST OF THE CROSSING.

NOW..  $v_B = v_A + v_{B/A}$

THEN AT  $t=3 \text{ MIN}$  HAVE..



$$r_{B/A}^2 = 3.3^2 + 2.4^2$$

$$-2(3.3)(2.4)\cos 25^\circ$$

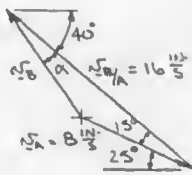
$$\text{OR } r_{B/A} = 2.96 \text{ km}$$

# 11.123 CONTINUED

NOTE THAT  $\vec{N}_{B/A}$  AND  $\vec{Q}_{B/A}$  MUST BE PARALLEL TO THE TOP SURFACE OF BLOCK A.

(a) HAVE..  $\vec{N}_B = \vec{N}_A + \vec{N}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN. NOTE THAT BECAUSE A IS MOVING DOWNWARD, B MUST BE MOVING UPWARD RELATIVE TO A.



HAVE..

$$N_B^2 = B^2 + 16^2 - 2(B)(16)\cos 15^\circ$$

$$\text{OR } N_B = 8.5278 \frac{\text{IN}}{\text{S}}$$

$$\text{AND } \frac{B}{\sin \alpha} = \frac{8.5278}{\sin 15^\circ}$$

$$\text{OR } \alpha = 14.05^\circ$$

$$\therefore N_B = 8.53 \frac{\text{IN}}{\text{S}} \angle 54.1^\circ$$

(b) THE SAME TECHNIQUE THAT WAS USED TO DETERMINE  $\vec{N}_B$  CAN BE USED TO DETERMINE  $\vec{Q}_B$ . AN ALTERNATIVE METHOD IS AS FOLLOWS.

HAVE..  $\vec{Q}_B = \vec{Q}_A + \vec{Q}_{B/A}$

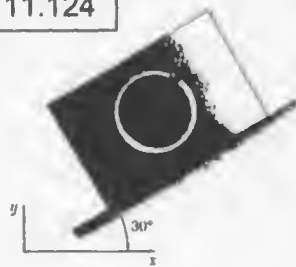
$$= (6\hat{i}) + 12(-\cos 15^\circ \hat{i} + \sin 15^\circ \hat{j}) +$$

$$= -(5.5911 \frac{\text{IN}}{\text{S}})\hat{i} + (3.1058 \frac{\text{IN}}{\text{S}})\hat{j}$$

$$\text{OR } \vec{Q}_B = 6.40 \frac{\text{IN}}{\text{S}} \angle 54.1^\circ$$

\* NOTE THE ORIENTATION OF THE COORDINATE AXES ON THE SKETCH OF THE SYSTEM

# 11.124



GIVEN:  $N_{P/A} = 200 \frac{\text{MM}}{\text{S}}$

$$N_A = 120 \frac{\text{MM}}{\text{S}} \angle 30^\circ$$

FIN: (a)  $\vec{v}_P$  WHEN  $\theta = 30^\circ$

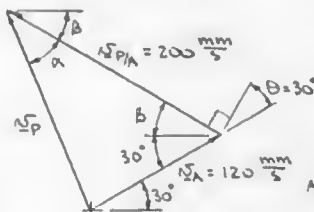
(b)  $\vec{v}_P$  WHEN  $\theta = 135^\circ$

NOTE: RATHER THAN APPLY THE SAME METHOD OF SOLUTION TWICE, TWO EQUALLY APPLICABLE TECHNIQUES WILL BE USED.

(a) METHOD 1.

HAVE..  $\vec{v}_P = \vec{v}_A + \vec{v}_{P/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FIRST NOTE..

$$\beta = 90^\circ - (30^\circ + 30^\circ) = 30^\circ$$

THEN..

$$v_P^2 = 120^2 + 200^2$$

$$- 2(120)(200)\cos 60^\circ$$

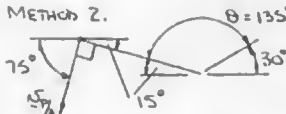
$$\text{OR } v_P = 174.356 \frac{\text{MM}}{\text{S}}$$

$$\text{AND } \frac{120}{\sin \alpha} = \frac{174.356}{\sin 60^\circ}$$

$$\text{OR } \alpha = 36.6^\circ$$

$$\therefore \vec{v}_P = 174.4 \frac{\text{MM}}{\text{S}} \angle 66.6^\circ$$

(b) METHOD 2.



(CONTINUED)

# 11.124 CONTINUED

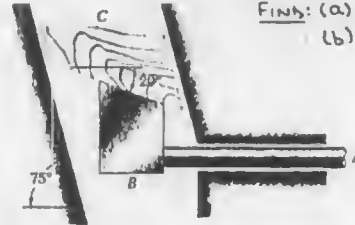
HAVE..  $\vec{N}_P = \vec{N}_A + \vec{N}_{P/A}$

$$= 120(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + 200(-\cos 75^\circ \hat{i} - \sin 75^\circ \hat{j})$$

$$= (52.159 \frac{\text{MM}}{\text{S}})\hat{i} - (133.185 \frac{\text{MM}}{\text{S}})\hat{j}$$

$$\text{OR } \vec{N}_P = 143.0 \frac{\text{MM}}{\text{S}} \angle 68.6^\circ$$

# 11.125



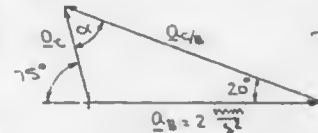
GIVEN:  $\vec{Q}_B = 2 \frac{\text{MM}}{\text{S}} \rightarrow$ ;  $(\vec{v}_B)_0 = (\vec{v}_E)_0 = 0$

FIN: (a)  $\vec{Q}_C$

(b)  $\vec{N}_C$  AT  $t = 10 \text{ S}$

(a) HAVE..  $\vec{Q}_C = \vec{Q}_B + \vec{Q}_{C/B}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FIRST NOTE..  $\alpha = 180^\circ - (20^\circ + 105^\circ)$

$$= 55^\circ$$

$$\text{THEN- } \frac{Q_C}{\sin 20^\circ} = \frac{2}{\sin 55^\circ}$$

$$Q_C = 0.83506 \frac{\text{MM}}{\text{S}}$$

$$\therefore \vec{Q}_C = 0.835 \frac{\text{MM}}{\text{S}} \angle 75^\circ$$

(b) FOR UNIFORMLY ACCELERATED MOTION..

$$N_C = (v_B)_0 + a_C t$$

$$\text{AT } t = 10 \text{ S: } N_C = (0.83506 \frac{\text{MM}}{\text{S}^2})(10 \text{ S}) = 8.3506 \frac{\text{MM}}{\text{S}}$$

$$\text{OR } \vec{N}_C = 8.35 \frac{\text{MM}}{\text{S}} \angle 75^\circ$$

# 11.126



GIVEN:  $\vec{Q}_A = 1.2 \frac{\text{M}}{\text{S}} \rightarrow$ ;  $(\vec{v}_B)_0 = 0$

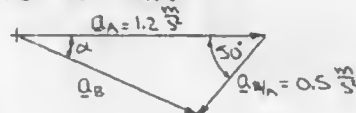
$$\vec{Q}_{B/A} = 0.5 \frac{\text{M}}{\text{S}} \angle 50^\circ$$

FIN: (a)  $\vec{Q}_B$

(b)  $\vec{v}_B$  AT  $t = 2 \text{ S}$

(a) HAVE..  $\vec{Q}_B = \vec{Q}_A + \vec{Q}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



$$\text{HAVE.. } Q_B^2 = 1.2^2 + 0.5^2 - 2(1.2)(0.5)\cos 50^\circ$$

$$\text{OR } Q_B = 0.95846 \frac{\text{M}}{\text{S}}$$

$$\text{AND } \frac{0.5}{\sin \alpha} = \frac{0.95846}{\sin 50^\circ}$$

$$\text{OR } \alpha = 23.6^\circ$$

$$\therefore \vec{Q}_B = 0.958 \frac{\text{M}}{\text{S}} \angle 23.6^\circ$$

(b) FOR UNIFORMLY ACCELERATED MOTION..

$$v_B = (v_B)_0 + a_B t$$

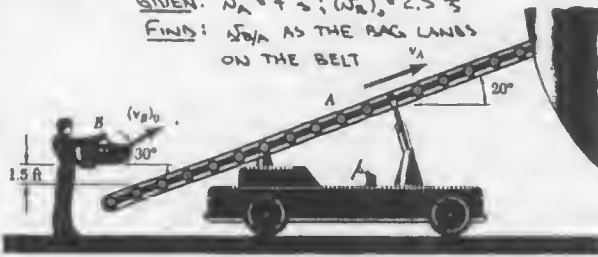
$$\text{AT } t = 2 \text{ S: } v_B = (0.95846 \frac{\text{M}}{\text{S}^2})(2 \text{ S}) = 1.91692 \frac{\text{M}}{\text{S}}$$

$$\text{OR } \vec{v}_B = 1.917 \frac{\text{M}}{\text{S}} \angle 23.6^\circ$$

11.127

GIVEN:  $\vec{v}_A = 4 \frac{\text{ft}}{\text{s}}$ ;  $(\vec{v}_B)_0 = 2.5 \frac{\text{ft}}{\text{s}}$

FIND:  $\vec{v}_{B/A}$  AS THE BAG LANDS ON THE BELT



FIRST DETERMINE THE VELOCITY OF THE BAG AS IT LANDS ON THE BELT. NOW..

$$[(\vec{v}_B)_x]_0 = (\vec{v}_B)_0 \cos 30^\circ = (2.5 \frac{\text{ft}}{\text{s}}) \cos 30^\circ$$

$$[(\vec{v}_B)_y]_0 = (\vec{v}_B)_0 \sin 30^\circ = (2.5 \frac{\text{ft}}{\text{s}}) \sin 30^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0 + (\vec{v}_B)_x t \quad (\vec{v}_B)_x = [(\vec{v}_B)_x]_0 = 2.5 \cos 30^\circ$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + [(\vec{v}_B)_y]_0 t - \frac{1}{2} g t^2 \quad (\vec{v}_B)_y = [(\vec{v}_B)_y]_0 - g t = 2.5 \sin 30^\circ - g t$$

THE EQUATION OF THE LINE COINCIDENT WITH THE TOP SURFACE OF THE BELT IS

$$y = x \tan 20^\circ$$

THUS, WHEN THE BAG REACHES THE BELT..

$$1.5 + (2.5 \sin 30^\circ) t - \frac{1}{2} g t^2 = [(2.5 \cos 30^\circ) t] \tan 20^\circ$$

$$\text{OR } \frac{1}{2} (32.2) t^2 + 2.5 (\cos 30^\circ \tan 20^\circ - \sin 30^\circ) t - 1.5 = 0$$

$$\text{OR } 16.1 t^2 - 0.46198 t - 1.5 = 0$$

$$\text{SOLVING } \dots t = 0.31992 \text{ s AND } t = -0.29122 \text{ s (REJECT)}$$

THE VELOCITY  $\vec{v}_B$  OF THE BAG AS IT LANDS ON THE BELT IS THEN..

$$\vec{v}_B = (2.5 \cos 30^\circ) \hat{i} + [2.5 \sin 30^\circ - 32.2(0.31992)] \hat{j} = (2.1651 \frac{\text{ft}}{\text{s}}) \hat{i} - (9.0514 \frac{\text{ft}}{\text{s}}) \hat{j}$$

$$\text{FINALLY } \dots \vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad \text{OR } \vec{v}_{B/A} = (2.1651 \hat{i} - 9.0514 \hat{j}) - 4(\cos 20^\circ \hat{i} + \sin 20^\circ \hat{j}) = -(1.59367 \frac{\text{ft}}{\text{s}}) \hat{i} - (10.4195 \frac{\text{ft}}{\text{s}}) \hat{j}$$

$$\text{OR } \vec{v}_{B/A} = 10.54 \frac{\text{ft}}{\text{s}} \angle 81.3^\circ$$

11.128 CONTINUED

$$\text{THEN } \vec{v}_C = -(1.38567 \frac{\text{ft}}{\text{s}}) \hat{i} - (18.5237 \frac{\text{ft}}{\text{s}}) \hat{j} = 18.9209 \frac{\text{ft}}{\text{s}} \angle 78.239^\circ$$

$$(a) \text{ HAVE } \dots \vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$$

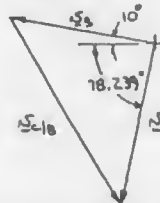
IF  $\vec{v}_{C/B}$  IS VERTICAL, THEN  $(\vec{v}_{C/B})_x = 0$  WHICH IMPLIES  $(\vec{v}_C)_x = (\vec{v}_B)_x$

$$\therefore -\vec{v}_B \cos 10^\circ = -3.8567$$

$$\text{OR } \vec{v}_B = 3.92 \frac{\text{ft}}{\text{s}} \angle 10^\circ$$

$$(b) \text{ HAVE } \dots \vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FOR  $\vec{v}_{C/B}$  TO BE MINIMUM,  $\vec{v}_{C/B}$  MUST BE PERPENDICULAR TO  $\vec{v}_B$ .

$$\therefore \vec{v}_B = 18.9209 \cos 88.239^\circ$$

$$\text{OR } \vec{v}_B = 0.581 \frac{\text{ft}}{\text{s}} \angle 10^\circ$$

11.129

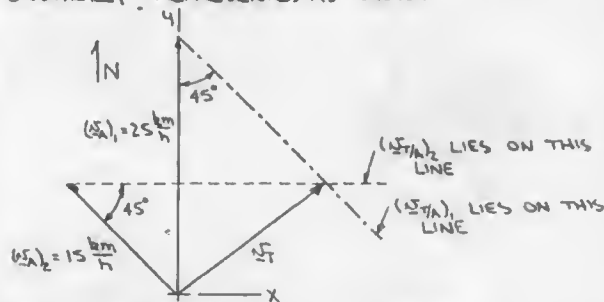
GIVEN:  $(\vec{v}_A)_1 = 25 \frac{\text{km}}{\text{h}} \angle 45^\circ$ ,  $(\vec{v}_{T/A})_1 \angle 45^\circ$

$$(\vec{v}_A)_2 = 15 \frac{\text{km}}{\text{h}} \angle 45^\circ, (\vec{v}_{T/A})_2 \rightarrow$$

FIND:  $\vec{v}_T$ , WHERE  $\vec{v}_T$  IS CONSTANT

$$\text{HAVE } \dots \vec{v}_T = \vec{v}_A + \vec{v}_{T/A}$$

USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



FROM THE DIAGRAM..

$$(\vec{v}_T)_x = 25 - 15 \sin 45^\circ = 14.3934 \frac{\text{km}}{\text{h}}$$

$$(\vec{v}_T)_y = 15 \sin 45^\circ = 10.6066 \frac{\text{km}}{\text{h}}$$

$$\therefore \vec{v}_T = 17.88 \frac{\text{km}}{\text{h}} \angle 36.4^\circ$$

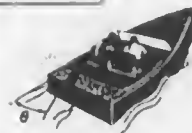
11.130

GIVEN:  $(\vec{v}_B)_1 = 5 \frac{\text{km}}{\text{h}} \angle 50^\circ$ ,  $(\vec{v}_{W/B})_1 \angle 50^\circ$

$$(\vec{v}_B)_2 = 20 \frac{\text{km}}{\text{h}} \rightarrow$$

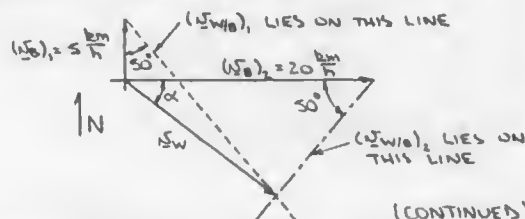
$$(\vec{v}_{W/B})_2 \angle 50^\circ$$

FIND:  $\vec{v}_W$



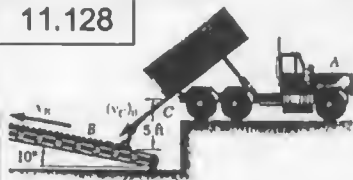
$$\text{HAVE } \dots \vec{v}_W = \vec{v}_B + \vec{v}_{W/B}$$

USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



(CONTINUED)

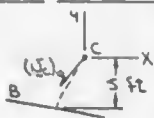
11.128



GIVEN:  $(\vec{v}_C)_0 = 16 \frac{\text{ft}}{\text{s}} \angle 50^\circ$

FIND: (a)  $\vec{v}_B$  IF  $\vec{v}_{C/B}$  IS VERTICAL

(b)  $\vec{v}_B$  IF  $\vec{v}_{C/B} = (\vec{v}_{C/B})_{\text{MIN}}$



FIRST DETERMINE THE VELOCITY OF THE COAL AS IT LANDS ON THE BELT. NOW..

$$[(\vec{v}_C)_x]_0 = (\vec{v}_C)_0 \cos 50^\circ = (16 \frac{\text{ft}}{\text{s}}) \cos 50^\circ$$

$$[(\vec{v}_C)_y]_0 = (\vec{v}_C)_0 \sin 50^\circ = (16 \frac{\text{ft}}{\text{s}}) \sin 50^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$(\vec{v}_C)_x = [(\vec{v}_C)_x]_0 = 16 \cos 50^\circ = 10.1268 \frac{\text{ft}}{\text{s}}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$(\vec{v}_C)_y = [(\vec{v}_C)_y]_0 - g(4 - y_0) \quad (g = 32.2 \frac{\text{ft}}{\text{s}^2})$$

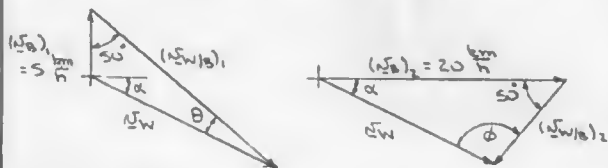
$$\text{AT THE BELT: } (\vec{v}_C)_y = (10.1268 \frac{\text{ft}}{\text{s}}) - 2(32.2)(-5)$$

$$\text{OR } (\vec{v}_C)_y = 18.5237 \frac{\text{ft}}{\text{s}}$$

(CONTINUED)

# 11.130 CONTINUED

WITH  $N_W$  NOW DEFINED, THE ABOVE DIAGRAM IS REDRAWN FOR THE TWO CASES FOR CLARITY.



NOTING THAT

$$\theta = 180^\circ - (50^\circ + 90^\circ + \alpha) = 40^\circ - \alpha$$

$$\phi = 180^\circ - (50^\circ + \alpha) = 130^\circ - \alpha$$

HAVE  $\frac{N_W}{\sin 50^\circ} = \frac{5}{\sin (40^\circ - \alpha)}$   $\frac{N_W}{\sin 50^\circ} = \frac{20}{\sin (130^\circ - \alpha)}$

THEREFORE  $\frac{5}{\sin (40^\circ - \alpha)} = \frac{20}{\sin (130^\circ - \alpha)}$

OR  $\sin 130^\circ \cos \alpha - \cos 130^\circ \sin \alpha = 4(\sin 40^\circ \cos \alpha - \cos 40^\circ \sin \alpha)$

OR  $\tan \alpha = \frac{\sin 130^\circ - 4 \sin 40^\circ}{\cos 130^\circ - 4 \cos 40^\circ}$

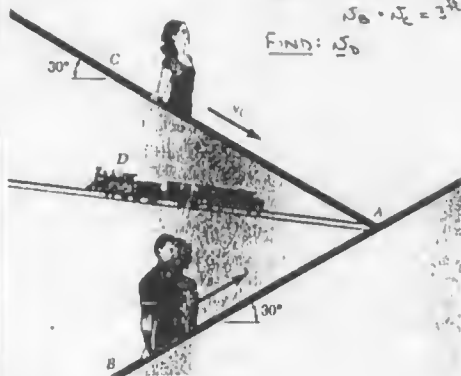
OR  $\alpha = 25.964^\circ$

THEN  $N_W = \frac{5 \sin 50^\circ}{\sin (40^\circ - 25.964^\circ)} = 15.79 \frac{\text{km}}{\text{h}}$   
 $\therefore N_W = 15.79 \frac{\text{km}}{\text{h}} \nearrow 26.0^\circ$

# 11.131

GIVEN:  $N_{D/B} \nearrow 22^\circ$ ;  $N_{D/C} \nearrow 23^\circ$ ;  
 $N_B = N_C = 3 \frac{\text{ft}}{\text{s}}$

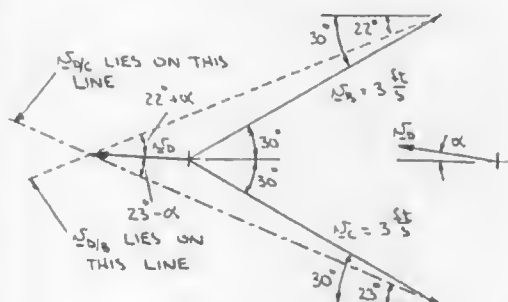
FIND:  $N_D$



HAVE  $N_D = N_B + N_{D/B}$

$N_D = N_C + N_{D/C}$

THE GRAPHICAL REPRESENTATIONS OF THESE EQUATIONS ARE THEN AS SHOWN.



(CONTINUED)

# 11.131 CONTINUED

THEN  $\frac{N_D}{\sin B^\circ} = \frac{3}{\sin (22^\circ + \alpha)}$   $\frac{N_D}{\sin 7^\circ} = \frac{3}{\sin (23^\circ - \alpha)}$

EQUATING THE EXPRESSIONS FOR  $\frac{N_D}{3}$  ..

$$\frac{\sin B^\circ}{\sin (22^\circ + \alpha)} = \frac{\sin 7^\circ}{\sin (23^\circ - \alpha)}$$

OR  $\sin B^\circ (\sin 23^\circ \cos \alpha - \cos 23^\circ \sin \alpha) = \sin 7^\circ (\sin 22^\circ \cos \alpha + \cos 22^\circ \sin \alpha)$

OR  $\tan \alpha = \frac{\sin B^\circ \sin 23^\circ - \sin 7^\circ \sin 22^\circ}{\sin B^\circ \cos 23^\circ + \sin 7^\circ \cos 22^\circ}$

OR  $\alpha = 2.0728^\circ$

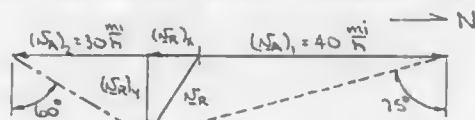
THEN  $N_D = \frac{3 \sin B^\circ}{\sin (22^\circ + 2.0728^\circ)} = 1.024 \frac{\text{ft}}{\text{s}}$   
 $\therefore N_D = 1.024 \frac{\text{ft}}{\text{s}} \nearrow 2.07^\circ$

# 11.132

GIVEN:  $(N_A)_1 = 40 \frac{\text{mi}}{\text{h}}$  N,  $(N_A)_2 \nearrow 75^\circ$ ;  
 $(N_A)_1 = 30 \frac{\text{mi}}{\text{h}}$  S,  $(N_A)_2 \searrow 60^\circ$  WITH THE VERTICAL

FIND:  $N_R$

HAVE  $N_R = (N_A)_1 + (N_A)_2$   $N_R = (N_A)_1 + (N_A)_2$   
 THE GRAPHICAL REPRESENTATIONS OF THESE EQUATIONS ARE THEN AS SHOWN. NOTE THAT THE LINE OF ACTION OF  $(N_A)_2$  MUST BE DIRECTED AS SHOWN SO THAT THE SECOND VELOCITY DIAGRAM 'CLOSES.'



$(N_A)_1$  LIES ON THIS LINE  $(N_A)_2$  LIES ON THIS LINE

FROM THE DIAGRAM..  $(N_R)_y = [40 + (N_A)_x] \tan 15^\circ$   
 AND  $(N_R)_y = [30 - (N_A)_x] \tan 30^\circ$

EQUATING THE EXPRESSIONS FOR  $(N_R)_y$  ..

$$[40 + (N_A)_x] \tan 15^\circ = [30 - (N_A)_x] \tan 30^\circ$$

OR  $(N_A)_x = 7.8109 \frac{\text{mi}}{\text{h}}$

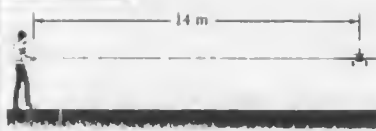
THEN  $(N_R)_y = (40 + 7.8109) \tan 15^\circ = 12.8109 \frac{\text{mi}}{\text{h}}$   
 $\therefore N_R = 15.00 \frac{\text{mi}}{\text{h}} \nearrow 30.6^\circ$

# 11.133

GIVEN:  $S = 18 \frac{\text{m}}{\text{s}}$

$p = 14 \text{ m}$

FIND:  $a_n$



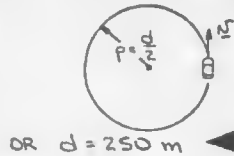
HAVE  $a_n = \frac{S^2}{p}$   
 $= \frac{(18 \frac{\text{m}}{\text{s}})^2}{14 \text{ m}}$

OR  $a_n = 23.1 \frac{\text{m}}{\text{s}^2}$

11.134

GIVEN: CIRCULAR TRACK OF DIAMETER  $d$   
 FIND: (a)  $d$  WHEN  $v = 72 \frac{\text{km}}{\text{h}}$ ,  $a_n = 3.2 \frac{\text{m}}{\text{s}^2}$   
 (b)  $v$  WHEN  $d = 180 \text{ m}$ ,  $a_n = 0.69$

(a) FIRST NOTE...  $v = 72 \frac{\text{km}}{\text{h}} = 20 \frac{\text{m}}{\text{s}}$   
 Now...  $a_n = \frac{v^2}{r}$   
 OR  $\frac{d}{2} = \frac{(20 \frac{\text{m}}{\text{s}})^2}{3.2 \frac{\text{m}}{\text{s}^2}}$



OR  $d = 250 \text{ m}$

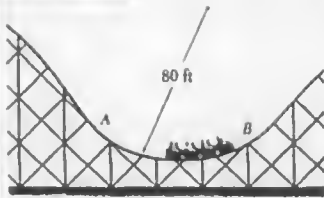
(b) HAVE  $a_n = \frac{v^2}{r}$

THEN...  $v^2 = (0.69 \frac{\text{m}}{\text{s}^2})(\frac{1}{2} \times 180 \text{ m})$   
 OR  $v = 23.016 \frac{\text{m}}{\text{s}}$

OR  $v = 82.9 \frac{\text{km}}{\text{h}}$

11.135

GIVEN:  $(a_n)_{AB} \leq 3g$   
 FIND:  $(v_{\text{max}})_{AB}$



HAVE...  $a_n = \frac{v^2}{r}$

THEN...  $(v_{\text{max}})_{AB}^2 = (3 \times 32.2 \frac{\text{ft}}{\text{s}^2})(80 \text{ ft})$   
 OR  $(v_{\text{max}})_{AB} = 87.909 \frac{\text{ft}}{\text{s}}$

OR  $(v_{\text{max}})_{AB} = 59.9 \frac{\text{mi}}{\text{h}}$

11.136



GIVEN:  $[(a_c)_n]_A = 26 \frac{\text{in}}{\text{s}^2}$   
 $[(a_c)_n]_B = 267 \frac{\text{in}}{\text{s}^2}$   
 B ROLLS ON A  
 FIND:  $d_B$

FIRST NOTE THAT "ROLLING WITHOUT SLIPPING"  
 IMPLIES  $(v_c)_A = (v_c)_B = v_c$

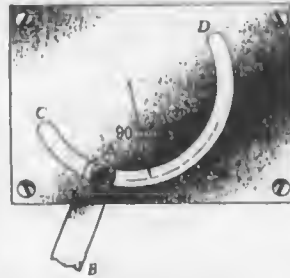
NOW...  $[(a_c)_n]_A = \frac{v_c^2}{r_A}$  AND  $[(a_c)_n]_B = \frac{v_c^2}{r_B}$

WHERE  $r_B = \frac{d_B}{2}$

THEN...  $r_A [(a_c)_n]_A = [(a_c)_n]_B (\frac{d_B}{2})$

SUBSTITUTING...  $(2.6 \text{ in.})(26 \frac{\text{in}}{\text{s}^2}) = (267 \frac{\text{in}}{\text{s}^2})(\frac{d_B}{2})$   
 OR  $d_B = 0.506 \text{ in.}$

11.137



GIVEN:  $(v_A)_0 = 0$ ;  
 $(a_A)_t = 20 \frac{\text{mm}}{\text{s}^2}$   
 FIND: (a)  $a_A$  AT  $t = 0$   
 (b)  $a_A$  AT  $t = 2 \text{ s}$

(a) AT  $t = 0$ ,  $v_A = 0$  WHICH IMPLIES  $(a_A)_n = 0$   
 $\therefore a_A = (a_A)_t$

OR  $a_A = 20 \frac{\text{mm}}{\text{s}^2}$

(b) HAVE UNIFORMLY ACCELERATED MOTION...

$\therefore v_A = (v_A)_0 + (a_A)_t t$

AT  $t = 2 \text{ s}$ ,  $v_A = (20 \frac{\text{mm}}{\text{s}^2})(2 \text{ s}) = 40 \frac{\text{mm}}{\text{s}}$

NOW...  $(a_A)_n = \frac{v_A^2}{r_A} = \frac{(40 \frac{\text{mm}}{\text{s}})^2}{90 \text{ mm}} = 17.778 \frac{\text{mm}}{\text{s}^2}$

FINALLY...  $a_A^2 = (a_A)_t^2 + (a_A)_n^2$   
 $= (20)^2 + (17.778)^2$

OR  $a_A = 26.8 \frac{\text{mm}}{\text{s}^2}$

11.138

GIVEN:  $d = 250 \text{ mm}$ ;  $v_0 = 45 \frac{\text{m}}{\text{s}}$ ;  $a_t = \text{CONSTANT}$ ;  
 AT  $t = 9 \text{ s}$ ,  $v = 0$   
 FIND:  $t$  WHEN  $a = 40 \frac{\text{m}}{\text{s}^2}$

HAVE UNIFORMLY DECELERATED MOTION...

$\therefore v = v_0 + a_t t$

AT  $t = 9 \text{ s}$ :  $0 = 45 \frac{\text{m}}{\text{s}} + a_t (9 \text{ s})$

OR  $a_t = -5 \frac{\text{m}}{\text{s}^2}$

NOW...  $a^2 = a_t^2 + a_n^2$

WHEN  $a = 40 \frac{\text{m}}{\text{s}^2}$ :  $40^2 = (-5)^2 + a_n^2$

HAVE...  $a_n = \frac{v^2}{r}$  OR  $a_n = 39.686 \frac{\text{m}}{\text{s}^2}$

THEN  $v^2 = (39.686 \frac{\text{m}}{\text{s}^2})(0.125 \text{ m})$

OR  $v = 2.227 \frac{\text{m}}{\text{s}}$

FINALLY...  $2.227 \frac{\text{m}}{\text{s}} = 45 \frac{\text{m}}{\text{s}} + (-5 \frac{\text{m}}{\text{s}^2})t$

OR  $t = 8.55 \text{ s}$

11.139

GIVEN:  $d = 420 \text{ ft}$ ;  $a_t = \text{CONSTANT}$ ;  $v_f = 14 \frac{\text{ft}}{\text{s}}$ ;  
 $v_c = 24 \frac{\text{ft}}{\text{s}}$ ;  $45_{12} = 95 \text{ ft}$   
 FIND:  $a$  AT  $t = 2 \text{ s}$

HAVE UNIFORMLY ACCELERATED MOTION...

$\therefore v_c^2 = v_f^2 + 2a_t \Delta s_{12}$

SUBSTITUTING...  $(24 \frac{\text{ft}}{\text{s}})^2 = (14 \frac{\text{ft}}{\text{s}})^2 + 2a_t (95 \text{ ft})$

OR  $a_t = 2 \frac{\text{ft}}{\text{s}^2}$

ALSO...  $v = v_f + a_t t$

AT  $t = 2 \text{ s}$ :  $v = 14 \frac{\text{ft}}{\text{s}} + (2 \frac{\text{ft}}{\text{s}^2})(2 \text{ s}) = 18 \frac{\text{ft}}{\text{s}}$

NOW...  $a_n = \frac{v^2}{r}$

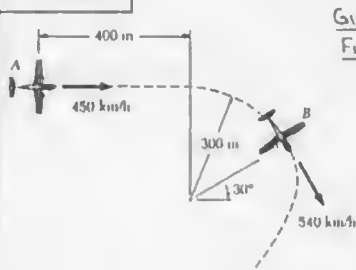
AT  $t = 2 \text{ s}$ :  $a_n = \frac{(18 \frac{\text{ft}}{\text{s}})^2}{210 \text{ ft}} = 1.54286 \frac{\text{ft}}{\text{s}^2}$

FINALLY...  $a^2 = a_t^2 + a_n^2$

AT  $t = 2 \text{ s}$ :  $a^2 = 2^2 + 1.54286^2$

OR  $a = 2.53 \frac{\text{ft}}{\text{s}^2}$

11.140

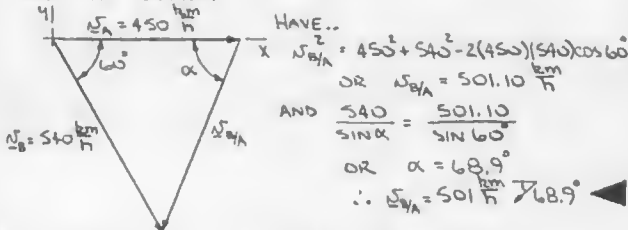


GIVEN:  $a_A = 8 \frac{m}{s^2}$ ,  $(a_B)_t = -3 \frac{m}{s^2}$   
 FIND: (a)  $\vec{v}_{B/A}$   
 (b)  $\vec{a}_{B/A}$

FIRST NOTE..  $v_A = 450 \frac{km}{h}$   $v_B = 540 \frac{km}{h} = 150 \frac{m}{s}$

(a) HAVE..  $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



(b) FIRST NOTE..  $a_A = 8 \frac{m}{s^2} \rightarrow (a_B)_t = 3 \frac{m}{s^2} \nearrow 60^\circ$   
 NOW..  $(a_B)_n = \frac{v_B^2}{r} = \frac{(150 \frac{m}{s})^2}{300 m}$  OR  $(a_B)_n = 75 \frac{m}{s^2} \nearrow 30^\circ$

THEN..  $\vec{a}_B = (a_B)_t + (a_B)_n$   
 $= 3(-\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) + 75(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$   
 $= -(66.452 \frac{m}{s^2}) \hat{i} - (34.902 \frac{m}{s^2}) \hat{j}$

FINALLY..  $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$   
 OR  $\vec{a}_{B/A} = (-66.452 \hat{i} - 34.902 \hat{j}) - (8 \hat{i})$   
 $= -(74.452 \frac{m}{s^2}) \hat{i} - (34.902 \frac{m}{s^2}) \hat{j}$   
 OR  $\vec{a}_{B/A} = 82.2 \frac{m}{s^2} \nearrow 25.1^\circ$

11.141



GIVEN:  $a_{\text{STRAIGHT}} = a_t = \text{CONSTANT}$   
 AT  $t=0$ , CAR ENTERS EXIT RAMP; FOR  $t > 10 s$ ,  
 $v = 20 \frac{m}{s}$ ,  $a = \frac{1}{4} a_t$ .  
 FIND:  $a_{\text{MAX}}$

FIRST NOTE..  $v_{10} = 20 \frac{m}{s} = \frac{88 \frac{ft}{s}}{5}$

WHILE THE CAR IS ON THE STRAIGHT PORTION OF THE HIGHWAY

$$a = a_{\text{STRAIGHT}} = a_t$$

AND FOR THE CIRCULAR EXIT RAMP

$$a = \sqrt{a_t^2 + a_n^2}$$

WHERE  $a_n = \frac{v^2}{r}$

BY OBSERVATION,  $a_{\text{MAX}}$  OCCURS WHEN  $v$  IS MAXIMUM, WHICH IS AT  $t=0$  WHEN THE CAR FIRST ENTERS THE RAMP.

FOR UNIFORMLY DECELERATED MOTION

$$v = v_0 + a_t t$$

(CONTINUED)

11.141 CONTINUED

AND AT  $t = 10 s$ :  $v = \text{CONSTANT} \Rightarrow a = a_n = \frac{v_0^2}{r}$   
 $a = \frac{1}{4} a_t$   
 THEN  $a_t = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_0^2}{r} = \frac{(88 \frac{ft}{s})^2}{560 \frac{ft}}$

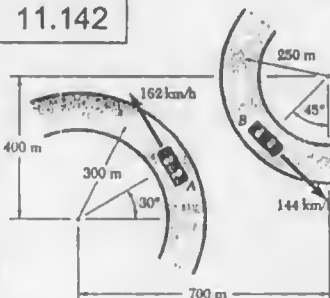
$$\text{OR } a_t = -6.1460 \frac{ft}{s^2}$$

(THE CAR IS DECELERATING; HENCE, THE MINUS SIGN).

THEN AT  $t = 10 s$ :  $\frac{88 \frac{ft}{s}}{5} = v_0 + (-6.1460 \frac{ft}{s^2})(10 s)$   
 OR  $v_0 = 90.793 \frac{ft}{s}$

THEN AT  $t=0$ :  $a_{\text{MAX}} = \sqrt{a_t^2 + (\frac{v_0^2}{r})^2}$   
 $= \sqrt{(-6.1460 \frac{ft}{s^2})^2 + (\frac{(90.793 \frac{ft}{s})^2}{560 \frac{ft}})^2}$   
 OR  $a_{\text{MAX}} = 15.95 \frac{ft}{s^2}$

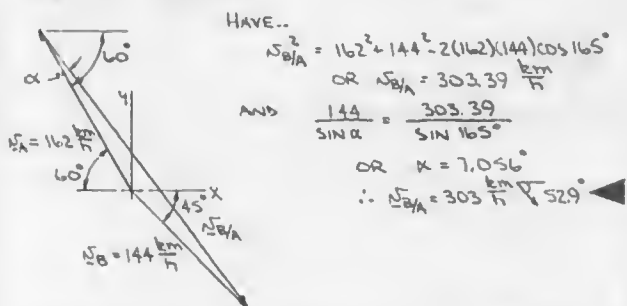
11.142



GIVEN:  $(a_A)_t = -7 \frac{m}{s^2}$   
 $(a_B)_t = 2 \frac{m}{s^2}$   
 FIND: (a)  $\vec{v}_{B/A}$   
 (b)  $\vec{a}_{B/A}$

FIRST NOTE..  $v_A = 162 \frac{km}{h} = 45 \frac{m}{s}$   $v_B = 144 \frac{km}{h} = 40 \frac{m}{s}$   
 (a) HAVE..  $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



(b) FIRST NOTE..  $(a_A)_t = 7 \frac{m}{s^2} \nearrow 60^\circ$   $(a_B)_t = 2 \frac{m}{s^2} \nearrow 45^\circ$

NOW..  $a_n = \frac{v^2}{r}$

THEN..  $(a_A)_n = \frac{(45 \frac{m}{s})^2}{300 m}$   $(a_B)_n = \frac{(40 \frac{m}{s})^2}{250 m}$

OR  $(a_A)_n = 6.75 \frac{m}{s^2} \nearrow 30^\circ$   $(a_B)_n = 6.40 \frac{m}{s^2} \nearrow 45^\circ$

NOTING THAT  $\vec{a} = \vec{a}_t + \vec{a}_n$

HAVE..  $\vec{a}_A = 7(\cos 60^\circ \hat{i} - \sin 60^\circ \hat{j}) + 6.75(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$   
 $= -(2.3457 \frac{m}{s^2}) \hat{i} - (9.4372 \frac{m}{s^2}) \hat{j}$

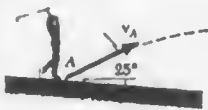
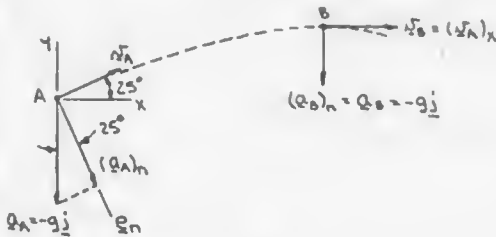
AND  $\vec{a}_B = 2(\cos 45^\circ \hat{i} - \sin 45^\circ \hat{j}) + 6.40(\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$   
 $= (5.9397 \frac{m}{s^2}) \hat{i} + (3.1113 \frac{m}{s^2}) \hat{j}$

FINALLY..  $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

OR  $\vec{a}_{B/A} = (5.9397 \hat{i} + 3.1113 \hat{j}) - (-2.3457 \hat{i} - 9.4372 \hat{j})$   
 $= (8.2854 \frac{m}{s^2}) \hat{i} + (12.5485 \frac{m}{s^2}) \hat{j}$

OR  $\vec{a}_{B/A} = 15.04 \frac{m}{s^2} \nearrow 56.6^\circ$

11.143

GIVEN:  $v_A = 50 \frac{\text{m}}{\text{s}}$ FIND: (a)  $p$  AT POINT A(b)  $p$  AT THE HIGHEST POINT OF THE TRAJECTORY

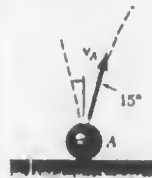
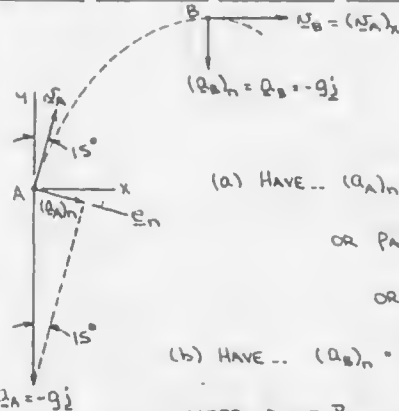
(a) HAVE..  $(a_n)_n = \frac{v_A^2}{p_A}$   
 OR  $p_A = \frac{(50 \frac{\text{m}}{\text{s}})^2}{(9.81 \frac{\text{m}}{\text{s}^2}) \cos 25^\circ}$

(b) HAVE..  $(a_n)_n = \frac{v_B^2}{p_B}$  OR  $p_A = 281 \text{ m}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT  $v_B = (v_A)_x = v_A \cos 25^\circ$   
 THEN..  $p_B = \frac{[(50 \frac{\text{m}}{\text{s}}) \cos 25^\circ]^2}{9.81 \frac{\text{m}}{\text{s}^2}}$

OR  $p_B = 209 \text{ m}$

11.145

GIVEN:  $v_A = 7.5 \frac{\text{ft}}{\text{s}}$ FIND: (a)  $p$  AT POINT A(b)  $p$  AT THE HIGHEST POINT OF THE TRAJECTORY

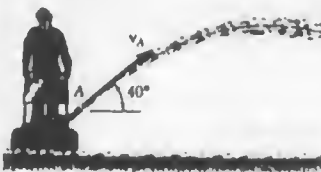
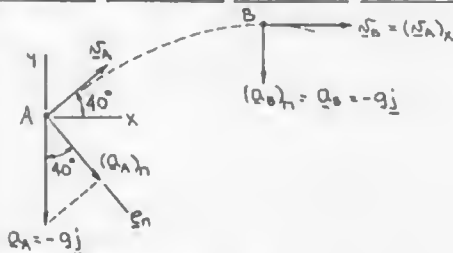
(a) HAVE..  $(a_n)_n = \frac{v_A^2}{p_A}$   
 OR  $p_A = \frac{(7.5 \frac{\text{ft}}{\text{s}})^2}{(32.2 \frac{\text{ft}}{\text{s}^2}) \sin 15^\circ}$   
 OR  $p_A = 6.75 \text{ ft}$

(b) HAVE..  $(a_n)_n = \frac{v_B^2}{p_B}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT  $v_B = (v_A)_x = v_A \sin 15^\circ$   
 THEN..  $p_B = \frac{[(7.5 \frac{\text{ft}}{\text{s}}) \sin 15^\circ]^2}{32.2 \frac{\text{ft}}{\text{s}^2}}$

OR  $p_B = 0.1170 \text{ ft}$

11.144

GIVEN:  $p_A = 8.5 \text{ m}$ FIND: (a)  $v_A$ (b)  $p$  AT THE HIGHEST POINT OF THE TRAJECTORY

(a) HAVE..  $(a_n)_n = \frac{v_A^2}{p_A}$   
 OR  $v_A^2 = (9.81 \cos 40^\circ)(8.5 \text{ m})$   
 $= 63.8766 \frac{\text{m}^2}{\text{s}^2}$  OR  $v_A = 7.99 \frac{\text{m}}{\text{s}} \angle 40^\circ$

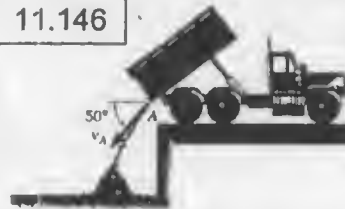
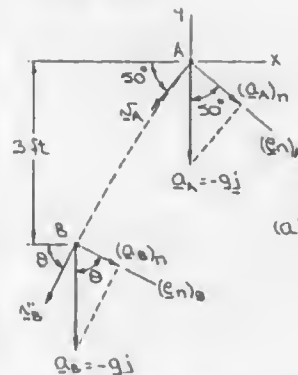
(b) HAVE..  $(a_n)_n = \frac{v_B^2}{p_B}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT  $v_B = (v_A)_x = v_A \cos 40^\circ$   
 THEN..

$p_B = \frac{(63.8766 \frac{\text{m}^2}{\text{s}^2}) \cos^2 40^\circ}{9.81 \frac{\text{m}}{\text{s}^2}}$

OR  $p_B = 3.82 \text{ m}$

11.146

GIVEN:  $v_A = 6 \frac{\text{ft}}{\text{s}}$ FIND: (a)  $p$  AT POINT A(b)  $p$  AT THE POINT ON THE TRAJECTORY 3 ft BELOW A

(a) HAVE..  $(a_n)_n = \frac{v_A^2}{p_A}$   
 OR  $p_A = \frac{(6 \frac{\text{ft}}{\text{s}})^2}{(32.2 \frac{\text{ft}}{\text{s}^2}) \cos 50^\circ}$   
 OR  $p_A = 1.739 \text{ ft}$

(b) HORIZONTAL MOTION (UNIFORM)

$(v_B)_x = (v_A)_x = (6 \frac{\text{ft}}{\text{s}}) \cos 50^\circ = 3.8567 \frac{\text{ft}}{\text{s}}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

HAVE..  $v_y^2 = (v_A)_y^2 - 2g(y - y_A)$  ( $g = 32.2 \frac{\text{ft}}{\text{s}^2}$ )

WHERE  $(v_A)_y = (6 \frac{\text{ft}}{\text{s}}) \sin 50^\circ = 4.5963 \frac{\text{ft}}{\text{s}}$

AT POINT B,  $y = -3 \text{ ft}$ :  $(v_B)_y^2 = (4.5963 \frac{\text{ft}}{\text{s}})^2 - 2(32.2 \frac{\text{ft}}{\text{s}^2})(-3 \text{ ft})$

OR  $(v_B)_y = 14.6399 \frac{\text{ft}}{\text{s}}$

THEN..  $v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{(3.8567)^2 + (14.6399)^2} = 15.1394 \frac{\text{ft}}{\text{s}}$

AND  $\tan \theta = \frac{v_y}{v_x} = \frac{14.6399}{3.8567}$  OR  $\theta = 75.24^\circ$

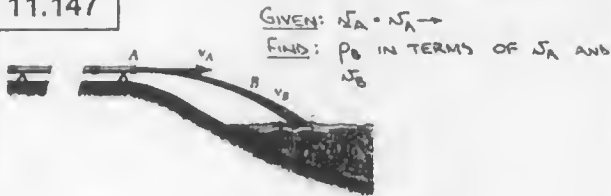
(CONTINUED)



# 11.146 CONTINUED

Now...  $(a_B)_n = \frac{v_B^2}{\rho_B}$   
 OR  $\rho_B = \frac{(15.1394 \frac{ft}{s})^2}{(32.2 \frac{ft}{s^2}) \cos 75.24^\circ}$   
 OR  $\rho_B = 27.9 \text{ ft}$

# 11.147



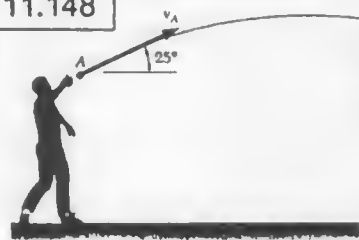
GIVEN:  $v_A = v_B \rightarrow$   
 FIND:  $\rho_B$  IN TERMS OF  $v_A$  AND  $v_B$



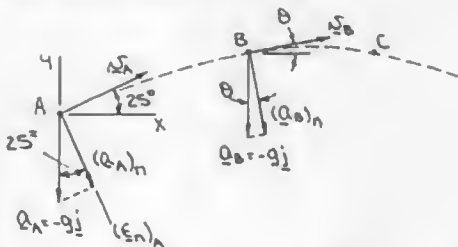
HAVE...  $(a_B)_n = \frac{v_B^2}{\rho_B}$

WHERE  $(a_B)_n = a_B \cos \theta = g \cos \theta$   
 NOTING THAT THE HORIZONTAL MOTION IS UNIFORM,  
 HAVE...  $(v_B)_x = v_A$   
 WHERE  $(v_B)_x = v_B \cos \theta$   
 $\therefore \cos \theta = \frac{v_A}{v_B}$   
 THEN --  $\rho_B = \frac{v_B^2}{g (\frac{v_A}{v_B})}$  OR  $\rho_B = \frac{v_B^3}{g v_A}$

# 11.148



GIVEN:  $v_A = 20 \frac{m}{s}$   
 FIND:  $\rho$  AT THOSE POINTS WHERE  $\rho = \frac{2}{3} \rho_A$



ASSUME THAT POINTS B AND C ARE THE POINTS OF INTEREST, WHERE  $y_B = y_C$  AND  $v_B = v_C$ . NOW...  
 $(a_B)_n = \frac{v_B^2}{\rho_B}$   
 OR  $\rho_B = \frac{v_B^2}{g \cos 25^\circ}$   
 THEN  $\rho_B = \frac{2}{3} \rho_A = \frac{2}{3} \frac{v_A^2}{g \cos 25^\circ}$

(CONTINUED)

# 11.148 CONTINUED

HAVE  $(a_B)_n = \frac{v_B^2}{\rho_B}$  WHERE  $(a_B)_n = g \cos \theta$   
 SO THAT  $\frac{2}{3} \frac{v_A^2}{g \cos 25^\circ} = \frac{v_B^2}{g \cos \theta}$   
 OR  $v_B^2 = \frac{2}{3} \frac{\cos \theta}{\cos 25^\circ} v_A^2$  (1)

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM, HAVE...  
 $(v_A)_x = (v_B)_x$   
 WHERE  $(v_A)_x = v_A \cos 25^\circ$   $(v_B)_x = v_B \cos \theta$   
 THEN  $v_A \cos 25^\circ = v_B \cos \theta$   
 OR  $\cos \theta = \frac{v_A}{v_B} \cos 25^\circ$

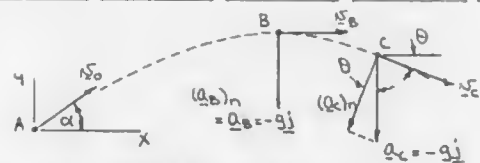
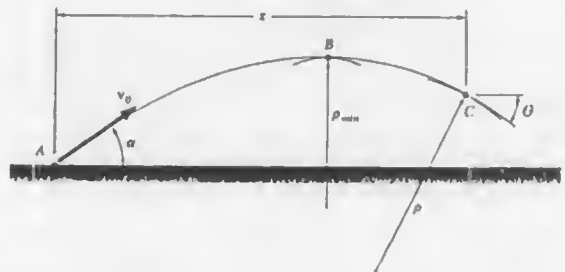
SUBSTITUTING FOR  $\cos \theta$  IN EQ. (1), HAVE..

$v_B^2 = \frac{2}{3} \left( \frac{v_A \cos 25^\circ}{v_B \cos \theta} \right)^2 \frac{v_B^2}{\cos 25^\circ}$   
 OR  $v_B^3 = \frac{2}{3} v_A^3 = \frac{2}{3} (20 \frac{m}{s})^3$   
 OR  $v_B = v_C = 18.17 \frac{m}{s}$

# 11.149

GIVEN: THE INITIAL VELOCITY  $v_0$  AND THE TRAJECTORY OF THE PROJECTILE AS SHOWN

SHOW: (a)  $\rho_B = \rho_{min}$ , WHERE  $y_B = y_{max}$   
 (b)  $\rho_C = \rho_{min} / \cos^2 \theta$



FOR THE ARBITRARY POINT C HAVE..

$(a_C)_n = \frac{v_C^2}{\rho_C}$   
 OR  $\rho_C = \frac{v_C^2}{g \cos \theta}$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM, HAVE..

WHERE  $(v_A)_x = (v_C)_x$   
 $v_0 \cos \alpha = v_C \cos \theta$   
 THEN  $v_C = \frac{\cos \alpha}{\cos \theta} v_0$

SO THAT  $\rho_C = \frac{1}{g \cos \theta} \left( \frac{\cos \alpha}{\cos \theta} v_0 \right)^2 = \frac{v_0^2 \cos^2 \alpha}{g \cos^3 \theta}$

(a) IN THE EXPRESSION FOR  $\rho_C$ ,  $v_0$ ,  $\alpha$ , AND  $g$  ARE CONSTANTS, SO THAT  $\rho_C$  IS MINIMUM WHERE  $\cos \theta$  IS MAXIMUM. BY OBSERVATION, THIS OCCURS AT POINT B WHERE  $\theta = 0$ .

$\therefore \rho_{min} = \rho_B = \frac{v_0^2 \cos^2 \alpha}{g}$  Q.E.D.

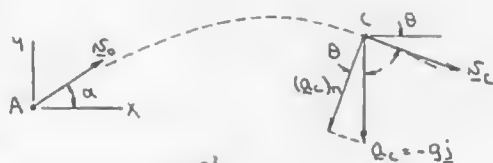
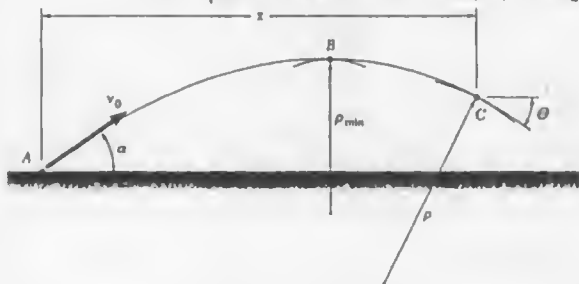
(b)  $\rho_C = \frac{1}{\cos^3 \theta} \left( \frac{v_0^2 \cos^2 \alpha}{g} \right)$

OR  $\rho_C = \frac{\rho_{min}}{\cos^3 \theta}$  Q.E.D.

11.150

GIVEN: THE INITIAL VELOCITY  $v_0$  AND THE TRAJECTORY OF THE PROJECTILE AS SHOWN

FIND:  $P_c$  IN TERMS OF  $x, v_0, \alpha$ , AND  $g$



HAVE...  $(a_c)_n = \frac{v_c^2}{R}$   
OR  $R = \frac{v_c^2}{g \cos \theta}$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM, HAVE  $(v_A)_x = (v_c)_x$   $x = v_0 \cos \alpha \cdot t = (v_0 \cos \alpha) t$   
WHERE  $(v_A)_x = v_0 \cos \alpha$   $(v_c)_x = v_c \cos \theta$   
THEN  $v_0 \cos \alpha = v_c \cos \theta$  AND  $(v_c)_x = v_0 \cos \alpha$  (1)  
OR  $\cos \theta = \frac{v_0}{v_c} \cos \alpha$

SO THAT  $R = \frac{v_c^2}{g v_0 \cos \alpha}$

FOR THE UNIFORMLY ACCELERATED VERTICAL MOTION HAVE

$(v_c)_y = (v_0)_y - gt = v_0 \sin \alpha - gt$

FROM ABOVE...  $x = (v_0 \cos \alpha) t$  OR  $t = \frac{x}{v_0 \cos \alpha}$

THEN...  $(v_c)_y = v_0 \sin \alpha - g \frac{x}{v_0 \cos \alpha}$  (2)

NOW...  $v_c^2 = (v_c)_x^2 + (v_c)_y^2$

SUBSTITUTING FOR  $(v_c)_x$  [Eq. (1)] AND  $(v_c)_y$  [Eq. (2)]

$v_c^2 = (v_0 \cos \alpha)^2 + (v_0 \sin \alpha - g \frac{x}{v_0 \cos \alpha})^2$   
 $= v_0^2 (1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2 x^2}{v_0^4 \cos^2 \alpha})$

OR  $v_c^2 = v_0^2 (1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2 x^2}{v_0^4 \cos^2 \alpha})^{1/2}$

FINALLY, SUBSTITUTING INTO THE EXPRESSION FOR  $R$ , OBTAIN--

$P = \frac{v_0^2}{g \cos \alpha} (1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2 x^2}{v_0^4 \cos^2 \alpha})^{3/2}$

\* 11.151

GIVEN:  $\mathbf{r} = (Rt \cos \omega t) \mathbf{i} + ct \mathbf{j} + (Rt \sin \omega t) \mathbf{k}$

FIND:  $P$  AT  $t=0$

HAVE...  $\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos \omega t - \omega t \sin \omega t) \mathbf{i} + c \mathbf{j} + R(\sin \omega t + \omega t \cos \omega t) \mathbf{k}$

AND...  $\mathbf{a} = \frac{d\mathbf{v}}{dt} = R(-\omega \sin \omega t - \omega \sin \omega t - \omega^2 t \cos \omega t) \mathbf{i} + R(\omega \cos \omega t + \omega \cos \omega t - \omega^2 t \sin \omega t) \mathbf{k}$

(CONTINUED)

11.151 CONTINUED

OR  $\mathbf{a} = \omega R [(-2 \sin \omega t + \omega t \cos \omega t) \mathbf{i} + (2 \cos \omega t - \omega t \sin \omega t) \mathbf{k}]$

NOW...  $v^2 = R^2 (\cos^2 \omega t - \omega t \sin \omega t)^2 + c^2 + R^2 (\sin^2 \omega t + \omega t \cos \omega t)^2$   
 $= R^2 (1 + \omega^2 t^2) + c^2$

THEN  $v = [R^2 (1 + \omega^2 t^2) + c^2]^{1/2}$

AND  $\frac{dv}{dt} = \frac{R^2 \omega t}{[R^2 (1 + \omega^2 t^2) + c^2]^{1/2}}$

NOW...  $a^2 = a_t^2 + a_n^2 = (\frac{dv}{dt})^2 + (\frac{v^2}{P})^2$

AT  $t=0$ :  $\frac{dv}{dt} = 0$

$\mathbf{a} = \omega R (2 \mathbf{k})$  OR  $a = 2\omega R$

$v^2 = R^2 + c^2$

THEN, WITH  $\frac{dv}{dt} = 0$ , HAVE...  $a = \frac{v^2}{P}$

OR  $2\omega R = \frac{R^2 + c^2}{P}$

OR  $P = \frac{R^2 + c^2}{2\omega R}$

\* 11.152

GIVEN:  $\mathbf{r} = (At \cos t) \mathbf{i} + (A\sqrt{t^2+1}) \mathbf{j} + (Bt \sin t) \mathbf{k}$ ,  $r=3t$ ,  $t=3$

$A=3, B=1$

FIND:  $P$  AT  $t=0$

WITH  $A=3, B=1$  HAVE...

$\mathbf{r} = (3t \cos t) \mathbf{i} + (3\sqrt{t^2+1}) \mathbf{j} + (t \sin t) \mathbf{k}$

NOW...  $\mathbf{v} = \frac{d\mathbf{r}}{dt} = (3 \cos t - t \sin t) \mathbf{i} + (\frac{3t}{\sqrt{t^2+1}}) \mathbf{j} + (\sin t + t \cos t) \mathbf{k}$

AND...  $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t \cos t) \mathbf{i} + 3[\frac{t^2+1-t}{t^2+1}] \mathbf{j} + (\cos t + \cos t - t \sin t) \mathbf{k}$   
 $= -3(2 \sin t + t \cos t) \mathbf{i} + 3[\frac{1}{(t^2+1)^{3/2}}] \mathbf{j} + (2 \cos t - t \sin t) \mathbf{k}$

THEN...  $v^2 = 9(\cos t - t \sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t \cos t)^2$

EXPANDING AND SIMPLIFYING YIELDS...

$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^2 + t) \sin 2t$

THEN  $v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^2 + t) \sin 2t]^{1/2}$

AND

$\frac{dv}{dt} = \frac{4t^3 + 38t + 8t \cos 2t \sin t + 4t^3 \sin^2 t + 2t^4 \sin t \cos t - 8(2t+1) \sin 2t + 2(t^2+t) \cos 2t}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^2 + t) \sin 2t]^{1/2}}$

NOW...  $a^2 = a_t^2 + a_n^2 = (\frac{dv}{dt})^2 + (\frac{v^2}{P})^2$

AT  $t=0$ :  $\mathbf{a} = 3 \mathbf{j} + 2 \mathbf{k}$  OR  $a = \sqrt{13} \frac{ft}{s^2}$

$\frac{dv}{dt} = 0$

$v^2 = 9(\frac{t^2}{3})^2$

THEN, WITH  $\frac{dv}{dt} = 0$ , HAVE...  $a = \frac{v^2}{P}$

OR  $P = \frac{9 \frac{ft^2}{s^2}}{\sqrt{13} \frac{ft}{s^2}}$

OR  $P = 2.50 \text{ ft}$

11.153

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $g_{\text{VENUS}} = 8.53 \frac{\text{m}}{\text{s}^2}$   
 $R_{\text{VENUS}} = 6161 \text{ km}$

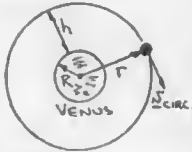
FIND:  $v_{\text{circ}}$  WHEN  $h = 160 \text{ km}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $v_{\text{circ}} = R \sqrt{\frac{g}{r}}$  WHERE  $r = R + h$   
 FOR THE GIVEN DATA..

$$v_{\text{circ}} = 6161 \text{ km} \sqrt{\frac{8.53 \text{ m/s}^2}{(6161 + 160) \times 10^3 \text{ m}}} = \frac{3600 \text{ s}}{1 \text{ h}}$$

$$\text{OR } v_{\text{circ}} = 25.8 \times 10^3 \frac{\text{km}}{\text{h}}$$



11.154

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $g_{\text{MARS}} = 3.83 \frac{\text{m}}{\text{s}^2}$   
 $R_{\text{MARS}} = 3332 \text{ km}$

FIND:  $v_{\text{circ}}$  WHEN  $h = 160 \text{ km}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $v_{\text{circ}} = R \sqrt{\frac{g}{r}}$  WHERE  $r = R + h$   
 FOR THE GIVEN DATA..

$$v_{\text{circ}} = 3332 \text{ km} \sqrt{\frac{3.83 \text{ m/s}^2}{(3332 + 160) \times 10^3 \text{ m}}} = \frac{3600 \text{ s}}{1 \text{ h}}$$

$$\text{OR } v_{\text{circ}} = 12.56 \times 10^3 \frac{\text{km}}{\text{h}}$$



11.155

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $g_{\text{JUPITER}} = 26.0 \frac{\text{m}}{\text{s}^2}$   
 $R_{\text{JUPITER}} = 69893 \text{ km}$

FIND:  $v_{\text{circ}}$  WHEN  $h = 160 \text{ km}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $v_{\text{circ}} = R \sqrt{\frac{g}{r}}$  WHERE  $r = R + h$   
 FOR THE GIVEN DATA..

$$v_{\text{circ}} = 69893 \text{ km} \sqrt{\frac{26.0 \text{ m/s}^2}{(69893 + 160) \times 10^3 \text{ m}}} = \frac{3600 \text{ s}}{1 \text{ h}}$$

$$\text{OR } v_{\text{circ}} = 153.3 \times 10^3 \frac{\text{km}}{\text{h}}$$



11.156

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $d_{\text{sun}} = 864,000 \text{ mi}$   
 $g_{\text{sun}} = 900 \frac{\text{ft}}{\text{s}^2}$   
 $[(v_{\text{mean}})_{\text{orbit}}]_{\text{EARTH}} = 66,600 \frac{\text{mi}}{\text{h}}$

FIND:  $r_{\text{EARTH}}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $r = g \left( \frac{R}{v^2} \right)^2$  WHERE  $R = \frac{1}{2} d$   
 FOR THE GIVEN DATA..

$$r_{\text{EARTH}} = (900 \frac{\text{ft}}{\text{s}^2}) \left( \frac{\frac{1}{2} \times 864,000 \text{ mi}}{66,600 \text{ mi/h}} \right)^2 = \frac{1 \text{ mi}}{5280 \text{ ft}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)^2$$

$$\text{OR } r_{\text{EARTH}} = 92.9 \times 10^6 \text{ mi}$$



11.157

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $d_{\text{sun}} = 864,000 \text{ mi}$   
 $g_{\text{sun}} = 900 \frac{\text{ft}}{\text{s}^2}$   
 $[(v_{\text{mean}})_{\text{orbit}}]_{\text{SATURN}} = 21,580 \frac{\text{mi}}{\text{h}}$

FIND:  $r_{\text{SATURN}}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $r = g \left( \frac{R}{v^2} \right)^2$  WHERE  $R = \frac{1}{2} d$   
 FOR THE GIVEN DATA..

$$r_{\text{SATURN}} = (900 \frac{\text{ft}}{\text{s}^2}) \left( \frac{\frac{1}{2} \times 864,000 \text{ mi}}{21,580 \text{ mi/h}} \right)^2 = \frac{1 \text{ mi}}{5280 \text{ ft}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)^2$$

$$\text{OR } r_{\text{SATURN}} = 885 \times 10^6 \text{ mi}$$



11.158

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $R_{\text{EARTH}} = 6370 \text{ km}$   
 $h = 590 \text{ km}$

FIND:  $t_{\text{orbit}}$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $v = R \sqrt{\frac{g}{r}}$  WHERE  $r = R + h$

THE CIRCUMFERENCE S OF THE CIRCULAR ORBIT IS EQUAL TO

$$S = 2\pi r$$

ASSUMING THAT THE SPEED OF THE TELESCOPE IS CONSTANT, HAVE

$$S = v t_{\text{orbit}}$$

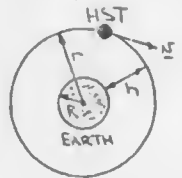
SUBSTITUTING FOR S AND v..

$$2\pi r = R \sqrt{\frac{g}{r}} t_{\text{orbit}}$$

$$\text{OR } t_{\text{orbit}} = \frac{2\pi}{R} \frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370 \text{ km}} \frac{[(6370 + 590) \text{ km}]^{3/2}}{[9.81 \times 10^{-3} \text{ m/s}^2]^{1/2}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$$\text{OR } t_{\text{orbit}} = 1.606 \text{ h}$$



11.159

GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $R_{\text{MARS}} = 2071 \text{ mi}$ ;  $h = 180 \text{ mi}$   
 $(t_{\text{orbit}})_2 = 1.1 (t_{\text{orbit}})_1$

FIND:  $h_2$

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$

OR  $v = R \sqrt{\frac{g}{r}}$  WHERE  $r = R + h$

THE CIRCUMFERENCE S OF A CIRCULAR ORBIT IS EQUAL TO

$$S = 2\pi r$$

ASSUMING THAT THE SPEED OF THE SATELLITE IN EACH ORBIT IS CONSTANT, HAVE

$$S = v t_{\text{orbit}}$$

SUBSTITUTING FOR S AND v..

$$2\pi r = R \sqrt{\frac{g}{r}} t_{\text{orbit}}$$

$$\text{OR } t_{\text{orbit}} = \frac{2\pi}{R} \frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R} \frac{(R + h)^{3/2}}{\sqrt{g}}$$

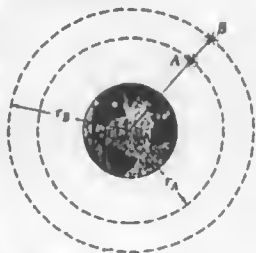
(CONTINUED)



# 11.159 CONTINUED

Now ..  $(t_{\text{orbit}})_2 = 1.1 (t_{\text{orbit}})_1$   
 OR  $\frac{2\pi (R+h_2)^{3/2}}{R \sqrt{g}} = 1.1 \frac{2\pi (R+h_1)^{3/2}}{R \sqrt{g}}$   
 OR  $h_2 = (1.1)^{2/3} (R+h_1) - R$   
 $= (1.1)^{2/3} (2071+180) \text{ mi} - (2071 \text{ mi})$   
 OR  $h_2 = 328 \text{ mi}$

# 11.160



GIVEN:  $a_n = g \frac{R^2}{r^2}$ ;  $h_A = 120 \text{ mi}$ ,  
 $h_B = 200 \text{ mi}$ ;  $R_{\text{EARTH}} = 3960 \text{ mi}$ ;  
 AT  $t=0$ , A AND B  
 ALIGNED AS SHOWN  
 FIND:  $t$  WHEN A AND B ARE  
 NEXT RADIIALLY ALIGNED

HAVE..  $a_n = g \frac{R^2}{r^2}$  AND  $a_n = \frac{v^2}{r}$   
 THEN  $g \frac{R^2}{r^2} = \frac{v^2}{r}$  OR  $v = R \sqrt{\frac{g}{r}}$  WHERE  $r = R+h$

THE CIRCUMFERENCE  $S$  OF A CIRCULAR ORBIT IS  
 EQUAL TO  $S = 2\pi r$   
 ASSUMING THAT THE SPEEDS OF THE SATELLITES  
 ARE CONSTANT, HAVE

$S = v t_{\text{orbit}}$   
 SUBSTITUTING FOR  $S$  AND  $v$ ..  
 $2\pi r = R \sqrt{\frac{g}{r}} t_{\text{orbit}}$   
 OR  $t_{\text{orbit}} = \frac{2\pi r^{3/2}}{R \sqrt{g}} = \frac{2\pi (R+h)^{3/2}}{R \sqrt{g}}$

NOW  $h_B > h_A \Rightarrow (t_{\text{orbit}})_B > (t_{\text{orbit}})_A$   
 NEXT LET TIME  $t_{\text{TOTAL}}$  BE THE TIME AT WHICH THE  
 SATELLITES ARE NEXT RADIIALLY ALIGNED. THEN, IF  
 IN TIME  $t_{\text{TOTAL}}$  SATELLITE B COMPLETES  $N$   
 ORBITS, SATELLITE A MUST COMPLETE  $(N+1)$  ORBITS.  
 THUS,

$t_{\text{TOTAL}} = N (t_{\text{orbit}})_B = (N+1) (t_{\text{orbit}})_A$   
 OR  $N \left[ \frac{2\pi (R+h_B)^{3/2}}{R \sqrt{g}} \right] = (N+1) \left[ \frac{2\pi (R+h_A)^{3/2}}{R \sqrt{g}} \right]$   
 OR  $N = \frac{(R+h_B)^{3/2}}{(R+h_B)^{3/2} - (R+h_A)^{3/2}} = \frac{1}{\left( \frac{R+h_B}{R+h_A} \right)^{3/2} - 1}$   
 $= \frac{1}{\left( \frac{3960+200}{3960+120} \right)^{3/2} - 1} = 33.835 \text{ ORBITS}$

THEN  $t_{\text{TOTAL}} = N (t_{\text{orbit}})_B = N \frac{2\pi (R+h_B)^{3/2}}{R \sqrt{g}}$   
 $= 33.835 \frac{2\pi}{3960 \text{ mi}} \left[ \frac{(3960+200) \text{ mi}}{32.2 \frac{\text{ft}}{\text{s}^2} \times \frac{1 \text{ mi}}{5280 \text{ ft}}} \right]^{3/2}$   
 $\times \frac{1 \text{ h}}{3600 \text{ s}}$   
 OR  $t_{\text{TOTAL}} = 51.2 \text{ h}$

## ALTERNATIVE SOLUTION

FROM ABOVE HAVE  $(t_{\text{orbit}})_B > (t_{\text{orbit}})_A$   
 THUS, WHEN THE SATELLITES ARE NEXT RADIIALLY  
 ALIGNED, THE ANGLES  $\theta_A$  AND  $\theta_B$  SWEEP OUT  
 (CONTINUED)

# 11.160 CONTINUED

BY RADIAL LINES DRAWN TO THE SATELLITES MUST  
 DIFFER BY  $2\pi$ . THAT IS,

$$\theta_A = \theta_B + 2\pi$$

FOR A CIRCULAR ORBIT..  $S = r\theta$

FROM ABOVE..  $S = vt$  AND  $v = R \sqrt{\frac{g}{r}}$

THEN  $\theta = \frac{S}{r} = \frac{vt}{r} = \frac{1}{r} (R \sqrt{\frac{g}{r}}) t = \frac{R \sqrt{g}}{r^{3/2}} t = \frac{R \sqrt{g}}{(R+h)^{3/2}} t$

AT TIME  $t_{\text{TOTAL}}$ :  $\frac{R \sqrt{g}}{(R+h_A)^{3/2}} t_{\text{TOTAL}} = \frac{R \sqrt{g}}{(R+h_B)^{3/2}} t_{\text{TOTAL}} + 2\pi$

OR  $t_{\text{TOTAL}} = \frac{2\pi}{R \sqrt{g} \left[ \frac{1}{(R+h_A)^{3/2}} - \frac{1}{(R+h_B)^{3/2}} \right]}$   
 $= \frac{2\pi}{(3960 \text{ mi}) \left( 32.2 \frac{\text{ft}}{\text{s}^2} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \right)^{3/2} \left[ \frac{1}{(3960+120) \text{ mi}} - \frac{1}{(3960+200) \text{ mi}} \right]}$   
 $= \frac{1 \text{ h}}{3600 \text{ s}}$  OR  $t_{\text{TOTAL}} = 51.2 \text{ h}$

# 11.161

GIVEN:  $r = 3(2-e^{-t})$ ,  $\theta = 4(t-2e^{-t})$   $r$  in m,  
 $t$  in s,  $\theta$  in RAD

FIND: (a)  $\vec{v}$  AND  $\vec{a}$  AT  $t=0$

(b)  $\vec{v}$  AND  $\vec{a}$  AS  $t \rightarrow \infty$ ; THE  
 FINAL PATH OF THE PARTICLE

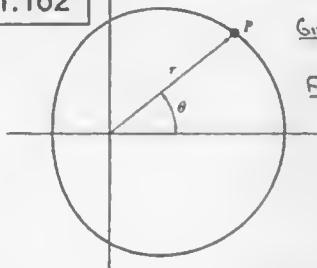
HAVE..  $r = 3(2-e^{-t})$   $\theta = 4(t-2e^{-t})$   
 THEN  $\dot{r} = 3e^{-t}$   $\dot{\theta} = 4(1-2e^{-t})$   
 AND  $\ddot{r} = -3e^{-t}$   $\ddot{\theta} = 8e^{-t}$   
 NOW..  $\vec{v} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta = 3e^{-t} \vec{e}_r + 12(2-e^{-t})(1-2e^{-t}) \vec{e}_\theta$   
 AND  $\vec{a} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \vec{e}_\theta$   
 $= [-3e^{-t} - 48(2-e^{-t})(1-2e^{-t})^2] \vec{e}_r$   
 $+ [24(2-e^{-t})e^{-t} + 24e^{-t}(1-2e^{-t})] \vec{e}_\theta$

(a) AT  $t=0$ :  $\vec{v} = 3 \vec{e}_r + 12(2-1)(1-2) \vec{e}_\theta$   
 OR  $\vec{v} = (3 \frac{\text{m}}{\text{s}}) \vec{e}_r - (12 \frac{\text{m}}{\text{s}}) \vec{e}_\theta$   
 $\vec{a} = [-3 - 48(2-1)(1-2)^2] \vec{e}_r$   
 $+ [24(2-1) + 24(1-2)] \vec{e}_\theta$   
 OR  $\vec{a} = -(51 \frac{\text{m}}{\text{s}^2}) \vec{e}_r$

(b) AS  $t \rightarrow \infty$ :  $\vec{v} = (0) \vec{e}_r + 12(2-0)(1-0) \vec{e}_\theta$   
 OR  $\vec{v} = (24 \frac{\text{m}}{\text{s}}) \vec{e}_\theta$   
 $\vec{a} = [0 - 48(2-0)(1-0)^2] \vec{e}_r + (0+0) \vec{e}_\theta$   
 OR  $\vec{a} = -(96 \frac{\text{m}}{\text{s}^2}) \vec{e}_r$

AS  $t \rightarrow \infty$ ,  $r \rightarrow 6 \text{ m}$ .. A CONSTANT. THUS, THE  
 FINAL PATH IS A CIRCLE OF RADIUS  $6 \text{ m}$ .  
 NOTE THAT THE SPEED OF THE PARTICLE IS  
 CONSTANT ( $24 \frac{\text{m}}{\text{s}}$ ); THUS, THE TRANSVERSE  
 (TANGENTIAL) COMPONENT OF THE ACCELERATION IS  
 ZERO.

11.162



GIVEN:  $r = b(2 + \cos \pi t)$ ,  
 $\theta = \pi t$   $t = s$ ,  $\theta$  - RAD  
 FIND: (a)  $\vec{N}$  AND  $\vec{Q}$  AT  
 $t = 2s$   
 (b)  $\theta$  FOR WHICH  
 $N = N_{\max}$

HAVE...  $r = b(2 + \cos \pi t)$   $\theta = \pi t$   
 THEN  $\dot{r} = -\pi b \sin \pi t$   $\dot{\theta} = \pi$   
 AND  $\ddot{r} = -\pi^2 b \cos \pi t$   $\ddot{\theta} = 0$   
 NOW...  $\vec{N} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta = -\pi b \sin \pi t \vec{e}_r - \pi b(2 + \cos \pi t) \vec{e}_\theta$   
 AND  $\vec{Q} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \vec{e}_\theta$   
 $= [-\pi^2 b \cos \pi t - \pi^2 b(2 + \cos \pi t)] \vec{e}_r$   
 $+ (0 - 2\pi^2 b \sin \pi t) \vec{e}_\theta$

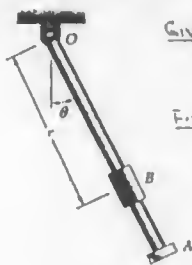
$$= -2\pi^2 b [(1 + \cos \pi t) \vec{e}_r + (\sin \pi t) \vec{e}_\theta]$$

(a) AT  $t = 2s$ :  $\vec{N} = -10 \vec{e}_r + \pi b(2 + 1) \vec{e}_\theta$   
 OR  $N = 3\pi b \vec{e}_\theta$   
 $\vec{Q} = -2\pi^2 b [(1 + 1) \vec{e}_r + (0) \vec{e}_\theta]$   
 OR  $\vec{Q} = -4\pi^2 b \vec{e}_r$

(b) HAVE...  $N = \pi b \sqrt{(-\sin \pi t)^2 + (2 + \cos \pi t)^2}$   
 $= \pi b \sqrt{5 + 4 \cos \pi t}$   $\theta = \pi t$   
 $= \pi b \sqrt{5 + 4 \cos \theta}$

BY OBSERVATION,  $N = N_{\max}$  WHEN  $\cos \theta = 1$   
 OR  $\theta = 2n\pi$ ,  $N = 0, 1, 2, \dots$

11.163

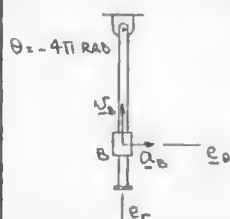


GIVEN:  $\theta = \pi(4t^2 - 8t)$ ,  
 $r = 10 + 6 \sin \pi t$   $\theta$  - RAD,  
 $t = s$ ,  $r$  - IN.  
 FIND: (a)  $\vec{N}$  AT  $t = 1s$   
 (b)  $\vec{Q}$  AT  $t = 1s$   
 (c)  $\vec{Q}_{B/OA}$

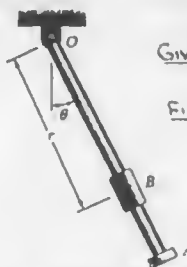
HAVE...  $r = 10 + 6 \sin \pi t$   $\theta = \pi(4t^2 - 8t)$   
 THEN  $\dot{r} = 6\pi \cos \pi t$   $\dot{\theta} = 8\pi(t - 1)$   
 AND  $\ddot{r} = -6\pi^2 \sin \pi t$   $\ddot{\theta} = 8\pi$   
 AT  $t = 1s$ :  $r = 10$  IN.  $\theta = -4\pi$  RAD  
 $\dot{r} = -6\pi$  IN/s  $\dot{\theta} = 0$   
 $\ddot{r} = 0$   $\ddot{\theta} = 8\pi$  RAD/s<sup>2</sup>

(a) HAVE...  $\vec{N} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta$   
 SO THAT  $\vec{N} = -6\pi \frac{\text{IN}}{s} \vec{e}_r$   
 (b) HAVE...  $\vec{Q} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \vec{e}_\theta$   
 $= (10)(8\pi) \vec{e}_\theta$   
 OR  $\vec{Q} = (80\pi \frac{\text{IN}}{s^2}) \vec{e}_\theta$

(c) HAVE...  $\vec{Q}_{B/OA} = \ddot{r}$   
 SO THAT  $\vec{Q}_{B/OA} = 0$



11.164



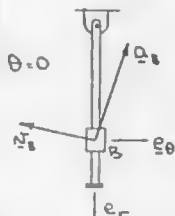
GIVEN:  $r = \frac{25}{t+4}$ ,  $\theta = \frac{2}{\pi} \sin \pi t$   
 $\theta$  - RAD,  $t = s$ ,  $r$  - IN.  
 FIND: (a)  $\vec{N}$  AT  $t = 1s$   
 (b)  $\vec{Q}$  AT  $t = 1s$   
 (c)  $\vec{Q}_{B/OA}$  AT  $t = 1s$

HAVE...  $r = \frac{25}{t+4}$   $\theta = \frac{2}{\pi} \sin \pi t$   
 THEN  $\dot{r} = -\frac{25}{(t+4)^2}$   $\dot{\theta} = 2 \cos \pi t$   
 AND  $\ddot{r} = \frac{50}{(t+4)^3}$   $\ddot{\theta} = -2\pi \sin \pi t$

AT  $t = 1s$ :  $r = 5$  IN.  $\theta = 0$   
 $\dot{r} = -1$  IN/s  $\dot{\theta} = 2$  RAD/s  
 $\ddot{r} = 0.4$  IN/s<sup>2</sup>  $\ddot{\theta} = 0$

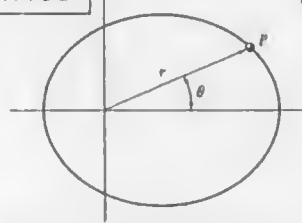
(a) HAVE...  $\vec{N} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta = (-1) \vec{e}_r + (5)(2) \vec{e}_\theta$   
 OR  $\vec{N} = -1 \frac{\text{IN}}{s} \vec{e}_r - (10 \frac{\text{IN}}{s}) \vec{e}_\theta$

(b) HAVE...  $\vec{Q} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \vec{e}_\theta$   
 $= [0.4 - (5)(2^2)] \vec{e}_r + [0 + 2(-1)(2)] \vec{e}_\theta$   
 OR  $\vec{Q} = -19.6 \frac{\text{IN}}{s^2} \vec{e}_r - (4 \frac{\text{IN}}{s}) \vec{e}_\theta$



(c) HAVE  $\vec{Q}_{B/OA} = \ddot{r}$   
 SO THAT  $\vec{Q}_{B/OA} = (0.4 \frac{\text{IN}}{s^2}) \vec{e}_r$

11.165



GIVEN:  $r = \frac{2}{2 - \cos \pi t}$ ,  $\theta = \pi t$   
 $r$  - M,  $t = s$ ,  $\theta$  - RAD  
 FIND: (a)  $\vec{N}$  AND  $\vec{Q}$  AT  
 $t = 0$   
 (b)  $\vec{N}$  AND  $\vec{Q}$  AT  
 $t = 0.5s$

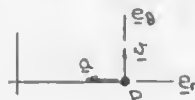
HAVE...  $r = \frac{2}{2 - \cos \pi t}$   $\theta = \pi t$   
 THEN  $\dot{r} = \frac{-2\pi \sin \pi t}{(2 - \cos \pi t)^2}$   $\dot{\theta} = \pi$   
 AND  $\ddot{r} = -2\pi \frac{\pi \cos \pi t (2 - \cos \pi t) - \sin \pi t (2\pi \sin \pi t)}{(2 - \cos \pi t)^3}$   $\ddot{\theta} = 0$   
 $= -2\pi^2 \frac{2 \cos \pi t - 1 - \sin^2 \pi t}{(2 - \cos \pi t)^3}$

(a) AT  $t = 0$ :  $r = 2$  M  $\theta = 0$   
 $\dot{r} = 0$   $\dot{\theta} = \pi$  RAD/s  
 $\ddot{r} = -2\pi^2 \frac{M}{s^2}$   $\ddot{\theta} = 0$

NOW...  $\vec{N} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta = (2)(\pi) \vec{e}_\theta$   
 OR  $\vec{N} = (2\pi \frac{M}{s}) \vec{e}_\theta$

AND...  $\vec{Q} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \vec{e}_\theta$   
 $= [-2\pi^2 - (2)(\pi^2)] \vec{e}_r$   
 OR  $\vec{Q} = -(4\pi^2 \frac{M}{s^2}) \vec{e}_r$

$\theta = 0$



(CONTINUED)

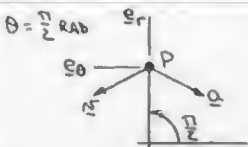
# 11.165 CONTINUED

(b) At  $t=0.5$ :  $r=1$  m  $\theta = \frac{\pi}{2}$  rad  
 $\dot{r} = -\frac{2\pi}{(2)^2} = -\frac{\pi}{2} \frac{\text{m}}{\text{s}}$   $\dot{\theta} = \pi \frac{\text{rad}}{\text{s}}$   
 $\ddot{r} = -2\pi^2 \frac{-1-1}{(2)^2} = \frac{\pi^2}{2} \frac{\text{m}}{\text{s}^2}$   $\ddot{\theta} = 0$

Now..  $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = (-\frac{\pi}{2})\underline{e}_r + (1)(\pi)\underline{e}_\theta$   
 OR  $\underline{v} = -(\frac{\pi}{2}\frac{\text{m}}{\text{s}})\underline{e}_r + (\pi\frac{\text{m}}{\text{s}})\underline{e}_\theta$

And..  $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$   
 $= [\frac{\pi^2}{2} - (1)(\pi)^2]\underline{e}_r + [2(-\frac{\pi}{2})(\pi)]\underline{e}_\theta$

OR  $\underline{a} = (-\frac{\pi^2}{2}\frac{\text{m}}{\text{s}^2})\underline{e}_r - \pi^2\frac{\text{m}}{\text{s}^2}\underline{e}_\theta$



# 11.166

GIVEN:  $r = 2a \cos \theta$ ,  $\theta = \frac{1}{2}bt^2$

FIND: (a)  $\underline{v}$  AND  $\underline{a}$

(b)  $\rho$ ; PATH OF THE PARTICLE

(a) HAVE..  $r = 2a \cos \theta$   $\theta = \frac{1}{2}bt^2$

THEN  $\dot{r} = -2a\dot{\theta} \sin \theta$   $\dot{\theta} = bt$

AND  $\ddot{r} = -2a(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)$   $\ddot{\theta} = b$

SUBSTITUTING FOR  $\dot{\theta}$  AND  $\ddot{\theta}$

$\dot{r} = -2abt \sin \theta$

$\ddot{r} = -2ab(\sin \theta + bt^2 \cos \theta)$

Now..  $\underline{v}_r = \dot{r}$   $\underline{v}_\theta = r\dot{\theta}$

$= -2abt \sin \theta$   $= 2abt \cos \theta$

THEN..  $\underline{v} = \sqrt{\underline{v}_r^2 + \underline{v}_\theta^2} = 2abt[(\sin \theta)^2 + (\cos \theta)^2]^{1/2}$   
 OR  $\underline{v} = 2abt$

ALSO..  $\underline{a}_r = \ddot{r} - r\dot{\theta}^2 = -2ab(\sin \theta + bt^2 \cos \theta) - 2abt^2 \cos \theta$   
 $= -2ab(\sin \theta + 2bt^2 \cos \theta)$

AND  $\underline{a}_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2ab \cos \theta - 4abt^2 \sin \theta$   
 $= 2ab(\cos \theta - 2bt^2 \sin \theta)$

THEN..  $\underline{a} = \sqrt{\underline{a}_r^2 + \underline{a}_\theta^2}$   
 $= 2ab[(\sin \theta + 2bt^2 \cos \theta)^2 + (\cos \theta - 2bt^2 \sin \theta)^2]^{1/2}$   
 OR  $\underline{a} = 2ab\sqrt{1+4b^2t^4}$

(b) Now..  $\underline{a}^2 = \underline{a}_r^2 + \underline{a}_\theta^2 = (\frac{dv}{dt})^2 + (\frac{v^2}{\rho})^2$

THEN..  $\frac{dv}{dt} = \frac{d}{dt}(2abt) = 2ab$

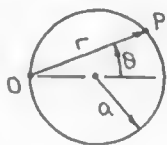
SO THAT  $(2ab\sqrt{1+4b^2t^4})^2 = (2ab)^2 + \frac{v^2}{\rho^2}$   
 OR  $4a^2b^2(1+4b^2t^4) = 4a^2b^2 + \frac{v^2}{\rho^2}$

OR  $\underline{a}_n = 4ab^2t^2$

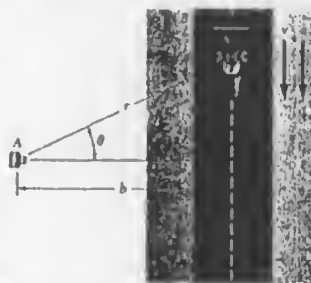
FINALLY..  $\underline{a}_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{(2abt)^2}{4ab^2t^2}$

OR  $\rho = a$

SINCE THE RADIUS OF CURVATURE IS A CONSTANT, THE PATH IS A CIRCLE OF RADIUS  $a$ .

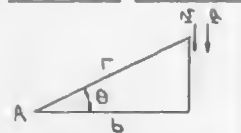


# 11.167 and 11.168



GIVEN: THE RECTILINEAR MOTION OF A RACE CAR AS SHOWN

HAVE..  $r = \frac{b}{\cos \theta}$   
 THEN  $\dot{r} = \frac{b\dot{\theta} \sin \theta}{\cos^2 \theta}$



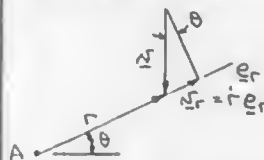
11.167 FIND:  $\underline{v}$  IN TERMS OF  $b$ ,  $\theta$ , AND  $\dot{\theta}$

HAVE..  $\underline{v}^2 = \underline{v}_r^2 + \underline{v}_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$   
 $= (\frac{b\dot{\theta} \sin \theta}{\cos^2 \theta})^2 + (\frac{b\dot{\theta}}{\cos \theta})^2$   
 $= \frac{b^2\dot{\theta}^2}{\cos^2 \theta} (\frac{\sin^2 \theta}{\cos^2 \theta} + 1) = \frac{b^2\dot{\theta}^2}{\cos^4 \theta}$   
 OR  $\underline{v} = \pm \frac{b\dot{\theta}}{\cos^2 \theta}$

FOR THE POSITION OF THE CAR SHOWN,  $\theta$  IS DECREASING; THUS, THE NEGATIVE ROOT IS CHOSEN.

$\therefore \underline{v} = -\frac{b\dot{\theta}}{\cos^2 \theta}$

ALTERNATIVE SOLUTION



FROM THE DIAGRAM..

$\dot{r} = -\underline{v} \sin \theta$

OR  $\frac{b\dot{\theta} \sin \theta}{\cos^2 \theta} = -\underline{v} \sin \theta$

OR  $\underline{v} = -\frac{b\dot{\theta}}{\cos^2 \theta}$

11.168 FIND:  $\underline{a}$  IN TERMS OF  $b$ ,  $\theta$ ,  $\dot{\theta}$ , AND  $\ddot{\theta}$

FOR RECTILINEAR MOTION  $\underline{a} = \frac{dv}{dt}$

FROM THE SOLUTION TO PROBLEM 11.167

$\underline{v} = -\frac{b\dot{\theta}}{\cos^2 \theta}$

THEN  $\underline{a} = \frac{d}{dt}(-\frac{b\dot{\theta}}{\cos^2 \theta}) = -b \frac{\ddot{\theta} \cos^2 \theta - \dot{\theta}(2\dot{\theta} \cos \theta \sin \theta)}{\cos^4 \theta}$

OR  $\underline{a} = -\frac{b}{\cos^4 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta)$

ALTERNATIVE SOLUTION

FROM ABOVE..  $r = \frac{b}{\cos \theta}$   $\dot{r} = \frac{b\dot{\theta} \sin \theta}{\cos^2 \theta}$

THEN..  $\ddot{r} = b \frac{(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)(\cos^3 \theta) - (\dot{\theta} \sin \theta)(-2\dot{\theta} \cos \theta \sin \theta)}{\cos^4 \theta}$

$= b [\frac{\ddot{\theta} \sin \theta}{\cos^3 \theta} + \frac{\dot{\theta}^2 (1 + \sin^2 \theta)}{\cos^3 \theta}]$

Now..  $\underline{a}^2 = \underline{a}_r^2 + \underline{a}_\theta^2$   
 WHERE  $\underline{a}_r = \ddot{r} - r\dot{\theta}^2 = b [\frac{\ddot{\theta} \sin \theta}{\cos^3 \theta} + \frac{\dot{\theta}^2 (1 + \sin^2 \theta)}{\cos^3 \theta}] - \frac{b\dot{\theta}^2}{\cos^3 \theta}$   
 $= \frac{b}{\cos^3 \theta} (\ddot{\theta} \sin \theta + \frac{2\dot{\theta}^2 \sin^2 \theta}{\cos^2 \theta})$

(CONTINUED)

# 11.168 CONTINUED

$$a_r = \frac{b \sin \theta}{\cos^2 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta)$$

AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b\ddot{\theta}}{\cos \theta} + 2 \frac{b\dot{\theta}^2 \sin \theta}{\cos^2 \theta}$

$$= \frac{b \cos \theta}{\cos^2 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta)$$

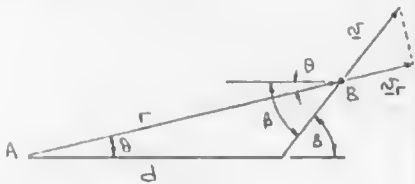
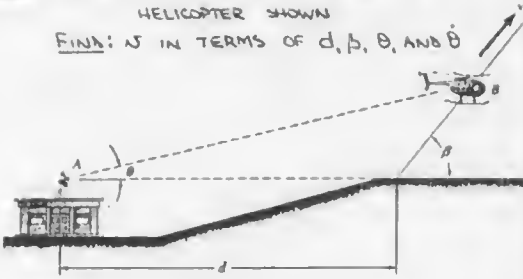
THEN  $a = \pm \frac{b}{\cos^2 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta) [(\sin \theta)^2 + (\cos \theta)^2]^{1/2}$

FOR THE POSITION OF THE CAR SHOWN,  $\ddot{\theta}$  IS NEGATIVE;  
FOR  $a$  TO BE POSITIVE, THE NEGATIVE ROOT IS CHOSEN.

$$\therefore a = -\frac{b}{\cos^2 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta)$$

# 11.169

GIVEN: STRAIGHT LINE TRAJECTORY OF THE HELICOPTER SHOWN  
FIND:  $N$  IN TERMS OF  $d, \beta, \theta$ , AND  $\dot{\theta}$



FROM THE DIAGRAM..

$$\frac{r}{\sin(180^\circ - \beta)} = \frac{d}{\sin(\beta - \theta)}$$

OR  $d \sin \beta = r (\sin \beta \cos \theta - \cos \beta \sin \theta)$

OR  $r = d \frac{\tan \beta}{\tan \beta \cos \theta - \sin \theta}$

THEN  $\dot{r} = d \tan \beta \frac{-(\tan \beta \sin \theta - \cos \theta) \dot{\theta}}{(\tan \beta \cos \theta - \sin \theta)^2}$

$$= d \dot{\theta} \tan \beta \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2}$$

FROM THE DIAGRAM

$N_f = N \cos(\beta - \theta)$  WHERE  $N_f = \dot{r}$

THEN  $d \dot{\theta} \tan \beta \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2} = N (\cos \beta \cos \theta + \sin \beta \sin \theta)$

$$= N \cos \beta (\tan \beta \sin \theta + \cos \theta)$$

OR  $N = \frac{d \dot{\theta} \tan \beta \sec \beta}{(\tan \beta \cos \theta - \sin \theta)^2}$

ALTERNATIVE SOLUTION

HAVE..  $N^2 = N_r^2 + N_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

USING THE EXPRESSIONS FOR  $r$  AND  $\dot{r}$  FROM ABOVE..

$$N^2 = \left[ d \dot{\theta} \tan \beta \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2} \right]^2 + \left( d \dot{\theta} \frac{\tan \beta}{\tan \beta \cos \theta - \sin \theta} \right)^2$$

(CONTINUED)

# 11.169 CONTINUED

OR  $N = \pm \frac{d \dot{\theta} \tan \beta}{(\tan \beta \cos \theta - \sin \theta)} \left[ \frac{(\tan \beta \sin \theta + \cos \theta)^2}{(\tan \beta \cos \theta - \sin \theta)^2} + 1 \right]^{1/2}$

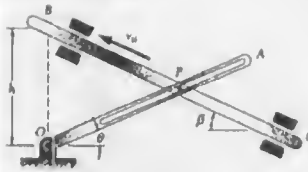
$$= \pm \frac{d \dot{\theta} \tan \beta}{(\tan \beta \cos \theta - \sin \theta)} \left[ \frac{\tan^2 \beta + 1}{(\tan \beta \cos \theta - \sin \theta)^2} \right]^{1/2}$$

NOTE THAT AS  $\theta$  INCREASES, THE HELICOPTER MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

$\therefore N = \frac{d \dot{\theta} \tan \beta \sec \beta}{(\tan \beta \cos \theta - \sin \theta)^2}$

# \* 11.170

GIVEN:  $N_o = \text{CONSTANT}$   
FIND:  $\dot{\theta}$  IN TERMS OF  $N_o, h, \beta$ , AND  $\theta$



FROM THE DIAGRAM..

$$\frac{r}{\sin(90^\circ - \beta)} = \frac{h}{\sin(\beta - \theta)}$$

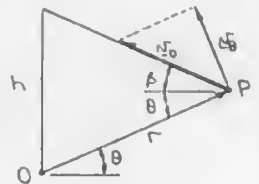
OR  $r (\sin \beta \cos \theta + \cos \beta \sin \theta) = h \cos \beta$

OR  $r = \frac{h}{\tan \beta \cos \theta + \sin \theta}$

ALSO..  $N_\theta = N_o \sin(\beta - \theta)$  WHERE  $N_\theta = r\dot{\theta}$

THEN  $\frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} = N_o (\sin \beta \cos \theta + \cos \beta \sin \theta)$

OR  $\dot{\theta} = \frac{N_o \cos \beta (\tan \beta \cos \theta + \sin \theta)^2}{h}$



ALTERNATIVE SOLUTION

FROM ABOVE..  $r = \frac{h}{\tan \beta \cos \theta + \sin \theta}$

THEN  $\dot{r} = h \frac{\tan \beta \sin \theta - \cos \theta}{(\tan \beta \cos \theta + \sin \theta)^2} \dot{\theta}$

NOW..  $N_o^2 = N_r^2 + N_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

OR  $N_o^2 = \left[ h \dot{\theta} \frac{\tan \beta \sin \theta - \cos \theta}{(\tan \beta \cos \theta + \sin \theta)^2} \right]^2 + \left( \frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} \right)^2$

OR  $N_o = \pm \frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} \left[ \frac{(\tan \beta \sin \theta - \cos \theta)^2}{(\tan \beta \cos \theta + \sin \theta)^2} + 1 \right]^{1/2}$

$= \pm \frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} \left[ \frac{\tan^2 \beta + 1}{(\tan \beta \cos \theta + \sin \theta)^2} \right]^{1/2}$

NOTE THAT AS  $\theta$  INCREASES, MEMBER BC MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

$\therefore \dot{\theta} = \frac{N_o \cos \beta}{h} (\tan \beta \cos \theta + \sin \theta)^2$

11.171



GIVEN:  $\theta_1 = 60^\circ$ ,  $\theta_2 = 35^\circ$ ,  
 $\Delta t_{12} = 0.5 \text{ s}$ ,  
 $b = 25 \text{ m}$   
 FIND:  $N_{\text{AVE}}$

FROM THE DIAGRAM...

$$\Delta r_{12} = 25 \tan 60^\circ - 25 \tan 35^\circ$$

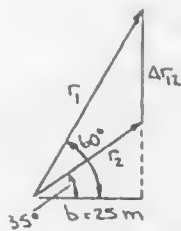
$$= 25.796 \text{ m}$$

NOW..  $N_{\text{AVE}} = \frac{\Delta r_{12}}{\Delta t_{12}}$

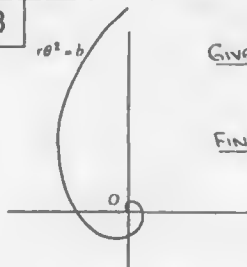
$$= \frac{25.796 \text{ m}}{0.5 \text{ s}}$$

$$= 51.592 \frac{\text{m}}{\text{s}}$$

OR  $N_{\text{AVE}} = 185.7 \frac{\text{km}}{\text{h}}$



11.173



GIVEN: A PARTICLE MOVES  
 ALONG THE SPIRAL  
 SHOWN  
 FIND:  $N$  IN TERMS OF  $b$ ,  
 $\theta$ , AND  $\dot{\theta}$

HAVE..  $r = \frac{b}{\theta}$

THEN  $\dot{r} = -\frac{b}{\theta^2} \dot{\theta}$

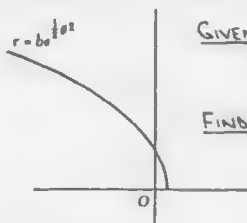
NOW..  $N^2 = \dot{r}^2 + N_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$$= \left(-\frac{b}{\theta^2} \dot{\theta}\right)^2 + \left(\frac{b}{\theta} \dot{\theta}\right)^2$$

$$= \left(\frac{b\dot{\theta}}{\theta^2}\right)^2 \left(\frac{1}{\theta^2} + 1\right)$$

OR  $N = \frac{b\dot{\theta}}{\theta^2} \sqrt{1 + \theta^2}$

11.174



GIVEN: A PARTICLE MOVES  
 ALONG THE SPIRAL  
 SHOWN  
 FIND:  $N$  IN TERMS OF  $b$ ,  
 $\theta$ , AND  $\dot{\theta}$

HAVE..  $r = b\theta^{1/2}$

THEN  $\dot{r} = b\dot{\theta} \frac{1}{2} \theta^{-1/2}$

NOW  $N^2 = \dot{r}^2 + N_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

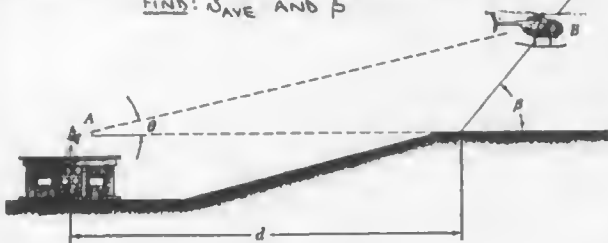
$$= \left(b\dot{\theta} \frac{1}{2} \theta^{-1/2}\right)^2 + \left(b\theta^{1/2} \dot{\theta}\right)^2$$

$$= \left(b\dot{\theta} \frac{1}{2} \theta^{-1/2}\right)^2 (\theta^2 + 1)$$

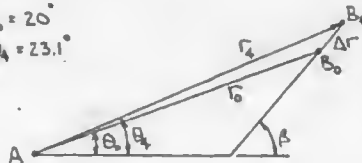
OR  $N = b\dot{\theta} \frac{1}{2} \theta^{-1/2} \sqrt{1 + \theta^2}$

11.172

GIVEN: AT  $t = 0$ ,  $r_{AB} = 3000 \text{ ft}$ ,  $\theta = 20^\circ$   
 AT  $t = 4 \text{ s}$ ,  $r_{AB} = 3320 \text{ ft}$ ,  $\theta = 23.1^\circ$   
 FIND:  $N_{\text{AVE}}$  AND  $\beta$



HAVE..  $r_0 = 3000 \text{ ft}$ ,  $\theta_0 = 20^\circ$   
 $r_4 = 3320 \text{ ft}$ ,  $\theta_4 = 23.1^\circ$



FROM THE DIAGRAM...

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1^\circ - 20^\circ)$$

$$\text{OR } \Delta r = 362.70 \text{ ft}$$

NOW..  $N_{\text{AVE}} = \frac{\Delta r}{\Delta t}$

$$= \frac{362.70 \text{ ft}}{4 \text{ s}}$$

$$= 90.675 \frac{\text{ft}}{\text{s}}$$

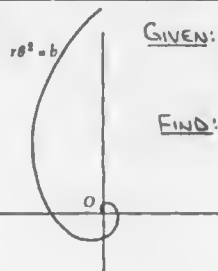
OR  $N_{\text{AVE}} = 61.8 \frac{\text{mi}}{\text{h}}$

ALSO..  $\Delta r \cos \beta = r_4 \cos \theta_4 - r_0 \cos \theta_0$

OR  $\cos \beta = \frac{3320 \cos 23.1^\circ - 3000 \cos 20^\circ}{362.70}$

OR  $\beta = 49.7^\circ$

11.175



GIVEN: A PARTICLE MOVES ALONG  
 THE SPIRAL SHOWN;  
 $\dot{\theta} = \omega = \text{CONSTANT}$   
 FIND:  $a$  IN TERMS OF  $b$ ,  $\theta$ ,  
 AND  $\omega$

HAVE..  $r = \frac{b}{\theta}$

THEN  $\dot{r} = -\frac{b\dot{\theta}}{\theta^2} = -\frac{b\omega}{\theta^2}$

AND  $\ddot{r} = -\frac{b\omega^2}{\theta^3} = -\frac{b\omega^2}{\theta^3}$

NOW..  $a^2 = \dot{r}^2 + a_\theta^2 = (\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2$

$$= \left(-\frac{b\omega^2}{\theta^3} - \frac{b}{\theta^2} \omega^2\right)^2 + \left[2\left(-\frac{b\omega}{\theta^2}\right)\omega\right]^2$$

$$= \left(\frac{b\omega^2}{\theta^3}\right)^2 \left[(b - \theta^2)^2 + (-4\theta^2)^2\right]$$

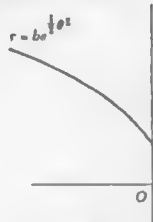
$$= \left(\frac{b\omega^2}{\theta^3}\right)^2 (36 + 4\theta^2 + \theta^4)$$

OR  $a = \frac{b\omega^2}{\theta^3} \sqrt{36 + 4\theta^2 + \theta^4}$



11.176

$$r = b e^{\frac{1}{2}\theta^2}$$



GIVEN: A PARTICLE MOVES  
ALONG THE SPIRAL  
SHOWN;  $\dot{\theta} = \omega = \text{CONSTANT}$   
FIND:  $a$  IN TERMS OF  $b$ ,  
 $\theta$ , AND  $\omega$

HAVE..  $r = b e^{\frac{1}{2}\theta^2}$ THEN  $\dot{r} = b \theta \dot{\theta} e^{\frac{1}{2}\theta^2} = b \omega \theta e^{\frac{1}{2}\theta^2}$ AND  $\ddot{r} = b \omega (\dot{\theta} e^{\frac{1}{2}\theta^2} + \theta^2 \dot{\theta} e^{\frac{1}{2}\theta^2}) = b \omega^2 e^{\frac{1}{2}\theta^2} (1 + \theta^2)$ 

$$\begin{aligned} \text{NOW.. } a^2 &= \dot{r}^2 + a_\theta^2 = (\dot{r} - r \dot{\theta}^2)^2 + (r \ddot{\theta})^2 \\ &= [b \omega^2 e^{\frac{1}{2}\theta^2} (1 + \theta^2) - b \omega^2 e^{\frac{1}{2}\theta^2}]^2 + (2 b \omega^2 \theta e^{\frac{1}{2}\theta^2})^2 \\ &= (b \omega^2 e^{\frac{1}{2}\theta^2})^2 (\theta^4 + 4\theta^2) \\ \text{OR } a &= b \omega^2 e^{\frac{1}{2}\theta^2} \sqrt{4 + \theta^2} \end{aligned}$$

11.177



GIVEN:  $\dot{\phi} = \dot{\psi}$ ,  $\dot{\phi} = \text{CONSTANT}$   
SHOW:  $\dot{r} = h \dot{\phi} \sin \theta$

FROM THE DIAGRAM..

$$r^2 = d^2 + h^2 - 2dh \cos \phi$$

THEN..  $2r \dot{r} = 2dh \dot{\phi} \sin \phi$ 

$$\text{NOW.. } \frac{r}{\sin \phi} = \frac{d}{\sin \theta}$$

$$\text{OR } r = \frac{d \sin \phi}{\sin \theta}$$

SUBSTITUTING FOR  $r$  IN THE EXPRESSION FOR  $\dot{r}$ 

$$\left( \frac{d \sin \phi}{\sin \theta} \right) \dot{r} = dh \dot{\phi} \sin \phi$$

$$\text{OR } \dot{r} = h \dot{\phi} \sin \theta \quad \text{Q.E.D.}$$

ALTERNATIVE SOLUTION

FIRST NOTE..  $\alpha = 180^\circ - (\phi + \theta)$ NOW..  $\dot{r} = \dot{r}_r + \dot{r}_\theta = \dot{r}_r + r \dot{\theta}$ 

WITH B AS THE ORIGIN..

$$\dot{r}_r = d \dot{\phi} \quad (d = \text{CONSTANT} \Rightarrow \dot{d} = 0)$$

WITH O AS THE ORIGIN..

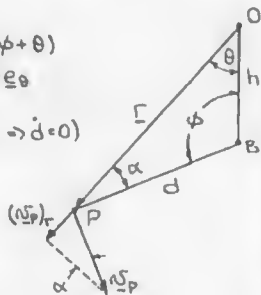
$$(\dot{r}_r)_r = \dot{r}$$

WHERE  $(\dot{r}_r)_r = \dot{r}_r \sin \alpha$ THEN  $\dot{r} = d \dot{\phi} \sin \alpha$ 

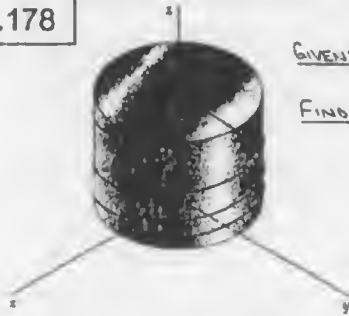
$$\text{NOW.. } \frac{h}{\sin \alpha} = \frac{d}{\sin \theta}$$

$$\text{OR } d \sin \alpha = h \sin \theta$$

$$\text{THEN.. } \dot{r} = h \dot{\phi} \sin \theta \quad \text{Q.E.D.}$$



11.178



GIVEN:  $R = A$ ,  $\theta = 2\pi t$ ,  
 $z = \frac{1}{2} A t^2$   
FIND:  $v$  AND  $a$

HAVE..  $R = A$  $\theta = 2\pi t$  $z = \frac{1}{2} A t^2$ THEN  $\dot{R} = 0$  $\dot{\theta} = 2\pi$  $\dot{z} = \frac{1}{2} A t$ AND  $\ddot{R} = 0$  $\ddot{\theta} = 0$  $\ddot{z} = \frac{1}{2} A$ 

$$\begin{aligned} \text{NOW.. } v^2 &= \dot{R}^2 + \dot{\theta}^2 + \dot{z}^2 = (\dot{R})^2 + (R \dot{\theta})^2 + (\dot{z})^2 \\ &= 0 + (A \cdot 2\pi)^2 + \left(\frac{1}{2} A t\right)^2 \\ &= A^2 (4\pi^2 + \frac{1}{4} t^2) \end{aligned}$$

$$\text{OR } v = \frac{1}{2} A \sqrt{16\pi^2 + t^2}$$

$$\begin{aligned} a^2 &= \dot{v}_R^2 + \dot{v}_\theta^2 + \dot{v}_z^2 = (\dot{R} - R \dot{\theta}^2)^2 + (R \ddot{\theta})^2 + (\ddot{z})^2 \\ &= [-A(2\pi)^2]^2 + 0 + \left(\frac{1}{2} A\right)^2 \\ &= A^2 (16\pi^4 + \frac{1}{4}) \end{aligned}$$

$$\text{OR } a = \frac{1}{2} A \sqrt{16\pi^4 + 1}$$

11.179

GIVEN:  $R = \frac{A}{t+1}$ ,  $\theta = Bt$ ,  $z = \frac{Ct}{t+1}$

FIND: (a)  $v$  AND  $a$  AT  $t=0$ (b)  $v$  AND  $a$  AS  $t \rightarrow \infty$ HAVE..  $R = \frac{A}{t+1}$  $\theta = Bt$  $z = \frac{Ct}{t+1}$ THEN  $\dot{R} = -\frac{A}{(t+1)^2}$  $\dot{\theta} = B$  $\dot{z} = C \left( \frac{(t+1) - t}{(t+1)^2} \right)$ AND  $\ddot{R} = \frac{2A}{(t+1)^3}$  $\ddot{\theta} = 0$  $\ddot{z} = -\frac{2C}{(t+1)^3}$ 

$$\begin{aligned} \text{NOW.. } v^2 &= (\dot{R})^2 + (\dot{\theta})^2 + (\dot{z})^2 = (\dot{R})^2 + (R \dot{\theta})^2 + (\dot{z})^2 \\ \text{AND } a^2 &= (\ddot{R})^2 + (\ddot{\theta})^2 + (\ddot{z})^2 = (\ddot{R} - R \dot{\theta}^2)^2 + (R \ddot{\theta})^2 + (\ddot{z})^2 \end{aligned}$$

(a) AT  $t=0$ :  $R = A$  $\dot{R} = -A$  $\dot{\theta} = B$  $\dot{z} = C$  $\ddot{R} = 2A$  $\ddot{z} = -2C$ THEN..  $v^2 = (-A)^2 + (AB)^2 + (C)^2$ 

$$\text{OR } v = \sqrt{(1+B^2)A^2 + C^2}$$

$$\begin{aligned} \text{AND } a^2 &= (2A - AB^2)^2 + [2(-A)(B)]^2 + (-2C)^2 \\ &= 4A^2 \left[ (1 - \frac{1}{2} B^2)^2 + B^2 + \frac{C^2}{A^2} \right] \\ &= 4 \left[ (1 + \frac{1}{4} B^4) A^2 + C^2 \right] \end{aligned}$$

$$\text{OR } a = 2 \sqrt{(1 + \frac{1}{4} B^4) A^2 + C^2}$$

(b) AS  $t \rightarrow \infty$ :  $R = 0$  $\dot{R} = 0$  $\dot{\theta} = B$  $\dot{z} = 0$  $\ddot{R} = 0$  $\ddot{\theta} = 0$  $\ddot{z} = 0$  $\therefore v = 0$ AND  $a = 0$

# \* 11.180

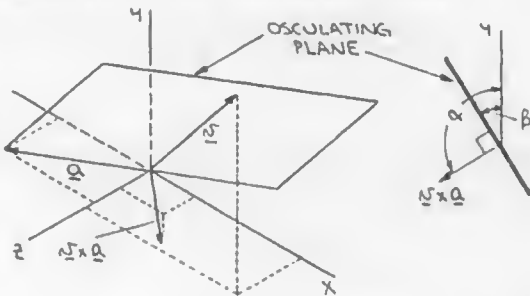
GIVEN:  $\underline{r} = (Rt \cos \omega_N t) \underline{i} + ct \underline{j} + (Rt \sin \omega_N t) \underline{k}$   
FIND: THE ANGLE THAT THE OSCULATING PLANE FORMS WITH THE Y AXIS

FIRST NOTE THAT THE VECTORS  $\underline{r}$  AND  $\underline{Q}$  LIE IN THE OSCULATING PLANE.

NOW..  $\underline{r} = (Rt \cos \omega_N t) \underline{i} + ct \underline{j} + (Rt \sin \omega_N t) \underline{k}$   
 THEN  $\underline{v} = \frac{d\underline{r}}{dt} = R(\cos \omega_N t - \omega_N t \sin \omega_N t) \underline{i} + c \underline{j} + R(\sin \omega_N t + \omega_N t \cos \omega_N t) \underline{k}$

AND  $\underline{Q} = \frac{d\underline{v}}{dt} = R(-\omega_N \sin \omega_N t - \omega_N \sin \omega_N t - \omega_N^2 t \cos \omega_N t) \underline{i} + R(\omega_N \cos \omega_N t + \omega_N \cos \omega_N t - \omega_N^2 t \sin \omega_N t) \underline{k}$   
 $= \omega_N R[-(2 \sin \omega_N t + \omega_N t \cos \omega_N t) \underline{i} + (2 \cos \omega_N t - \omega_N t \sin \omega_N t) \underline{k}]$

IT THEN FOLLOWS THAT THE VECTOR  $(\underline{v} \times \underline{Q})$  IS PERPENDICULAR TO THE OSCULATING PLANE.



$(\underline{v} \times \underline{Q}) = \omega_N R \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ R(\cos \omega_N t - \omega_N t \sin \omega_N t) & c & R(\sin \omega_N t + \omega_N t \cos \omega_N t) \\ -2 \sin \omega_N t + \omega_N t \cos \omega_N t & 0 & 2 \cos \omega_N t - \omega_N t \sin \omega_N t \end{vmatrix}$   
 $= \omega_N R \{ c(2 \cos \omega_N t - \omega_N t \sin \omega_N t) \underline{j} + R[-(\sin \omega_N t + \omega_N t \cos \omega_N t)(2 \sin \omega_N t + \omega_N t \cos \omega_N t) - (\cos \omega_N t - \omega_N t \sin \omega_N t)(2 \cos \omega_N t - \omega_N t \sin \omega_N t)] \underline{k} + c(2 \sin \omega_N t + \omega_N t \cos \omega_N t) \underline{i} \}$   
 $= \omega_N R [c(2 \cos \omega_N t - \omega_N t \sin \omega_N t) \underline{j} - R(2 + \omega_N^2 t^2) \underline{k} + c(2 \sin \omega_N t + \omega_N t \cos \omega_N t) \underline{i}]$

THE ANGLE  $\alpha$  FORMED BY THE VECTOR  $(\underline{v} \times \underline{Q})$  AND THE Y AXIS IS FOUND FROM..

$$\cos \alpha = \frac{(\underline{v} \times \underline{Q}) \cdot \underline{j}}{|\underline{v} \times \underline{Q}| |\underline{j}|}$$

WHERE  $|\underline{j}| = 1$   
 $(\underline{v} \times \underline{Q}) \cdot \underline{j} = -\omega_N R^2 (2 + \omega_N^2 t^2)$   
 $|\underline{v} \times \underline{Q}| = \omega_N R [c^2 (2 \cos \omega_N t - \omega_N t \sin \omega_N t)^2 + R^2 (2 + \omega_N^2 t^2)^2 + c^2 (2 \sin \omega_N t + \omega_N t \cos \omega_N t)^2]^{1/2}$   
 $= \omega_N R [c^2 (4 + \omega_N^2 t^2) + R^2 (2 + \omega_N^2 t^2)^2]^{1/2}$

THEN  $\cos \alpha = \frac{-\omega_N R^2 (2 + \omega_N^2 t^2)}{\omega_N R [c^2 (4 + \omega_N^2 t^2) + R^2 (2 + \omega_N^2 t^2)^2]^{1/2}}$   
 $= \frac{-R(2 + \omega_N^2 t^2)}{[c^2 (4 + \omega_N^2 t^2) + R^2 (2 + \omega_N^2 t^2)^2]^{1/2}}$

(CONTINUED)

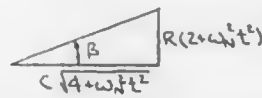
# 11.180 CONTINUED

THE ANGLE  $\beta$  THAT THE OSCULATING PLANE FORMS WITH THE Y AXIS (SEE THE ABOVE DIAGRAM) IS EQUAL TO

$$\beta = \alpha - 90^\circ$$

THEN  $\cos \alpha = \cos(\beta + 90^\circ) = -\sin \beta$

$$\therefore -\sin \beta = \frac{-R(2 + \omega_N^2 t^2)}{[c^2 (4 + \omega_N^2 t^2) + R^2 (2 + \omega_N^2 t^2)^2]^{1/2}}$$



THEN  $\tan \beta = \frac{R(2 + \omega_N^2 t^2)}{c \sqrt{4 + \omega_N^2 t^2}}$

OR  $\beta = \tan^{-1} \left[ \frac{R(2 + \omega_N^2 t^2)}{c \sqrt{4 + \omega_N^2 t^2}} \right]$

# \* 11.181

GIVEN:  $\underline{r} = (At \cos t) \underline{i} + (A \sqrt{t^2 + 1}) \underline{j} + (Bt \sin t) \underline{k}$   
 $\underline{r} = ft, t = s; A = 3, B = 1$

FIND: (a) DIRECTION OF  $\underline{e}_b$  AT  $t = 0$   
 (b) DIRECTION OF  $\underline{e}_b$  AT  $t = \frac{\pi}{2} s$

FIRST NOTE THAT  $\underline{e}_b$  IS GIVEN BY

$$\underline{e}_b = \frac{\underline{v} \times \underline{Q}}{|\underline{v} \times \underline{Q}|}$$

NOW..  $\underline{r} = (3t \cos t) \underline{i} + (3\sqrt{t^2 + 1}) \underline{j} + (t \sin t) \underline{k}$

THEN  $\underline{v} = \frac{d\underline{r}}{dt} = 3(\cos t - t \sin t) \underline{i} + \frac{3t}{\sqrt{t^2 + 1}} \underline{j} + (\sin t + t \cos t) \underline{k}$

AND  $\underline{Q} = \frac{d\underline{v}}{dt} = 3(-\sin t - \sin t - t \cos t) \underline{i} + 3 \frac{(t^2 + 1) - t(\frac{t}{\sqrt{t^2 + 1}})}{t^2 + 1} \underline{j} + (\cos t + \cos t - t \sin t) \underline{k}$   
 $= -3(2 \sin t + t \cos t) \underline{i} + \frac{3}{(t^2 + 1)^{3/2}} \underline{j} + (2 \cos t - t \sin t) \underline{k}$

(a) AT  $t = 0$ :  $\underline{v} = (3 \frac{0}{1}) \underline{j} = 3 \underline{j}$ ,  $\underline{Q} = (3 \frac{0}{1}) \underline{j} + (2 \frac{0}{1}) \underline{k} = 3 \underline{j}$

THEN  $\underline{v} \times \underline{Q} = 3 \underline{j} \times (3 \underline{j} + 2 \underline{k}) = 3(-2 \underline{i} + 3 \underline{k})$

AND  $|\underline{v} \times \underline{Q}| = 3 \sqrt{(-2)^2 + (3)^2} = 3\sqrt{13}$

THEN  $\underline{e}_b = \frac{3(-2 \underline{i} + 3 \underline{k})}{3\sqrt{13}} = \frac{1}{\sqrt{13}} (-2 \underline{i} + 3 \underline{k})$

$\therefore \cos \theta_x = 0$ ,  $\cos \theta_y = -\frac{2}{\sqrt{13}}$ ,  $\cos \theta_z = \frac{3}{\sqrt{13}}$   
 OR  $\theta_x = 90^\circ$ ,  $\theta_y = 123.7^\circ$ ,  $\theta_z = 33.7^\circ$

(b) AT  $t = \frac{\pi}{2} s$ :  $\underline{v} = (-\frac{3\pi}{2} \frac{0}{1}) \underline{i} + (\frac{3\pi}{\sqrt{(\frac{\pi}{2})^2 + 1}} \frac{0}{1}) \underline{j} + (1 \frac{0}{1}) \underline{k} = -\frac{3\pi}{2} \underline{i} + \frac{3\pi}{\sqrt{5/4}} \underline{j}$

$\underline{Q} = -(6 \frac{0}{1}) \underline{i} + [\frac{24}{(\frac{\pi^2}{4} + 4)^{3/2}} \frac{0}{1}] \underline{j} - (\frac{\pi}{2} \frac{0}{1}) \underline{k} = -6 \underline{i} - \frac{\pi}{2} \underline{k}$

THEN..  $\underline{v} \times \underline{Q} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -\frac{3\pi}{2} & \frac{3\pi}{(\frac{\pi^2}{4} + 4)^{1/2}} & 1 \\ -6 & \frac{24}{(\frac{\pi^2}{4} + 4)^{3/2}} & -\frac{\pi}{2} \end{vmatrix}$   
 $= -[\frac{3\pi^2}{2(\frac{\pi^2}{4} + 4)^{1/2}} + \frac{24}{(\frac{\pi^2}{4} + 4)^{3/2}}] \underline{j} - (6 + \frac{3\pi^2}{4}) \underline{i} + [-\frac{36\pi}{(\frac{\pi^2}{4} + 4)^{3/2}} + \frac{18\pi}{(\frac{\pi^2}{4} + 4)^{3/2}}] \underline{k}$   
 $= -4.4398 \underline{i} - 13.4022 \underline{j} + 12.9945 \underline{k}$

AND  $|\underline{v} \times \underline{Q}| = [(-4.4398)^2 + (-13.4022)^2 + (12.9945)^2]^{1/2} = 19.18829$

THEN..  $\underline{e}_b = \frac{1}{19.18829} (-4.4398 \underline{i} - 13.4022 \underline{j} + 12.9945 \underline{k})$

$\therefore \cos \theta_x = -\frac{4.4398}{19.18829}$ ,  $\cos \theta_y = -\frac{13.4022}{19.18829}$ ,  $\cos \theta_z = \frac{12.9945}{19.18829}$

OR  $\theta_x = 103.4^\circ$ ,  $\theta_y = 134.3^\circ$ ,  $\theta_z = 47.4^\circ$

11.182

GIVEN:  $x = 2t^3 - 15t^2 + 24t + 4$   $x$  in,  $t$  in s  
 FIND: (a)  $t$  WHEN  $v = 0$   
 (b)  $x$  AND TOTAL DISTANCE TRAVELED WHEN  $a = 0$

HAVE ..  $x = 2t^3 - 15t^2 + 24t + 4$   
 THEN  $v = \frac{dx}{dt} = 6t^2 - 30t + 24$   
 AND  $a = \frac{dv}{dt} = 12t - 30$

(a) WHEN  $v = 0$ :  $6t^2 - 30t + 24 = 0$   
 OR  $(t-1)(t-4) = 0$   
 OR  $t = 1$  s AND  $t = 4$  s

(b) WHEN  $a = 0$ :  $12t - 30 = 0$  OR  $t = 2.5$  s  
 AT  $t = 2.5$  s:  $x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$   
 OR  $x_{2.5} = 1.5$  m

FIRST OBSERVE THAT  $0 \leq t < 1$  s  $v > 0$   
 $1 \leq t \leq 2.5$  s  $v < 0$

NOW.. AT  $t = 0$ :  $x_0 = 4$  m  
 $t = 1$  s:  $x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4 = 15$  m



THEN..  $x_1 - x_0 = 15 - 4 = 11$  m  
 $|x_{2.5} - x_1| = |1.5 - 15| = 13.5$  m  
 $\therefore$  TOTAL DISTANCE TRAVELED =  $(11 + 13.5)$  m =  $24.5$  m

11.183

GIVEN:  $a = -60x^{-1.5}$   $a$  in  $\frac{m}{s^2}$ ,  $x$  in m; AT  $t = 0$ ,  
 $v = 0$ ,  $x = 4$  m  
 FIND: (a)  $v$  WHEN  $x = 2$  m  
 (b)  $v$  WHEN  $x = 1$  m  
 (c)  $v$  WHEN  $x = 0.1$  m

HAVE..  $v \frac{dv}{dx} = a = -60x^{-1.5}$   
 WHEN  $x = 4$  m,  $v = 0$ :  $\int_0^v v dv = \int_4^x (-60x^{-1.5}) dx$   
 OR  $\frac{1}{2}v^2 = 120[x^{-0.5}]_4^x$   
 OR  $v^2 = 240(\frac{1}{\sqrt{x}} - \frac{1}{2})$

(a) WHEN  $x = 2$  m:  $v^2 = 240(\frac{1}{\sqrt{2}} - \frac{1}{2})$   
 OR  $v = -7.05 \frac{m}{s}$

(b) WHEN  $x = 1$  m:  $v^2 = 240(1 - \frac{1}{2})$   
 OR  $v = -10.95 \frac{m}{s}$

(c) WHEN  $x = 0.1$  m:  $v^2 = 240(\frac{1}{\sqrt{0.1}} - \frac{1}{2})$   
 OR  $v = -25.3 \frac{m}{s}$

11.184

GIVEN:  $v = v_0 - kx$   $v$  in  $\frac{ft}{s}$ ,  $x$  in ft; AT  $t = 0$ ,  
 $x = 0$ ,  $v_0 = 900 \frac{ft}{s}$ ; WHEN  $x = 4$  in.,  
 $v = 0$   
 FIND: (a)  $a$  AT  $t = 0$   
 (b)  $t$  WHEN  $x = 3.9$  in.

FIRST NOTE .. WHEN  $x = \frac{4}{12}$  ft,  $v = 0$ :  $0 = (900 \frac{ft}{s}) - k(\frac{4}{12} ft)$   
 OR  $k = 2700 \frac{1}{s}$

(a) HAVE ..  $v = v_0 - kx$   
 THEN  $a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$   
 (CONTINUED)

11.184 CONTINUED

OR  $a = -k(v_0 - kx)$   
 AT  $t = 0$ :  $a = -2700 \frac{1}{s}(900 \frac{ft}{s} - 0)$  OR  $a_0 = -2.43 \times 10^6 \frac{ft}{s^2}$

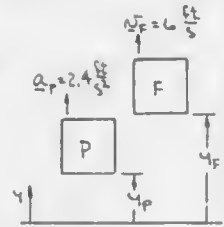
(b) HAVE ..  $\frac{dv}{dx} = \frac{v}{v_0 - kx}$   
 AT  $t = 0$ ,  $x = 0$ :  $\int_0^v \frac{dx}{v_0 - kx} = \int_0^t dt$   
 OR  $-\frac{1}{k} \{ \ln(v_0 - kx) \}_0^x = t$   
 OR  $t = \frac{1}{k} \ln(\frac{v_0}{v_0 - kx}) = \frac{1}{k} \ln(\frac{1}{1 - \frac{k}{v_0}x})$   
 WHEN  $x = 3.9$  in.:  $t = \frac{1}{2700 \frac{1}{s}} \ln[ \frac{1}{1 - \frac{2700 \frac{1}{s}}{900 \frac{ft}{s}} (\frac{3.9}{12} ft) } ]$   
 OR  $t = 1.36 \times 10^{-3}$  s

11.185

GIVEN:  $v_F = 6 \frac{ft}{s}$ ; AT  $t = 0$ ,  $y_F = y_P = 0$ ; FOR  
 $t \leq 4$  s,  $v_P = 0$ ;  $a_P = 2.4 \frac{ft}{s^2}$   
 FIND: (a)  $t$  AND  $y$  WHEN  $y_F = y_P$   
 (b)  $v_P$  WHEN  $y_F = y_P$

(a) FOR  $t \leq 4$ :  $y_F = (v_F)_0^0 + v_F t$   
 $t \geq 4$ :  $y_P = (v_P)_0^0 + (a_P)_0^4(t-4) + \frac{1}{2}a_P(t-4)^2$   
 WHEN  $y_F = y_P$ :  $(6 \frac{ft}{s})t = \frac{1}{2}(2.4 \frac{ft}{s^2})(t-4)^2$   
 EXPANDING AND SIMPLIFYING..  
 $t^2 - 13t + 16 = 0$   
 SOLVING..  $t = 1.376$  s AND  $t = 11.623$  s  
 MOST REQUIRE  $t > 4$  s  $\therefore t = 11.62$  s

AT  $t = 11.623$  s:  $y_F = (6 \frac{ft}{s})(11.623 \text{ s})$   
 OR  $y_F = y_P = 69.7$  ft  
 (b) FOR  $t \geq 4$  s:  $v_P = (v_P)_0^0 + a_P(t-4)$   
 AT  $t = 11.623$  s:  $v_P = (2.4 \frac{ft}{s^2})(11.623 - 4)$   
 OR  $v_P = 18.30 \frac{ft}{s}$



11.186

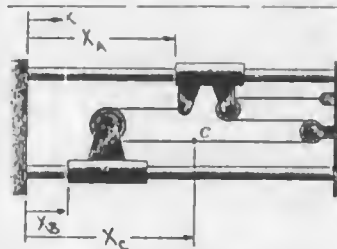
GIVEN:  $v_B = 150 \frac{mm}{s}$   
 FIND: (a)  $v_A$   
 (b)  $v_C$   
 (c)  $v_{C/B}$



(a) FROM THE DIAGRAM HAVE..  
 $(x_A - x_B) + (-x_B) + 2(-x_A) = \text{CONSTANT}$   
 THEN..  $v_A + 2v_B = 0$   
 SUBSTITUTING..  
 $v_A + 2(-150 \frac{mm}{s}) = 0$   
 OR  $v_A = 300 \frac{mm}{s}$

(b) FROM THE DIAGRAM HAVE..  
 $(x_A - x_B) + (x_C - x_B) = \text{CONSTANT}$   
 THEN..  $v_A - 2v_B + v_C = 0$   
 SUBSTITUTING..  
 $300 \frac{mm}{s} - 2(-150 \frac{mm}{s}) + v_C = 0$   
 OR  $v_C = -600 \frac{mm}{s}$

(c) HAVE..  $v_{C/B} = v_C - v_B$   
 $= -600 \frac{mm}{s} - (-150 \frac{mm}{s})$   
 OR  $v_{C/B} = -450 \frac{mm}{s}$



11.187

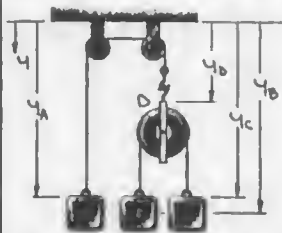


GIVEN:  $\dot{y}_A, \dot{y}_B, \dot{y}_C$  CONSTANTS;

$$\dot{y}_{AC} = 300 \frac{\text{mm}}{\text{s}} \uparrow$$

$$\dot{y}_{BA} = 200 \frac{\text{mm}}{\text{s}} \downarrow$$

FIND:  $\dot{y}_A, \dot{y}_B$ , AND  $\dot{y}_C$



FROM THE DIAGRAM...

CABLE 1:  $y_A + y_B = \text{CONSTANT}$

THEN..  $\dot{y}_A + \dot{y}_B = 0$  (1)

CABLE 2:  $(y_B - y_D) + (y_C - y_D) = \text{CONSTANT}$

THEN..  $\dot{y}_B + \dot{y}_C - 2\dot{y}_D = 0$  (2)

COMBINING EQS. (1) AND (2) TO ELIMINATE  $\dot{y}_D$ ..

$$2\dot{y}_A + \dot{y}_B + \dot{y}_C = 0 \quad (3)$$

$$\text{NOW -- } \dot{y}_{AC} = \dot{y}_A - \dot{y}_C = -300 \frac{\text{mm}}{\text{s}} \quad (4)$$

$$\text{AND } \dot{y}_{B/A} = \dot{y}_B - \dot{y}_A = 200 \frac{\text{mm}}{\text{s}} \quad (5)$$

THEN (3) + (4) - (5)  $\Rightarrow$

$$(2\dot{y}_A + \dot{y}_B + \dot{y}_C) + (\dot{y}_A - \dot{y}_C) - (\dot{y}_B - \dot{y}_A) = (-300) - (200)$$

$$\text{OR } \dot{y}_A = 125 \frac{\text{mm}}{\text{s}} \uparrow$$

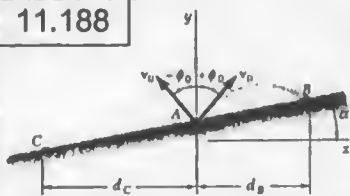
$$\text{AND USING EQ(5), } \dot{y}_B - (-125) = 200$$

$$\text{OR } \dot{y}_B = 75 \frac{\text{mm}}{\text{s}} \downarrow$$

$$\text{EQ. (4) -- } -125 - \dot{y}_C = -300$$

$$\text{OR } \dot{y}_C = 175 \frac{\text{mm}}{\text{s}} \downarrow$$

11.188



GIVEN:  $\dot{y}_0 = 30 \frac{\text{ft}}{\text{s}}, \phi_0 = 40^\circ, \alpha = 10^\circ$

FIND:  $d_B$  AND  $d_C$

FIRST NOTE..

$$(\dot{y}_0)_x = \dot{y}_0 \sin \phi = (30 \frac{\text{ft}}{\text{s}}) \sin \phi$$

$$(\dot{y}_0)_y = \dot{y}_0 \cos \phi = (30 \frac{\text{ft}}{\text{s}}) \cos \phi$$

ALSO, ALONG INCLINE CAB..

$$y = x \tan 10^\circ$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0^0 + (\dot{y}_0)_x t = (30 \sin \phi) t \quad \text{OR } t = \frac{x}{30 \sin \phi}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0^0 + (\dot{y}_0)_y t - \frac{1}{2} g t^2 = (30 \cos \phi) t - \frac{1}{2} g t^2$$

SUBSTITUTING FOR  $t$ ..

$$y = (30 \cos \phi) \left( \frac{x}{30 \sin \phi} \right) - \frac{1}{2} g \left( \frac{x}{30 \sin \phi} \right)^2$$

$$= \frac{x}{\tan \phi} - \frac{g}{1800 \sin^2 \phi} x^2 \quad (g = 32.2 \frac{\text{ft}}{\text{s}^2})$$

$$\text{AT B: } \phi = 40^\circ, x = d_B: d_B \tan 10^\circ = \frac{d_B}{\tan 40^\circ} - \frac{32.2}{1800 \sin^2 40^\circ} d_B^2$$

$$\text{OR } d_B = 23.5 \text{ ft}$$

$$\text{AT C: } \phi = -40^\circ, x = -d_C: -d_C \tan 10^\circ = \frac{-d_C}{\tan(-40^\circ)} - \frac{32.2}{1800 \sin^2(-40^\circ)} d_C^2$$

$$\text{OR } d_C = 31.6 \text{ ft}$$

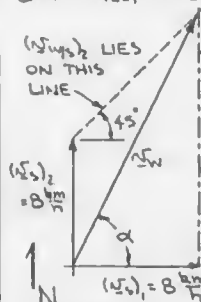
11.189

GIVEN:  $(\dot{y}_s)_1 = 8 \frac{\text{km}}{\text{h}} \rightarrow, (\dot{y}_s)_{1/2} \uparrow$   
 $(\dot{y}_s)_2 = 8 \frac{\text{km}}{\text{h}} \uparrow, (\dot{y}_s)_{2/2} \angle 45^\circ$

FIND:  $\dot{y}_W$ , WHERE  $\dot{y}_W$  IS CONSTANT

HAVE..  $\dot{y}_W = \dot{y}_s + \dot{y}_{s/W}$

USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



$(\dot{y}_s)_{1/2}$  LIES ON THIS LINE

FROM THE DIAGRAM..

$$\dot{y}_W^2 = (8)^2 + (8+8)^2$$

$$\text{OR } \dot{y}_W = 17.89 \frac{\text{km}}{\text{h}}$$

$$\text{AND } \tan \alpha = \frac{16}{8}$$

$$\text{OR } \alpha = 63.4^\circ$$

$$\therefore \dot{y}_W = 17.89 \frac{\text{km}}{\text{h}} \angle 63.4^\circ$$

11.190

GIVEN:  $p = 1500 \text{ ft}; \dot{y}_1 = 45 \frac{\text{mi}}{\text{h}}, \dot{y}_2 = 30 \frac{\text{mi}}{\text{h}}, \Delta s_{12} = 750 \text{ ft}; a_t = \text{CONSTANT}$

FIND:  $a$  WHEN  $\Delta s = 500 \text{ ft}$

FIRST NOTE..  $\dot{y}_1 = 45 \frac{\text{mi}}{\text{h}} = 66 \frac{\text{ft}}{\text{s}}$

$$\dot{y}_2 = 30 \frac{\text{mi}}{\text{h}} = 44 \frac{\text{ft}}{\text{s}}$$

HAVE UNIFORMLY DECELERATED MOTION..

$$\therefore \dot{y}^2 = \dot{y}_1^2 + 2a_t(s - s_1)$$

$$\text{WHEN } \dot{y} = \dot{y}_2: (44 \frac{\text{ft}}{\text{s}})^2 = (66 \frac{\text{ft}}{\text{s}})^2 + 2a_t(750 \text{ ft})$$

$$\text{OR } a_t = -1.61333 \frac{\text{ft}}{\text{s}^2}$$

THEN WHEN  $\Delta s = 500 \text{ ft}$ :

$$\dot{y}^2 = (66 \frac{\text{ft}}{\text{s}})^2 + 2(-1.61333 \frac{\text{ft}}{\text{s}^2})(500 \text{ ft})$$

$$= 2742.67 \frac{\text{ft}^2}{\text{s}^2}$$

$$\text{NOW.. } a_n = \frac{\dot{y}^2}{p} = \frac{2742.67 \frac{\text{ft}^2}{\text{s}^2}}{1500 \text{ ft}} = 1.82845 \frac{\text{ft}}{\text{s}^2}$$

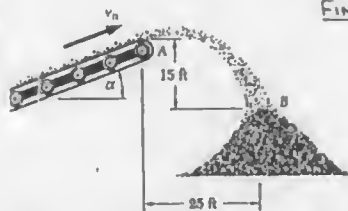
$$\text{FINALLY.. } a^2 = a_t^2 + a_n^2 = (-1.61333 \frac{\text{ft}}{\text{s}^2})^2 + (1.82845 \frac{\text{ft}}{\text{s}^2})^2$$

$$\text{OR } a = 2.44 \frac{\text{ft}}{\text{s}^2}$$

11.191

GIVEN:  $\dot{y}_0 = 24 \frac{\text{ft}}{\text{s}}$

FIND:  $\alpha$



FIRST NOTE..  $(\dot{y}_0)_x = \dot{y}_0 \cos \alpha = (24 \frac{\text{ft}}{\text{s}}) \cos \alpha$

$$(\dot{y}_0)_y = \dot{y}_0 \sin \alpha = (24 \frac{\text{ft}}{\text{s}}) \sin \alpha$$

HORIZONTAL MOTION (UNIFORM)

$$x = x_0^0 + (\dot{y}_0)_x t = (24 \cos \alpha) t$$

$$\text{AT POINT B: } 25 = (24 \cos \alpha) t$$

$$\text{OR } t_B = \frac{25}{24 \cos \alpha}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0^0 + (\dot{y}_0)_y t - \frac{1}{2} g t^2 = (24 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 32.2 \frac{\text{ft}}{\text{s}^2})$$

$$\text{AT POINT B: } -15 = (24 \sin \alpha) t_B - \frac{1}{2} g t_B^2$$

SUBSTITUTING FOR  $t_B$ ..

(CONTINUED)

# 11.191 CONTINUED

$$-15 = (24 \sin \alpha) \left( \frac{25}{24 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{25}{24 \cos \alpha} \right)^2$$

$$\text{OR } -3 = 5 \tan \alpha - \frac{125g}{1152 \cos^2 \alpha}$$

$$\text{NOW... } \frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$$

$$\text{THEN... } -3 = 5 \tan \alpha - \frac{125 \times 32.2}{1152} (1 + \tan^2 \alpha)$$

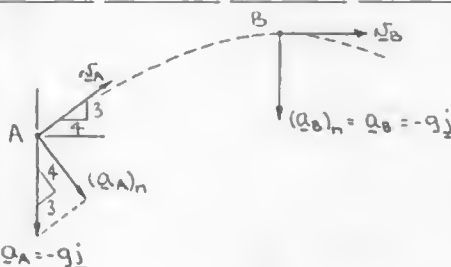
$$\text{OR } 3.4939 \tan^2 \alpha - 5 \tan \alpha + 0.49392 = 0$$

$$\text{SOLVING... } \tan \alpha = 0.106746 \text{ AND } \tan \alpha = 1.32432$$

$$\text{THEN... } \alpha = 6.09^\circ \text{ AND } \alpha = 52.9^\circ$$

# 11.192

**GIVEN:**  $P_A = 25 \text{ m}$   
**FINN:** (a)  $\mathcal{U}_A$   
 (b)  $P_B$ , WHERE  
 $y_B = y_{\text{max}}$



$$(a) \text{ HAVE... } (a_A)_n = \frac{\mathcal{U}_A^2}{P_A}$$

$$\text{OR } \mathcal{U}_A^2 = \left[ \frac{4}{3} (9.81 \frac{\text{m}}{\text{s}^2}) \right] (25 \text{ m})$$

$$\text{OR } \mathcal{U}_A = 14.0071 \frac{\text{m}}{\text{s}}$$

$$\therefore \mathcal{U}_A = 14.01 \frac{\text{m}}{\text{s}} \angle 36.9^\circ$$

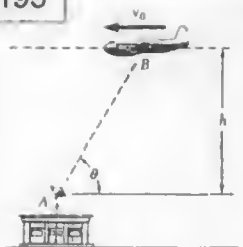
$$(b) \text{ HAVE... } (a_B)_n = \frac{\mathcal{U}_B^2}{P_B}$$

$$\text{WHERE } \mathcal{U}_B = (\mathcal{U}_A)_x = \frac{4}{5} \mathcal{U}_A$$

$$\text{THEN... } P_B = \frac{\left( \frac{4}{5} \times 14.0071 \frac{\text{m}}{\text{s}} \right)^2}{9.81 \frac{\text{m}}{\text{s}^2}}$$

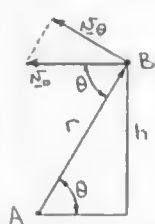
$$\text{OR } P_B = 12.80 \text{ m}$$

# 11.193



**GIVEN:**  $\mathcal{U}_0 = \mathcal{U}_B \leftarrow$ ,  $\mathcal{U}_0 = \text{CONSTANT}$

**FINN:**  $\dot{\theta}$  AND  $\ddot{\theta}$  IN  
 TERMS OF  $\mathcal{U}_0$ ,  $h$ ,  
 AND  $\theta$



FROM THE DIAGRAM

$$r = \frac{h}{\sin \theta} \quad \mathcal{U}_B = \mathcal{U}_0 \sin \theta$$

$$\text{NOW... } \mathcal{U}_B = r \dot{\theta}$$

SUBSTITUTING FOR  $\mathcal{U}_B$  AND  $r$ ..

$$\mathcal{U}_0 \sin \theta = \left( \frac{h}{\sin \theta} \right) \dot{\theta}$$

(CONTINUED)

# 11.193 CONTINUED

$$\text{OR } \dot{\theta} = \frac{\mathcal{U}_0}{h} \sin^2 \theta$$

$$\text{HAVE } \dot{\theta} = \frac{\mathcal{U}_0}{h} \sin^2 \theta$$

$$\text{THEN... } \ddot{\theta} = \frac{\mathcal{U}_0}{h} (2 \dot{\theta} \sin \theta \cos \theta)$$

SUBSTITUTING FOR  $\dot{\theta}$ ..

$$\ddot{\theta} = \frac{\mathcal{U}_0}{h} (2 \sin \theta \cos \theta) \left( \frac{\mathcal{U}_0}{h} \sin^2 \theta \right)$$

$$\text{OR } \ddot{\theta} = 2 \frac{\mathcal{U}_0^2}{h^2} \sin^3 \theta \cos \theta$$

# ALTERNATIVE SOLUTIONS

$$\text{HAVE... } r = \frac{h}{\sin \theta}$$

$$\text{THEN } \dot{r} = - \frac{h \cos \theta}{\sin^2 \theta} \dot{\theta}$$

$$\text{NOW... } \mathcal{U}_r^2 + \mathcal{U}_\theta^2 = (\dot{r})^2 + (r \dot{\theta})^2$$

$$\begin{aligned} \text{OR } \mathcal{U}_0^2 &= \left( - \frac{h \cos \theta}{\sin^2 \theta} \dot{\theta} \right)^2 + \left( \frac{h}{\sin \theta} \dot{\theta} \right)^2 \\ &= \left( \frac{h \dot{\theta}}{\sin \theta} \right)^2 \left( \frac{\cos^2 \theta}{\sin^2 \theta} + 1 \right) \\ &= \left( \frac{h \dot{\theta}}{\sin^2 \theta} \right)^2 \end{aligned}$$

$$\text{OR } \dot{\theta} = \pm \frac{\mathcal{U}_0}{h} \sin^2 \theta$$

NOTE THAT AS  $\theta$  INCREASES, THE AIRPLANE MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

$$\therefore \dot{\theta} = \frac{\mathcal{U}_0}{h} \sin^2 \theta$$

$$\text{HAVE... } \mathcal{Q} = \mathcal{Q}_r + \mathcal{Q}_\theta$$

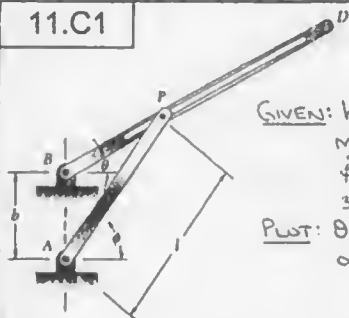
$$\text{NOW } \mathcal{U}_0 = \text{CONSTANT} \Rightarrow \mathcal{Q} = 0$$

$$\therefore \mathcal{Q}_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0$$

$$\text{OR } \ddot{\theta} = -2 \frac{\left( - \frac{h \cos \theta}{\sin^2 \theta} \cdot \frac{\mathcal{U}_0}{h} \sin^2 \theta \right) \left( \frac{\mathcal{U}_0}{h} \sin^2 \theta \right)}{\frac{h}{\sin \theta}}$$

$$\text{OR } \ddot{\theta} = 2 \frac{\mathcal{U}_0^2}{h^2} \sin^3 \theta \cos \theta$$

## 11.C1



GIVEN: WHITWORTH QUICK-RETURN MECHANISM SHOWN;  
 $\dot{\phi} = 1 \frac{\text{RAD}}{\text{s}}$ ;  $l = 4 \text{ IN.}$ ;  $b = 2.5 \text{ IN.}$ ,  
 $3.0 \text{ IN.}$ ,  $3.5 \text{ IN.}$

PLST:  $\theta$  vs.  $\phi$  AND  $\dot{\theta}$  vs.  $\phi$  FOR ONE REVOLUTION OF ROD AB

## ANALYSIS

HAVE..  $\frac{b}{\sin(\phi - \theta)} = \frac{l}{\sin(90^\circ - \theta)}$

OR  $b \cos \theta = l (\sin \phi \cos \theta - \cos \phi \sin \theta)$

OR  $b = l (\sin \phi - \cos \phi \tan \theta)$

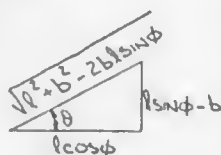
OR  $\tan \theta = \frac{l \sin \phi - b}{l \cos \phi}$  (1)

THEN  $\sec^2 \theta \dot{\theta} = \frac{(l \cos \phi \dot{\phi}) (l \cos \phi) - (l \sin \phi - b) (-l \sin \phi \dot{\phi})}{(l \cos \phi)^2}$

OR  $\dot{\theta} = \cos^2 \theta \frac{l^2 - b l \sin \phi}{(l \cos \phi)^2} \dot{\phi}$

USING EQ. (1) ..

$\cos \theta = \frac{l \cos \phi}{\sqrt{l^2 + b^2 - 2 b l \sin \phi}}$



THEN..  $\dot{\theta} = \left[ \frac{(l \cos \phi)^2}{l^2 + b^2 - 2 b l \sin \phi} \right] \cdot \frac{l^2 - b l \sin \phi}{(l \cos \phi)^2} \dot{\phi}$   
 $= l \frac{l - b \sin \phi}{l^2 + b^2 - 2 b l \sin \phi} \dot{\phi}$  (2)

NOTE: FOR  $0 \leq \phi < \tan^{-1} \left( \frac{b}{\sqrt{l^2 - b^2}} \right)$

EQ. (1)  $\Rightarrow -90^\circ \leq \theta < 0$

THUS, FOR THESE VALUES OF  $\phi$  MUST USE

$\theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 360^\circ$

WHEN PLOTTING THE GRAPH.

SIMILARLY,

FOR  $90^\circ < \phi < 270^\circ$ , EQ. (1)  $\Rightarrow -90^\circ < \theta < 90^\circ$

$\therefore \theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 180^\circ$

FOR  $270^\circ < \phi \leq 360^\circ$ , EQ. (1)  $\Rightarrow -90^\circ < \theta \leq 0$

$\therefore \theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 360^\circ$

## OUTLINE OF PROGRAM

INPUT VALUE OF b

CONSTRUCT BORDER FOR GRAPH OF  $\theta$  vs.  $\phi$ ;

LABEL AXES

FOR VALUES OF  $\phi$  FROM 0 TO  $360^\circ$  IN INCREMENTS OF  $1^\circ$

COMPUTE  $\theta$ :

FOR  $0 \leq \phi < \tan^{-1} \left( \frac{b}{\sqrt{l^2 - b^2}} \right)$ ,

$\theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 360^\circ$

FOR  $\tan^{-1} \left( \frac{b}{\sqrt{l^2 - b^2}} \right) \leq \phi < 90^\circ$

(CONTINUED)

## 11.C1 continued

$\theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right)$

FOR  $\phi = 90^\circ$ ,  $\theta = 90^\circ$

FOR  $90^\circ < \phi < 270^\circ$ ,  $\theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 180^\circ$

FOR  $\phi = 270^\circ$ ,  $\theta = 270^\circ$

FOR  $270^\circ < \phi \leq 360^\circ$ ,  $\theta = \tan^{-1} \left( \frac{l \sin \phi - b}{l \cos \phi} \right) + 360^\circ$

PLOT ( $\phi$ ,  $\theta$ )

(CONSTRUCT BORDER FOR GRAPH OF  $\dot{\theta}$  vs.  $\phi$ ;

LABEL AXES

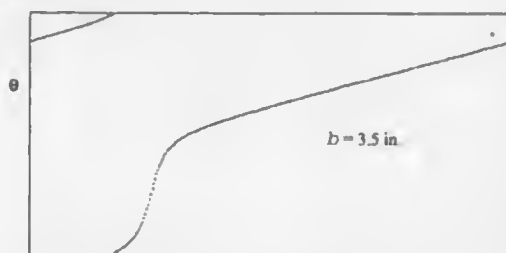
FOR VALUES OF  $\phi$  FROM 0 TO  $360^\circ$  IN

INCREMENTS OF  $1^\circ$

COMPUTE  $\dot{\theta}$ :  $\dot{\theta} = 4 \frac{l - b \sin \phi}{l^2 + b^2 - 2 b l \sin \phi}$

PLOT ( $\phi$ ,  $\dot{\theta}$ )

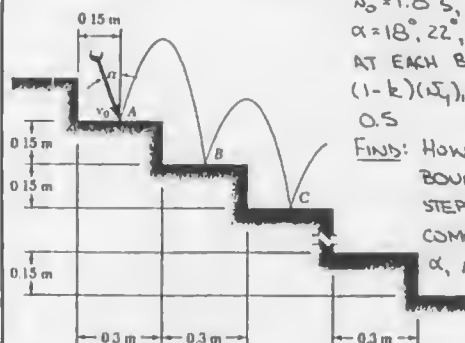
## PROGRAM OUTPUT



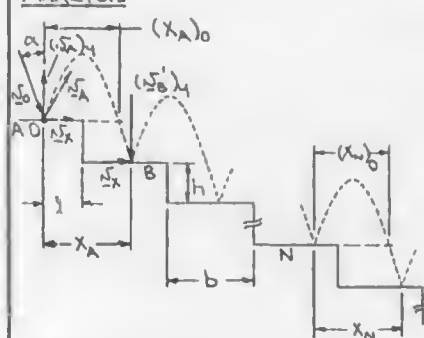
## 11.C2

GIVEN: EIGHT STEPS AS SHOWN;  
 $S_0 = 1.8 \text{ m}$ ,  $2.4 \text{ m}$ ,  $3.0 \text{ m}$ ;  
 $\alpha = 18^\circ, 22^\circ, 26^\circ$ ;  $S_x = \text{CONSTANT}$ ;  
 AT EACH BOUNCE,  $(S_y)_{\text{final}} = (1-k)(S_y)_{\text{initial}}$ ,  $k = 0.4$ ,  
 $0.5$

FIND: HOW THE BALL BOUNCES DOWN THE STEPS FOR EACH COMBINATION OF  $S_0$ ,  $\alpha$ , AND  $k$



## ANALYSIS



(CONTINUED)

## 11.C2 continued

FIRST NOTE..  $N_x = N_0 \sin \alpha$   $(N_A)_y = (1-k)N_0 \cos \alpha$   
WITH THE ORIGIN OF A RECTANGULAR  
COORDINATE SYSTEM AT POINT O..

HORIZONTAL MOTION (UNIFORM)

$$x = x_0^0 + N_x t \quad \text{OR} \quad t = \frac{x}{N_x}$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0^0 + (N_A)_y t - \frac{1}{2} g t^2 \quad N_y = (N_A)_y - g t$$

SUBSTITUTING FOR  $t$ ..

$$y = \frac{(N_A)_y}{N_x} x - \frac{1}{2} g \frac{x^2}{N_x^2} \quad N_y = (N_A)_y - g \frac{x}{N_x}$$

CONSIDER THE MOTION OF THE BALL AFTER IT  
LANDS ON A GIVEN STEP

1. DETERMINE IF THE BALL BOUNCES TWICE  
ON STEP A:

$$\text{ON STEP A, } y=0: \quad 0 = \frac{(N_A)_y}{N_x} (x_A)_0 - \frac{1}{2} g \frac{(x_A)_0^2}{N_x^2}$$

$$\text{OR } (x_A)_0 = \frac{2}{g} N_x (N_A)_y$$

$\therefore$  IF  $(x_A)_0 < l$ , THE BALL BOUNCES TWICE  
ON STEP A.

IN GENERAL, THE BALL BOUNCES TWICE ON  
STEP N ( $N=A, B, C, \dots, H$ ) IF

$$(x_N)_0 < l + (N-1)b = \sum_{j=A}^{N-1} x_j$$

WHERE  $(x_N)_0 = \frac{2}{g} N_x (N_A)_y$

AND  $x_N$  AND  $(N_A)_y$  ARE GIVEN BELOW.

2. DETERMINE IF THE BALL BOUNCES ON STEP B:

$$\text{ON STEP B, } y=-h: \quad -h = \frac{(N_A)_y}{N_x} x_A - \frac{1}{2} g \frac{x_A^2}{N_x^2}$$

SOLVING FOR  $x_A$  AND TAKING THE POSITIVE

ROOT ( $x_A > 0$ ), HAVE..

$$x_A = \frac{\frac{(N_A)_y}{N_x} + \left\{ \left[ -\frac{(N_A)_y}{N_x} \pm \sqrt{\left( \frac{(N_A)_y}{N_x} \right)^2 - 4 \left( \frac{1}{2} g \right) (-h)} \right]^{1/2}}{2 \left( \frac{1}{2} g \right)}$$

$$= \frac{N_x}{g} \left\{ (N_A)_y + \sqrt{[(N_A)_y]^2 + 2gh} \right\}$$

$\therefore$  IF  $x_A \leq l+b$ , THE BALL BOUNCES ON  
STEP B.

IN GENERAL, AFTER THE BALL BOUNCES ON  
STEP N, IT NEXT BOUNCES ON STEP  $i$  IF

$$\sum_{j=A}^N x_N \leq l + (i-1)b$$

$$\text{WHERE } x_N = \frac{N_x}{g} \left\{ (N_A)_y + \sqrt{[(N_A)_y]^2 + 2g[(i-N)h]} \right\}$$

FINALLY, IF THE BALL BOUNCES ON STEP B, HAVE  
USING THE EXPRESSION DERIVED ABOVE FOR  $N_y$ .

$$(N'_B)_y = (N_A)_y - g \frac{x_A}{N_x}$$

NOTING THAT  $(N'_B)_y < 0$  AND THAT THE  
MAGNITUDE OF THE VERTICAL COMPONENT  $(N_B)_y$   
OF THE VELOCITY AFTER THE BOUNCE IS

$$(N_B)_y = (1-k) \left[ g \frac{x_A}{N_x} - (N_A)_y \right]$$

HAVE IN GENERAL..

$$(N_N)_y = (1-k) \left[ g \frac{x_{N-1}}{N_x} - (N_{N-1})_y \right]$$

(CONTINUED)

## 11.C2 continued

### OUTLINE OF PROGRAM

FOR INITIAL ANGLES  $\alpha$ :  $\alpha = 18^\circ, 22^\circ, 26^\circ$

FOR VALUES OF  $k$ :  $k = 0.4, 0.5$

FOR INITIAL VELOCITIES  $N_0$ :  $N_0 = 1.8 \frac{m}{s}, 2.4 \frac{m}{s}, 3.0 \frac{m}{s}$

FOR EACH COMBINATION OF  $\alpha$ ,  $k$ , AND  $N_0$

COMPUTE  $N_x$  AND  $(N_A)_y$ :

$$N_x = N_0 \sin \alpha \quad (N_A)_y = (1-k)N_0 \cos \alpha$$

SET INITIAL CONDITIONS:  $N=1, i=2, x_{TOTAL}=0$

WHERE  $1, 2, 3, \dots, 8$  CORRESPOND TO STEPS  
A, B, C, ..., H AND  $x_{TOTAL}$  IS THE SUM OF  
THE HORIZONTAL DISTANCES BETWEEN  
SUCCESSIVE POINTS OF IMPACT.

DETERMINE IF THE BALL BOUNCES TWICE ON  
STEP N:

$$\text{IF } \frac{2}{g} N_x (N_A)_y \leq 0.15 + (N-1)(0.3) - x_{TOTAL}$$

PRINT: "BALL FIRST BOUNCES TWICE  
ON STEP N."

CONSIDER THE NEXT COMBINATION OF  $\alpha$ ,  $k$ ,  
AND  $N_0$ .

DETERMINE THE NEXT STEP ON WHICH THE  
BALL BOUNCES

UPDATE  $x_{TOTAL}$ :  $x_{TOTAL} = x_{TOTAL} + x_N$

$$\text{WHERE } x_N = \frac{N_x}{g} \left\{ (N_A)_y + \sqrt{[(N_A)_y]^2 + 0.3g(i-N)} \right\}$$

DETERMINE IF THE BALL BOUNCES ON  
CONSECUTIVE STEPS

IF  $x_{TOTAL} > 0.15 + (i-1)(0.3)$  AND  
 $i \leq 8$  PRINT: "BALL MISSES STEP  $i$ ."

RESET  $x_{TOTAL}$ :  $x_{TOTAL} = x_{TOTAL} - x_N$

UPDATE  $i$ :  $i = i+1$

IF  $i < 8$ , COMPUTE NEW  $x_N$  AND  
 $x_{TOTAL}$  AND REPEAT CHECK

IF  $i \geq 8$ , CONSIDER THE NEXT  
COMBINATION OF  $\alpha$ ,  $k$ , AND  $N_0$

DETERMINE HOW THE BALL BOUNCES  
DOWN THE REMAINING STEPS

IF  $N \geq 8$  PRINT: "BALL CONTINUES  
TO BOUNCE DOWN THE STEPS."

IF  $N < 8$ , UPDATE VALUES FOR  
THE NEXT STEP:

$$N_y: (N_i)_y = (1-k) \left[ g \frac{x_N}{N_x} - (N_{N-1})_y \right]$$

$$N: N=i$$

$$i: i=i+1$$

### PROGRAM OUTPUT

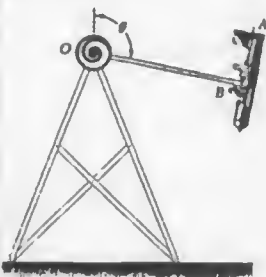
$\alpha$	$k$	$v_0$	
$18^\circ$	40%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball first bounces twice on step C
		3.0 m/s	Ball misses step D
			Ball continues to bounce down the steps
$22^\circ$	40%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball first bounces twice on step B
		3.0 m/s	Ball first bounces twice on step H
			Ball continues to bounce down the steps

(CONTINUED)

## 11.C2 continued

	3.0 m/s	Ball misses step B	
		Ball misses step E	
		Ball misses step G	
50%	1.8 m/s	Ball first bounces	twice on step A
	2.4 m/s	Ball first bounces	twice on step C
	3.0 m/s	Ball misses step C	
		Ball misses step H	
26% 40%	1.8 m/s	Ball first bounces	twice on step B
	2.4 m/s	Ball misses step D	
		Ball misses step G	
	3.0 m/s	Ball misses step B	
		Ball misses step D	
		Ball misses step F	
		Ball misses step H	
50%	1.8 m/s	Ball first bounces	twice on step A
	2.4 m/s	Ball continues to bounce down the	steps
	3.0 m/s	Ball misses step B	
		Ball misses step E	
		Ball misses step G	

## 11.C3



**GIVEN:**  $L_{0B} = 10 \text{ m}$ ;  $a_{\text{drag}} = -kV^2$ ,  $k = 0.2 \times 10^{-4} \text{ m}^{-1}$ ,  $4 \times 10^{-2} \text{ m}^{-1}$ ;  $\theta_0 = 70^\circ, 100^\circ, 130^\circ$

**FIND:**  $V_{\text{max}}$  AND THE FIRST TWO VALUES OF  $\theta$  FOR WHICH  $V=0$  FOR EACH COMBINATION OF  $\theta_0$  AND  $k$

### ANALYSIS

IN THE TANGENTIAL DIRECTION, THE TANGENTIAL COMPONENT OF THE ACCELERATION OF THE AIRPLANE IS

$$a_t = g \sin(180^\circ - \theta) - kV^2 = g \sin \theta - kV^2$$

RECALLING THAT  $a_t = \frac{dV}{dt}$  HAVE  $\frac{dV}{dt} = g \sin \theta - kV^2$

NOW, SINCE  $r = \text{CONSTANT}$ , HAVE  $V = r\dot{\theta}$  THEREFORE, THE DIFFERENTIAL EQUATIONS

$$\frac{dV}{dt} = g \sin \theta - kV^2$$

$$\frac{d\theta}{dt} = \frac{1}{r} V$$

DEFINE THE MOTION OF THE AIRPLANE.

### OUTLINE OF PROGRAM

INPUT VALUE OF  $k$

INPUT VALUE OF  $\theta_0$

CASE 1: DETERMINE THE VALUE OF THE VELOCITY AT THE SPECIFIED ANGLE  $\theta_f$

INPUT  $\theta_f$

USE, FOR EXAMPLE, THE MODIFIED EULER METHOD (SECOND-ORDER RUNGE-KUTTA METHOD -- SEE CHAPRA AND CANALE, NUMERICAL METHODS FOR ENGINEERS, 2d (CONTINUED))

## 11.C3 continued

ED, MCGRAW-HILL, 1988.) WITH A STEP SIZE  $\Delta t = 0.008 \text{ s}$  TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dV}{dt} = \begin{cases} g \sin \theta - kV^2 & \theta \leq 180^\circ \\ -g \sin \theta - kV^2 & \theta > 180^\circ \end{cases}$$

$$\frac{d\theta}{dt} = \frac{1}{r} V$$

UNTIL  $\theta'_1 \leq \theta \leq \theta'_2$ , WHERE  $\theta'_1$  AND  $\theta'_2$  ARE THE VALUES OF  $\theta$  AT THE MIDPOINT AND END, RESPECTIVELY, OF THE FINAL TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE FINAL VELOCITY  $V_f$ :

$$V_f = V'_1 + \frac{\theta_f - \theta'_1}{\theta'_2 - \theta'_1} (V'_2 - V'_1)$$

PRINT THE VALUES OF  $k$ ,  $\theta_0$ ,  $\theta_f$ , AND  $V_f$

CASE 2: DETERMINE THE VALUE OF  $\theta$  FOR WHICH THE VELOCITY IS FIRST ZERO

USE THE MODIFIED EULER METHOD WITH A STEP SIZE  $\Delta t = 0.008 \text{ s}$  TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dV}{dt} = \begin{cases} g \sin \theta - kV^2 & \theta_0, \theta_1 < 180^\circ \text{ OR } \theta_0, \theta_1 > 180^\circ \\ -g \sin \theta - kV^2 & \theta_0 < 180^\circ, \theta_1 > 180^\circ \text{ OR } \theta_0 > 180^\circ, \theta_1 < 180^\circ \end{cases}$$

$$\frac{d\theta}{dt} = \frac{1}{r} V$$

WHERE  $\theta_1$  IS THE VALUE OF  $\theta$  AT THE BEGINNING OF A TIME INTERVAL, UNTIL  $V_2 < 0$ , WHERE  $V_2$  IS THE VELOCITY AT THE END OF A TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE FINAL ANGLE  $\theta_f$ :

$$\theta_f = \theta'_1 + \frac{0 - V'_1}{V'_2 - V'_1} (\theta_2 - \theta'_1)$$

PRINT THE VALUES OF  $k$ ,  $\theta_0$ , AND  $\theta_f$

### SUMMARY OF PROGRAM OUTPUT

Maximum velocity attained for a release angle  $\theta_0$

	$V_{\text{max}}, \text{ m/s}$			
$\theta_0$	$k = 0$	$k = 2 \times 10^{-4} \text{ m}^{-1}$	$k = 4 \times 10^{-2} \text{ m}^{-1}$	$k = 0, \text{ theory}$
$70^\circ$	16.23	16.19	11.67	16.23
$100^\circ$	12.73	12.71	9.78	12.73
$130^\circ$	8.37	8.36	6.97	8.37

First  $[(\theta)_1]$  and second  $[(\theta)_2]$  rest positions for a release angle  $(\theta_0)$

	$k = 0$		$k = 2 \times 10^{-4} \text{ m}^{-1}$		$k = 4 \times 10^{-2} \text{ m}^{-1}$	
$\theta_0$	$(\theta)_1$	$(\theta)_2$	$(\theta)_1$	$(\theta)_2$	$(\theta)_1$	$(\theta)_2$
$70^\circ$	290.0°	70.0°	289.2°	71.6°	229.4°	146.7°
$100^\circ$	260.0°	100.0°	259.7°	100.6°	223.7°	149.3°
$130^\circ$	230.0°	130.0°	229.9°	130.2°	213.6°	154.6°



## 11.C4

GIVEN: CAR TRAVELING ON AN EXIT RAMP;  $v_0 = 60 \frac{\text{mi}}{\text{h}}$ ,  $v_{\text{FINAL}} = 0$ ;  $|a_{\text{MAX}}| = 10 \frac{\text{ft}}{\text{s}^2}$ ; RAMP IS EITHER STRAIGHT OR CURVED ( $p = 800 \text{ ft}$ );  $\frac{dv}{dt}$  IS EITHER CONSTANT OR VARIES LINEARLY DURING TIME INTERVALS OF 1 S

FIND:  $t_{\text{STOP}}$  AND DISTANCE TRAVELED ON THE RAMP FOR EACH COMBINATION OF RAMP TYPE AND  $\frac{dv}{dt}$

ANALYSIS

CASE 1: STRAIGHT RAMP,  $\frac{dv}{dt} = \text{CONSTANT}$

FOR THIS UNIFORMLY DECELERATED RECTILINEAR MOTION HAVE--

$$\frac{dv}{dt} = a = -10 \frac{\text{ft}}{\text{s}^2}$$

THEN  $v = v_0 + (-10)t$

$$\text{AND } v^2 = v_0^2 + 2(-10)(x - x_0)$$

NOTING THAT  $a$  IS CONSTANT AND  $v_{\text{FINAL}} = 0$ , HAVE

$$t_{\text{STOP}} = \frac{v_0}{10} \quad (\text{s})$$

$$x_{\text{TOTAL}} = \frac{v_0^2}{20} \quad (\text{ft})$$

WHERE  $t_{\text{STOP}}$  AND  $x_{\text{TOTAL}}$  ARE THE TIME FOR THE CAR TO COME TO REST AND THE TOTAL DISTANCE TRAVELED BY THE CAR ON THE RAMP, RESPECTIVELY. ALSO,  $v_0 = 60 \frac{\text{mi}}{\text{h}} = 88 \frac{\text{ft}}{\text{s}}$

CASE 2: STRAIGHT RAMP,  $\frac{dv}{dt}$  LINEARLY VARYING

HAVE  $a = \frac{dv}{dt}$

AND ASSUMING THAT FOR ANY TIME INTERVAL

$$a_1 = 0 \quad a_2 = -10 \frac{\text{ft}}{\text{s}^2}$$

HAVE

$$\frac{dv}{dt} = a = -\frac{10}{\Delta t}(t - t_1) \quad (\frac{\text{ft}}{\text{s}^2})$$

$$\text{AT } t = t_1, v = v_1: \int_{v_1}^v dv = \int_{t_1}^t -\frac{10}{\Delta t}(t - t_1) dt$$

$$\text{OR } v = v_1 - \frac{5}{\Delta t}(t - t_1)^2 \quad (1)$$

NOW--

$$\text{AT } t = t_1, x = x_1: \int_{x_1}^x dx = \int_{t_1}^t [v_1 - \frac{5}{\Delta t}(t - t_1)^2] dt$$

$$\text{OR } x = x_1 + v_1(t - t_1) - \frac{5}{3\Delta t}(t - t_1)^3 \quad (2)$$

FOR  $\Delta t = 1 \text{ s}$  AND WHEN  $t = t_2$ , HAVE--

$$(1) \Rightarrow v_2 = v_1 - 5 \quad (\frac{\text{ft}}{\text{s}})$$

$$(2) \Rightarrow x_2 = x_1 + v_1 - \frac{5}{3} \quad (\text{ft})$$

FOR THE FINAL TIME INTERVAL ( $\Delta t_{\text{FINAL}} < 1 \text{ s}$ ),

$v = 0$  AT  $t = t_{\text{FINAL}}$ . THEN, ASSUMING  $t_1 = 0$

(FOR CONVENIENCE) HAVE--

$$(1) \Rightarrow 0 = v_1 - \frac{5}{\Delta t}(t_{\text{FINAL}}) \quad \Delta t = 1 \text{ s}$$

$$\text{OR } t_{\text{FINAL}} = \sqrt{\frac{v_1^2}{5}} \quad (\text{s})$$

$$\text{AND } (2) \Rightarrow x_{\text{FINAL}} = x_1 + v_1 t_{\text{FINAL}} - \frac{5}{3} t_{\text{FINAL}}^3 \quad (\text{ft})$$

WHERE  $x_{\text{FINAL}}$  IS THE TOTAL DISTANCE,  $t_{\text{FINAL}}$  IS THE TIME DURATION OF THE FINAL TIME

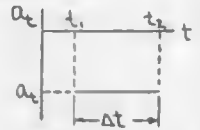
(CONTINUED)

## 11.C4 continued

INTERVAL, AND  $v_1$  AND  $x_1$  ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

CASE 3: CURVED RAMP,  $\frac{dv}{dt} = \text{CONSTANT}$

HAVE--  $a_t = \frac{dv}{dt} = \text{CONSTANT}$



$$\text{NOW-- } a^2 = a_t^2 + a_n^2 = a_t^2 + \left(\frac{v^2}{p}\right)^2$$

WHERE  $p = 800 \text{ ft}$  AND  $|a_{\text{MAX}}| = 10 \frac{\text{ft}}{\text{s}^2}$

FOR EACH TIME INTERVAL,  $a_t$  IS CONSTANT AND  $a_n$  IS MAXIMUM AT TIME  $t_1$  SINCE THE VELOCITY DECREASES FROM  $t_1$  TO  $t_2$ .

$$\therefore a_{\text{MAX}}^2 = a_t^2 + \left(\frac{v_1^2}{p}\right)^2$$

$$\text{OR } a_t = -\sqrt{a_{\text{MAX}}^2 - \frac{v_1^4}{p^2}} \quad (\frac{\text{ft}}{\text{s}^2})$$

FOR EACH TIME INTERVAL

NOW--  $a_t = \text{CONSTANT}$  (UNIF. ACCEL. MOTION)

$$\text{THEN-- } v = v_1 + a_t(t - t_1) \quad (3)$$

$$\text{AND } x = x_1 + v_1(t - t_1) + \frac{1}{2}a_t(t - t_1)^2 \quad (4)$$

FOR  $\Delta t = 1 \text{ s}$  AND WHEN  $t = t_2$ , HAVE--

$$a_t = -\sqrt{a_{\text{MAX}}^2 - \frac{v_1^4}{p^2}} \quad (\frac{\text{ft}}{\text{s}^2})$$

$$(3) \Rightarrow v_2 = v_1 + a_t \quad (\frac{\text{ft}}{\text{s}})$$

$$(4) \Rightarrow x_2 = x_1 + v_1 + \frac{1}{2}a_t \quad (\text{ft})$$

FOR THE FINAL TIME INTERVAL,  $v = 0$  AT  $t = t_{\text{FINAL}}$

THEN, ASSUMING  $t_1 = 0$  HAVE--

$$a_t = -\sqrt{a_{\text{MAX}}^2 - \frac{v_1^4}{p^2}} \quad (\frac{\text{ft}}{\text{s}^2})$$

$$(3) \Rightarrow 0 = v_1 + a_t(t_{\text{FINAL}})$$

$$\text{OR } t_{\text{FINAL}} = \frac{v_1}{a_t} \quad (\text{s})$$

$$(4) \Rightarrow x_{\text{FINAL}} = x_1 + v_1 t_{\text{FINAL}} + \frac{1}{2}a_t t_{\text{FINAL}}^2 \quad (\text{ft})$$

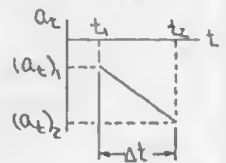
WHERE  $v_1$  AND  $x_1$  ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

CASE 4: CURVED RAMP,  $\frac{dv}{dt}$  LINEARLY VARYING

ASSUMING FOR ANY TIME INTERVAL ( $a_t$ )<sub>1</sub> = 0

HAVE--

$$a_t = \frac{(a_t)_2}{\Delta t}(t - t_1)$$



$$\text{NOW-- } \frac{dv}{dt} = a_t = \frac{(a_t)_2}{\Delta t}(t - t_1)$$

$$\text{AT } t = t_1, v = v_1: \int_{v_1}^v dv = \int_{t_1}^t \frac{(a_t)_2}{\Delta t}(t - t_1) dt$$

$$\text{OR } v = v_1 + \frac{(a_t)_2}{2\Delta t}(t - t_1)^2 \quad (5)$$

ALSO,  $\frac{dx}{dt} = v$

$$\text{AT } t = t_1, x = x_1: \int_{x_1}^x dx = \int_{t_1}^t [v_1 + \frac{(a_t)_2}{2\Delta t}(t - t_1)^2] dt$$

$$\text{OR } x = x_1 + v_1(t - t_1) + \frac{(a_t)_2}{6\Delta t}(t - t_1)^3 \quad (6)$$

$$\text{NOW-- } a^2 = a_t^2 + a_n^2 = a_t^2 + \left(\frac{v^2}{p}\right)^2$$

(CONTINUED)

# 11.C4 continued

WHERE  $p = 800 \text{ ft}$  AND  $|a_{\text{max}}| = 10 \text{ ft/s}^2$ .

NOW, FOR ANY TIME INTERVAL,

$(a_n)_{\text{max}}$  OCCURS AT  $t = t_1$  (WHEN THE VELOCITY IS MAXIMUM)

$(a_t)_{\text{max}}$  OCCURS AT  $t = t_2$

$(a_n)_{\text{max}} < a_{\text{max}}$  AT ALL TIMES (NOTE..  $\frac{v_0^2}{p} < 10 \text{ ft/s}^2$ )

$\therefore$  ASSUME  $a = a_{\text{max}}$  AT  $t = t_2$ . THEN..

$$a_{\text{max}} = (a_t)_2 + \left(\frac{v_2^2}{p}\right) \quad (7)$$

FOR  $\Delta t = 1 \text{ s}$  AND WHEN  $t = t_2$ , HAVE

$$(5) \Rightarrow v_2 = v_1 + \frac{1}{2}(a_t)_2$$

$$\text{OR } (a_t)_2 = 2(v_2 - v_1) \quad (8)$$

$$(6) \Rightarrow x_2 = x_1 + v_1 + \frac{1}{2}(a_t)_2 \quad (9)$$

COMBINING Eqs. (7) AND (8) TO ELIMINATE  $(a_t)_2$ ..

$$a_{\text{max}} = [2(v_2 - v_1)]^2 + \frac{v_2^4}{p^2}$$

$$\text{OR } \frac{v_2^4}{p^2} + 4v_2^2 - 8v_1v_2 + (4v_1^2 - a_{\text{max}}) = 0$$

-- A QUARTIC EQUATION WHICH DEFINES  $v_2$ .

FOR THE FINAL TIME INTERVAL,  $v=0$  AT  $t = t_{\text{FINAL}}$ . THEN, ASSUMING  $t_1 = 0$  HAVE..

$$\text{Eq. (8): } (a_t)_2 = 2(v_2 - v_1) \quad \text{WHERE } v_2 < 0$$

$$(5) \Rightarrow 0 = v_1 + \frac{1}{2}(a_t)_2 t_{\text{FINAL}}^2$$

$$\text{OR } t_{\text{FINAL}} = \sqrt{\frac{-2v_1}{(a_t)_2}} \quad (5)$$

$$(6) \Rightarrow x_{\text{FINAL}} = x_1 + v_1 t_{\text{FINAL}} + \frac{1}{2}(a_t)_2 t_{\text{FINAL}}^2 \quad (6)$$

WHERE  $v_1$  AND  $x_1$  ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

## OUTLINE OF PROGRAM

INPUT INITIAL VELOCITY  $v_1$

CONSIDER EACH CASE:

CASE 1: STRAIGHT RAMP,  $\frac{dv}{dt} = \text{CONSTANT}$

$$\text{COMPUTE TIME } t_{\text{STOP}}: t_{\text{STOP}} = \frac{v_1}{10}$$

$$\text{COMPUTE DISTANCE } x_{\text{TOTAL}}: x_{\text{TOTAL}} = \frac{v_1^2}{20}$$

PRINT THE VALUES OF  $t_{\text{STOP}}$  AND  $x_{\text{TOTAL}}$

CASE 2: STRAIGHT RAMP,  $\frac{dv}{dt}$  LINEARLY VARYING

FOR EACH SUCCESSIVE TIME INTERVAL

$$\text{COMPUTE } v_2: v_2 = v_1 - 5$$

WHILE  $v_2 > 0$

$$\text{UPDATE DISTANCE } x: x = x_1 + v_1 - \frac{5}{3}$$

UPDATE TIME AND SPEED:

$$t = t + 1; v_1 = v_2$$

FOR THE FINAL TIME INTERVAL

$$\text{COMPUTE } t_{\text{FINAL}}: t_{\text{FINAL}} = \sqrt{\frac{2}{3}v_1}$$

$$\text{COMPUTE TIME } t_{\text{STOP}}: t_{\text{STOP}} = t + t_{\text{FINAL}}$$

$$\text{COMPUTE DISTANCE } x_{\text{TOTAL}}: x_{\text{TOTAL}} = x_1 + v_1 t_{\text{FINAL}} - \frac{5}{3} t_{\text{FINAL}}^2$$

PRINT THE VALUES OF  $t_{\text{STOP}}$  AND  $x_{\text{TOTAL}}$

CASE 3: CURVED RAMP,  $\frac{dv}{dt} = \text{CONSTANT}$

FOR EACH SUCCESSIVE TIME INTERVAL

(CONTINUED)

# 11.C4 continued

$$\text{COMPUTE } a_t: a_t = -(100 - \frac{v_1^4}{64 \times 10^4})^{1/2}$$

$$\text{COMPUTE } v_2: v_2 = v_1 + a_t$$

WHILE  $v_2 > 0$

$$\text{UPDATE DISTANCE } x: x = x_1 + v_1 + \frac{1}{2} a_t$$

UPDATE TIME AND SPEED:

$$t = t + 1; v_1 = v_2$$

FOR THE FINAL TIME INTERVAL

$$\text{COMPUTE } a_t: a_t = -(100 - \frac{v_1^4}{64 \times 10^4})^{1/2}$$

$$\text{COMPUTE } t_{\text{FINAL}}: t_{\text{FINAL}} = -\frac{v_1}{a_t}$$

$$\text{COMPUTE TIME } t_{\text{STOP}}: t_{\text{STOP}} = t + t_{\text{FINAL}}$$

COMPUTE DISTANCE  $x_{\text{TOTAL}}$ :

$$x_{\text{TOTAL}} = x_1 + v_1 t_{\text{FINAL}} + \frac{1}{2} a_t t_{\text{FINAL}}^2$$

PRINT THE VALUES OF  $t_{\text{STOP}}$  AND  $x_{\text{TOTAL}}$

CASE 4: CURVED RAMP,  $\frac{dv}{dt}$  LINEARLY VARYING

FOR EACH SUCCESSIVE TIME INTERVAL

SOLVE THE EQUATION

$$\frac{v_2^4}{64 \times 10^4} + 4v_2^2 - 8v_1v_2 + (4v_1^2 - 100) = 0$$

FOR  $v_2$  USING NEWTON'S METHOD

(SEE, FOR EXAMPLE, CHAPRA AND

CANALE, NUMERICAL METHODS FOR

ENGINEERS, 2d ED., MCGRAW-HILL,

1988.)

WHILE  $v_2 > 0$

$$\text{COMPUTE } (a_t)_2: (a_t)_2 = 2(v_2 - v_1)$$

$$\text{UPDATE DISTANCE } x: x = x_1 + v_1 + \frac{1}{2}(a_t)_2$$

UPDATE TIME AND SPEED:

$$t = t + 1; v_1 = v_2; v_2 = 0$$

FOR THE FINAL TIME INTERVAL

$$\text{COMPUTE } (a_t)_2: (a_t)_2 = 2(v_2 - v_1)$$

$$\text{COMPUTE } t_{\text{FINAL}}: t_{\text{FINAL}} = \left[-2 \frac{v_1}{(a_t)_2}\right]^{1/2}$$

$$\text{COMPUTE TIME } t_{\text{STOP}}: t_{\text{STOP}} = t + t_{\text{FINAL}}$$

COMPUTE DISTANCE  $x_{\text{TOTAL}}$ :

$$x_{\text{TOTAL}} = x_1 + v_1 t_{\text{FINAL}} + \frac{1}{2}(a_t)_2 t_{\text{FINAL}}^2$$

PRINT THE VALUES OF  $t_{\text{STOP}}$  AND  $x_{\text{TOTAL}}$

## PROGRAM OUTPUT

For a straight highway and a constant rate of change of the speed,

time to stop = 8.80 s

distance traveled = 387.2 ft

For a straight highway and a uniformly varying rate of change of the speed,

time to stop = 17.77 s

distance traveled = 789.2 ft

For a curved highway and a constant rate of change of the speed,

time to stop = 11.29 s

distance traveled = 581.4 ft

For a curved highway and a uniformly varying rate of change of the speed,

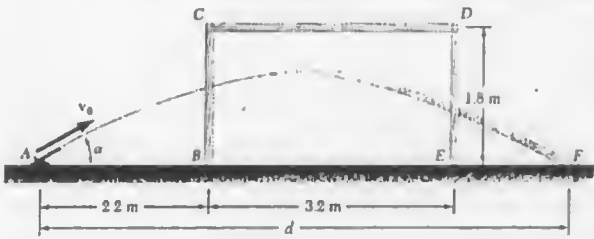
time to stop = 20.71 s

distance traveled = 1015.3 ft

# 11.C5

GIVEN:  $v_0 = 10 \frac{ft}{s}$ ;  $\alpha = 20^\circ$  TO  $80^\circ$  IN  $5^\circ$  INCREMENTS

FIND: (a)  $d$  FOR EACH VALUE OF  $\alpha$   
(b)  $d_{max}$  AND  $\alpha$  WHEN  $d = d_{max}$

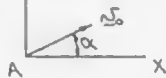


## ANALYSIS

HORIZONTAL MOTION (UNIFORM)  $y$

$$x = x_0^0 + (v_0 \cos \alpha) t$$

$$\text{OR } t = \frac{x}{v_0 \cos \alpha}$$



VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0^0 + (v_0 \sin \alpha) t - \frac{1}{2} g t^2 \quad v_y = v_0 \sin \alpha - g t$$

SUBSTITUTING FOR  $t$ ..

$$y = (\tan \alpha) x - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2 \alpha}$$

AT POINT F,  $x = d$  AND  $y = 0$ :

$$0 = (\tan \alpha) d - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2 \alpha}$$

$$\text{OR } d = \frac{v_0^2}{g} \sin 2\alpha$$

AT THE MAXIMUM THEORETICAL HEIGHT  $y_{max}$  OF THE WATER,  $v_y = 0$ . THEN..

$$0 = v_0 \sin \alpha - g t_{y_{max}} \quad \text{OR } t_{y_{max}} = \frac{v_0 \sin \alpha}{g}$$

$$\text{THEN } y_{max} = (v_0 \sin \alpha) \left( \frac{v_0 \sin \alpha}{g} \right) - \frac{1}{2} g \left( \frac{v_0 \sin \alpha}{g} \right)^2$$

$$= \frac{1}{2} \frac{v_0^2}{g} \sin^2 \alpha$$

$$\text{AND } x_{y_{max}} = (v_0 \cos \alpha) \left( \frac{v_0 \sin \alpha}{g} \right)$$

$$= \frac{v_0^2}{2g} \sin 2\alpha$$

IF THE WATER HITS THE ARBOR,  $y = 1.8$  m AT THE POINT OF IMPACT. THE CORRESPONDING VALUE OF  $x$  IS THEN..

$$1.8 = (\tan \alpha) x - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2 \alpha}$$

$$\text{OR } x_{arbor} = \frac{\tan \alpha \pm \sqrt{(-\tan \alpha)^2 - \frac{3.6g}{v_0^2 \cos^2 \alpha}}}{\frac{g}{v_0^2 \cos^2 \alpha}}$$

WHERE THE (+) AND (-) SIGNS CORRESPOND TO THE WATER HITTING THE ARBOR FROM ABOVE AND FROM BELOW, RESPECTIVELY.

## OUTLINE OF PROGRAM

INPUT MINIMUM AND MAXIMUM VALUES OF  $\alpha$   
INPUT SIZE OF INCREMENT OF  $\alpha$   
FOR EACH VALUE OF  $\alpha$

COMPUTE  $y$  AT  $x = 2.2$  m:

$$y_{2.2} = 2.2 \tan \alpha - \frac{0.0242g}{\cos^2 \alpha}$$

COMPUTE  $y$  AT  $x = 5.4$  m:

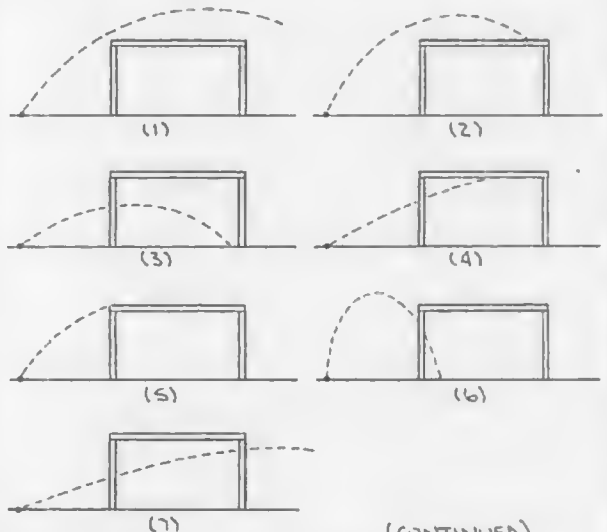
$$y_{5.4} = 5.4 \tan \alpha - \frac{0.1458g}{\cos^2 \alpha}$$

(CONTINUED)

# 11.C5 continued

- (1) IF  $y_{2.2} > 1.8$  m AND  $y_{5.4} > 1.8$  m  
COMPUTE  $d$ :  $d = \frac{100}{g} \sin 2\alpha$   
PRINT THE VALUES OF  $\alpha$  AND  $d$   
NEXT VALUE OF  $\alpha$
- (2) IF  $y_{2.2} > 1.8$  m AND  $y_{5.4} \leq 1.8$  m  
COMPUTE  $(x_{arbor})_{above}$ :  
$$(x_{arbor})_{above} = \frac{100 \cos \alpha}{g} \left( \sin \alpha + \sqrt{\sin^2 \alpha - 0.036g} \right)$$
  
PRINT THE VALUES OF  $\alpha$  AND  $(x_{arbor})_{above}$   
NEXT VALUE OF  $\alpha$
- COMPUTE  $y_{max}$ :  $y_{max} = \frac{50}{g} \sin^2 \alpha$   
COMPUTE  $x_{y_{max}}$ :  $x_{y_{max}} = \frac{50}{g} \sin 2\alpha$
- (3) IF  $y_{max} < 1.8$  m  
COMPUTE  $d$ :  $d = \frac{100}{g} \sin 2\alpha$   
PRINT THE VALUES OF  $\alpha$  AND  $d$   
NEXT VALUE OF  $\alpha$
- (4) IF  $2.2 \text{ m} \leq x_{y_{max}} \leq 5.4 \text{ m}$   
COMPUTE  $(x_{arbor})_{below}$ :  
$$(x_{arbor})_{below} = \frac{100 \cos \alpha}{g} \left( \sin \alpha - \sqrt{\sin^2 \alpha - 0.036g} \right)$$
  
PRINT THE VALUES OF  $\alpha$  AND  $(x_{arbor})_{below}$   
NEXT VALUE OF  $\alpha$
- (5) IF  $y_{2.2} = 1.8$  m  
PRINT "THE WATER HITS THE ARBOR AT CORNER C."  
NEXT VALUE OF  $\alpha$
- (6), (7) IF  $x_{y_{max}} < 2.2$  m OR IF  $y_{5.4} < 1.8$  m  
AND  $x_{y_{max}} > 5.4$  m  
COMPUTE  $d$ :  $d = \frac{100}{g} \sin 2\alpha$   
PRINT THE VALUES OF  $\alpha$  AND  $d$   
NEXT VALUE OF  $\alpha$

THE SEVEN POSSIBLE TRAJECTORIES TESTED FOR IN THE PROGRAM ARE ILLUSTRATED BELOW.



(CONTINUED)

## 11.C5 continued

### PROGRAM OUTPUT

(a)

For  $\alpha = 20.00^\circ$ , the water hits the ground at  $d = 6.552$  m  
 For  $\alpha = 25.00^\circ$ , the water hits the ground at  $d = 7.809$  m  
 For  $\alpha = 30.00^\circ$ , the water hits the ground at  $d = 8.828$  m  
 For  $\alpha = 35.00^\circ$ , the water hits the ground at  $d = 9.579$  m  
 For  $\alpha = 40.00^\circ$ , the water hits the top of the arbor from  
 below at  $x = 3.106$  m  
 For  $\alpha = 45.00^\circ$ , the water hits the top of the arbor from  
 below at  $x = 2.335$  m  
 For  $\alpha = 50.00^\circ$ , the water hits the ground at  $d = 10.039$  m  
 For  $\alpha = 55.00^\circ$ , the water hits the ground at  $d = 9.579$  m  
 For  $\alpha = 60.00^\circ$ , the water hits the ground at  $d = 8.828$  m  
 For  $\alpha = 65.00^\circ$ , the water hits the ground at  $d = 7.809$  m  
 For  $\alpha = 70.00^\circ$ , the water hits the ground at  $d = 6.552$  m  
 For  $\alpha = 75.00^\circ$ , the water hits the top of the arbor from  
 above at  $x = 4.557$  m  
 For  $\alpha = 80.00^\circ$ , the water hits the top of the arbor from  
 above at  $x = 3.133$  m

(b)

For  $\alpha = 46.20^\circ$ , the water hits the top of the arbor from  
 below at  $x = 2.202$  m  
 For  $\alpha = 46.21^\circ$ , the water hits the top of the arbor from  
 below at  $x = 2.201$  m  
 For  $\alpha = 46.22^\circ$ , the water hits the top of the arbor from  
 below at  $x = 2.200$  m  
 For  $\alpha = 46.23^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.24^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.25^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.26^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.27^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.28^\circ$ , the water hits the ground at  $d = 10.184$  m  
 For  $\alpha = 46.29^\circ$ , the water hits the ground at  $d = 10.183$  m  
 For  $\alpha = 46.30^\circ$ , the water hits the ground at  $d = 10.183$  m

## 12.1

GIVEN:  $g = 9.7807(1 + 0.0053 \sin^2 \phi) \text{ m/s}^2$ ;  
 $m = 2 \text{ kg}$

FIND: (a)  $m$  AND  $W$  AT  $\phi = 0$

(b)  $m$  AND  $W$  AT  $\phi = 45^\circ$

(c)  $m$  AND  $W$  AT  $\phi = 60^\circ$

FIRST NOTE THAT AT ALL LATITUDES

$$m = 2.000 \text{ kg}$$

NOW ..  $g = 9.7807(1 + 0.0053 \sin^2 \phi) \text{ m/s}^2$   
 AND  $W = mg$

THEN ..

(a)  $\phi = 0$ :  $W = 2 \text{ kg} \times 9.7807(1 + 0.0053 \sin^2 0) \frac{\text{m}}{\text{s}^2}$

$$\text{OR } W = 19.56 \text{ N}$$

(b)  $\phi = 45^\circ$ :  $W = 2 \text{ kg} \times 9.7807(1 + 0.0053 \sin^2 45^\circ) \frac{\text{m}}{\text{s}^2}$

$$\text{OR } W = 19.61 \text{ N}$$

(c)  $\phi = 60^\circ$ :  $W = 2 \text{ kg} \times 9.7807(1 + 0.0053 \sin^2 60^\circ) \frac{\text{m}}{\text{s}^2}$

$$\text{OR } W = 19.64 \text{ N}$$

## 12.2

GIVEN:  $g = 12.3 \frac{\text{ft}}{\text{s}^2}$ ;  $m = 50 \text{ lb}$

FIND: (a)  $m$  (lb)

(b)  $m$  ( $\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}$ )

(c)  $W$  (lb)

(a) GIVEN ..

(b) HAVE ..  $m = 50 \text{ lb} \times \frac{1 \text{ lb} \cdot \text{s}^2 / \text{ft}}{32.2 \text{ lb} \cdot \text{s}^2 / \text{ft}}$

$$= 1.55280 \frac{\text{lb} \cdot \text{s}^2}{\text{ft}}$$

$$\text{OR } m = 1.553 \frac{\text{lb} \cdot \text{s}^2}{\text{ft}}$$

(c) HAVE ..  $W = mg$

$$= 1.55280 \frac{\text{lb} \cdot \text{s}^2}{\text{ft}} \times 12.3 \frac{\text{ft}}{\text{s}^2}$$

$$\text{OR } W = 19.10 \text{ lb}$$

## 12.3

GIVEN:  $m = 200 \text{ kg}$ ;  $v = 23.4 \times 10^3 \text{ km/h}$

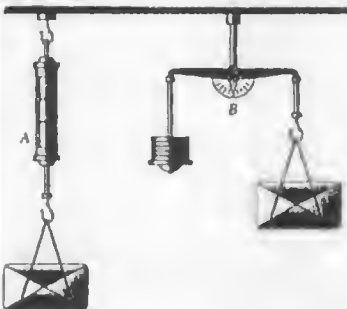
FIND:  $L$

FIRST NOTE ..  $v = 23.4 \times 10^3 \frac{\text{km}}{\text{h}} = 6500 \frac{\text{m}}{\text{s}}$

NOW ..  $L = mv = 200 \text{ kg} \times 6500 \frac{\text{m}}{\text{s}}$

$$\text{OR } L = 1.30 \times 10^6 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

## 12.4



GIVEN: LEVER ARMS OF  
 SCALE B ARE OF  
 EQUAL LENGTH;  
 WHEN  $a_E = 4 \frac{\text{ft}}{\text{s}^2}$ ,  
 $R_A = 14.1 \text{ lb}$

FIND: (a)  $W$

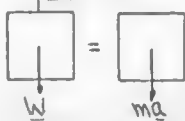
(b)  $R_A$  AND  $m_B$

WHEN

$$a_E = 4 \frac{\text{ft}}{\text{s}^2}$$

(a) WHEN THE ELEVATOR IS MOVING DOWNWARDS HAVE ..

$$R_A = 14.1 \text{ lb}$$



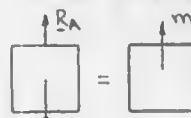
$$+\uparrow \Sigma F_y = ma: W - 14.1 \text{ lb} = \frac{W}{32.2 \frac{\text{ft}}{\text{s}^2}} = 4 \frac{\text{ft}}{\text{s}^2}$$

$$\text{OR } W = 16.10 \text{ lb}$$

(CONTINUED)

## 12.4 CONTINUED

(b) WHEN THE ELEVATOR IS MOVING UPWARDS HAVE ..



$$W = 16.10 \text{ lb}$$

$$\text{OR } R_A = 18.10 \text{ lb}$$

NOW OBSERVE THAT BECAUSE THE LEVER ARMS OF  
 SCALE B ARE EQUAL,

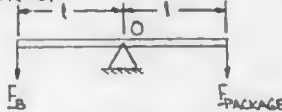
$$m_B = m \quad (m = m_{\text{PACKAGE}})$$

REGARDLESS OF THE ACCELERATION OF THE  
 ELEVATOR. THEN ..

$$m_B = m = \frac{W}{g} = \frac{16.10 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}$$

$$\text{OR } m_B = 0.500 \frac{\text{lb} \cdot \text{s}^2}{\text{ft}}$$

\* PROOF

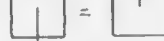


$$+\uparrow \Sigma M_O = 0: R_B - F_{\text{PACKAGE}} = 0$$

$$\text{OR } F_B = F_{\text{PACKAGE}}$$

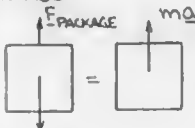
NEXT CONSIDER THE MASS  $m_B$  AND THE PACKAGE  
 FOR AN ARBITRARY ACCELERATION  $a$  OF THE  
 ELEVATOR HAVE ..

MASS B:  $F_B$   $m_B a$



$$W_{\text{MASS B}} = m_B g$$

PACKAGE:



$$W = mg$$

$$+\uparrow \Sigma F_y = ma: F_{\text{PACKAGE}} - mg = ma \quad (2)$$

SUBTRACTING EQ. (2) FROM EQ. (1) AND RECALLING  
 $F_B = F_{\text{PACKAGE}} ..$

$$-m_B g - (-mg) = m_B a - ma$$

$$\text{OR } -(m_B - m)g = (m_B - m)a$$

SINCE, IN GENERAL,  $a \neq g$ , IT FOLLOWS THAT

$$m_B = m \quad \text{Q.E.D.}$$

## 12.5

GIVEN: A PUCK WITH AN INITIAL VELOCITY  
 $v_0$ ; AT  $t = 9 \text{ s}$ ,  $v = 0$ ,  $x = 30 \text{ m}$

FIND: (a)  $v_0$

(b)  $\mu_k$  BETWEEN THE PUCK AND  
 THE ICE

(a) ASSUME UNIFORMLY DECELERATED MOTION.

$$\text{THEN } v = v_0 + at$$

$$\text{AT } t = 9 \text{ s: } 0 = v_0 + a(9) \quad \text{OR } a = -\frac{v_0}{9}$$

$$\text{ALSO .. } v^2 = v_0^2 + 2a(x - x_0)$$

$$\text{AT } t = 9 \text{ s: } 0 = v_0^2 + 2a(30)$$

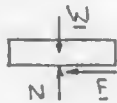
(CONTINUED)

## 12.5 CONTINUED

SUBSTITUTING FOR  $a$ ...  $0 = v_s^2 + 2(-\frac{v_s}{g})(30) = 0$   
OR  $v_s = 6.6667 \frac{m}{s}$  OR  $v_s = 6.67 \frac{m}{s}$

AND  $a = -\frac{6.6667}{g} = -0.74074 \frac{m}{s^2}$

(b)



HAVE...  $\sum F_y = 0: N - W = 0$   
OR  $N = W = mg$

SLIDING:  $F = \mu_k N$   
 $= \mu_k mg$

$\sum F_x = ma: -F = ma$   
OR  $-\mu_k mg = ma$

OR  $\mu_k = -\frac{a}{g} = -\frac{-0.74074 \frac{m}{s^2}}{9.81 \frac{m}{s^2}}$

OR  $\mu_k = 0.0755$

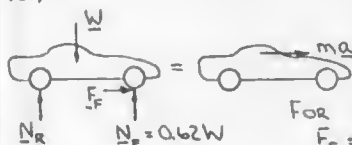
## 12.6

GIVEN: AN AUTOMOBILE INITIALLY AT REST;  
 $\mu_s = 0.80$  BETWEEN THE TIRES AND  
THE PAVEMENT

FIND: (a)  $v_{MAX}$  WHEN  $x = 400$  m FOR FRONT-  
WHEEL DRIVE,  $W_{FRONT}/W = 0.62$

(b)  $v_{MAX}$  WHEN  $x = 400$  m FOR REAR-  
WHEEL DRIVE,  $W_{REAR}/W = 0.43$

(a)



FOR MAXIMUM ACCELERATION...

$F_F = F_{MAX} = \mu_s N_F = 0.8(0.62W)$   
 $= 0.496W = 0.496mg$

NOW...  $\sum F_x = ma: F_F = ma$   
OR  $0.496mg = ma$

THEN  $a = 0.496(9.81 \frac{m}{s^2}) = 4.86576 \frac{m}{s^2}$

SINCE  $a$  IS CONSTANT, HAVE...

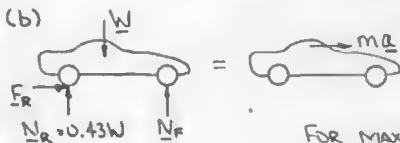
$v^2 = v_0^2 + 2a(x - x_0)$

WHEN  $x = 400$  m:  $v_{MAX}^2 = 2(4.86576 \frac{m}{s^2})(400 \text{ m})$

OR  $v_{MAX} = 62.391 \frac{m}{s}$

OR  $v_{MAX} = 225 \frac{km}{h}$

(b)



FOR MAXIMUM ACCELERATION...

$F_R = F_{MAX} = \mu_s N_R = 0.8(0.43W)$   
 $= 0.344W = 0.344mg$

NOW...  $\sum F_x = ma: F_R = ma$

OR  $0.344mg = ma$

THEN  $a = 0.344(9.81 \frac{m}{s^2}) = 3.37464 \frac{m}{s^2}$

SINCE  $a$  IS CONSTANT, HAVE...

$v^2 = v_0^2 + 2a(x - x_0)$

WHEN  $x = 400$  m:  $v_{MAX}^2 = 2(3.37464 \frac{m}{s^2})(400 \text{ m})$

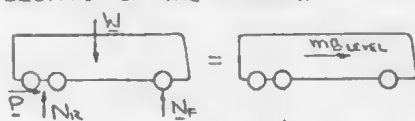
OR  $v_{MAX} = 51.959 \frac{m}{s}$

OR  $v_{MAX} = 187.1 \frac{km}{h}$

## 12.7

GIVEN: (a)  $\theta_{LEVEL} = 3 \frac{ft}{s^2}$ ;  $\theta_{UPGRADE} = 7^\circ$ ;  
 $(v_0)_{UPGRADE} = 60 \frac{mi}{h}$ ;  $P = \text{CONSTANT}$   
FIND:  $x_{UPGRADE}$  WHEN  $v = 50 \frac{mi}{h}$

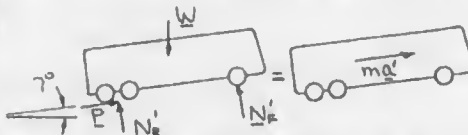
FIRST CONSIDER WHEN THE BUS IS ON THE LEVEL  
SECTION OF THE HIGHWAY.



$a_{LEVEL} = 3 \frac{ft}{s^2}$

HAVE...  $\sum F_x = ma: P = \frac{W}{g} a_{LEVEL}$

NOW CONSIDER WHEN THE BUS IS ON THE UPGRADE.



HAVE...  $\sum F_x = ma: P - W \sin 7^\circ = \frac{W}{g} a'$

SUBSTITUTING FOR  $P$ ...  $\frac{W}{g} a_{LEVEL} - W \sin 7^\circ = \frac{W}{g} a'$

OR  $a' = a_{LEVEL} - g \sin 7^\circ = (3 - 32.2 \sin 7^\circ) \frac{ft}{s^2}$   
 $= -0.92419 \frac{ft}{s^2}$

FOR THE UNIFORMLY DECELERATED MOTION...

$v^2 = (v_0)_{UPGRADE}^2 + 2a'(x_{UPGRADE} - x_0)$

NOTING THAT  $60 \frac{mi}{h} = 88 \frac{ft}{s}$ , THEN WHEN

$v = 50 \frac{mi}{h} (= \frac{5}{6} v_0)$ , HAVE...

$(\frac{5}{6} \cdot 88 \frac{ft}{s})^2 = (88 \frac{ft}{s})^2 + 2(-0.92419 \frac{ft}{s^2}) x_{UPGRADE}$

OR  $x_{UPGRADE} = 1280.16 \text{ ft}$

OR  $x_{UPGRADE} = 0.242 \text{ mi}$

## 12.8

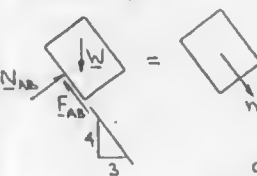


GIVEN:  $a_{AB} = 18 \frac{ft}{s^2}$ ;

$(\mu_k)_{AB} = (\mu_k)_{BC} = \mu_k$

FIND:  $a_{BC}$

FIRST CONSIDER THE MOTION OF THE PACKAGE ON  
SECTION AB.



$\sum F_y = 0: N_{AB} - \frac{3}{5}W = 0$

OR  $N_{AB} = \frac{3}{5}W$

SLIDING:  $F_{AB} = \mu_k N_{AB}$   
 $= \frac{3}{5} \mu_k W$

$\sum F_x = ma: \frac{4}{5}W - F_{AB} = ma_{AB}$   
OR  $\frac{4}{5}W - \frac{3}{5} \mu_k W = \frac{W}{g} a_{AB}$

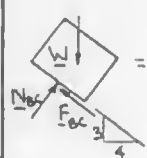
THEN  $\mu_k = \frac{5}{3} (\frac{4}{5} - \frac{18 \frac{ft}{s^2}}{32.2 \frac{ft}{s^2}})$

$= 0.40166$

NOW CONSIDER SECTION BC.

(CONTINUED)

## 12.8 CONTINUED



$$\begin{aligned}
 +\sum F_y = 0: N_{bc} - \frac{4}{5}W &= 0 \\
 \text{OR } N_{bc} &= \frac{4}{5}W \\
 \text{SLIDING: } F_{bc} &= \mu_k N_{bc} \\
 &= \frac{4}{5}\mu_k W \\
 +\sum F_x = ma: \frac{3}{5}W - F_{bc} &= ma_{bc} \\
 \text{OR } \frac{3}{5}W - \frac{4}{5}\mu_k W &= \frac{W}{g}a_{bc} \\
 \text{OR } a_{bc} &= (32.2 \frac{\text{ft}}{\text{s}^2}) \left( \frac{3}{5} - \frac{4}{5} \times 0.40166 \right) \\
 \text{OR } a_{bc} &= 8.97 \frac{\text{ft}}{\text{s}^2} \angle 36.9^\circ
 \end{aligned}$$

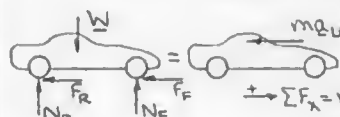
## 12.9

**GIVEN:** AN AUTOMOBILE'S BRAKING DISTANCE,  $x_{br}$ , FROM 90 km/h ON LEVEL PAVEMENT IS 50 m

**FIND:** (a)  $x_{br}$  FROM 90 km/h FOR A 5° INCLINE - UP  
(b)  $x_{br}$  FROM 90 km/h FOR A 3% INCLINE - DOWN

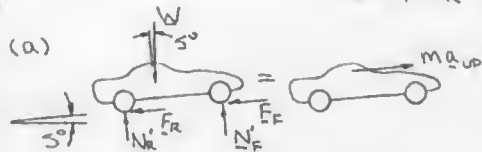
FIRST CONSIDER BRAKING ON LEVEL PAVEMENT. ASSUMING UNIFORMLY DECELERATED MOTION, HAVE..

$$\begin{aligned}
 v^2 &= (v_0)^2 + 2a_{\text{LEVEL}}(x - x_0) \\
 \text{NOTING THAT } 90 \text{ km/h} &= 25 \text{ m/s HAVE..} \\
 0 &= (25 \frac{\text{m}}{\text{s}})^2 + 2a_{\text{LEVEL}}(50 \text{ m}) \\
 \text{OR } a_{\text{LEVEL}} &= -6.25 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$



$$\begin{aligned}
 +\sum F_x = ma: -(F_f + F_r) &= -ma_{\text{LEVEL}} \\
 \text{OR } F_f + F_r &= (6.25 \text{ m}) N
 \end{aligned}$$

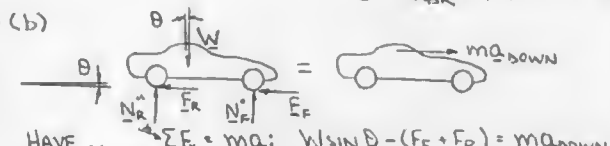
(a)



ASSUMING THAT THE BRAKING FORCE ( $F_f + F_r$ ) IS INDEPENDENT OF THE GRADE, HAVE..

$$\begin{aligned}
 +\sum F_x = ma: -(F_f + F_r) - W \sin 5^\circ &= ma_{\text{UP}} \\
 \text{OR } -6.25 \text{ m} - mg \sin 5^\circ &= ma_{\text{UP}} \\
 \text{THEN } a_{\text{UP}} &= -(6.25 + 9.81 \sin 5^\circ) = -7.1050 \frac{\text{m}}{\text{s}^2} \\
 \text{FINALLY.. } v^2 &= (v_0)^2 + 2a_{\text{UP}}(x_{br} - x_0) \\
 \text{SUBSTITUTING.. } 0 &= (25 \frac{\text{m}}{\text{s}})^2 + 2(-7.1050 \frac{\text{m}}{\text{s}^2})x_{br} \\
 \text{OR } x_{br} &= 44.0 \text{ m}
 \end{aligned}$$

(b)



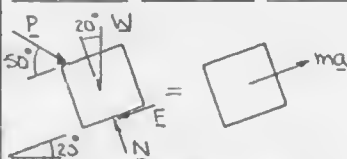
$$\begin{aligned}
 \text{HAVE.. } +\sum F_x = ma: W \sin \theta - (F_f + F_r) &= ma_{\text{DOWN}} \\
 \text{NOW.. } \tan \theta &= 0.03 \Rightarrow \theta \text{ SMALL} \Rightarrow \sin \theta \approx \tan \theta \\
 \text{THEN.. } mg \tan \theta - 6.25 \text{ m} &= ma_{\text{DOWN}} \\
 \text{OR } a_{\text{DOWN}} &= 9.81(0.03) - 6.25 = -5.9557 \frac{\text{m}}{\text{s}^2} \\
 \text{FINALLY.. } v^2 &= (v_0)^2 + 2a_{\text{DOWN}}(x_{br} - x_0) \\
 \text{SUBSTITUTING.. } 0 &= (25 \frac{\text{m}}{\text{s}})^2 + 2(-5.9557 \frac{\text{m}}{\text{s}^2})x_{br} \\
 \text{OR } x_{br} &= 52.5 \text{ m}
 \end{aligned}$$

## 12.10



**GIVEN:**  $m = 20 \text{ kg}$ ;  $\mu_s = 0$ ;  
AT  $t = 10 \text{ s}$ ,  $\Delta x = 5 \text{ m}$ ;  
 $\mu_s = 0.4$ ,  $\mu_k = 0.3$

**FIND:** P



FIRST OBSERVE THAT THE PACKAGE IS UNIFORMLY ACCELERATED SINCE ALL OF THE FORCES ARE CONSTANT. THEN..

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\begin{aligned}
 \text{AT } t = 10 \text{ s: } 5 \text{ m} &= \frac{1}{2} a (10 \text{ s})^2 \\
 \text{OR } a &= 0.10 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{NOW.. } +\sum F_y = 0: N - W \cos 20^\circ - P \sin 50^\circ &= 0 \\
 \text{OR } N &= mg \cos 20^\circ + P \sin 50^\circ
 \end{aligned}$$

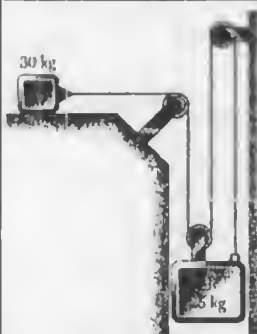
$$\begin{aligned}
 \text{SLIDING: } F &= \mu_k N \\
 &= \mu_k (mg \cos 20^\circ + P \sin 50^\circ)
 \end{aligned}$$

$$+\sum F_x = ma: P \cos 50^\circ - W \sin 20^\circ - F = ma$$

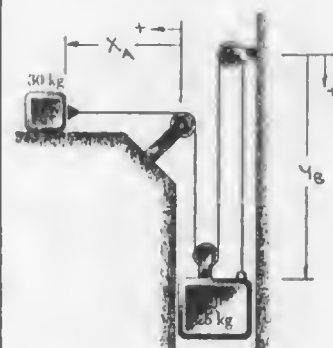
$$\text{THEN.. } P \cos 50^\circ - mg \sin 20^\circ - \mu_k (mg \cos 20^\circ + P \sin 50^\circ) = ma$$

$$\begin{aligned}
 \text{OR } P &= \frac{m[a + g(\sin 20^\circ + \mu_k \cos 20^\circ)]}{\cos 50^\circ - \mu_k \sin 50^\circ} \\
 &= \frac{20 \text{ kg} [0.10 \frac{\text{m}}{\text{s}^2} + 9.81 \frac{\text{m}}{\text{s}^2} (\sin 20^\circ + 0.3 \cos 20^\circ)]}{\cos 50^\circ - 0.3 \sin 50^\circ} \\
 \text{OR } P &= 301 \text{ N}
 \end{aligned}$$

## 12.11 and 12.12



**GIVEN:** BLOCKS A AND B AND THE PULLEY/ CABLE SYSTEM, WHICH IS OF NEGLIGIBLE MASS, SHOWN;  
 $(v_A)_0 = (v_B)_0 = 0$



FROM THE DIAGRAM..

$$\begin{aligned}
 x_A + 3y_B &= \text{CONSTANT} \\
 \text{THEN.. } v_A + 3v_B &= 0 \\
 \text{AND } a_A + 3a_B &= 0 \\
 \text{OR } a_A &= -3a_B \quad (1)
 \end{aligned}$$

(CONTINUED)

# 12.11 and 12.12 CONTINUED

12.11 GIVEN:  $\mu_A = 0$   
 FIND: (a)  $a_A$  AND  $a_B$   
 (b)  $T$

(a)

A:

$$\sum F_x = m_A a_A \quad -T = m_A a_A$$

Using Eq. (1)  $T = 3m_A a_B$

B:

$$\sum F_y = m_B a_B \quad W_B - 3T = m_B a_B$$

SUBSTITUTING FOR T...

$$m_B g - 3(3m_A a_B) = m_B a_B$$

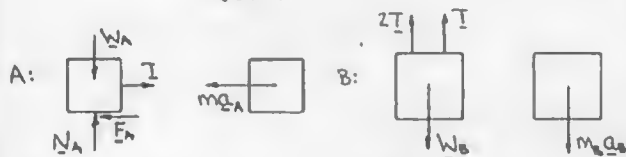
$$m_B g - 9m_A a_B = m_B a_B$$

$$a_B = \frac{g}{1 + 9 \frac{m_A}{m_B}} = \frac{9.81 \frac{m}{s^2}}{1 + 9 \frac{30 \text{ kg}}{25 \text{ kg}}} = 0.831 \frac{m}{s^2}$$

THEN  $a_A = 2.49 \frac{m}{s^2}$   
 AND  $a_B = 0.831 \frac{m}{s^2}$

(b) HAVE  $T = 3 \cdot 30 \text{ kg} \cdot 0.831 \frac{m}{s^2}$   
 OR  $T = 74.8 \text{ N}$

12.12 GIVEN:  $(\mu_s)_A = 0.25$ ,  $(\mu_k)_A = 0.20$   
 FIND: (a)  $a_A$  AND  $a_B$   
 (b)  $T$



FIRST DETERMINE IF THE BLOCKS WILL MOVE.  
 WITH  $a_A = a_B = 0$ , HAVE..

B:  $\sum F_y = 0: W_B - 3T = 0$  OR  $T = \frac{1}{3} m_B g$

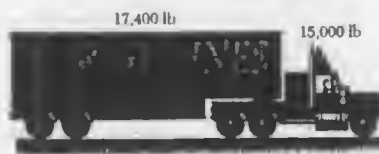
A:  $\sum F_x = 0: F_A - T = 0$   
 THEN  $F_A = \frac{1}{3} \cdot 25 \text{ kg} \cdot 9.81 \frac{m}{s^2} = 81.75 \text{ N}$   
 $\sum F_y = 0: W_A - N_A = 0$  OR  $N_A = m_A g$   
 ALSO..  $(F_A)_{\max} = (\mu_s)_A N_A = (\mu_s)_A m_A g$   
 $= 0.25 \cdot 30 \text{ kg} \cdot 9.81 \frac{m}{s^2}$   
 $= 73.575 \text{ N}$

$\therefore F_A > (F_A)_{\max}$  WHICH IMPLIES THAT THE BLOCKS WILL MOVE.

(a) A:  $\sum F_y = 0: W_A - N_A = 0$  OR  $N_A = m_A g$   
 SLIDING:  $F_A = (\mu_k)_A N_A = 0.20 m_A g$   
 $\sum F_x = m_A a_A: F_A - T = m_A a_A$   
 USING EQ. (1)  $T = 0.20 m_A g + 3m_A a_B$   
 B:  $\sum F_y = m_B a_B: W_B - 3T = m_B a_B$   
 OR  $m_B g - 3(0.20 m_A g + 3m_A a_B) = m_B a_B$   
 OR  $a_B = \frac{g(1 - 0.6 \frac{m_A}{m_B})}{1 + 9 \frac{m_A}{m_B}} = \frac{(9.81 \frac{m}{s^2})(1 - 0.6 \frac{30 \text{ kg}}{25 \text{ kg}})}{1 + 9 \frac{30 \text{ kg}}{25 \text{ kg}}} = 0.23278 \frac{m}{s^2}$   
 THEN..  $a_A = 0.698 \frac{m}{s^2}$   
 AND  $a_B = 0.233 \frac{m}{s^2}$   
 (b) HAVE  $T = (30 \text{ kg})(0.20 \cdot 9.81 + 3 \cdot 0.23278) \frac{m}{s^2}$  OR  $T = 79.8 \text{ N}$

# 12.13

GIVEN: AT  $t=0$ ,  $v = 60 \frac{mi}{h}$ , BRAKES ARE APPLIED;  $(F_{BR})_{TRAC} = 3600 \text{ lb}$ ,  $(F_{BR})_{TRL} = 13,700 \text{ lb}$   
 FIND: (a)  $\Delta x$  WHEN  $v = 0$   
 (b)  $P_{HITCH}$



(a)

$$\sum F_x = m a: -(F_{BR})_{TRAC} - (F_{BR})_{TRL} = \frac{W_{TOTAL}}{g} a$$

OR  $a = -\frac{32.2 \frac{ft}{s^2}}{(15,000 + 17,400) \text{ lb}} (3600 + 13,700) \text{ lb}$   
 $= -17.1932 \frac{ft}{s^2}$

FOR UNIFORMLY DECELERATED MOTION..  
 $v^2 = v_0^2 + 2a(x - x_0)$   $v_0 = 60 \frac{mi}{h} = 88 \frac{ft}{s}$   
 WHEN  $v = 0: 0 = (88 \frac{ft}{s})^2 + 2(-17.1932 \frac{ft}{s^2})(\Delta x)$   
 OR  $\Delta x = 225 \text{ ft}$

(b)

$$\sum F_x = m_{TRL} a: -(F_{BR})_{TRL} + P_{HITCH} = \frac{W_{TRL}}{g} a$$

THEN  $P_{HITCH} = 13,700 \text{ lb} + \frac{17,400 \text{ lb}}{32.2 \frac{ft}{s^2}} (-17.1932 \frac{ft}{s^2})$   
 OR  $P_{HITCH} = 4410 \text{ lb (T)}$

# 12.14

GIVEN: TRACTOR-TRAILER OF PROBLEM 12.13 WITH A SECOND TRAILER..  
 $(W)_{TRL2} = 24,900 \text{ lb}$ ,  $(F_{BR})_{TRL2} = 12,900 \text{ lb}$ ;  
 AT  $t=0$ ,  $v = 60 \frac{mi}{h}$ , BRAKES ARE APPLIED;  $(F_{BR})_{TRAC} = 3600 \text{ lb}$ ,  $(F_{BR})_{TRL1} = 13,700 \text{ lb}$   
 FIND: (a)  $\Delta x$  WHEN  $v = 0$   
 (b)  $(P_{HITCH})_{TRAC}$

(a)

$$\sum F_x = m a: -(F_{BR})_{TRAC} - (F_{BR})_{TRL1} - (F_{BR})_{TRL2} = \frac{W_{TOTAL}}{g} a$$

OR  $a = -\frac{32.2 \frac{ft}{s^2}}{(15,000 + 17,400 + 24,900) \text{ lb}} (3600 + 13,700 + 12,900) \text{ lb}$   
 $= -16.9710 \frac{ft}{s^2}$

(CONTINUED)



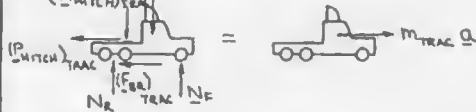
## 12.14 CONTINUED

FOR UNIFORMLY DECELERATED MOTION..

$$v^2 = v_0^2 + 2a(x - x_0) \quad v_0 = 60 \frac{\text{ft}}{\text{s}} = 88 \frac{\text{ft}}{\text{s}}$$

WHEN  $v = 0$ :  $0 = (88 \frac{\text{ft}}{\text{s}})^2 + 2(-16.9710 \frac{\text{ft}}{\text{s}^2})(\Delta x)$   
OR  $\Delta x = 228 \text{ ft}$

(b) 15,000 lb

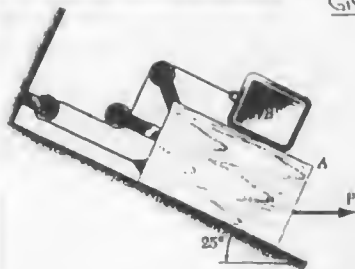


$$\sum F_x = m_{\text{TRAC}} a: -(F_B)_{\text{TRAC}} - (P_{\text{HITCH}})_{\text{TRAC}} = \frac{W_{\text{TRAC}}}{g} a$$

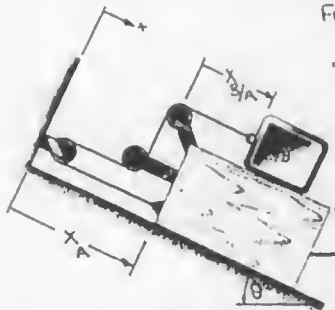
$$\text{THEN } (P_{\text{HITCH}})_{\text{TRAC}} = -3600 \text{ lb} - \frac{15,000 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} (-16.9710 \frac{\text{ft}}{\text{s}^2})$$

$$\text{OR } (P_{\text{HITCH}})_{\text{TRAC}} = 4310 \text{ lb (T)}$$

## 12.15 and 12.16



GIVEN:  $m_A = 40 \text{ kg}$ ,  $m_B = 8 \text{ kg}$ ;  
 $\mu_s = 0.20$ ,  $\mu_k = 0.15$



FROM THE DIAGRAM..

$$z x_A + x_{B/A} = \text{CONSTANT}$$

$$\text{THEN } z \dot{x}_A + \dot{x}_{B/A} = 0$$

$$\text{AND } z a_A + a_{B/A} = 0$$

NOW..

$$a_B = a_A + a_{B/A}$$

THEN

$$a_B = a_A + (-2a_A)$$

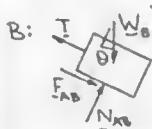
$$\text{OR } a_B = -a_A \quad (1)$$

12.15 GIVEN:  $P = 0$ ,  $\theta = 25^\circ$

FIND: (a)  $a_B$

(b)  $T$

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF  $\theta$ . THUS, SEEK THE VALUE OF  $\theta$  FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.



$$\sum F_y = 0: N_{AB} - W_B \cos \theta = 0$$

$$\text{OR } N_{AB} = m_B g \cos \theta$$

$$\text{NOW.. } F_{AB} = \mu_s N_{AB}$$

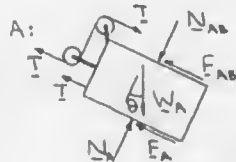
$$= 0.2 m_B g \cos \theta$$

$$\sum F_x = 0: -T + F_{AB} + W_B \sin \theta = 0$$

$$\text{OR } T = m_B g (0.2 \cos \theta + \sin \theta)$$

(CONTINUED)

## 12.15 and 12.16 CONTINUED



$$\sum F_y = 0: N_{AB} - N_{AB} - W_A \cos \theta = 0$$

$$\text{OR } N_{AB} = (m_A + m_B) g \cos \theta$$

$$\text{NOW.. } F_{AB} = \mu_s N_{AB}$$

$$= 0.2 (m_A + m_B) g \cos \theta$$

$$\sum F_x = 0: -T - F_{AB} - F_{AB} + W_A \sin \theta = 0$$

$$\text{OR } T = m_A g \sin \theta - 0.2 (m_A + m_B) g \cos \theta$$

$$- 0.2 m_B g \cos \theta$$

$$= g [m_A \sin \theta - 0.2 (m_A + 2m_B) \cos \theta]$$

EQUATING THE TWO EXPRESSIONS FOR T...

$$m_B g (0.2 \cos \theta + \sin \theta) = g [m_A \sin \theta - 0.2 (m_A + 2m_B) \cos \theta]$$

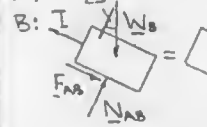
$$\text{OR } B(0.2 + \tan \theta) = [40 \tan \theta - 0.2(40 + 2 \cdot 8)]$$

$$\text{OR } \tan \theta = 0.4$$

$$\text{OR } \theta = 21.8^\circ$$

FOR IMPENDING MOTION, SINCE  $\theta < 25^\circ$ , THE BLOCKS WILL MOVE. NOW CONSIDER THE MOTION OF THE BLOCKS.

(a)



$$\sum F_y = 0: N_{AB} - W_B \cos 25^\circ = 0$$

$$\text{OR } N_{AB} = m_B g \cos 25^\circ$$

$$\text{SLIDING: } F_{AB} = \mu_k N_{AB}$$

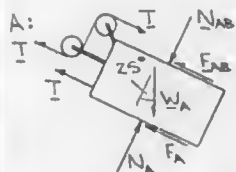
$$= 0.15 m_B g \cos 25^\circ$$

$$\sum F_x = m_B a_B: -T + F_{AB} + W_B \sin 25^\circ = m_B a_B$$

$$\text{OR } T = m_B [g(0.15 \cos 25^\circ + \sin 25^\circ) - a_B]$$

$$= 8[9.81(0.15 \cos 25^\circ + \sin 25^\circ) - a_B]$$

$$= 8(5.47952 - a_B) \quad (N)$$



$$\sum F_y = 0: N_{AB} - N_{AB} - W_A \cos 25^\circ = 0$$

$$\text{OR } N_{AB} = (m_A + m_B) g \cos 25^\circ$$

$$\text{SLIDING: } F_{AB} = \mu_k N_{AB} = 0.15 (m_A + m_B) g \cos 25^\circ$$

$$\sum F_x = m_A a_A: -T - F_{AB} - F_{AB} + W_A \sin 25^\circ = m_A a_A$$

SUBSTITUTING AND USING EQ. (1)..

$$T = m_A g \sin 25^\circ - 0.15 (m_A + m_B) g \cos 25^\circ - 0.15 m_B g \cos 25^\circ$$

$$- m_A (-a_B)$$

$$= g [m_A \sin 25^\circ - 0.15 (m_A + 2m_B) \cos 25^\circ] + m_A a_B$$

$$= 9.81 [40 \sin 25^\circ - 0.15 (40 + 2 \cdot 8) \cos 25^\circ] + 40 a_B$$

$$= 91.15202 + 40 a_B \quad (N)$$

EQUATING THE TWO EXPRESSIONS FOR T..

$$8(5.47952 - a_B) = 91.15202 + 40 a_B$$

$$\text{OR } a_B = -0.98575 \frac{\text{m}}{\text{s}^2}$$

$$\therefore a_B = 0.986 \frac{\text{m}}{\text{s}^2} \uparrow 25^\circ$$

(b) HAVE..  $T = 8[5.47952 - (-0.98575)]$

$$\text{OR } T = 51.7 \text{ N}$$

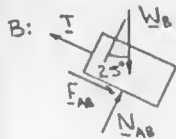
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## 12.16 CONTINUED

12.16 GIVEN:  $P = 40 \text{ N} \rightarrow$ ,  $\theta = 25^\circ$

FIND: (a)  $a_B$   
(b)  $T$

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF  $P$ . THUS, SEEK THE VALUE OF  $P$  FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.



$$+\uparrow \Sigma F_y = 0: N_{AB} - W_B \cos 25^\circ = 0$$

$$\text{OR } N_{AB} = m_B g \cos 25^\circ$$

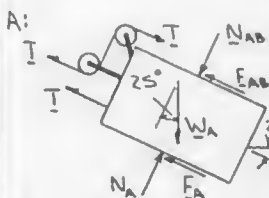
$$\text{NOW } F_{AB} = \mu_s N_{AB} = 0.2 m_B g \cos 25^\circ$$

$$+\uparrow \Sigma F_x = 0: -T + F_{AB} + W_B \sin 25^\circ = 0$$

$$\text{OR } T = 0.2 m_B g \cos 25^\circ + m_B g \sin 25^\circ$$

$$= (8 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(0.2 \cos 25^\circ + \sin 25^\circ)$$

$$= 47.39249 \text{ N}$$



$$+\uparrow \Sigma F_y = 0: N_A - N_{AB} - W_A \cos 25^\circ + P \sin 25^\circ = 0$$

$$\text{OR } N_A = (m_A + m_B) g \cos 25^\circ - P \sin 25^\circ$$

$$\text{NOW } F_A = \mu_s N_A$$

$$\text{OR } F_A = 0.2 [(m_A + m_B) g \cos 25^\circ - P \sin 25^\circ]$$

$$+\uparrow \Sigma F_x = 0: -T - F_A - F_{AB} + W_A \sin 25^\circ + P \cos 25^\circ = 0$$

$$\text{OR } -T - 0.2 [(m_A + m_B) g \cos 25^\circ - P \sin 25^\circ] - 0.2 m_B g \cos 25^\circ + m_A g \sin 25^\circ + P \cos 25^\circ = 0$$

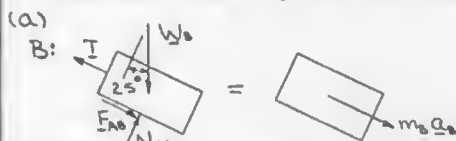
$$\text{OR } P(0.2 \sin 25^\circ + \cos 25^\circ) - T + 0.2 [(m_A + 2m_B) g \cos 25^\circ] - m_A g \sin 25^\circ$$

$$\text{THEN } P(0.2 \sin 25^\circ + \cos 25^\circ) = 47.39249 \text{ N}$$

$$+ 9.81 \frac{\text{m}}{\text{s}^2} \{0.2 [(40 + 2 \cdot 8) \cos 25^\circ - 40 \sin 25^\circ] \text{ kg}\}$$

$$\text{OR } P = -19.04 \text{ N FOR IMPENDING MOTION.}$$

SINCE  $P < 40 \text{ N}$ , THE BLOCKS WILL MOVE. NOW CONSIDER THE MOTION OF THE BLOCKS.



$$+\uparrow \Sigma F_y = 0: N_{AB} - W_B \cos 25^\circ = 0$$

$$\text{OR } N_{AB} = m_B g \cos 25^\circ$$

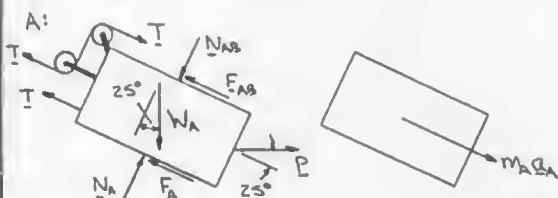
$$\text{SLIDING: } F_{AB} = \mu_k N_{AB} = 0.15 m_B g \cos 25^\circ$$

$$+\uparrow \Sigma F_x = m_B a_B: -T + F_{AB} + W_B \sin 25^\circ = m_B a_B$$

$$\text{OR } T = m_B [g(0.15 \cos 25^\circ + \sin 25^\circ) - a_B]$$

$$= 8 [9.81(0.15 \cos 25^\circ + \sin 25^\circ) - a_B]$$

$$= 8(5.47952 - a_B) \text{ (N)}$$



(CONTINUED)

## 12.16 CONTINUED

$$+\uparrow \Sigma F_y = 0: N_A - N_{AB} - W_A \cos 25^\circ + P \sin 25^\circ = 0$$

$$\text{OR } N_A = (m_A + m_B) g \cos 25^\circ - P \sin 25^\circ$$

$$\text{SLIDING: } F_A = \mu_k N_A = 0.15 [(m_A + m_B) g \cos 25^\circ - P \sin 25^\circ]$$

$$+\uparrow \Sigma F_x = m_A a_A: -T - F_A - F_{AB} + W_A \sin 25^\circ + P \cos 25^\circ = m_A a_A$$

SUBSTITUTING AND USING EQ. (1)...

$$T = m_A g \sin 25^\circ - 0.15 [(m_A + m_B) g \cos 25^\circ - P \sin 25^\circ]$$

$$- 0.15 m_B g \cos 25^\circ + P \cos 25^\circ - m_A (-a_B)$$

$$= g [m_A \sin 25^\circ - 0.15 (m_A + 2m_B) \cos 25^\circ]$$

$$+ P(0.15 \sin 25^\circ + \cos 25^\circ) + m_A a_B$$

$$= 9.81 [40 \sin 25^\circ - 0.15 (40 + 2 \cdot 8) \cos 25^\circ]$$

$$+ 40(0.15 \sin 25^\circ + \cos 25^\circ) + 40 a_B$$

$$= 129.94004 + 40 a_B \text{ (N)}$$

EQUATING THE TWO EXPRESSIONS FOR  $T$ ...

$$8(5.47952 - a_B) = 129.94004 + 40 a_B$$

$$\text{OR } a_B = -1.79383 \frac{\text{m}}{\text{s}^2}$$

$$\therefore a_B = 1.794 \frac{\text{m}}{\text{s}^2} \angle 25^\circ$$

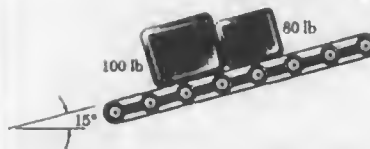
$$(b) \text{ HAVE } T = 8[5.47952 - (-1.79383)]$$

$$\text{OR } T = 58.2 \text{ N}$$

## 12.17

GIVEN: AT  $t = 0$ ,  $v_A = v_B = 0$ , BELT BEGINS TO MOVE  $\rightarrow$  SO THAT SLIPPING OF BOTH BOXES OCCURS;  $(\mu_k)_A = 0.30$ ,  $(\mu_k)_B = 0.32$

FIND:  $a_A$  AND  $a_B$



ASSUME THAT  $a_B > a_A$  SO THAT THE NORMAL FORCE  $N_{AB}$  BETWEEN THE BOXES IS ZERO.

A:  $+\uparrow \Sigma F_y = 0: N_A - W_A \cos 15^\circ = 0$   
OR  $N_A = W_A \cos 15^\circ$   
SLIPPING:  $F_A = (\mu_k)_A N_A = 0.3 W_A \cos 15^\circ$

$$+\uparrow \Sigma F_x = m_A a_A: F_A - W_A \sin 15^\circ = m_A a_A$$

$$\text{OR } 0.3 W_A \cos 15^\circ - W_A \sin 15^\circ = \frac{W_A}{g} a_A$$

$$\text{OR } a_A = (32.2 \frac{\text{ft}}{\text{s}^2})(0.3 \cos 15^\circ - \sin 15^\circ) = 0.997 \frac{\text{ft}}{\text{s}^2}$$

B:  $+\uparrow \Sigma F_y = 0: N_B - W_B \cos 15^\circ = 0$   
OR  $N_B = W_B \cos 15^\circ$   
SLIPPING:  $F_B = (\mu_k)_B N_B = 0.32 W_B \cos 15^\circ$

$$+\uparrow \Sigma F_x = m_B a_B: F_B - W_B \sin 15^\circ = m_B a_B$$

$$\text{OR } 0.32 W_B \cos 15^\circ - W_B \sin 15^\circ = \frac{W_B}{g} a_B$$

$$\text{OR } a_B = (32.2 \frac{\text{ft}}{\text{s}^2})(0.32 \cos 15^\circ - \sin 15^\circ) = 1.619 \frac{\text{ft}}{\text{s}^2}$$

$a_B > a_A \Rightarrow$  ASSUMPTION IS CORRECT

$$\therefore a_A = 0.997 \frac{\text{ft}}{\text{s}^2} \angle 15^\circ$$

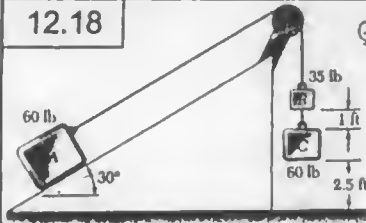
$$a_B = 1.619 \frac{\text{ft}}{\text{s}^2} \angle 15^\circ$$

NOTE: IF IT IS ASSUMED THAT THE BOXES REMAIN IN CONTACT ( $N_{AB} \neq 0$ ), THEN (CONTINUED)

## 12.17 CONTINUED

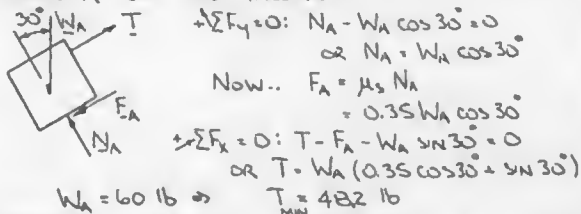
$a_A = a_B$  AND FIND  $(\Sigma F_x = ma)$   
 A:  $0.3W_A \cos 15^\circ - W_A \sin 15^\circ - N_{AB} = \frac{W_A}{g} a$   
 B:  $0.32W_B \cos 15^\circ - W_B \sin 15^\circ - N_{AB} = \frac{W_B}{g} a$   
 SOLVING YIELDS  $a = 1.273 \frac{\text{ft}}{\text{s}^2}$  AND  $N_{AB} = -0.859 \text{ lb}$ ,  
 WHICH CONTRADICTS THE ASSUMPTION.

## 12.18



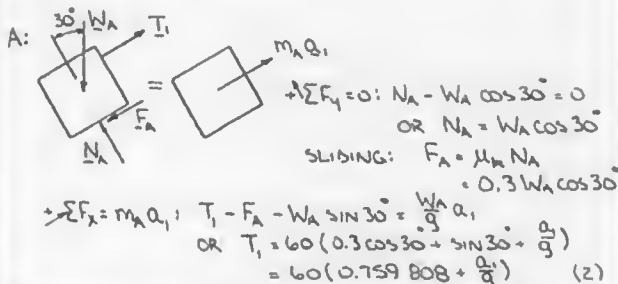
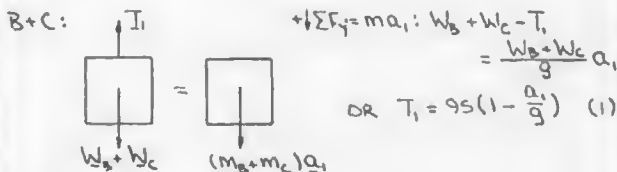
GIVEN:  $\mu_s = 0.35, \mu_k = 0.30$ ;  
 $t = 0, v = 0$   
 FIND: (a)  $(\dot{x}_A)_{\text{MAX}}$   
 (b)  $\Delta x_A$  WHEN  
 $\dot{x}_A = 0$

FIRST DETERMINE THE COMBINED MINIMUM WEIGHT OF BLOCKS B AND C FOR IMPENDING MOTION OF PACKAGE A UP THE INCLINE.



THEREFORE, SINCE  $T_{\text{MIN}}$  IS LESS THAN  $T_{B+C}$  (95 lb), PACKAGE A WILL MOVE UP THE INCLINE WHEN BLOCKS B AND C ARE RELEASED.

(a) "MOTION 1" - A, B, AND C MOVE TOGETHER THROUGH 2.5 ft.



EQUATING THE TWO EXPRESSIONS FOR  $T_2$ ..

$$95(1 - \frac{a_1}{32.2}) = 60(0.759808 + \frac{a_1}{32.2})$$

$$\text{OR } a_1 = 10.2648 \frac{\text{ft}}{\text{s}^2}$$

"MOTION 2" - C IS AT REST, A AND B MOVE TOGETHER THROUGH 1 ft. FOR THIS CASE, EQS. (1) AND (2) BECOME..

$$T_2 = 35(1 - \frac{a_2}{g}) \quad (1')$$

(CONTINUED)

## 12.18 continued

$$T_2 = 60(0.759808 + \frac{a_2}{g}) \quad (2')$$

$$\text{THEN } 35(1 - \frac{a_2}{32.2}) = 60(0.759808 + \frac{a_2}{32.2})$$

$$\text{OR } a_2 = -3.5889 \frac{\text{ft}}{\text{s}^2}$$

$\therefore$  SINCE  $a_2 < 0$ , A BEGINS TO DECELERATE AFTER BLOCK C REACHES THE GROUND; THUS,  $(\dot{x}_A)_{\text{MAX}}$  OCCURS AT THE END OF "MOTION 1." FOR THE UNIFORMLY ACCELERATION OF "MOTION 1," HAVE..

$$v_A^2 = (v_A^0)^2 + 2a_1(x - x_0)$$

$$\text{WHEN } \Delta x = 2.5 \text{ ft: } (\dot{x}_A)_{\text{MAX}}^2 = 2(10.2648 \frac{\text{ft}}{\text{s}^2})(2.5 \text{ ft})$$

$$\text{OR } (\dot{x}_A)_{\text{MAX}} = 7.1641 \frac{\text{ft}}{\text{s}} \quad (\dot{x}_A)_{\text{MAX}} = 7.16 \frac{\text{ft}}{\text{s}} \quad \Delta 30^\circ$$

(b) FIRST NOTE THAT AT THE END OF "MOTION 2," THE SPEED OF PACKAGE A IS..

$$(v_A^2)_2 = (v_A^1)^2 + 2a_2 \Delta x_2$$

$$= (7.1641 \frac{\text{ft}}{\text{s}})^2 + 2(-3.5889 \frac{\text{ft}}{\text{s}^2})(1 \text{ ft})$$

$$\text{OR } (v_A)_2 = 6.6443 \frac{\text{ft}}{\text{s}}$$

"MOTION 3" - B AND C ARE AT REST, A CONTINUES UP THE INCLINE AND FINALLY COMES TO REST.

FOR THIS CASE,  $T = 0$  SO THAT EQ (2) BECOMES

$$60(0.759808 + \frac{a_3}{g}) = 0 \quad (2'')$$

$$\text{THEN.. } a_3 = -0.759808(32.2) = -24.466 \frac{\text{ft}}{\text{s}^2}$$

$$\text{THEN.. } v_A^2 = (v_A^1)^2 + 2a_3(x - x_0)_3$$

$$\text{WHEN } v_A = 0: 0 = (6.6443 \frac{\text{ft}}{\text{s}})^2 + 2(-24.466 \frac{\text{ft}}{\text{s}^2})\Delta x_3$$

$$\text{OR } \Delta x_3 = 0.9022 \text{ ft}$$

THE TOTAL DISTANCE  $\Delta x_A$  TRAVELED BY A UP THE INCLINE BEFORE COMING TO REST IS THEN..

$$\Delta x_A = \Delta x_1 + \Delta x_2 + \Delta x_3 = (2.5 + 1 + 0.9022) \text{ ft}$$

$$\text{OR } \Delta x_A = 4.40 \text{ ft}$$

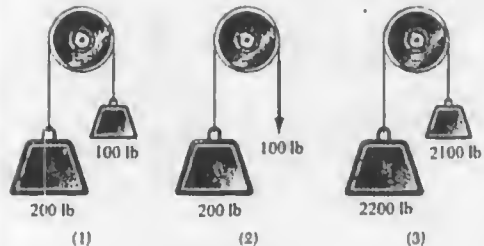
## 12.19

GIVEN: THE THREE SYSTEMS SHOWN;  $v_0 = 0$   
 FIND (FOR EACH SYSTEM):

(a)  $a_A$

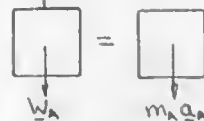
(b)  $\dot{x}_A$  WHEN  $\Delta x_A = 10 \text{ ft}$

(c)  $t$  WHEN  $\dot{x}_A = 20 \frac{\text{ft}}{\text{s}}$

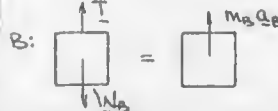


### SYSTEM 1

(a)  $\Sigma F_y = m_A a_A: W_A - T = \frac{W_A}{g} a_A$   
 OR  $T = 200(1 - \frac{a_A}{g})$



B:  $\Sigma F_y = m_B a_B: T - W_B = \frac{W_B}{g} a_B$   
 OR  $T = 100(1 + \frac{a_B}{g})$



(CONTINUED)

# 12.19 continued

EQUATING THE TWO EXPRESSIONS FOR T AND NOTING THAT  $|a_A| = |a_B| \dots$

$$200(1 - \frac{a_A}{g}) = 100(1 + \frac{a_A}{g})$$

$$\text{OR } a_A = \frac{1}{3}g = \frac{1}{3}(32.2 \frac{\text{ft}}{\text{s}^2}) = 10.7333 \frac{\text{ft}}{\text{s}^2} \downarrow$$

$$\therefore a_A = 10.73 \frac{\text{ft}}{\text{s}^2} \downarrow$$

(b) HAVE  $\dots v_A^2 = (v_A^0)^2 + 2a_A(y - y_0)$   
 WHEN  $\Delta y_A = 10 \text{ ft}$ :  $v_A^2 = 2(10.7333 \frac{\text{ft}}{\text{s}^2})(10 \text{ ft})$   
 OR  $v_A = 14.65 \frac{\text{ft}}{\text{s}} \downarrow$

(c) HAVE  $\dots v_A = (v_A^0)^2 + a_A t$   
 WHEN  $v_A = 20 \frac{\text{ft}}{\text{s}}$ :  $20 \frac{\text{ft}}{\text{s}} = (10.7333 \frac{\text{ft}}{\text{s}^2})t$   
 OR  $t = 1.863 \text{ s}$

## SYSTEM 2

(a) A:  $T = 100 \text{ lb}$   
 $\uparrow \Sigma F_y = m_A a_A: W_A - T = \frac{W_A}{g} a_A$   
 OR  $\dots a_A = (32.2)(1 - \frac{100}{200})$   
 OR  $a_A = 16.1 \frac{\text{ft}}{\text{s}^2} \downarrow$

(b) HAVE  $\dots v_A^2 = (v_A^0)^2 + 2a_A(y - y_0)$   
 WHEN  $\Delta y_A = 10 \text{ ft}$ :  $v_A^2 = 2(16.1 \frac{\text{ft}}{\text{s}^2})(10 \text{ ft})$   
 OR  $v_A = 17.94 \frac{\text{ft}}{\text{s}} \downarrow$

(c) HAVE  $\dots v_A = (v_A^0)^2 + a_A t$   
 WHEN  $v_A = 20 \frac{\text{ft}}{\text{s}}$ :  $20 \frac{\text{ft}}{\text{s}} = (16.1 \frac{\text{ft}}{\text{s}^2})t$   
 OR  $t = 1.242 \text{ s}$

## SYSTEM 3

(a) A:  $\uparrow \Sigma F_y = m_A a_A: W_A - T = \frac{W_A}{g} a_A$   
 OR  $T = 2200(1 - \frac{a_A}{g})$

B:  $\uparrow \Sigma F_y = m_B a_B: T - W_B = \frac{W_B}{g} a_B$   
 OR  $T = 2100(1 + \frac{a_B}{g})$

EQUATING THE TWO EXPRESSIONS FOR T AND NOTING THAT  $|a_A| = |a_B| \dots$

$$2200(1 - \frac{a_A}{g}) = 2100(1 + \frac{a_A}{g})$$

$$\text{OR } a_A = \frac{1}{43}g = \frac{1}{43}(32.2 \frac{\text{ft}}{\text{s}^2}) = 0.74884 \frac{\text{ft}}{\text{s}^2} \downarrow$$

$$\therefore a_A = 0.749 \frac{\text{ft}}{\text{s}^2} \downarrow$$

(b) HAVE  $\dots v_A^2 = (v_A^0)^2 + 2a_A(y - y_0)$   
 WHEN  $\Delta y_A = 10 \text{ ft}$ :  $v_A^2 = 2(0.74884 \frac{\text{ft}}{\text{s}^2})(10 \text{ ft})$   
 OR  $v_A = 3.87 \frac{\text{ft}}{\text{s}} \downarrow$

(c) HAVE  $\dots v_A = (v_A^0)^2 + a_A t$   
 WHEN  $v_A = 20 \frac{\text{ft}}{\text{s}}$ :  $20 \frac{\text{ft}}{\text{s}} = (0.74884 \frac{\text{ft}}{\text{s}^2})t$   
 OR  $t = 26.7 \text{ s}$

# 12.20



GIVEN:  $a_B = \text{CONSTANT}$ ;  
 $m_B = 3 \text{ kg}$ ; MOTION OF B IS IMPENDING;  
 $\mu_s = 0.30, \mu_k = 0.25$   
 FIND: (a)  $a_{EL}$  WHEN  $a_{EL} \uparrow$  AND  $N_{AB} = N_{BC} = 2W_B$   
 (b)  $N_{AB}$  AND  $N_{BC}$  WHEN  $a_{EL} = 2.0 \frac{\text{m}}{\text{s}^2} \uparrow$

FIRST OBSERVE THAT BECAUSE B IS NOT MOVING RELATIVE TO A AND TO C THAT  $a_B = a_{EL}$ .

(a) HAVE  $\dots F = \mu_s N$   
 $= 0.30(2W_B)$   
 $= 0.6W_B = 0.6m_B g$   
 FOR  $a_{EL}$  TO BE  $\uparrow$ , THE NET VERTICAL FORCE MUST BE  $\uparrow$ , WHICH REQUIRES THAT THE FRICTIONAL FORCES BE ACTING AS SHOWN. IT THEN FOLLOWS THAT THE IMPENDING MOTION OF B RELATIVE TO A AND C IS DOWNWARD. THEN  $\dots$   
 $\uparrow \Sigma F_y = m_B a_{EL}: 2F - W_B = m_B a_{EL}$   
 OR  $2(0.6m_B g) - m_B g = m_B a_{EL}$   
 OR  $a_{EL} = 0.2 \times 9.81 \frac{\text{m}}{\text{s}^2}$   
 OR  $a_{EL} = 1.962 \frac{\text{m}}{\text{s}^2} \uparrow$

(b) HAVE  $\dots F = \mu_s N$   
 $= 0.30 N$   
 NOW OBSERVE THAT BECAUSE THE DIRECTION OF THE IMPENDING MOTION IS UNKNOWN, THE DIRECTIONS OF THE FRICTIONAL FORCES IS ALSO UNKNOWN (ALTHOUGH  $F_{NET}$  MUST BE DOWNWARD).  
 $\uparrow \Sigma F_y = m_B a_{EL}: \pm 2F - W_B = -m_B |a_{EL}|$   
 OR  $\pm 2F = m_B (g - |a_{EL}|)$   
 $= 3 \text{ kg} \times (9.81 - 2) \frac{\text{m}}{\text{s}^2}$   
 SINCE THE MAGNITUDE OF F MUST BE POSITIVE, IT THEN FOLLOWS THAT F  $\uparrow$  AND THAT THE IMPENDING MOTION OF B RELATIVE TO A AND C IS DOWNWARD. FINALLY  $\dots$   
 $2(0.30 N) = 3 \text{ kg} \times (9.81 - 2) \frac{\text{m}}{\text{s}^2}$   
 OR  $N_{AB} = N_{BC} = 39.1 \text{ N}$

## 12.21



GIVEN: At  $t=0, v=0$ ; FOR  
 $0 < t \leq 1.3 \text{ s}$ ,  $a_{\text{BELT}} = 2 \frac{\text{m}}{\text{s}^2} \rightarrow$ ;  
 FOR  $t > 1.3 \text{ s}$ ,  $a_{\text{BELT}} = 0 \leftarrow$ ;  
 WHEN  $\Delta x_{\text{BELT}} = 2.2 \text{ m}$ ,  
 $v_{\text{BELT}} = 0$ ;  $\mu_s = 0.35$ ,  $\mu_k = 0.25$   
 FIND: (a)  $a_2$   
 (b)  $\Delta x_{\text{PACKAGE/BELT}}$  WHEN  
 $v_{\text{BELT}} = 0$

(a) FOR THE UNIFORMLY ACCELERATED MOTION OF A POINT ON THE BELT HAVE..

1  $\rightarrow$  2:  $x_{12} = x_1^0 + v_1^0 t + \frac{1}{2} a_1 t^2$   
 $= \frac{1}{2} (2 \frac{\text{m}}{\text{s}^2}) (1.3 \text{ s})^2$   
 $= 1.69 \text{ m}$   
 AND  $v_2 = v_1^0 + a_1 t_{12}$   
 $= (2 \frac{\text{m}}{\text{s}^2}) (1.3 \text{ s})$   
 $= 2.6 \frac{\text{m}}{\text{s}}$

2  $\rightarrow$  3:  $\Delta x_2^0 = v_2^2 + 2 a_2 (x_3 - x_2)$   
 $0 = (2.6 \frac{\text{m}}{\text{s}})^2 + 2 a_2 (2.2 - 1.69) \text{ m}$   
 OR  $a_2 = -6.62745 \frac{\text{m}}{\text{s}^2}$

$\therefore a_2 = 6.63 \frac{\text{m}}{\text{s}^2}$

(b) NOW CONSIDER THE PACKAGE FOR EACH PORTION OF THE MOTION

1  $\rightarrow$  2  
 $+ \Sigma F_y = 0: N - W = 0$   
 OR  $N = W$   
 NOW..  $F_{\text{MAX}} = \mu_s N = 0.35 W$   
 ASSUME THAT THE PACKAGE DOES NOT SLIP NOR IS IN IMPENDING MOTION RELATIVE TO THE BELT.  
 THEN  $F_{12} < F_{\text{MAX}}$  ( $a_{\text{PACK}})_1 = a_1$   
 AND  $\therefore \Sigma F_x = m a_1: F_{12} = m a_1 = \frac{m}{g} \cdot \frac{W}{g}$   
 $= W (\frac{2 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}})$   
 $= 0.204 W$

$\therefore F_{12} (0.204 W) < F_{\text{MAX}} (0.35 W) \Rightarrow$  ASSUMPTION IS CORRECT (NO SLIPPING) SO THAT  
 $(x_{\text{PACKAGE/BELT}})_{12} = 0$

2  $\rightarrow$  3  
 $+ \Sigma F_y = 0: N - W = 0$   
 OR  $N = W$   
 NOW..  $F_{\text{MAX}} = \mu_k N = 0.25 W$   
 REPEATING THE ABOVE ASSUMPTION IMPLIES  
 $F < F_{\text{MAX}}$  ( $a_{\text{PACK}})_2 = a_2$   
 THEN  $\Sigma F_x = m a_2: -F_{23} = m a_2 = \frac{m}{g} \cdot \frac{W}{g}$   
 $= W (\frac{-6.62745 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}})$   
 OR  $F_{23} = 0.676 W$

$\therefore F_{23} (0.676 W) > F_{\text{MAX}} (0.25 W) \Rightarrow$  ASSUMPTION IS INCORRECT, SO THAT THE PACKAGE SLIPS ON THE BELT AS THE BELT COMES TO REST.

(CONTINUED)

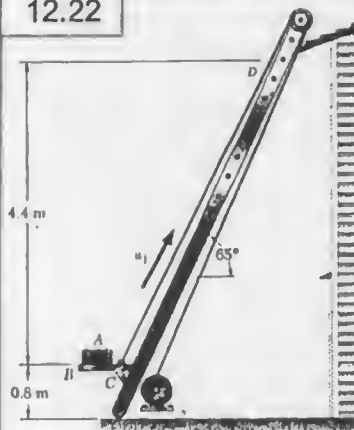
## 12.21 continued

THEN.. SLIPPING:  $F_{23} = \mu_k N$   
 $= 0.25 m g$   
 $\therefore \Sigma F_x = m (a_{\text{PACK}})_2: -F_{23} = m (a_{\text{PACK}})_2$   
 OR  $-0.25 m g = m (a_{\text{PACK}})_2$   
 OR  $(a_{\text{PACK}})_2 = -0.25 (9.81 \frac{\text{m}}{\text{s}^2}) = -2.4525 \frac{\text{m}}{\text{s}^2}$   
 NOW..  $(a_{\text{PACK}})_2 = a_2 + (a_{\text{PACK/BELT}})_2$   
 OR  $(a_{\text{PACK/BELT}})_2 = -2.4525 \frac{\text{m}}{\text{s}^2} - (-6.62745 \frac{\text{m}}{\text{s}^2})$   
 $= 4.17495 \frac{\text{m}}{\text{s}^2}$

FOR THE BELT..  $v_2^0 = v_2 + a_2 t_{23}$   
 OR  $0 = 2.6 \frac{\text{m}}{\text{s}} + (-6.62745 \frac{\text{m}}{\text{s}^2}) t_{23}$   
 OR  $t_{23} = 0.39231 \text{ s}$   
 THEN..  $(x_{\text{PACK/BELT}})_{23} = x_2^0 + v_2^0 t_{23} + \frac{1}{2} (a_{\text{PACK/BELT}})_2 t_{23}^2$   
 $= \frac{1}{2} (4.17495 \frac{\text{m}}{\text{s}^2}) (0.39231 \text{ s})^2$   
 $= 0.321 \text{ m}$

FINALLY..  $x_{\text{PACKAGE/BELT}} = (x_{\text{PACK/BELT}})_{12} + (x_{\text{PACK/BELT}})_{23}$   
 OR  $x_{\text{PACKAGE/BELT}} = 0.321 \text{ m}$

## 12.22



GIVEN:  $(v_{BC})_0 = 0$ ,  
 $(v_{BC})_0 = 0$ ;  
 BC MOVES  
 CONSTANT  
 ACCELERATIONS  
 $a_1$  AND  $a_2$ ;  
 $\mu_s = 0.30$   
 FIND:  $(a_1)_{\text{MAX}}$  AND  
 $(a_2)_{\text{MAX}}$  IF  
 SLIDING OF  
 SHINGLES A IS  
 NOT TO OCCUR

$a_1$ :  
 $F_1 = \mu_s N_1 = 0.3 N_1$   
 THEN..  $F_1 = \mu_s N_1 = 0.3 N_1$   
 $+ \Sigma F_y = m a_y: N_1 - W_A = m a_1 \sin 65^\circ$   
 OR  $N_1 = m a_1 (g + a_1 \sin 65^\circ)$   
 $+ \Sigma F_x = m a_x: F_1 = m a_1 \cos 65^\circ$   
 THEN..  $0.3 [m a_1 (g + a_1 \sin 65^\circ)] = m a_1 \cos 65^\circ$   
 OR  $a_1 = \frac{0.3 (9.81 \frac{\text{m}}{\text{s}^2})}{\cos 65^\circ - 0.3 \sin 65^\circ}$   
 OR  $(a_1)_{\text{MAX}} = 19.53 \frac{\text{m}}{\text{s}^2} \Delta 65^\circ$

$a_2$ :  

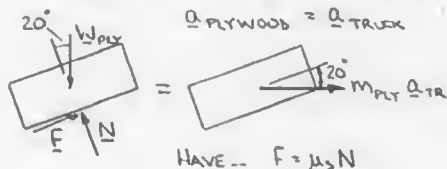
 REQUIRE..  $F_2 = \mu_s N_2 = 0.3 N_2$   
 $+ \Sigma F_y = m a_y: N_2 - W_A = m a_2 \sin 65^\circ$   
 OR  $N_2 = m a_2 (g + a_2 \sin 65^\circ)$   
 $+ \Sigma F_x = m a_x: -F_2 = -m a_2 \cos 65^\circ$   
 THEN..  $0.3 [m a_2 (g + a_2 \sin 65^\circ)] = m a_2 \cos 65^\circ$   
 OR  $a_2 = \frac{0.3 (9.81 \frac{\text{m}}{\text{s}^2})}{\cos 65^\circ + 0.3 \sin 65^\circ}$   
 OR  $(a_2)_{\text{MAX}} = 4.24 \frac{\text{m}}{\text{s}^2} \Delta 65^\circ$

## 12.23



GIVEN:  $t = 0$ ,  $v = 0$ ;  $\mu_k = 0.40$ ,  
 $\mu_s = 0.30$   
 FIND: (a)  $a_{\text{TRUCK}}_{\text{MIN}}$  SO THAT  
 PLYWOOD SLIDES  
 (b)  $a_{\text{TRUCK}}$  SO THAT  
 $\Delta x_{\text{PLYWOOD/TRUCK}} = 2 \text{ m}$   
 AT  $t = 0.9 \text{ s}$

(a) SEEK THE VALUE OF  $a_{\text{TRUCK}}$  SO THAT  
 RELATIVE MOTION OF THE PLYWOOD WITH  
 RESPECT TO THE TRUCK IS IMPENDING. NOTE..



$$\text{HAVE.. } F = \mu_s N = 0.4 N \quad (1)$$

$$+\Sigma F_y = m_{\text{PLY}} a_y: N - W_{\text{PLY}} \cos 20^\circ = -m_{\text{PLY}} a_{\text{TR}} \sin 20^\circ$$

$$+\Sigma F_x = m_{\text{PLY}} a_x: F - W_{\text{PLY}} \sin 20^\circ = m_{\text{PLY}} a_{\text{TR}} \cos 20^\circ$$

$$\text{OR } F = m_{\text{PLY}} (g \sin 20^\circ + a_{\text{TR}} \cos 20^\circ)$$

SUBSTITUTING INTO EQ. (1)...

$$m_{\text{PLY}} (g \sin 20^\circ + a_{\text{TR}} \cos 20^\circ) = 0.4 m_{\text{PLY}} (g \cos 20^\circ - a_{\text{TR}} \sin 20^\circ)$$

$$\text{OR } a_{\text{TR}} = \frac{g(0.4 \cos 20^\circ - \sin 20^\circ)}{\cos 20^\circ + 0.4 \sin 20^\circ} = (9.81 \frac{\text{m}}{\text{s}^2}) \frac{0.4 - \tan 20^\circ}{1 + 0.4 \tan 20^\circ}$$

$$\text{OR } (a_{\text{TRUCK}})_{\text{MIN}} = 0.309 \frac{\text{m}}{\text{s}^2} \rightarrow$$

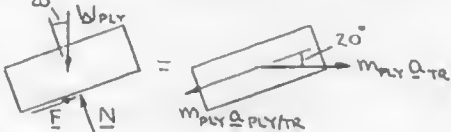
(b) FIRST NOTE THAT BECAUSE ALL OF THE  
 FORCES ARE CONSTANT, THE ACCELERATIONS  
 ARE ALSO CONSTANT, THEN..

$$x_{\text{PLY/TR}} = (x_{\text{PLY/TR}})_0 + v_0 t + \frac{1}{2} a_{\text{PLY/TR}} t^2$$

$$\text{AT } t = 0.9 \text{ s: } 2 \text{ m} = \frac{1}{2} a_{\text{PLY/TR}} (0.9 \text{ s})^2$$

$$\text{OR } a_{\text{PLY/TR}} = 4.93827 \frac{\text{m}}{\text{s}^2} \rightarrow 20^\circ$$

NOW..  $a_{\text{PLY}} = a_{\text{TR}} + a_{\text{PLY/TR}}$   
 THEN



$$\text{HAVE.. } F = \mu_k N = 0.3 N \quad (1)$$

$$+\Sigma F_x = m_{\text{PLY}} a_x: F - W_{\text{PLY}} \sin 20^\circ = m_{\text{PLY}} (a_{\text{TR}} \cos 20^\circ - a_{\text{PLY/TR}})$$

$$\text{OR } F = m_{\text{PLY}} (g \sin 20^\circ + a_{\text{TR}} \cos 20^\circ - a_{\text{PLY/TR}})$$

$$+\Sigma F_y = m_{\text{PLY}} a_y: N - W_{\text{PLY}} \cos 20^\circ = -m_{\text{PLY}} a_{\text{TR}} \sin 20^\circ$$

$$\text{OR } N = m_{\text{PLY}} (g \cos 20^\circ - a_{\text{TR}} \sin 20^\circ)$$

SUBSTITUTING INTO EQ. (1)...

$$m_{\text{PLY}} (g \sin 20^\circ + a_{\text{TR}} \cos 20^\circ - a_{\text{PLY/TR}}) = 0.3 [m_{\text{PLY}} (g \cos 20^\circ - a_{\text{TR}} \sin 20^\circ)]$$

$$\text{OR } a_{\text{TR}} = \frac{g(0.3 \cos 20^\circ - \sin 20^\circ) + a_{\text{PLY/TR}}}{\cos 20^\circ + 0.3 \sin 20^\circ}$$

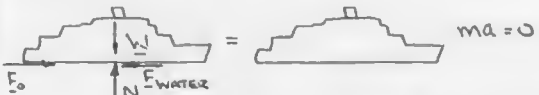
$$= \frac{(9.81 \frac{\text{m}}{\text{s}^2}) (0.3 \cos 20^\circ - \sin 20^\circ) + 4.93827 \frac{\text{m}}{\text{s}^2}}{\cos 20^\circ + 0.3 \sin 20^\circ}$$

$$\text{OR } a_{\text{TRUCK}} = 4.17 \frac{\text{m}}{\text{s}^2} \rightarrow$$

## 12.24

GIVEN: SHIP OF WEIGHT  $W$  HAVING A  
 PROPULSIVE FORCE  $F_0$ ; AT  $t = 0$ ,  
 $v = v_0$  ( $v_{\text{MAX}}$ ), FORWARD, ENGINES ARE  
 REVERSED;  $F_{\text{WATER}} \propto v^2$   
 FIND:  $x$  WHEN  $v = 0$

FIRST CONSIDER WHEN THE SHIP IS MOVING  
 FORWARD.



$$\text{LET } F_{\text{WATER}} = kv^2 \quad \text{WHERE } k \text{ IS A CONSTANT}$$

$$+\Sigma F_x = 0: F_0 - kv_0^2 = 0 \quad \text{OR } k = \frac{F_0}{v_0^2}$$

NOW CONSIDER WHEN THE SHIP IS DECELERATING.



$$+\Sigma F_x = ma: -F_0 - F_{\text{WATER}} = \frac{W}{g} a$$

$$\text{OR } a = -\frac{g}{W} (F_0 + \frac{F_0}{v_0^2} v^2) = -\frac{g}{v_0^2} \frac{F_0}{W} (v_0^2 + v^2)$$

$$\text{NOW.. } v \frac{dv}{dx} = a = -\frac{g}{v_0^2} \frac{F_0}{W} (v_0^2 + v^2)$$

$$\text{AT } t = 0, x = 0, v = v_0$$

$$\int_0^x dx = -\frac{v_0^2}{g} \frac{W}{F_0} \int_{v_0}^0 \frac{v dv}{v^2 + v_0^2}$$

$$\text{OR } x = -\frac{v_0^2}{g} \frac{W}{F_0} \left[ \frac{1}{2} \ln(v^2 + v_0^2) \right]_{v_0}^0 = -\frac{1}{2} \frac{v_0^2}{g} \frac{W}{F_0} \ln \frac{v_0^2}{v_0^2 + v_0^2}$$

$$\text{OR } x = \frac{1}{2} \frac{W}{g} \frac{v_0^2}{F_0} \ln 2$$

## 12.25

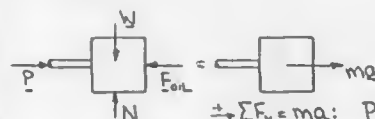
GIVEN: CONSTANT FORCE  $P$ ;

PISTON AND ROD OF

MASS  $m$ ;  $F_{\text{OIL}} = kv$ ;

AT  $t = 0$ ,  $x = 0$ ,  $v = 0$

SHOW:  $f(x, v, t) = 0$  IS LINEAR  
 IN  $x$ ,  $v$ , AND  $t$



$$+\Sigma F_x = ma: P - F_{\text{OIL}} = ma \quad \text{OR } a = \frac{1}{m} (P - kv)$$

$$\text{NOW.. } \frac{dv}{dt} = a = \frac{1}{m} (P - kv)$$

$$\text{AT } t = 0, v = 0: \int_0^t dt = m \int_0^v \frac{dv}{P - kv}$$

$$\text{OR } t = m \left[ -\frac{1}{k} \ln(P - kv) \right]_0^v$$

$$\text{OR } t = -\frac{m}{k} \ln \frac{P - kv}{P} \quad (1)$$

$$\text{ALSO.. } v \frac{dv}{dx} = a = \frac{1}{m} (P - kv)$$

$$\text{AT } x = 0, v = 0: \int_0^x dx = m \int_0^v \frac{v dv}{P - kv}$$

$$\text{OR } x = m \left\{ \int_0^v \left[ -\frac{1}{k} + \frac{P}{k(P - kv)} \right] dv \right\}$$

$$= m \left[ -\frac{v}{k} + \frac{P}{k} \ln \frac{P - kv}{P} \right]_0^v$$

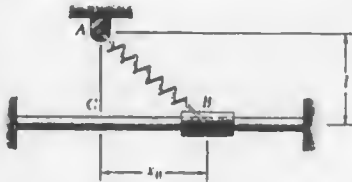
$$= -m \left( \frac{v}{k} + \frac{P}{k} \ln \frac{P - kv}{P} \right)$$

$$\text{USING EQ. (1).. } x = -\frac{mv}{k} + \frac{P}{k} t$$

$$\text{OR } xk + mv - Pt = 0$$

WHICH IS LINEAR IN  $x$ ,  $v$ , AND  $t$ .

## 12.26

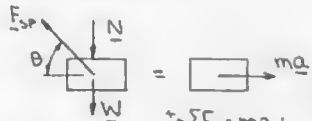


GIVEN: SPRING CONSTANT  $k$ ,  
 $m_{col}$ ,  $m$ ;  $(x_{sp})_{uns} = l$ ;  
 AT  $t=0$ ,  $x=x_0$ ,  $v=0$   
 FIND:  $v$  AT C

FIRST NOTE..

$$F_{sp} = k [L_{AB} - (x_{sp})_{uns}]$$

$$= k (\sqrt{x^2 + l^2} - l)$$



WHERE  $\cos \theta = \frac{x}{\sqrt{x^2 + l^2}}$

THEN..  $a = -\frac{k}{m} (\sqrt{x^2 + l^2} - l) \left( \frac{x}{\sqrt{x^2 + l^2}} \right) = -\frac{k}{m} \left( x - \frac{lx}{\sqrt{x^2 + l^2}} \right)$

NOW..  $v \frac{dv}{dx} = a = -\frac{k}{m} \left( x - \frac{lx}{\sqrt{x^2 + l^2}} \right)$

AT  $x=x_0$ ,  $v=0$ :  $\int_0^{(v_{col})} v dv = -\frac{k}{m} \int_{x_0}^0 \left( x - \frac{lx}{\sqrt{x^2 + l^2}} \right) dx$

$$\text{OR } \frac{1}{2} (v_{col})^2 = -\frac{k}{m} \left[ \frac{1}{2} x^2 - l \sqrt{x^2 + l^2} \right]_{x_0}^0$$

$$= -\frac{k}{m} \left\{ (-l^2) - \left[ \frac{1}{2} x_0^2 - l \sqrt{x_0^2 + l^2} \right] \right\}$$

$$= \frac{1}{2} \frac{k}{m} \left[ (x_0^2 + l^2) - 2l \sqrt{x_0^2 + l^2} + l^2 \right]$$

$$= \frac{1}{2} \frac{k}{m} (\sqrt{x_0^2 + l^2} - l)^2$$

OR  $(v_{col})_c = \sqrt{\frac{k}{m} (\sqrt{x_0^2 + l^2} - l)}$

## 12.27

GIVEN: AUTOMOBILE WEIGHING 2700 lb, FRONT-WHEEL DRIVE,  $W_{FR} = 0.62W$ ;  $\mu_s = 0.70$ ,  $D = 0.0125^2$  D=1b,  $v = \frac{54}{5}$ , AT  $t=0$ ,  $x=0$ ,  $v=0$

FIND:  $v_{max}$  WHEN  $x = 0.25$  mi



$N_R = 0.38W$   $N_F = 0.62W$

$F = F_{max}$  FOR  $v = v_{max}$   
 $\therefore F = \mu_s N_F = 0.70 (0.62W)$   
 $= 0.434W$

$\sum F_x = ma$ :  $F - D = \frac{W}{g} a$   
 OR  $a = \frac{g}{W} (0.434W - 0.0125^2)$   
 $= 0.002 \frac{g}{W} (217W - 6.5^2)$

NOW..  $v \frac{dv}{dx} = a = 0.002 \frac{g}{W} (217W - 6.5^2)$

AT  $x=0$ ,  $v=0$ :  $\int_0^v v dv = \int_0^x \frac{v dv}{217W - 6.5^2}$

OR  $0.002 \frac{g}{W} x = -\frac{1}{12} \ln(217W - 6.5^2) = -\frac{1}{12} \ln \left( \frac{217W - 6.5^2}{217W} \right)$

OR  $\frac{217W - 6.5^2}{217W} = e^{-0.024 \frac{g}{W} x}$

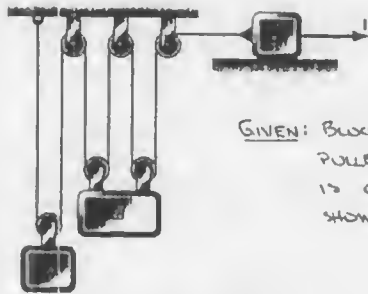
OR  $v = \left[ \frac{217}{6} W (1 - e^{-0.024 \frac{g}{W} x}) \right]^{1/2}$

WHEN  $x = 0.25$  mi = 1320 ft:

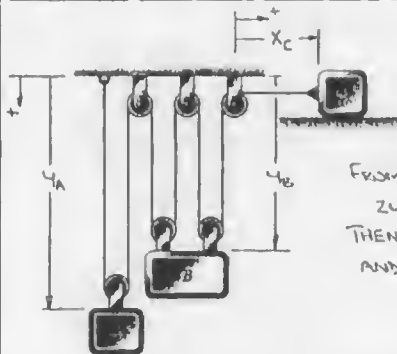
$v_{max} = \left[ \frac{217}{6} (2700) (1 - e^{-0.024 \frac{32.2}{2700} \cdot 1320}) \right]^{1/2} = 175.285 \frac{ft}{s}$

OR  $v_{max} = 119.5 \frac{mi}{h}$

## 12.28 and 12.29



GIVEN: BLOCKS A, B, AND C AND THE PULLEY/CABLE SYSTEM, WHICH IS OF NEGLIGIBLE MASS, SHOWN



FROM THE DIAGRAM..

$2y_A + 4y_B + y_C = \text{CONSTANT}$

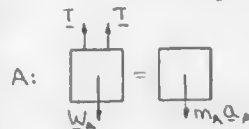
THEN..  $2\dot{y}_A + 4\dot{y}_B + \dot{y}_C = 0$

AND  $2a_A + 4a_B + a_C = 0$  (1)

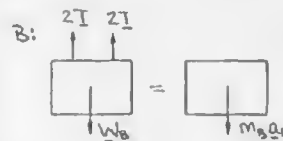
## 12.28

GIVEN:  $m_A = 4$  kg,  $m_B = 10$  kg,  $m_C = 2$  kg;  
 $P = 0$

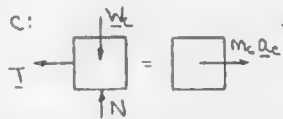
FIND: (a)  $a_A$ ,  $a_B$ , AND  $a_C$   
 (b)  $T$



$\sum F_y = m_A a_A$ :  $W_A - 2T = m_A a_A$   
 OR  $a_A = \frac{1}{m_A} (m_A g - 2T)$   
 $= g - \frac{2}{4} T$   
 $= g - \frac{1}{2} T$



$\sum F_y = m_B a_B$ :  $W_B - 4T = m_B a_B$   
 OR  $a_B = \frac{1}{m_B} (m_B g - 4T)$   
 $= g - \frac{4}{10} T$   
 $= g - \frac{2}{5} T$



$\sum F_x = m_C a_C$ :  $-T = m_C a_C$   
 OR  $a_C = -\frac{1}{2} T$

SUBSTITUTING THE EXPRESSIONS FOR  $a_A$ ,  $a_B$ , AND  $a_C$  INTO EQ. (1) ..

$2(g - \frac{1}{2} T) + 4(g - \frac{2}{5} T) + (-\frac{1}{2} T) = 0$

OR  $T = \frac{60}{31} g = \frac{60}{31} (9.81) = 18.9871$  N

(a) THEN..  $a_A = 9.81 - \frac{1}{2} (18.9871)$

OR  $a_A = 0.316 \frac{m}{s^2}$

$a_B = 9.81 - \frac{2}{5} (18.9871)$

OR  $a_B = 2.22 \frac{m}{s^2}$

$a_C = -\frac{1}{2} (18.9871)$

OR  $a_C = -9.49 \frac{m}{s^2}$

(b) HAVE..

$T = 18.99$  N

(CONTINUED)

## 12.29 continued

12.29. **GIVEN:**  $m_A = 8 \text{ kg}$ ,  $m_B = 16 \text{ kg}$ ,  $m_C = 10 \text{ kg}$ ;  
 $\mu_s = 0.30$ ,  $\mu_k = 0.20$ ; AT  $t = 0, 15 = 0$ ;  
 AT  $t = 0.8 \text{ s}$ ,  $\Delta y_B = 2 \text{ m}$

**FIND:** (a)  $a_A$ ,  $a_B$ , AND  $a_C$

(b)  $T$

(c)  $P$

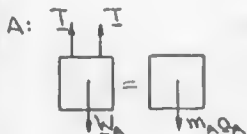
(a) FIRST NOTE THAT BECAUSE ALL OF THE FORCES ARE CONSTANT, ALL OF THE ACCELERATIONS ARE CONSTANT. THEN..

$$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

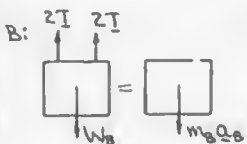
$$\text{AT } t = 0.8 \text{ s: } 2 \text{ m} = \frac{1}{2} a_B (0.8 \text{ s})^2$$

$$\text{OR } a_B = 6.25 \frac{\text{m}}{\text{s}^2}$$

$$\therefore a_B = 6.25 \frac{\text{m}}{\text{s}^2} \downarrow$$



$$\begin{aligned} +\uparrow \Sigma F_y &= m_A a_A: W_A - 2T = m_A a_A \\ \text{OR } m_A g - 2T &= m_A a_A \\ \text{OR } 8g - 2T &= 8a_A \quad (2) \end{aligned}$$



$$\begin{aligned} +\uparrow \Sigma F_y &= m_B a_B: W_B - 4T = m_B a_B \\ \text{OR } m_B g - 4T &= m_B a_B \\ \text{OR } 16g - 4T &= 16a_B \quad (3) \end{aligned}$$

COMPARING EQS (2) AND (3), IT FOLLOWS THAT  $a_A = a_B$

$$\therefore a_A = 6.25 \frac{\text{m}}{\text{s}^2} \downarrow$$

SUBSTITUTING INTO EQ. (1)..

$$2(6.25) + 4(6.25) + a_C = 0$$

$$a_C = -37.5 \frac{\text{m}}{\text{s}^2}$$

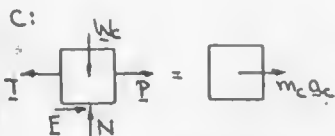
$$\therefore a_C = 37.5 \frac{\text{m}}{\text{s}^2} \leftarrow$$

(b) FROM EQ (2)..  $T = 4(g - a_A)$

$$= 4(9.81 - 6.25)$$

$$\text{OR } T = 14.24 \text{ N}$$

(c)



$$+\uparrow \Sigma F_y = 0: W_C - N = 0$$

$$\text{OR } N = m_C g$$

$$\text{SLIDING: } F = \mu_k N = 0.2 m_C g$$

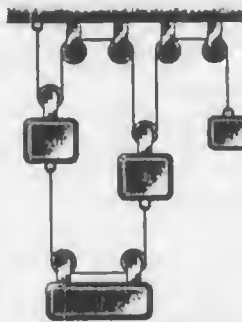
$$+\rightarrow \Sigma F_x = m_C a_C: P + F - T = m_C a_C$$

$$\text{OR } P = T + m_C (a_C - 0.2g)$$

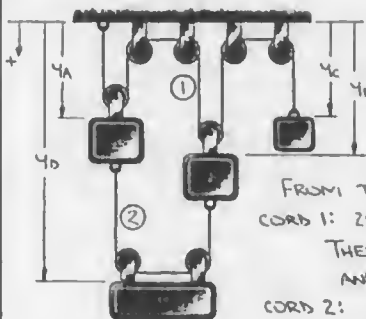
$$= 14.24 \text{ N} + (10 \text{ kg})(-37.5 - 0.2 \cdot 9.81) \frac{\text{m}}{\text{s}^2}$$

$$\text{OR } P = 380 \text{ N} \leftarrow$$

## 12.30 and 12.31



**GIVEN:** BLOCKS A, B, C, AND D AND THE PULLEY/CABLE SYSTEM, WHICH IS OF NEGLECTIBLE WEIGHT, SHOWN;  $W_A = W_B = 20 \text{ lb}$ ,  $W_C = 14 \text{ lb}$ ,  $W_D = 16 \text{ lb}$



**NOTE:** AS SHOWN, THE SYSTEM IS IN EQUILIBRIUM.

FROM THE DIAGRAM..

$$\text{CORD 1: } 2y_A + 2y_B + y_C = \text{CONSTANT}$$

$$\text{THEN.. } 2\Delta y_A + 2\Delta y_B + \Delta y_C = 0$$

$$\text{AND } 2a_A + 2a_B + a_C = 0 \quad (1)$$

$$\text{CORD 2: } (y_D - y_A) + (y_D - y_B) = \text{CONSTANT}$$

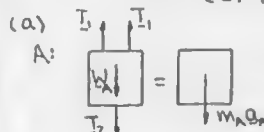
$$\text{THEN.. } 2\Delta y_D - \Delta y_A - \Delta y_B = 0$$

$$\text{AND } 2a_D - a_A - a_B = 0 \quad (2)$$

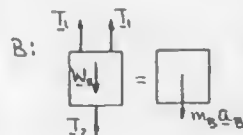
12.30 **GIVEN:**  $(F_D)_{\text{EXT}} = 24 \text{ lb} \downarrow$

**FIND:** (a)  $a_A$ ,  $a_B$ ,  $a_C$ , AND  $a_D$

(b)  $T_1 (= T_{\text{ABC}})$



$$\begin{aligned} +\uparrow \Sigma F_y &= m_A a_A: W_A - 2T_1 + T_2 = \frac{W_A}{g} a_A \\ \text{OR } 20 - 2T_1 + T_2 &= \frac{20}{g} a_A \quad (3) \end{aligned}$$



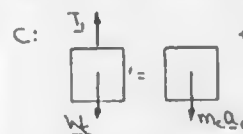
$$\begin{aligned} +\uparrow \Sigma F_y &= m_B a_B: W_B - 2T_1 + T_2 = \frac{W_B}{g} a_B \\ \text{OR } 20 - 2T_1 + T_2 &= \frac{20}{g} a_B \quad (4) \end{aligned}$$

**NOTE:** EQS. (3) AND (4)  $\Rightarrow$

$$a_A = a_B$$

THEN.. EQ (1)  $\Rightarrow a_C = -4a_A$

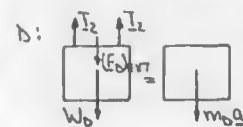
$$\text{EQ (2)} \Rightarrow a_D = a_A$$



$$+\uparrow \Sigma F_y = m_C a_C: W_C - T_1 = \frac{W_C}{g} a_C$$

$$\text{OR } T_1 = 14 \left(1 - \frac{a_C}{g}\right)$$

$$= 14 \left(1 + \frac{4a_A}{g}\right) \quad (5)$$



$$+\uparrow \Sigma F_y = m_D a_D: W_D - 2T_2 + (F_D)_{\text{EXT}} = \frac{W_D}{g} a_D$$

$$\text{OR } T_2 = \frac{1}{2} \left[ 16 \left(1 - \frac{a_D}{g}\right) + 24 \right]$$

$$= 20 - 8 \frac{a_D}{g}$$

SUBSTITUTING FOR  $T_1$  [EQ. (5)] AND  $T_2$  [EQ. (6)] IN EQ. (3)..

$$20 - 2 \cdot 14 \left(1 + \frac{4a_A}{g}\right) + \left(20 - 8 \frac{a_A}{g}\right) = \frac{20}{g} a_A$$

$$\text{OR } a_A = \frac{3}{35} g = \frac{3}{35} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} = 2.76 \frac{\text{ft}}{\text{s}^2}$$

(CONTINUED)



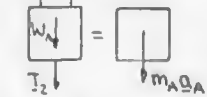
# 12.30 and 12.31 continued

AND  $a_c = -4(2.76 \frac{ft}{s^2})$  OR  $a_c = 11.04 \frac{ft}{s^2}$   
 (b) SUBSTITUTING INTO Eq. (5)..  
 $T_1 = 14(1 + \frac{4+2.76}{32.2})$  OR  $T_1 = 18.80 \text{ lb}$

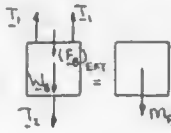
12.31 GIVEN:  $(F_B)_{EXT} = 10 \text{ lb}$ ; AT  $t=0, v=0$   
 FIND: (a)  $v_{B/A}$  AT  $t=3 \text{ s}$   
 (b)  $a_{C/D}$  AT  $t=3 \text{ s}$

FIRST DETERMINE THE ACCELERATIONS OF BLOCKS A, C, AND D.

A:  $\sum F_y = m_A a_A: W_A - 2T_1 + T_2 = \frac{W_A}{g} a_A$   
 OR  $20 - 2T_1 + T_2 = \frac{20}{g} a_A$  (3)



B:  $\sum F_y = m_B a_B: W_B - 2T_1 + T_2 + (F_B)_{EXT} = \frac{W_B}{g} a_B$   
 OR  $20 - 2T_1 + T_2 + 10 = \frac{20}{g} a_B$  (4)

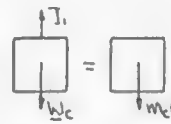


FORMING (3)-(4)  $\Rightarrow -10 = \frac{20}{g} (a_A - a_B)$   
 OR  $a_B = a_A + \frac{1}{2}g$

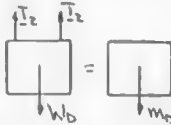
THEN.. EQ. (1):  $2a_A + 2(a_A + \frac{1}{2}g) \cdot a_c = 0$   
 OR  $a_c = -4a_A - g$

EQ. (2):  $2a_B - a_A - (a_A + \frac{1}{2}g) \cdot 0$   
 OR  $a_B = a_A + \frac{1}{2}g$

C:  $\sum F_y = m_C a_c: W_C - T_1 = \frac{W_C}{g} a_c$   
 OR  $T_1 = 14(1 - \frac{a_c}{g})$   
 $= 14[1 - \frac{1}{g}(-4a_A - g)]$   
 $= 28(1 + 2\frac{a_A}{g})$  (5)



D:  $\sum F_y = m_D a_D: W_D - 2T_2 = \frac{W_D}{g} a_D$   
 OR  $T_2 = \frac{1}{2} \cdot 16(1 - \frac{a_D}{g})$   
 $= 8[1 - \frac{1}{g}(a_A + \frac{1}{2}g)]$   
 $= 8(\frac{1}{2} - \frac{a_A}{g})$  (6)



SUBSTITUTING FOR  $T_1$  [EQ. (5)] AND  $T_2$  [EQ. (6)] IN EQ. (3)...

$20 - 2[28(1 + 2\frac{a_A}{g})] + 8(\frac{1}{2} - \frac{a_A}{g}) = \frac{20}{g} a_A$   
 OR  $a_A = -\frac{3}{16}g = -\frac{3}{16}(32.2 \frac{ft}{s^2}) = -6.90 \frac{ft}{s^2}$

THEN..  $a_c = -4(-6.90 \frac{ft}{s^2}) - 32.2 \frac{ft}{s^2} = -4.60 \frac{ft}{s^2}$   
 $a_D = -6.90 \frac{ft}{s^2} - \frac{1}{2}(32.2 \frac{ft}{s^2}) = 1.15 \frac{ft}{s^2}$

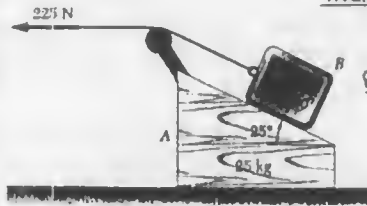
NOTE: HAVE UNIFORMLY ACCELERATED MOTION, SO THAT  $v = v_0 + at$

(a) HAVE..  $v_{B/A} = v_B - v_A$   
 OR  $v_{B/A} = a_B t - a_A t$   
 $= [1.15 - (-6.90)] \frac{ft}{s^2} \cdot 3 \text{ s}$   
 OR  $v_{B/A} = 24.2 \frac{ft}{s}$

(b) HAVE..  $a_{C/D} = a_C - a_D$   
 OR  $a_{C/D} = a_C t - a_D t$   
 $= (-4.60 - 1.15) \frac{ft}{s^2} \cdot 3 \text{ s}$   
 OR  $a_{C/D} = 17.25 \frac{ft}{s^2}$

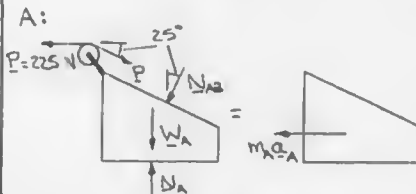
# 12.32

GIVEN: BLOCKS A AND B AND THE 225 N FORCE SHOWN  
 FIND: (a)  $a_A$   
 (b)  $a_{B/A}$



(a) FIRST NOTE..  $a_B = a_A + a_{B/A}$  WHERE  $a_{B/A}$  IS DIRECTED ALONG THE INCLINED SURFACE OF A.

B:  $\sum F_x = m_B a_B: P - W_B \sin 25^\circ = m_B a_B$   
 OR  $225 - 15g \sin 25^\circ = 15(a_A \cos 25^\circ + a_{B/A})$   
 OR  $15 \cdot 9.8 \sin 25^\circ = a_A \cos 25^\circ + a_{B/A}$  (1)  
 $\sum F_y = m_B a_y: N_{AB} - W_B \cos 25^\circ = -m_B a_A \sin 25^\circ$   
 OR  $N_{AB} = 15(g \cos 25^\circ - a_A \sin 25^\circ)$



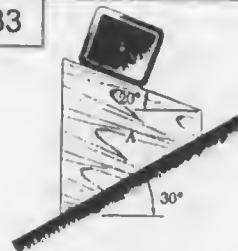
$\sum F_x = m_A a_A: P - P \cos 25^\circ + N_{AB} \sin 25^\circ = m_A a_A$   
 OR  $N_{AB} = [25a_A - 225(1 - \cos 25^\circ)] / \sin 25^\circ$

EQUATING THE TWO EXPRESSIONS FOR  $N_{AB}$ ..  
 $15(g \cos 25^\circ - a_A \sin 25^\circ) = \frac{25a_A - 225(1 - \cos 25^\circ)}{\sin 25^\circ}$   
 OR  $a_A = \frac{3(9.81) \cos 25^\circ \sin 25^\circ + 45(1 - \cos 25^\circ)}{5 + 3 \sin^2 25^\circ}$   
 $= 2.7979 \frac{m}{s^2}$

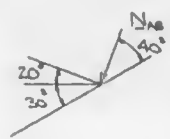
(b) FROM EQ. (1)..  
 $a_{B/A} = 15 - (9.81) \sin 25^\circ - 2.7979 \cos 25^\circ$   
 OR  $a_{B/A} = 8.32 \frac{m}{s^2} \nearrow 25^\circ$

# 12.33

GIVEN:  $m_A = 22 \text{ kg}$ ,  $m_B = 10 \text{ kg}$ ;  
 AT  $t=0, v=0$   
 FIND: (a)  $a_B$   
 (b)  $v_{B/A}$  AT  $t=0.5 \text{ s}$



(a) A:  $\sum F_x = m_A a_A: W_A \sin 30^\circ + N_{AB} \cos 40^\circ = m_A a_A$   
 OR  $N_{AB} = \frac{22(a_A - \frac{1}{2}g)}{\cos 40^\circ}$

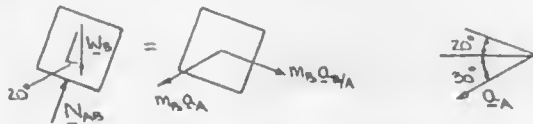


(CONTINUED)

## 12.33 continued

NOW NOTE:  $Q_B = Q_A + Q_{B/A}$  WHERE  $Q_{B/A}$  IS DIRECTED ALONG THE TOP SURFACE OF A.

B:



$$\sum F_y = m_B a_y: N_{AB} - W_B \cos 20^\circ = -m_B Q_A \sin 50^\circ$$

$$\text{OR } N_{AB} = 10(g \cos 20^\circ - Q_A \sin 50^\circ)$$

EQUATING THE TWO EXPRESSIONS FOR  $N_{AB}$ ..

$$\frac{22(Q_A - \frac{1}{3}g)}{\cos 40^\circ} = 10(g \cos 20^\circ - Q_A \sin 50^\circ)$$

$$\text{OR } Q_A = \frac{(9.81)(1.1 + \cos 20^\circ \cos 40^\circ)}{2.2 + \cos 40^\circ \sin 50^\circ} = 6.4061 \frac{m}{s^2}$$

$$\sum F_x = m_B a_x: W_B \sin 20^\circ = m_B Q_{B/A} - m_B Q_A \cos 50^\circ$$

$$\text{OR } Q_{B/A} = g \sin 20^\circ + Q_A \cos 50^\circ$$

$$= (9.81 \sin 20^\circ + 6.4061 \cos 50^\circ) \frac{m}{s^2}$$

$$= 7.4730 \frac{m}{s^2}$$

FINALLY..  $Q_B = Q_A + Q_{B/A}$

HAVE..  $Q_B^2 = 6.4061^2 + 7.4730^2$

$$- 2(6.4061)(7.4730) \cos 50^\circ$$

$$\text{OR } Q_B = 5.9447 \frac{m}{s^2}$$

AND  $\frac{7.4730}{\sin \alpha} = \frac{5.9447}{\sin 50^\circ}$

$$\text{OR } \alpha = 74.4^\circ$$

$$\therefore Q_B = 5.94 \frac{m}{s^2} \nearrow 15.6^\circ$$

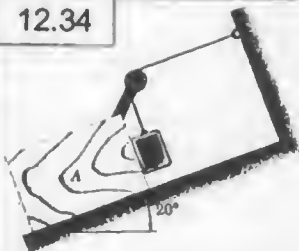
(b) NOTE: HAVE UNIFORMLY ACCELERATED MOTION, SO THAT  $v = u + at$

$$\text{NOW.. } v_{B/A} = v_B - v_A = Q_B t - Q_A t = Q_{B/A} t$$

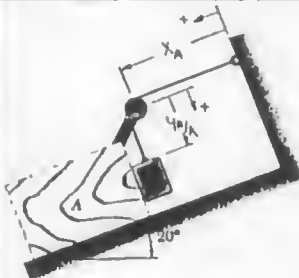
$$\text{AT } t = 0.55: v_{B/A} = 7.4730 \frac{m}{s^2} \times 0.55$$

$$\text{OR } v_{B/A} = 3.74 \frac{m}{s} \nearrow 20^\circ$$

## 12.34



GIVEN:  $W_A = 50 \text{ lb}$ ,  $W_B = 30 \text{ lb}$   
FIND:  $Q_A$  AND  $T$



FROM THE DIAGRAM..

$$x_A + y_{B/A} = \text{CONSTANT}$$

$$\text{THEN.. } v_A + v_{B/A} = 0$$

$$\text{AND } a_A + a_{B/A} = 0$$

$$\text{OR } a_{B/A} = -a_A \quad (1)$$

FIRST NOTE:  $Q_B = Q_A + Q_{B/A}$  WHERE  $Q_{B/A}$  IS DIRECTED ALONG THE SIDE OF A

(CONTINUED)

## 12.34 continued

B:

$$\sum F_x = m_B a_x: W_B \sin 20^\circ - N_{AB} = m_B Q_A$$

$$\text{OR } N_{AB} = W_B (\sin 20^\circ - \frac{Q_A}{g})$$

$$\sum F_y = m_B a_y: W_B \cos 20^\circ - T = m_B Q_{B/A}$$

USING EQ. (1)..

$$T = W_B (\cos 20^\circ + \frac{Q_A}{g})$$

A:

$$\sum F_x = m_A a_x: W_A \sin 20^\circ - N_{AB} - T = m_A Q_A \quad (2)$$

NOW SUBSTITUTE THE EXPRESSIONS FOR  $N_{AB}$  AND  $T$  INTO EQ. (2)..

$$50 \sin 20^\circ + 30 (\sin 20^\circ - \frac{Q_A}{g}) - 30 (\cos 20^\circ + \frac{Q_A}{g}) = 50 \frac{Q_A}{g}$$

$$\text{OR } Q_A = \frac{1}{11} (32.2 \frac{ft}{s^2}) (8 \sin 20^\circ - 3 \cos 20^\circ)$$

$$= -0.24272 \frac{ft}{s^2}$$

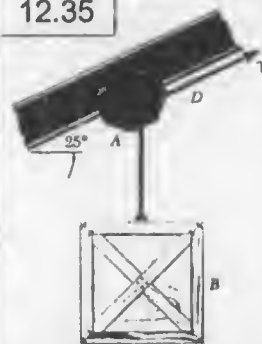
$$\therefore Q_A = 0.243 \frac{ft}{s^2} \triangleleft 20^\circ$$

USING THE ABOVE EXPRESSION FOR  $T$ ..

$$T = (30 \text{ lb}) (\cos 20^\circ + \frac{-0.24272 \frac{ft}{s^2}}{32.2 \frac{ft}{s^2}})$$

$$\text{OR } T = 28.0 \text{ lb}$$

## 12.35



GIVEN:  $W_A = 40 \text{ lb}$ ,  $W_B = 500 \text{ lb}$ ;  
 $Q_A = 1.2 \frac{ft}{s^2}$

FIND: (a)  $Q_{B/A}$   
(b)  $T_{CD}$

(a) FIRST NOTE:  $Q_B = Q_A + Q_{B/A}$  WHERE  $Q_{B/A}$  IS DIRECTED PERPENDICULAR TO CABLE AB

B:

$$\sum F_x = m_B a_x: 0 = -m_B Q_{B/A} \cos 25^\circ$$

$$\text{OR } Q_{B/A} = (1.2 \frac{ft}{s^2}) \cos 25^\circ$$

$$\text{OR } Q_{B/A} = 1.088 \frac{ft}{s^2} \triangleleft$$

(b)

FOR CRATE B..

$$\sum F_y = m_B a_y: T_{AB} - W_B = \frac{W_B}{g} Q_A \sin 25^\circ$$

$$\text{OR } T_{AB} = (500 \text{ lb}) [1 + \frac{(1.2 \frac{ft}{s^2}) \sin 25^\circ}{32.2 \frac{ft}{s^2}}]$$

$$= 507.87 \text{ lb}$$

FOR TROLLEY A..

$$\sum F_x = m_A a_x: T_{CD} - T_{AB} \sin 25^\circ - W_A \sin 25^\circ = \frac{W_A}{g} Q_A$$

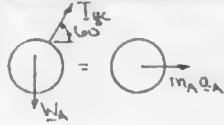
$$\text{OR } T_{CD} = (507.87 \text{ lb}) \sin 25^\circ + (40 \text{ lb}) (\sin 25^\circ + \frac{1.2 \frac{ft}{s^2}}{32.2 \frac{ft}{s^2}})$$

$$\text{OR } T_{CD} = 233 \text{ lb} \triangleleft$$

12.36



GIVEN:  $m_A = 7.1 \text{ kg}$ ;  $\dot{\theta} = \text{CONSTANT}$ ;  
 $p = 0.93 \text{ m}$ ,  $\theta = 60^\circ$   
 FIND: (a)  $T_{BC}$   
 (b)  $\dot{\theta}$



FIRST NOTE..  $a_A = a_n = \frac{\dot{\theta}^2 p}{p}$

$$(a) \uparrow \Sigma F_y = 0: T_{BC} \sin 60^\circ - W_A = 0$$

$$\text{OR } T_{BC} = \frac{7.1 \text{ kg} \cdot 9.81 \text{ m/s}^2}{\sin 60^\circ}$$

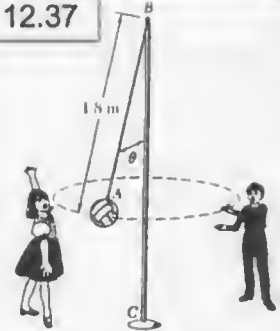
$$= 80.426 \text{ N}$$

$$(b) \uparrow \Sigma F_x = m_A a_n: T_{BC} \cos 60^\circ = m_A \frac{\dot{\theta}^2 p}{p}$$

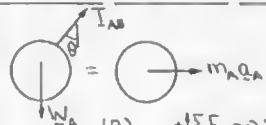
$$\text{OR } \dot{\theta}^2 = \frac{(80.426 \text{ N}) \cos 60^\circ}{7.1 \text{ kg} \cdot 0.93 \text{ m}}$$

$$\text{OR } \dot{\theta} = 2.30 \frac{\text{m}}{\text{s}}$$

12.37



GIVEN:  $m_A = 0.450 \text{ kg}$ ;  $\dot{\theta} = 4 \frac{\text{m}}{\text{s}}$   
 FIND: (a)  $\theta$   
 (b)  $T_{AB}$



FIRST NOTE..  $a_A = a_n = \frac{\dot{\theta}^2 p}{p}$   
 WHERE  $p = l_{AB} \sin \theta$

$$(a) \uparrow \Sigma F_y = 0: T_{AB} \cos \theta - W_A = 0$$

$$\text{OR } T_{AB} = \frac{m_A g}{\cos \theta}$$

$$\uparrow \Sigma F_x = m_A a_n: T_{AB} \sin \theta = m_A \frac{\dot{\theta}^2 p}{p}$$

SUBSTITUTING FOR  $T_{AB}$  AND  $p$ ..

$$\frac{m_A g}{\cos \theta} \sin \theta = m_A \frac{\dot{\theta}^2 l_{AB} \sin \theta}{l_{AB} \sin \theta}$$

$$1 - \cos^2 \theta = \frac{(4 \text{ m/s})^2}{1.8 \text{ m} \cdot 9.81 \frac{\text{m}}{\text{s}^2}} \cos \theta$$

$$\text{OR } \cos^2 \theta + 0.906105 \cos \theta - 1 = 0$$

$$\text{SOLVING.. } \cos \theta = 0.64479$$

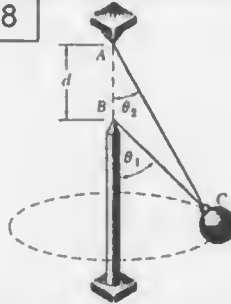
$$\text{OR } \theta = 49.9^\circ$$

$$(b) \text{ FROM ABOVE.. } T_{AB} = \frac{m_A g}{\cos \theta}$$

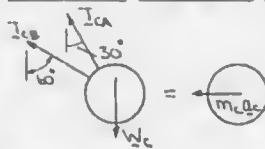
$$= \frac{0.450 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2}}{0.64479}$$

$$\text{OR } T_{AB} = 6.85 \text{ N}$$

12.38



GIVEN:  $L_{ACB} = 80 \text{ in.}$ ;  
 $\dot{\theta}_C = \text{CONSTANT}$ ;  
 $\theta_1 = 60^\circ$ ,  $\theta_2 = 30^\circ$ ;  
 $T_{CA} = T_{CB} = T$   
 FIND:  $\dot{\theta}_C$



FIRST NOTE..  $a_C = a_n = \frac{\dot{\theta}_C^2 p}{p}$

WHERE  $p = L_{AC} \sin 30^\circ + L_{BC} \sin 60^\circ$

$$\text{NOW.. } L_{AC} + L_{BC} = L_{ACB}$$

$$\text{OR } p \left( \frac{1}{\sin 30^\circ} + \frac{1}{\sin 60^\circ} \right) = 80 \text{ in.}$$

$$\text{OR } p = 25.359 \text{ in.}$$

$$\uparrow \Sigma F_y = 0: T_{CA} \cos 30^\circ + T_{CB} \cos 60^\circ - W_c = 0$$

$$\text{OR } T = \frac{m_c g}{\cos 30^\circ + \cos 60^\circ} = 0.73205 m_c g$$

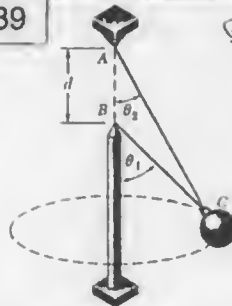
$$\uparrow \Sigma F_x = m_c a_n: T_{CA} \sin 30^\circ + T_{CB} \sin 60^\circ = m_c \frac{\dot{\theta}_C^2 p}{p}$$

$$\text{OR } 0.73205 m_c g (\sin 30^\circ + \sin 60^\circ) = m_c \frac{\dot{\theta}_C^2 p}{p}$$

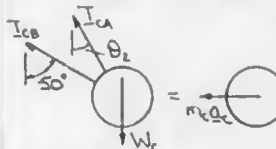
$$\text{OR } \dot{\theta}_C^2 = 0.73205 (32.2 \frac{\text{ft}}{\text{s}^2}) \left( \frac{25.359 \text{ in.}}{12 \text{ in./ft}} \right) (\sin 30^\circ + \sin 60^\circ)$$

$$\text{OR } \dot{\theta}_C = 8.25 \frac{\text{ft}}{\text{s}}$$

12.39



GIVEN:  $W_C = 12 \text{ lb}$ ;  $\dot{\theta}_C = \text{CONSTANT}$ ;  
 $T_{CA} = T_{CB} = 7.6 \text{ lb}$ ;  
 $\theta_1 = 50^\circ$ ,  $d = 30 \text{ in.}$   
 FIND: (a)  $\theta_2$   
 (b)  $\dot{\theta}_C$



$$(a) \uparrow \Sigma F_y = 0: T_{CA} \cos \theta_2 + T_{CB} \cos 50^\circ - W_C = 0$$

$$\text{OR } (7.6 \text{ lb}) (\cos \theta_2 + \cos 50^\circ) = 12 \text{ lb}$$

$$\text{OR } \theta_2 = 20.584^\circ$$

$$\therefore \theta_2 = 20.6^\circ$$

$$(b) \text{ FIRST NOTE.. } a_C = a_n = \frac{\dot{\theta}_C^2 p}{p}$$

$$\text{WHERE } p = d \tan \theta_1 \text{ AND } p = (d + l) \tan \theta_2$$

$$\text{THEN } p = \left( d + \frac{d}{\tan \theta_1} \right) \tan \theta_2$$

$$\text{OR } p = \frac{d}{\frac{1}{\tan \theta_2} - \frac{1}{\tan \theta_1}} = \frac{30 \text{ in.}}{\frac{1}{\tan 20.584^\circ} - \frac{1}{\tan 50^\circ}}$$

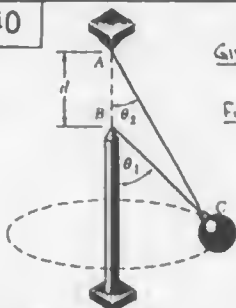
$$= 16.4508 \text{ in.}$$

$$\uparrow \Sigma F_x = m_c a_n: T_{CA} \sin \theta_2 + T_{CB} \sin 50^\circ = \frac{W_C}{g} \frac{\dot{\theta}_C^2 p}{p}$$

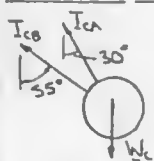
$$\text{OR } \dot{\theta}_C^2 = \frac{7.6 \text{ lb}}{12 \text{ lb}} \left( 32.2 \frac{\text{ft}}{\text{s}^2} \right) \left( \frac{16.4508 \text{ in.}}{12 \text{ in./ft}} \right) (\sin 20.584^\circ + \sin 50^\circ)$$

$$\text{OR } \dot{\theta}_C = 5.59 \frac{\text{ft}}{\text{s}}$$

12.40



GIVEN:  $m_c = 7 \text{ kg}$ ;  $\theta_1 = 55^\circ$ ;  $\theta_2 = 30^\circ$ ;  
 $d = 1.4 \text{ m}$ ;  $\omega_c = \text{CONSTANT}$   
 FIND: RANGE OF VALUES OF  $\omega_c$   
 SO THAT WIRES AC AND BC BOTH REMAIN TAUT

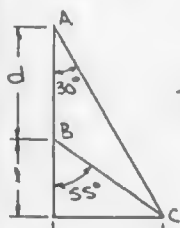


FIRST NOTE..  $a_c = a_n = \frac{\omega_c^2 p}{p}$   
 WHERE

$$p = d \tan 55^\circ \text{ AND } p = (d + h) \tan 30^\circ$$

$$\text{THEN } p = \left( d + \frac{p}{\tan 55^\circ} \right) \tan 30^\circ$$

$$\text{OR } p = \frac{1.4 \text{ m}}{\tan 30^\circ - \tan 55^\circ} = 1.35680 \text{ m}$$



$$\sum F_x = m_c a_c: T_{CA} \sin 30^\circ + T_{CB} \sin 55^\circ = m_c \frac{\omega_c^2}{p}$$

$$\sum F_y = 0: T_{CA} \cos 30^\circ + T_{CB} \cos 55^\circ - W_c = 0$$

$$\text{OR } T_{CA} \cos 30^\circ + T_{CB} \cos 55^\circ = m_c g \quad (2)$$

CASE 1:  $T_{CA} \rightarrow 0$ ; Eq. (2)  $\Rightarrow T_{CB} = \frac{m_c g}{\cos 55^\circ}$

SUBSTITUTING INTO EQ. (1).. $\frac{m_c g}{\cos 55^\circ} \sin 55^\circ = m_c \frac{\omega_c^2}{p}$

$$\text{OR } (\omega_c^2)_{\text{CA}=0} = (1.35680 \text{ m})(9.81 \text{ m/s}^2) \tan 55^\circ$$

$$\text{OR } (\omega_c)_{\text{CA}=0} = 4.36 \text{ m/s}$$

NOW FORM  $(\cos 30^\circ)(1) - (\sin 30^\circ)(2)$ ..

$$T_{CB} \sin 55^\circ \cos 30^\circ - T_{CB} \cos 55^\circ \sin 30^\circ = m_c \frac{\omega_c^2}{p} \cos 30^\circ - m_c g \sin 30^\circ$$

$$\text{OR } T_{CB} \sin 25^\circ = m_c \frac{\omega_c^2}{p} \cos 30^\circ - m_c g \sin 30^\circ$$

$\therefore (\omega_c)_{\text{MAX}}$  OCCURS WHEN  $T_{CB} = (T_{CB})_{\text{MAX}}$ , WHICH OCCURS WHEN  $T_{CA} = 0$ .

$\therefore (\omega_c)_{\text{MAX}} = 4.36 \text{ m/s}$  AND WIRE AC WILL BE TAUT IF  $\omega_c < 4.36 \text{ m/s}$ .

CASE 2:  $T_{CB} \rightarrow 0$ ; Eq. (2)  $\Rightarrow T_{CA} = \frac{m_c g}{\cos 30^\circ}$

SUBSTITUTING INTO EQ. (1).. $\frac{m_c g}{\cos 30^\circ} \sin 30^\circ = m_c \frac{\omega_c^2}{p}$

$$\text{OR } (\omega_c^2)_{\text{CB}=0} = (1.35680 \text{ m})(9.81 \text{ m/s}^2) \tan 30^\circ$$

$$\text{OR } (\omega_c)_{\text{CB}=0} = 2.77 \text{ m/s}$$

NOW FORM  $(\cos 55^\circ)(1) - (\sin 55^\circ)(2)$ ..

$$T_{CA} \sin 30^\circ \cos 55^\circ - T_{CA} \cos 30^\circ \sin 55^\circ = m_c \frac{\omega_c^2}{p} \cos 55^\circ - m_c g \sin 55^\circ$$

$$\text{OR } -T_{CA} \sin 25^\circ = m_c \frac{\omega_c^2}{p} \cos 55^\circ - m_c g \sin 55^\circ$$

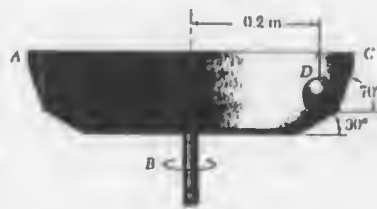
$\therefore (\omega_c)_{\text{MIN}}$  OCCURS WHEN  $T_{CA} = (T_{CA})_{\text{MAX}}$ , WHICH OCCURS WHEN  $T_{CB} = 0$ .

$\therefore (\omega_c)_{\text{MIN}} = 2.77 \text{ m/s}$  AND WIRE BC WILL BE TAUT IF  $\omega_c > 2.77 \text{ m/s}$ .

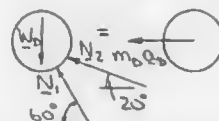
$\therefore$  BOTH WIRES ARE TAUT WHEN  $2.77 \text{ m/s} < \omega_c < 4.36 \text{ m/s}$

12.41

GIVEN:  $m_b = 0.1 \text{ kg}$ ;  $\omega_b = \text{CONSTANT}$   
 FIND: RANGE OF VALUES OF  $\omega_b$  SO THAT NEITHER OF THE NORMAL FORCES EXCEEDS 1.1 N



FIRST NOTE -  $a_b = a_n = \frac{\omega_b^2}{p}$   
 WHERE  $p = 0.2 \text{ m}$



$$\sum F_x = m_b a_b: N_1 \cos 60^\circ + N_2 \cos 20^\circ = m_b \frac{\omega_b^2}{p} \quad (1)$$

$$\sum F_y = 0: N_1 \sin 60^\circ + N_2 \sin 20^\circ - W_b = 0$$

$$\text{OR } N_1 \sin 60^\circ + N_2 \sin 20^\circ = m_b g \quad (2)$$

CASE 1:  $N_1$  IS MAXIMUM

LET  $N_1 = 1.1 \text{ N}$

Eq. (2).. $(1.1 \text{ N}) \sin 60^\circ + N_2 \sin 20^\circ = (0.1 \text{ kg})(9.81 \text{ m/s}^2)$

OR  $N_2 = 0.082954 \text{ N}$

$\therefore (N_2)_{(N_1)_{\text{MAX}}} < 1.1 \text{ N}$ .. O.K.

Eq. (1) ..

$$(\omega_b^2)_{(N_1)_{\text{MAX}}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}} (1.1 \cos 60^\circ + 0.082954 \cos 20^\circ) \text{ N}$$

$$\text{OR } (\omega_b)_{(N_1)_{\text{MAX}}} = 1.121 \text{ m/s}$$

NOW FORM  $(\sin 20^\circ)(1) - (\cos 20^\circ)(2)$ ..

$$N_1 \cos 60^\circ \sin 20^\circ - N_1 \sin 60^\circ \cos 20^\circ = m_b \frac{\omega_b^2}{p} \sin 20^\circ - m_b g \cos 20^\circ$$

$$\text{OR } -N_1 \sin 40^\circ = m_b \frac{\omega_b^2}{p} \sin 20^\circ - m_b g \cos 20^\circ$$

$\therefore (\omega_b)_{\text{MIN}}$  OCCURS WHEN  $N_1 = (N_1)_{\text{MAX}}$

$$\therefore (\omega_b)_{\text{MIN}} = 1.121 \text{ m/s}$$

CASE 2:  $N_2$  IS MAXIMUM

LET  $N_2 = 1.1 \text{ N}$

Eq. (2).. $N_1 \sin 60^\circ + (1.1 \text{ N}) \sin 20^\circ = (0.1 \text{ kg})(9.81 \text{ m/s}^2)$

OR  $N_1 = 0.69834 \text{ N}$

$\therefore (N_1)_{(N_2)_{\text{MAX}}} < 1.1 \text{ N}$ .. O.K.

Eq. (1) ..

$$(\omega_b^2)_{(N_2)_{\text{MAX}}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}} (0.69834 \cos 60^\circ + 1.1 \cos 20^\circ) \text{ N}$$

$$\text{OR } (\omega_b)_{(N_2)_{\text{MAX}}} = 1.663 \text{ m/s}$$

NOW FORM  $(\sin 60^\circ)(1) - (\cos 60^\circ)(2)$ ..

$$N_2 \cos 20^\circ \sin 60^\circ - N_2 \sin 20^\circ \cos 60^\circ = m_b \frac{\omega_b^2}{p} \sin 60^\circ - m_b g \cos 60^\circ$$

$$\text{OR } N_2 \cos 40^\circ = m_b \frac{\omega_b^2}{p} \sin 60^\circ - m_b g \cos 60^\circ$$

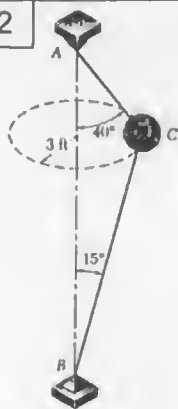
$\therefore (\omega_b)_{\text{MAX}}$  OCCURS WHEN  $N_2 = (N_2)_{\text{MAX}}$

$$\therefore (\omega_b)_{\text{MAX}} = 1.663 \text{ m/s}$$

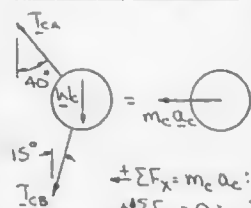
$\therefore$  FOR  $N_1, N_2 \leq 1.1 \text{ N}$

$$1.121 \text{ m/s} \leq \omega_b \leq 1.663 \text{ m/s}$$

12.42



GIVEN:  $W_C = 12 \text{ lb}$ ;  $\dot{\phi} = \text{CONSTANT}$ ;  
 $0 < T_{CA}, T_{CB} \leq 26 \text{ lb}$   
 FIND: RANGE OF VALUES OF  $\dot{\phi}$



FIRST NOTE..  $a_c = a_n = \frac{v^2}{\rho}$   
 WHERE  $\rho = 3 \text{ ft}$

$$\pm \sum F_x = m_c a_c: T_{CA} \sin 40^\circ + T_{CB} \sin 15^\circ = \frac{W_C}{g} \frac{v^2}{\rho} \quad (1)$$

$$\pm \sum F_y = 0: T_{CA} \cos 40^\circ - T_{CB} \cos 15^\circ - W_C = 0 \quad (2)$$

NOTE THAT EQ. (2) IMPLIES THAT

- (a) WHEN  $T_{CB} = (T_{CB})_{\max}$ ,  $T_{CA} = (T_{CA})_{\max}$   
 (b) WHEN  $T_{CB} = (T_{CB})_{\min}$ ,  $T_{CA} = (T_{CA})_{\min}$

CASE 1:  $T_{CA}$  IS MAXIMUM

LET  $T_{CA} = 26 \text{ lb}$

$$\text{EQ. (2)} \dots (26 \text{ lb}) \cos 40^\circ - T_{CB} \cos 15^\circ - (12 \text{ lb}) = 0$$

$$\text{OR } T_{CB} = 8.1964 \text{ lb}$$

$$\therefore (T_{CB})_{\min} < 26 \text{ lb} \dots \text{OK} \quad [(T_{CB})_{\max} = 8.1964 \text{ lb}]$$

EQ. (1) ..

$$(\dot{\phi}^2)_{\max} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(3 \text{ ft})}{12 \text{ lb}} (26 \sin 40^\circ + 8.1964 \sin 15^\circ) \text{ lb}$$

$$\text{OR } (\dot{\phi})_{\max} = 12.31 \frac{\text{ft}}{\text{s}}$$

NOW FORM  $(\cos 15^\circ)(1) + (\sin 15^\circ)(2) \dots$

$$T_{CA} \sin 40^\circ \cos 15^\circ + T_{CB} \cos 40^\circ \sin 15^\circ = \frac{W_C}{g} \frac{v^2}{\rho} \cos 15^\circ + W_C \sin 15^\circ$$

$$\text{OR } T_{CA} \sin 55^\circ = \frac{W_C}{g} \frac{v^2}{\rho} \cos 15^\circ + W_C \sin 15^\circ \quad (3)$$

$\therefore (\dot{\phi})_{\max}$  OCCURS WHEN  $T_{CA} = (T_{CA})_{\max}$

$$\therefore (\dot{\phi})_{\max} = 12.31 \frac{\text{ft}}{\text{s}}$$

CASE 2:  $T_{CA}$  IS MINIMUM

BECAUSE  $(T_{CA})_{\min}$  OCCURS WHEN  $T_{CB} = (T_{CB})_{\min}$ ,  
 LET  $T_{CB} = 0$  (NOTE THAT WIRE BC WILL NOT BE TAUT).

$$\text{EQ. (2)} \dots T_{CA} \cos 40^\circ - (12 \text{ lb}) = 0$$

$$\text{OR } T_{CA} = 15.6649 \text{ lb} < 26 \text{ lb} \dots \text{OK}$$

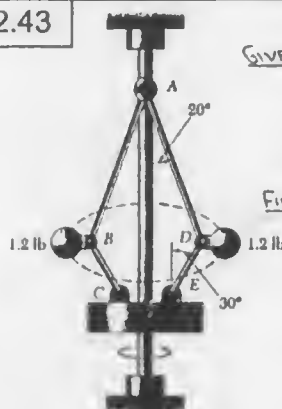
NOTE: EQ. (3) IMPLIES THAT WHEN  $T_{CA} = (T_{CA})_{\min}$ ,  
 $\dot{\phi} = (\dot{\phi})_{\min}$ . THEN..

$$\text{EQ. (1)} \dots (\dot{\phi}^2)_{\min} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(3 \text{ ft})}{12 \text{ lb}} (15.6649 \text{ lb}) \sin 40^\circ$$

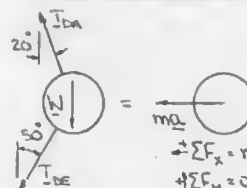
$$\text{OR } (\dot{\phi})_{\min} = 9.00 \frac{\text{ft}}{\text{s}}$$

$$\therefore 0 < T_{CA}, T_{CB} \leq 26 \text{ lb} \text{ WHEN } 9.00 \frac{\text{ft}}{\text{s}} < \dot{\phi} \leq 12.31 \frac{\text{ft}}{\text{s}}$$

12.43



GIVEN:  $\dot{\phi}_{\text{FLYBALL}} = \dot{\phi} = \text{CONSTANT}$ ;  
 $p = 6 \text{ in.}$ ;  $W_{AB}, W_{AC}$ ,  
 $W_{AD}$ , AND  $W_{DE}$   
 NEGLECTIBLE;  
 $0 \leq T_{AB}, T_{BC}, T_{AD}, T_{DE} \leq 17 \text{ lb}$   
 FIND: RANGE OF VALUES OF  $\dot{\phi}$



FIRST NOTE..  $a_c = a_n = \frac{v^2}{\rho}$   
 WHERE  $\rho = 0.5 \text{ ft}$

$$\pm \sum F_x = m a: T_{CA} \sin 20^\circ + T_{CB} \sin 30^\circ = \frac{W}{g} \frac{v^2}{\rho} \quad (1)$$

$$\pm \sum F_y = 0: T_{CA} \cos 20^\circ - T_{CB} \cos 30^\circ - W = 0 \quad (2)$$

NOTE THAT EQ. (2) IMPLIES THAT

- (a) WHEN  $T_{CB} = (T_{CB})_{\max}$ ,  $T_{CA} = (T_{CA})_{\max}$   
 (b) WHEN  $T_{CB} = (T_{CB})_{\min}$ ,  $T_{CA} = (T_{CA})_{\min}$

CASE 1:  $T_{CA}$  IS MAXIMUM

LET  $T_{CA} = 17 \text{ lb}$

$$\text{EQ. (2)} \dots (17 \text{ lb}) \cos 20^\circ - T_{CB} \cos 30^\circ - (1.2 \text{ lb}) = 0$$

$$\text{OR } T_{CB} = 17.06 \text{ lb} \dots \text{UNACCEPTABLE } (> 17 \text{ lb})$$

NOW LET  $T_{CB} = 17 \text{ lb}$

$$\text{EQ. (2)} \dots T_{CA} \cos 20^\circ - (17 \text{ lb}) \cos 30^\circ - (1.2 \text{ lb}) = 0$$

$$\text{OR } T_{CA} = 16.9443 \text{ lb} \dots \text{OK } (< 17 \text{ lb})$$

$$\therefore (T_{CA})_{\max} = 16.9443 \text{ lb} \quad (T_{CB})_{\max} = 17 \text{ lb}$$

EQ. (1) ..

$$(\dot{\phi}^2)_{\max} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(0.5 \text{ ft})}{1.2 \text{ lb}} (16.9443 \sin 20^\circ + 17 \sin 30^\circ) \text{ lb}$$

$$\text{OR } \dot{\phi}_{\max} = 13.85 \frac{\text{ft}}{\text{s}}$$

NOW FORM  $(\cos 30^\circ)(1) + (\sin 30^\circ)(2) \dots$

$$T_{CA} \sin 20^\circ \cos 30^\circ + T_{CB} \cos 20^\circ \sin 30^\circ = \frac{W}{g} \frac{v^2}{\rho} \cos 30^\circ + W \sin 30^\circ$$

$$\text{OR } T_{CA} \sin 50^\circ = \frac{W}{g} \frac{v^2}{\rho} \cos 30^\circ + W \sin 30^\circ \quad (3)$$

$\therefore \dot{\phi}_{\min}$  OCCURS WHEN  $T_{CA} = (T_{CA})_{\min}$

$$\therefore \dot{\phi}_{\min} = 13.85 \frac{\text{ft}}{\text{s}}$$

CASE 2:  $T_{CA}$  IS MINIMUM

BECAUSE  $(T_{CA})_{\min}$  OCCURS WHEN  $T_{CB} = (T_{CB})_{\min}$ ,  
 LET  $T_{CB} = 0$ .

$$\text{EQ. (2)} \dots T_{CA} \cos 20^\circ - (1.2 \text{ lb}) = 0$$

$$\text{OR } T_{CA} = 1.27701 \text{ lb} < 17 \text{ lb} \dots \text{OK}$$

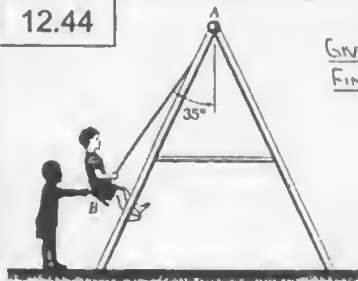
NOTE: EQ. (3) IMPLIES THAT WHEN  $T_{CA} = (T_{CA})_{\min}$ ,  
 $\dot{\phi} = \dot{\phi}_{\min}$ . THEN..

$$\text{EQ. (1)} \dots (\dot{\phi}^2)_{\min} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(0.5 \text{ ft})}{1.2 \text{ lb}} (1.27701 \text{ lb}) \sin 20^\circ$$

$$\text{OR } \dot{\phi}_{\min} = 2.42 \frac{\text{ft}}{\text{s}}$$

$$\therefore 0 \leq T_{AB}, T_{BC}, T_{AD}, T_{DE} \leq 17 \text{ lb} \text{ WHEN } 2.42 \frac{\text{ft}}{\text{s}} \leq \dot{\phi} \leq 13.85 \frac{\text{ft}}{\text{s}}$$

12.44



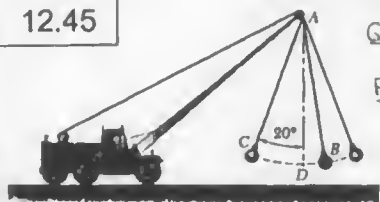
GIVEN:  $m = 22 \text{ kg}$   
 FIND: (a)  $T_{BA}$  WHEN  $F_c = F_c$   
 (b)  $T_{BA}$  AT  $t = 0$  WHEN  $F_c = 0$

NOTE: THE FACTORS OF  $\frac{1}{2}$  ARE INCLUDED IN THE FOLLOWING FREE-BODY DIAGRAMS BECAUSE THERE ARE TWO ROPES AND ONLY ONE IS CONSIDERED.

(a) FOR THE SWING AT REST...  
 $\Sigma F_y = 0: T_{BA} \cos 35^\circ - \frac{1}{2}W = 0$   
 OR  $T_{BA} = \frac{22 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2}}{2 \cos 35^\circ}$   
 OR  $T_{BA} = 131.7 \text{ N}$

(b) AT  $t = 0$ ,  $v = 0$  SO THAT  $a_n = \frac{v^2}{r} = 0$   
 $\Sigma F_n = 0: T_{BA} - \frac{1}{2}W \cos 35^\circ = 0$   
 OR  $T_{BA} = \frac{1}{2}(22 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2}) \cos 35^\circ$   
 OR  $T_{BA} = 88.4 \text{ N}$

12.45



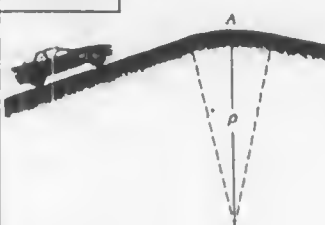
GIVEN:  $m_B = 60 \text{ kg}$ ,  $L_{AB} = 15 \text{ m}$ ,  
 $(v_B)_B = 4.2 \frac{\text{m}}{\text{s}}$   
 FIND: (a)  $T_{BA}$  AT C  
 (b)  $T_{BA}$  AT D

(a) AT C, THE TOP OF THE SWING,  $v_B = 0$ ; THUS  $a_n = \frac{v_B^2}{L_{AB}} = 0$

$\Sigma F_n = 0: T_{BA} - W_B \cos 20^\circ = 0$   
 OR  $T_{BA} = (60 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) \cos 20^\circ$   
 OR  $T_{BA} = 553 \text{ N}$

(b)  $\Sigma F_n = ma_n: T_{BA} - W_B = m_B \frac{(v_B)_B^2}{L_{AB}}$   
 OR  $T_{BA} = (60 \text{ kg}) \left[ 9.81 \frac{\text{m}}{\text{s}^2} + \frac{(4.2 \frac{\text{m}}{\text{s}})^2}{15 \text{ m}} \right]$   
 OR  $T_{BA} = 659 \text{ N}$

12.46

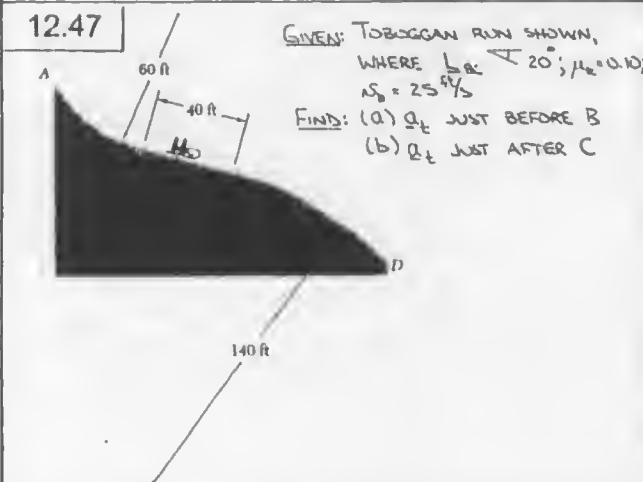


GIVEN: ROAD WITH RADIUS OF CURVATURE  $p$  AS SHOWN  
 FIND: (a)  $p$  FOR  $W_{car} = 2400 \text{ lb}$ ,  $v_A = 100 \frac{\text{mi}}{\text{h}}$ ,  $N_{road} = 0$   
 (b)  $N$  FOR  $W = 160 \text{ lb}$ ,  $v_A = 50 \frac{\text{mi}}{\text{h}}$

(a) NOTE:  $100 \frac{\text{mi}}{\text{h}} = 146.667 \frac{\text{ft}}{\text{s}}$   
 $\Sigma F_n = ma_n: W_{car} = m_{car} \frac{v_A^2}{p}$   
 OR  $p = \frac{(146.667 \frac{\text{ft}}{\text{s}})^2}{32.2 \frac{\text{ft}}{\text{s}^2}} = 668.05 \text{ ft}$   
 OR  $p = 668 \text{ ft}$

(b) NOTE:  $v$  IS CONSTANT  $\Rightarrow a_t = 0$ ; SO  $\frac{v}{r} = 73.333 \frac{\text{ft}}{\text{s}}$   
 $\Sigma F_n = ma_n: W - N = m \frac{v_A^2}{p}$   
 OR  $N = (160 \text{ lb}) \left[ 1 - \frac{(73.333 \frac{\text{ft}}{\text{s}})^2}{(32.2 \frac{\text{ft}}{\text{s}^2})(668.05 \text{ ft})} \right]$   
 OR  $N = 120.0 \text{ lb}$

12.47



GIVEN: TOBOGGAN RUN SHOWN, WHERE  $L_{AC} = 20^\circ$ ;  $\mu_k = 0.10$ ,  $(v_B)_B = 25 \frac{\text{ft}}{\text{s}}$   
 FIND: (a)  $Q_t$  JUST BEFORE B  
 (b)  $Q_t$  JUST AFTER C

(a) NOTE: JUST BEFORE B,  $p_B = 60 \text{ ft}$   
 $\Sigma F_n = ma_n: N - W \cos 20^\circ = m \frac{v_B^2}{p_B}$   
 OR  $N = W (\cos 20^\circ + \frac{v_B^2}{g p_B})$

SLIDING:  $F = \mu_k N$   
 $= \mu_k W (\cos 20^\circ + \frac{v_B^2}{g p_B})$   
 $\Sigma F_t = ma_t: W \sin 20^\circ - F = m a_t$   
 OR  $a_t = g (\sin 20^\circ - \mu_k \cos 20^\circ) - \mu_k \frac{v_B^2}{p_B}$  (CONTINUED)

# 12.47 continued

THEN..  $a_t = (32.2 \frac{ft}{s^2})(\sin 20^\circ - 0.1 \cos 20^\circ) - 0.1 \frac{(25 \frac{ft}{s})^2}{60 ft}$

OR  $a_t = 6.95 \frac{ft}{s^2} \nabla 20^\circ$

(b) IT IS FIRST NECESSARY TO DETERMINE  $a_t$ .  
FOR SECTION BC..



$\sum F_y = 0: N_{bc} - W \cos 20^\circ = 0$   
OR  $N_{bc} = W \cos 20^\circ$

SLIDING:  $F_{bc} = \mu_k N_{bc} = \mu_k W \cos 20^\circ$

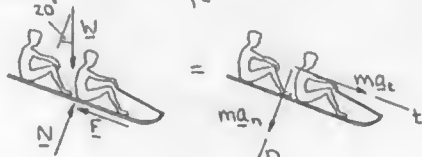
$\sum F_x = m a_t: W \sin 20^\circ - F_{bc} = m a_t$   
OR  $a_t = g(\sin 20^\circ - \mu_k \cos 20^\circ)$   
 $= (32.2 \frac{ft}{s^2})(\sin 20^\circ - 0.1 \cos 20^\circ)$   
 $= 7.9872 \frac{ft}{s^2}$

FOR THIS UNIFORMLY ACCELERATED MOTION HAVE..

$v_c^2 = v_b^2 + 2 a_{bc} \Delta x_{bc}$   
 $= (25 \frac{ft}{s})^2 + 2(7.9872 \frac{ft}{s^2})(40 ft)$

OR  $v_c = 35.552 \frac{ft}{s}$

NOW.. JUST AFTER C,  $r = 140 ft$



$\sum F_n = m a_n: W \cos 20^\circ - N = \frac{W v_c^2}{r}$   
OR  $N = W(\cos 20^\circ - \frac{v_c^2}{g r})$

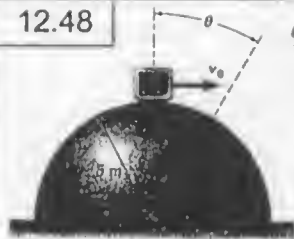
SLIDING:  $F = \mu_k N$   
 $= \mu_k W(\cos 20^\circ - \frac{v_c^2}{g r})$

$\sum F_t = m a_t: W \sin 20^\circ - F = \frac{W v_c^2}{r}$   
OR  $a_t = g(\sin 20^\circ - \mu_k \cos 20^\circ) + \mu_k \frac{v_c^2}{r}$

NOTE:  $g(\sin 20^\circ - \mu_k \cos 20^\circ) = a_{bc}$   
THEN..  $a_t = 7.9872 \frac{ft}{s^2} + 0.1 \frac{(35.552 \frac{ft}{s})^2}{140 ft}$

OR  $a_t = 8.89 \frac{ft}{s^2} \nabla 20^\circ$

# 12.48

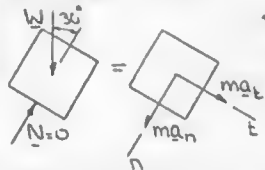


GIVEN:  $m = 0.5 kg$ ; AT  $t = 0$ ,  
 $v = v_0$ ; WHEN  $\theta = 30^\circ$ ,  
 $N \rightarrow 0$

FIND: (a)  $v_0$

(b) FORCE EXERTED ON  
THE SURFACE BY  
THE BLOCK WHEN  
 $v = v_0$

(a) WHEN  $\theta = 30^\circ$ ..

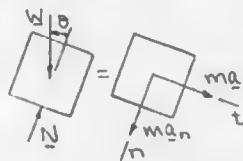


$\sum F_n = m a_n: W \cos 30^\circ = m \frac{v_0^2}{r}$   
OR  $v_0^2 = g r \cos 30^\circ$

(CONTINUED)

# 12.48 continued

FOR  $0 < \theta < 30^\circ$



$\sum F_t = m a_t: W \sin \theta = m a_t$   
OR  $a_t = g \sin \theta$

NOW..  $a_t = v \frac{dv}{ds}$  AND  $ds = r d\theta$

THEN  $\int_{v_0}^v v \frac{dv}{ds} = a_t = g \sin \theta$   
AT  $\theta = 0$ ,  $v = v_0$ :  $\int_{v_0}^v v dv = g \int_0^{30^\circ} \sin \theta d\theta$

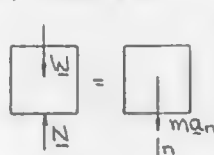
OR  $\frac{1}{2} [v^2]_{v_0}^v = g [-\cos \theta]_{0^\circ}^{30^\circ}$

OR  $v^2 - v_0^2 = 2 g (1 - \cos 30^\circ)$

THEN  $v_0^2 = g \cos 30^\circ - 2 g (1 - \cos 30^\circ)$   
 $= g (3 \cos 30^\circ - 2)$   
 $= (1.5 m)(9.81 \frac{m}{s^2})(3 \cos 30^\circ - 2)$   
 $= 8.80069 \frac{m^2}{s^2}$

OR  $v_0 = 2.97 \frac{m}{s}$

(b) WHEN  $\theta = 0$ ..



$\sum F_n = m a_n: W - N = m \frac{v_0^2}{r}$   
OR  $N = m(g - \frac{v_0^2}{r})$

$= (0.5 kg)(9.81 \frac{m}{s^2} - \frac{8.80069 \frac{m^2}{s^2}}{1.5 m})$

$= 1.971 N$

$\therefore$  THE FORCE EXERTED ON THE SURFACE BY THE BLOCK  
IS  $1.971 N \nmid$

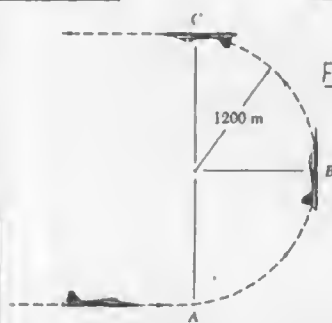
# 12.49

GIVEN:  $m = 54 kg$ ;  $a_t = \text{CONSTANT}$

$(W_{app})_A = 1680 N$

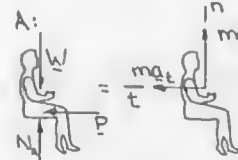
$(W_{app})_C = 350 N$

FIND:  $(F_{pilot})_B$



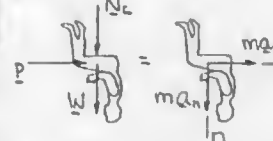
FIRST NOTE THAT THE PILOT'S APPARENT WEIGHT IS  
EQUAL TO THE VERTICAL FORCE THAT SHE EXERTS  
ON THE SEAT OF THE JET TRAINER.

AT A:



$\sum F_n = m a_n: N_A - W = m \frac{v_A^2}{r}$   
OR  $v_A^2 = (1200 m)(\frac{1680 N}{54 kg} - 9.81 \frac{m}{s^2})$   
 $= 25561.3 \frac{m^2}{s^2}$

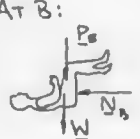
AT C:



$\sum F_n = m a_n: N_C + W = m \frac{v_C^2}{r}$   
OR  $v_C^2 = (1200 m)(\frac{350 N}{54 kg} + 9.81 \frac{m}{s^2})$   
 $= 19549.8 \frac{m^2}{s^2}$   
(CONTINUED)


# 12.49 continued

SINCE  $a_t = \text{CONSTANT}$ , HAVE FROM A TO C..  
 $v_C^2 = v_A^2 + 2a_t \Delta s_{AC}$   
 OR  $19.549.8 \frac{m^2}{s^2} = 2.5561.3 \frac{m^2}{s^2} + 2a_t(\pi \cdot 1200 m)$   
 OR  $a_t = -0.79730 \frac{m}{s^2}$   
 THEN FROM A TO B..  
 $v_B^2 = v_A^2 + 2a_t \Delta s_{AB}$   
 $= 2.5561.3 \frac{m^2}{s^2} + 2(-0.79730 \frac{m}{s^2})(\frac{\pi}{2} \cdot 1200 m)$   
 $= 22.555 \frac{m^2}{s^2}$

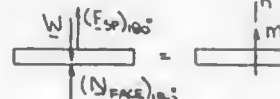
AT B:  
  
 $\pm \Sigma F_n = ma_n; N_b = m \frac{v_B^2}{r}$   
 OR  $N_b = 54 kg \frac{22.555 \frac{m^2}{s^2}}{1200 m}$   
 OR  $N_b = 1014.98 N \leftarrow$   
 $\pm \Sigma F_t = ma_t; W + P_c = m a_t$   
 OR  $P_c = (54 kg)(0.79730 - 9.81) \frac{m}{s^2}$   
 OR  $P_c = 486.69 N \downarrow$   
 FINALLY..  $(F_{PILOT})_b = \sqrt{N_b^2 + P_c^2} = \sqrt{(1014.98)^2 + (486.69)^2}$   
 $= 1126 N$   
 OR  $(F_{PILOT})_b = 1126 N \searrow 25.6^\circ \leftarrow$

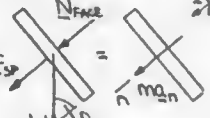
# 12.50

GIVEN:  $W_B = 0.5 lb$ ;  
 $\dot{\theta} = \text{CONSTANT}$ ; WHEN  
 $\theta = 180^\circ$ ,  $N_{FACE} = 0.8 lb$   
 FIND: RANGE OF VALUES  
 OF  $\theta$  SO THAT  
 $N_{FACE} = 0$



FIRST NOTE THAT  $\dot{\theta} = \text{CONSTANT} \Rightarrow a_B = \text{CONSTANT} \Rightarrow a_t = 0$   
 WHEN  $\theta = 180^\circ$ :

  
 $\pm \Sigma F_n = ma_n; (N_{FACE})_{180^\circ} + (F_{cp})_{180^\circ} - W = m \frac{v_B^2}{r_{max}}$

FOR AN ARBITRARY VALUE OF  $\theta$ :  
 $\pm \Sigma F_n = ma_n; N_{FACE} + F_{cp} + W \cos \theta = m \frac{v_B^2}{r}$   


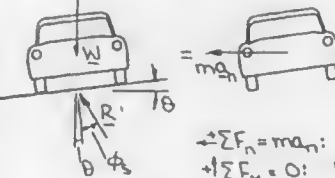
NOW.. AS BLOCK B LOSES CONTACT WITH THE CAVITY AT A,  $N_{FACE} \rightarrow 0$ ,  $F_{cp} \approx (F_{cp})_{180^\circ}$ ,  $P \approx P_{max}$   
 THEN..  $(F_{cp})_{180^\circ} + W \cos \theta = (N_{FACE})_{180^\circ} + (F_{cp})_{180^\circ} - W (= m \frac{v_B^2}{r_{max}})$   
 OR  $\cos \theta = \frac{(N_{FACE})_{180^\circ}}{W} - 1 = \frac{0.8 lb}{0.5 lb} - 1 = 0.6$   
 OR  $\theta = \pm 53.1^\circ$

$\therefore$  BLOCK B IS NOT IN CONTACT WITH THE FACE OF THE CAVITY AT END A WHEN  $-53.1^\circ \leq \theta \leq 53.1^\circ \leftarrow$

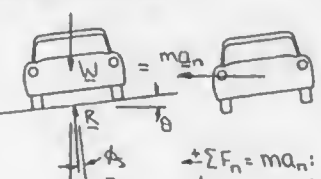
# 12.51

GIVEN: CAR TRAVELING AT A CONSTANT SPEED  
 $v$  ON A ROAD BANKED AT AN ANGLE  
 $\theta$   
 FIND: RANGE OF VALUES OF  $v$  SO THAT THE  
 CAR DOES NOT SKID;  $v = f(r, \theta, \phi_s)$

CASE 1:  $v = v_{MAX}$

NOTE:  $R = F + N$   
  
 $\pm \Sigma F_n = ma_n; R \sin(\theta + \phi_s) = m \frac{v_{MAX}^2}{r}$  (1)  
 $\pm \Sigma F_t = 0; R \cos(\theta + \phi_s) - W = 0$  (2)  
 OR  $R \cos(\theta + \phi_s) = mg$  (2)  
 FORMING (1) / (2) ..  $\frac{R \sin(\theta + \phi_s)}{R \cos(\theta + \phi_s)} = \frac{m \frac{v_{MAX}^2}{r}}{mg}$   
 OR  $v_{MAX} = \sqrt{rg \tan(\theta + \phi_s)}$

CASE 2:  $v = v_{MIN}$

NOTE:  $R = F + N$   
  
 $\pm \Sigma F_n = ma_n; R \sin(\theta - \phi_s) = m \frac{v_{MIN}^2}{r}$  (3)  
 $\pm \Sigma F_t = 0; R \cos(\theta - \phi_s) - W = 0$  (4)  
 OR  $R \cos(\theta - \phi_s) = mg$  (4)  
 FORMING (3) / (4) ..  $\frac{R \sin(\theta - \phi_s)}{R \cos(\theta - \phi_s)} = \frac{m \frac{v_{MIN}^2}{r}}{mg}$   
 OR  $v_{MIN} = \sqrt{rg \tan(\theta - \phi_s)}$   
 $\therefore$  FOR THE CAR NOT TO SKID..  
 $\sqrt{rg \tan(\theta - \phi_s)} \leq v \leq \sqrt{rg \tan(\theta + \phi_s)} \leftarrow$

# 12.52

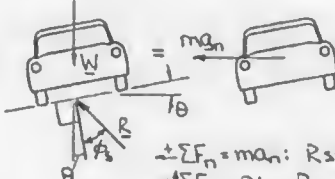
GIVEN:  $v = 95 \frac{km}{h}$ ;  $r = 40 m$ ;  $\mu_s = 0.70$   
 FIND: (a)  $\Delta s$  FOR NO SKIDDING WHEN  $\theta = 10^\circ$   
 (b)  $\Delta s$  FOR NO SKIDDING WHEN  $\theta = -5^\circ$



FIRST NOTE..  $\tan \phi_s = 0.70 (= \mu_s)$   
 OR  $\phi_s = 34.992^\circ$

ALSO, REQUIRING THAT THE SPEED OF THE CAR BE DECREASED TO AVOID SKIDDING, IMPLIES THAT IMPENDING SKIDDING IS "OUTWARD."

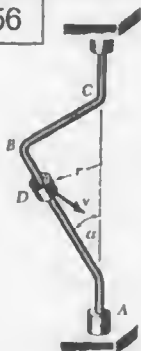
(a)  $\theta = 10^\circ$

  
 $\pm \Sigma F_n = ma_n; R \sin(\theta + \phi_s) = m \frac{v^2}{r}$  (1)  
 $\pm \Sigma F_t = 0; R \cos(\theta + \phi_s) - W = 0$  (2)  
 OR  $R \cos(\theta + \phi_s) = mg$  (2)  
 FORMING (1) / (2) ..  $\frac{R \sin(\theta + \phi_s)}{R \cos(\theta + \phi_s)} = \frac{m \frac{v^2}{r}}{mg}$   
 (CONTINUED)





12.56



GIVEN:  $m_D = 0.2 \text{ kg}$ ;  $\alpha = 30^\circ$ ;  $r = 0.6 \text{ m}$ ,  
 $\mu_s = 0.30$ ;  $\dot{\theta}_{ABC} = \text{CONSTANT}$   
 FIND: RANGE OF VALUES OF  $\dot{\theta}$   
 SO THAT COLLAR D DOES NOT SLIDE ON THE ROD

CASE 1:  $N = N_{\text{MIN}}$ , IMPENDING MOTION DOWNWARD

$$\begin{aligned} \sum F_x = m a_x: N - W \sin 30^\circ &= m \frac{v^2}{r} \cos 30^\circ \\ \text{OR } N &= m(g \sin 30^\circ + \frac{v^2}{r} \cos 30^\circ) \\ \sum F_y = m a_y: F - W \cos 30^\circ &= -m \frac{v^2}{r} \sin 30^\circ \\ \text{OR } F &= m(g \cos 30^\circ - \frac{v^2}{r} \sin 30^\circ) \end{aligned}$$

Now..  $F = \mu_s N$

THEN..  $W(g \cos 30^\circ - \frac{v^2}{r} \sin 30^\circ) = \mu_s m(g \sin 30^\circ + \frac{v^2}{r} \cos 30^\circ)$

$$\begin{aligned} \text{OR } v^2 &= g r \frac{1 - \mu_s \tan 30^\circ}{\mu_s + \tan 30^\circ} \\ &= (9.81 \frac{\text{m}}{\text{s}^2})(0.6 \text{ m}) \frac{1 - 0.3 \tan 30^\circ}{0.3 + \tan 30^\circ} \end{aligned}$$

$$\text{OR } v_{\text{MIN}} = 2.36 \frac{\text{m}}{\text{s}}$$

CASE 2:  $N = N_{\text{MAX}}$ , IMPENDING MOTION UPWARD

$$\begin{aligned} \sum F_x = m a_x: N - W \sin 30^\circ &= m \frac{v^2}{r} \cos 30^\circ \\ \text{OR } N &= m(g \sin 30^\circ + \frac{v^2}{r} \cos 30^\circ) \\ \sum F_y = m a_y: F + W \cos 30^\circ &= m \frac{v^2}{r} \sin 30^\circ \\ \text{OR } F &= m(-g \cos 30^\circ + \frac{v^2}{r} \sin 30^\circ) \end{aligned}$$

Now..  $F = \mu_s N$

THEN  $m(-g \cos 30^\circ + \frac{v^2}{r} \sin 30^\circ) = \mu_s m(g \sin 30^\circ + \frac{v^2}{r} \cos 30^\circ)$

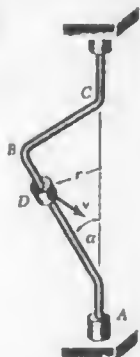
$$\begin{aligned} \text{OR } v^2 &= g r \frac{1 + \mu_s \tan 30^\circ}{\tan 30^\circ - \mu_s} \\ &= (9.81 \frac{\text{m}}{\text{s}^2})(0.6 \text{ m}) \frac{1 + 0.3 \tan 30^\circ}{\tan 30^\circ - 0.3} \end{aligned}$$

$$\text{OR } v_{\text{MAX}} = 4.99 \frac{\text{m}}{\text{s}}$$

FOR THE COLLAR NOT TO SLIDE..

$$2.36 \frac{\text{m}}{\text{s}} \leq v \leq 4.99 \frac{\text{m}}{\text{s}}$$

12.57



GIVEN:  $W_D = 0.6 \text{ lb}$ ;  $r = 8 \text{ in.}$ ,  
 $\dot{\theta} = 10 \frac{\text{RAD}}{\text{s}}$  (CONSTANT);  
 COLLAR D DOES NOT SLIDE ON THE ROD

FIND: (a)  $(\mu_s)_{\text{MIN}}$  WHEN  $\alpha = 15^\circ$   
 (b)  $(\mu_s)_{\text{MIN}}$  WHEN  $\alpha = 45^\circ$

12.57 continued

FIRST NOTE THAT  $v = r \dot{\theta}_{ABC} = (\frac{8}{12} \text{ ft})(10 \frac{\text{RAD}}{\text{s}}) = \frac{20}{3} \frac{\text{ft}}{\text{s}}$   
 AND THAT REQUIRING  $\mu_s = (\mu_s)_{\text{MIN}}$  IMPLIES THAT  
 SLIDING OF COLLAR D IS IMPENDING. ALSO,  
 $\mu_s = \tan \phi_s$

NOW CONSIDER THE TWO POSSIBLE CASES OF  
 IMPENDING MOTION.

CASE 1: IMPENDING MOTION DOWNWARD

$$\begin{aligned} \sum F_x = m a_x: N - W \sin \alpha &= \frac{W}{g} \frac{v^2}{r} \cos \alpha \\ \text{OR } N &= W(\sin \alpha + \frac{v^2}{g r} \cos \alpha) \\ \sum F_y = m a_y: F - W \cos \alpha &= -\frac{W}{g} \frac{v^2}{r} \sin \alpha \\ \text{OR } F &= W(\cos \alpha - \frac{v^2}{g r} \sin \alpha) \end{aligned}$$

Now..  $F = \mu_s N$

THEN..  $W(\cos \alpha - \frac{v^2}{g r} \sin \alpha) = \mu_s W(\sin \alpha + \frac{v^2}{g r} \cos \alpha)$

$$\begin{aligned} \text{OR } \frac{v^2}{g r} &= \frac{1 - \mu_s \tan \alpha}{\tan \alpha + \mu_s} = \frac{1 - \tan \phi_s \tan \alpha}{\tan \alpha + \tan \phi_s} \\ &= \frac{1}{\tan(\alpha + \phi_s)} \end{aligned}$$

CASE 2: IMPENDING MOTION UPWARD

$$\begin{aligned} \sum F_x = m a_x: N - W \sin \alpha &= \frac{W}{g} \frac{v^2}{r} \cos \alpha \\ \text{OR } N &= W(\sin \alpha + \frac{v^2}{g r} \cos \alpha) \\ \sum F_y = m a_y: F + W \cos \alpha &= \frac{W}{g} \frac{v^2}{r} \sin \alpha \\ \text{OR } F &= W(-\cos \alpha + \frac{v^2}{g r} \sin \alpha) \end{aligned}$$

Now..  $F = \mu_s N$

THEN..  $W(-\cos \alpha + \frac{v^2}{g r} \sin \alpha) = \mu_s W(\sin \alpha + \frac{v^2}{g r} \cos \alpha)$

$$\begin{aligned} \text{OR } \frac{v^2}{g r} &= \frac{1 + \mu_s \tan \alpha}{\tan \alpha - \mu_s} = \frac{1 + \tan \phi_s \tan \alpha}{\tan \alpha - \tan \phi_s} \\ &= \frac{1}{\tan(\alpha - \phi_s)} \end{aligned}$$

Now..  $\frac{g r}{v^2} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(\frac{8}{12} \text{ ft})}{(\frac{20}{3} \frac{\text{ft}}{\text{s}})^2} = 0.483$

THEN  $\tan(\alpha \pm \phi_s) = 0.483$

OR  $\alpha \pm \phi_s = 25.781^\circ$ ,  $\phi_s \geq 0$

AND WHERE THE "+" CORRESPONDS TO IMPENDING  
 MOTION DOWNWARD AND THE "-" TO IMPENDING  
 MOTION UPWARD.

(a)  $\alpha = 15^\circ$ : HAVE  $15^\circ \pm \phi_s = 25.781^\circ$   
 $\phi_s \geq 0 \Rightarrow "+"$  SO THAT  $\phi_s = 10.781^\circ$

THEN  $(\mu_s)_{\text{MIN}} = \tan 10.781^\circ$

OR  $(\mu_s)_{\text{MIN}} = 0.1904$ , MOTION IMPENDING DOWNWARD

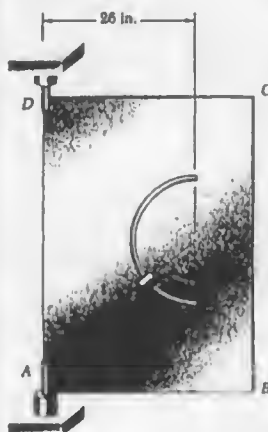
(b)  $\alpha = 45^\circ$ : HAVE  $45^\circ \pm \phi_s = 25.781^\circ$   
 $\phi_s \geq 0 \Rightarrow "-"$  SO THAT  $\phi_s = 19.219^\circ$

THEN  $(\mu_s)_{\text{MIN}} = \tan 19.219^\circ$

OR  $(\mu_s)_{\text{MIN}} = 0.349$ , MOTION IMPENDING UPWARD

(CONTINUED)

## 12.58



GIVEN:  $F = 10 \text{ in}$ ,  $\phi_{ABCD} = 14 \frac{\text{rad}}{\text{s}}$ ;  
 $W_E = 0.8 \text{ lb}$ ;  $\mu_s = 0.35$ ,  
 $\mu_k = 0.25$

C FIND: (a)  $F$  AND IF THE BLOCK SLIDES IN THE SLOT AT  $t=0$  WHEN  $\theta = 80^\circ$   
 (b)  $F$  AND IF THE BLOCK SLIDES IN THE SLOT AT  $t=0$  WHEN  $\theta = 40^\circ$

FIRST NOTE..  $\rho = \frac{1}{2}(26 - 10 \sin \theta) \text{ ft}$

$$\text{THEN } a_n = \frac{v_c^2}{\rho} = \frac{(\dot{\phi} \rho)^2}{\rho} = \rho (\dot{\phi}_{ABCD})^2 = \left[ \frac{1}{2}(26 - 10 \sin \theta) \text{ ft} \right] \left( 14 \frac{\text{rad}}{\text{s}} \right)^2$$

$$= \frac{98}{3} (13.5 \sin \theta) \frac{\text{ft}}{\text{s}^2}$$

ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.

$$\sum F_x = m a_x: N + W \cos \theta = m \frac{v_c^2}{\rho} \sin \theta$$

$$\text{OR } N = W(-\cos \theta + \frac{v_c^2}{g \rho} \sin \theta)$$

$$\sum F_y = m a_y: -F + W \sin \theta = m \frac{v_c^2}{\rho} \cos \theta$$

$$\text{OR } F = W(\sin \theta + \frac{v_c^2}{g \rho} \cos \theta)$$

(a) HAVE  $\theta = 80^\circ$ .. THEN

$$N = (0.8 \text{ lb}) \left[ -\cos 80^\circ + \frac{1}{32.2 \frac{\text{ft}}{\text{s}^2}} \cdot \frac{98}{3} (13.5 \sin 80^\circ) \frac{\text{ft}}{\text{s}^2} \cdot \sin 80^\circ \right]$$

$$= 6.3159 \text{ lb}$$

$$F = (0.8 \text{ lb}) \left[ \sin 80^\circ + \frac{1}{32.2 \frac{\text{ft}}{\text{s}^2}} \cdot \frac{98}{3} (13.5 \sin 80^\circ) \frac{\text{ft}}{\text{s}^2} \cdot \cos 80^\circ \right]$$

$$= 1.92601 \text{ lb}$$

$$\text{NOW.. } F_{\max} = \mu_s N = 0.35(6.3159 \text{ lb})$$

$$= 2.2106 \text{ lb}$$

$\therefore$  THE BLOCK DOES NOT SLIDE IN THE SLOT AND

$$F = 1.926 \text{ lb} \angle 80^\circ$$

(b) HAVE  $\theta = 40^\circ$ .. THEN

$$N = (0.8 \text{ lb}) \left[ -\cos 40^\circ + \frac{1}{32.2 \frac{\text{ft}}{\text{s}^2}} \cdot \frac{98}{3} (13.5 \sin 40^\circ) \frac{\text{ft}}{\text{s}^2} \cdot \sin 40^\circ \right]$$

$$= 4.4924 \text{ lb}$$

$$F = (0.8 \text{ lb}) \left[ \sin 40^\circ + \frac{1}{32.2 \frac{\text{ft}}{\text{s}^2}} \cdot \frac{98}{3} (13.5 \sin 40^\circ) \frac{\text{ft}}{\text{s}^2} \cdot \cos 40^\circ \right]$$

$$= 6.5984 \text{ lb}$$

NOW..  $F_{\max} = \mu_s N$  FROM WHICH IT FOLLOWS THAT  $F > F_{\max}$

$\therefore$  BLOCK E WILL SLIDE IN THE SLOT

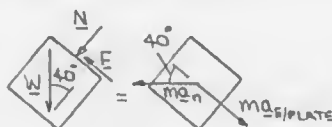
AND  $Q_E = Q_n + Q_{E/\text{PLATE}}$

$$= Q_n + (Q_{E/\text{PLATE}})_t + (Q_{E/\text{PLATE}})_n$$

AT  $t=0$ , THE BLOCK IS AT REST RELATIVE TO THE PLATE. THUS,  $(Q_{E/\text{PLATE}})_n = 0$  AT  $t=0$ , SO THAT  $Q_{E/\text{PLATE}}$  MUST BE DIRECTED TANGENTIALLY TO THE SLOT.

(CONTINUED)

## 12.58 continued



$$\sum F_x = m a_x: N + W \cos 40^\circ = m \frac{v_c^2}{\rho} \sin 40^\circ$$

$$\text{OR } N = W(-\cos 40^\circ + \frac{v_c^2}{g \rho} \sin 40^\circ) \quad (\text{AS ABOVE})$$

$$= 4.4924 \text{ lb}$$

$$\text{SLIDING: } F = \mu_k N = 0.25(4.4924 \text{ lb})$$

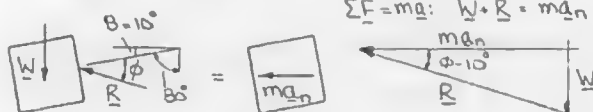
$$= 1.123 \text{ lb}$$

NOTING THAT  $F$  AND  $Q_{E/\text{PLATE}}$  MUST BE DIRECTED AS SHOWN (IF THEIR DIRECTIONS ARE REVERSED, THEN  $\sum F_x$  IS  $\searrow$  WHILE  $m a_x$  IS  $\nearrow$ ), HAVE  $\therefore$  THE BLOCK SLIDES DOWNWARD IN THE SLOT AND

$$F = 1.123 \text{ lb} \angle 40^\circ$$

## ALTERNATIVE SOLUTIONS

(a) ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.



$$\text{THEN.. } \tan(\phi - 10^\circ) = \frac{W}{m a_n} = \frac{W}{\frac{W \sqrt{v_c^2}}{g \rho}} = \frac{g}{\rho (\dot{\phi}_{ABCD})^2}$$

$$= \frac{32.2 \frac{\text{ft}}{\text{s}^2}}{\frac{98}{3} (13.5 \sin 80^\circ) \frac{\text{ft}}{\text{s}^2}} \quad (\text{FROM ABOVE})$$

$$\text{OR } \phi - 10^\circ = 6.9588^\circ$$

$$\text{AND } \phi = 16.9588^\circ$$

NOW..  $\tan \phi_s \cdot \mu_s \quad \mu_s = 0.35$

SO THAT  $\phi_s = 19.29^\circ$

$\therefore 0 < \phi < \phi_s \Rightarrow$  BLOCK DOES NOT SLIDE AND  $R$  IS DIRECTED AS SHOWN.

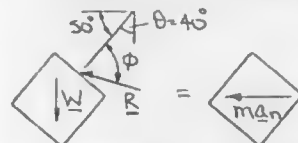
$$\text{NOW.. } F = R \sin \phi \quad \text{AND } R = \frac{W}{\sin(\phi - 10^\circ)}$$

$$\text{THEN.. } F = (0.8 \text{ lb}) \frac{\sin 16.9588^\circ}{\sin 6.9588^\circ} = 1.926 \text{ lb}$$

$\therefore$  THE BLOCK DOES NOT SLIDE IN THE SLOT AND

$$F = 1.926 \text{ lb} \angle 80^\circ$$

(b) ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.



$\sum F = m a: W - R = m a_n$   
 FROM PART (a) (ABOVE), IT THEN FOLLOWS THAT

$$\tan(\phi - 50^\circ) = \frac{g}{\rho (\dot{\phi}_{ABCD})^2} = \frac{32.2 \frac{\text{ft}}{\text{s}^2}}{\frac{98}{3} (13.5 \sin 40^\circ) \frac{\text{ft}}{\text{s}^2}}$$

$$\text{OR } \phi - 50^\circ = 5.752^\circ$$

$$\text{AND } \phi = 55.752^\circ$$

NOW  $\phi_s = 19.29^\circ$  SO THAT  $\phi > \phi_s$

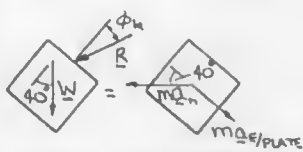
$\therefore$  THE BLOCK WILL SLIDE IN THE SLOT

AND THEN  $\phi = \phi_k$  WHERE  $\tan \phi_k = \mu_k \quad \mu_k = 0.25$   
 OR  $\phi_k = 14.0362^\circ$

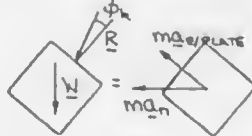
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## 12.58 continued

TO DETERMINE IN WHICH DIRECTION THE BLOCK WILL SLIDE, CONSIDER THE FREE-BODY DIAGRAMS FOR THE TWO POSSIBLE CASES.



SLIDING DOWNWARD



SLIDING UPWARD

NOW..  $\Sigma F = ma$ :  $W + R = ma_n + mg \sin \theta$   
FROM THE DIAGRAMS IT CAN BE CONCLUDED THAT THIS EQUATION CAN BE SATISFIED ONLY IF THE BLOCK IS SLIDING DOWNWARD. THEN..

$$\Sigma F_x = ma_x: W \cos 40^\circ + R \cos \phi_R = m \frac{v^2}{\rho} \sin 40^\circ$$

$$\text{NOW.. } F = R \sin \phi_R$$

$$\text{THEN.. } W \cos 40^\circ + \frac{F}{\tan \phi_R} = \frac{W v^2}{g \rho} \sin 40^\circ$$

$$\text{OR } F = \mu_s W (-\cos 40^\circ + \frac{v^2}{g \rho} \sin 40^\circ)$$

$$= 1.123 \text{ lb (SEE THE FIRST SOLUTION)}$$

$\therefore$  THE BLOCK SLIDES DOWNWARD IN THE SLOT AND

$$F = 1.123 \text{ lb } \Delta 40^\circ$$

## 12.59



GIVEN:  $d = 0.225 \text{ m}$ ;  $v_0 = 0$ ,  
 $a_t = 4 \frac{\text{m}}{\text{s}^2}$ ;  $m = 1.6 \times 10^6 \text{ kg}$

FIND: (a)  $v$  AT  $t = 35$   
(b)  $F_{\text{TUFT}}$  AT  $t = 35$

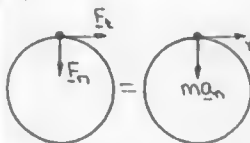
(a)  $a_t = \text{CONSTANT} \Rightarrow$  UNIFORMLY ACCELERATION MOTION

THEN..  $v = v_0 + a_t t$

$$\text{AT } t = 35: v = (4 \frac{\text{m}}{\text{s}^2})(35)$$

$$\text{OR } v = 12 \frac{\text{m}}{\text{s}}$$

(b)



$$\Sigma F_x = ma_x: F_t = ma_t$$

$$\text{OR } F_t = (1.6 \times 10^6 \text{ kg})(4 \frac{\text{m}}{\text{s}^2})$$

$$= 6.4 \times 10^6 \text{ N}$$

$$\Sigma F_n = ma_n: F_n = m \frac{v^2}{\rho}$$

$$\text{AT } t = 35: F_n = (1.6 \times 10^6 \text{ kg}) \frac{(12 \frac{\text{m}}{\text{s}})^2}{(0.225 \text{ m})}$$

$$= 2.048 \times 10^3 \text{ N}$$

$$\text{FINALLY.. } F_{\text{TUFT}} = \sqrt{F_t^2 + F_n^2}$$

$$= \sqrt{(6.4 \times 10^6 \text{ N})^2 + (2.048 \times 10^3 \text{ N})^2}$$

$$\text{OR } F_{\text{TUFT}} = 2.05 \times 10^3 \text{ N}$$

## 12.60



GIVEN:  $v_0 = 0$ ,  $(a_0)_t = 0.24 \frac{\text{m}}{\text{s}^2}$   
TRUNK B BEGINS TO  
SLIDE AT  $t = 10 \text{ s}$

FIND:  $\mu_s$

FIRST NOTE THAT  $(a_0)_t = \text{CONSTANT}$  IMPLIES UNIFORMLY ACCELERATED MOTION.

$$\therefore v = v_0 + (a_0)_t t$$

$$\text{AT } t = 10 \text{ s: } v = (0.24 \frac{\text{m}}{\text{s}^2})(10 \text{ s}) = 2.4 \frac{\text{m}}{\text{s}}$$

TOP VIEW



IN THE PLANE OF THE  
TURNABLE..

$$\Sigma F = m a_0: F = m(a_0)_t + m(a_0)_n$$

$$\text{THEN.. } F = m \sqrt{(a_0)_t^2 + (a_0)_n^2}$$

$$= m \sqrt{(a_0)_t^2 + (\frac{v^2}{\rho})^2}$$

$$+\Sigma F_y = 0: N - W = 0$$

$$\text{OR } N = m g$$

$$\text{AT } t = 10 \text{ s: } F = \mu_s N = \mu_s m g$$

THEN..

$$\mu_s m g = m \sqrt{(a_0)_t^2 + (\frac{v^2}{\rho})^2}$$

$$\text{OR } \mu_s = \frac{1}{9.81 \frac{\text{m}}{\text{s}^2}} \left\{ (0.24 \frac{\text{m}}{\text{s}^2})^2 + \left( \frac{(2.4 \frac{\text{m}}{\text{s}})^2}{2.5 \text{ m}} \right)^2 \right\}^{1/2}$$

$$\text{OR } \mu_s = 0.236$$

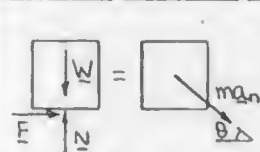
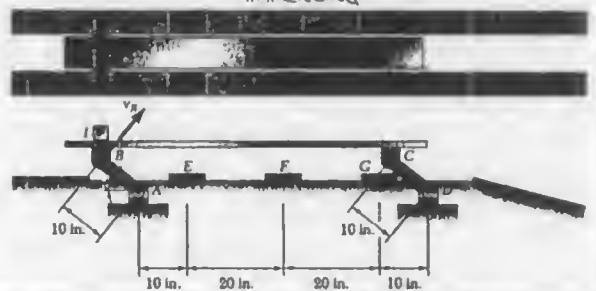
## 12.61

GIVEN: PARALLEL-LINK MECHANISM ABCD;

$$v_B = 2.2 \frac{\text{ft}}{\text{s}}$$

FIND: (a)  $(\mu_s)_{\text{MIN}}$  IF COMPONENTS ARE NOT  
TO SLIDE

(b)  $\theta$  FOR WHICH SLIDING IS  
IMPENDING



$$+\Sigma F_x = ma_x: F = \frac{W}{g} \frac{v^2}{\rho} \cos \theta$$

$$+\Sigma F_y = ma_y: N - W = -\frac{W}{g} \frac{v^2}{\rho} \sin \theta$$

$$\text{OR } N = W(1 - \frac{v^2}{g \rho} \sin \theta)$$

$$\text{NOW.. } F_{\text{MAX}} = \mu_s N = \mu_s W(1 - \frac{v^2}{g \rho} \sin \theta)$$

AND FOR THE COMPONENT NOT TO SLIDE

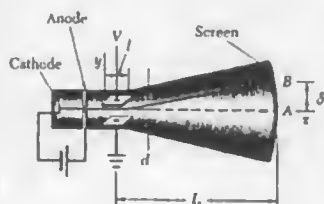
$$F \leq F_{\text{MAX}}$$

$$\text{OR } \frac{W}{g} \frac{v^2}{\rho} \cos \theta \leq \mu_s W(1 - \frac{v^2}{g \rho} \sin \theta)$$

(CONTINUED)



# 12.63 and 12.64



GIVEN:  $v_x = v_0$  (= CONSTANT)  
 $(F_y)_{\text{PLATE}} = \frac{eV}{d}$

FIRST NOTE THAT THE HORIZONTAL COMPONENT OF THE VELOCITY OF AN ELECTRON IS A CONSTANT ( $v_0$ ) REGARDLESS OF THE VALUE OF THE POTENTIAL V. THEN..

$$l = v_0 t + \frac{1}{2} a_x t^2$$

THE TIME  $t_{\text{PLATE}}$  FOR AN ELECTRON TO TRAVEL BETWEEN THE PLATES IS THEN..

$$l = v_0 (t_{\text{PLATE}})$$

$$\text{OR } t_{\text{PLATE}} = \frac{l}{v_0}$$

AND THE TIME  $t_{\text{SCREEN}}$  TO TRAVEL FROM THE END OF THE PLATES TO THE SCREEN IS..

$$(L - \frac{1}{2}l) = v_0 (t_{\text{SCREEN}})$$

$$\text{OR } t_{\text{SCREEN}} = \frac{L - \frac{1}{2}l}{v_0}$$

NEXT CONSIDER THE VERTICAL MOTION OF AN ELECTRON AS IT MOVES BETWEEN THE PLATES.

$$\uparrow \Sigma F_y = m a_y; (F_y)_{\text{PLATE}} = m a_y$$

$$\text{OR } a_y = \frac{eV}{md}$$

THEN, FOR THE UNIFORMLY ACCELERATED MOTION IN THE y DIRECTION HAVE

$$v_y = (v_y)_0 + a_y t$$

$$y = y_0 + (v_y)_0 t + \frac{1}{2} a_y t^2$$

$$= \left(\frac{eV}{md}\right)t$$

$$= \frac{1}{2} \left(\frac{eV}{md}\right)t^2$$

AT THE END OF THE PLATES..

$$(v_y)_1 = \left(\frac{eV}{md}\right)\left(\frac{l}{v_0}\right)$$

$$y_1 = \frac{1}{2} \left(\frac{eV}{md}\right)\left(\frac{l}{v_0}\right)^2$$

$$= \frac{eVl^2}{2mdv_0^2}$$

12.63 FIND:  $\delta$  IN TERMS OF  $V, v_0, l, m, d, l, L$

FIRST NOTE THAT THE VELOCITY OF AN ELECTRON IS CONSTANT AFTER IT LEAVES THE PLATES.



THEN, FROM THE END OF THE PLATES TO THE SCREEN..

$$y = y_1 + (v_y)_1 t = \left(\frac{eVl^2}{2mdv_0^2}\right) + \left(\frac{eVl}{mdv_0}\right)t$$

$$\text{AT THE SCREEN: } \delta = \frac{eVl^2}{2mdv_0^2} + \left(\frac{eVl}{mdv_0}\right)\left(\frac{L - \frac{1}{2}l}{v_0}\right)$$

$$\text{OR } \delta = \frac{eVlL}{mdv_0^2}$$

(CONTINUED)

# 12.64 continued

12.64 GIVEN: AT  $x=l, \left(\frac{d}{l} - y\right)_{\text{MIN}} = 0.05d$   
 FIND:  $\left(\frac{d}{l}\right)_{\text{MIN}}$  IN TERMS OF  $e, m, v_0, V$



AT  $x=l$ , HAVE  
 $y = y_1 = \frac{eVl^2}{2mdv_0^2}$

$$\text{AND } \frac{d}{l} - y_1 \geq 0.05d$$

$$\text{OR } 0.45d \geq \frac{eVl^2}{2mdv_0^2}$$

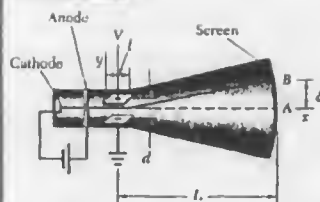
$$\text{OR } \frac{d}{l} \geq \frac{eV}{0.9mv_0^2}$$

THE MINIMUM VALUE OF  $\frac{d}{l}$  IS THEN

$$\left(\frac{d}{l}\right)_{\text{MIN}} = \left(\frac{eV}{0.9mv_0^2}\right)^{1/2}$$

$$\text{OR } \left(\frac{d}{l}\right)_{\text{MIN}} = \frac{1.054}{v_0} \sqrt{\frac{eV}{m}}$$

# 12.65



GIVEN:  $v_x = v_0$  (= CONSTANT),  
 $(F_y)_{\text{PLATE}} = \frac{eV}{d}$ ;  
 $L' = 0.6L, d' = 0.8d$ ,  
 $\delta, V$ , AND  $v_0$   
 UNCHANGED

FIND:  $\delta'$

FROM THE SOLUTION TO PROBLEM 12.63 HAVE

$$\delta = \frac{eVlL}{mdv_0^2}$$

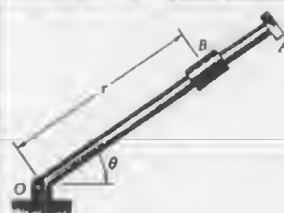
THEN, SINCE  $\delta$  IS UNCHANGED, HAVE

$$(\delta)_{\text{MODIFIED}} = (\delta)_{\text{ORIGINAL}}: \frac{eVl'L'}{md'v_0^2} = \frac{eVlL}{mdv_0^2}$$

$$\text{OR } \frac{l'(0.6L)}{0.8d} = \frac{lL}{d}$$

$$\text{OR } l' = 1.333l$$

# 12.66 and 12.67



GIVEN:  $m_B = 0.2 \text{ kg}$ ;  
 $r = 250 + 150 \sin \pi t$ ,  
 $\theta = \pi(4t^2 - 8t)$   
 $r$  - mm,  $t$  - s,  $\theta$  - rad

HAVE  $r = (0.25 + 0.15 \sin \pi t) \text{ m}$   
 THEN  $\dot{r} = (0.15 \pi \cos \pi t) \frac{\text{m}}{\text{s}}$   
 AND  $\ddot{r} = -(0.15 \pi^2 \sin \pi t) \frac{\text{m}}{\text{s}^2}$

$\theta = \pi(4t^2 - 8t) \text{ rad}$   
 $\dot{\theta} = \pi(8t - 8) \frac{\text{rad}}{\text{s}}$   
 $\ddot{\theta} = 8\pi \frac{\text{rad}}{\text{s}^2}$

12.66 FIND: (a)  $F_r$  AND  $F_\theta$  AT  $t=0$   
 (b)  $F_r$  AND  $F_\theta$  AT  $t=0.5 \text{ s}$

(a) AT  $t=0$ :  $r = 0.25 \text{ m}$   
 $\dot{r} = 0.15 \pi \frac{\text{m}}{\text{s}}$   
 $\ddot{r} = 0$

$\dot{\theta} = -8\pi \frac{\text{rad}}{\text{s}}$   
 $\ddot{\theta} = 8\pi \frac{\text{rad}}{\text{s}^2}$

(CONTINUED)

## 12.66 and 12.67 continued

Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.25 \text{ m})(-8\pi \frac{\text{RAD}}{\text{s}})^2 = -16\pi^2 \frac{\text{m}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.25 \text{ m})(8\pi \frac{\text{RAD}}{\text{s}^2}) + 2(0.15\pi \frac{\text{m}}{\text{s}})(-8\pi \frac{\text{RAD}}{\text{s}})$   
 $= \pi(2 - 2.4\pi) \frac{\text{m}}{\text{s}^2}$

FINALLY..  $F_r = m a_r = (0.2 \text{ kg})(-16\pi^2 \frac{\text{m}}{\text{s}^2})$   
 OR  $F_r = -31.6 \text{ N}$   $\blacktriangleleft$   
 $F_\theta = m a_\theta = (0.2 \text{ kg})(\pi(2 - 2.4\pi) \frac{\text{m}}{\text{s}^2})$   
 OR  $F_\theta = -3.48 \text{ N}$   $\blacktriangleleft$

(b) At  $t = 0.5 \text{ s}$ :  $r = 0.40 \text{ m}$   
 $\dot{r} = 0$   $\dot{\theta} = -4\pi \frac{\text{RAD}}{\text{s}}$   
 $\ddot{r} = -0.15\pi^2 \frac{\text{m}}{\text{s}^2}$   $\ddot{\theta} = 8\pi \frac{\text{RAD}}{\text{s}^2}$

Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = (-0.15\pi^2 \frac{\text{m}}{\text{s}^2}) - (0.40 \text{ m})(-4\pi \frac{\text{RAD}}{\text{s}})^2$   
 $= -6.55\pi^2 \frac{\text{m}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.40 \text{ m})(8\pi \frac{\text{RAD}}{\text{s}^2}) + 0 = 3.20\pi \frac{\text{m}}{\text{s}^2}$

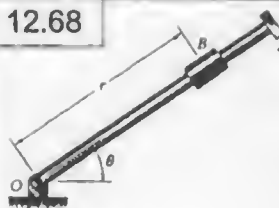
FINALLY..  $F_r = m a_r = (0.2 \text{ kg})(-6.55\pi^2 \frac{\text{m}}{\text{s}^2})$   
 OR  $F_r = -12.93 \text{ N}$   $\blacktriangleleft$   
 $F_\theta = m a_\theta = (0.2 \text{ kg})(3.20\pi \frac{\text{m}}{\text{s}^2})$   
 OR  $F_\theta = 2.01 \text{ N}$   $\blacktriangleleft$

## 12.67 FIND: $F_r$ AND $F_\theta$ AT $t = 1.5 \text{ s}$

At  $t = 1.5 \text{ s}$ :  $r = 0.10 \text{ m}$   
 $\dot{r} = 0$   $\dot{\theta} = 4\pi \frac{\text{RAD}}{\text{s}}$   
 $\ddot{r} = 0.15\pi^2 \frac{\text{m}}{\text{s}^2}$   $\ddot{\theta} = 8\pi \frac{\text{RAD}}{\text{s}^2}$   
 Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = (0.15\pi^2 \frac{\text{m}}{\text{s}^2}) - (0.10 \text{ m})(4\pi \frac{\text{RAD}}{\text{s}})^2$   
 $= -1.45\pi^2 \frac{\text{m}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.10 \text{ m})(8\pi \frac{\text{RAD}}{\text{s}^2}) + 0 = 0.8\pi \frac{\text{m}}{\text{s}^2}$

FINALLY..  $F_r = m a_r = (0.2 \text{ kg})(-1.45\pi^2 \frac{\text{m}}{\text{s}^2})$   
 OR  $F_r = -2.86 \text{ N}$   $\blacktriangleleft$   
 $F_\theta = m a_\theta = (0.2 \text{ kg})(0.8\pi \frac{\text{m}}{\text{s}^2})$   
 OR  $F_\theta = 0.503 \text{ N}$   $\blacktriangleleft$

## 12.68



GIVEN:  $W_B = 5 \text{ lb}$ ;  
 $r = \frac{10}{t+4}$ ,  $\theta = \frac{2}{\pi} \sin \pi t$   
 $r = \text{ft}$ ,  $t = \text{s}$ ,  $\theta = \text{RAD}$   
 FIND: (a)  $F_r$  AND  $F_\theta$  AT  $t = 1 \text{ s}$   
 (b)  $F_r$  AND  $F_\theta$  AT  $t = 6 \text{ s}$

HAVE..  $r = \frac{10}{t+4} \text{ ft}$   $\theta = (\frac{2}{\pi} \sin \pi t) \frac{\text{RAD}}{\text{s}}$   
 THEN..  $\dot{r} = -\frac{10}{(t+4)^2} \frac{\text{ft}}{\text{s}}$   $\dot{\theta} = (2 \cos \pi t) \frac{\text{RAD}}{\text{s}}$   
 AND  $\ddot{r} = \frac{20}{(t+4)^3} \frac{\text{ft}}{\text{s}^2}$   $\ddot{\theta} = -(2\pi \sin \pi t) \frac{\text{RAD}}{\text{s}^2}$

(a) At  $t = 1 \text{ s}$ :  $r = 2 \text{ ft}$   
 $\dot{r} = -0.4 \frac{\text{ft}}{\text{s}}$   $\dot{\theta} = -2 \frac{\text{RAD}}{\text{s}}$   
 $\ddot{r} = 0.16 \frac{\text{ft}}{\text{s}^2}$   $\ddot{\theta} = 0$   
 Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = (0.16 \frac{\text{ft}}{\text{s}^2}) - (2 \text{ ft})(-2 \frac{\text{RAD}}{\text{s}})^2$   
 $= -7.84 \frac{\text{ft}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.4 \frac{\text{ft}}{\text{s}})(-2 \frac{\text{RAD}}{\text{s}}) = 1.6 \frac{\text{ft}}{\text{s}^2}$

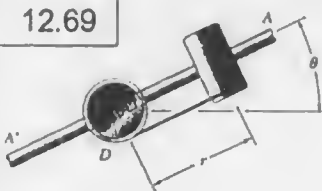
FINALLY..  $F_r = m a_r = \frac{5 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}(-7.84 \frac{\text{ft}}{\text{s}^2})$   
 OR  $F_r = -1.217 \text{ lb}$   $\blacktriangleleft$   
 $F_\theta = m a_\theta = \frac{5 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}(1.6 \frac{\text{ft}}{\text{s}^2})$   
 OR  $F_\theta = 0.248 \text{ lb}$   $\blacktriangleleft$   
 (CONTINUED)

## 12.68 continued

(b) At  $t = 6 \text{ s}$ :  $r = 1 \text{ ft}$   
 $\dot{r} = -0.1 \frac{\text{ft}}{\text{s}}$   $\dot{\theta} = 2 \frac{\text{RAD}}{\text{s}}$   
 $\ddot{r} = 0.02 \frac{\text{ft}}{\text{s}^2}$   $\ddot{\theta} = 0$   
 Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = (0.02 \frac{\text{ft}}{\text{s}^2}) - (1 \text{ ft})(2 \frac{\text{RAD}}{\text{s}})^2 = -3.98 \frac{\text{ft}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.1 \frac{\text{ft}}{\text{s}})(2 \frac{\text{RAD}}{\text{s}}) = -0.4 \frac{\text{ft}}{\text{s}^2}$

FINALLY..  $F_r = m a_r = \frac{5 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}(-3.98 \frac{\text{ft}}{\text{s}^2})$   
 OR  $F_r = -0.618 \text{ lb}$   $\blacktriangleleft$   
 $F_\theta = m a_\theta = \frac{5 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}(-0.4 \frac{\text{ft}}{\text{s}^2})$   
 OR  $F_\theta = -0.0621 \text{ lb}$   $\blacktriangleleft$

## 12.69



GIVEN:  $\dot{\theta} = ct$ ,  $c = \text{CONSTANT}$ ,  
 $\dot{r} = -k$ ; AT  $t = 0$ ,  $r = r_0$   
 FIND: (a)  $T$  IN TERMS OF  
 $m, c, k, r_0, t$   
 (b)  $Q$ , FORCE  
 EXERTED ON B  
 BY ARM AA'

### KINEMATICS

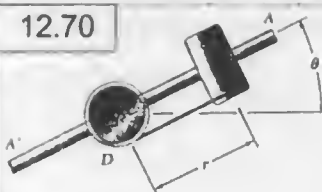
HAVE..  $\frac{dr}{dt} = \dot{r} = -k$   
 AT  $t = 0$ ,  $r = r_0$ :  $\int_{r_0}^r dr = \int_0^t -k dt$   
 OR  $r = r_0 - kt$   
 ALSO..  $\ddot{r} = 0$   $\ddot{\theta} = c$

Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (r_0 - kt)(ct)^2 = -c^2(r_0 - kt)t^2$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (r_0 - kt)(c) + 2(-k)(ct)$   
 $= c(r_0 - 3kt)$

### KINETICS

(a)  $\sum F_r = m a_r$ :  $-T = m[-c^2(r_0 - kt)t^2]$   
 OR  $T = mc^2(r_0 - kt)t^2$   $\blacktriangleleft$   
 (b)  $\sum F_\theta = m a_\theta$ :  $Q = m[c(r_0 - 3kt)]$   
 OR  $Q = mc(r_0 - 3kt)$   $\blacktriangleleft$

## 12.70



GIVEN:  $m_B = 3 \text{ kg}$ ;  $\dot{\theta} = 0.75 t$   
 $\dot{r} = \frac{\text{RAD}}{\text{s}}$ ,  $t = \text{s}$ ;  
 $\dot{r} = 0.5 \frac{\text{m}}{\text{s}}$ ; AT  $t = 0$ ,  
 $r = 0$   
 FIND:  $t$  WHEN  $T = Q$ ,  
 WHERE  $Q$  IS THE  
 FORCE ON B FROM AA'

### KINEMATICS

HAVE..  $\frac{dr}{dt} = \dot{r} = 0.5 \frac{\text{m}}{\text{s}}$   
 AT  $t = 0$ ,  $r = 0$ :  $\int_0^r dr = \int_0^t 0.5 dt$   
 OR  $r = (0.5t) \text{ m}$   
 ALSO..  $\ddot{r} = 0$   $\ddot{\theta} = (0.75 t) \frac{\text{RAD}}{\text{s}^2}$


Now..  $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - [(0.5t) \text{ m}][(0.75t) \frac{\text{RAD}}{\text{s}}]^2$   
 $= -(0.28125 t^3) \frac{\text{m}}{\text{s}^2}$   
 AND  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = [(0.5t) \text{ m}](0.75 \frac{\text{RAD}}{\text{s}^2})$   
 $+ 2(0.5 \frac{\text{m}}{\text{s}})(0.75t \frac{\text{RAD}}{\text{s}})$   
 $= (1.125 t) \frac{\text{m}}{\text{s}^2}$

(CONTINUED)



## 12.70 continued

### KINETICS



$$\begin{aligned} \sum F_r = m a_r: -T &= (3 \text{ kg})(-0.28125 \text{ t}^3) \frac{\text{m}}{\text{s}^2} \\ \text{OR } T &= (0.84375 \text{ t}^3) \text{ N} \\ \sum F_\theta = m a_\theta: Q &= (3 \text{ kg})(1.125 \text{ t}^3) \frac{\text{m}}{\text{s}^2} \\ \text{OR } Q &= (3.375 \text{ t}^3) \text{ N} \end{aligned}$$

NOW REQUIRE THAT:  $T = Q$   
 OR  $(0.84375 \text{ t}^3) \text{ N} = (3.375 \text{ t}^3) \text{ N}$   
 OR  $t^2 = 4.000$

OR  $t = 2.00 \text{ s}$

## 12.71 continued

$$\sum F_r: F_r = Q \cos \theta$$

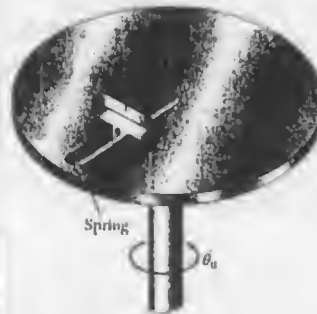
$$\text{OR } Q = (5.76 \tan^2 \theta \sec \theta) \frac{1}{\cos \theta}$$

$$\text{OR } Q = (5.76 \text{ N}) \tan^2 \theta \sec^2 \theta \rightarrow$$

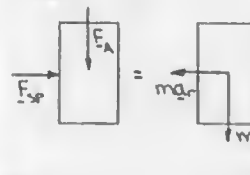
## 12.72

GIVEN:  $\dot{\theta}_0 = 15 \frac{\text{RAD}}{\text{s}}$ ;  $W_B = 0.5 \text{ lb}$ ,  
 $k = 4 \frac{\text{lb}}{\text{ft}}$ ; WHEN  $r = 0$ ,  
 $x_{sp} = 0$ ;  $\ddot{r} = -40 \frac{\text{ft}}{\text{s}^2}$ ,  
 $F_A = 2 \text{ lb}$

FIND: (a)  $r$   
 (b)  $N_r$



FIRST NOTE: WHEN  $r = 0$ ,  $x_{sp} = 0 \Rightarrow F_{sp} = k r$   
 AND  $\dot{\theta} = \dot{\theta}_0 = 15 \frac{\text{RAD}}{\text{s}}$   
 THEN  $\ddot{\theta} = 0$



$$\begin{aligned} \sum F_r = m a_r: -F_{sp} &= \frac{W_B}{g} (\ddot{r} - r \dot{\theta}^2) \\ \text{OR } -k r &= \frac{W_B}{g} (\ddot{r} - r \dot{\theta}_0^2) \\ \text{OR } r &= \frac{\ddot{r}}{\dot{\theta}_0^2 - \frac{g}{W_B}} \end{aligned}$$

THEN..  $r = \frac{-40 \text{ ft/s}^2}{(15 \frac{\text{RAD}}{\text{s}})^2 - \frac{(32.2 \text{ ft/s}^2)(4 \text{ lb/ft})}{0.5 \text{ lb}}}$  OR  $r = 1.227 \text{ ft}$

(b)  $\sum F_\theta = m a_\theta: F_A = \frac{W_B}{g} (r \dot{\theta}_0^2 + 2 \dot{r} \dot{\theta})$

NOW..  $N_r = \dot{r}$   
 THEN  $N_r = \frac{g F_A}{2 W_B \dot{\theta}_0} = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})(2 \text{ lb})}{2(0.5 \text{ lb})(15 \frac{\text{RAD}}{\text{s}})}$

OR  $N_r = 4.29 \frac{\text{ft}}{\text{s}}$

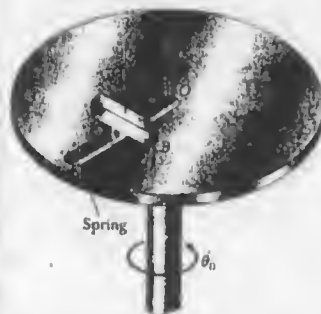
## \* 12.73

GIVEN:  $\dot{\theta}_0 = 12 \frac{\text{RAD}}{\text{s}}$ ;  $W_B = 8.05 \text{ oz}$ ;  
 WHEN  $r = 0$ ,  $x_{sp} = 0$ ;  
 AT  $t = 0$ ,  $\dot{r} = 0$ ,  $r = 15 \text{ in.}$

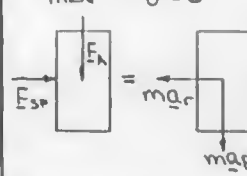
FIND: (a)  $r$  AND  $F_A$  AT  $t = 0.1 \text{ s}$ ,  
 $k = 2.25 \frac{\text{lb}}{\text{ft}}$

(b)  $r$  AND  $F_A$  AT  $t = 0.1 \text{ s}$ ,  
 $k = 3.25 \frac{\text{lb}}{\text{ft}}$

WHERE  $F_A$  IS THE  
 HORIZONTAL FORCE  
 EXERTED ON THE  
 SLIDER BY THE DISK



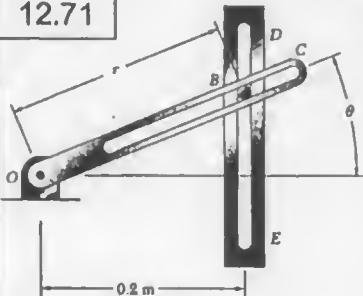
FIRST NOTE: WHEN  $r = 0$ ,  $x_{sp} = 0 \Rightarrow F_{sp} = k r$   
 AND  $\dot{\theta} = \dot{\theta}_0 = 12 \frac{\text{RAD}}{\text{s}}$   
 THEN  $\ddot{\theta} = 0$



$$\begin{aligned} \sum F_r = m a_r: -F_{sp} &= \frac{W_B}{g} (\ddot{r} - r \dot{\theta}^2) \\ \text{OR } \ddot{r} + \left( \frac{k}{W_B} - \dot{\theta}_0^2 \right) r &= 0 \quad (1) \\ \sum F_\theta = m a_\theta: F_A &= \frac{W_B}{g} (r \dot{\theta}_0^2 + 2 \dot{r} \dot{\theta}) \quad (2) \end{aligned}$$

(CONTINUED)

## 12.71



GIVEN:  $m_B = 0.1 \text{ kg}$ ;  
 $\dot{\theta} = \dot{\theta}_0 = 12 \frac{\text{RAD}}{\text{s}}$   
 FIND: (a)  $F_r$  AND  $F_\theta$  ON  
 PIN B  
 (b)  $P$  AND  $Q$ ,  
 WHERE  $P$  IS  
 DUE TO  $OC$   
 AND  $Q$  IS DUE  
 TO  $DE$

### KINEMATICS

FROM THE DRAWING OF THE SYSTEM HAVE..

$$r = \frac{0.2}{\cos \theta} \text{ m}$$

THEN  $\dot{r} = (0.2 \frac{\sin \theta}{\cos^2 \theta} \dot{\theta}) \frac{\text{m}}{\text{s}}$   $\dot{\theta} = 12 \frac{\text{RAD}}{\text{s}}$

AND  $\ddot{r} = 0.2 \frac{\cos \theta (\cos^2 \theta) - \sin \theta (-2 \cos \theta \sin \theta)}{\cos^4 \theta} \ddot{\theta}$   
 $= (0.2 \frac{1 + \sin^2 \theta}{\cos^3 \theta} \ddot{\theta}) \frac{\text{m}}{\text{s}^2}$

SUBSTITUTING FOR  $\ddot{\theta}$ ..

$$\dot{r} = 0.2 \frac{\sin \theta}{\cos^2 \theta} (12) = (2.4 \frac{\sin \theta}{\cos^2 \theta}) \frac{\text{m}}{\text{s}}$$

$$\ddot{r} = 0.2 \frac{1 + \sin^2 \theta}{\cos^3 \theta} (12)^2 = (28.8 \frac{1 + \sin^2 \theta}{\cos^3 \theta}) \frac{\text{m}}{\text{s}^2}$$

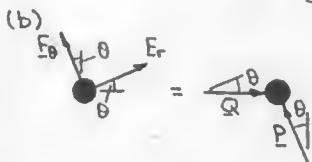
NOW..  $a_r = \ddot{r} - r \dot{\theta}^2 = (28.8 \frac{1 + \sin^2 \theta}{\cos^3 \theta}) - (\frac{0.2}{\cos \theta}) (12)^2$   
 $= (57.6 \frac{\sin^2 \theta}{\cos^3 \theta}) \frac{\text{m}}{\text{s}^2}$

AND  $a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 2(2.4 \frac{\sin \theta}{\cos^2 \theta})(12)$   
 $= (57.6 \frac{\sin \theta}{\cos^2 \theta}) \frac{\text{m}}{\text{s}^2}$

### KINETICS

(a) HAVE..  $F_r = m_B a_r = (0.1 \text{ kg})(57.6 \frac{\sin^2 \theta}{\cos^3 \theta}) \frac{\text{m}}{\text{s}^2}$   
 OR  $F_r = (5.76 \text{ N}) \tan^2 \theta \sec \theta$

AND  $F_\theta = m_B a_\theta = (0.1 \text{ kg})(57.6 \frac{\sin \theta}{\cos^2 \theta}) \frac{\text{m}}{\text{s}^2}$   
 OR  $F_\theta = (5.76 \text{ N}) \tan \theta \sec \theta$



NOW..  $\sum F_y: F_\theta \cos \theta + F_r \sin \theta = P \cos \theta$   
 OR  $P = 5.76 \tan \theta \sec \theta + (5.76 \tan^2 \theta \sec \theta) \tan \theta$   
 OR  $P = (5.76 \text{ N}) \tan \theta \sec^2 \theta \quad \forall \theta$

(CONTINUED)



## 12.73 continued

(a)  $k = 2.25 \text{ lb/ft}$

SUBSTITUTING THE GIVEN VALUES INTO EQ (1)...

$$\ddot{r} + \left[ \frac{2.25 \text{ lb/ft} \cdot 32.2 \text{ ft/s}^2}{8.05 \text{ lb}} - \left( 12 \frac{\text{rad}}{\text{s}} \right)^2 \right] r = 0$$

OR  $\ddot{r} = 0$

THEN...  $\frac{dr}{dt} = \dot{r} = 0$  AND AT  $t=0$ ,  $\dot{r} = 0$ :

$$\therefore \int_0^0 \dot{r} dt = \int_0^0 (0) dt$$

OR  $\dot{r} = 0$

AND...  $\frac{dr}{dt} = \dot{r} = 0$  AND AT  $t=0$ ,  $r_0 = 1.25 \text{ ft}$

$$\therefore \int_{r_0}^r dr = \int_0^0 (0) dt$$

OR  $r = r_0$

$$\therefore r = 1.25 \text{ ft}$$

NOTE:  $\dot{r} = 0$  IMPLIES THAT THE SLIDER REMAINS AT ITS INITIAL RADIAL POSITION.

WITH  $\dot{r} = 0$ , EQ. (2) IMPLIES

$$F_H = 0$$

(b)  $k = 3.25 \text{ lb/ft}$

SUBSTITUTING THE GIVEN VALUES INTO EQ (1)...

$$\ddot{r} + \left[ \frac{3.25 \text{ lb/ft} \cdot 32.2 \text{ ft/s}^2}{8.05 \text{ lb}} - \left( 12 \frac{\text{rad}}{\text{s}} \right)^2 \right] r = 0$$

OR  $\ddot{r} + 64r = 0$

NOW...  $\ddot{r} = \frac{d}{dt}(\dot{r})$   $\dot{r} = v_r$   $\frac{d}{dt} = \frac{dr}{dt} \frac{d}{dr} = v_r \frac{d}{dr}$

THEN  $\ddot{r} = v_r \frac{dv_r}{dr}$

SO THAT  $v_r \frac{dv_r}{dr} + 64r = 0$

AT  $t=0$ ,  $v_r = 0$ ,  $r = r_0$ :  $\int_0^0 v_r dv_r = -64 \int_{r_0}^r r dr$

OR  $v_r^2 = -64(r^2 - r_0^2)$

NOW...  $v_r = \frac{dr}{dt} = 8 \sqrt{r_0^2 - r^2}$

AT  $t=0$ ,  $r = r_0$ :  $\int_{r_0}^r \frac{dr}{\sqrt{r_0^2 - r^2}} = \int_0^t 8 dt = 8t$

LET  $r = r_0 \sin \phi$ ,  $dr = r_0 \cos \phi d\phi$

THEN  $\int_0^{\pi/2} \sin'(\phi) \frac{r_0 \cos \phi d\phi}{\sqrt{r_0^2 - r_0^2 \sin^2 \phi}} = 8t$

OR  $\int_0^{\pi/2} \sin'(\phi) d\phi = 8t$

OR  $\sin'(\phi) - \frac{\pi}{2} = 8t$

OR  $r = r_0 \sin(8t + \frac{\pi}{2}) = r_0 \cos 8t = (1.25 \text{ ft}) \cos 8t$

THEN  $\dot{r} = -(10 \frac{\text{ft}}{\text{s}}) \sin 8t$

FINALLY... AT  $t = 0.1 \text{ s}$ :

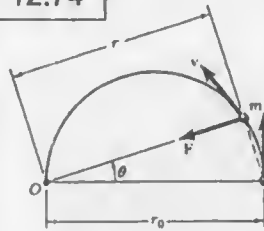
$$r = (1.25 \text{ ft}) \cos(8 \times 0.1)$$

OR  $r = 0.871 \text{ ft}$

EQ (2)...  $F_A = \frac{(8.05/16) \text{ lb}}{32.2 \text{ ft/s}^2} \cdot 2 \cdot [-(10 \frac{\text{ft}}{\text{s}}) \sin(8 \times 0.1)] (12 \frac{\text{rad}}{\text{s}})^2$

OR  $F_A = -2.69 \text{ lb}$

## 12.74



GIVEN: CENTRAL FORCE  $F$  AND SEMICIRCULAR PATH SHOWN; AT  $t=0$ ,  $\theta=0$ ,  $v = v_0$ ,  $r = r_0$   
SHOW:  $v = \frac{v_0}{\cos^2 \theta}$

HAVE -  $v = \dot{r} e_r + r \dot{\theta} e_\theta$

SO THAT  $v^2 = \dot{r}^2 + r^2 \dot{\theta}^2$  (1)

FROM THE DIAGRAM...  $r = r_0 \cos \theta$

THEN  $\dot{r} = -(r_0 \sin \theta) \dot{\theta}$

SUBSTITUTING INTO EQ (1)...

$$v^2 = (-r_0 \dot{\theta} \sin \theta)^2 + (r_0 \cos \theta)^2 \dot{\theta}^2 = r_0^2 (\sin^2 \theta + \cos^2 \theta) \dot{\theta}^2$$

OR  $v = r_0 \dot{\theta}$  (2)

AT  $t=0$ :  $v_0 = r_0 \dot{\theta}_0$

FROM EQ. (12.27):

$$r^2 \dot{\theta} = r_0^2 \dot{\theta}_0$$

$$= r_0 v_0$$

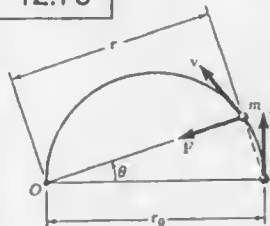
$$\text{OR } \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{(r_0 \cos \theta)^2} = \frac{v_0}{r_0 \cos^2 \theta}$$

SUBSTITUTING FOR  $\dot{\theta}$  IN EQ. (2)...

$$v = r_0 \left( \frac{v_0}{r_0 \cos^2 \theta} \right)$$

OR  $v = \frac{v_0}{\cos^2 \theta}$  Q.E.D.

## 12.75



GIVEN: CENTRAL FORCE  $F$  AND SEMICIRCULAR PATH SHOWN; AT  $t=0$ ,  $\theta=0$ ,  $v = v_0$ ,  $r = r_0$   
FIND: (a)  $F_t$  WHEN  $\theta=0$   
(b)  $F_t$  WHEN  $\theta=45^\circ$

FROM THE DIAGRAM...  $r = r_0 \cos \theta$

THEN  $\dot{r} = -(r_0 \sin \theta) \dot{\theta}$

NOW...  $v = \dot{r} e_r + r \dot{\theta} e_\theta$

SO THAT AT  $t=0$ ...  $v_0 = r_0 \dot{\theta}_0$

FROM EQ. (12.27):  $r^2 \dot{\theta} = r_0^2 \dot{\theta}_0$

$$\text{OR } \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{(r_0 \cos \theta)^2} = \frac{v_0}{r_0 \cos^2 \theta}$$

FROM PROBLEM 12.74:  $v = \frac{v_0}{\cos^2 \theta}$

$$\begin{aligned} \text{NOW... } a_t &= \frac{dv}{dt} = \frac{d}{dt} \left( \frac{v_0}{\cos^2 \theta} \right) = v_0 \frac{2 \cos \theta \sin \theta}{\cos^4 \theta} \dot{\theta} \\ &= 2 v_0 \frac{\sin \theta}{\cos^3 \theta} \cdot \left( \frac{v_0}{r_0 \cos^2 \theta} \right) \\ &= 2 \frac{v_0^2}{r_0} \frac{\sin \theta}{\cos^5 \theta} \end{aligned}$$

FINALLY...  $F_t = m a_t = 2m \frac{v_0^2}{r_0} \frac{\sin \theta}{\cos^5 \theta}$

(a) WHEN  $\theta=0$   $F_t = 0$

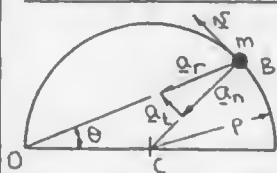
(b) WHEN  $\theta=45^\circ$ :  $F_t = 2m \frac{v_0^2}{r_0} \frac{\sin 45^\circ}{\cos^5 45^\circ}$

OR  $F_t = 8m \frac{v_0^2}{r_0}$

(CONTINUED)

## 12.75 continued

### ALTERNATIVE SOLUTION



FIRST NOTE THAT TRIANGLE OBC IS AN ISOSCELES TRIANGLE.

$$\therefore \angle OBC = \theta$$

FOR CENTRAL FORCE MOTION  $a_\theta = 0$

$$\therefore a = a_r + a_\theta = a_r$$

NOW...  $a = a_t + a_n$  OR  $a_t + a_n = a_r$   
FROM THE ABOVE DIAGRAM...

$$a_t = a_n \tan \theta$$

$$\text{WHERE } a_n = \frac{v^2}{r} \quad r = \frac{r_0}{2}$$

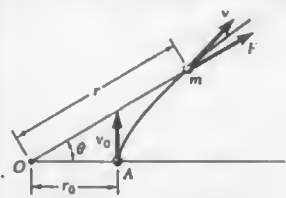
AND FROM PROBLEM 12.74

$$v = \frac{v_0}{\cos^2 \theta}$$

$$\text{THEN } a_t = \frac{(v_0^2 / \cos^4 \theta)^2}{\frac{r_0}{2}} \cdot \frac{\sin \theta}{\cos^3 \theta} = 2 \frac{v_0^2}{r_0} \frac{\sin \theta}{\cos^5 \theta}$$

$$\text{FINALLY... } F_t = m a_t = 2m \frac{v_0^2}{r_0} \frac{\sin \theta}{\cos^5 \theta} \quad (\text{AS ABOVE})$$

## 12.76 and 12.77



GIVEN: CENTRAL FORCE  $F$  AND  
PATH SHOWN;  
 $r = \frac{r_0}{\sqrt{\cos 2\theta}}$ ; AT  $t=0$ ,  
 $\theta=0$ ,  $v=v_0$ ,  $r=r_0$

$$\text{HAVE } r = \frac{r_0}{\sqrt{\cos 2\theta}} = r_0 (\cos 2\theta)^{-1/2}$$

$$\text{THEN } \dot{r} = r_0 \left[ -\frac{1}{2} (\cos 2\theta)^{-3/2} \right] (-2 \sin 2\theta) \dot{\theta} \\ = r_0 \frac{\sin 2\theta}{\cos^{3/2} 2\theta} \dot{\theta}$$

$$\text{NOW... } \dot{r} = \dot{r} e_r + r \dot{\theta} e_\theta$$

$$\text{SO THAT AT } t=0, \dot{r} = r_0 \dot{\theta}_0$$

$$\text{FROM EQ. (12.27): } r \dot{\theta} = r_0 \dot{\theta}_0$$

$$\text{OR } \dot{\theta} = \frac{r_0 \dot{\theta}_0}{r} = \frac{r_0 \dot{\theta}_0}{\left( \frac{r_0}{\sqrt{\cos 2\theta}} \right)^2} = \frac{\dot{\theta}_0}{r_0} \cos 2\theta$$

12.76 FIND:  $N_r$  AND  $N_\theta$  AS FUNCTIONS OF  $\theta$

$$\text{HAVE... } N_r = \dot{r} = r_0 \frac{\sin 2\theta}{\cos^{3/2} 2\theta} = \frac{v_0^2}{r_0} \cos 2\theta$$

$$\text{OR } N_r = v_0^2 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

$$\text{AND } N_\theta = r \dot{\theta} = \frac{r_0}{\sqrt{\cos 2\theta}} \cdot \frac{v_0^2}{r_0} \cos 2\theta$$

$$\text{OR } N_\theta = v_0^2 \sqrt{\cos 2\theta}$$

12.77 SHOW: (a)  $N \propto r$  AND  $F \propto r$

(b)  $p \propto r^3$

(a) FROM THE SOLUTION TO PROBLEM 12.76 HAVE

$$N_r = v_0^2 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

$$N_\theta = v_0^2 \sqrt{\cos 2\theta}$$

(CONTINUED)

## 12.77 continued

$$\text{NOW... } N^2 = N_r^2 + N_\theta^2 \\ = \left( v_0^2 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \right)^2 + \left( v_0^2 \sqrt{\cos 2\theta} \right)^2 \\ = v_0^2 \left( \frac{\sin^2 2\theta}{\cos 2\theta} + \cos 2\theta \right) = \frac{v_0^2}{\cos 2\theta}$$

$$\text{OR } N = \frac{v_0}{\sqrt{\cos 2\theta}}$$

$$\text{RECALLING THAT } r = \frac{r_0}{\sqrt{\cos 2\theta}}$$

$$\text{IT FOLLOWS THAT } N = \frac{v_0}{r_0} r \quad \text{OR } N \propto r \text{ Q.E.D.}$$

$$\text{NOW... } \dot{r} = N_r = v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \quad \text{AND } r = \frac{r_0}{\sqrt{\cos 2\theta}}$$

$$\text{COMBINING... } \dot{r} = \frac{v_0}{r_0} r \sin 2\theta$$

$$\text{THEN... } \ddot{r} = \frac{d}{dt} \left( \frac{v_0}{r_0} r \sin 2\theta \right) = \frac{v_0}{r_0} \left[ \dot{r} \sin 2\theta + r (2 \cos 2\theta) \dot{\theta} \right]$$

NOTING THAT  $r \dot{\theta} = v_0$  HAVE...

$$\ddot{r} = \frac{v_0}{r_0} \left[ \left( v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \right) \sin 2\theta + 2 \left( v_0 \sqrt{\cos 2\theta} \right) \cos 2\theta \right] \\ = \frac{v_0^2}{r_0} \frac{1 + \cos^2 2\theta}{\sqrt{\cos 2\theta}} \\ = \frac{v_0^2}{r_0} (1 + \cos^2 2\theta) r$$

$$\text{NOW... } a_r = \ddot{r} - r \dot{\theta}^2 \quad \dot{\theta} = \frac{v_0}{r_0} \cos 2\theta \quad (\text{FROM ABOVE})$$

$$= \frac{v_0^2}{r_0} (1 + \cos^2 2\theta) r - r \left( \frac{v_0}{r_0} \cos 2\theta \right)^2$$

$$= \frac{v_0^2}{r_0} r$$

FINALLY...  $\vec{F} = \vec{F}_r + \vec{F}_\theta$  AND FOR CENTRAL FORCE MOTION,  $F_\theta = 0$ . THEN...

$$F = F_r = m a_r = m \frac{v_0^2}{r_0} r$$

OR  $F \propto r$  Q.E.D.

(b) FIRST NOTE...  $N = \frac{v_0}{r_0} r$  (PART a)

AND  $a_\theta = 0$  (CENTRAL FORCE MOTION)

$$\text{NOW... } a_t = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{v_0}{r_0} r \right) = \frac{v_0}{r_0} \dot{r} \\ = \frac{v_0}{r_0} \left( v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \right) \quad (\text{FROM PART a})$$

$$= \frac{v_0^2}{r_0} r \sin 2\theta$$

$$\text{HAVE... } a^2 = a_t^2 + a_n^2 = a_r^2 + a_\theta^2 \quad a_r = \frac{v_0^2}{r_0} r \quad (\text{PART a})$$

$$\text{SO THAT } a_n^2 = \left( \frac{v_0^2}{r_0} r \right)^2 - \left( \frac{v_0^2}{r_0} r \sin 2\theta \right)^2$$

$$= \frac{v_0^2}{r_0^2} r^2 \cos^2 2\theta \quad r = \frac{r_0}{\sqrt{\cos 2\theta}}$$

$$= \frac{v_0^2}{r_0^2} r^2 \cos^2 2\theta$$

$$\text{OR } a_n = \frac{v_0^2}{r_0} r$$

$$\text{FINALLY... } a_n = \frac{v_0^2}{r_0} r \quad N = \frac{v_0}{r_0} r \quad (\text{FROM PART a})$$

$$\text{OR } \frac{N^2}{r} = \left( \frac{v_0}{r_0} r \right)^2$$

$$\text{OR } p = \frac{1}{r_0^2} r^3$$

OR  $p \propto r^3$  Q.E.D.

12.78

GIVEN: A PLANET OF RADIUS  $R$  AND OF DENSITY  $\rho$ ; MOON HAVING ORBITAL RADIUS  $r = 2R$

SHOW:  $T = (24\pi/G\rho)^{1/2}$

HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $F = F_n = ma_n = m \frac{v^2}{r}$

THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $v^2 = \frac{GM}{r}$

FOR THE PLANET..  $M = \rho V = \rho \left( \frac{4}{3} \pi R^3 \right)$

THEN  $v^2 = \frac{G}{r} \left( \rho \frac{4}{3} \pi R^3 \right) = \frac{4}{3} \pi G \rho \frac{R^3}{r}$

THE TIME  $T$  FOR THE MOON TO COMPLETE ONE FULL REVOLUTION IS

$$T = \frac{2\pi r}{v} = 2\pi r \left( \frac{3}{4\pi G \rho R^3} \right)^{1/2}$$

$$= \sqrt{\frac{3\pi}{G\rho}} \left( \frac{r}{R} \right)^{1/2}$$

FOR  $r = 2R$ ..  $T = \sqrt{\frac{3\pi}{G\rho}} \left( \frac{2R}{R} \right)^{1/2}$

OR  $T = \sqrt{\frac{24\pi}{G\rho}}$  Q.E.D.



12.79

GIVEN: A PLANET OF RADIUS  $R$  HAVING AN ACCELERATION OF GRAVITY  $g$  AT ITS SURFACE;  $T$ , THE ORBITAL PERIOD OF A MOON

SHOW:  $r = f(R, g, T)$ , WHERE  $r$  IS THE ORBITAL RADIUS OF THE MOON

FIND:  $g$  FOR JUPITER;  $R = 71492$  km,

$T_{\text{EUROPA}} = 3.551$  DAYS,

$r_{\text{EUROPA}} = 670.9 \times 10^3$  km

HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $F = F_n = ma_n = m \frac{v^2}{r}$

THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $v^2 = \frac{GM}{r}$

NOW  $GM = gR^2$  [Eq. (12.30)]

SO THAT  $v^2 = \frac{gR^2}{r}$  OR  $v = R \sqrt{\frac{g}{r}}$

FOR ONE ORBIT..  $T = \frac{2\pi r}{v} = \frac{2\pi r}{R \sqrt{\frac{g}{r}}}$

OR  $r = \left( \frac{gT^2 R^3}{4\pi^2} \right)^{1/3}$  Q.E.D.

SOLVING FOR  $g$ ..  $g = 4\pi^2 \frac{r^3}{T^2 R^3}$

AND NOTING THAT  $T = 3.551$  DAYS  $= 306806$  s,

THEN

$$g_{\text{JUPITER}} = 4\pi^2 \frac{r_{\text{EUROPA}}^3}{T_{\text{EUROPA}}^2 R_{\text{JUP}}^3}$$

$$= 4\pi^2 \frac{(670.9 \times 10^3 \text{ m})^3}{(306806 \text{ s})^2 (71492 \times 10^3 \text{ m})^3}$$

OR  $g_{\text{JUPITER}} = 24.8 \frac{\text{m}}{\text{s}^2}$

NOTE:  $g_{\text{JUPITER}} \approx 2.53 g_{\text{EARTH}}$



12.80

GIVEN: SATELLITE IN A GEOSYNCHRONOUS EARTH ORBIT;  $T = 23.934$  h

FIND: (a) ALTITUDE  $h$  OF THE SATELLITE  
(b) VELOCITY  $v$  OF THE SATELLITE

FIRST NOTE..  $T = 23.934 \text{ h} = 86.1624 \times 10^3 \text{ s}$

AND  $R_{\text{EARTH}} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$R_{\text{EARTH}} = 6.37 \times 10^6 \text{ m}$

(a) FROM THE SOLUTION TO PROBLEM

12.79 HAVE

$$r = \left( \frac{gT^2 R^3}{4\pi^2} \right)^{1/3}$$

NOW..  $h = r - R$

THEN.. SI:  $h = \left[ \frac{9.81 \frac{\text{m}}{\text{s}^2} \cdot (86.1624 \times 10^3 \text{ s})^2 \cdot (6.37 \times 10^6 \text{ m})^3}{4\pi^2} \right]^{1/3}$

$$= 6.37 \times 10^6 \text{ m}$$

$$= (42.145 - 6.37) \times 10^6 \text{ m}$$

OR  $h = 35.77 \times 10^3 \text{ km}$

U.S. UNITS:  $h = \left[ \frac{32.2 \frac{\text{ft}}{\text{s}^2} \cdot (86.1624 \times 10^3 \text{ s})^2 \cdot (20.9088 \times 10^6 \text{ ft})^3}{4\pi^2} \right]^{1/3}$

$$= 20.9088 \times 10^6 \text{ ft}$$

$$= (138.3343 - 20.9088) \times 10^6 \text{ ft}$$

OR  $h = 22,240 \text{ mi}$

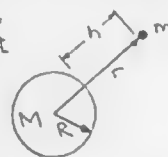
(b) HAVE..  $v = \frac{2\pi r}{T}$

THEN.. SI:  $v = 2\pi \frac{42.145 \times 10^6 \text{ m}}{86.1624 \times 10^3 \text{ s}}$

OR  $v = 3070 \frac{\text{m}}{\text{s}}$

U.S. UNITS:  $v = 2\pi \frac{138.3343 \times 10^6 \text{ ft}}{86.1624 \times 10^3 \text{ s}}$

OR  $v = 10,090 \frac{\text{ft}}{\text{s}}$



12.81

GIVEN:  $r_{\text{MOON}} = 238,910$  mi,  $T_{\text{MOON}} = 27.32$  DAYS

FIND: MASS  $M$  OF THE EARTH

HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $F = F_n = ma_n = m \frac{v^2}{r}$

THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $M = \frac{r}{G} v^2$

NOW..  $v = \frac{2\pi r}{T}$

SO THAT  $M = \frac{r}{G} \left( \frac{2\pi r}{T} \right)^2 = \frac{1}{G} \left( \frac{2\pi}{T} \right)^2 r^3$

NOTING THAT  $T = 27.32$  DAYS  $= 2.3604 \times 10^6$  s

AND  $r = 238,910 \text{ mi} = 1.26144 \times 10^9 \text{ ft}$

HAVE..  $M = \frac{1}{34.4 \times 10^9 \frac{\text{ft}^3}{\text{lb} \cdot \text{s}^2}} \left( \frac{2\pi}{2.3604 \times 10^6 \text{ s}} \right)^2 (1.26144 \times 10^9 \text{ ft})^3$

OR  $M = 4.13 \times 10^{21} \frac{\text{lb} \cdot \text{s}^2}{\text{ft}}$



# 12.82

GIVEN: ALTITUDE  $h = 380$  km OF SPACECRAFT IN ORBIT ABOUT MARS;  $\rho_{\text{MARS}} = 3.94 \text{ Mg/m}^3$   
 $R_{\text{MARS}} = 3397 \text{ km}$

FIND: (a) TIME  $\tau$  OF ONE ORBIT  
 (b) VELOCITY  $v$  OF THE SPACECRAFT

(a) FROM THE SOLUTION TO PROBLEM 12.78 HAVE

$$\tau = \sqrt{\frac{3\pi}{G\rho}} \left(\frac{r}{R}\right)^{3/2}$$

WHERE  $r = R + h = (3397 + 380) \text{ km} = 3777 \text{ km}$

$$\text{THEN.. } \tau = \left[ \frac{3\pi}{(66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2})(3.94 \times 10^3 \frac{\text{kg}}{\text{m}^3})} \right]^{1/2} \left( \frac{3777 \text{ km}}{3397 \text{ km}} \right)^{3/2}$$

$$= 7019.5 \text{ s} \quad \text{OR } \tau = 1 \text{ h } 57 \text{ min}$$

(b) HAVE  $v = \frac{2\pi r}{\tau}$

$$= \frac{2\pi (3777 \times 10^3 \text{ m})}{7019.5 \text{ s}}$$

$$\text{OR } v = 3380 \frac{\text{m}}{\text{s}}$$



# 12.84

GIVEN: FOR THE MOONS JULIET AND TITANIA OF URANUS,  $\tau_J = 0.4931 \text{ DAYS}$ ,

$\tau_T = 8.706 \text{ DAYS}$ ,  $r_J = 49,000 \text{ mi}$

FIND: (a) MASS  $M$  OF URANUS  
 (b)  $r_T$

HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $F = F_n = m a_n = m \frac{v^2}{r}$

THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $M = \frac{r v^2}{G}$

NOW  $v = \frac{2\pi r}{\tau}$

SO THAT  $M = \frac{r}{G} \left( \frac{2\pi r}{\tau} \right)^2 = \frac{1}{G} \left( \frac{2\pi}{\tau} \right)^2 r^3$  (1)

NOW..  $\tau_J = 0.4931 \text{ DAYS} = 42,604 \text{ s}$

AND  $r_J = 49,000 \text{ mi} = 211.2 \times 10^6 \text{ ft}$

(a) USING EQ. (1)..

$$M = \frac{1}{G} \left( \frac{2\pi}{\tau_J} \right)^2 r_J^3 = \frac{1}{34.4 \times 10^{-9} \frac{\text{ft}^3}{\text{lb} \cdot \text{s}^2}} \left( \frac{2\pi}{42,604 \text{ s}} \right)^2 (211.2 \times 10^6 \text{ ft})^3$$

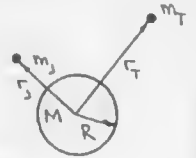
$$\text{OR } M = 5.96 \times 10^{24} \frac{\text{lb}}{\text{ft}^3}$$

(b) REWRITING EQ. (1)..

$$\frac{MG}{4\pi^2} = \frac{r^3}{\tau^2} \quad \text{AND THEN} \quad \frac{r_T^3}{\tau_T^2} = \frac{r_J^3}{\tau_J^2}$$

$$\text{OR } r_T = \left( \frac{8.706 \text{ DAYS}}{0.4931 \text{ DAYS}} \right)^{2/3} (49,000 \text{ mi})$$

$$\text{OR } r_T = 271.2 \times 10^3 \text{ mi}$$



# 12.83

GIVEN: ALTITUDE  $h_s = 3400$  km OF SATELLITE IN ORBIT ABOUT SATURN;  $v_s = 24.45 \frac{\text{km}}{\text{s}}$ ;  
 FOR MOON ATLAS,  $r = 137.6 \times 10^3 \text{ km}$ ,  
 $\tau_{\text{ATLAS}} = 0.6019 \text{ DAYS}$

FIND: (a) RADIUS  $R$  OF SATURN  
 (b) MASS  $M$  OF SATURN

HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $F = F_n = m a_n = m \frac{v^2}{r}$

THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $GM = r v^2$

EQ. (12.29):  $g = \frac{GM}{R^2}$

AND THEN  $g R^2 = r v^2$

$$\text{OR } v = R \sqrt{\frac{g}{r}} \quad (1) \quad \text{AND} \quad R \sqrt{g} = v \sqrt{r} = v \sqrt{R+h} \quad (2)$$

$$\text{NOW.. } \tau = \frac{2\pi r}{v} = \frac{2\pi r}{R \sqrt{\frac{g}{r}}} \quad [\text{USING EQ. (1)}]$$

$$\text{OR } R \sqrt{g} = \frac{2\pi r^{3/2}}{\tau} \quad (3)$$

(a) USING EQS. (2) AND (3)..

$$R_{\text{SATURN}} \sqrt{g_{\text{SATURN}}} = v_s \sqrt{R+h_s} = \frac{2\pi r_A^{3/2}}{\tau_A}$$

$$\text{OR } R = \left( \frac{2\pi r_A^{3/2}}{v_s \tau_A} \right)^2 - h_s$$

NOTING THAT  $\tau_A = 0.6019 \text{ DAYS} = 52.0042 \times 10^3 \text{ s}$

$$\text{HAVE.. } R = \left[ \frac{2\pi (137.6 \times 10^3 \text{ m})^{3/2}}{(24.45 \times 10^3 \frac{\text{m}}{\text{s}})(52.0042 \times 10^3 \text{ s})} \right]^2 - 3400 \times 10^3 \text{ m}$$

$$= 60.273 \times 10^6 \text{ m}$$

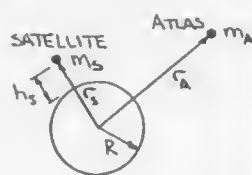
$$\text{OR } R = 60.3 \times 10^3 \text{ km}$$

(b) FROM ABOVE..  $GM = r v^2$

THEN..  $M = \frac{v_s^2 r_s}{G} = \frac{v_s^2 (R+h_s)}{G}$

$$= \frac{(24.45 \times 10^3 \frac{\text{m}}{\text{s}})^2 (60.273 \times 10^6 + 3400 \times 10^3 \text{ m})}{66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}}$$

$$\text{OR } M = 570 \times 10^{24} \text{ kg}$$



# 12.85

GIVEN: SPACECRAFT OF WEIGHT  $W = 1200 \text{ lb}$ ;  
 $h_E = 2800 \text{ mi}$ ;  $m_{\text{MOON}} = 0.01230 M_{\text{EARTH}}$ ,  
 $R_{\text{MOON}} = 1080 \text{ mi}$

FIND: (a) GRAVITATIONAL FORCE  $F$  ON THE SPACECRAFT, EARTH ORBIT

(b)  $r_M$ ,  $\tau_E = \tau_M$

(c)  $g_{\text{MOON}}$

FIRST NOTE THAT  $R_E = 3960 \text{ mi}$

THEN  $r_E = R_E + h_E = (3960 + 2800) \text{ mi} = 6760 \text{ mi}$

(a) HAVE..  $F = G \frac{Mm}{r^2}$  [Eq. (12.28)]

AND  $GM = g R^2$  [Eq. (12.29)]

THEN..  $F = g R^2 \frac{m}{r^2} = W \left( \frac{R}{r} \right)^2$

$$\text{FOR THE EARTH ORBIT.. } F = (1200 \text{ lb}) \left( \frac{3960 \text{ mi}}{6760 \text{ mi}} \right)^2$$

$$\text{OR } F = 412 \text{ lb}$$

(b) FROM THE SOLUTION TO PROBLEM 12.81 HAVE

$$M = \frac{1}{G} \left( \frac{2\pi}{\tau} \right)^2 r^3$$

$$\text{THEN } \tau = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\text{NOW.. } \tau_E = \tau_M \Rightarrow \frac{2\pi r_E^{3/2}}{\sqrt{GM_E}} = \frac{2\pi r_M^{3/2}}{\sqrt{GM_M}} \quad (1)$$

$$\text{OR } r_M = \left( \frac{M_M}{M_E} \right)^{1/3} r_E = (0.01230)^{1/3} (6760 \text{ mi})$$

$$\text{OR } r_M = 1560 \text{ mi}$$

(c) HAVE..  $GM = g R^2$  [Eq. (12.29)]

SUBSTITUTING INTO EQ. (1)

$$\frac{2\pi r_E^{3/2}}{R_E \sqrt{g_E}} = \frac{2\pi r_M^{3/2}}{R_M \sqrt{g_M}}$$

(CONTINUED)



# 12.85 continued

$$\text{OR } g_M = \left(\frac{R_E}{R_M}\right)^2 \left(\frac{M_M}{M_E}\right) g_E = \left(\frac{R_E}{R_M}\right)^2 \left(\frac{M_M}{M_E}\right) g_E$$

USING THE RESULTS OF PART (b). THEN..

$$g_M = \left(\frac{3960 \text{ mi}}{1080 \text{ mi}}\right)^2 (0.01230) (32.2 \frac{\text{ft}}{\text{s}^2})$$

$$\text{OR } g_{\text{MOON}} = 5.32 \frac{\text{ft}}{\text{s}^2}$$

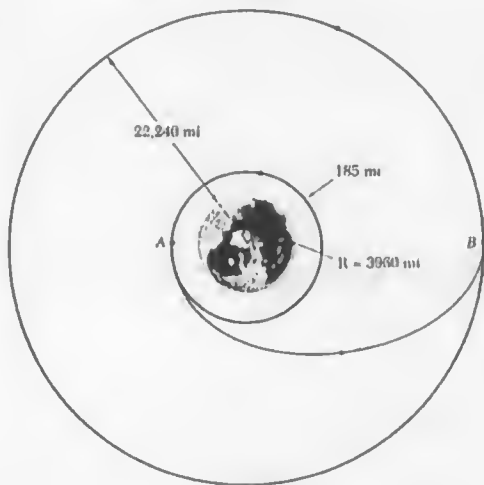
NOTE:  $g_{\text{MOON}} \approx \frac{1}{6} g_{\text{EARTH}}$

# 12.86

GIVEN: CIRCULAR ORBITS AND ELLIPTIC TRANSFER ORBIT AB SHOWN;

$$\Delta v_B = 4810 \frac{\text{ft}}{\text{s}}$$

FIND: (a)  $(v_B)_{\text{TR}}$   
(b)  $\Delta v_A$



FIRST NOTE..  $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$   
 $r_A = (3960 + 185) \text{ mi} = 4145 \text{ mi} = 21.8856 \times 10^6 \text{ ft}$   
 $r_B = (3960 + 22,240) \text{ mi} = 26,200 \text{ mi} = 138.336 \times 10^6 \text{ ft}$

FOR A CIRCULAR ORBIT..  $\Sigma F_n = m a_n$ ;  $F = m \frac{v^2}{r}$   
 EQ. (12.28):  $F = G \frac{Mm}{r^2}$   
 THEN  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$   
 OR  $v^2 = \frac{GM}{r}$

EQ. (12.29):  $GM = gR^2$   
 SO THAT  $v^2 = \frac{gR^2}{r}$  FOR A CIRCULAR ORBIT

THEN..  $(v_A)_{\text{CIRC}}^2 = \frac{32.2 \frac{\text{ft}}{\text{s}^2} \times (20.9088 \times 10^6 \text{ ft})^2}{21.8856 \times 10^6 \text{ ft}}$

OR  $(v_A)_{\text{CIRC}} = 25,362 \frac{\text{ft}}{\text{s}}$

AND  $(v_B)_{\text{CIRC}}^2 = \frac{32.2 \frac{\text{ft}}{\text{s}^2} \times (20.9088 \times 10^6 \text{ ft})^2}{138.336 \times 10^6 \text{ ft}}$

OR  $(v_B)_{\text{CIRC}} = 10,088 \frac{\text{ft}}{\text{s}}$

(a) HAVE..  $(v_B)_{\text{CIRC}} = (v_B)_{\text{TR}} + \Delta v_B$   
 OR  $(v_B)_{\text{TR}} = (10,088 - 4810) \frac{\text{ft}}{\text{s}} = 5278 \frac{\text{ft}}{\text{s}}$

OR  $(v_B)_{\text{TR}} = 5280 \frac{\text{ft}}{\text{s}}$

(b) CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT  $r_A m (v_A)_{\text{TR}} = r_B m (v_B)_{\text{TR}}$

OR  $(v_A)_{\text{TR}} = \frac{26,200 \text{ mi}}{4145 \text{ mi}} \times 5278 \frac{\text{ft}}{\text{s}}$   
 $= 33,362 \frac{\text{ft}}{\text{s}}$

(CONTINUED)

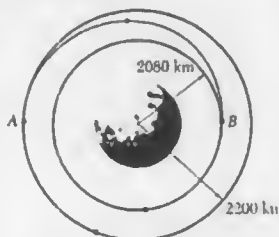
# 12.86 continued

NOW..  $(v_A)_{\text{TR}} = (v_A)_{\text{CIRC}} + \Delta v_A$   
 OR  $\Delta v_A = (33,362 - 25,362) \frac{\text{ft}}{\text{s}}$   
 OR  $\Delta v_A = 8000 \frac{\text{ft}}{\text{s}}$

# 12.87

GIVEN: CIRCULAR ORBITS ABOUT THE MOON AND ELLIPTIC TRANSFER ORBIT AB AS SHOWN;  $\Delta v_A = -26.3 \frac{\text{m}}{\text{s}}$ ;  
 $m_{\text{MOON}} = 73.49 \times 10^{21} \text{ kg}$

FIND: (a)  $(v_B)_{\text{TR}}$   
(b)  $\Delta v_B$



FOR A CIRCULAR ORBIT..  $\Sigma F_n = m a_n$ ;  $F = m \frac{v^2}{r}$

EQ. (12.28):  $F = G \frac{Mm}{r^2}$

THEN..  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR  $v^2 = \frac{GM}{r}$

THEN..  $(v_A)_{\text{CIRC}}^2 = \frac{66.73 \times 10^{-12} \frac{\text{kg} \cdot \text{m}^3}{\text{s}^2} \times 73.49 \times 10^{21} \text{ kg}}{2200 \times 10^3 \text{ m}}$

OR  $(v_A)_{\text{CIRC}} = 1493.0 \frac{\text{m}}{\text{s}}$

AND  $(v_B)_{\text{CIRC}}^2 = \frac{66.73 \times 10^{-12} \frac{\text{kg} \cdot \text{m}^3}{\text{s}^2} \times 73.49 \times 10^{21} \text{ kg}}{2080 \times 10^3 \text{ m}}$

OR  $(v_B)_{\text{CIRC}} = 1535.5 \frac{\text{m}}{\text{s}}$

(a) HAVE..  $(v_A)_{\text{TR}} = (v_A)_{\text{CIRC}} + \Delta v_A = (1493.0 - 26.3) \frac{\text{m}}{\text{s}}$   
 $= 1466.7 \frac{\text{m}}{\text{s}}$

CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT  $r_A m (v_A)_{\text{TR}} = r_B m (v_B)_{\text{TR}}$

OR  $(v_B)_{\text{TR}} = \frac{2200 \text{ km}}{2080 \text{ km}} \times 1466.7 \frac{\text{m}}{\text{s}} = 1551.3 \frac{\text{m}}{\text{s}}$

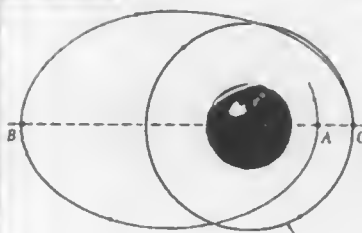
OR  $(v_B)_{\text{TR}} = 1551 \frac{\text{m}}{\text{s}}$

(b) NOW..  $(v_B)_{\text{CIRC}} = (v_B)_{\text{TR}} + \Delta v_B$   
 OR  $\Delta v_B = (1535.5 - 1551.3) \frac{\text{m}}{\text{s}}$

OR  $\Delta v_B = -15.8 \frac{\text{m}}{\text{s}}$

# 12.88

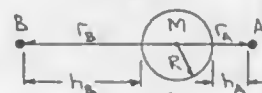
GIVEN: CIRCULAR ORBIT ABOUT VENUS AND ELLIPTIC TRANSFER ORBITS AB AND BC;  $r = 6420 \text{ km}$ ;  
 $v_A = 7420 \frac{\text{m}}{\text{s}}$ ;  $h_A = 288 \text{ km}$   
 $\Delta v_B = 24.5 \frac{\text{m}}{\text{s}}$ ;  
 $\Delta v_E = -264 \frac{\text{m}}{\text{s}}$ ;  
 $m_{\text{VENUS}} = 4.869 \times 10^{24} \text{ kg}$   
 $R_{\text{VENUS}} = 6052 \text{ km}$



Circular orbit

FIND: (a)  $(v_B)_{\text{TRAB}}$   
(b)  $h_B$

FIRST NOTE..  $r_A = R + h_A$   
 $= (6052 + 288) \text{ km}$   
 $= 6340 \text{ km}$



FOR A CIRCULAR ORBIT..  $\Sigma F_n = m a_n$ ;  $F = m \frac{v^2}{r}$

EQ. (12.28):  $F = G \frac{Mm}{r^2}$

THEN..  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$  OR  $v^2 = \frac{GM}{r}$

THEN  $(v_E)_{\text{CIRC}}^2 = \frac{66.73 \times 10^{-12} \frac{\text{kg} \cdot \text{m}^3}{\text{s}^2} \times 4.869 \times 10^{24} \text{ kg}}{6420 \times 10^3 \text{ m}}$

(CONTINUED)

# 12.88 continued

OR  $(\omega_c)_{circ} = 7114.0 \frac{m}{s}$   
 NOW..  $(\omega_c)_{circ} = (\omega_c)_{trac} + \Delta\omega_c$   
 OR  $(\omega_c)_{trac} = [7114.0 - (264)] \frac{m}{s} = 7378.0 \frac{m}{s}$

(a) CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT.. AB:  $r_A m (\omega_A) = r_B m (\omega_B)_{trac}$  (1)  
 BC:  $r_B m (\omega_B)_{trac} = r_C m (\omega_c)_{trac}$  (2)

THEN (2)  $\Rightarrow \frac{r_B (\omega_B)_{trac}}{r_B (\omega_B)_{trac}} = \frac{r_C (\omega_c)_{trac}}{r_A (\omega_A)}$   
 NOW..  $(\omega_B)_{trac} = (\omega_B)_{trac} + \Delta\omega_B$   
 THEN..  $\frac{(\omega_B)_{trac} + \Delta\omega_B}{(\omega_B)_{trac}} = \frac{r_C (\omega_c)_{trac}}{r_A (\omega_A)}$

OR  $(\omega_B)_{trac} = \frac{\Delta\omega_B}{\frac{r_C (\omega_c)_{trac}}{r_A (\omega_A)} - 1} = \frac{24.5 \frac{m}{s}}{\frac{6420 \text{ km} \cdot 7378.0 \frac{m}{s}}{6340 \text{ km} \cdot 7420 \frac{m}{s}} - 1} = 3557.7 \frac{m}{s}$   
 OR  $(\omega_B)_{trac} = 3560 \frac{m}{s}$

(b) FROM EQ. (1)..  
 $r_B = \frac{\omega_A}{(\omega_B)_{trac}} r_A = \frac{7420 \frac{m}{s}}{3557.7 \frac{m}{s}} \cdot 6340 \text{ km} = 13223 \text{ km}$   
 NOW..  $r_B = R + h_B$   
 OR  $h_B = (13223 - 6052) \text{ km}$   
 OR  $h_B = 7170 \text{ km}$

# 12.89 continued

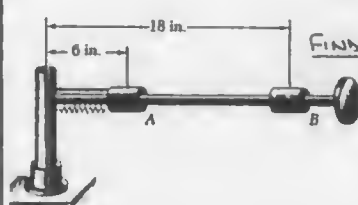
CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT..  
 BC:  $r_B m (\omega_B)_{trac} = r_C m (\omega_c)_{trac}$  (1)  
 (D):  $r_C m (\omega_c)_{trac} = r_A m (\omega_A)_{trac}$  (2)

FROM EQ (1)..  $(\omega_c)_{trac} = \frac{r_B (\omega_B)_{trac}}{r_C} = \frac{4140 \text{ mi}}{4289 \text{ mi}} \cdot 25,657 \frac{ft}{s} = 24,766 \frac{ft}{s}$   
 NOW..  $(\omega_c)_{trac} = (\omega_c)_{trac} + \Delta\omega_c = (24,766 + 260) \frac{ft}{s} = 25,026 \frac{ft}{s}$   
 FROM EQ. (2)..  $(\omega_A)_{trac} = \frac{r_C (\omega_c)_{trac}}{r_A} = \frac{4289 \text{ mi}}{4340 \text{ mi}} \cdot 25,026 \frac{ft}{s} = 24,732 \frac{ft}{s}$   
 FINALLY..  $(\omega_A)_{circ} = (\omega_A)_{trac} + \Delta\omega_A$   
 OR  $\Delta\omega_A = (24,785 - 24,732) \frac{ft}{s}$   
 OR  $\Delta\omega_A = 53 \frac{ft}{s}$

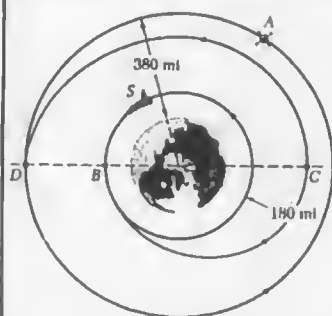
# 12.90

GIVEN:  $r_0 = r_A$ ,  $\dot{\theta}_0 = 16 \frac{rad}{s}$   
 $(\omega_{collar})_0 = 0$ ;  $k = 2 \text{ lb/ft}$ ;  
 NEGLECT FRICTION AND  
 MASS;  $W = 3 \text{ lb}$

FINDB: (a)  $(a_r)_r$  AND  $(a_r)_\theta$   
 (b)  $(a_{collar/rod})_A$   
 (c)  $(\omega_B)_\theta$



# 12.89



GIVEN: CIRCULAR ORBITS A AND B ABOUT THE EARTH AND ELLIPTIC TRANSFER ORBITS BC AND CD;  
 $\Delta\omega_B = 280 \frac{ft}{s}$ ;  $\Delta\omega_C = 260 \frac{ft}{s}$ ;  
 $r_C = 4289 \text{ mi}$

FINDB:  $\Delta\omega_D$

FIRST NOTE..  $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$   
 $r_A = (3960 + 380) \text{ mi} = 4340 \text{ mi} = 22.9152 \times 10^6 \text{ ft}$   
 $r_B = (3960 + 180) \text{ mi} = 4140 \text{ mi} = 21.8592 \times 10^6 \text{ ft}$


FOR A CIRCULAR ORBIT..  $\Sigma F_r = m a_r$ ;  $F = m \frac{v^2}{r}$   
 EQ. (12.28):  $F = G \frac{Mm}{r^2}$   
 THEN..  $G \frac{Mm}{r^2} = m \frac{v^2}{r}$   
 OR  $v^2 = \frac{GM}{r} = \frac{g R^2}{r}$  USING EQ. (12.29)

THEN..  $(\omega_A)_{circ}^2 = \frac{32.2 \frac{ft}{s^2} \times (20.9088 \times 10^6 \text{ ft})^2}{22.9152 \times 10^6 \text{ ft}}$   
 OR  $(\omega_A)_{circ} = 24,785 \frac{ft}{s}$   
 AND  $(\omega_B)_{circ}^2 = \frac{32.2 \frac{ft}{s^2} \times (20.9088 \times 10^6 \text{ ft})^2}{21.8592 \times 10^6 \text{ ft}}$   
 OR  $(\omega_B)_{circ} = 25,377 \frac{ft}{s}$

HAVE..  $(\omega_B)_{trac} = (\omega_B)_{circ} + \Delta\omega_B = (25,377 + 280) \frac{ft}{s} = 25,657 \frac{ft}{s}$

(CONTINUED)

FIRST NOTE..  $F_{sp} = k(r - r_0)$



(a)  $F_\theta = 0$  AND AT A,  $F_r = -F_{sp} = 0$   
 $\therefore (a_r)_r = 0$   
 $(a_r)_\theta = 0$

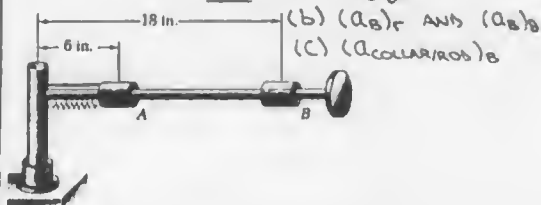
(b)  $\Sigma F_r = m a_r$ ;  $-F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$   
 NOTING THAT  $a_{collar/rod} = \ddot{r}$ , HAVE AT A..  
 $0 = m[a_{collar/rod} - (6 \text{ in.})(16 \frac{rad}{s})^2]$   
 OR  $(a_{collar/rod})_A = 1536 \frac{in.}{s^2}$

(c) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.  
 $\therefore r_A m (\omega_A)_\theta = r_B m (\omega_B)_\theta$  WHERE  $(\omega_A)_\theta = r_A \dot{\theta}_0$   
 THEN..  $(\omega_B)_\theta = \frac{r_A}{r_B} \dot{\theta}_0 = \frac{6 \text{ in.}}{18 \text{ in.}} \cdot (16 \frac{rad}{s}) = 5.33 \frac{rad}{s}$   
 OR  $(\omega_B)_\theta = 32.0 \frac{in.}{s}$

## 12.91

GIVEN:  $r = r_A$ ,  $\dot{\theta}_0 = 12 \frac{\text{rad}}{\text{s}}$ ,  $(x_{B0})_0 = 0$ ;  
 $k = 2 \text{ lb/in}$ ; NEGLECT FRICTION AND  
 $m_{\text{rod}}$ ;  $W = 3 \text{ lb}$

FIND: (a)  $(\dot{x}_B)_0$



FIRST NOTE..  $F_{\text{sp}} = k(r - r_A)$

AT B:  $(F_{\text{sp}})_B = 2 \frac{\text{lb}}{\text{in}} \cdot (18 - 6) \text{ in.} = 24 \text{ lb}$



(a) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

$\therefore r_A m (\dot{x}_A)_0 = r_B m (\dot{x}_B)_0$  WHERE  $(\dot{x}_A)_0 = r_A \dot{\theta}_0$

THEN..  $(\dot{x}_B)_0 = \frac{6 \text{ in.}}{18 \text{ in.}} [(6 \text{ in.})(12 \frac{\text{rad}}{\text{s}})]$

OR  $(\dot{x}_B)_0 = 24.0 \frac{\text{in.}}{\text{s}}$

(b) HAVE..  $F_B = 0$

NOW..  $\sum F_r = m a_r$ :  $-(F_{\text{sp}})_B = \frac{W}{g} (\dot{x}_B)_0$

OR  $(\dot{x}_B)_0 = -\frac{24 \text{ lb}}{3 \text{ lb}} = -32.2 \frac{\text{ft}}{\text{s}} = -21.46 \frac{\text{ft}}{\text{s}} = -257.6 \frac{\text{in.}}{\text{s}}$

OR  $(\dot{x}_B)_0 = -258 \frac{\text{in.}}{\text{s}}$

(c) HAVE..  $a_r = \ddot{r} - r \dot{\theta}^2$

NOW..  $a_{\text{collar/rod}} = \ddot{r}$  AND  $\dot{\theta}_B = \frac{(\dot{x}_B)_0}{r_B}$

THEN.. AT B:  $(a_{\text{collar/rod}})_B = -257.6 \frac{\text{in.}}{\text{s}} + 18 \text{ in.} \cdot \left( \frac{24.0 \frac{\text{in.}}{\text{s}}}{18 \text{ in.}} \right)^2$

OR  $(a_{\text{collar/rod}})_B = -226 \frac{\text{in.}}{\text{s}^2}$

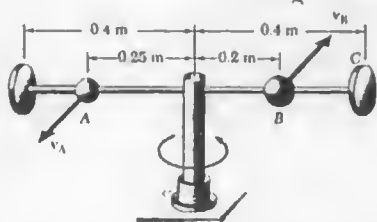
## 12.92

GIVEN:  $m_A = 0.2 \text{ kg}$ ,  $m_B = 0.4 \text{ kg}$ ,  $m_{\text{rod}} = 0$ ;  
 $(\dot{x}_A)_0 = 2.5 \frac{\text{m}}{\text{s}}$ ; NEGLECT FRICTION;  
 AT  $t = 0$ , BALL B BEGINS TO MOVE FROM B TO C

FIND: (a)  $(\dot{x}_B)_0$  AND  $(\dot{x}_B)_0$  AT  $t = 0$

(b)  $a_{B/\text{rod}}$  AT  $t = 0$

(c)  $\dot{x}_A$  WHEN BALL B IS AT C



(a) WHEN THE PIN HOLDING BALL B IS REMOVED, THERE ARE THEN NO HORIZONTAL FORCES ACTING ON THE BALL. THEREFORE,

AT  $t = 0$ ,  $F_r = 0$  AND  $F_B = 0$

(CONTINUED)

## 12.92 continued

SO THAT

$[(a_B)_r]_0 = 0$

$[(a_B)_\theta]_0 = 0$

(b) HAVE..  $a_r = \ddot{r} - r \dot{\theta}^2$

NOW..  $a_{B/\text{rod}} = \ddot{r}$  AND  $\dot{\theta} = \frac{\dot{x}_A}{r_A}$

THEN, AT  $t = 0$ ..  $(a_{B/\text{rod}})_0 - (r_B)_0 \left( \frac{(\dot{x}_A)_0}{r_A} \right)^2 = 0$

OR  $(a_{B/\text{rod}})_0 = 0.2 \text{ m} \cdot \left( \frac{2.5 \frac{\text{m}}{\text{s}}}{0.25 \text{ m}} \right)^2$

OR  $(a_{B/\text{rod}})_0 = 20.0 \frac{\text{m}}{\text{s}^2}$

(c) NOW,  $F_r = 0$  AND  $F_B = 0$  WHILE B IS MOVING FROM ITS INITIAL TO ITS FINAL POSITION. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED. THUS..

$r_A m_A (\dot{x}_A)_0 + (r_B)_0 m_B (\dot{x}_B)_0 = r_A m_A \dot{x}_A' + r_B m_B \dot{x}_B'$

WHERE  $(\cdot)'$  DENOTES THE STATE WHEN BALL B IS AT C. NOW..

$(\dot{x}_B)_0 = (r_B)_0 \dot{\theta}_0 = (r_B)_0 \left( \frac{(\dot{x}_A)_0}{r_A} \right)$

AND  $\dot{x}_B' = r_B' \dot{\theta}' = r_B' \left( \frac{\dot{x}_A'}{r_A} \right)$

THEN..  $r_A m_A (\dot{x}_A)_0 + (r_B)_0 m_B \left( \frac{(r_B)_0}{r_A} (\dot{x}_A)_0 \right) = r_A m_A \dot{x}_A' + r_B m_B \left( \frac{r_B}{r_A} \dot{x}_A' \right)$

OR  $\left\{ 1 + \frac{m_B}{m_A} \left( \frac{r_B}{r_A} \right)^2 \right\} (\dot{x}_A)_0 = \left\{ 1 + \frac{m_B}{m_A} \left( \frac{r_B}{r_A} \right)^2 \right\} \dot{x}_A'$

SUBSTITUTING..

$\left\{ 1 + \frac{0.4 \text{ kg}}{0.2 \text{ kg}} \left( \frac{0.2 \text{ m}}{0.25 \text{ m}} \right)^2 \right\} (2.5 \frac{\text{m}}{\text{s}}) = \left\{ 1 + \frac{0.4 \text{ kg}}{0.2 \text{ kg}} \left( \frac{0.4 \text{ m}}{0.25 \text{ m}} \right)^2 \right\} \dot{x}_A'$

OR  $\dot{x}_A' = 0.931 \frac{\text{m}}{\text{s}}$

## 12.93

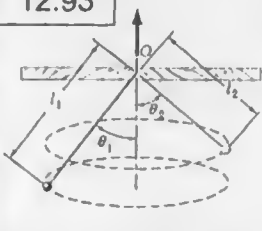
GIVEN: INITIAL STATE OF THE BALL DEFINED BY  $\theta_1, \theta_1$  AND THE FINAL STATE DEFINED BY  $\theta_2, \theta_2$

FIND: (a) RELATION AMONG

$\theta_1, \theta_1, \theta_2$ , AND  $\theta_2$

(b)  $\theta_2$  WHEN  $\theta_1 = 0.8 \text{ m}$ ,

$\theta_2 = 0.6 \text{ m}$ ,  $\theta_1 = 35^\circ$



(a) FOR STATE 1 OR 2..

$\sum F_y = 0$ :  $T \cos \theta - W = 0$

OR  $T = \frac{mg}{\cos \theta}$

$\sum F_r = m a_r$ :  $T \sin \theta = m \frac{v^2}{r}$

WHERE  $r = l \sin \theta$

THEN  $\left( \frac{mg}{\cos \theta} \right) \sin \theta = m \frac{v^2}{l \sin \theta}$

OR  $v^2 = gl \sin \theta \tan \theta$

IT THEN FOLLOWS THAT

$\frac{v_1^2}{v_2^2} = \frac{l_2 \sin \theta_2 \tan \theta_2}{l_1 \sin \theta_1 \tan \theta_1}$  (1)

NOW..  $\sum M_o = 0 \Rightarrow H_y = \text{CONSTANT}$

THUS..  $r_1 m_1 \dot{\theta}_1 = r_2 m_2 \dot{\theta}_2$

OR  $\frac{l_1 \sin \theta_1}{l_2 \sin \theta_2} = \frac{l_1 \sin \theta_1}{l_2 \sin \theta_2}$  (2)

COMBINING EQS. (1) AND (2)..  $\left( \frac{l_1 \sin \theta_1}{l_2 \sin \theta_2} \right)^2 = \frac{l_2 \sin \theta_2 \tan \theta_2}{l_1 \sin \theta_1 \tan \theta_1}$

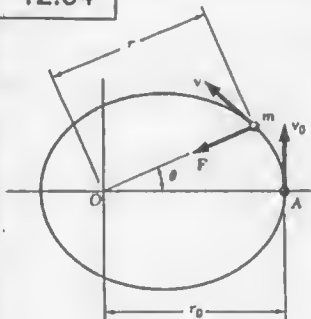
OR  $\theta_1^3 \sin^3 \theta_1 \tan \theta_1 = \theta_2^3 \sin^3 \theta_2 \tan \theta_2$

(b) HAVE..  $(0.8 \text{ m})^3 \sin^3 35^\circ \tan 35^\circ = (0.6 \text{ m})^3 \sin^3 \theta_2 \tan \theta_2$

OR  $\sin^3 \theta_2 \tan \theta_2 = 0.313197$

OR  $\theta_2 = 43.6^\circ$

12.94



GIVEN: PARTICLE OF MASS  $m$  MOVING UNDER THE CENTRAL FORCE  $F$  ALONG THE ELLIPSE  $r = r_0(2 - \cos\theta)$ ; AT  $t = 0, \dot{\theta} = \dot{\theta}_0$   
 SHOW:  $F \propto \frac{1}{r^2}$  USING EQ. (12.37)

HAVE  $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$  EQ. (12.37)

WHERE  $u = \frac{1}{r}$  AND  $mh^2 = \text{CONSTANT}$

$\therefore F \propto u^2 \left( \frac{d^2u}{d\theta^2} + u \right)$

NOW  $u = \frac{1}{r} = \frac{1}{r_0(2 - \cos\theta)}$

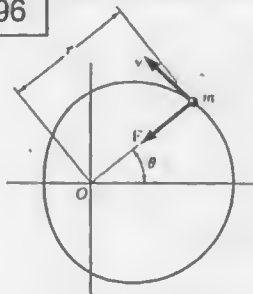
THEN  $\frac{du}{d\theta} = \frac{d}{d\theta} \left[ \frac{1}{r_0(2 - \cos\theta)} \right] = \frac{1}{r_0} \sin\theta$

AND  $\frac{d^2u}{d\theta^2} = \frac{1}{r_0} \cos\theta$

THEN  $F \propto \left( \frac{1}{r} \right)^2 \left[ \left( \frac{1}{r_0} \cos\theta \right) + \frac{1}{r_0(2 - \cos\theta)} \right] = \frac{2}{r_0} \frac{1}{r^2}$   
 $\therefore F \propto \frac{1}{r^2}$  Q.E.D.

NOTE:  $F > 0$  IMPLIES THAT  $F$  IS ATTRACTIVE.

12.96



GIVEN: PARTICLE OF MASS  $m$  MOVING UNDER THE CENTRAL FORCE  $F$  ALONG THE CARDIOID  $r = \frac{r_0}{2}(1 + \cos\theta)$   
 SHOW:  $F \propto \frac{1}{r^4}$  USING EQ. (12.37)

HAVE  $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$  EQ. (12.37)

WHERE  $u = \frac{1}{r}$  AND  $mh^2 = \text{CONSTANT}$

$\therefore F \propto u^2 \left( \frac{d^2u}{d\theta^2} + u \right)$

NOW  $u = \frac{1}{r} = \frac{2}{r_0} \frac{1}{1 + \cos\theta}$

THEN  $\frac{du}{d\theta} = \frac{d}{d\theta} \left( \frac{2}{r_0} \frac{1}{1 + \cos\theta} \right) = \frac{2}{r_0} \frac{\sin\theta}{(1 + \cos\theta)^2}$

AND  $\frac{d^2u}{d\theta^2} = \frac{2}{r_0} \frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta[2(1 + \cos\theta)(-\sin\theta)]}{(1 + \cos\theta)^4}$

$= \frac{2}{r_0} \frac{1 + \cos\theta + \sin^2\theta}{(1 + \cos\theta)^3} = \frac{2}{r_0} \left[ \frac{1}{(1 + \cos\theta)^3} + \frac{1 - \cos^2\theta}{(1 + \cos\theta)^3} \right]$

$= \frac{2}{r_0} \frac{2 - \cos\theta}{(1 + \cos\theta)^2} = \frac{2}{r_0} \left( \frac{r_0}{2r} \right)^2 \left[ 2 - \left( \frac{2r}{r_0} - 1 \right) \right]$

$= \frac{2}{r_0} \frac{1}{r^2} \left( 3 - \frac{2r}{r_0} \right)$

THEN  $F \propto \left( \frac{1}{r} \right)^2 \left[ \frac{2}{r_0} \left( 3 - \frac{2r}{r_0} \right) + \frac{2}{r_0} \frac{1}{r^2} \right] = \frac{2}{r_0} \frac{1}{r^4}$   
 $\therefore F \propto \frac{1}{r^4}$  Q.E.D.

NOTE:  $F > 0$  IMPLIES THAT  $F$  IS ATTRACTIVE.

12.95

GIVEN: PARTICLE OF MASS  $m$  MOVING UNDER A CENTRAL FORCE  $F$  ALONG THE PATH  $r = r_0 \sin\theta$

SHOW:  $F \propto \frac{1}{r^3}$  USING EQ. (12.37)

HAVE  $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$  EQ. (12.37)

WHERE  $u = \frac{1}{r}$  AND  $mh^2 = \text{CONSTANT}$

$\therefore F \propto u^2 \left( \frac{d^2u}{d\theta^2} + u \right)$

NOW  $u = \frac{1}{r} = \frac{1}{r_0 \sin\theta}$

THEN  $\frac{du}{d\theta} = \frac{d}{d\theta} \left( \frac{1}{r_0 \sin\theta} \right) = -\frac{1}{r_0} \frac{\cos\theta}{\sin^2\theta}$

AND  $\frac{d^2u}{d\theta^2} = -\frac{1}{r_0} \left[ \frac{-\sin\theta(\sin^2\theta) - \cos\theta(2\sin\theta \cos\theta)}{\sin^4\theta} \right]$

$= \frac{1}{r_0} \frac{1 + \cos^2\theta}{\sin^3\theta}$

THEN  $F \propto \left( \frac{1}{r} \right)^2 \left( \frac{1}{r_0} \frac{1 + \cos^2\theta}{\sin^3\theta} + \frac{1}{r_0 \sin\theta} \right)$

$= \frac{1}{r_0} \frac{1}{r^2} \left( \frac{1 + \cos^2\theta}{\sin^3\theta} + \frac{\sin^2\theta}{\sin^3\theta} \right)$

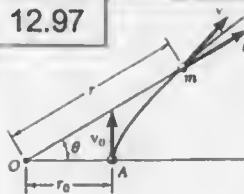
$= \frac{2}{r_0} \frac{1}{r^2} \frac{1}{\sin^3\theta} \quad \sin^3\theta = \left( \frac{r}{r_0} \right)^3$

$= \frac{2r_0^2}{r^3}$

$\therefore F \propto \frac{1}{r^3}$  Q.E.D.

NOTE:  $F > 0$  IMPLIES THAT  $F$  IS ATTRACTIVE.

12.97



GIVEN: PARTICLE OF MASS  $m$  MOVING UNDER THE CENTRAL FORCE  $F$  ALONG THE PATH  $r = r_0 / \sqrt{\cos 2\theta}$ ; AT  $t = 0, \dot{\theta} = \dot{\theta}_0$

SHOW:  $F \propto r$  USING EQ. (12.37)

HAVE  $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$  EQ. (12.37)

WHERE  $u = \frac{1}{r}$  AND  $mh^2 = \text{CONSTANT}$

$\therefore F \propto u^2 \left( \frac{d^2u}{d\theta^2} + u \right)$

NOW  $u = \frac{1}{r} = \frac{1}{r_0} \sqrt{\cos 2\theta}$

THEN  $\frac{du}{d\theta} = \frac{d}{d\theta} \left( \frac{1}{r_0} \sqrt{\cos 2\theta} \right) = -\frac{1}{r_0} \frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$

AND  $\frac{d^2u}{d\theta^2} = -\frac{1}{r_0} \frac{2\cos 2\theta(\cos 2\theta) - \sin 2\theta(-\sin 2\theta/\sqrt{\cos 2\theta})}{\cos 2\theta}$

$= -\frac{1}{r_0} \frac{1 + \cos^2 2\theta}{(\cos 2\theta)^{3/2}} = -\frac{1}{r_0} \left( \frac{r_0}{r} \right)^3 \left[ 1 + \left( \frac{r_0}{r} \right)^4 \right]$

$= -\frac{r_0^3}{r^3} \left[ 1 + \left( \frac{r_0}{r} \right)^4 \right]$

THEN  $F \propto \left( \frac{1}{r} \right)^2 \left\{ -\frac{r_0^3}{r^3} \left[ 1 + \left( \frac{r_0}{r} \right)^4 \right] + \frac{1}{r} \right\} = -\frac{r_0}{r}$

$\therefore F \propto r$  Q.E.D.

NOTE:  $F < 0$  IMPLIES THAT  $F$  IS REPULSIVE.



12.98

GIVEN: PARABOLIC TRAJECTORY OF GALILEO SPACECRAFT ABOUT THE EARTH;  
MINIMUM ALTITUDE = 960 km

FIND:  $N_{\text{MAX}}$

FIRST NOTE..  $R = 6.37 \times 10^6 \text{ m}$

$$\text{SO THAT } r_0 = (6.37 \times 10^6 + 960 \times 10^3) \text{ m} = 7.33 \times 10^6 \text{ m}$$

NOW..  $N_{\text{MAX}} = N_0$

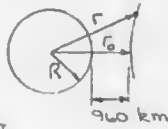
AND FROM PAGE 709 OF THE TEXT

$$N_0 = \sqrt{\frac{2GM}{r_0}} = \sqrt{\frac{2gR^2}{r_0}} \text{ USING EQ. (12.30)}$$

$$\text{THEN.. } N_{\text{MAX}} = \left[ \frac{2 \times 9.81 \text{ m/s}^2 \times (6.37 \times 10^6 \text{ m})^2}{7.33 \times 10^6 \text{ m}} \right]^{1/2}$$

$$= 10421.7 \frac{\text{m}}{\text{s}}$$

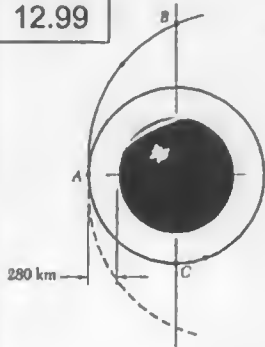
$$\text{OR } N_{\text{MAX}} = 10.42 \frac{\text{km}}{\text{s}}$$



12.99

GIVEN: PARABOLIC APPROACH TRAJECTORY AND CIRCULAR ORBIT ABOUT VENUS;  $M_{\text{VENUS}} = 4.87 \times 10^{24} \text{ kg}$   
 $R = 6052 \text{ km}$

FIND: (a)  $(N_A)_{\text{PAR}}$   
(b)  $|\Delta N_A|$



FIRST NOTE..  $r_A = (6052 + 280) \text{ km} = 6332 \text{ km}$

(a) FROM PAGE 709 OF THE TEXT, THE VELOCITY AT THE POINT OF CLOSEST APPROACH ON A PARABOLIC TRAJECTORY IS GIVEN BY

$$N_0 = \sqrt{\frac{2GM}{r_0}}$$

$$\text{THUS, } (N_A)_{\text{PAR}} = \left[ \frac{2 \times 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \times 4.87 \times 10^{24} \text{ kg}}{6332 \times 10^3 \text{ m}} \right]^{1/2}$$

$$= 10131.4 \frac{\text{m}}{\text{s}}$$

$$\text{OR } (N_A)_{\text{PAR}} = 10.13 \frac{\text{km}}{\text{s}}$$

(b) HAVE..  $(N_A)_{\text{CIRC}} = (N_A)_{\text{PAR}} + \Delta N_A$

$$\text{NOW.. } (N_A)_{\text{CIRC}} = \sqrt{\frac{GM}{r_0}} \text{ EQ. (12.44)}$$

$$= \frac{1}{\sqrt{2}} (N_A)_{\text{PAR}}$$

$$\text{THEN.. } \Delta N_A = \frac{1}{\sqrt{2}} (N_A)_{\text{PAR}} - (N_A)_{\text{PAR}}$$

$$= \left( \frac{1}{\sqrt{2}} - 1 \right) (10131.4 \frac{\text{km}}{\text{s}})$$

$$= -2.97 \frac{\text{km}}{\text{s}}$$

$$\therefore |\Delta N_A| = 2.97 \frac{\text{km}}{\text{s}}$$

12.100

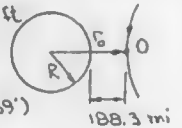
GIVEN: TRAJECTORY OF GALILEO SPACECRAFT ABOUT THE EARTH; AT THE POINT OF CLOSEST APPROACH,  $N = 46.2 \times 10^3 \frac{\text{ft}}{\text{s}}$ , ALTITUDE = 188.3 mi

FIND:  $\epsilon$  AT POINT OF CLOSEST APPROACH

FIRST NOTE..  $R = 3960 \text{ mi} = 20,9088 \times 10^3 \text{ ft}$

$$\text{AND } r_0 = (3960 + 188.3) \text{ mi} = 4148.3 \text{ mi} = 21,9030 \times 10^3 \text{ ft}$$

$$\text{HAVE.. } \frac{1}{r} = \frac{GM}{h^2} (1 + \epsilon \cos \theta) \text{ EQ. (12.39')}$$



AT POINT O,  $r = r_0$ ,  $\theta = 0$ ,  $h = h_0 = r_0 N_0$

ALSO..  $GM = gR^2$  EQ. (12.30)

$$\text{THEN.. } \frac{1}{r_0} = \frac{gR^2}{(r_0 N_0)^2} (1 + \epsilon)$$

$$\text{OR } \epsilon = \frac{r_0 N_0^2}{gR^2} - 1 = \frac{(21,9030 \times 10^3 \text{ ft})(46.2 \times 10^3 \frac{\text{ft}}{\text{s}})^2}{(32.2 \frac{\text{ft}}{\text{s}^2})(20,9088 \times 10^3 \text{ ft})^2} - 1$$

$$\text{OR } \epsilon = 2.32$$

12.101

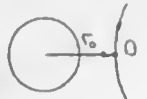
GIVEN: TRAJECTORY OF GALILEO SPACECRAFT ABOUT IO; AT THE POINT OF CLOSEST APPROACH,  $r_0 = 1750 \text{ mi}$ ,  $N_0 = 49.4 \times 10^3 \frac{\text{ft}}{\text{s}}$ ;  $M_{\text{IO}} = 0.01496 M_{\text{EARTH}}$

FIND:  $\epsilon$  AT POINT OF CLOSEST APPROACH

FIRST NOTE..  $r_0 = 1750 \text{ mi} = 9.24 \times 10^6 \text{ ft}$

$$R_{\text{EARTH}} = 3960 \text{ mi} = 20,9088 \times 10^3 \text{ ft}$$

$$\text{HAVE.. } \frac{1}{r} = \frac{GM}{h^2} (1 + \epsilon \cos \theta) \text{ EQ. (12.39')}$$



AT POINT O,  $r = r_0$ ,  $\theta = 0$ ,  $h = h_0 = r_0 N_0$

ALSO..  $GM_{\text{IO}} = G(0.01496 M_{\text{EARTH}})$   
 $= 0.01496 g R_{\text{EARTH}}^2$  USING EQ. (12.30)

$$\text{THEN.. } \frac{1}{r_0} = \frac{0.01496 g R_{\text{EARTH}}^2}{(r_0 N_0)^2} (1 + \epsilon)$$

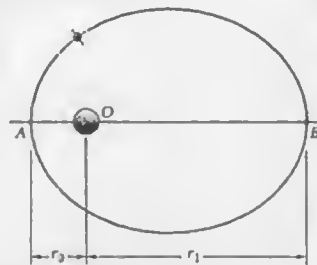
$$\text{OR } \epsilon = \frac{r_0 N_0^2}{0.01496 g R_{\text{EARTH}}^2} - 1 = \frac{(9.24 \times 10^6 \text{ ft})(49.4 \times 10^3 \frac{\text{ft}}{\text{s}})^2}{0.01496 (32.2 \frac{\text{ft}}{\text{s}^2})(20,9088 \times 10^3 \text{ ft})^2} - 1$$

$$\text{OR } \epsilon = 106.1$$

12.102

GIVEN: ELLIPTIC ORBIT OF A SATELLITE ABOUT A PLANET OF MASS M

DERIVE:  $\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$



$$\text{HAVE.. } \frac{1}{r} = \frac{GM}{h^2} (1 + \epsilon \cos \theta) \text{ EQ. (12.39')}$$

$$\text{NOW.. AT A: } r = r_0, \theta = 0; \therefore \frac{1}{r_0} = \frac{GM}{h^2} (1 + \epsilon) \quad (1)$$

$$\text{AT B: } r = r_1, \theta = 180^\circ; \therefore \frac{1}{r_1} = \frac{GM}{h^2} (1 - \epsilon) \quad (2)$$

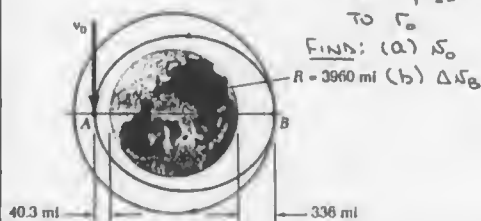
$$\text{THEN } (1) + (2) \Rightarrow \frac{1}{r_0} + \frac{1}{r_1} = \frac{GM}{h^2} [(1 + \epsilon) + (1 - \epsilon)]$$

$$\text{OR } \frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad \text{Q.E.D.}$$

## 12.103

GIVEN: ELLIPTIC AND CIRCULAR ORBITS OF THE SPACE SHUTTLE ABOUT THE EARTH;  $\vec{v}_0$  PERPENDICULAR TO  $\vec{r}_0$

FIND: (a)  $\vec{v}_0$   
(b)  $\Delta \vec{v}_B$



FIRST NOTE...  $r_A = (3960 + 40.3) \text{ mi} = 4000.3 \text{ mi}$   
 $= 21.1216 \times 10^6 \text{ ft}$

$r_B = (3960 + 336) \text{ mi} = 4296 \text{ mi}$   
 $= 22.6829 \times 10^6 \text{ ft}$

$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

(a) FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

NOW..  $h = h_A = r_A v_0$   $GM = gR^2$  (Eq. (12.30))

$$\text{THEN.. } \frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR^2}{(r_A v_0)^2}$$

$$\text{OR } v_0 = \frac{R}{r_A} \left( \frac{2g}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2}$$

$$= \frac{3960 \text{ mi}}{4000.3 \text{ mi}} \left( \frac{2 \times 32.2 \text{ ft/s}^2}{\frac{1}{21.1216 \times 10^6 \text{ ft}} + \frac{1}{22.6829 \times 10^6 \text{ ft}}} \right)^{1/2}$$

$$= 26,272 \frac{\text{ft}}{\text{s}}$$

$$\text{OR } v_0 = 26.3 \times 10^3 \frac{\text{ft}}{\text{s}}$$

(b) FOR THE ELLIPTIC ORBIT AB HAVE-

$$h = h_A = h_B: r_A v_0 = r_B (v_B)_{AB}$$

$$\text{THEN.. } (v_B)_{AB} = \frac{4000.3 \text{ mi}}{4296 \text{ mi}} \times 26,272 \frac{\text{ft}}{\text{s}}$$

$$= 24,464 \frac{\text{ft}}{\text{s}}$$

FOR THE CIRCULAR ORBIT, USE EQ. (12.44)

$$(v_B)_{\text{CIRC}} = \sqrt{\frac{gR^2}{r_B}} = 20.9088 \times 10^6 \text{ ft} \left( \frac{32.2 \text{ ft/s}^2}{22.6829 \times 10^6 \text{ ft}} \right)^{1/2}$$

$$= 24,912 \frac{\text{ft}}{\text{s}}$$

FINALLY..  $(v_B)_{\text{CIRC}} = (v_B)_{AB} + \Delta v_B$

$$\text{OR } \Delta v_B = (24,912 - 24,464) \frac{\text{ft}}{\text{s}}$$

$$\text{OR } \Delta v_B = 448 \frac{\text{ft}}{\text{s}}$$

## 12.104 continued

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

$$\text{NOW.. } h = h_A = r_A (v_A)_{AB}$$

$$= [R(1+\alpha)] (\beta v_0)$$

THEN..

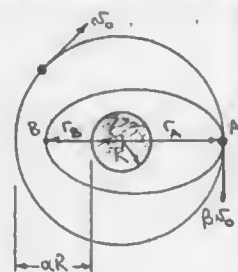
$$\frac{1}{R(1+\alpha)} + \frac{1}{r_B} = \frac{2v_0^2 R(1+\alpha)}{[R(1+\alpha)]^2 (\beta v_0)^2}$$

$$= \frac{2}{\beta^2 R(1+\alpha)}$$

NOW..  $\beta_{\text{MIN}}$  CORRESPONDS TO  $r_B \rightarrow R$ . THEN..

$$\frac{1}{R(1+\alpha)} + \frac{1}{R} = \frac{2}{\beta_{\text{MIN}}^2 R(1+\alpha)}$$

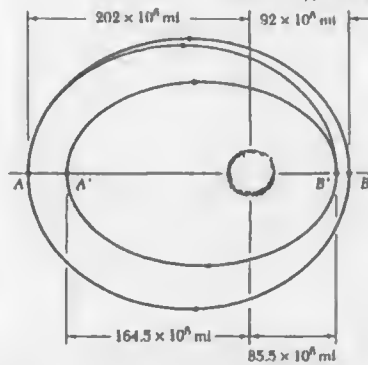
$$\text{OR } \beta_{\text{MIN}} = \sqrt{\frac{2}{2+\alpha}}$$



## 12.105

GIVEN: ELLIPTIC ORBITS AB AND A'B' OF A SPACECRAFT ABOUT THE SUN AND THE ELLIPTIC TRANSFER ORBIT AB';  $M_{\text{SUN}} = (332.8 \times 10^3) M_{\text{EARTH}}$

FIND: (a)  $\vec{v}_A$  (ON AB)  
(b)  $|\Delta \vec{v}_A|$  AND  $|\Delta \vec{v}_B|$



FIRST NOTE..  $R_{\text{EARTH}} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$$r_A = 202 \times 10^6 \text{ mi} = 1066.56 \times 10^9 \text{ ft}$$

$$r_B = 92 \times 10^6 \text{ mi} = 485.76 \times 10^9 \text{ ft}$$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT THE SUN..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{\text{SUN}}}{h^2}$$

(a) FOR THE ELLIPTIC ORBIT AB HAVE..

$$r_1 = r_A, r_2 = r_B, h = h_A = r_A v_A$$

$$\text{ALSO.. } GM_{\text{SUN}} = G(332.8 \times 10^3) M_{\text{EARTH}}$$

$$= gR_{\text{EARTH}}^2 (332.8 \times 10^3) \text{ USING EQ. (12.30)}$$

$$\text{THEN.. } \frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR_{\text{EARTH}}^2 (332.8 \times 10^3)}{(r_A v_A)^2}$$

$$\text{OR } v_A = \frac{R_{\text{EARTH}}}{r_A} \left( \frac{665.69 \times 10^3}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2}$$

$$= \frac{3960 \text{ mi}}{202 \times 10^6 \text{ mi}} \left( \frac{665.6 \times 10^3 \times 32.2 \text{ ft/s}^2}{\frac{1}{1066.56 \times 10^9 \text{ ft}} + \frac{1}{485.76 \times 10^9 \text{ ft}}} \right)^{1/2}$$

$$= 52,431 \frac{\text{ft}}{\text{s}}$$

$$\text{OR } v_A = 52.4 \times 10^3 \frac{\text{ft}}{\text{s}}$$

(CONTINUED)

## 12.104

GIVEN: A PLANET OF RADIUS  $R$  AND A SPACE PROBE IN A CIRCULAR ORBIT ABOUT THE PLANET AT AN ALTITUDE  $\alpha R$  AND HAVING A VELOCITY  $\vec{v}_0$ ; ELLIPTIC ORBIT, WHERE  $\vec{v} = \beta \vec{v}_0$ ,  $\beta < 1$

FIND:  $\beta_{\text{MIN}}$  SO THAT THE PROBE DOES NOT CRASH

FOR THE CIRCULAR ORBIT..  $\vec{v}_0 = \sqrt{\frac{GM}{r_A}}$  [Eq. (12.44)]

WHERE  $r_A = R + \alpha R = R(1+\alpha)$

THEN..  $GM = v_0^2 R(1+\alpha)$

(CONTINUED)

# 12.105 continued

(b) FROM PART (a) HAVE

$$2GM_{\text{SUN}} = (r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)$$

THEN, FOR ANY OTHER ELLIPTIC ORBIT ABOUT THE SUN HAVE --

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{(r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)}{h^2}$$

FOR THE ELLIPTIC TRANSFER ORBIT AB HAVE --

$$r_1 = r_A, r_2 = r_B, h = h_{\text{TR}} = r_A (v_A)_{\text{TR}}$$

$$\text{THEN -- } \frac{1}{r_A} + \frac{1}{r_B} = \frac{(r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)}{[r_A (v_A)_{\text{TR}}]^2}$$

$$\begin{aligned} \text{OR } (v_A)_{\text{TR}} &= v_A \left( \frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2} = v_A \left( \frac{1 + \frac{r_A}{r_B}}{1 + \frac{r_A}{r_B}} \right)^{1/2} \\ &= (52,431 \frac{\text{ft}}{\text{s}}) \left( \frac{1 + \frac{202}{92}}{1 + \frac{202}{85.5}} \right)^{1/2} \\ &= 51,113 \frac{\text{ft}}{\text{s}} \end{aligned}$$

$$\text{NOW -- } h_{\text{TR}} = (h_A)_{\text{TR}} = (h_B)_{\text{TR}} : r_A (v_A)_{\text{TR}} = r_B (v_B)_{\text{TR}}$$

$$\begin{aligned} \text{THEN } (v_B)_{\text{TR}} &= \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \times 51,113 \frac{\text{ft}}{\text{s}} \\ &= 120,758 \frac{\text{ft}}{\text{s}} \end{aligned}$$

FOR THE ELLIPTIC ORBIT A'B' HAVE --

$$r_1 = r_A', r_2 = r_B', h = r_B' (v_B')$$

$$\text{THEN -- } \frac{1}{r_A'} + \frac{1}{r_B'} = \frac{(r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)}{(r_B' v_B')^2}$$

$$\begin{aligned} \text{OR } v_B' &= v_A \frac{r_A}{r_B'} \left( \frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2} \\ &= (52,431 \frac{\text{ft}}{\text{s}}) \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \left( \frac{\frac{1}{202 \times 10^6} + \frac{1}{92 \times 10^6}}{\frac{1}{164.5 \times 10^6} + \frac{1}{85.5 \times 10^6}} \right)^{1/2} \\ &= 116,862 \frac{\text{ft}}{\text{s}} \end{aligned}$$

$$\text{FINALLY -- } (v_A)_{\text{TR}} = v_A + \Delta v_A$$

$$\text{OR } \Delta v_A = (51,113 - 52,431) \frac{\text{ft}}{\text{s}} \quad \text{OR } |\Delta v_A| = 1318 \frac{\text{ft}}{\text{s}}$$

AND

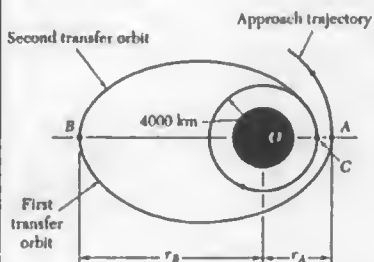
$$\begin{aligned} v_B' &= (v_B)_{\text{TR}} + \Delta v_B \\ \text{OR } \Delta v_B &= (116,862 - 120,758) \frac{\text{ft}}{\text{s}} \\ &= -3896 \frac{\text{ft}}{\text{s}} \end{aligned}$$

$$\text{OR } |\Delta v_B| = 3900 \frac{\text{ft}}{\text{s}}$$

# 12.106

GIVEN: PARABOLIC APPROACH TRAJECTORY, ELLIPTIC TRANSFER ORBITS AB AND BC, AND CIRCULAR ORBIT OF A SPACE PROBE ABOUT MARS;  
 $r_A = 9 \times 10^3 \text{ km}$ ,  $r_B = 180 \times 10^3 \text{ km}$ ;  
 $M_{\text{MARS}} = 0.1074 M_{\text{EARTH}}$

FIND: (a)  $|\Delta v_A|$   
 (b)  $|\Delta v_B|$   
 (c)  $|\Delta v_C|$



(CONTINUED)

# 12.106 continued

(a) FOR THE PARABOLIC APPROACH TRAJECTORY, POINT A IS THE POINT OF CLOSEST APPROACH. THEN, FROM PAGE 709 OF THE TEXT HAVE

$$(v_A)_{\text{PAR}} = \sqrt{\frac{2GM_{\text{MARS}}}{r_A}}$$

$$\begin{aligned} \text{NOW -- } GM_{\text{MARS}} &= G(0.1074 M_{\text{EARTH}}) \\ &= 0.1074 g_{\text{EARTH}}^2 \quad \text{USING EQ. (12.30)} \end{aligned}$$

$$\begin{aligned} \text{THEN -- } (v_A)_{\text{PAR}} &= \sqrt{r_{\text{EARTH}} \left( \frac{2 \times 0.1074 g}{r_A} \right)^{1/2}} \\ &= (6.37 \times 10^6 \text{ m}) \left( \frac{0.2148 \times 9.81 \text{ m/s}^2}{9 \times 10^3 \text{ m}} \right)^{1/2} \\ &= 3082.3 \frac{\text{m}}{\text{s}} \end{aligned}$$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT MARS --

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{\text{MARS}}}{h^2} \quad (1)$$

$$\text{FROM ABOVE -- } 2GM_{\text{MARS}} = r_A [(v_A)_{\text{PAR}}]^2$$

THEN -- FOR THE ELLIPTIC TRANSFER ORBIT AB --

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A [(v_A)_{\text{PAR}}]^2}{h_{\text{AB}}^2}$$

$$\text{WHERE } h_{\text{AB}} = (h_A)_{\text{AB}} = r_A (v_A)_{\text{AB}}$$

$$\text{THEN -- } \frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A [(v_A)_{\text{PAR}}]^2}{[r_A (v_A)_{\text{AB}}]^2}$$

$$\begin{aligned} \text{OR } (v_A)_{\text{AB}} &= \frac{(v_A)_{\text{PAR}}}{v_A} \left( \frac{1}{r_A} + \frac{1}{r_B} \right)^{1/2} = (v_A)_{\text{PAR}} \left( \frac{1}{1 + \frac{r_A}{r_B}} \right)^{1/2} \\ &= (3082.3 \frac{\text{m}}{\text{s}}) \left( \frac{1}{1 + \frac{9 \times 10^3 \text{ km}}{180 \times 10^3 \text{ km}}} \right)^{1/2} \\ &= 3008.0 \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\text{FINALLY -- } (v_A)_{\text{AB}} = (v_A)_{\text{PAR}} + \Delta v_A$$

$$\text{OR } \Delta v_A = (3008.0 - 3082.3) \frac{\text{m}}{\text{s}} \quad \text{OR } |\Delta v_A| = 74.3 \frac{\text{m}}{\text{s}}$$

(b) FOR THE ELLIPTIC TRANSFER ORBIT AB --

$$h_{\text{AB}} = (h_A)_{\text{AB}} = (h_B)_{\text{AB}} : r_A (v_A)_{\text{AB}} = r_B (v_B)_{\text{AB}}$$

$$\begin{aligned} \text{THEN -- } (v_B)_{\text{AB}} &= \frac{9 \times 10^3 \text{ km}}{180 \times 10^3 \text{ km}} \times 3008.0 \frac{\text{m}}{\text{s}} \\ &= 150.40 \frac{\text{m}}{\text{s}} \end{aligned}$$

NOW APPLY EQ. (1) TO THE SECOND ELLIPTIC TRANSFER ORBIT BC AND USE

$$h_{\text{BC}} = r_B (v_B)_{\text{BC}}$$

$$\text{THEN -- } \frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B [(v_B)_{\text{AB}}]^2}{[r_B (v_B)_{\text{BC}}]^2}$$

$$\begin{aligned} \text{OR } (v_B)_{\text{BC}} &= \frac{(v_B)_{\text{AB}}}{v_B} \left( \frac{1}{r_B} + \frac{1}{r_C} \right)^{1/2} \\ &= \frac{3082.3 \frac{\text{m}}{\text{s}}}{180 \times 10^3 \text{ km}} \left( \frac{1}{180 \times 10^3 \text{ km}} + \frac{1}{4 \times 10^3 \text{ km}} \right)^{1/2} \\ &= 101.62 \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\text{FINALLY -- } (v_B)_{\text{BC}} = (v_B)_{\text{AB}} + \Delta v_B$$

$$\text{OR } \Delta v_B = (101.62 - 150.40) \frac{\text{m}}{\text{s}} \quad \text{OR } |\Delta v_B| = 48.8 \frac{\text{m}}{\text{s}}$$

(c) FOR THE ELLIPTIC TRANSFER ORBIT BC --

$$h_{\text{BC}} = (h_B)_{\text{BC}} = (h_C)_{\text{BC}} : r_B (v_B)_{\text{BC}} = r_C (v_C)_{\text{BC}}$$

$$\begin{aligned} \text{THEN -- } (v_C)_{\text{BC}} &= \frac{180 \times 10^3 \text{ km}}{4 \times 10^3 \text{ km}} \times 101.62 \frac{\text{m}}{\text{s}} \\ &= 4572.9 \frac{\text{m}}{\text{s}} \end{aligned}$$

FOR THE CIRCULAR ORBIT HAVE --

$$(v_C)_{\text{CIRC}} = \sqrt{\frac{GM_{\text{MARS}}}{r_C}} \quad [\text{EQ. (12.44)}]$$

(CONTINUED)

## 12.106 continued

RECALLING FROM PART (a) THAT  $(v_A)_{par} = \sqrt{\frac{2GM_{MARS}}{r_A}}$   
HAVE

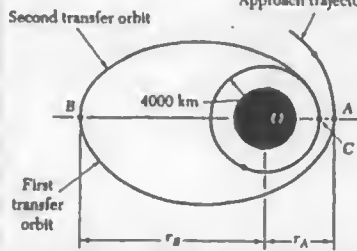
$$(v_c)_{circ} = (v_A)_{par} \left( \frac{r_A}{2r_c} \right)^{1/2} \\ = (3082.3 \frac{m}{s}) \left( \frac{9 \times 10^3 km}{2 \times 4 \times 10^3 km} \right)^{1/2} \\ = 3269.3 \frac{m}{s}$$

FINALLY ..  $(v_c)_{circ} = (v_c)_{ec} + \Delta v_c$   
OR  $\Delta v_c = (3269.3 - 4572.9) \frac{m}{s}$   
OR  $|\Delta v_c| = 1304 \frac{m}{s}$  ◀

## 12.107

GIVEN: PARABOLIC APPROACH TRAJECTORY, ELLIPTIC TRANSFER ORBITS AB AND BC, AND CIRCULAR ORBIT OF A SPACE PROBE ABOUT MARS;  
 $M_{MARS} = 0.1074 M_{EARTH}$ ;  $r_A = 9 \times 10^3 km$ ,  
Approach trajectory  $\Delta v_A = -440 \frac{m}{s}$

FIND: (a)  $r_B$   
(b)  $|\Delta v_B|$  AND  $|\Delta v_c|$



(a) FOR THE PARABOLIC APPROACH TRAJECTORY, POINT A IS THE POINT OF CLOSEST APPROACH. THEN, FROM PAGE 703 OF THE TEXT HAVE

$$(v_A)_{par} = \sqrt{\frac{2GM_{MARS}}{r_A}}$$

NOW ..  $GM_{MARS} = G(0.1074 M_{EARTH})$   
 $= 0.1074 g R_{EARTH}^2$  USING EQ.(12.30)

THEN ..  $(v_A)_{par} = R_{EARTH} \left( \frac{2 \times 0.1074 g}{r_A} \right)^{1/2}$   
 $= (6.37 \times 10^6 m) \left( \frac{0.2148 \times 9.81 m/s^2}{9 \times 10^6 m} \right)^{1/2}$   
 $= 3082.3 \frac{m}{s}$

NOW ..  $(v_A)_{AB} = (v_A)_{par} + \Delta v_A = (3082.3 - 440) \frac{m}{s}$   
 $= 2642.3 \frac{m}{s}$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT MARS..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{MARS}}{h^2} \quad (1)$$

FROM ABOVE ..  $2GM_{MARS} = r_A [(v_A)_{par}]^2$

THEN.. FOR THE ELLIPTIC TRANSFER ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A [(v_A)_{par}]^2}{h_{AB}^2}$$

WHERE  $h_{AB} = (h_A)_{AB} = r_A (v_A)_{AB}$

THEN ..  $\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A [(v_A)_{par}]^2}{[r_A (v_A)_{AB}]^2}$   
 $= \left[ \frac{(v_A)_{par}}{(v_A)_{AB}} \right]^2 \frac{1}{r_A}$

OR  $\frac{1}{r_B} = \frac{1}{r_A} \left\{ \left[ \frac{(v_A)_{par}}{(v_A)_{AB}} \right]^2 - 1 \right\} = \frac{1}{9 \times 10^3 km} \left[ \left( \frac{3082.3 \frac{m}{s}}{2642.3 \frac{m}{s}} \right)^2 - 1 \right]$

OR  $r_B = 24.946 \times 10^3 km$

OR  $r_B = 24.9 \times 10^3 km$  ◀

(b) FOR THE ELLIPTIC TRANSFER ORBIT AB..

$h_{AB} = (h_A)_{AB} = (h_B)_{AB}$ ;  $r_A (v_A)_{AB} = r_B (v_B)_{AB}$   
(CONTINUED)

## 12.107 continued

THEN ..  $(v_B)_{AB} = \frac{9 \times 10^3 km}{24.946 \times 10^3 km} \times 2642.3 \frac{m}{s}$   
 $= 953.3 \frac{m}{s}$

NOW APPLY EQ.(1) TO THE SECOND ELLIPTIC TRANSFER ORBIT BC AND USE

$$h_{BC} = r_B (v_B)_{BC}$$

THEN ..  $\frac{1}{r_B} + \frac{1}{r_c} = \frac{r_B [(v_B)_{par}]^2}{[r_B (v_B)_{BC}]^2}$

OR  $(v_B)_{BC} = \frac{(v_A)_{par}}{r_B} \left( \frac{r_A}{\frac{1}{r_B} + \frac{1}{r_c}} \right)^{1/2}$   
 $= \frac{3082.3 \frac{m}{s}}{24.946 \times 10^3 km} \left( \frac{9 \times 10^3 km}{\frac{1}{24.946 \times 10^3 km} + \frac{1}{9 \times 10^3 km}} \right)^{1/2}$   
 $= 688.2 \frac{m}{s}$

THEN ..  $(v_B)_{BC} = (v_B)_{AB} + \Delta v_B$

OR  $\Delta v_B = (688.2 - 953.3) \frac{m}{s}$   
OR  $|\Delta v_B| = 265 \frac{m}{s}$  ◀

NOW.. FOR THE ELLIPTIC TRANSFER ORBIT BC..

$$h_{BC} = (h_B)_{BC} = (h_c)_{BC} = r_B (v_B)_{BC} = r_c (v_c)_{BC}$$

THEN ..  $(v_c)_{BC} = \frac{24.946 \times 10^3 km}{4 \times 10^3 km} \times 688.2 \frac{m}{s}$   
 $= 4292.0 \frac{m}{s}$

FOR THE CIRCULAR ORBIT HAVE ..

$$(v_c)_{circ} = \sqrt{\frac{GM_{MARS}}{r_c}} \quad [\text{EQ. (12.41)}]$$

RECALLING FROM PART (a) THAT  $(v_A)_{par} = \sqrt{\frac{2GM_{MARS}}{r_A}}$   
HAVE

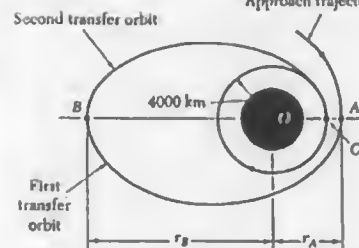
$$(v_c)_{circ} = (v_A)_{par} \left( \frac{r_A}{2r_c} \right)^{1/2} \\ = (3082.3 \frac{m}{s}) \left( \frac{9 \times 10^3 km}{2 \times 4 \times 10^3 km} \right)^{1/2} \\ = 3269.3 \frac{m}{s}$$

FINALLY ..  $(v_c)_{circ} = (v_c)_{BC} + \Delta v_c$   
OR  $\Delta v_c = (3269.3 - 4292.0) \frac{m}{s}$

OR  $|\Delta v_c| = 1023 \frac{m}{s}$  ◀

## 12.108

GIVEN: ELLIPTIC TRANSFER ORBIT AB OF PROBLEM 12.106;  $r_A = 9 \times 10^3 km$ ,  
 $r_B = 180 \times 10^3 km$   
Approach trajectory FIND:  $t_{AB}$



FROM THE SOLUTION TO PROBLEM 12.106 HAVE

$$(v_A)_{AB} = 3008.0 \frac{m}{s}$$

FROM EQ. (12.45) IT FOLLOWS THAT

$$t_{AB} = \frac{1}{2} (T_{\text{ELLIPSE}})_{AB} = \frac{\pi a_{AB}}{h_{AB}}$$

WHERE  $a = \frac{1}{2} (r_A + r_B) = \frac{1}{2} (9 \times 10^3 + 180 \times 10^3) = 94.5 \times 10^3 km$

AND  $b = \sqrt{r_A r_B} = \sqrt{(9 \times 10^3)(180 \times 10^3)} = 40.249 \times 10^3 km$

ALSO..  $h_{AB} = r_A (v_A)_{AB} = (9 \times 10^3 m) (3008.0 \frac{m}{s}) = 27.072 \times 10^9 \frac{m^2}{s}$

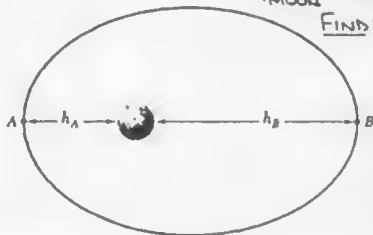
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# 12.108 continued

THEN..  $t_{AB} = \frac{\pi(94.5 \times 10^6 \text{ m})(40.249 \times 10^6 \text{ m})}{27.072 \times 10^9 \frac{\text{m}^2}{\text{s}}}$   
 $= 441.384 \times 10^3 \text{ s}$   
 OR  $t_{AB} = 122 \text{ h } 36 \text{ min } 24 \text{ s}$

# 12.109

GIVEN: ELLIPTIC ORBIT OF THE CLEMENTINE SPACECRAFT ABOUT THE MOON;  
 $h_A = 400 \text{ km}$ ,  $h_B = 2940 \text{ km}$ ;  
 $R_{\text{MOON}} = 1737 \text{ km}$ ,  
 $M_{\text{MOON}} = 0.01230 M_{\text{EARTH}}$   
 FIND: PERIODIC TIME  $\tau$



FIRST NOTE..

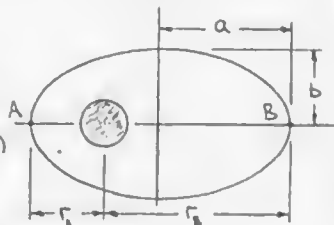
$$r_A = (1737 + 400) = 2137 \text{ km}$$

$$r_B = (1737 + 2940) = 4677 \text{ km}$$

NOW..  $\tau = \frac{2\pi ab}{h}$  EQ. (12.45)

WHERE  $a = \frac{1}{2}(r_A + r_B)$   
 $= \frac{1}{2}(2137 + 4677) \text{ km}$   
 $= 3407 \text{ km}$

AND  $b = \sqrt{r_A r_B}$



FROM THE SOLUTION TO PROBLEM 12.102 HAVE..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM_{\text{MOON}}}{h^2}$$

NOW..  $GM_{\text{MOON}} = G(0.01230 M_{\text{EARTH}})$   
 $= 0.01230 g R_{\text{EARTH}}^2$  USING EQ. (12.30)

THEN..  $h^2 = \frac{2(0.01230 g R_{\text{EARTH}}^2)}{\frac{1}{r_A} + \frac{1}{r_B}} = \frac{0.01230 g R_{\text{EARTH}}^2}{\frac{r_A + r_B}{r_A r_B}}$

$$= \frac{b^2}{2a} (2 \times 0.01230 g R_{\text{EARTH}}^2)$$

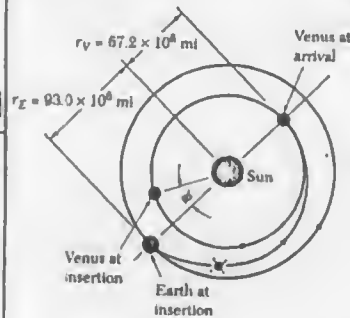
OR  $h = b R_{\text{EARTH}} \left( \frac{0.01230 g}{a} \right)^{1/2}$

THEN..  $\tau = \frac{2\pi ab}{b R_{\text{EARTH}} \left( \frac{0.01230 g}{a} \right)^{1/2}} = \frac{2\pi a^{3/2}}{R_{\text{EARTH}} (0.01230 g)^{1/2}}$   
 $= \frac{2\pi (3407 \times 10^3 \text{ m})^{3/2}}{(6.37 \times 10^6 \text{ m})(0.01230 \times 9.81 \text{ m/s}^2)^{1/2}}$   
 $= 17.8571 \times 10^3 \text{ s}$

OR  $\tau = 4 \text{ h } 57 \text{ min } 37 \text{ s}$

# 12.110

GIVEN: ORBITS OF VENUS AND THE EARTH AND THE ELLIPTIC TRANSFER ORBIT OF A SPACE PROBE;  
 $M_{\text{SUN}} = 332.8 M_{\text{EARTH}}$   
 FIND:  $\phi$ , THE RELATIVE POSITION OF VENUS WITH RESPECT TO THE EARTH AT THE TIME OF INSERTION



FIRST DETERMINE THE TIME  $t_{\text{PROBE}}$  FOR THE PROBE TO TRAVEL FROM THE EARTH TO VENUS. NOW..

$$t_{\text{PROBE}} = \frac{1}{2} \tau_{\text{TR}}$$

WHERE  $\tau_{\text{TR}}$  IS THE PERIODIC TIME OF THE ELLIPTIC TRANSFER ORBIT. APPLYING KEPLER'S THIRD LAW TO THE ORBITS ABOUT THE SUN OF THE EARTH AND THE PROBE OBTAIN..

$$\frac{\tau_{\text{TR}}^2}{\tau_{\text{EARTH}}^2} = \frac{a_{\text{TR}}^3}{a_{\text{EARTH}}^3}$$

WHERE  $a_{\text{TR}} = \frac{1}{2}(r_E + r_V) = \frac{1}{2}(93 \times 10^6 + 67.2 \times 10^6) \text{ mi}$   
 $= 80.1 \times 10^6 \text{ mi}$

AND  $a_{\text{EARTH}} \approx r_E$  (NOTE:  $e_{\text{EARTH}} = 0.0167$ )

THEN..  $t_{\text{PROBE}} = \frac{1}{2} \left( \frac{a_{\text{TR}}}{r_E} \right)^{3/2} \tau_{\text{EARTH}}$   
 $= \frac{1}{2} \left( \frac{80.1 \times 10^6 \text{ mi}}{93.0 \times 10^6 \text{ mi}} \right)^{3/2} (365.25 \text{ DAYS})$   
 $= 145.977 \text{ DAYS}$   
 $= 12.6124 \times 10^6 \text{ s}$

IN TIME  $t_{\text{PROBE}}$ , VENUS TRAVELS THROUGH THE ANGLE  $\theta_V$  GIVEN BY

$$\theta_V = \omega_V t_{\text{PROBE}} = \frac{v_V}{r_V} t_{\text{PROBE}}$$

ASSUMING THAT THE ORBIT OF VENUS IS CIRCULAR (NOTE:  $e_{\text{VENUS}} = 0.0068$ ), THEN, FOR A CIRCULAR ORBIT..

$$v_V = \sqrt{\frac{GM_{\text{SUN}}}{r_V}} \quad [\text{EQ. (12.44)}]$$

NOW..  $GM_{\text{SUN}} = G(332.8 \times 10^3 M_{\text{EARTH}})$   
 $= 332.8 \times 10^3 (g R_{\text{EARTH}}^2)$  USING EQ. (12.30)

THEN..  $\theta_V = \frac{t_{\text{PROBE}}}{r_V} \left[ \frac{332.8 \times 10^3 (g R_{\text{EARTH}}^2)}{r_V} \right]^{1/2}$   
 $= t_{\text{PROBE}} R_{\text{EARTH}} \frac{(332.8 \times 10^3)^{1/2}}{r_V^{1/2}}$

WHERE  $R_{\text{EARTH}} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$   
 AND  $r_V = 67.2 \times 10^6 \text{ mi} = 354.816 \times 10^9 \text{ ft}$

THEN..  $\theta_V = (12.6124 \times 10^6 \text{ s})(20.9088 \times 10^6 \text{ ft}) \frac{(332.8 \times 10^3 \times 32.2 \text{ ft/s}^2)^{1/2}}{(354.816 \times 10^9 \text{ ft})^{1/2}}$   
 $= 4.0845 \text{ RAD}$   
 $= 234.02^\circ$

FINALLY..  $\phi = \theta_V - 180^\circ$   
 $= 234.02^\circ - 180^\circ$

OR  $\phi = 54.0^\circ$

12.111

GIVEN: ELLIPTIC ORBIT ABOUT THE SUN OF THE COMET HYAKUTAKE;  $E = 0.999887$ ,  $r_{\min} = 0.230 R_E$ ;  $\dot{r}_E = a_{\text{EARTH}}$  FOR THE EARTH'S ORBIT ABOUT THE SUN

FIND:  $T$  FOR THE COMET

USING EQ. (12.39') HAVE FOR ANY ELLIPTIC ORBIT ABOUT THE SUN--

$$\frac{1}{r} = \frac{GM_{\text{SUN}}}{h^2} (1 + E \cos \theta)$$

AT A,  $\theta = 0$ :

$$\frac{1}{r_0} = \frac{GM_{\text{SUN}}}{h^2} (1 + E) \quad (1)$$

AT B,  $\theta = 180^\circ$ :  $\frac{1}{r_1} = \frac{GM_{\text{SUN}}}{h^2} (1 - E) \quad (2)$

$$\text{FORMING } \frac{(1)}{(2)} \Rightarrow \frac{\frac{1}{r_0}}{\frac{1}{r_1}} = \frac{1+E}{1-E} \quad \text{OR } r_1 = \frac{1+E}{1-E} r_0$$

$$\text{NOW.. } a = \frac{1}{2}(r_0 + r_1) = \frac{1}{2}(r_0 + \frac{1+E}{1-E} r_0) = \frac{r_0}{1-E}$$

APPLYING KEPLER'S THIRD LAW TO THE ORBITS ABOUT THE SUN OF THE EARTH AND THE COMET HAVE--

$$\frac{T_{\text{COMET}}^2}{T_{\text{EARTH}}^2} = \frac{a_{\text{COMET}}^3}{a_{\text{EARTH}}^3}$$

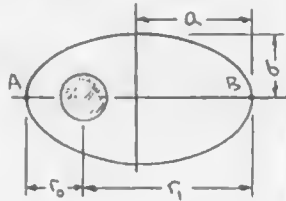
$$\text{FROM ABOVE.. } a_{\text{COMET}} = \frac{(r_0)_{\text{COMET}}}{1-E_{\text{COMET}}} = \frac{(r_{\min})_{\text{COMET}}}{1-E_{\text{COMET}}} = \frac{0.230 R_E}{1-E_{\text{COMET}}}$$

AND  $a_{\text{EARTH}} = R_E$

$$\text{THEN.. } \frac{T_{\text{COMET}}}{T_{\text{EARTH}}} = \left( \frac{\frac{0.230 R_E}{1-E_{\text{COMET}}}}{R_E} \right)^3 = \left( \frac{0.230}{1-E_{\text{COMET}}} \right)^3$$

$$\text{OR } T_{\text{COMET}} = \left( \frac{0.230}{1-0.999887} \right)^{3/2} (1 \text{ yr})$$

$$\text{OR } T_{\text{COMET}} = 91.8 \times 10^3 \text{ yr} \quad \blacktriangleleft$$

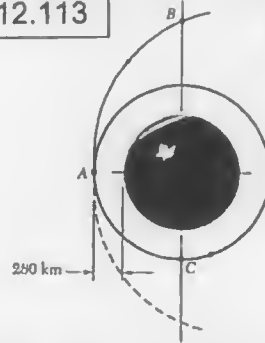


12.112 continued

$$\begin{aligned} \text{AREA SWEEPED OUT} &= A_{\text{BAC}} = \frac{2}{3}(r_A)(r_C) = \frac{2}{3}r_A^2 \\ \text{NOW.. } \frac{dA}{dt} &= \frac{1}{2}h, \text{ WHERE } h = \text{CONSTANT} \\ \text{THEN } A &= \frac{1}{2}ht \quad \text{OR } t_{\text{BC}} = \frac{2A_{\text{BAC}}}{h} \quad h = r_A v_A \\ &= \frac{2 \times \frac{2}{3}r_A^2}{r_A v_A} = \frac{4}{3} \frac{r_A}{v_A} \\ &= \frac{4}{3} \frac{7530 \text{ km}}{10.42 \text{ km/s}} \\ &= 3751.8 \text{ s} \end{aligned}$$

$$\text{OR } t_{\text{BC}} = 1 \text{ h } 2 \text{ min } 32 \text{ s} \quad \blacktriangleleft$$

12.113



GIVEN: PARABOLIC APPROACH TRAJECTORY AND CIRCULAR ORBIT ABOUT VENUS OF A SPACE PROBE;  $R = 6052 \text{ km}$ ,  $M_{\text{VENUS}} = 4.87 \times 10^{24} \text{ kg}$

FIND:  $t_{\text{BC}}$

FROM THE SOLUTION TO PROBLEM 12.99 HAVE..

$$(v_A)_{\text{PAR}} = 10131.4 \frac{\text{m}}{\text{s}}$$

$$\text{AND } (v_A)_{\text{CIRC}} = \frac{1}{2}(v_A)_{\text{PAR}} = 7164.0 \frac{\text{m}}{\text{s}}$$

$$\text{ALSO, } r_A = (6052 + 280) \text{ km} = 6332 \text{ km}$$

FOR THE PARABOLIC TRAJECTORY BA HAVE

$$\frac{1}{r} = \frac{GM_V}{h_{\text{BA}}^2} (1 + E \cos \theta) \quad [\text{Eq. (12.39')}]$$

WHERE  $E = 1$ . NOW--

$$\text{AT A, } \theta = 0: \frac{1}{r_A} = \frac{GM_V}{h_{\text{BA}}^2} (1 + 1) \quad \text{OR } r_A = \frac{h_{\text{BA}}^2}{2GM_V}$$

$$\text{AT B, } \theta = -90^\circ: \frac{1}{r_B} = \frac{GM_V}{h_{\text{BA}}^2} (1 + 0) \quad \text{OR } r_B = \frac{h_{\text{BA}}^2}{GM_V}$$

$$\therefore r_B = 2r_A$$

AS THE PROBE TRAVELS FROM B TO A, THE AREA SWEEPED OUT IS THE SEMIPARABOLIC AREA DEFINED BY VERTEX A AND POINT B. THUS,

$$(\text{AREA SWEEPED OUT})_{\text{BA}} = A_{\text{BA}} = \frac{2}{3}(r_A)(r_B) = \frac{4}{3}r_A^2$$

NOW..  $\frac{dA}{dt} = \frac{1}{2}h$ , WHERE  $h = \text{CONSTANT}$

$$\begin{aligned} \text{THEN } A &= \frac{1}{2}ht \quad \text{OR } t_{\text{BA}} = \frac{2A_{\text{BA}}}{h_{\text{BA}}} \quad h_{\text{BA}} = r_A v_A \\ &= \frac{2 \times \frac{4}{3}r_A^2}{r_A v_A} = \frac{8}{3} \frac{r_A}{v_A} \\ &= \frac{8}{3} \frac{6332 \times 10^3 \text{ m}}{10131.4 \text{ m/s}} \\ &= 1666.63 \text{ s} \end{aligned}$$

FOR THE CIRCULAR TRAJECTORY AC,

$$t_{\text{AC}} = \frac{\frac{1}{2}r_A}{(v_A)_{\text{CIRC}}} = \frac{\frac{1}{2} \times 6332 \times 10^3 \text{ m}}{7164.0 \text{ m/s}} = 1388.37 \text{ s}$$

$$\begin{aligned} \text{FINALLY.. } t_{\text{BC}} &= t_{\text{BA}} + t_{\text{AC}} \\ &= (1666.63 + 1388.37) \text{ s} \\ &= 3055.0 \text{ s} \end{aligned}$$

$$\text{OR } t_{\text{BC}} = 50 \text{ min } 55 \text{ s} \quad \blacktriangleleft$$

12.112

GIVEN: PARABOLIC TRAJECTORY OF THE GALILEO SPACECRAFT ABOUT THE EARTH;  $v_A = 10.42 \frac{\text{km}}{\text{s}}$

FIND:  $t_{\text{BC}}$



$$\text{HAVE.. } \frac{1}{r} = \frac{GM}{h^2} (1 + E \cos \theta) \quad \text{Eq. (12.39')}$$

FOR A PARABOLIC TRAJECTORY,  $E = 1$

$$\text{NOW.. AT A, } \theta = 0: \frac{1}{r_A} = \frac{GM}{h^2} (1 + 1) \quad \text{OR } r_A = \frac{h^2}{2GM}$$

$$\text{AT C, } \theta = 90^\circ: \frac{1}{r_C} = \frac{GM}{h^2} (1 + 0) \quad \text{OR } r_C = \frac{h^2}{GM}$$

$$\therefore r_C = 2r_A$$

AS THE SPACECRAFT TRAVELS FROM B TO C, THE AREA SWEEPED OUT IS THE PARABOLIC AREA BAC. THUS,

(CONTINUED)

12.114

GIVEN: CIRCULAR ORBIT OF RADIUS  $nR$  OF A SPACE PROBE HAVING VELOCITY  $v_0$  ABOUT A PLANET OF RADIUS  $R$ ; AT POINT A, VELOCITY IS REDUCED TO  $\beta v_0$  ( $\beta < 1$ ) SO THAT PROBE IMPACTS AT POINT B

FIND:  $\angle AOB$  IN TERMS OF  $n$  AND  $\beta$

HAVE FOR THE CIRCULAR ORBIT

$$v_0 = \sqrt{\frac{GM}{nR}} \quad [\text{Eq. (12.44)}]$$

$$\text{OR } GM = nR v_0^2$$

FOR THE ELLIPTIC ORBIT ABC

$$\frac{1}{r} = \frac{GM}{h^2} (1 + \epsilon \cos \theta) \quad [\text{Eq. (12.39)}]$$

WHERE  $h_{ABC} = (h_A)_{ABC} = r_A (v_A)_{ABC} = (nR)(\beta v_0)$

$$\text{THEN } \frac{1}{r} = \frac{nR v_0^2}{(nR \beta v_0)^2} (1 + \epsilon \cos \theta) = \frac{1}{nR \beta^2} (1 + \epsilon \cos \theta)$$

NOTING THAT POINT C IS THE PERIGEE OF THE ELLIPTIC IMPACT TRAJECTORY SO THAT ANGLE  $\theta$  IS DEFINED AS SHOWN, HAVE..

$$\text{AT A, } \theta = 180^\circ: \frac{1}{nR} = \frac{1}{nR \beta^2} (1 - \epsilon) \quad \text{OR } \epsilon = 1 - \beta^2$$

$$\text{AT B: } \frac{1}{R} = \frac{1}{nR \beta^2} (1 + \epsilon \cos \theta) = \frac{1}{nR \beta^2} [1 + (1 - \beta^2) \cos \theta]$$

$$\text{OR } \cos \theta = \frac{n\beta^2 - 1}{1 - \beta^2}$$

NOW..  $\angle AOB = 180^\circ - \theta$

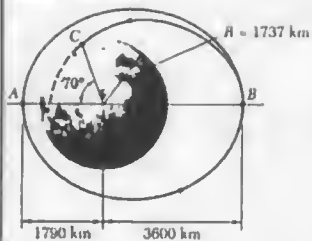
$$\text{SO THAT } \cos(180^\circ - \angle AOB) = \frac{n\beta^2 - 1}{1 - \beta^2}$$

$$\text{OR } -\cos(\angle AOB) = \frac{n\beta^2 - 1}{1 - \beta^2}$$

$$\text{OR } \angle AOB = \cos^{-1} \frac{1 - n\beta^2}{1 - \beta^2}$$

12.115

GIVEN: ELLIPTIC ORBIT AND ELLIPTIC IMPACT TRAJECTORY OF LUNAR ORBITER Z;  
 $M_{\text{MOON}} = 0.01230 M_{\text{EARTH}}$   
 FIND:  $|\Delta v_B|$  FOR IMPACT AT POINT C



FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM_{\text{MOON}}}{h_{AB}^2}$$

$$\text{WHERE } h_{AB} = (h_B)_{AB} = r_B (v_B)_{AB}$$

$$\text{AND } GM_{\text{MOON}} = G(0.01230 M_{\text{EARTH}}) = 0.01230 g R_{\text{EARTH}}^2 \quad \text{USING EQ. (12.30)}$$

$$\text{THEN } \frac{1}{r_A} + \frac{1}{r_B} = \frac{2(0.01230 g R_{\text{EARTH}}^2)}{[r_B (v_B)_{AB}]^2} \quad (\text{CONTINUED})$$

12.115 continued

$$\text{OR } (v_B)_{AB} = \frac{R_{\text{EARTH}}}{r_B} \left( \frac{0.0246 g}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2} = \frac{6.37 \times 10^6 \text{ m}}{3600 \times 10^3 \text{ m}} \left( \frac{0.0246 \times 9.81 \text{ m/s}^2}{\frac{1}{1790 \times 10^3 \text{ m}} + \frac{1}{3600 \times 10^3 \text{ m}}} \right)^{1/2} = 950.43 \text{ m/s}$$

FOR THE ELLIPTIC IMPACT TRAJECTORY HAVE..

$$\frac{1}{r} = \frac{GM_{\text{MOON}}}{h_{BC}^2} + \epsilon \cos \theta \quad [\text{Eq. (12.39)}]$$

$$\text{WHERE } h_{BC} = (h_B)_{BC} = r_B (v_B)_{BC}$$

NOTING THAT POINT B IS THE APOGEE OF THIS TRAJECTORY, HAVE

$$\text{AT B, } \theta = 180^\circ: \frac{1}{r_B} = \frac{GM_{\text{MOON}}}{h_{BC}^2} - \epsilon$$

$$\text{OR } \epsilon = \frac{GM_{\text{MOON}}}{h_{BC}^2} - \frac{1}{r_B}$$

$$\text{AT C, } \theta = -70^\circ: \frac{1}{r} = \frac{GM_{\text{MOON}}}{h_{BC}^2} + \epsilon \cos(-70^\circ)$$

$$\text{OR } \epsilon = \frac{1}{\cos 70^\circ} \left( \frac{1}{r} - \frac{GM_{\text{MOON}}}{h_{BC}^2} \right)$$

$$\text{THEN } \frac{GM_{\text{MOON}}}{h_{BC}^2} - \frac{1}{r_B} = \frac{1}{\cos 70^\circ} \left( \frac{1}{r} - \frac{GM_{\text{MOON}}}{h_{BC}^2} \right)$$

$$\text{OR } h_{BC}^2 = \frac{GM_{\text{MOON}} (1 + \cos 70^\circ)}{\frac{1}{r} + \frac{\cos 70^\circ}{r_B}}$$

$$\text{OR } (v_B)_{BC} = \frac{R_{\text{EARTH}}}{r_B} \left[ \frac{0.01230 g (1 + \cos 70^\circ)}{\frac{1}{r} + \frac{\cos 70^\circ}{r_B}} \right]^{1/2}$$

$$(v_B)_{BC} = \frac{6.37 \times 10^6 \text{ m}}{3600 \times 10^3 \text{ m}} \left[ \frac{0.01230 (9.81 \text{ m/s}^2) (1 + \cos 70^\circ)}{\frac{1}{1737 \times 10^3 \text{ m}} + \frac{\cos 70^\circ}{3600 \times 10^3 \text{ m}}} \right]^{1/2} = 869.43 \text{ m/s}$$

$$\text{FINALLY.. } (v_B)_{BC} = (v_B)_{AB} + \Delta v_B$$

$$\text{OR } \Delta v_B = (869.43 - 950.43) \text{ m/s}$$

$$\text{OR } |\Delta v_B| = 81.0 \text{ m/s}$$

12.116

GIVEN: HYPERBOLIC TRAJECTORY OF A PROBE,  $\epsilon = 1.031$ ; ALTITUDE AT B = 450 km,  $v_B = 82.9^\circ$ ; FOR JUPITER  $R = 71.492 \times 10^3 \text{ km}$ ,  $M = 1.9 \times 10^{27} \text{ kg}$

FIND: (a)  $\angle AOB$   
 (b)  $v_B$



FIRST NOTE..  $r_B = (71.492 \times 10^3 + 450) \text{ km} = 71.942 \times 10^3 \text{ km}$

$$(a) \text{ HAVE.. } \frac{1}{r} = \frac{GM_J}{h^2} (1 + \epsilon \cos \theta) \quad [\text{Eq. (12.39)}]$$

$$\text{AT A, } \theta = 0: \frac{1}{r_A} = \frac{GM_J}{h^2} (1 + \epsilon)$$

$$\text{OR } \frac{h^2}{GM_J} = r_A (1 + \epsilon)$$

$$\text{AT B, } \theta = \theta_B = \angle AOB: \frac{1}{r_B} = \frac{GM_J}{h^2} (1 + \epsilon \cos \theta_B)$$

$$\text{OR } \frac{h^2}{GM_J} = r_B (1 + \epsilon \cos \theta_B)$$

$$\text{THEN.. } r_A (1 + \epsilon) = r_B (1 + \epsilon \cos \theta_B)$$

$$\text{OR } \cos \theta_B = \frac{1}{\epsilon} \left[ \frac{r_A}{r_B} (1 + \epsilon) - 1 \right]$$

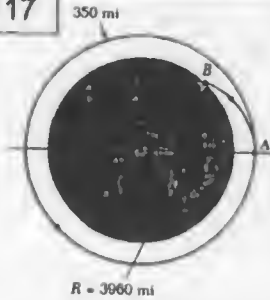
$$= \frac{1}{1.031} \left[ \frac{70.8 \times 10^3 \text{ km}}{71.942 \times 10^3 \text{ km}} (1 + 1.031) - 1 \right] = 0.96873 \quad (\text{CONTINUED})$$

# 12.116 continued

OR  $\theta_B = 14.3661^\circ$   
 (b) FROM ABOVE --  $h^2 = GM_1 r_A (1 + e \cos \theta_B)$   
 WHERE  $h = \frac{1}{m} |\mathbf{r}_B \times m \mathbf{v}_B| = r_B v_B \sin \phi$   
 $\phi = (\theta_B + 82.9^\circ) = 97.2661^\circ$

THEN --  
 $(r_B v_B \sin \phi)^2 = GM_1 r_B (1 + e \cos \theta_B)$   
 OR  $v_B = \frac{1}{\sin \phi} \left[ \frac{GM_1}{r_B} (1 + e \cos \theta_B) \right]^{1/2}$   
 $= \frac{1}{\sin 97.2661^\circ} \left\{ \frac{66.73 \times 10^{-12} \frac{m^3}{kg \cdot s^2} \cdot 1.9 \times 10^{17} kg}{71.942 \times 10^6 m} \right\}^{1/2}$   
 $= [1 + (1.031)(0.96873)]^{1/2}$   
 OR  $v_B = 59.8 \frac{km}{s}$

# 12.117



GIVEN: CIRCULAR ORBIT  
 AND THE ELLIPTIC  
 DESCENT TRAJECTORY  
 OF A SPACE  
 SHUTTLE;  
 $\Delta v_A = -500 \frac{ft}{s}$ ;  
 ALTITUDE AT B  
 $= 75 \text{ mi}$   
 FIND:  $\Delta AOB$

FIRST NOTE..  $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$   
 $r_A = (3960 + 350) \text{ mi} = 4310 \text{ mi}$   
 $= 22.7568 \times 10^6 \text{ ft}$   
 $r_B = (3960 + 75) \text{ mi} = 4035 \text{ mi}$

FOR THE CIRCULAR ORBIT HAVE

$$v_{circ} = \sqrt{\frac{gR^2}{r_A}} \quad [\text{Eq. (12.44)}]$$

$$= 20.9088 \times 10^6 \text{ ft} \left( \frac{32.2 \frac{ft}{s^2}}{22.7568 \times 10^6 \text{ ft}} \right)^{1/2}$$

$$= 24,871 \frac{ft}{s}$$

NOW..  $(v_A)_B = v_{circ} + \Delta v_A = (24,871 - 500) \frac{ft}{s}$   
 $= 24,371 \frac{ft}{s}$

FOR THE ELLIPTIC TRAJECTORY HAVE..

$$\frac{1}{r} = \frac{GM}{h^2} + C \cos \theta \quad [\text{Eq. (12.39)}]$$

NOTING THAT POINT A IS AT THE APOGEE OF  
 THIS TRAJECTORY, HAVE..

AT A,  $\theta = 180^\circ$ :  $\frac{1}{r_A} = \frac{GM}{h^2} - C$   
 OR  $C = \frac{GM}{h^2} - \frac{1}{r_A}$

AT B,  $\theta = \theta_B = 180^\circ - \Delta AOB$ :  $\frac{1}{r_B} = \frac{GM}{h^2} + C \cos \theta_B$   
 OR  $C = \frac{1}{\cos \theta_B} \left( \frac{1}{r_B} - \frac{GM}{h^2} \right)$

THEN..  $\frac{GM}{h^2} - \frac{1}{r_A} = \frac{1}{\cos \theta_B} \left( \frac{1}{r_B} - \frac{GM}{h^2} \right)$

OR  $\cos \theta_B = \frac{\frac{1}{r_B} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{r_A}}$

NOW..  $h = (h_A)_{AB} = r_A (v_A)_{AB}$   
 AND  $GM = gR^2$  Eq. (12.30)

FROM ABOVE --  $gR^2 = r_A (v_{circ})^2$  [Eq. (12.44)]

THEN..  $\frac{GM}{h^2} = \frac{r_A (v_{circ})^2}{[r_A (v_A)_{AB}]^2} = \frac{1}{r_A} \left[ \frac{v_{circ}}{(v_A)_{AB}} \right]^2$

(CONTINUED)

# 12.117 continued

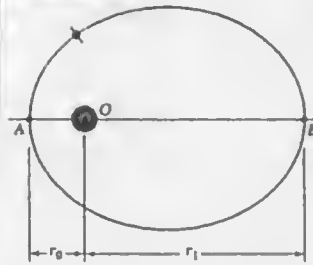
SO THAT  $\cos \theta_B = \frac{\frac{1}{r_B} - \frac{1}{r_A} \left[ \frac{v_{circ}}{(v_A)_{AB}} \right]^2}{\frac{1}{r_A} \left[ \frac{v_{circ}}{(v_A)_{AB}} \right]^2 - \frac{1}{r_A}} = \frac{\frac{1}{r_B} - \left[ \frac{v_{circ}}{(v_A)_{AB}} \right]^2}{\left[ \frac{v_{circ}}{(v_A)_{AB}} \right]^2 - 1}$   
 $= \frac{\frac{4310 \text{ mi}}{4035 \text{ mi}} - \left( \frac{24,871 \frac{ft}{s}}{24,371 \frac{ft}{s}} \right)^2}{\left( \frac{24,871 \frac{ft}{s}}{24,371 \frac{ft}{s}} \right)^2 - 1}$   
 $= 0.64411$

OR  $\theta_B = 49.901^\circ$

FINALLY..  $\Delta AOB = 180^\circ - 49.901^\circ$

OR  $\Delta AOB = 130.1^\circ$

# 12.118



GIVEN: ELLIPTIC ORBIT OF A  
 SATELLITE AS SHOWN

SHOW:  $\frac{1}{p} = \frac{1}{2} \left( \frac{1}{r_0} + \frac{1}{r_1} \right)$

WHERE  $p = r_A = r_B$

FROM THE SOLUTION TO PROBLEM 12.10Z HAVE..

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad \text{WHERE } h = h_A = r_0 v_A$$

CONSIDER THE SATELLITE AT POINT A..

$$\sum \mathbf{F}_n = m(\mathbf{a}_n)_n \Rightarrow \Sigma F_n = m a_n: F_A = m \frac{v_A^2}{p}$$

NOW..  $F_A = G \frac{Mm}{r_0^2}$  [Eq. (12.2B)]

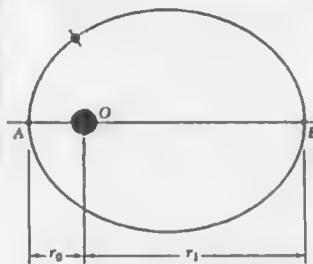
THEN..  $m \frac{v_A^2}{p} = G \frac{Mm}{r_0^2}$

OR  $GM = \frac{p}{r_0^2} (v_A^2 r_0^2) = \frac{p}{r_0^2} h^2$

FINALLY..  $\frac{1}{r_0} + \frac{1}{r_1} = \frac{2 \left( \frac{p}{h^2} h^2 \right)}{h^2}$

OR  $\frac{1}{p} = \frac{1}{2} \left( \frac{1}{r_0} + \frac{1}{r_1} \right)$  Q.E.D.

# 12.119



GIVEN: ELLIPTIC ORBIT OF A  
 SATELLITE AS SHOWN;  
 FOR COMET  
 HYAKUTAKE,  $r_0 = 0.230 R_E$

$e = 0.999887$

$R_E = 149.6 \times 10^6 \text{ km}$

FIND: (a)  $e$  IN TERMS OF

$r_0$  AND  $r_1$

(b)  $r_1$  FOR COMET

HYAKUTAKE

(a) HAVE..  $\frac{1}{r} = \frac{GM}{h^2} (1 + e \cos \theta)$  Eq. (12.39')

AT A,  $\theta = 0$ :  $\frac{1}{r_0} = \frac{GM}{h^2} (1 + e)$  OR  $\frac{h^2}{GM} = r_0 (1 + e)$

AT B,  $\theta = 180^\circ$ :  $\frac{1}{r_1} = \frac{GM}{h^2} (1 - e)$  OR  $\frac{h^2}{GM} = r_1 (1 - e)$

THEN..  $r_0 (1 + e) = r_1 (1 - e)$

OR  $e = \frac{r_1 - r_0}{r_1 + r_0}$

(b) FROM ABOVE..  $r_1 = \frac{1 + e}{1 - e} r_0$

WHERE  $r_0 = 0.230 R_E$

(CONTINUED)



### 12.119 continued

THEN..  $r_1 = \frac{1+0.999887}{1-0.999887} \cdot 0.230(149.6 \times 10^9 \text{ m})$

OR  $r_1 = 609 \times 10^{12} \text{ m}$

NOTE:  $r_1 \approx 4070 R_E$  OR  $r_1 \approx 0.064$  LIGHT YEARS

### 12.120

GIVEN: ELLIPTIC ORBIT OF SEMIMAJOR AXIS  $a$  AND ECCENTRICITY  $e$  OF A SATELLITE ABOUT A PLANET OF MASS  $M$

SHOW:  $h = \sqrt{GMa(1-e^2)}$

HAVE..  $\frac{1}{r} = \frac{GM}{h^2}(1+e \cos \theta)$  EQ. (12.39)

AT A,  $\theta = 0$ :  $\frac{1}{r_0} = \frac{GM}{h^2}(1+e)$

OR  $r_0 = \frac{h^2}{GM(1+e)}$

AT B,  $\theta = 180^\circ$ :  $\frac{1}{r_1} = \frac{GM}{h^2}(1-e)$

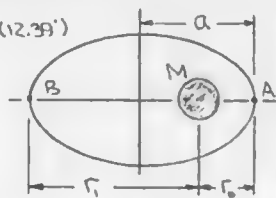
OR  $r_1 = \frac{h^2}{GM(1-e)}$

THEN..  $r_0 + r_1 = \frac{h^2}{GM} \frac{1}{1+e} + \frac{h^2}{GM} \frac{1}{1-e} = \frac{h^2}{GM} \frac{2}{1-e^2}$

NOW..  $a = \frac{1}{2}(r_0 + r_1)$

SO THAT  $2a = \frac{h^2}{GM} \frac{2}{1-e^2}$

OR  $h = \sqrt{GMa(1-e^2)}$  Q.E.D.



### 12.121

GIVEN: TWO ELLIPTIC ORBITS OF SEMIMAJOR AXES  $a_1$  AND  $a_2$  ABOUT A BODY OF MASS  $M$ ; PERIODIC TIMES  $T_1$  AND  $T_2$  OF TWO SATELLITES IN THE ELLIPTIC ORBITS

DERIVE: KEPLER'S THIRD LAW ( $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$ ) USING EQS. (12.39) AND (12.45)

CONSIDER THE ELLIPTIC ORBIT OF SATELLITE 1. NOW

$\frac{1}{r} = \frac{GM}{h^2}(1+e \cos \theta)$  EQ. (12.39)

THEN, FOR ORBIT 1..

AT A,  $\theta = 0$ :  $\frac{1}{(r_A)_1} = \frac{GM}{h_1^2}(1+e_1)$

AT B,  $\theta = 180^\circ$ :  $\frac{1}{(r_B)_1} = \frac{GM}{h_1^2}(1-e_1)$

THEN..  $\frac{1}{(r_A)_1} + \frac{1}{(r_B)_1} = \left(\frac{GM}{h_1^2}(1+e_1)\right) + \left(\frac{GM}{h_1^2}(1-e_1)\right)$

OR  $\frac{(r_A)_1 + (r_B)_1}{(r_A)_1(r_B)_1} = \frac{2GM}{h_1^2}$

NOW  $a_1 = \frac{1}{2}[(r_A)_1 + (r_B)_1]$   $b_1 = \sqrt{(r_A)_1(r_B)_1}$

THEN..  $\frac{2a_1}{b_1^2} = \frac{2GM}{h_1^2}$

OR  $h_1 = b_1 \sqrt{\frac{GM}{a_1}}$

NOW..  $T = \frac{2\pi ab}{h}$  EQ. (12.45)

FOR ORBIT 1..  $T_1 = \frac{2\pi a_1 b_1}{b_1 \sqrt{\frac{GM}{a_1}}} = \frac{2\pi}{\sqrt{GM}} a_1^{3/2}$

(CONTINUED)

### 12.121 continued

SIMILARLY, FOR THE ORBIT OF SATELLITE 2..

$T_2 = \frac{2\pi}{\sqrt{GM}} a_2^{3/2}$

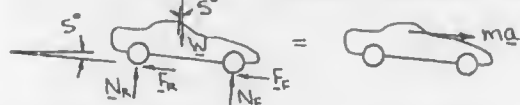
THEN..  $\frac{T_1}{T_2} = \frac{\frac{2\pi}{\sqrt{GM}} a_1^{3/2}}{\frac{2\pi}{\sqrt{GM}} a_2^{3/2}}$

OR  $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$  Q.E.D.

### 12.122

GIVEN: AUTOMOBILE OF WEIGHT 3000 lb MOVING DOWN A  $5^\circ$  INCLINE;  $v_0 = 50 \frac{\text{mi}}{\text{h}}$ ; AT  $t=0$ ,  $F_{\text{BRAKE}} = 1200 \text{ lb}$  IS APPLIED

FIND:  $x$  WHEN  $v=0$



HAVE..  $\sum F_x = ma$ :  $W \sin 5^\circ - (F_f + F_b) = \frac{W}{g} a$

WHERE  $F_f + F_b = F_{\text{BRAKE}}$

THEN..  $a = (32.2 \frac{\text{ft}}{\text{s}^2})(\sin 5^\circ - \frac{1200 \text{ lb}}{3000 \text{ lb}}) = -10.0736 \frac{\text{ft}}{\text{s}^2}$

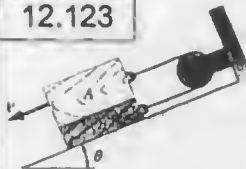
FOR THIS UNIFORMLY DECELERATED MOTION HAVE..

$v^2 = v_0^2 + 2a(x - x_0)$

WHERE  $v_0 = 50 \frac{\text{mi}}{\text{h}} = 73.333 \frac{\text{ft}}{\text{s}}$

THEN WHEN  $v=0$ ..  $0 = (73.333 \frac{\text{ft}}{\text{s}})^2 + 2(-10.0736 \frac{\text{ft}}{\text{s}^2})x$  OR  $x = 267 \text{ ft}$

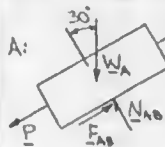
### 12.123



GIVEN:  $m_A = 30 \text{ kg}$ ,  $m_B = 15 \text{ kg}$ ;  $\mu_s = 0.15$ ,  $\mu_k = 0.10$ ;  $\theta = 30^\circ$ ,  $P = 250 \text{ N}$

FIND: (a)  $a$  (b)  $T$

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF  $P$ . THUS, SEEK THE VALUE OF  $P$  FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.

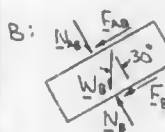


$\sum F_y = 0$ :  $N_{AB} - W_A \cos 30^\circ = 0$  OR  $N_{AB} = m_A g \cos 30^\circ$

NOW..  $F_{AB} = \mu_s N_{AB} = 0.15 m_A g \cos 30^\circ$

$\sum F_x = 0$ :  $T - P + F_{AB} - W_A \sin 30^\circ = 0$  OR  $T = P + m_A g (\sin 30^\circ - 0.15 \cos 30^\circ)$

SUBSTITUTING..  $T = P + (30 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(\sin 30^\circ - 0.15 \cos 30^\circ) = (P + 108.919) \text{ N}$



$\sum F_y = 0$ :  $N_B - N_{AB} - W_B \cos 30^\circ = 0$  OR  $N_B = g \cos 30^\circ (m_A + m_B)$

NOW..  $F_B = \mu_s N_B = 0.15 g \cos 30^\circ (m_A + m_B)$

$\sum F_x = 0$ :  $T - F_{AB} - F_B - W_B \sin 30^\circ = 0$

OR  $T = m_B g \sin 30^\circ + 0.15 m_A g \cos 30^\circ + 0.15 g \cos 30^\circ (m_A + m_B)$

(CONTINUED)

# 12.123 continued

$$\begin{aligned} \text{OR } T &= g [m_B \sin 30^\circ + 0.15 (2m_A + m_B) \cos 30^\circ] \\ &= (9.81 \frac{\text{m}}{\text{s}^2}) [(15 \text{ kg}) \sin 30^\circ + 0.15 (2 \cdot 30 + 15) \text{ kg} \cdot \cos 30^\circ] \\ &= 169.152 \text{ N} \end{aligned}$$

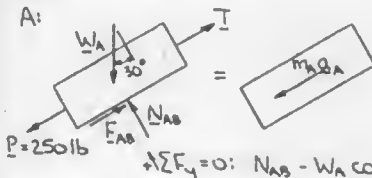
$$\text{THEN -- } 169.152 \text{ N} = (P + 108.919) \text{ N}$$

OR  $P = 60.2 \text{ N}$  FOR IMPENDING MOTION OF A DOWNWARD. SINCE  $P < 250 \text{ N}$ , THE BLOCKS WILL MOVE, WITH A MOVING DOWNWARD.

NOW CONSIDER THE MOTION OF THE BLOCKS.

(a)

A:



$$\sum F_y = 0: N_{AB} - W_A \cos 30^\circ = 0$$

$$\text{OR } N_{AB} = m_A g \cos 30^\circ$$

$$\text{SLIDING: } F_{AB} = \mu_k N_{AB}$$

$$= 0.1 m_A g \cos 30^\circ$$

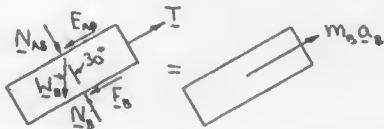
$$\sum F_x = m_A a_A: T + P - F_{AB} + W_A \sin 30^\circ = m_A a_A$$

$$\text{OR } T = P + m_A g (\sin 30^\circ - 0.1 \cos 30^\circ) - m_A a_A$$

SUBSTITUTING...

$$\begin{aligned} T &= 250 \text{ N} + (30 \text{ kg}) [(9.81 \frac{\text{m}}{\text{s}^2}) \sin 30^\circ - 0.1 \cos 30^\circ] - a_A \\ &= (371.663 - 30 a_A) \text{ N} \quad (1) \end{aligned}$$

B:



$$\sum F_y = 0: N_B - N_{AB} - W_B \cos 30^\circ = 0$$

$$\text{OR } N_B = g \cos 30^\circ (m_A + m_B)$$

$$\text{SLIDING: } F_B = \mu_k N_B$$

$$= 0.1 g \cos 30^\circ (m_A + m_B)$$

$$\sum F_x = m_B a_B: T - F_{AB} - F_B - W_B \sin 30^\circ = m_B a_B$$

$$\text{OR } T = m_B g \sin 30^\circ + 0.1 m_A g \cos 30^\circ + 0.1 g \cos 30^\circ (m_A + m_B) + m_B a_B$$

$$\begin{aligned} &= g [m_B \sin 30^\circ + 0.1 (2m_A + m_B) \cos 30^\circ] + m_B a_B \\ &= (9.81 \frac{\text{m}}{\text{s}^2}) [(15 \text{ kg}) \sin 30^\circ + 0.1 (2 \cdot 30 + 15) \text{ kg} \cdot \cos 30^\circ] \\ &\quad + (15 \text{ kg}) a_B \end{aligned}$$

$$= (137.293 + 15 a_B) \text{ N} \quad (2)$$

EQUATING THE EXPRESSIONS FOR T [Eqs. (1) AND

(2)] AND NOTING THAT  $a_A = a_B$  --

$$371.663 - 30 a_A = 137.293 + 15 a_A$$

$$\text{OR } a_A = 5.2082 \frac{\text{m}}{\text{s}^2}$$

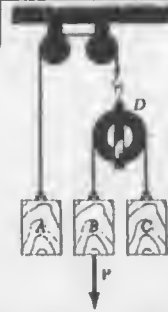
$$\therefore a_A = 5.21 \frac{\text{m}}{\text{s}^2} \nearrow 30^\circ$$

(b) SUBSTITUTING INTO Eq. (1) --

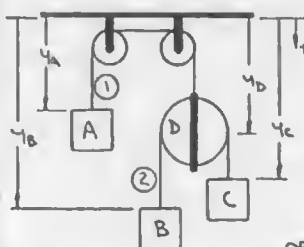
$$T = (371.663 - 30 \cdot 5.2082) \text{ N}$$

$$\text{OR } T = 215 \text{ N}$$

# 12.124



GIVEN:  $W_A = 20 \text{ lb}$ ,  
 $W_B = W_C = 10 \text{ lb}$ ;  
 AT  $t = 0$ ,  $v = 0$ ; AT  
 $t = 2 \text{ s}$ ,  $\Delta y_B = 8 \text{ ft}$ !  
 FIND: (a) P  
 (b)  $T_{AB}$



FROM THE DIAGRAM..

CORP 1:  $y_A + y_B = \text{CONSTANT}$

THEN..  $\Delta y_A + \Delta y_B = 0$

AND  $a_A + a_B = 0$

CORP 2:  $(y_B - y_D) + (y_C - y_D) = \text{CONSTANT}$

THEN..  $\Delta y_B + \Delta y_C - 2 \Delta y_D = 0$

AND  $a_B + a_C - 2 a_D = 0$

OR..  $2 a_A + a_B + a_C = 0 \quad (1)$

NOW.. HAVE UNIFORMLY ACCELERATED MOTION BECAUSE ALL OF THE FORCES ARE CONSTANT. THEN..

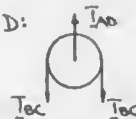
$$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$\text{AT } t = 2 \text{ s}, \Delta y_B = 8 \text{ ft: } 8 \text{ ft} = \frac{1}{2} a_B (2 \text{ s})^2$$

$$\text{OR } a_B = 4 \frac{\text{ft}}{\text{s}^2} \downarrow$$

(a)

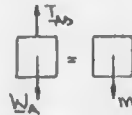
PULLEY D:



$$\sum F_y = 0: 2 T_{AD} - T_{BC} = 0$$

$$\text{OR } T_{AD} = T_{BC}$$

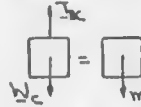
BLOCK A:



$$\sum F_y = m_A a_A: W_A - T_{AB} = \frac{W_A}{g} a_A$$

$$\text{OR } a_A = g (1 - \frac{T_{AB}}{W_A})$$

BLOCK C:



$$\sum F_y = m_C a_C: W_C - T_{BC} = \frac{W_C}{g} a_C$$

$$\text{OR } a_C = g (1 - \frac{T_{BC}}{W_C})$$

SUBSTITUTING THE EXPRESSIONS FOR  $a_A$  AND  $a_C$  INTO

$$\text{Eq. (1) -- } 2g (1 - \frac{T_{AB}}{W_A}) + a_B + g (1 - \frac{T_{BC}}{W_C}) = 0$$

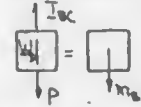
$$\text{OR } (\frac{2}{W_A} + \frac{1}{W_C}) T_{AB} = 3 + \frac{a_B}{g}$$

$$\text{THEN.. } (\frac{2}{20 \text{ lb}} + \frac{1}{2 \cdot 10 \text{ lb}}) T_{AB} = 3 + \frac{4 \frac{\text{ft}}{\text{s}^2}}{32.2 \frac{\text{ft}}{\text{s}^2}}$$

$$\text{OR } T_{AB} = 20.828 \text{ lb}$$

$$\text{AND THEN } T_{BC} = 10.414 \text{ lb}$$

BLOCK B:



$$\sum F_y = m_B a_B: P + W_B - T_{BC} = \frac{W_B}{g} a_B$$

$$\text{OR } P = T_{BC} + W_B (\frac{a_B}{g} - 1)$$

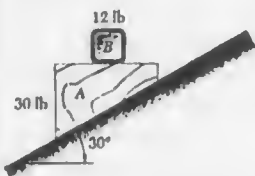
$$\text{SUBSTITUTING -- } P = 10.414 \text{ lb} + (10 \text{ lb}) (\frac{4 \frac{\text{ft}}{\text{s}^2}}{32.2 \frac{\text{ft}}{\text{s}^2}} - 1)$$

$$\text{OR } P = 1.656 \text{ lb}$$

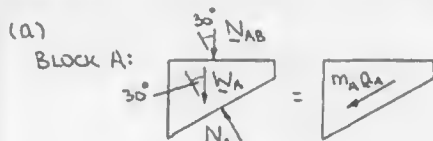
(b) HAVE FROM ABOVE..

$$T_{AB} = 20.8 \text{ lb}$$

## 12.125



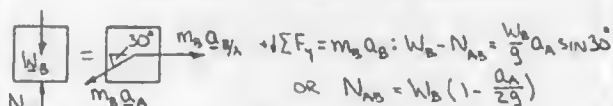
**GIVEN:** BLOCKS A AND B AS SHOWN; AT  $t=0$ ,  $v=0$ ; NEGLECT FRICTION  
**FIND:** (a)  $a_A$  AT  $t=0$   
 (b)  $a_{B/A}$  AT  $t=0$



$$\sum F_x = m_A a_A: N_{AB} \sin 30^\circ + W_A \sin 30^\circ = \frac{W_A}{g} a_A$$

$$\text{OR } N_{AB} = W_A \left( \frac{a_A}{g \sin 30^\circ} - 1 \right) = W_A \left( \frac{2a_A}{g} - 1 \right)$$

**BLOCK B:** FIRST NOTE THAT  $a_B = a_A + a_{B/A}$  WHERE  $a_{B/A}$  IS DIRECTED PARALLEL TO THE TOP SURFACE OF BLOCK A.



$$\sum F_y = m_B a_B: W_B - N_{AB} = \frac{W_B}{g} a_B \sin 30^\circ$$

$$\text{OR } N_{AB} = W_B \left( 1 - \frac{a_B}{2g} \right)$$

THEN..  $W_A \left( \frac{2a_A}{g} - 1 \right) = W_B \left( 1 - \frac{a_B}{2g} \right)$

$$\text{OR } (30 \text{ lb}) \left( \frac{2a_A}{g} - 1 \right) = (12 \text{ lb}) \left( 1 - \frac{a_B}{2g} \right)$$

$$\text{OR } a_A = \frac{7}{11} g = \frac{7}{11} (32.2 \text{ ft/s}^2) = 20.49 \text{ ft/s}^2$$

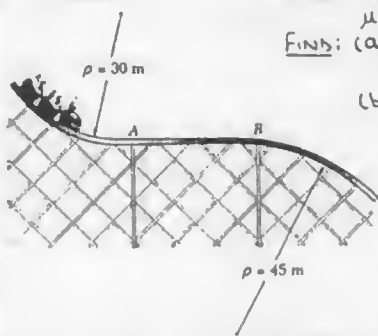
$$\therefore a_A = 20.49 \text{ ft/s}^2 \nearrow 30^\circ$$

(b) FOR BLOCK B..  $\sum F_x = m_B a_x: 0 = m_B a_{B/A} - m_B a_A \cos 30^\circ$

$$\text{OR } a_{B/A} = (20.49 \text{ ft/s}^2) \cos 30^\circ$$

$$\text{OR } a_{B/A} = 17.75 \text{ ft/s}^2 \rightarrow$$

## 12.126

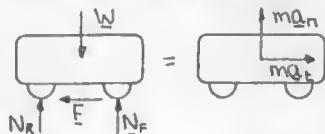


**GIVEN:**  $v_0 = 72 \frac{\text{km}}{\text{h}}$ ; SLIDING:  
 $\mu_k = 0.25$

**FIND:** (a)  $|a_t|$  IF THE CAR IS ALMOST AT A  
 (b)  $|a|$  IF THE CAR IS BETWEEN A AND B  
 (c)  $|a_t|$  IF THE CAR IS JUST PAST B

**FIRST NOTE..**  $v_0 = 72 \frac{\text{km}}{\text{h}} = 20 \frac{\text{m}}{\text{s}}$

(a) HAVE JUST BEFORE A..  $r_A = 30 \text{ m}$



$$\sum F_n = m a_n: (N_F + N_R) - W = m \frac{v_0^2}{r_A}$$

$$\text{OR } (N_F + N_R) = m \left( g + \frac{v_0^2}{r_A} \right)$$

SLIDING:  $F = \mu_k (N_F + N_R) = 0.25 m \left( g + \frac{v_0^2}{r_A} \right)$

$$\sum F_t = m a_t: -F = m a_t$$

$$\text{OR } -0.25 m \left( g + \frac{v_0^2}{r_A} \right) = m a_t$$

$$\text{OR } a_t = -0.25 \left[ 9.81 \frac{\text{m}}{\text{s}^2} + \frac{(20 \text{ m/s})^2}{30 \text{ m}} \right]$$

$$\text{OR } |a_t| = 5.79 \frac{\text{m}}{\text{s}^2}$$

(CONTINUED)

## 12.126 continued

(b)

$$\sum F_y = 0: (N_F + N_R) - W = 0$$

$$\text{OR } (N_F + N_R) = mg$$

SLIDING:  $F = \mu_k (N_F + N_R) = 0.25 mg$

$$\sum F_x = m a: -F = m a$$

$$\text{OR } -0.25 mg = m a$$

$$\text{OR } a = -0.25 (9.81 \text{ m/s}^2)$$

$$\text{OR } |a| = 2.45 \frac{\text{m}}{\text{s}^2}$$

(c) HAVE JUST PAST B..  $r_B = 45 \text{ m}$

$$\sum F_n = m a_n: W - (N_F + N_R) = m \frac{v_0^2}{r_B}$$

$$\text{OR } (N_F + N_R) = m \left( g - \frac{v_0^2}{r_B} \right)$$

SLIDING:  $F = \mu_k (N_F + N_R) = 0.25 m \left( g - \frac{v_0^2}{r_B} \right)$

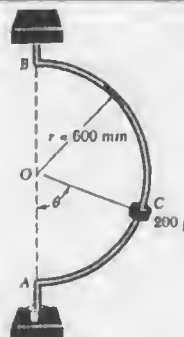
$$\sum F_t = m a_t: -F = m a_t$$

$$\text{OR } -0.25 m \left( g - \frac{v_0^2}{r_B} \right) = m a_t$$

$$\text{OR } a_t = -0.25 \left[ 9.81 \frac{\text{m}}{\text{s}^2} - \frac{(20 \text{ m/s})^2}{45 \text{ m}} \right]$$

$$\text{OR } |a_t| = 0.230 \frac{\text{m}}{\text{s}^2}$$

## 12.127



**GIVEN:** 0.2-kg COLLAR C  
 ON ROD AB;  
 $\dot{\theta} = 6 \text{ rad/s}$

**FIND:**  $(\mu_s)_{\text{min}}$  IF C IS NOT TO SLIDE ON AB WHEN  
 (a)  $\theta = 90^\circ$   
 (b)  $\theta = 75^\circ$   
 (c)  $\theta = 45^\circ$

**FIRST NOTE..**  $v_C = (r \sin \theta) \dot{\theta} = (0.6 \text{ m}) (6 \frac{\text{rad}}{\text{s}}) \sin \theta$

$$= (3.6 \frac{\text{m}}{\text{s}}) \sin \theta$$

(a) WITH  $\theta = 90^\circ$ ,  $v_C = 3.6 \frac{\text{m}}{\text{s}}$

$$\sum F_y = 0: F - W_C = 0$$

$$\text{OR } F = m_C g$$

Now..  $F = \mu_s N$

$$\text{OR } N = \frac{1}{\mu_s} m_C g$$

$$\sum F_n = m_C a_n: N = m_C \frac{v_C^2}{r}$$

$$\text{OR } \frac{1}{\mu_s} m_C g = m_C \frac{v_C^2}{r}$$

$$\text{OR } \mu_s = \frac{g r}{v_C^2} = \frac{(9.81 \frac{\text{m}}{\text{s}^2}) (0.6 \text{ m})}{(3.6 \frac{\text{m}}{\text{s}})^2}$$

$$\text{OR } (\mu_s)_{\text{min}} = 0.454$$

THE DIRECTION OF THE IMPENDING MOTION IS

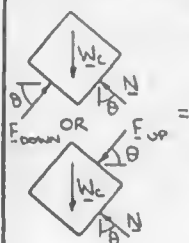
DOWNWARD

(b) AND (c)

FIRST OBSERVE THAT FOR AN ARBITRARY VALUE OF  $\theta$  IT IS NOT KNOWN WHETHER THE IMPENDING MOTION WILL BE UPWARD OR DOWNWARD. TO CONSIDER BOTH POSSIBILITIES FOR EACH VALUE OF  $\theta$ , LET  $F_{\text{DOWN}}$  CORRESPOND TO IMPENDING MOTION DOWNWARD (CONTINUED)

# 12.127 continued

$F_{up}$  CORRESPONDS TO IMPENDING MOTION UPWARD  
THEN, WITH THE TOP SIGN CORRESPONDING TO  $F_{down}$ , HAVE..



$$+\uparrow \Sigma F_y = 0: N \cos \theta + F \sin \theta - W_c = 0$$

Now..  $F = \mu_s N$   
THEN  $N \cos \theta + \mu_s N \sin \theta - m_c g = 0$   
OR  $N = \frac{m_c g}{\cos \theta + \mu_s \sin \theta}$   
AND  $F = \frac{\mu_s m_c g}{\cos \theta + \mu_s \sin \theta}$

$$+\uparrow \Sigma F_x = m_c a_n: N \sin \theta - F \cos \theta = m_c \frac{v_c^2}{\rho}$$

SUBSTITUTING FOR N AND F..

$$\frac{m_c g}{\cos \theta + \mu_s \sin \theta} \sin \theta - \frac{\mu_s m_c g}{\cos \theta + \mu_s \sin \theta} \cos \theta = m_c \frac{v_c^2}{r \sin \theta}$$

$$\text{OR } \frac{\tan \theta}{1 + \mu_s \tan \theta} - \frac{\mu_s}{1 + \mu_s \tan \theta} = \frac{v_c^2}{g r \sin \theta}$$

$$\text{OR } \mu_s = \pm \frac{\tan \theta - \frac{v_c^2}{g r \sin \theta}}{1 + \frac{v_c^2}{g r \sin \theta} \tan \theta}$$

$$\text{Now.. } \frac{v_c^2}{g r \sin \theta} = \frac{[(3.6 \frac{m}{s}) \sin \theta]^2}{(9.81 \frac{m}{s^2})(0.6 m) \sin \theta} = 2.2018 \sin \theta$$

$$\text{Then.. } \mu_s = \pm \frac{\tan \theta - 2.2018 \sin \theta}{1 + 2.2018 \sin \theta \tan \theta}$$

$$(b) \theta = 75^\circ$$

$$\mu_s = \pm \frac{\tan 75^\circ - 2.2018 \sin 75^\circ}{1 + 2.2018 \sin 75^\circ \tan 75^\circ} = \pm 0.1796$$

THEN.. DOWNWARD:  $\mu_s = +0.1796$

UPWARD:  $\mu_s < 0$  .. NOT POSSIBLE

$$\therefore (\mu_s)_{min} = 0.1796$$

THE DIRECTION OF THE IMPENDING MOTION IS DOWNWARD

$$(c) \theta = 45^\circ$$

$$\mu_s = \pm \frac{\tan 45^\circ - 2.2018 \sin 45^\circ}{1 + 2.2018 \sin 45^\circ \tan 45^\circ} = \pm (-0.218)$$

THEN.. DOWNWARD:  $\mu_s < 0$  .. NOT POSSIBLE

UPWARD:  $\mu_s = 0.218$

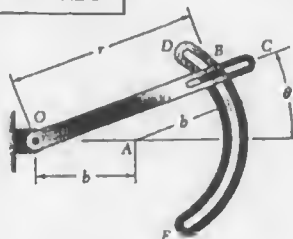
$$\therefore (\mu_s)_{min} = 0.218$$

THE DIRECTION OF THE IMPENDING MOTION IS UPWARD

NOTE: WHEN  $\tan \theta - 2.2018 \sin \theta = 0$   
OR  $\theta = 62.988^\circ$

$\mu_s = 0$ . THUS, FOR THIS VALUE OF  $\theta$  FRICTION IS NOT NECESSARY TO PREVENT THE COLLAR FROM SLIDING ON THE ROD.

# 12.128



GIVEN:  $W_c = \frac{1}{2} \text{ lb}$ ,  $b = 20 \text{ in.}$   
WHEN  $\theta = 20^\circ$ ,  $\dot{\theta} = 15 \frac{\text{rad}}{\text{s}}$   
 $\ddot{\theta} = 250 \frac{\text{rad}}{\text{s}^2}$

FIND: (a)  $F_c$  AND  $F_b$  AT B WHEN  $\theta = 20^\circ$   
(b)  $P$  AND  $Q$  WHEN  $\theta = 20^\circ$ , WHERE  $P$  IS DUE TO AC AND  $Q$  IS DUE TO DE

## KINEMATICS

FROM THE DRAWING OF THE SYSTEM HAVE-

$$r = 2b \cos \theta$$

$$\text{THEN.. } \dot{r} = -2b \sin \theta \dot{\theta}$$

$$\text{AND } \ddot{r} = -2b (\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)$$

$$\text{Now.. } a_r = \ddot{r} - r \dot{\theta}^2 = -2b (\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) - (2b \cos \theta) \dot{\theta}^2$$

$$= -2b (\ddot{\theta} \sin \theta + 2 \dot{\theta}^2 \cos \theta)$$

$$= -2 \left( \frac{20}{12} \text{ ft} \right) \left[ \left( 250 \frac{\text{rad}}{\text{s}^2} \right) \sin 20^\circ + 2 \left( 15 \frac{\text{rad}}{\text{s}} \right)^2 \cos 20^\circ \right]$$

$$= -1694.56 \frac{\text{ft}}{\text{s}^2}$$

$$\text{AND } a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = (2b \cos \theta) \ddot{\theta} + 2(-2b \sin \theta) \dot{\theta}$$

$$= 2b (\ddot{\theta} \cos \theta - 2 \dot{\theta}^2 \sin \theta)$$

$$= 2 \left( \frac{20}{12} \text{ ft} \right) \left[ \left( 250 \frac{\text{rad}}{\text{s}^2} \right) \cos 20^\circ - 2 \left( 15 \frac{\text{rad}}{\text{s}} \right)^2 \sin 20^\circ \right]$$

$$= 270.05 \frac{\text{ft}}{\text{s}^2}$$

## KINETICS

$$(a) \text{ HAVE.. } F_r = m a_r = \frac{\frac{1}{2} \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} (-1694.56 \frac{\text{ft}}{\text{s}^2}) = -13.1565 \text{ lb}$$

$$\text{OR } F_r = -13.16 \text{ lb} \blacktriangleleft$$

$$\text{AND } F_\theta = m a_\theta = \frac{\frac{1}{2} \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} (270.05 \frac{\text{ft}}{\text{s}^2}) = 2.0967 \text{ lb}$$

$$\text{OR } F_\theta = 2.10 \text{ lb} \blacktriangleleft$$

(b)

$$+\uparrow \Sigma F_r: -F_r = -Q \cos 20^\circ$$

$$\text{OR } Q = \frac{F_r}{\cos 20^\circ} = \frac{13.1565 \text{ lb}}{\cos 20^\circ} = 14.0009 \text{ lb}$$

$$+\uparrow \Sigma F_\theta: F_\theta = P - Q \sin 20^\circ$$

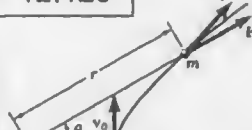
$$\text{OR } P = (2.0967 + 14.0009 \sin 20^\circ) \text{ lb}$$

$$= 6.89 \text{ lb}$$

$$\therefore P = 6.89 \text{ lb} \blacktriangleup 70^\circ$$

$$Q = 14.00 \text{ lb} \blacktriangledown 40^\circ$$

# 12.129



GIVEN: CENTRAL FORCE  $F$  AND PATH SHOWN;  
 $r = r_0 / \cos \theta$ ; AT  $t = 0$ ,  
 $v = v_0$ ,  $\theta = 0$ ,  $\dot{r} = 0$   
FIND:  $v_r$  AND  $v_\theta$  AS FUNCTIONS OF  $\theta$

$$\text{HAVE.. } r = \frac{r_0}{\cos \theta}$$

$$\text{THEN.. } \dot{r} = \frac{r_0 \sin \theta}{\cos^2 \theta} \dot{\theta}$$

$$\text{Now.. } v = \dot{r} e_r + r \dot{\theta} e_\theta$$

$$\text{SO THAT AT } t = 0 \dots v_0 = v_0 e_\theta$$

$$\text{FROM EQ. (12.27): } r^2 \dot{\theta} = r_0^2 \dot{\theta}_0$$

$$\text{OR } \dot{\theta} = \frac{r_0 v_0}{r^2} = r_0 v_0 \left( \frac{\cos \theta}{r_0} \right)^2 = \frac{v_0}{r_0} \cos^2 \theta$$

$$\text{THEN } \dot{r} = \frac{r_0 \sin \theta}{\cos^2 \theta} \left( \frac{v_0}{r_0} \cos^2 \theta \right) = v_0 \sin \theta$$

(CONTINUED)

# 12.129 continued

$$\text{Now } v_r = \dot{r}$$

$$\text{AND } v_\theta = r\dot{\theta} = \frac{r_0}{\cos 2\theta} = \frac{v_0}{\cos^2 2\theta} \quad \text{OR } v_r = 2v_0 \sin 2\theta$$

$$\text{OR } v_\theta = v_0 \cos 2\theta$$

# 12.130

GIVEN: RADIUS  $r$  OF THE MOON'S ORBIT;  
RADIUS  $R$  OF THE EARTH; THE  
ACCELERATION OF GRAVITY  $g$  AT  
THE EARTH'S SURFACE; THE  
PERIODIC TIME  $\tau$  OF THE MOON

SHOW:  $\tau = f(R, g, \tau)$

FIND:  $\tau$  KNOWING THAT  $\tau = 27.3$  DAYS

$$\text{HAVE -- } F = G \frac{Mm}{r^2} \quad [\text{Eq. (12.28)}]$$

$$\text{AND } F = F_n = ma_n = m \frac{v^2}{r}$$

$$\text{THEN } G \frac{Mm}{r^2} = m \frac{v^2}{r}$$

$$\text{OR } v^2 = \frac{GM}{r}$$

$$\text{Now } GM = gR^2 \quad \text{Eq. (12.30)}$$

$$\text{SO THAT } v^2 = \frac{gR^2}{r} \quad \text{OR } v = R \sqrt{\frac{g}{r}}$$

$$\text{FOR ONE ORBIT -- } \tau = \frac{2\pi r}{v} = \frac{2\pi r}{R \sqrt{\frac{g}{r}}}$$

$$\text{OR } \tau = \left( \frac{g \tau^2 R^3}{4\pi^2} \right)^{1/3} \quad \text{Q.E.D.}$$

$$\text{Now -- } \tau = 27.3 \text{ DAYS} = 2.35872 \times 10^6 \text{ s}$$

$$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$$

$$\underline{\text{S1:}} \quad \tau = \left[ \frac{9.81 \frac{\text{m}}{\text{s}^2} \cdot (2.35872 \times 10^6 \text{ s})^2 \cdot (6.37 \times 10^6 \text{ m})^2}{4\pi^2} \right]^{1/3}$$

$$= 382.81 \times 10^6 \text{ m}$$

$$\text{OR } \tau = 383 \times 10^3 \text{ km}$$

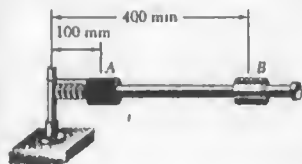
U.S. CUSTOMARY UNITS:

$$\tau = \left[ \frac{32.2 \frac{\text{ft}}{\text{s}^2} \cdot (2.35872 \times 10^6 \text{ s})^2 \cdot (20.9088 \times 10^6 \text{ ft})^2}{4\pi^2} \right]^{1/3}$$

$$= 1256.52 \times 10^6 \text{ ft}$$

$$\text{OR } \tau = 238 \times 10^3 \text{ mi}$$

# 12.131



GIVEN:  $m = 0.25 \text{ kg}$ ;  $k = 6 \frac{\text{N}}{\text{m}}$ ,  
 $(L_0)_{sp} = 0.5 \text{ m}$ ; AT  $t = 0$ ,  
 $\dot{\theta}_0 = 16 \frac{\text{rad}}{\text{s}}$ , COLLAR IS  
AT A; NEGLECT  
FRICTION AND  $m_{rod}$

FIND: (a)  $(v_\theta)_B$   
(b)  $(a_B)_r$  AND  $(a_B)_\theta$   
(c)  $(a_{collar/rod})_B$

FIRST NOTE --  $F_{sp} = k[(L_0)_{sp} - r]$

$$\text{AT B: } (F_{sp})_B = 6 \frac{\text{N}}{\text{m}} (0.5 - 0.4) \text{ m} = 0.6 \text{ N}$$



(a) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL  
FORCE ACTING ON THE COLLAR IS DUE TO THE  
(CONTINUED)

# 12.131 continued

SPRING. THUS, ANGULAR MOMENTUM ABOUT THE  
SHAFT IS CONSERVED.

$$\therefore r_A m (v_A)_\theta = r_B m (v_B)_\theta \quad \text{WHERE } (v_A)_\theta = r_A \dot{\theta}$$

$$\text{THEN -- } (v_B)_\theta = \frac{0.1 \text{ m}}{0.4 \text{ m}} \left[ (0.1 \text{ m}) (16 \frac{\text{rad}}{\text{s}}) \right]$$

$$\text{OR } (v_B)_\theta = 0.400 \frac{\text{m}}{\text{s}}$$

(b) HAVE  $F_\theta = 0$

$$\text{Now -- } \sum F_r = ma_r: (F_{sp})_B = m (a_B)_r$$

$$\text{OR } (a_B)_r = \frac{0.6 \text{ N}}{0.25 \text{ kg}}$$

$$\text{OR } (a_B)_r = 2.40 \frac{\text{m}}{\text{s}^2}$$

(c) HAVE --  $a_r = \ddot{r} - r\dot{\theta}^2$

$$\text{Now -- } a_{collar/rod} = \ddot{r} \quad \text{AND } \dot{\theta}_B = \frac{(v_B)_\theta}{r_B}$$

$$\text{THEN -- AT B: } (a_{collar/rod})_B = 2.40 \frac{\text{m}}{\text{s}^2} + (0.4 \text{ m}) \left( \frac{0.400 \frac{\text{m}}{\text{s}}}{0.4 \text{ m}} \right)^2$$

$$\text{OR } (a_{collar/rod})_B = 2.80 \frac{\text{m}}{\text{s}^2}$$

# 12.132

GIVEN: TRAJECTORY OF THE VOYAGER I

SPACECRAFT ABOUT SATURN; AT THE  
POINT OF CLOSEST APPROACH,  
 $r = 185 \times 10^3 \text{ km}$ ,  $v = 21.0 \text{ km/s}$ ; FOR  
THE CIRCULAR ORBIT OF THE MOON  
TETHYS,  $r = 295 \times 10^3 \text{ km}$ ,  
 $v = 11.35 \times 10^3 \text{ km/s}$

FIND:  $\epsilon$  AT THE POINT OF CLOSEST  
APPROACH OF VOYAGER I

FOR A CIRCULAR ORBIT

$$v = \sqrt{\frac{GM}{r}} \quad \text{Eq. (12.44)}$$

FOR THE ORBIT OF TETHYS --

$$GM = r_T v_T^2$$

FOR VOYAGER'S TRAJECTORY HAVE --

$$\frac{1}{r} = \frac{GM}{h^2} (1 + \epsilon \cos \theta)$$

WHERE  $h = r_0 v_0$

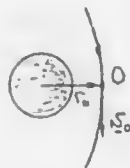
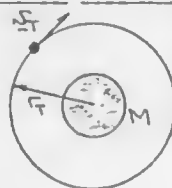
AT  $\theta$ ,  $r = r_0$ ,  $\theta = 0$

$$\text{THEN -- } \frac{1}{r_0} = \frac{GM}{(r_0 v_0)^2} (1 + \epsilon)$$

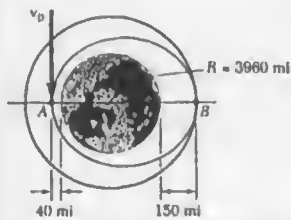
$$\text{OR } \epsilon = \frac{r_0 v_0^2}{GM} - 1 = \frac{r_0 v_0^2}{r_T v_T^2} - 1$$

$$= \frac{185 \times 10^3 \text{ km}}{295 \times 10^3 \text{ km}} \cdot \left( \frac{21.0 \text{ km/s}}{11.35 \text{ km/s}} \right)^2 - 1$$

$$\text{OR } \epsilon = 1.147$$



## 12.133



GIVEN: ELLIPTIC AND CIRCULAR ORBITS OF THE SHUTTLE COLUMBIA ABOUT THE EARTH

FIND: (a)  $t_{AB}$   
(b)  $\tau_{CIRC}$

FIRST NOTE..  $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$   
 $r_A = (3960 + 40) \text{ mi} = 4000 \text{ mi} = 21.120 \times 10^6 \text{ ft}$   
 $r_B = (3960 + 150) \text{ mi} = 4110 \text{ mi} = 21.7008 \times 10^6 \text{ ft}$

(a) THE PERIODIC TIME  $\tau$  OF AN ELLIPTIC ORBIT IS  
 $\tau = \frac{2\pi ab}{h} \quad [\text{Eq. (12.45)}]$

$$\therefore t_{AB} = \frac{1}{2} \tau = \frac{\pi ab}{h_{AB}}$$

WHERE  $a = \frac{1}{2}(r_A + r_B) = \frac{1}{2}(21.120 \times 10^6 + 21.7008 \times 10^6) \text{ ft}$   
 $= 21.4104 \times 10^6 \text{ ft}$   
 $b = \sqrt{r_A r_B} = [(21.120 \times 10^6 \text{ ft})(21.7008 \times 10^6 \text{ ft})]^{1/2}$   
 $= 21.4084 \times 10^6 \text{ ft}$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h_{AB}^2}$$

NOW..  $GM = gR^2 \quad [\text{Eq. (12.30)}]$

SO THAT

$$h_{AB} = \left( \frac{2gR^2}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2} = \left[ \frac{2(32.2 \frac{\text{ft}}{\text{s}^2})(20.9088 \times 10^6 \text{ ft})^2}{\frac{1}{21.120 \times 10^6 \text{ ft}} + \frac{1}{21.7008 \times 10^6 \text{ ft}}} \right]^{1/2}$$

$$= 548.95 \times 10^9 \frac{\text{ft}^2}{\text{s}}$$

FINALLY..  $t_{AB} = \frac{\pi(21.4104 \times 10^6 \text{ ft})(21.4084 \times 10^6 \text{ ft})}{548.95 \times 10^9 \text{ ft}^2/\text{s}}$   
 $= 2623.2 \text{ s}$

OR  $t_{AB} = 43 \text{ MIN } 43 \text{ S}$

(b) FOR THE CIRCULAR ORBIT

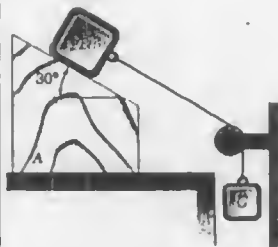
$$\tau_{CIRC} = \frac{2\pi r_B}{v_{CIRC}}$$

WHERE  $v_{CIRC} = \sqrt{\frac{gR^2}{r_B}} \quad [\text{Eq. (12.44)}]$

THEN..  $\tau_{CIRC} = \frac{2\pi r_B^{3/2}}{R \sqrt{g}} = \frac{2\pi(21.7008 \times 10^6 \text{ ft})^{3/2}}{(20.9088 \times 10^6 \text{ ft})(32.2 \frac{\text{ft}}{\text{s}^2})^{1/2}}$   
 $= 5353.5 \text{ s}$

OR  $\tau_{CIRC} = 1 \text{ h } 29 \text{ MIN } 13 \text{ S}$

## 12.C1



GIVEN:  $m_A = 20 \text{ kg}$ ,  $m_B = 10 \text{ kg}$ ,  
 $m_C = 2 \text{ kg}$ ;  $t = 0, 15, 0$ ;  
 $\mu \geq 0$   
 FIND:  $a_A$  AND  $a_{B/A}$  FOR  
 $\mu \geq 0$  USING  $\Delta \mu = 0.01$   
 WHILE  $a_A > 0$  AND  
 $\Delta \mu = 0.1$  WHILE  $a_{B/A} > 0$

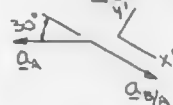
## ANALYSIS

## KINEMATICS

HAVE..  $a_B = a_A + a_{B/A}$

WHERE  $a_{B/A}$  IS DIRECTED ALONG THE INCLINED SURFACE OF A. THEN..

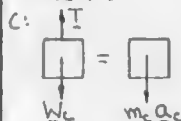
$$a_B = a_A(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) + a_{B/A} \hat{i}$$



ALSO, 'SINCE THE CORD IS OF CONSTANT LENGTH

$$a_C = (a_B)_{\hat{i}} = a_{B/A} - a_A \cos 30^\circ$$

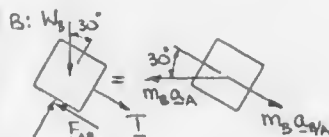
## KINETICS



$$+\uparrow \Sigma F_y = m_C a_C: T - W_C = m_C a_C$$

$$\text{OR } T = m_C(g - a_C)$$

$$= m_C(g - a_{B/A} + a_A \cos 30^\circ) \quad \dots (2)$$



$$N_{AB} + \Sigma F_{x'} = m_B a_{x'}: T - F_{AB} + W_B \sin 30^\circ = m_B a_{B/A} - m_B a_A \cos 30^\circ$$

$$\text{OR } T - F_{AB} + 10g \sin 30^\circ = 10 a_{B/A} - 10 a_A \cos 30^\circ \quad (3)$$

$$+\uparrow \Sigma F_{y'} = m_B a_{y'}: N_{AB} - W_B \cos 30^\circ = -m_B a_A \sin 30^\circ$$

$$\text{OR } N_{AB} = 10g \cos 30^\circ - 10 a_A \sin 30^\circ \quad (4)$$

SLIDING:  $F_{AB} = \mu N_{AB}$

$$\text{OR } F_{AB} = 10\mu(g \cos 30^\circ - a_A \sin 30^\circ) \quad (5)$$

SUBSTITUTING EQS. (2) AND (5) INTO EQ. (3)..

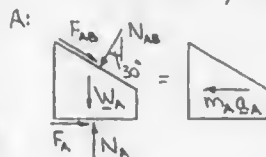
$$2(g - a_{B/A} + a_A \cos 30^\circ) - 10\mu(g \cos 30^\circ - a_A \sin 30^\circ) + 10g \sin 30^\circ = 10 a_{B/A} - 10 a_A \cos 30^\circ$$

$$\text{OR } g(1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ) = 6 a_{B/A} - a_A(5\mu \sin 30^\circ + 6 \cos 30^\circ) \quad (6)$$

NOTE: BLOCK A WILL NOT MOVE ( $a_A = 0$ ) BEFORE BLOCKS B AND C WILL NOT MOVE ( $a_{B/A} = a_B = 0$ ). THEREFORE, THE SYSTEM WILL REMAIN AT REST WHEN

$$g(1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ) = 0$$

$$\text{OR } \mu \geq 0.808 \text{ FOR NO MOTION}$$



$$+\uparrow \Sigma F_x = m_A a_A: N_{AB} \sin 30^\circ - F_A - F_{AB} \cos 30^\circ = m_A a_A$$

$$\text{OR } N_{AB}(\sin 30^\circ - \mu \cos 30^\circ) - F_A = 20 a_A \quad (7)$$

(CONTINUED)

## 12.C1 continued

$$\uparrow \Sigma F_y = 0: N_A - N_{AB} \cos 30^\circ - F_{AB} \sin 30^\circ - W_A = 0$$

$$\text{OR } N_A = N_{AB} (\cos 30^\circ + \mu \sin 30^\circ) + 20g \quad (8)$$

$$\text{SLIDING: } F_A = \mu N_A$$

$$\text{OR } F_A = \mu N_{AB} (\cos 30^\circ + \mu \sin 30^\circ) + 20\mu g \quad (9)$$

SUBSTITUTING EQ. (9) INTO EQ. (7)...

$$N_{AB} (\sin 30^\circ - \mu \cos 30^\circ) - \mu N_{AB} (\cos 30^\circ + \mu \sin 30^\circ) - 20\mu g = 20a_A$$

$$\text{OR } N_{AB} [(1-\mu^2) \sin 30^\circ - 2\mu \cos 30^\circ] - 20\mu g = 20a_A$$

SUBSTITUTING FOR  $N_{AB}$  [EQ. (4)]...

$$(10g \cos 30^\circ - 10a_A \sin 30^\circ) [(1-\mu^2) \sin 30^\circ - 2\mu \cos 30^\circ] - 20\mu g = 20a_A$$

$$\text{LET } A = (1-\mu^2) \sin 30^\circ - 2\mu \cos 30^\circ$$

$$\text{THEN.. } g(A \cos 30^\circ - 2\mu) = (2 + A \sin 30^\circ) a_A$$

$$\text{OR } a_A = \frac{A \cos 30^\circ - 2\mu}{2 + A \sin 30^\circ} g \quad (10)$$

NOTE: BLOCK A WILL REMAIN AT REST WHEN

$$g(A \cos 30^\circ - 2\mu) = 0$$

$$\text{OR } [(1-\mu^2) \sin 30^\circ - 2\mu \cos 30^\circ] \cos 30^\circ - 2\mu = 0$$

$$\text{OR } (\frac{1}{2} \sin 60^\circ) \mu^2 + 2(1 + \cos^2 30^\circ) \mu - \frac{1}{2} \sin 60^\circ = 0$$

$$\text{OR } \mu \geq 0.12188 \text{ FOR BLOCK A TO REMAIN AT REST}$$

NOW.. REWRITE EQ. (6) AS

$$a_{B/A} = \frac{1}{6} [g(1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ) + a_A(5\mu \sin 30^\circ + 6 \cos 30^\circ)] \quad (11)$$

WHICH REDUCES TO

$$a_{B/A} = \frac{9}{6} (1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ) \quad (12)$$

WHEN  $a_A = 0$

### OUTLINE OF PROGRAM

INPUT INITIAL VALUE OF  $\mu$ :  $\mu = 0$

COMPUTE A:  $A = (1-\mu^2) \sin 30^\circ - 2\mu \cos 30^\circ$

COMPUTE  $a_A$ :  $a_A = \frac{A \cos 30^\circ - 2\mu}{2 + A \sin 30^\circ} g$

WHILE  $a_A > 0$

COMPUTE  $a_{B/A}$ :

$$a_{B/A} = \frac{1}{6} [g(1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ) + a_A(5\mu \sin 30^\circ + 6 \cos 30^\circ)]$$

PRINT THE VALUES OF  $\mu$ ,  $a_A$ , AND  $a_{B/A}$

UPDATE  $\mu$ :  $\mu = \mu + 0.01$

INCREASE  $\mu$  TO THE NEXT TENTH:

$$\mu = \frac{1}{10} [\text{INTEGER VALUE}(10\mu)] + 0.1$$

COMPUTE  $a_{B/A}$ :

$$a_{B/A} = \frac{9}{6} (1 - 5\mu \cos 30^\circ + 5 \sin 30^\circ)$$

WHILE  $a_{B/A} > 0$

PRINT THE VALUES OF  $\mu$  AND  $a_{B/A}$

UPDATE  $\mu$ :  $\mu = \mu + 0.1$

(CONTINUED)

## 12.C1 continued

### PROGRAM OUTPUT

$\mu$	accel. of A, m/s <sup>2</sup>	accel. of B wrt A, m/s <sup>2</sup>
0.00	1.888	7.358
0.01	1.742	7.167
0.02	1.594	6.975
0.03	1.445	6.780
0.04	1.295	6.582
0.05	1.143	6.382
0.06	0.989	6.179
0.07	0.833	5.973
0.08	0.676	5.764
0.09	0.518	5.553
0.10	0.357	5.339
0.11	0.195	5.122
0.12	0.031	4.901

For those values of  $\mu$  for which the wedge is at rest

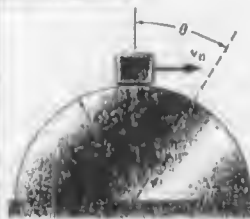
$\mu$	accel. of B wrt A, m/s <sup>2</sup>
0.20	4.307
0.30	3.599
0.40	2.891
0.50	2.183
0.60	1.475
0.70	0.767
0.80	0.059

## 12.C2

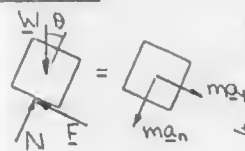
GIVEN:  $W = 11b$ ,  $\gamma = 10 \frac{ft}{s^2}$ ;

$$0 \leq \mu_k \leq 0.4$$

FIND:  $\theta$  AT WHICH THE BLOCK LEAVES THE SURFACE;  
 $\mu = 0, 0.05, 0.10, \dots, 0.4$



### ANALYSIS



$$\uparrow \Sigma F_n = ma_n: W \cos \theta - N = m \frac{v^2}{r}$$

$$\text{OR } N = m(g \cos \theta - \frac{v^2}{r})$$

$$\text{SLIDING: } F = \mu_k N$$

$$= \mu_k m(g \cos \theta - \frac{v^2}{r})$$

$$\uparrow \Sigma F_t = ma_t: W \sin \theta - F = ma_t$$

$$\text{OR } a_t = g \sin \theta - \mu_k \frac{v^2}{r}$$

$$\text{SUBSTITUTING FOR } F.. a_t = g(\sin \theta - \mu_k \cos \theta) + \mu_k \frac{v^2}{r}$$

$$\text{NOW.. } a_t = \frac{dv}{dt} \dots \frac{dv}{dt} = g(\sin \theta - \mu_k \cos \theta) + \mu_k \frac{v^2}{r} \quad (1)$$

$$\text{ALSO.. } v = r \dot{\theta} \text{ OR } \frac{dv}{dt} = \frac{1}{r} \frac{d}{dt} v^2 \quad (2)$$

THUS, DIFFERENTIAL EQUATIONS (1) AND (2)

DEFINE THE MOTION OF THE BLOCK.

AS THE BLOCK LEAVES THE SURFACE,  $N \rightarrow 0$ .

THUS,  $g \cos \theta - \frac{v^2}{r} = 0$

DEFINES THE VALUE OF  $\theta$  AT WHICH THE BLOCK LEAVES THE SURFACE.

### OUTLINE OF PROGRAM

FOR EACH VALUE OF  $\mu_k$

DEFINE THE INITIAL VALUES OF  $v$  AND  $\theta$

USE THE MODIFIED EULER METHOD (SEE THE

SOLUTION TO PROBLEM 11.C3) WITH A STEP

(CONTINUED)

## 12.C2 continued

SIZE  $\Delta t = 0.01$  S TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dV}{dt} = g(\sin\theta - \mu_k \cos\theta) + \mu_k \frac{V^2}{p}$$

$$\frac{d\theta}{dt} = \dot{\theta} V$$

WHERE  $p = S \dot{s}$ .

COMPUTE  $N_1$  AND  $N_2$ :  $N_1 = \cos\theta_1 - \frac{V_1^2}{g p_1^2}$   
 $N_2 = \cos\theta_2 - \frac{V_2^2}{g p_2^2}$

WHERE  $\theta_1$  AND  $V_1$  ARE THE VALUES OF  $\theta$  AND THE VELOCITY, RESPECTIVELY, AT THE BEGINNING OF A TIME INTERVAL, AND  $\theta_2$  AND  $V_2$  ARE THE VALUES AT THE END OF THE TIME INTERVAL.

IF  $N_2 > 0$ , UPDATE  $V$  AND  $\theta$ :  $V_2 = V_1$ ;  $\theta_2 = \theta_1$   
 IF  $N_2 < 0$ , USE LINEAR INTERPOLATION TO DETERMINE THE VALUE OF  $\theta$  AT WHICH  $N = 0$ :

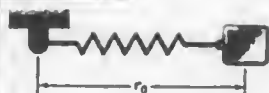
$$\theta = \theta_1 + \frac{0 - N_1}{N_2 - N_1} (\theta_2 - \theta_1)$$

PRINT THE VALUES OF  $\mu$  AND  $\theta$

### PROGRAM OUTPUT

$\mu$	$\theta$
0.00	29.11°
0.05	29.61°
0.10	30.16°
0.15	30.72°
0.20	31.33°
0.25	31.96°
0.30	32.63°
0.35	33.35°
0.40	34.11°

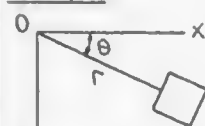
## 12.C3



GIVEN: BLOCK OF MASS  $m$  AND SPRING OF CONSTANT  $k$ ; AT  $t=0$ ,  $V=0$  AND THE SPRING IS HORIZONTAL AND UNSTRETCHED

FIND: (a)  $r$  AND  $V$  WHEN THE BLOCK PASSES UNDER THE PIVOT O  
 (b)  $r/m$  WHEN  $r_0 = 1$  m SO THAT  $V \rightarrow$  WHEN THE BLOCK PASSES UNDER O

### ANALYSIS



FIRST NOTE..  $r = \sqrt{x^2 + y^2}$   
 $\cos\theta = \frac{x}{r}$   $\sin\theta = \frac{y}{r}$   
 $F_{sp} = k(r - r_0)$

$$\sum F_x = m a_x: -F_{sp} \cos\theta = m a_x$$

$$\text{OR } a_x = -\frac{k}{m}(r - r_0) \cos\theta$$

$$\frac{dV_x}{dt} = -\frac{k}{m}(\sqrt{x^2 + y^2} - r_0) \left(\frac{x}{\sqrt{x^2 + y^2}}\right) \quad (1)$$

$$+\sum F_y = m a_y: W - F_{sp} \sin\theta = m a_y$$

$$\text{OR } a_y = g - \frac{k}{m}(r - r_0) \sin\theta$$

$$\text{OR } \frac{dV_y}{dt} = g - \frac{k}{m}(\sqrt{x^2 + y^2} - r_0) \left(\frac{y}{\sqrt{x^2 + y^2}}\right) \quad (2)$$

(CONTINUED)

## 12.C3 continued

ALSO..  $\frac{dx}{dt} = V_x \quad (3)$   $\frac{dy}{dt} = V_y \quad (4)$

THEREFORE, DIFFERENTIAL EQUATIONS (1)-(4) DEFINE THE MOTION OF THE MASS.

NOW..  $V = \sqrt{V_x^2 + V_y^2}$  AND  $\theta_V = \tan^{-1} \frac{V_y}{V_x}$

DEFINE THE MAGNITUDE AND DIRECTION, RESPECTIVELY, OF THE VELOCITY.

### OUTLINE OF PROGRAM

INPUT VALUE OF  $k/m$

INPUT UNSTRETCHED LENGTH OF THE SPRING  $r_0$

INPUT SYSTEM OF UNITS

DEFINE THE INITIAL CONDITIONS:

$$x_1 = r_0, y_1 = 0; (V_x)_1 = 0, (V_y)_1 = 0$$

USE THE MODIFIED EULER METHOD (SEE THE SOLUTION TO PROBLEM 11.C3) WITH A STEP SIZE  $\Delta t = 0.001$  S TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dV_x}{dt} = -\frac{k}{m}(\sqrt{x^2 + y^2} - r_0) \left(\frac{x}{\sqrt{x^2 + y^2}}\right)$$

$$\frac{dV_y}{dt} = g - \frac{k}{m}(\sqrt{x^2 + y^2} - r_0) \left(\frac{y}{\sqrt{x^2 + y^2}}\right)$$

$$\frac{dx}{dt} = V_x$$

$$\frac{dy}{dt} = V_y$$

WHEN  $x_1 > 0$  AND  $x_2 < 0$

$$\text{COMPUTE } r_1 \text{ AND } r_2: r_1 = \sqrt{x_1^2 + y_1^2} \quad r_2 = \sqrt{x_2^2 + y_2^2}$$

$$\text{COMPUTE } V_1 \text{ AND } \theta_{V1}: V_1 = \sqrt{(V_x)_1^2 + (V_y)_1^2}$$

$$(\theta_V)_1 = \tan^{-1} \left( \frac{(V_y)_1}{(V_x)_1} \right)$$

$$\text{COMPUTE } V_2 \text{ AND } \theta_{V2}: V_2 = \sqrt{(V_x)_2^2 + (V_y)_2^2}$$

$$(\theta_V)_2 = \tan^{-1} \left( \frac{(V_y)_2}{(V_x)_2} \right)$$

WHERE  $( )_1$  AND  $( )_2$  DENOTE VALUES AT THE BEGINNING AND END, RESPECTIVELY, OF A TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE VALUES OF  $r$ ,  $V$ , AND  $\theta_V$  AT  $x=0$ :

$$r = r_1 + \frac{0 - x_1}{x_2 - x_1} (r_2 - r_1)$$

$$V = V_1 + \frac{0 - x_1}{x_2 - x_1} (V_2 - V_1)$$

$$\theta_V = (\theta_V)_1 + \frac{0 - x_1}{x_2 - x_1} [(\theta_V)_2 - (\theta_V)_1]$$

PRINT THE VALUES OF  $k/m$ ,  $r_0$ ,  $r$ ,  $V$ , AND  $\theta_V$

### PROGRAM OUTPUT

(a)  $k/m = 15.00 / s^2$   
 Unstretched length of the spring = 1 m

$x_1 = 0.001$  m  $x_2 = -0.002$  m  
 $r = 2.765$  m  
 $v = 2.740$  m/s  
 Angle  $v$  forms with the horizontal =  $-6.19^\circ$

$k/m = 20.00 / s^2$   
 Unstretched length of the spring = 1 m

$x_1 = 0.001$  m  $x_2 = -0.002$  m  
 $r = 2.372$  m  
 $v = 2.983$  m/s  
 Angle  $v$  forms with the horizontal =  $0.93^\circ$

(CONTINUED)





## 12.C4 continued

### OUTLINE OF PROGRAM

INPUT VALUE OF  $\dot{\phi}$

CONSIDER CASES 1 AND 4

FOR VALUES OF  $\theta$  FROM 0 TO 179° IN INCREMENTS OF 1°

COMPUTE  $f_1(\theta)$ :

$$f_1(\theta) = (67.62 - 13\dot{\phi}^2) \cos \theta - (4.55\dot{\phi}^2 + 193.2) \sin \theta + 1.75\dot{\phi}^2 \sin^2 \theta + 2.5\dot{\phi}^2 \sin 2\theta$$

COMPUTE  $f_1(\theta+1^\circ)$

COMPUTE  $f_1(\theta) \cdot f_1(\theta+1^\circ)$  TO DETERMINE IF A ROOT LIES BETWEEN  $\theta$  AND  $(\theta+1^\circ)$

IF  $f_1(\theta) \cdot f_1(\theta+1^\circ) \leq 0$ , SOLVE  $f_1(\theta)$  FOR  $\theta$  USING NEWTON'S METHOD (SEE THE SOLUTION TO PROBLEM 11.C4)  
PRINT THE VALUE OF  $\theta_{\text{ROOT}}$  AND WHETHER  $F_{\text{MAX}} - F$  AT  $\theta$  IS  $\geq$  OR  $\leq 0$

CONSIDER CASES 2 AND 3

FOR VALUES OF  $\theta$  FROM 0 TO 179° IN INCREMENTS OF 1°

COMPUTE  $f_2(\theta)$ :

$$f_2(\theta) = -(67.62 + 13\dot{\phi}^2) \cos \theta + (4.55\dot{\phi}^2 - 193.2) \sin \theta - 1.75\dot{\phi}^2 \sin^2 \theta + 2.5\dot{\phi}^2 \sin 2\theta$$

COMPUTE  $f_2(\theta+1^\circ)$

COMPUTE  $f_2(\theta) \cdot f_2(\theta+1^\circ)$

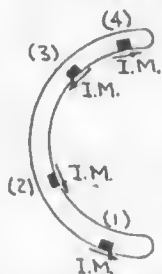
IF  $f_2(\theta) \cdot f_2(\theta+1^\circ) \leq 0$ , SOLVE  $f_2(\theta)$  FOR  $\theta$  USING NEWTON'S METHOD  
PRINT THE VALUE OF  $\theta_{\text{ROOT}}$  AND WHETHER  $F_{\text{MAX}} - F$  AT  $\theta$  IS  $\geq$  OR  $\leq 0$

### PROGRAM OUTPUT

(a) Rate of rotation = 2 rad/s  
At  $\theta = 4^\circ$ ,  $F(\text{max}) - F \gg 0$   
 $\theta(1) = 4.68^\circ$   
  
At  $\theta = 148^\circ$ ,  $F(\text{max}) - F \ll 0$   
 $\theta(3) = 148.57^\circ$

(b) Rate of rotation = 14 rad/s  
At  $\theta = 115^\circ$ ,  $F(\text{max}) - F \gg 0$   
 $\theta(4) = 115.91^\circ$   
  
At  $\theta = 77^\circ$ ,  $F(\text{max}) - F \ll 0$   
 $\theta(2) = 77.63^\circ$

NOTE: IN THE ABOVE OUTPUT, THE  $i$  IN  $\theta(i)$  DENOTES THE CASE FOR WHICH MOTION IS IMPENDING.



## 12.C5

GIVEN: TWO POINTS ON THE TRAJECTORY OF A SPACECRAFT:  $\theta_1$  AND  $\theta_2$  OR  $r_2$  AND THE RADIAL DISTANCE TO AND THE VELOCITY AT THE APOGEE OR THE PERIGEE

FIND: TIME  $t$  FOR THE SPACECRAFT TO TRAVEL BETWEEN THE POINTS

(a) B AND C OF PROB. 12.115;

$$\dot{r}_B = 869.4 \frac{\text{m}}{\text{s}}$$

(b) A AND B OF PROB. 12.117;

$$\dot{r}_A = 24,371 \frac{\text{ft}}{\text{s}}$$

### ANALYSIS

HAVE...  $\frac{1}{r} = \frac{GM}{h^2} (1 + e \cos \theta)$  [Eq. (12.39')]

WHERE  $h = r_{\text{APOGEE}} \dot{r}_{\text{APOGEE}} = r_{\text{PERIGEE}} \dot{r}_{\text{PERIGEE}} = r_{\text{AP}} \dot{r}_{\text{AP}}$

$$\theta_{\text{APOGEE}} = 180^\circ \quad \theta_{\text{PERIGEE}} = 0$$

$$GM = G \left( \frac{M}{M_{\text{EARTH}}} \right) M_{\text{EARTH}} = \left( \frac{M}{M_{\text{EARTH}}} \right) g R_{\text{EARTH}}^2$$

THEN...  $\frac{1}{r_{\text{AP}}} = \frac{\left( \frac{M}{M_{\text{EARTH}}} \right) g R_{\text{EARTH}}^2}{(r_{\text{AP}} \dot{r}_{\text{AP}})^2} (1 + e \cos \theta_{\text{AP}})$

OR  $e = \frac{1}{\cos \theta_{\text{AP}}} \left[ \frac{r_{\text{AP}} \dot{r}_{\text{AP}}^2}{\left( \frac{M}{M_{\text{EARTH}}} \right) g R_{\text{EARTH}}^2} - 1 \right]$

THUS, THE ECCENTRICITY OF THE TRAJECTORY CAN BE DETERMINED.

FROM PAGE 698 OF THE TEXT HAVE..

$$\frac{dA}{dt} = \frac{1}{2} h$$

WHERE  $h$  IS A CONSTANT. THEN..

$$t = \frac{2}{h} \int_{\theta_1}^{\theta_2} dA$$

WHERE  $dA = \frac{1}{2} (r) (r d\theta)$

$$\text{OR } dA = \frac{1}{2} r^2 d\theta$$

$$\text{AND } t = \frac{1}{h} \int r^2 d\theta$$

WHERE  $r$  IS GIVEN BY EQ. (12.39').



### OUTLINE OF PROGRAM

SET VALUE OF  $\Delta\theta$ :  $\Delta\theta = 0.05^\circ$

INPUT UNITS AND CONSTANTS

INPUT WHETHER VALUES ARE KNOWN AT THE APOGEE OR THE PERIGEE

SET VALUE OF  $\theta_{\text{AP}}$ :  $\theta_{\text{AP}} = 0$  (PERIGEE)

$$\theta_{\text{AP}} = 180^\circ \text{ (APOGEE)}$$

INPUT THE DISTANCE  $r_{\text{AP}}$  TO AND THE VELOCITY  $\dot{r}_{\text{AP}}$  AT THE APOGEE OR THE PERIGEE

INPUT THE VALUE OF  $\theta_1$  FOR THE FIRST POINT ON THE TRAJECTORY

INPUT WHETHER THE SECOND POINT ON THE TRAJECTORY IS DETERMINED BY THE VALUE OF  $\theta_2$  (CASE 1) OR BY THE VALUE OF THE RADIAL DISTANCE  $r_2$  (CASE 2)

INPUT  $M/M_{\text{EARTH}}$

COMPUTE THE ECCENTRICITY  $e$  OF THE TRAJECTORY:

$$e = \frac{1}{\cos \theta_{\text{AP}}} \left[ \frac{r_{\text{AP}} \dot{r}_{\text{AP}}^2}{\left( \frac{M}{M_{\text{EARTH}}} \right) g R_{\text{EARTH}}^2} - 1 \right]$$

(CONTINUED)

## 12.C5 continued

CASE 1:

INPUT THE VALUE OF  $\theta_2$

IF  $\theta_2 < \theta_1$ , SET  $\Delta\theta = -\Delta\theta$

FOR VALUES OF  $\theta$  FROM  $\theta_1$  TO  $\theta_2 - \Delta\theta$  IN INCREMENTS OF  $\Delta\theta$

UPDATE AREA A:

$$A = A + \frac{|\Delta\theta|}{2} \left[ \frac{\left( \frac{M}{M_{\text{EARTH}}} \right) g_{\text{EARTH}}^2}{(\dot{r}_{\text{AP}} \dot{\theta}_{\text{AP}})^2} (1 + \epsilon \cos \theta) \right]^{-2}$$

COMPUTE TIME  $t$ :  $t = \frac{2A}{\dot{r}_{\text{AP}} \dot{\theta}_{\text{AP}}}$

PRINT THE VALUES OF  $\dot{r}_{\text{AP}}$ ,  $\dot{\theta}_{\text{AP}}$ ,  $\theta_1$ ,  $\theta_2$ , AND  $t$

CASE 2:

INPUT THE VALUE OF  $r_2$

SET THE INITIAL VALUE OF  $\theta$ :  $\theta = \theta_1$

WHILE  $r > r_2$  IF  $r_1 > r_2$  OR WHILE  $r < r_2$  IF

$r_1 < r_2$

$$\text{COMPUTE } r: r = \left[ \frac{\left( \frac{M}{M_{\text{EARTH}}} \right) g_{\text{EARTH}}^2}{(\dot{r}_{\text{AP}} \dot{\theta}_{\text{AP}})^2} (1 + \epsilon \cos \theta) \right]^{-1}$$

UPDATE AREA A:  $A = A + \frac{1}{2} r^2 \Delta\theta$

UPDATE  $\theta$ :  $\theta = \theta + \Delta\theta$

COMPUTE TIME  $t$ :  $t = \frac{2A}{\dot{r}_{\text{AP}} \dot{\theta}_{\text{AP}}}$

PRINT THE VALUES OF  $\dot{r}_{\text{AP}}$ ,  $\dot{\theta}_{\text{AP}}$ ,  $\theta_1$ ,  $r_2$ , AND  $t$

### PROGRAM OUTPUT

(a)

The radial distance to and the velocity at the apogee are, respectively, 3600 km and .8694 km/s

$\theta_1 = 180^\circ$      $\theta_2 = 290^\circ$

Time  $t = 1 \text{ h } 10 \text{ min } 29 \text{ s}$

(b)

The radial distance to and the velocity at the apogee are, respectively, 4310 mi and 24371 ft/s

$\theta_1 = 180^\circ$      $r_2 = 4035 \text{ mi}$

Time  $t = 0 \text{ h } 33 \text{ min } 30 \text{ s}$

13.1

GIVEN: MASS OF SATELLITE,  $m = 1500 \text{ kg}$   
 SPEED OF SATELLITE,  $v = 22.9 \times 10^3 \text{ km/h}$   
 FIND: KINETIC ENERGY,  $T$

$$v = 22.9 \times 10^3 \text{ km/h} = 6.36 \times 10^3 \text{ m/s}$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} (1500 \text{ kg}) (6.36 \times 10^3 \text{ m/s})^2$$

$$T = 30.337 \times 10^9 \text{ N}\cdot\text{m}$$

NOTE: ACCELERATION OF GRAVITY  
 HAS NO EFFECT ON THE MASS  $T = 30.3 \text{ GJ}$   
 OF THE SATELLITE.

13.2

GIVEN: WEIGHT OF SATELLITE,  $W = 870 \text{ lb}$   
 SPEED OF SATELLITE,  $v = 12,500 \text{ mi/h}$   
 FIND: KINETIC ENERGY,  $T$

$$v = (12,500 \text{ mi/h}) (h/3600 \text{ s}) (5280 \text{ ft/mi})$$

$$v = 18,333 \text{ ft/s}$$

$$\text{MASS OF SATELLITE} = (870 \text{ lb}) (32.2 \text{ ft/s}^2)$$

$$m = 27.019 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} (27.019) (18,333)^2$$

$$T = 4.5405 \times 10^9 \text{ lb}\cdot\text{ft}$$

NOTE: ACCELERATION OF GRAVITY HAS  
 NO EFFECT ON THE MASS OF THE  
 SATELLITE  $T = 4.54 \times 10^9 \text{ lb}\cdot\text{ft}$

13.3

GIVEN: WEIGHT OF STONE,  $W = 5 \text{ lb}$   
 VELOCITY OF STONE,  $v = 80 \text{ ft/s}$   
 ACCELERATION OF GRAVITY ON THE  
 MOON,  $g_m = 5.31 \text{ ft/s}^2$

FIND: (a) KINETIC ENERGY,  $T$   
 HEIGHT  $h$ , FROM WHICH STONE  
 WAS DROPPED  
 (b)  $T$  AND  $h$  ON THE MOON

(a) ON THE EARTH

$$T = \frac{1}{2} m v^2 = \frac{1}{2} \left( \frac{5 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (80 \text{ ft/s})^2$$

$$T = 496.89 \text{ lb}\cdot\text{ft}$$

$$T = 497 \text{ lb}\cdot\text{ft}$$

$$T_1 + U_{1-2} = T_2 \quad T_1 = 0, U_{1-2} = W h = (5 \text{ lb}) (h), T_2 = 497 \text{ lb}\cdot\text{ft}$$

$$W h = T_2 \quad h = \frac{5 \text{ lb}}{497 \text{ lb}\cdot\text{ft}} = 99.4 \text{ ft}$$

$$h = 99.4 \text{ ft}$$

(b) ON THE MOON

MASS IS UNCHANGED

THUS  $T$  IS UNCHANGED

$$T = 497 \text{ lb}\cdot\text{ft}$$

WEIGHT ON THE MOON IS:  $W_m = m g_m = \frac{(5 \text{ lb}) (5.31 \text{ ft/s}^2)}{(32.2 \text{ ft/s}^2)}$

$$W_m = 0.8245 \text{ lb}$$

$$h_m = \frac{T_2}{W_m} = \frac{(497 \text{ lb}\cdot\text{ft})}{(0.8245 \text{ lb})} = 602.7 \text{ ft}$$

$$h_m = 603 \text{ ft}$$

13.4

GIVEN: MASS OF STONE,  $m = 4 \text{ kg}$   
 VELOCITY OF STONE,  $v = 25 \text{ m/s}$   
 ACCELERATION OF GRAVITY  
 ON THE MOON,  $g_m = 1.62 \text{ m/s}^2$

FIND:

(a) KINETIC ENERGY,  $T$ 

HEIGHT  $h$ , FROM WHICH THE STONE  
 WAS DROPPED

13.4

continued

(b)  $T$  AND  $h$  ON THE MOON

(a) ON THE EARTH

$$T = \frac{1}{2} m v^2 = \frac{1}{2} (4 \text{ kg}) (25 \text{ m/s})^2 = 1250 \text{ N}\cdot\text{m}$$

$$T = 1250 \text{ J}$$

$$W = m g = (4 \text{ kg}) (9.81 \text{ m/s}^2) = 39.240 \text{ N}$$

$$T_1 + U_{1-2} = T_2 \quad T_1 = 0 \quad U_{1-2} = W h \quad T_2 = 1250 \text{ N}\cdot\text{m}$$

$$h = \frac{T_2}{W} = \frac{(1250 \text{ N}\cdot\text{m})}{(39.240 \text{ N})} = 31.855 \text{ m}$$

$$h = 31.9 \text{ m}$$

(b) ON THE MOON

MASS IS UNCHANGED,  $m = 4 \text{ kg}$ THUS  $T$  IS UNCHANGED

$$T = 1250 \text{ J}$$

WEIGHT ON THE MOON IS,  $W_m = m g_m = (4 \text{ kg}) (1.62 \text{ m/s}^2)$

$$W_m = 6.48 \text{ N}$$

$$h_m = \frac{T}{W_m} = \frac{(1250 \text{ N}\cdot\text{m})}{(6.48 \text{ N})} = 192.9 \text{ m}$$

$$h_m = 192.9 \text{ m}$$

13.5

GIVEN: DISTANCE  $d = 120 \text{ m}$  $\mu_s = 0.75$ , NO SLIPPING

60% OF WEIGHT ON FRONT WHEELS

40% OF WEIGHT ON REAR WHEELS

FIND: MAXIMUM THEORETICAL SPEED AT  
 120 M STARTING FROM REST  
 (a) FOR FRONT WHEEL DRIVE  
 (b) FOR REAR WHEEL DRIVE

(a) FRONT WHEEL DRIVE

SINCE 60% OF WEIGHT IS DISTRIBUTED ON FRONT  
 WHEELS, THE MAXIMUM FORCE TO MOVE THE CAR  
 IS  $F = \mu_s N = (0.75) (0.6 W) = 0.450 m g$

$$\text{FOR } 120 \text{ m } U_{1-2} = (0.450 m g) (120 \text{ m}) = 54 m g$$

$$T_1 = 0$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 54 m g = \frac{1}{2} m v_2^2$$

$$v_2^2 = (2) (54 g) = (108) (9.81 \text{ m/s}^2)$$

$$v_2^2 = 1059.5$$

$$v_2 = 32.55 \text{ m/s}$$

$$v_2 = 117.2 \text{ km/h}$$

(b) REAR WHEEL DRIVE

USE SAME SOLUTION AS FOR (a) EXCEPT THAT  
 40% WEIGHT IS DISTRIBUTED ON REAR WHEELS

$$F = \mu_s N = (0.75) (0.40 W) = 0.3 m g$$

$$\text{FOR } 120 \text{ m } U_{1-2} = (0.3 m g) (120 \text{ m}) = 36 m g$$

$$T_1 = 0$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 36 m g = \frac{1}{2} m v_2^2$$

$$v_2^2 = (2) (36 g) = (72) (9.81 \text{ m/s}^2) = 706.32$$

$$v_2 = 26.58 \text{ m/s}$$

$$v_2 = 95.7 \text{ km/h}$$

NOTE: THE CAR IS TREATED AS A PARTICLE IN THIS  
 PROBLEM. THE WEIGHT DISTRIBUTION IS ASSUMED  
 TO BE THE SAME FOR STATIC AND DYNAMIC  
 CONDITIONS. COMPARE WITH SAMPLE PROBLEM  
 16.1 WHERE THE VEHICLE IS TREATED AS  
 A RIGID BODY.

13.6



**GIVEN:** 1320 ft drag race track, car starts from rest cars' front wheels off the ground for first 60 ft wheels roll without slipping for remaining 1260 ft with 60% of weight on rear wheels  $\mu_k = 0.60$ ,  $\mu_s = 0.85$ , no air or rolling resistance

**FIND:** (a) speed of the car at end of first 60 ft  
(b) maximum theoretical speed at finish line

(a) **FIRST 60 ft:** REAR WHEELS SKID TO GENERATE THE MAXIMUM FORCE. SINCE ALL THE WEIGHT IS ON THE REAR WHEELS THIS FORCE IS:

$$F = \mu_k N = (0.60)(W)$$

$$T_1 = 0 \quad T_2 = \frac{1}{2} W \quad v_{60}^2$$

FOR FIRST 60 ft

$$U_{1-2} = (F)(60 \text{ ft}) = (0.6W)(60) = 36W$$

$$T_1 + U_{1-2} = T_2$$

$$36W = \frac{1}{2} W v_{60}^2$$

$$v_{60}^2 = 72$$

$$v_{60} = 8.485 \text{ ft/s}$$

$$v_{60} = 32.8 \text{ mi/h}$$

(b) **FOR 1320 ft** REAR WHEELS SKID FOR FIRST 60 ft AND ROLL WITH SLIDING IMPENDING FOR REMAINING 1260 ft WITH 60% OF THE WEIGHT ON THE REAR (DRIVE) WHEELS. THE MAXIMUM FORCE GENERATED IS:

FIRST 60 ft  $F_1 = (0.6W)$  AS IN (a)  
REMAINING 1260 ft  $F_2 = \mu_s N = (0.85)(0.60)(W) = 0.510W$

$$T_1 = 0 \quad T_2 = \frac{1}{2} W v_{1320}^2$$

$$U_{1-2} = (0.6W)(60) + (0.510W)(1260)$$

$$= (36 + 642.6)W = 678.6W$$

$$0 + 678.6W = \frac{1}{2} W v_{1320}^2$$

$$v_{1320}^2 = 8310.2$$

$$v_{1320} = 91.16 \text{ ft/s}$$

$$v_{1320} = 209.05 \text{ ft/s}$$

$$v_{1320} = 142.5 \text{ mi/h}$$

SEE NOTE FOR PROB. 13.5 FOR DISCUSSION OF WEIGHT DISTRIBUTION

13.7



**GIVEN:** 1320 ft drag race track, car starts from rest. cars' front wheels off the ground and rear wheels skid for first 60 ft speed at end of first 60 ft is 36 mi/h. wheels roll with slipping impending for remaining 1260 ft, with 75% of the weight on rear (drive) wheels.  $\mu_k = 0.80$   
NO AIR OR ROLLING RESISTANCE

13.7 continued

**FIND:** SPEED OF CAR AT END OF RACE

**FIRST 60 ft:** SINCE ALL THE CAR'S WEIGHT IS ON THE REAR WHEELS WHICH SKID, THE FORCE MOVING THE CAR IS

$$F = \mu_k N = (0.80)(W)$$

$$v_{60} = (36 \text{ mi/h})(88 \text{ ft/s}) / (60 \text{ mi/h})$$

$$v_{60} = 52.8 \text{ ft/s}$$

$$T_1 = 0 \quad T_2 = \frac{1}{2} m v_{60}^2 = \frac{1}{2} (W/g)(52.8 \text{ ft/s})^2 = (1393.9)(W/g)$$

$$U_{1-2} = (F)(60 \text{ ft}) = (0.80)(W)(60 \text{ ft})$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 60 \mu_k W = (1393.9)(W/g)$$

$$\mu_k = \frac{(1393.9)}{(60)(32.2)} = 0.72149$$

**FOR 1320 ft** FORCE MOVING THE CAR IS

FOR FIRST 60 ft,  $F_1 = (0.80)(W) = (0.72149)W$   
FOR REMAINING 1260 ft, WITH 75% OF WEIGHT ON REAR (DRIVE) WHEELS AND IMPENDING SLIDING

$$F_2 = (\mu_s)(0.75)W \quad \mu_s = \mu_k(0.80) = (0.72149)(0.80)$$

$$F_2 = (0.90186)(0.75)W = 0.6764W \quad \mu_s = 0.90186$$

$$T_1 = 0 \quad T_2 = \frac{1}{2} (W/g)(v_{1320})^2$$

$$U_{1-2} = (F_1)(60 \text{ ft}) + F_2(1260 \text{ ft})$$

$$= (0.72149)(W)(60 \text{ ft}) + (0.6764)(W)(1260 \text{ ft})$$

$$= 43.29W + 852.3W = 895.55W$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 895.55W = \frac{1}{2} (W/g)(v_{1320})^2$$

$$v_{1320}^2 = (2g)(895.55) = (2)(32.2 \text{ ft/s}^2)(895.55)$$

$$v_{1320}^2 = 57,673 \quad v_{1320} = 240.2 \text{ ft/s}$$

$$v_{1320} = 163.7 \text{ mi/h}$$

SEE NOTE FOR PROB. 13.5

13.8



**GIVEN:** 400 m drag race track, car starts from rest front wheels off the ground and rear wheels skid for first 20 m.

WHEELS ROLL WITH SLIDING IMPENDING FOR REMAINING 380 m, WITH 80% OF THE WEIGHT ON THE REAR DRIVE WHEELS. PEAK SPEED AT END OF THE RACE,  $v = 270 \text{ km/h}$   
 $\mu_k = 0.75$

**FIND:**

- (a) COEFFICIENT OF STATIC FRICTION,  $\mu_s$   
(b) SPEED AT THE END OF THE FIRST 20 m

(a) FORCE MOVING THE CAR FOR THE FIRST 20 m, WITH ALL OF THE WEIGHT ON THE REAR DRIVE WHEELS AND THE WHEELS SKIDING,

$$\mu_k = 0.75 \quad F_1 = \mu_k N = \mu_k W = (0.75)(0.80)mg$$

FORCE MOVING THE CAR FOR REMAINING 380 m WITH 80% OF THE WEIGHT ON THE REAR (DRIVE) WHEELS AND SLIDING IMPENDING (CONTINUED)

### 13.8 continued

$$F_2 = 4_3 (0.80)(W) = 4_3 (0.80)(W) = 4_3 (1.60)mg$$

$$T_1 = 0 \quad v_{400} = (270 \frac{km}{h}) (\frac{1000m}{km}) / (3600 \frac{s}{h})$$

$$v_{400} = 75 m/s$$

$$T_2 = \frac{1}{2} m v_{400}^2 = \frac{1}{2} m (75)^2 = 2812.5 m$$

$$U_{1-2} = F_1 (20m) + F_2 (380m)$$

$$U_{1-2} = (4_3) (675) mg (20m) + (4_3) (1.60) mg (380)$$

$$U_{1-2} = 15 4_3 mg + 304 4_3 mg = 319 4_3 mg$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 319 4_3 mg = 2812.5 m$$

$$4_3 = (2812.5) / (319)(9.81) = 0.8987$$

$$4_3 = 0.899$$

(b) FOR FIRST 20 m

$$4_k = (0.75)(4_3) = 0.6741$$

$$F_1 = 4_k N = (0.6741)(mg)$$

$$T_1 = 0 \quad T_2 = \frac{1}{2} m v_{20}^2$$

$$U_{1-2} = (0.6741)(mg)(20m) = 13.481 mg$$

$$0 + (13.481)(mg) = \frac{1}{2} m v_{20}^2$$

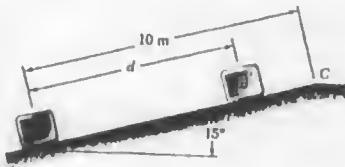
$$v_{20}^2 = (2)(13.481)(9.81) = 264.5$$

$$v_{20} = v_0 = 26 m/s$$

SEE NOTE FOR P13.5

$$v_{20} = 58.6 km/h$$

### 13.9



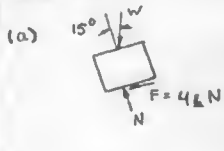
GIVEN:  $v_{ATC} = 0$

$$4_k = 0.12$$

FIND:

(a) INITIAL  $v$  AT A

(b)  $v$  AS PACKAGE RETURNS TO A



UP THE PLANE, FROM A TO C,  $v_C = 0$

$$T_A = \frac{1}{2} m v_A^2, \quad T_C = 0$$

$$U_{A-C} = (-W \sin 15^\circ - F)(10m)$$

$$\sum F = 0 \quad N - W \cos 15^\circ = 0$$

$$N = W \cos 15^\circ$$

$$F = 4_k N = 0.12 W \cos 15^\circ$$

$$U_{A-C} = -W (\sin 15^\circ + 0.12 \cos 15^\circ)(10m)$$

$$T_A + U_{A-C} = T_C \quad \frac{1}{2} m v_A^2 - W (\sin 15^\circ + 0.12 \cos 15^\circ)(10m)$$

$$v_A^2 = (2)(9.81)(\sin 15^\circ + 0.12 \cos 15^\circ)(10m)$$

$$v_A^2 = 73.5$$

$$v_A = 8.57 m/s$$

(b) DOWN THE PLANE FROM C TO A

$$T_C = 0 \quad T_A = \frac{1}{2} m v_A^2 \quad U_{C-A} = (W \sin 15^\circ - F) 10$$

(F REVERSES DIRECTION)

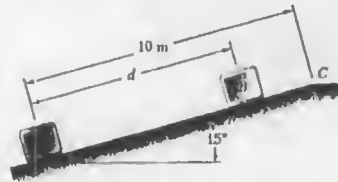
$$T_C + U_{C-A} = T_A \quad 0 + W (\sin 15^\circ - 0.12 \cos 15^\circ)(10m) = \frac{1}{2} m v_A^2$$

$$v_A^2 = (2)(9.81)(\sin 15^\circ - 0.12 \cos 15^\circ)(10m)$$

$$v_A^2 = 28.039$$

$$v_A = 5.30 m/s$$

### 13.10



GIVEN:  $v_{AT A} = 8 m/s$

$$4_k = 0.12$$

FIND:

(a) DISTANCE  $d$  PACKAGE MOVES UP THE PLANE

(b) VELOCITY  $v_A$  AS PACKAGE RETURNS TO A.

(a) UP THE PLANE FROM A TO B

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} m (8 m/s)^2 = 32 \frac{W}{g} \quad T_B = 0$$

$$U_{A-B} = (-W \sin 15^\circ - F)d \quad F = 4_k N = 0.12 N$$

$$\sum F = 0 \quad N - W \cos 15^\circ = 0 \quad N = W \cos 15^\circ$$

$$U_{A-B} = -W (\sin 15^\circ + 0.12 \cos 15^\circ)d = -Wd(0.3747)$$

$$T_A + U_{A-B} = T_B \quad 32 \frac{W}{g} - Wd(0.3743) = 0$$

$$d = (32) / (9.81)(0.3747)$$

$$d = 8.70 m$$

(b) DOWN THE PLANE FROM B TO A (F REVERSES DIRECTION)

$$T_A = \frac{1}{2} m v_A^2 \quad T_B = 0 \quad d = 8.72 m/s$$

$$U_{B-A} = (W \sin 15^\circ - F)d = W (\sin 15^\circ - 0.12 \cos 15^\circ)(8.70m)$$

$$U_{B-A} = 1.245 W$$

$$T_B + U_{B-A} = T_A$$

$$0 + 1.245 W = \frac{1}{2} m v_A^2$$

$$v_A^2 = (2)(9.81)(1.245) = 253.9$$

$$v_A = 4.94 m/s$$

$$v_A = 4.94 m/s$$

### 13.11

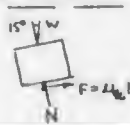


GIVEN: AT A,  $v = v_0$

FOR AB,  $4_k = 0.40$

AT B,  $v = 3 ft/s$

FIND:  $v_0$



$$T_A = \frac{1}{2} m v_0^2 \quad T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (3 ft/s)^2$$

$$T_B = 32 m$$

$$U_{A-B} = (W \sin 15^\circ - 4_k N)(20 ft)$$

$$\sum F = 0 \quad N - W \cos 15^\circ = 0$$

$$N = W \cos 15^\circ$$

$$U_{A-B} = W (\sin 15^\circ - 0.40 \cos 15^\circ)(20 ft)$$

$$U_{A-B} = -(2.551)(W) = -2.551 mg$$

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_0^2 - 2.551 mg = 32 m$$

$$v_0^2 = (2)(32 + (2.551)(32.2 ft/s^2))$$

$$v_0^2 = 228.29$$

$$v_0 = 15.11 ft/s$$

13.12



GIVEN: AT A,  $v = v_0$   
 AT B,  $v = 0$   
 FOR AB,  $\mu_k = 0.40$   
 FIND:  $v_0$

$$T_A = 2m v_0^2 \quad T_B = 0$$

$$U_{A-B} = (W \sin 15^\circ - \mu_k N)(20 \text{ ft})$$

$$\sum F = 0 \quad N - \cos 15^\circ = 0$$

$$N = W \cos 15^\circ$$

$$U_{A-B} = W(\sin 15^\circ - 0.40 \cos 15^\circ)(20 \text{ ft})$$

$$U_{A-B} = -(2.551)(W) = -2.551 mg$$

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_0^2 - 2.551 mg = 0$$

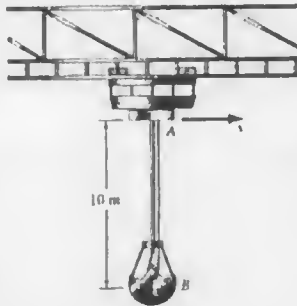
$$v_0^2 = (2)(2.551)(32.2 \text{ ft/s}^2)$$

$$v_0^2 = 164.28$$

$$v_0 = 12.82 \text{ ft/s}$$

DOWN TO THE LEFT

13.14



GIVEN: CRANE MOVES AT  
 VELOCITY  $v = 3 \text{ m/s}$   
 AND STOPS  
 SUDDENLY  
 FIND: MAXIMUM HORIZONTAL  
 DISTANCE  $d$  MOVED  
 BY THE BUCKET

REFER TO FREE BODY DIAGRAM IN P.13.13

$$v_1 = v = 3 \text{ m/s} \quad T_1 = \frac{1}{2} m v^2 = \frac{1}{2} m (3 \text{ m})^2 = 4.5 \text{ m}$$

$$T_2 = 0$$

$$U_{1-2} = -mgh$$

$$T_1 + U_{1-2} = T_2$$

$$4.5 \text{ m} - mgh = 0$$

$$h = \frac{4.5}{9.81} = 0.4587$$

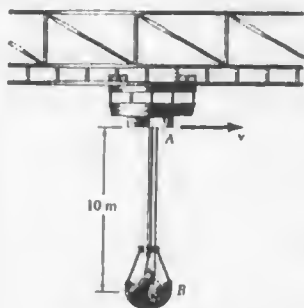
$$\overline{AB}^2 = (10)^2 = d^2 + y^2 = d^2 + (10 - 0.4587)^2$$

$$100 = d^2 + 91.04$$

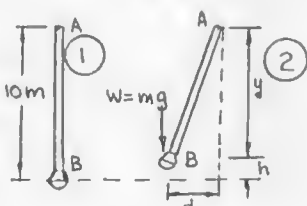
$$d^2 = 8.96$$

$$d = 2.99 \text{ m}$$

13.13



GIVEN: CRANEMOVES AT  
 VELOCITY  $v$  AND  
 STOPS SUDDENLY  
 BUCKET IS TO  
 SWING NO MORE  
 THAN 4 M  
 HORIZONTALLY  
 FIND: MAXIMUM ALLOWABLE  
 VELOCITY  $v$



$$v_1 = v$$

$$v_2 = 0$$

$$T_1 = \frac{1}{2} m v^2$$

$$T_2 = 0$$

$$U_{1-2} = -mgh \quad d = 4 \text{ m}$$

$$\overline{AB}^2 = (10 \text{ m})^2 = d^2 + y^2 = (4 \text{ m})^2 + y^2$$

$$y^2 = 100 - 16 = 84 \quad y = \sqrt{84}$$

$$h = 10 - y = 10 - \sqrt{84} = 0.8349 \text{ m}$$

$$U_{1-2} = -m(9.81)(0.8349) = -0.8190 \text{ m}$$

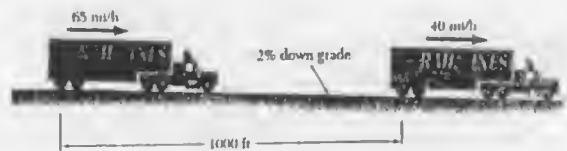
$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m v^2 - 0.8190 \text{ m} = 0$$

$$v^2 = (2)(0.8190) = 16.38$$

$$v = 4.05 \text{ m/s}$$

13.15



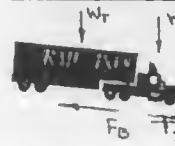
GIVEN: CAB WEIGHT,  $W_c = 4000 \text{ lb}$   
 TRAILER WEIGHT,  $W_T = 12,000 \text{ lb}$ , 2% GRADE  
 70% BRAKING FORCE SUPPLIED BY TRAILER  
 30% BRAKING FORCE SUPPLIED BY CAB

FIND:

(a) AVERAGE BRAKING FORCE TO SLOW DOWN  
 FROM 65 mi/h TO 40 mi/h AS SHOWN

(b) AVERAGE FORCE BETWEEN CAB AND TRAILER

(a) CAB-TRAILER SYSTEM



$$v_1 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$$

$$v_2 = 40 \text{ mi/h} = 58.67 \text{ ft/s}$$

$$T_1 = \frac{1}{2} (m_T + m_c) v_1^2 = \frac{1}{2} (m_T + m_c) (95.33 \text{ ft/s})^2$$

$$T_1 = (4,544) (m_T + m_c)$$

$$T_2 = \frac{1}{2} (m_T + m_c) v_2^2 = \frac{1}{2} (m_T + m_c) (58.67 \text{ ft/s})^2$$

$$T_2 = (1721) (m_T + m_c)$$

$$T_1 + U_{1-2} = T_2 \quad U_{1-2} = -1000 F_B + (W_T + W_c)(20 \text{ ft})$$

$$4544 (m_T + m_c) - 1000 F_B + (W_T + W_c)(20 \text{ ft}) = 1721 (m_T + m_c)$$

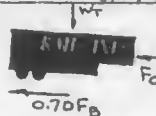
$$F_B = \left[ (4544 - 1721) \left( \frac{16,000}{32.2} \right) + (16,000)(20) \right] \left( \frac{1}{1000} \right) = 1722.7$$

$$F_B = 1723 \text{ lb}$$

(CONTINUED)

### 13.15 continued

(b) TRAILER CONSIDERED SEPARATELY



$$T_1 = \frac{1}{2} (m_T) (45.33)^2 = 4544 \text{ mT}$$

$$F_c T_2 = \frac{1}{2} (m_T) (58.67)^2 = 1721 \text{ mT}$$

$$T_1 + U_{1-2} = T_2 \quad 4544 \text{ mT} - 1000 (F_c + 0.70 F_B) + 20 W_T = 1721 \text{ mT}$$

FROM (a)  $F_B = 1722.7$

$$1000 F_c = (4544 - 1721) \left( \frac{12,000}{32.2} \right) - (100)(1722.7) + (20)(12,000)$$

$$F_c = (1052) - 12059 + 240 = 86.1 \text{ lb}$$

SEE NOTE FOR P13.5

$$F_c = 86.1 \text{ lb (C)}$$

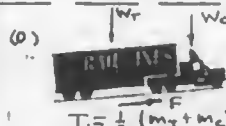
### 13.16



GIVEN: CAB WEIGHT,  $W_c = 4000 \text{ lb}$ ; TRAILER WEIGHT,  $W_T = 12,000 \text{ lb}$   
2% UP GRADE

FIND (a) AVERAGE FORCE ON THE WHEELS TO SPEED UP, F  
(b) AVERAGE FORCE IN THE COUPLING

(a) CAB-TRAILER SYSTEM



$$v_1 = 40 \text{ mi/h} = 58.67 \text{ ft/s}$$

$$v_2 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$$

$$T_1 = \frac{1}{2} (m_T + m_c) (v_1)^2 = \frac{1}{2} (m_T + m_c) (58.67)^2 = 1721 (m_T + m_c)$$

$$T_2 = \frac{1}{2} (m_T + m_c) (v_2)^2 = \frac{1}{2} (m_T + m_c) (95.33)^2 = 4544 (m_T + m_c)$$

$$T_1 + U_{1-2} = T_2 \quad U_{1-2} = (1000)(F) - (1000)(2/100)(W_T + W_c)$$


$$1721 (m_T + m_c) + 1000 F - 20 (W_T + W_c) = 4544 (m_T + m_c)$$

$$1000 F = (4544 - 1721) \left( \frac{16,000}{32.2} \right) + 20 (16,000)$$

$$F = 1403 + 320 = 1723 \text{ lb}$$

$$F = 1723 \text{ lb}$$

(b) TRAILER CONSIDERED SEPARATELY



$$T_1 = \frac{1}{2} (m_T) (58.67)^2 = 1721 \text{ mT}$$

$$T_2 = \frac{1}{2} (m_T) (95.33)^2 = 4544 \text{ mT}$$

$$T_1 + U_{1-2} = T_2 \quad 1721 \text{ mT} + 1000 F_c - (1000) \left( \frac{2}{100} \right) W_T = 4544 \text{ mT}$$

$$1000 F_c = (4544 - 1721) \left( \frac{12,000}{32.2} \right) + (20)(12,000)$$

$$F_c = 1052 + 240 = 1292 \text{ lb}$$

$$F_c = 1292 \text{ lb (T)}$$

SEE NOTE FOR P13.5

### 13.17

GIVEN: 2000 kg CAB

8000 kg TRAILER  
LEVEL GROUND.

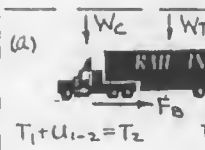
TRUCK COMES TO  
ASTOP IN 1200 M.  
60% OF BRAKING  
FORCE FROM TRAILER  
90% OF BRAKING  
FORCE FROM CAB

FIND: (a) AVERAGE BRAKING  
FORCE

(b) AVERAGE FORCE  
IN THE COUPLING



(a) TRAILER AND CAB



$$v_i = (90 \text{ km/h}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)$$

$$v_i = 25 \text{ m/s} \quad v_f = 0$$

$$T_1 + U_{1-2} = T_2 \quad T_1 = \frac{1}{2} (m_T + m_c) (v_i)^2 = \left( \frac{1}{2} \right) (10,000 \text{ kg}) (25 \text{ m/s})^2$$

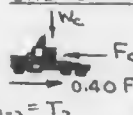
$$T_1 = 3125 \times 10^3 \text{ N-m} \quad T_2 = 0$$

$$3125 \times 10^3 - (1200 \text{ m}) (F_B) = 0$$

$$F_B = \frac{(3125 \times 10^3 \text{ N-m})}{(1200 \text{ m})} = 2604 \text{ N-m}$$

$$F_B = 2.60 \text{ kN}$$

(b) CAB CONSIDERED SEPARATELY



$$T_1 = \frac{1}{2} m_c (v_i)^2 = (1000) (25 \text{ m/s})^2$$

$$T_1 = 625 \times 10^3 \text{ N-m} \quad T_2 = 0$$

$$T_1 + U_{1-2} = T_2$$

$$F_B = 2604 \text{ N-m (FROM (a))}$$

$$625 \times 10^3 - (0.40)(2604)(1200) + (F_c)(1200) = 0$$

$$F_c = (0.40)(2604) - \frac{625}{1.2} = 1042 - 521$$

SEE NOTE FOR P13.5

$$F_c = 521 \text{ N (C)}$$

### 13.18

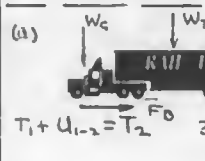
GIVEN: 2000 kg CAB, 8000 kg TRAILER  
AVERAGE BRAKING FORCE 3000 N  
LEVEL GROUND

FIND: (a) DISTANCE X, TO COME TO  
A STOP

(b) FORCE IN COUPLING,  $F_c$   
(TRAILER BRAKES ONLY)



(a) TRAILER AND CAB



$$v_i = 25 \text{ m/s}$$

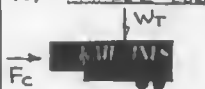
$$T_1 = \frac{1}{2} (m_T + m_c) (25)^2 = 3125 \times 10^3 \text{ J}$$

$$T_2 = 0 \quad U_{1-2} = F_B X$$

$$T_1 + U_{1-2} = T_2 \quad 3125 \times 10^3 - (3000) X = 0$$

$$X = 1042 \text{ m}$$

(b) TRAILER CONSIDERED SEPARATELY



$$T_1 = \frac{1}{2} m_T (25)^2 = (4000)(625)$$

$$T_1 = 2500 \times 10^3 \text{ J}$$

$$T_2 = 0$$

$$T_1 + U_{1-2} = T_2$$

$$2500 \times 10^3 - (F_c)(X) = 0$$

$$\text{FROM (a)} \quad X = 1042 \text{ m}$$

$$2500 \times 10^3 - F_c (1042) = 0$$

$$F_c = \frac{2500 \times 10^3}{1042} = 2399.2 \text{ N}$$

SEE NOTE FOR P13.5

$$F_c = 2.40 \text{ kN (C)}$$



13.19

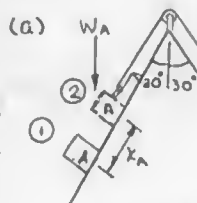
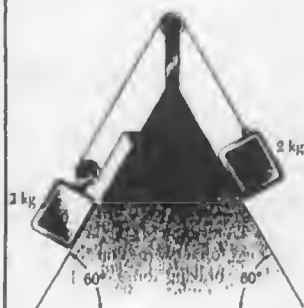
GIVEN:

BLOCKS RELEASED FROM REST; NO FRICTION

FIND:

(a) VELOCITY OF BLOCK B AFTER IT HAS MOVED 2 m.

(b) TENSION IN THE CABLE.

KINEMATICS  $x_B = 2x_A$   
 $v_B = 2v_A$ 

A AND B

ASSUME B MOVES DOWN

 $v_i = 0$   $T_i = 0$ 

$$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} (2 \text{ kg}) \left( \frac{v_B^2}{4} + v_B^2 \right)$$

$$T_2 = \frac{5}{4} v_B^2$$

$$U_{1-2} = -m_A g (\cos 30^\circ) (x_A) + m_B g (\cos 30^\circ) x_B$$

$$x_B = 2 \text{ m}$$

$$x_A = 1 \text{ m}$$

$$U_{1-2} = (2)(9.81) \left( \frac{\sqrt{3}}{2} \right) [-1 + 2]$$

$$U_{1-2} = 16.99 \text{ J}$$

SINCE WORK IS POSITIVE BLOCK B DOES MOVE DOWN

$$T_1 + U_{1-2} = T_2$$

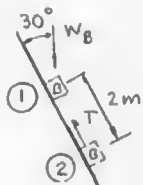
$$0 + 16.99 = \frac{5}{4} v_B^2$$

$$v_B^2 = 13.59$$

$$v_B = 3.69 \text{ m/s}$$

DOWN TO THE RIGHT

(b)



B ALONE

$$v_1 = 0$$

$$T_1 = 0$$

$$v_2 = 3.69 \text{ m/s (FROM (a))}$$

$$T_2 = \frac{1}{2} m_B v_2^2 = \frac{1}{2} (2) (3.69)^2 = 13.59 \text{ J}$$

$$U_{1-2} = (m_B g) (\cos 30^\circ) (x_B) - (T) (x_B)$$

$$U_{1-2} = [(2 \text{ kg}) (9.81 \text{ m/s}^2) \left( \frac{\sqrt{3}}{2} \right) - (T)] (2 \text{ m})$$

$$U_{1-2} = 33.98 - 2T$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 33.98 - 2T = 13.59$$

$$2T = 33.98 - 13.59 = 20.39$$

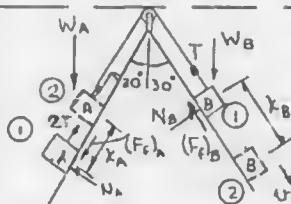
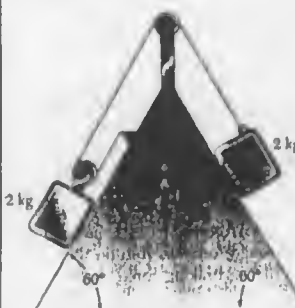
$$T = 10.19 \text{ N}$$

13.20

GIVEN:

BLOCKS RELEASED FROM REST; FRICTION  $\mu_s = 0.30$ ,  $\mu_k = 0.20$ 

FIND:

(a) VELOCITY OF BLOCK B AFTER IT HAS MOVED 2 m.  
(b) TENSION IN THE CABLE.

CHECK AT ① TO SEE IF BLOCKS MOVE. WITH MOTION IMPENDING AT B DOWNWARD DETERMINE REQUIRED FRICTION FORCE AT A FOR EQUILIBRIUM

BLOCK B

$$\sum F = N_B - (m_B g) (\sin 30^\circ) = 0$$

$$N_B = (2 \text{ g}) \left( \frac{1}{2} \right) = \text{g}$$

$$\sum F = T - (m_B g) (\cos 30^\circ) + (F_B)_f = 0$$

$$(F_B)_f = \mu_s N_B = (0.30) (\text{g})$$

$$T = (2 \text{ g}) \left( \frac{\sqrt{3}}{2} \right) - (0.30) \text{g}$$

$$T = (\sqrt{3} - 0.30) (\text{g})$$

BLOCK A

$$\sum F = N_A - (m_A g) (\sin 30^\circ) = 0$$

$$N_A = (2 \text{ g}) \left( \frac{1}{2} \right) = \text{g}$$

$$\sum F = 2T - (m_A g) (\cos 30^\circ) - (F_A)_f = 0$$

$$(F_A)_f = 2T - (2 \text{ g}) \left( \frac{\sqrt{3}}{2} \right)$$

SUBSTITUTE T FROM ① INTO ②

$$(F_A)_f = (2) (\sqrt{3} - 0.30) (\text{g}) - \sqrt{3} \text{g}$$

$$\text{REQ. FOR EQUIL } (F_A)_f = (\sqrt{3} - 0.60) \text{g} = 1.132 \text{ g}$$

MAX FRICTION THAT CAN BE DEVELOPED AT A =

$$\mu_s N_A = 0.30 \text{g}$$

SINCE  $0.30 \text{g} < 1.132 \text{g}$ ; BLOCKS MOVE

(a) A AND B

$$(F_A)_f = \mu_k N_B = (0.20) \text{g} \quad (F_A)_f = \mu_k N_A = (0.20) \text{g}$$

$$\text{KINEMATICS } x_B = 2x_A \quad v_B = 2v_A$$

$$v_1 = 0, T_1 = 0, T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} (2 \text{ kg}) \left( \frac{v_B^2}{4} + v_B^2 \right)$$

$$T_2 = \frac{5}{4} v_B^2$$

$$U_{1-2} = -m_A g (\cos 30^\circ) (x_A) + m_B g (\cos 30^\circ) x_B - (F_A)_f (x_A) - (F_B)_f (x_B)$$

$$x_B = 2 \text{ m}, x_A = 1 \text{ m}$$

$$U_{1-2} = [-(2 \text{ kg}) \left( \frac{\sqrt{3}}{2} \right) (1 \text{ m}) + (2 \text{ kg}) \left( \frac{\sqrt{3}}{2} \right) (2 \text{ m}) - (0.20) (1 \text{ m}) - (0.20) (2 \text{ m})] [9.81 \text{ m/s}^2]$$

$$U_{1-2} = [(1.732) - (0.6)] [9.81] = 11.105 \text{ J}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 11.105 = \frac{5}{4} v_B^2$$

$$v_B^2 = 8.88$$

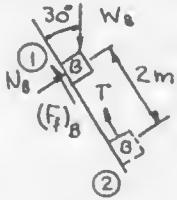
$$v_B = 2.98 \text{ m/s}$$

DOWN TO THE RIGHT

(CONTINUED)

# 13.20 continued

(b)



B ALONE

$$v_1 = 0 \quad T_1 = 0$$

$$v_2 = 2.98 \text{ m/s (FROM (a))}$$

$$T_2 = \frac{1}{2} m_B v_B^2 = \left(\frac{1}{2}\right)(2)(2.98)^2$$

$$N_B = m_B g \sin 30^\circ = 9 \text{ N}$$

$$T_2 = 8.88 \text{ J}$$

$$U_{1-2} = m_B g (\cos 30^\circ)(2) - (T_1)(2) - (F_f)_B(2)$$

$$U_{1-2} = (2 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{\sqrt{3}}{2}\right)(2\text{m}) - 2T - (0.2)(9 \text{ N})(2\text{m})$$

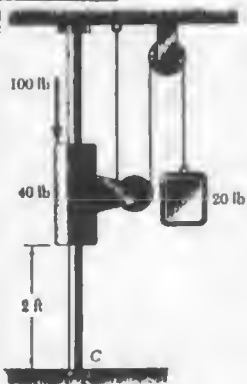
$$U_{1-2} = 2\sqrt{3}g - 2T - 0.6g$$

$$T_1 + U_{1-2} = T_2 \quad 0 + 2\sqrt{3}g - 2T - 0.4g = 8.88$$

$$2T = (2\sqrt{3} - 0.4)(g) - 8.88 = 21.179$$

$$T = 10.59 \text{ N}$$

## 13.21

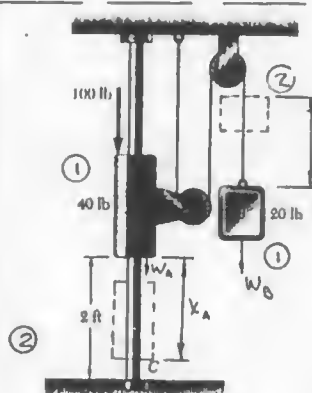


GIVEN:

SYSTEM AT REST WHEN 100 lb FORCE IS APPLIED TO. NO FRICTION. IGNORE PULLEYS MASS

FIND:

- (a) VELOCITY,  $v_A$  OF A JUST BEFORE IT HITS C  
(b)  $v_A$  IF COUNTERWEIGHT B IS REPLACED BY A 20-lb DOWNWARD FORCE



KINEMATICS

$$x_B = 2x_A$$

$$v_B = 2v_A$$

(a) BLOCKS A AND B

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2$$

$$T_2 = \frac{1}{2} (20 \text{ lb}/32.2 \text{ ft/s}^2)(2v_A)^2 + \frac{1}{2} (40 \text{ lb}/32.2 \text{ ft/s}^2)(v_A)^2$$

$$T_2 = (60/32.2)(v_A)^2$$

$$U_{1-2} = (100)(x_A) + (W_A)(x_A) - (W_B)(x_B)$$

$$U_{1-2} = (100 \text{ lb})(2 \text{ ft}) + (40 \text{ lb})(2 \text{ ft}) - (20 \text{ lb})(4 \text{ ft})$$

$$U_{1-2} = 200 + 80 - 80 = 200 \text{ lb}\cdot\text{ft}$$

(CONTINUED)

# 13.21 continued

$$T_1 + U_{1-2} = T_2$$

$$0 + 200 = (60/32.2) v_A^2$$

$$v_A^2 = 107.33$$

$$v_A = 10.36 \text{ ft/s}$$

(b) SINCE THE 20 lb WEIGHT AT B IS REPLACED BY A 20 lb FORCE THE KINETIC ENERGY AT ② IS  $T_2 = \frac{1}{2} m_A v_A^2 = \frac{1}{2} (40/g) v_A^2 \quad T_1 = 0$

THE WORK DONE IS THE SAME AS IN PART (a)

$$U_{1-2} = 200 \text{ lb}\cdot\text{ft}$$

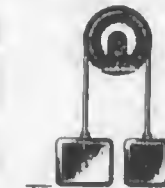
$$T_1 + U_{1-2} = T_2$$

$$0 + 200 = (20/g) v_A^2$$

$$v_A^2 = 32.2$$

$$v_A = 17.94 \text{ ft/s}$$

## 13.22



GIVEN:

$$m_A = 11 \text{ kg} \quad m_B = 5 \text{ kg}$$

$$h = 2 \text{ m}$$

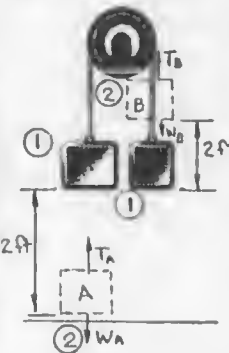
SYSTEM RELEASED FROM REST

$v_A = 3 \text{ m/s}$  JUST BEFORE HITTING THE GROUND

FIND:

(a) ENERGY,  $E_p$ , DISSIPATED IN FRICTION

(b) TENSION IN EACH PORTION OF CORD



(a)  $v_1 = 0 \quad T_1 = 0$  ENERGY DISSIPATED

$$v_2 = v_A = 3 \text{ m/s} = v_B$$

$$T_2 = \frac{1}{2} (m_A + m_B) v_2^2$$

$$T_2 = \left(\frac{16}{2} \text{ kg}\right)(3 \text{ m/s})^2 = 72 \text{ J}$$

$$U_{1-2} = m_A g(2) - m_B g(2) - E_p$$

$$U_{1-2} = (6 \text{ kg})(9.81 \text{ m/s}^2)(2 \text{ m}) - E_p$$

$$U_{1-2} = 117.72 - E_p$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 117.72 - E_p = 72$$

$$E_p = 117.72 - 72 = 45.7 \text{ J}$$

(b) BLOCK A

$$T_1 = 0 \quad T_2 = \frac{1}{2} m_A v_2^2 = \left(\frac{11}{2} \text{ kg}\right)(3 \text{ m/s})^2 = 49.5 \text{ J}$$

$$U_{1-2} = (m_A g - T_A)(2) = [(11 \text{ kg})(9.81 \text{ m/s}^2) - T_A](2 \text{ m})$$

$$U_{1-2} = 215.82 - 2T_A$$

$$T_1 + U_{1-2} = T_2 \quad 0 + 215.82 - 2T_A = 49.5$$

$$T_A = 83.2 \text{ N}$$

BLOCK B

$$T_1 = 0 \quad T_2 = \frac{1}{2} m_B v_2^2 = \left(\frac{5}{2} \text{ kg}\right)(3 \text{ m/s})^2 = 22.5 \text{ J}$$

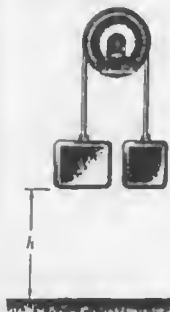
$$U_{1-2} = -m_B g(2) + T_B(2) = -(5 \text{ kg})(9.81 \text{ m/s}^2)(2 \text{ m}) + 2T_B$$

$$T_1 + U_{1-2} = T_2 \quad U_{1-2} = -98.1 + 2T_B$$

$$0 - 98.1 + 2T_B = 22.5$$

$$T_B = 60.3 \text{ N}$$

13.23



GIVEN:

$$W_A = 20 \text{ lb}; W_B = 8 \text{ lb}$$

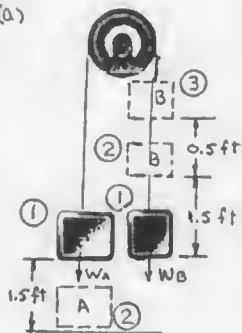
$$h = 1.5 \text{ ft}$$

SYSTEM RELEASED FROM REST  
BLOCK A HITS THE GROUND WITHOUT REBOUND  
BLOCK B REACHES A HEIGHT OF 3.5 ft

FIND:

- (a)  $v_A$  JUST BEFORE BLOCK A HITS THE GROUND  
(b) ENERGY,  $E_p$ , DISSIPATED BY THE PULLEY IN FRICTION

(a)



$v_B$  AT ② =  $v_A$  AT ② JUST BEFORE IMPACT  
FROM ② TO ③; BLOCK B

$$T_3 = 0$$

$$T_2 = \frac{1}{2} m_B v_B^2$$

$$T_2 = \frac{1}{2} (8 \text{ lb} / 32.2 \text{ ft/s}^2) v_B^2$$

$$T_2 = 0.1242 v_B^2$$

TENSION IN THE CORD IS ZERO

$$\text{THUS } U_{2-3} = (8 \text{ lb})(0.5 \text{ ft})$$

$$U_{2-3} = 4 \text{ lb}\cdot\text{ft}$$

$$T_2 + U_{2-3} = T_3$$

$$0.1242 v_B^2 = 4$$

$$v_B^2 = 32.2 = v_A^2$$

$$v_A = 5.68 \text{ ft/s}$$

(b) FROM ① TO ②

BLOCKS A AND B

$$T_1 = 0 \quad T_2 = \frac{1}{2} (m_A + m_B) v_2^2$$

JUST BEFORE IMPACT,  $v_2 = v_B = v_A = 5.68 \text{ ft/s}$

$$T_2 = \frac{1}{2} (28 \text{ lb} / 32.2 \text{ ft/s}^2) (5.68)^2$$

$$T_2 = 14 \text{ lb}\cdot\text{ft}$$

$$U_{1-2} = (W_A)(1.5) - (W_B)(1.5) - E_p$$

( $E_p$  = ENERGY DISSIPATED BY PULLEY)

$$U_{1-2} = (12 \text{ lb})(1.5 \text{ ft}) - E_p = 18 - E_p$$

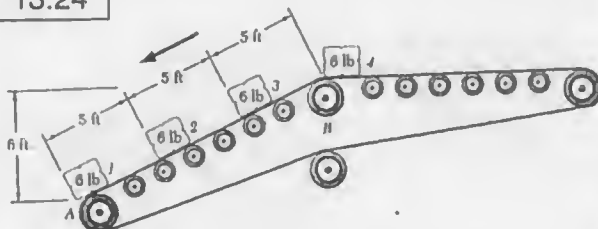
$$T_1 + U_{1-2} = T_2$$

$$0 + 18 - E_p = 14$$

$$-E_p = 14 - 18$$

$$E_p = 4.00 \text{ ft}\cdot\text{lb}$$

13.24



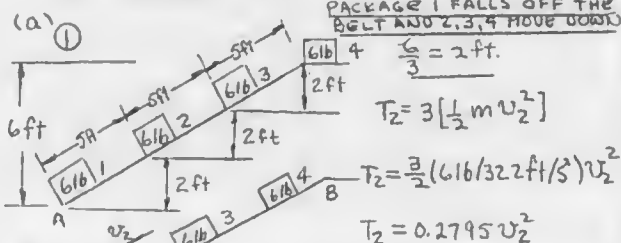
GIVEN:

CONVEYOR IS DISENGAGED, PACKAGES HELD BY FRICTION AND SYSTEM IS RELEASED FROM REST. NEGLECT MASS OF BELT AND ROLLERS. PACKAGE 1 LEAVES THE BELT AS PACKAGE 4 COMES ONTO THE BELT.

FIND:

- (a) VELOCITY OF PACKAGE 2 AS IT LEAVES THE BELT AT A  
(b) VELOCITY OF PACKAGE 3 AS IT LEAVES THE BELT AT A.

(a)



PACKAGE 1 FALLS OFF THE BELT AND 2, 3, 4 MOVE DOWN

$$\frac{6}{3} = 2 \text{ ft.}$$

$$T_2 = 3 \left[ \frac{1}{2} m v_2^2 \right]$$

$$T_2 = \frac{3}{2} (6 \text{ lb} / 32.2 \text{ ft/s}^2) v_2^2$$

$$T_2 = 0.2795 v_2^2$$

$$U_{1-2} = (3)(W)(2) = (3)(6 \text{ lb})(2 \text{ ft})$$

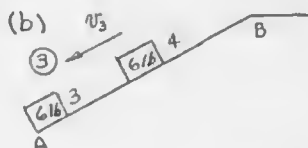
$$U_{1-2} = 36 \text{ lb}\cdot\text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 36 = 0.2795 v_2^2$$

$$v_2^2 = 128.8$$

$$v_2 = 11.35 \text{ ft/s}$$



PACKAGE 2 FALLS OFF THE BELT AND ITS ENERGY IS LOST TO THE SYSTEM AND 3 AND 4 MOVE DOWN 2 ft.

$$T_2' = (2) \left[ \frac{1}{2} m v_2^2 \right]$$

$$T_2' = (6 \text{ lb} / 32.2 \text{ ft/s}^2) (128.8)$$

$$T_2' = 24 \text{ lb}\cdot\text{ft}$$

$$T_3 = (2) \left[ \frac{1}{2} m v_3^2 \right]$$

$$T_3 = (6 \text{ lb} / 32.2 \text{ ft/s}^2) (v_3^2)$$

$$T_3 = 0.18634 v_3^2$$

$$U_{2-3} = (2)(W)(2) = (2)(6 \text{ lb})(2 \text{ ft})$$

$$U_{2-3} = 24 \text{ lb}\cdot\text{ft}$$

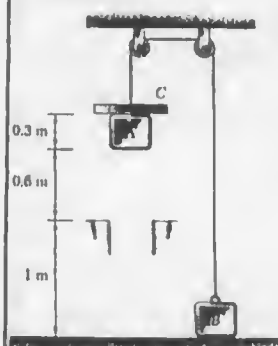
$$T_2' + U_{2-3} = T_3$$

$$24 + 24 = 0.18634 v_3^2$$

$$v_3^2 = 257.6$$

$$v_3 = 16.05 \text{ ft/s}$$

13.25

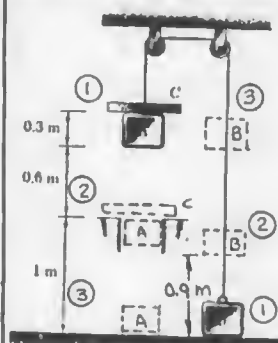


GIVEN:

$$m_A = 4 \text{ kg} \\ m_B = 5 \text{ kg} \\ m_C = 3 \text{ kg} \\ \text{SYSTEM RELEASED FROM REST}$$

FIND:

$v_A$ , JUST BEFORE IT STRIKES THE GROUND



POSITION 1 TO POSITION 2

$$v_1 = 0 \quad T_1 = 0$$

AT 2 BEFORE C IS REMOVED FROM THE SYSTEM

$$T_2 = \frac{1}{2} (m_A + m_B + m_C) v_2^2$$

$$T_2 = \frac{1}{2} (12 \text{ kg}) v_2^2 = 6 v_2^2$$

$$U_{1-2} = (m_A + m_C - m_B) g (0.9 \text{ m})$$

$$U_{1-2} = (4 + 3 - 5)(9.81 \text{ m/s}^2)(0.9 \text{ m})$$

$$U_{1-2} = 17.658 \text{ J}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 17.658 = 6 v_2^2$$

$$v_2^2 = 2.943$$

AT POSITION 2, COLLAR C IS REMOVED FROM THE SYSTEM

POSITION 2 TO POSITION 3

$$T_2' = \frac{1}{2} (m_A + m_B) v_2'^2 = (9 \text{ kg})(2.943)$$

$$T_2' = 13.244 \text{ J}$$

$$T_3 = \frac{1}{2} (m_A + m_B) (v_3)^2 = \frac{9}{2} v_3^2$$

$$U_{2-3} = (m_A - m_B)(g)(0.7 \text{ m}) = (-1 \text{ kg})(9.81 \text{ m/s}^2)(0.7 \text{ m})$$

$$U_{2-3} = -6.867 \text{ J}$$

$$T_2' + U_{2-3} = T_3$$

$$13.244 - 6.867 = 4.5 v_3^2$$

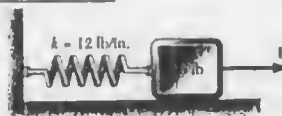
$$v_3^2 = 1.417$$

$$v_A = v_3 = 1.190 \text{ m/s}$$

$$v_A = 1.190 \text{ m/s}$$

13.26

GIVEN:



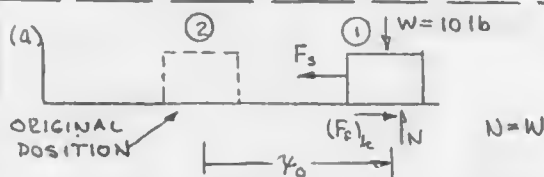
$$\mu_s = 0.60, \mu_k = 0.40$$

FORCE F IS SLOWLY APPLIED UNTIL THE TENSION IN THE SPRING IS 20 lb AND THEN RELEASED

FIND:

(a) VELOCITY OF BLOCK AS IT RETURNS TO ITS ORIGINAL POSITION

(b) THE MAXIMUM VELOCITY OF THE BLOCK

FIND INITIAL POSITION  $x_0$  OF THE BLOCK AT (1)

$$k = 12 \text{ lb/in} = 144 \text{ lb/ft}$$

$$\text{AT 1, } F_s = 20 \text{ lb} \quad F_s = k x_0 \quad 20 \text{ lb} = (144 \text{ lb/ft}) x_0$$

$$x_0 = 20/144 = 0.1389 \text{ ft}$$

$$T_1 = 0, \quad T_2 = \frac{1}{2} \left( \frac{W}{g} \right) v_2^2 = \left( \frac{1}{2} \right) (10 \text{ lb}/32.2 \text{ ft/s}^2) v_2^2$$

$$T_2 = 0.1553 v_2^2$$

$$U_{1-2} = \int_{x_0}^0 -F_s dx + (F_f)_k (x_0); \quad F_s = kx = 144x$$

$$(F_f)_k = \mu_k N$$

$$U_{1-2} = \left[ -\frac{144}{2} x^2 \right]_{x_0}^0 + (F_f)_k (x_0) \quad (F_f)_k = (0.4)(10) = 4 \text{ lb}$$

$$U_{1-2} = (72 \text{ lb/ft})(0.1389 \text{ ft})^2 + (4 \text{ lb})(-0.1389 \text{ ft})$$

$$U_{1-2} = 1.389 - 0.5556 = 0.8335 \text{ lb}\cdot\text{ft}$$

$$T_1 + U_{1-2} = T_2 \quad 0 + 0.8335 = 0.1553 v_2^2$$

$$v_2^2 = 5.367$$

$$v_2 = 2.32 \text{ ft/s}$$

AT ORIGINAL POSITION,  $v = 2.32 \text{ ft/s}$

(b) FOR ANY POSITION (2) AT A DISTANCE  $x$  TO THE RIGHT OF THE ORIGINAL POSITION (1)

$$T_1 = 0 \quad T_2 = \frac{1}{2} \left( \frac{W}{g} \right) (v_2)^2 = 0.1553 v_2^2$$

$$U_{1-2}' = \int_{x_0}^x -F_s dx + \int_{x_0}^x (F_f)_k dx \quad x_0 = 0.1389$$

$$U_{1-2}' = \left[ -\frac{144}{2} x^2 \right]_{x_0}^x + (F_f)_k (x - x_0) \quad (F_f)_k = 4 \text{ lb}$$

$$T_1 + U_{1-2}' = T_2 \quad 0 + (72 \text{ lb/ft}) \left[ (0.1389)^2 - x^2 \right] + (4 \text{ lb})(x - 0.1389) = 0.1553 v_2^2$$

$$\text{MAX } v_2 \text{ WHEN } \frac{dv_2}{dx} = 0$$

$$-144x + 4 = 0$$

$$\text{MAX } v_2 \text{ WHEN } x = 0.02778 \text{ m}$$

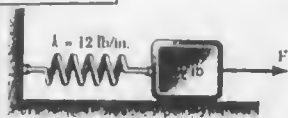
$$0.1553 v_{\text{max}}^2 = (72) [(0.1389)^2 - (0.02778)^2] + (4)(0.02778 - 0.1389)$$

$$0.1553 v_{\text{max}}^2 = 1.3336 - 0.4445 = 0.8891$$

$$v_{\text{max}}^2 = 5.725$$

$$v_{\text{max}} = 2.39 \text{ ft/s}$$

13.27

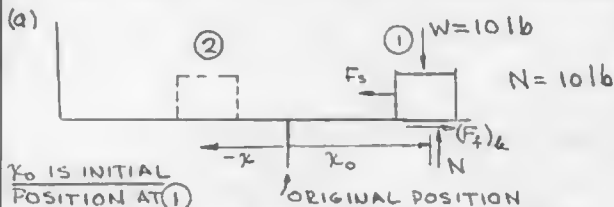


GIVEN:

$\mu_s = 0.60, \mu_k = 0.40$   
 FORCE  $F$  IS SLOWLY  
 APPLIED UNTIL THE  
 TENSION IN THE  
 SPRING IS 20 lb  
 AND THEN RELEASED

FIND:

- (a) DISTANCE THE BLOCK MOVES TO THE  
 LEFT BEFORE COMING TO A STOP  
 (b) WHETHER THE BLOCK THEN MOVES  
 BACK TO THE RIGHT.



BLOCK HAS A VELOCITY TO THE LEFT AS  
 IT REACHES ITS ORIGINAL POSITION  
 (SEE P 13.26)

$$k = 12 \text{ lb/in} = 144 \text{ lb/ft}$$

$$F_s = 144x$$

$$(F_f)_k = 4 \text{ lb}$$

$$(F_f)_s = (0.60)(10)$$

$$(F_f)_k = 4 \text{ lb}$$

$$T_1 = 0 \quad T_2 = 0$$

$$U_{1-2} = \int_{x_0}^{-x} -F_s dx + \int_{x_0}^{-x} (F_f)_k dx$$

$$U_{1-2} = -\frac{144}{2} x^2 \Big|_{x_0}^{-x} + (F_f)_k (-x - x_0)$$

$$U_{1-2} = -72(x^2 - x_0^2) - 4(x + x_0)$$

$$T_1 + U_{1-2} = T_2 \quad 0 - 72(x - x_0)(x + x_0) - 4(x + x_0) = 0$$

$$-72(x - x_0) - 4 = 0$$

$$-72x = 4 - 72x_0$$

$$\text{AT ① } F_s = 20 \text{ lb}$$

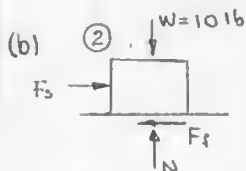
$$F_s = kx_0 = 144x_0$$

$$x_0 = \frac{20}{144} = 0.1389 \quad x = 0.0833 \text{ ft}$$

TOTAL DISTANCE MOVED TO  
 THE LEFT =  $x_0 + x$

$$x_0 + x = 0.1389 + 0.0833$$

$$x_0 + x = 0.222 \text{ ft}$$



FROM (a) WITH  $x = 0.0833 \text{ ft}$

$$F_s = (144)(0.0833) = 12 \text{ lb}$$

$$(F_f)_s = \mu_s N = (0.60)(10) = 6 \text{ lb}$$

SINCE  $F_s > (F_f)_s$

BLOCK MOVES TO THE RIGHT

13.28



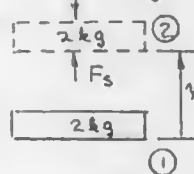
GIVEN:

3 kg BLOCK RESTS ON 2 kg BLOCK  
 WHICH IS NOT ATTACHED TO A  
 SPRING OF CONSTANT 40 N/m  
 UPPER BLOCK IS SUDDENLY  
 REMOVED

FIND:

- (a)  $v_{\text{max}}$  OF 2 kg BLOCK  
 (b) MAXIMUM HEIGHT,  $h$ ,  
 REACHED BY THE 2 kg BLOCK

(a)  $W = 2g \text{ N}$



AT THE INITIAL POSITION ①  
 THE FORCE IN THE SPRING  
 EQUALS THE WEIGHT OF  
 BOTH BLOCKS, I.E.  $5g \text{ N}$   
 THUS AT A DISTANCE  $x$   
 THE FORCE IN THE SPRING  
 IS  $F_s = 5g - kx$

$$F_s = 5g - 40x$$

MAX VELOCITY OF THE 2 kg BLOCK OCCURS  
 WHILE THE SPRING IS STILL IN CONTACT  
 WITH THE BLOCK.

$$T_1 = 0 \quad T_2 = \frac{1}{2} m v^2 = \left(\frac{1}{2}\right)(2 \text{ kg})(v^2) = v^2$$

$$U_{1-2} = \int_0^x (5g - 40x) dx - 2gx = 3gx - 20x^2$$

$$T_1 + U_{1-2} = T_2 \quad 0 + 3gx - 20x^2 = v^2 \quad (1)$$

$$\text{MAX } v \text{ WHEN } \frac{dv}{dx} = 0 = 3g - 40x$$

$$x(\text{MAX } v) = \frac{3g}{40} \text{ m}$$

SUBSTITUTE IN (1)

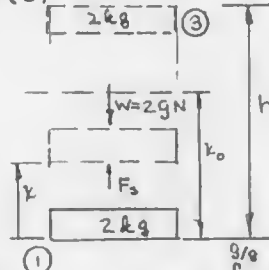
$$x(\text{MAX } v) = 0.7358 \text{ m}$$

$$v_{\text{MAX}}^2 = (3)(9.81)(0.7358) - (20)(0.7358)^2$$

$$= 21.65 - 10.83 = 10.83$$

$$v_{\text{MAX}} = 3.29 \text{ m/s}$$

(b)



$x_0$  = INITIAL COMPRESSION  
 OF THE SPRING

$$x_0 = \frac{(2g + 3g)}{40} = \frac{g}{8} \text{ m}$$

$$F_s = 5g - 40x$$

$$T_1 = 0 \quad T_3 = 0$$

$$U_{1-3} = \int_0^x (5g - 40x) dx - 2gx$$

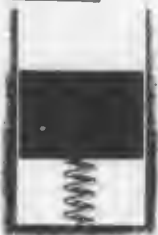
$$U_{1-3} = \frac{5g^2}{8} - \frac{20g^2}{64} - 2gh$$

$$T_1 + U_{1-3} = T_3 \quad 0 + \frac{20g^2}{64} - 2gh = 0$$

$$h = \frac{10g}{64} = \frac{(10)(9.81)}{64}$$

$$h = 1.533 \text{ m}$$

13.29



GIVEN:

3 kg BLOCK RESTS ON A 2 kg BLOCK WHICH IS ATTACHED TO A SPRING OF 40 N/m WHEN UPPER BLOCK IS SUDDENLY REMOVED

FIND:

- (a)  $v_{\max}$  OF 2 kg BLOCK  
(b) MAXIMUM HEIGHT  $h$  REACHED BY 2 kg BLOCK

(a) SEE SOLUTION TO (a) OF P13.28

$$v_{\max} = 3.29 \text{ m/s}$$

(b) REFER TO FIGURE IN (b) OF P13.28

$$T_1 = 0 \quad T_3 = 0$$

$$U_{1-3} = \int_0^h (5g - 40x) dx - 2gh$$

SINCE THE SPRING REMAINS ATTACHED TO THE 2 kg BLOCK THE INTEGRATION MUST BE CARRIED OUT THROUGHOUT THE TOTAL DISTANCE  $h$ .

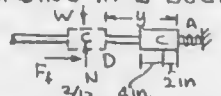
$$T_1 + U_{1-3} = T_2 \quad 0 + 5gh - 20h^2 - 2gh = 0$$

$$h = \frac{3g}{20} = \frac{(3)(9.81)}{20}$$

$$h = 1.472 \text{ m}$$

13.30 continued

(b) ASSUME THAT C DOES NOT REACH THE SPRING AT B BECAUSE OF FRICTION



$$N = W = 6 \text{ lb}$$

$$F_f = (0.35)(6 \text{ lb}) = 2.80 \text{ lb}$$

$$T_A = T_D = 0$$

$$U_{A-D} = \int_0^0 144x dx - F_f(y) = 2 - 2.80y$$

$$T_A + U_{A-D} = T_D \quad 0 + 2 - 2.80y = 0$$

$$y = 0.714 \text{ ft} = 8.57 \text{ in.}$$

THE COLLAR MUST TRAVEL  $16 - 6 + 2 = 12 \text{ in.}$  BEFORE IT ENGAGES THE SPRING AT B. SINCE  $y = 8.57 \text{ in.}$

IT STOPS BEFORE ENGAGING THE SPRING AT B

$$\text{TOTAL DISTANCE} = 8.57 \text{ in.}$$

13.31



GIVEN:

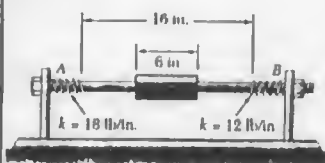
$$W_C = 6 \text{ lb.}$$

UPPER SPRING IS COMPRESSED 2 IN AND COLLAR C IS RELEASED

FIND:

- (a)  $y_m$ , THE MAXIMUM DEFLECTION OF THE LOWER SPRING  
(b)  $v_m$ , THE MAXIMUM VELOCITY OF THE COLLAR

13.30



GIVEN:

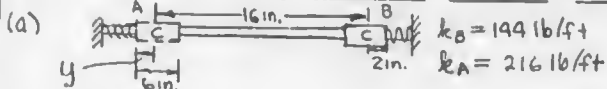
$$W_C = 6 \text{ lb}$$

COLLAR C COMPRESSES SPRING AT B 2 IN. AND IS RELEASED

FIND:

(a) DISTANCE TRAVELED BY COLLAR WITH NO FRICTION.

(b) SAME AS (a) WITH FRICTION,  $\mu_k = 0.35$



$$(a) \quad k_B = 144 \text{ lb/ft} \quad k_A = 216 \text{ lb/ft}$$

SINCE COLLAR C LEAVES THE SPRING AT B AND THERE IS NO FRICTION IT MUST ENGAGE THE SPRING AT A

$$T_A = 0 \quad T_B = 0$$

$$U_{A-B} = \int_0^{2/12} k_B x dx - \int_0^y k_A x dx$$

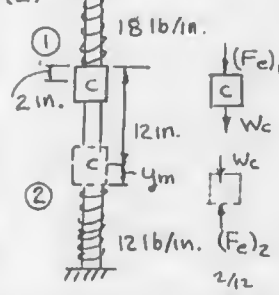
$$U_{A-B} = \left( \frac{144 \text{ lb/ft}}{2} \right) \left( \frac{2}{12} \text{ ft} \right)^2 - \left( \frac{216 \text{ lb/ft}}{2} \right) (y)^2$$

$$T_A + U_{A-B} = T_B \quad 0 + 2 - 108y^2 = 0$$

$$y = 0.1361 \text{ ft} = 1.633 \text{ in.}$$

$$\text{TOTAL DISTANCE} = 2 + 16 - (6 - 1.633) = 13.63 \text{ m.}$$

(a)



SPRING CONSTANTS

$$18 \text{ lb/in} = 216 \text{ lb/ft}$$

$$12 \text{ lb/in} = 144 \text{ lb/ft}$$

MAXIMUM DEFLECTION AT ② WHEN VELOCITY OF COLLAR C IS ZERO

$$v_2 = 0 \quad T_2 = 0$$

$$v_1 = 0 \quad T_1 = 0$$

$$U_{1-2} = U_e + U_g = \int_0^{2/12} (F_e)_1 dx - \int_0^{y_m} (F_e)_2 dx + W_C(1+y)$$

$$U_{1-2} = \left( \frac{216 \text{ lb/ft}}{2} \right) \left( \frac{1}{6} \text{ ft} \right)^2 - \left( \frac{144 \text{ lb/ft}}{2} \right) (y_m^2) + 6 \text{ lb}(1+y)$$

$$U_{1-2} = 3 - 72y_m^2 + 6 + 6y_m = -72y_m^2 + 6y_m + 6$$

$$T_1 + U_{1-2} = T_2 \quad 0 - 72y_m^2 + 6y_m + 6 = 0$$

$$y_m = \frac{1}{3} \text{ ft} = 4.00 \text{ in.}$$

(b) MAXIMUM VELOCITY OCCURS AS THE LOWER SPRING IS COMPRESSED A DISTANCE  $y_1$

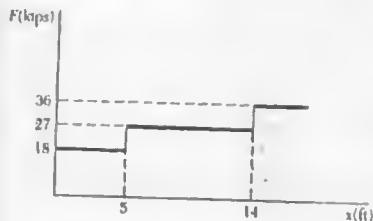
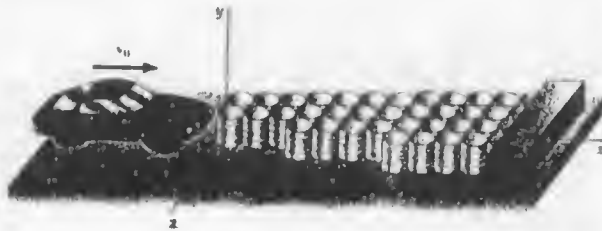
$$T_1 = 0 \quad T_2 = \frac{1}{2} m_C v^2 = \frac{1}{2} \left( \frac{6}{g} \right) v^2 = \left( \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) v^2$$

$$T_1 + U_{1-2} = T_2 \quad 0 - 72y^2 + 6y + 6 = (0.09317) v^2$$

$$\frac{dU}{dy} = 0 \quad (-144y + 6 = 0; y_1 = 0.041667 \text{ ft}$$

$$-0.125 + 0.250 + 6 = 0.09317 v_m^2; v_m = 8.11 \text{ ft/s} = 9.23 \text{ m/s}$$

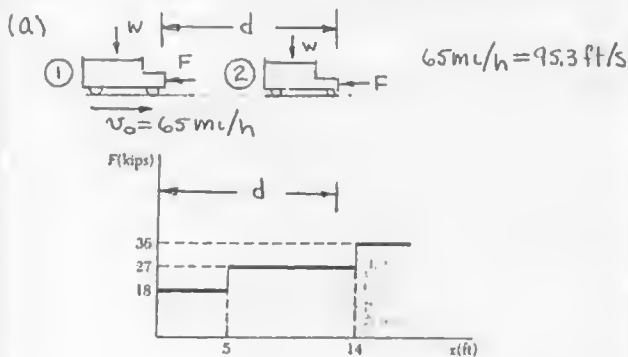
13.32



GIVEN:

$v_0 = 65 \text{ mi/h}$   
 FORCE FROM  
 CUSHION AS  
 SHOWN  
 NEGLECT  
 FRICTION  
 $W = 2250 \text{ lb}$

FIND:

(a) DISTANCE  $d$  FOR AUTOMOBILE TO COME TO REST(b) MAXIMUM DECELERATION,  $a_D$ ASSUME AUTO STOPS IN  $5 < d < 14 \text{ ft}$ 

$$v_1 = 95.33 \text{ ft/s} \quad T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} \left( \frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (95.33 \text{ ft/s})^2$$

$$v_2 = 0 \quad T_2 = 0 \quad T_1 = 317,530 \text{ lb} \cdot \text{ft} = 317.53 \text{ k} \cdot \text{ft}$$

$$U_{1-2} = (18 \text{ k})(5 \text{ ft}) + (27 \text{ k})(d - 5)$$

$$= 90 + 27d - 135 = 27d - 45 \text{ k} \cdot \text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$317.53 = 27d - 45$$

$$d = 13.43 \text{ ft}$$

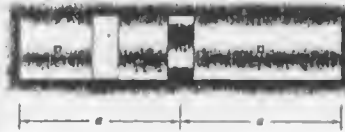
ASSUMPTION THAT  $d < 14 \text{ ft}$  IS OK(b) MAXIMUM DECELERATION OCCURS WHEN  $F$  IS LARGEST. FOR  $d = 13.43 \text{ ft}$ ,  $F = 27 \text{ k}$ .

$$\text{THUS } F = m a_D$$

$$(27,000 \text{ lb}) = \left( \frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (a_D)$$

$$a_D = 386 \text{ ft/s}^2$$

13.33

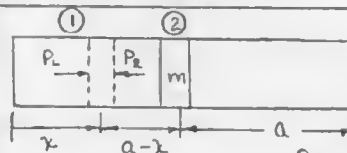


GIVEN:

PISTON AREA  $A$   
 PISTON MASS  $m$   
 INITIAL PRESSURE  $P$   
 PRESSURE VARIES  
 INVERSELY WITH  
 VOLUME. PISTON  
 MOVED  $a/2$  AND RELEASED

FIND:

VELOCITY OF THE PISTON AS IT RETURNS TO THE CENTER



PRESSURES VARY  
 INVERSELY AS  
 THE VOLUME

$$\frac{P_L}{P} = \frac{A a}{A x} \quad P_L = \frac{P a}{x}$$

$$\frac{P_R}{P} = \frac{A a}{A (2a - x)} \quad P_R = \frac{P a}{(2a - x)}$$

INITIALLY AT ①

$$v = 0 \quad x = \frac{a}{2}$$

$$T_1 = 0$$

$$\text{AT ②, } x = a, T_2 = \frac{1}{2} m v^2$$

$$U_{1-2} = \int_{a/2}^a (P_L - P_R) A dx = \int_{a/2}^a P a A \left[ \frac{1}{x} - \frac{1}{2a - x} \right] dx$$

$$U_{1-2} = p a A \left[ \ln x + \ln (2a - x) \right]_{a/2}^a$$

$$U_{1-2} = p a A \left[ \ln a + \ln a - \ln (a/2) - \ln (3a/2) \right]$$

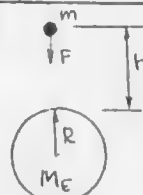
$$U_{1-2} = p a A \left[ \ln a^2 - \ln 3a^2/4 \right] = p a A \ln (4/3)$$

$$T_1 + U_{1-2} = T_2 \quad 0 + p a A \ln (4/3) = \frac{1}{2} m v^2$$

$$v^2 = \frac{2 p a A \ln (4/3)}{m} = 0.5754 \frac{p a A}{m}$$

$$v = 0.759 \sqrt{\frac{p a A}{m}}$$

13.34



GIVEN:

ACCELERATION  
 OF GRAVITY  $g_0$   
 AT EARTH'S SURFACE

FIND:

ACCELERATION OF GRAVITY  
 $g_h$  AT HEIGHT  $h$  ABOVE  
 THE EARTH'S SURFACE  
 IN TERMS OF  $g_0, h, R$ .

AND ERROR IN WEIGHT AT  $h$  IF WEIGHT  
 AT EARTH'S SURFACE IS USED FOR (a)  $h = 1 \text{ km}$   
 (b)  $h = 1000 \text{ km}$

$$F = \frac{G M_E m}{(h + R)^2} = \frac{G M_E m / R^2}{\left( \frac{h}{R} + 1 \right)^2} = m g_h$$

$$\text{AT EARTH'S SURFACE } (h = 0) \quad G M_E m / R^2 = m g_0$$

$$G M_E / R^2 = g_0 \quad g_h = \frac{G M_E / R^2}{\left( \frac{h}{R} + 1 \right)^2}$$

$$\text{THUS } g_h = \frac{g_0}{\left( \frac{h}{R} + 1 \right)^2}$$

(CONTINUED)

### 13.34 continued

$$R = 6370 \text{ km}$$

AT ALTITUDE  $h$ , TRUE WEIGHT  $F = m g_h = W_T$

ASSUMED WEIGHT  $W_0 = m g_0$

$$\text{ERROR} = E = \frac{W_0 - W_T}{W_0} = \frac{m g_0 - m g_h}{m g_0} = \frac{g_0 - g_h}{g_0}$$

$$g_h = \frac{g_0}{\left(\frac{h}{R} + 1\right)^2} \quad E = g_0 - \frac{g_0}{\left(1 + \frac{h}{R}\right)^2} = \left[1 - \frac{1}{\left(1 + \frac{h}{R}\right)^2}\right] g_0$$

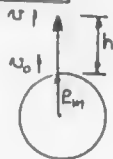
$$(a) h = 1 \text{ km} \quad P = 100E = 100 \left[1 - \frac{1}{\left(1 + \frac{1}{6370}\right)^2}\right]$$

$$P = 0.0314\%$$

$$(b) h = 1000 \text{ km} \quad P = 100 \left[1 - \frac{1}{\left(1 + \frac{1000}{6370}\right)^2}\right]$$

$$P = 25.3\%$$

### 13.35



GIVEN:

VELOCITY AT MOON'S SURFACE  $= v_0$

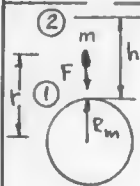
VELOCITY AT HEIGHT  $h = v$

RADIUS OF THE MOON,  $R_m$

ACCELERATION OF GRAVITY ON THE MOON'S SURFACE,  $g_m$

FIND:

FORMULA FOR  $h_m/h_u$ , WHERE  $h_m$  IS FOUND USING NEWTON'S LAW OF GRAVITATION AND  $h_u$  IS FOUND USING A UNIFORM GRAVITATIONAL FIELD



NEWTON'S LAW OF GRAVITATION

$$T_1 = \frac{1}{2} m v_0^2, \quad T_2 = \frac{1}{2} m v^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h} (-F) dr \quad F_n = \frac{m g_m R_m^2}{r^2}$$

$$U_{1-2} = -m g_m R_m^2 \int_{R_m}^{R_m+h} \frac{dr}{r^2}$$

$$U_{1-2} = m g_m R_m^2 \left( \frac{1}{R_m} - \frac{1}{R_m+h} \right)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m v_0^2 + m g_m \left( R_m - \frac{R_m}{1 + \frac{h}{R_m}} \right) = \frac{1}{2} m v^2$$

$$h_m = \frac{(v_0^2 - v^2)}{2 g_m} \left[ \frac{R_m}{R_m - \frac{v_0^2 - v^2}{2 g_m}} \right] \quad (1)$$

UNIFORM GRAVITATIONAL FIELD

$$T_1 = \frac{1}{2} m v_0^2, \quad T_2 = \frac{1}{2} m v^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h} (-F) dr = -m g_m (R_m + h - R_m) = -m g_m h$$

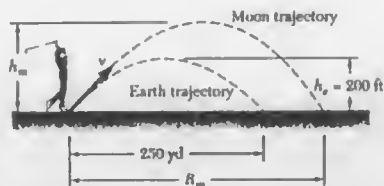
$$T_1 + U_{1-2} = T_2 \quad \frac{1}{2} m v_0^2 - m g_m h_u = \frac{1}{2} m v^2$$

$$h_u = \frac{(v_0^2 - v^2)}{2 g_m} \quad (2)$$

DIVIDE (1) BY (2)

$$\frac{h_m}{h_u} = \frac{1}{1 - \frac{(v_0^2 - v^2)}{2 g_m R_m}}$$

### 13.36



GIVEN:

EARTH TRAJECTORY AS SHOWN

MAGNITUDE AND DIRECTION OF  $v$  ON

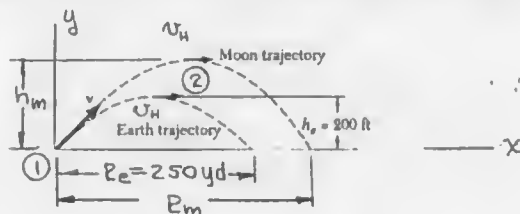
THE EARTH IS THE SAME ON THE MOON

TRAJECTORY IS A PARABOLA

$$g_m = 0.165 g_e$$

FIND:

RANGE  $R_m$  OF THE BALL ON THE MOON



SOLVE FOR  $h_m$

AT MAXIMUM HEIGHT THE TOTAL VELOCITY IS THE HORIZONTAL COMPONENT OF THE VELOCITY WHICH IS CONSTANT AND THE SAME IN BOTH CASES

$$T_1 = \frac{1}{2} m v^2 \quad T_2 = \frac{1}{2} m v_H^2$$

$$U_{1-2} = -m g_e h_e \quad \text{EARTH}$$

$$U_{1-2} = -m g_m h_m \quad \text{MOON}$$

$$\text{EARTH} \quad \frac{1}{2} m v^2 - m g_e h_e = \frac{1}{2} m v_H^2$$

$$\text{MOON} \quad \frac{1}{2} m v^2 - m g_m h_m = \frac{1}{2} m v_H^2$$

$$\text{SUBTRACTING} \quad -g_e h_e + g_m h_m = 0 \quad \frac{h_m}{h_e} = \frac{g_e}{g_m}$$

$$h_m = (200 \text{ ft}) \left( \frac{g_e}{0.165 g_e} \right) = 1212 \text{ ft}$$

$$\text{EQUATION OF A PARABOLA} \quad (y-h) = -C \left( x - \frac{R}{2} \right)^2$$

$$(y-h_e) = -C_e \left( x - \frac{R_e}{2} \right)^2 \quad \text{EARTH}$$

$$(y-h_m) = -C_m \left( x - \frac{R_m}{2} \right)^2 \quad \text{MOON}$$

AT  $x=0$ ,  $v$  IS THE SAME, THUS  $\frac{dy}{dx}$  IS THE SAME

$$\frac{dy}{dx} \bigg|_{x=0} = C_e R_e = C_m R_m$$

$$\frac{C_e}{C_m} = \frac{R_m}{R_e}$$

$$\text{AT } x=0, y=0 \quad h_e = C_e \frac{R_e^2}{4} \quad h_m = C_m \frac{R_m^2}{4}$$

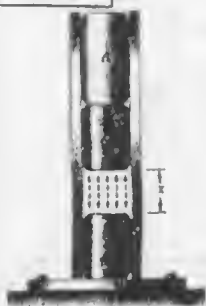
$$\frac{h_m}{h_e} = \frac{C_m R_m^2}{C_e R_e^2} = \frac{R_m}{R_e}$$

$$\frac{h_m}{h_e} = \frac{g_e}{g_m} = \frac{R_m}{R_e} \quad R_m = (g_e / 0.165 g_e) (250 \text{ yd})$$

$$R_m = 1515 \text{ yd}$$



13.37



GIVEN:

 $m_A = 300\text{-g}$  (NON MAGNETIC) $m_B = 200\text{-g}$  (MAGNETIC) $x = 4\text{ mm}$ , INITIALLY

REPELLING FORCE

BETWEEN BAND C IS

 $F = k/x^2$ 

BLOCK A IS SUDDENLY

REMOVED. NO AIR RESISTANCE

FIND:

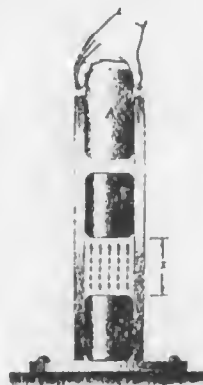
(a) MAXIMUM VELOCITY,

 $v_m$  OF B

(b) MAXIMUM ACCELERATION

 $a_m$  OF B

13.38



GIVEN:

 $W_B = 0.4\text{-lb}$  (MAGNETIC) $W_A = 0.6\text{-lb}$  (NON-MAGNETIC) $x = 0.15\text{ in.}$  INITIALLY

REPELLING FORCE

BETWEEN BAND C IS

 $F = k/x^2$ ; NO AIR RESISTANCE

BLOCK A IS PLACED ON

BLOCK B AND RELEASED

FIND:

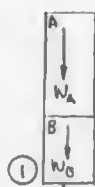
(a) MAXIMUM VELOCITY OF

A AND B

(b) MAXIMUM DEFLECTION OF

A AND B

(a)



②

 $x$ F =  $k/(4 \times 10^{-3} \text{ m})^2$ 

CALCULATE K

$$\Sigma F = (m_A + m_B)g - k/(4 \times 10^{-3} \text{ m})^2$$

$$k = (4 \times 10^{-3} \text{ m})(0.5 \text{ kg})(9.8 \text{ m/s}^2)$$

$$k = 8 \times 10^{-6} \text{ g N-m}$$

$$v_1 = 0 \quad T_1 = 0 \quad v_2 = v \quad T_2 = \frac{1}{2} m_B v^2 = 0.1 v^2 \text{ N-m}$$

$$U_{1-2} = \int_0^x (F - m_B g) dx$$

$$U_{1-2} = \int_{4 \times 10^{-3}}^x \left( \frac{8 \times 10^{-6} \text{ g}}{x^2} - 0.2 \text{ g} \right) dx$$

$$T_1 + U_{1-2} = T_2$$

$$0 + \int_{4 \times 10^{-3}}^x \left( \frac{8 \times 10^{-6} \text{ g}}{x^2} - 0.2 \text{ g} \right) dx = 0.1 v^2$$

$$\text{FOR MAX } v, \frac{d(0.1 v^2)}{dx} = 0$$

THUS

$$\frac{8 \times 10^{-6} \text{ g}}{x^2} - 0.2 \text{ g} = 0$$

$$\text{AT } v_{\text{MAX}}, x = 0.00632 \text{ m}$$

$$0 + \int_{0.004}^{0.00632} \left( \frac{8 \times 10^{-6} \text{ g}}{x^2} - 0.2 \text{ g} \right) dx = 0.1 v_{\text{MAX}}^2$$

$$0 + \left[ \frac{-(8 \times 10^{-6})}{x} - 0.2(0.00632) \right] = 0.1 v_{\text{MAX}}^2$$

$$0.004 \text{ m}$$

$$v_{\text{MAX}} = 0.1628 \text{ m/s}$$

$$v_H = 162.8 \text{ mm/s}$$

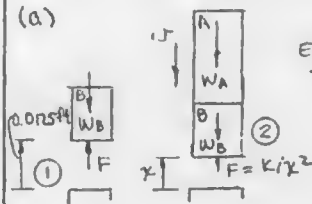
(b) MAXIMUM ACCELERATION AT  $x = 0.004 \text{ m}$ WHEN  $\Sigma F$  ARE THE GREATEST

$$\Sigma F = k/x^2 - W_B = m_B a$$

$$(8 \times 10^{-6})/(0.004)^2 - (0.2)(9.81) = (0.2) a_m$$

$$a_m = 14.72 \text{ m/s}^2$$

(a)



CALCULATE K

EQUILIBRIUM AT ①

$$\Sigma F = k/x^2 - W_B = 0$$

$$k = x^2 W_B$$

$$x = 0.15 \text{ in.} = 0.0125 \text{ ft}$$

$$k = (0.0125 \text{ ft})^2 (0.4 \text{ lb})$$

$$k = 0.0000625 \text{ ft}^2 \cdot \text{lb}$$

$$v_1 = 0 \quad T_1 = 0 \quad v_2 = v \quad T_2 = \frac{1}{2} (m_A + m_B) v^2$$

$$T_2 = \frac{1}{2} \left( \frac{1 \text{ lb}}{32.2 \text{ ft/s}^2} \right) v^2$$

$$T_2 = 0.01553 v^2$$

$$U_{1-2} = \int_{0.0125}^x [F - (W_A + W_B)] dx$$

$$T_1 + U_{1-2} = T_2 \quad 0 + \int_{0.0125}^x \left[ \frac{0.0000625}{x^2} - 1 \right] dx = 0.01553 v^2$$

$$\text{FOR MAX } v, \frac{d(0.01553 v^2)}{dx} = 0$$

$$\text{AT } v_m, \frac{0.0000625}{x^2} - 1 = 0 \quad x = 0.007906 \text{ ft}$$

$$0.007906$$

$$\int_{0.0125}^{0.007906} \left[ \frac{0.0000625}{x^2} - 1 \right] dx = 0.01553 v_m^2$$

$$0.0125 \left[ -\frac{0.0000625}{x} - x \right]_{0.0125}^{0.007906} = 0.01553 v_m^2$$

$$v_m^2 = 0.10876$$

$$v_m = 0.3298 \text{ ft/s}$$

$$v_m = 3.96 \text{ in/s}$$

(b) MAXIMUM DEFLECTION WHEN  $v = 0$ 

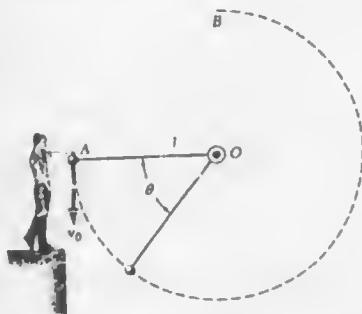
$$T_1 = 0 \quad T_2 = 0 \quad 0 + \int_{0.0125}^x \left[ \frac{0.0000625}{x^2} - 1 \right] dx = 0$$

$$-0.0000625 \left[ \frac{1}{x} - \frac{1}{0.0125} \right] - x + 0.0125 = 0$$

$$x = 0.005 \text{ ft}$$

$$\text{MAXIMUM DEFLECTION} = 0.0125 - 0.005 = 0.0075 \text{ ft} = 0.090 \text{ in.}$$

13.39

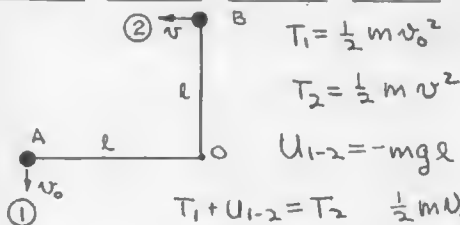


GIVEN:

 $v_0$ , AS SHOWN

FIND:

SMALLEST  $v_0$   
FOR THE  
SPHERE TO  
REACH B, IF  
(a) AO IS A  
ROPE  
(b) AO IS A  
ROD



$$T_1 = \frac{1}{2} m v_0^2$$

$$T_2 = \frac{1}{2} m v^2$$

$$U_{1-2} = -mgl$$

$$T_1 + U_{1-2} = T_2 \quad \frac{1}{2} m v_0^2 - mgl = \frac{1}{2} m v^2$$

$$v_0^2 = v^2 + 2gl$$

NEWTONS' LAW AT (2)

(a)  $mg = \frac{mv^2}{l}$  FOR MINIMUM  $v$ , TENSION IN THE CORD MUST BE ZERO.  
THUS,  $v^2 = gl$   
 $v_0^2 = v^2 + 2gl = 3gl$

(b)  $mg = N$   $N=0$   
 $F = mg$   $q_n = 0$  FORCE IN THE ROD CAN SUPPORT THE WEIGHT SO THAT  $v$  CAN BE ZERO  
THUS  $v_0^2 = 0 + 2gl$   
 $v_0 = \sqrt{2gl}$

13.40 continued

$$T_1 + U_{1-2} = T_2 \quad 12.5m + 2mg \sin \theta = \frac{1}{2} m v^2$$

$$25 + 4g \sin \theta = v^2 \quad (a)$$

NEWTONS' LAW AT (2)

$F = 2mg$   $ma_n = \frac{mv^2}{l}$   
 $mg + 2mg - mg \sin \theta = m \frac{v^2}{l} = m \frac{v^2}{2}$   
 $v^2 = 4g - 2g \sin \theta \quad (b)$

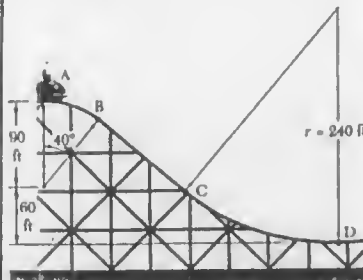
SUBSTITUTE FOR  $v^2$  FROM EQ (b) INTO EQ (a)

$$25 + 4g \sin \theta = 4g - 2g \sin \theta$$

$$\sin \theta = \frac{(4)(9.81) - 25}{(6)(9.81)} = 0.2419$$

$$\theta = 14.00^\circ$$

13.41



GIVEN:

$v_A = 0$   
WEIGHT OF  
CAR AND  
OCCUPANTS  
= 560 lb

FIND:

NORMAL FORCE  
 $N$ , AS CAR  
REACHES B

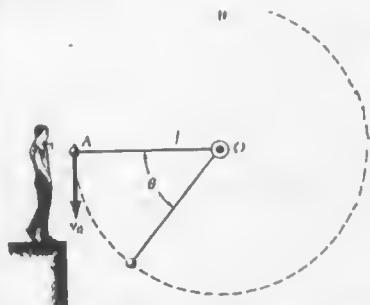
$v_A = 0$   $T_A = 0$   $T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} \left( \frac{560}{g} \right) v_B^2 = \frac{280}{g} v_B^2$   
 $U_{AB} = W(90)(1 - \cos 40^\circ)$   
 $U_{AB} = (560 \text{ lb})(90 \text{ ft})(0.234)$   
 $U_{AB} = 11791 \text{ ft} \cdot \text{lb}$   
 $T_A + U_{AB} = T_B$   $0 + 11791 = \frac{280}{g} v_B^2$   
 $v_B^2 = \frac{(11791 \text{ ft} \cdot \text{lb})(32.2 \text{ ft/s}^2)}{(280 \text{ lb})}$   
 $v_B^2 = 1356 \text{ ft}^2/\text{s}^2$

NEWTONS LAW AT B

$90 \text{ ft}$   $40^\circ$   $N$   $W = 560 \text{ lb}$   $ma_n = \frac{mv_B^2}{R}$   
 $N - W \cos 40^\circ = -\frac{mv_B^2}{R}$   $v_B^2 = 1356 \text{ ft}^2/\text{s}^2$   
 $N = (560 \text{ lb})(\cos 40^\circ) - \frac{(560 \text{ lb})(1356 \text{ ft}^2/\text{s}^2)}{(32.2 \text{ ft/s}^2)(90 \text{ ft})}$   
 $N = 429 - 262 = 167.0 \text{ lb}$

$$N = 167.0 \text{ lb}$$

13.40



GIVEN:

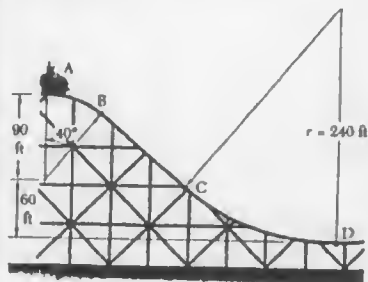
$v_0 = 5 \text{ m/s}$   
 $l = 2 \text{ m}$   
TENSION = 2W  
WHEN ROPE BREAKS

FIND:

VALUE OF  $\theta$   
WHEN ROPE  
BREAKS

(1)  $5 \text{ m/s}$   $2 \text{ m}$   $2 \sin \theta$   
 $T_1 = \frac{1}{2} m v_0^2 = \frac{1}{2} m (5)^2$   
 $T_1 = 12.5 \text{ m}$   
 $T_2 = \frac{1}{2} m v^2$   
 $U_{1-2} = mg(2) \sin \theta$

13.42



GIVEN:

$v_A = 0$ ,  
CAR AND  
OCCUPANTS  
WEIGH 560 lb

FIND:

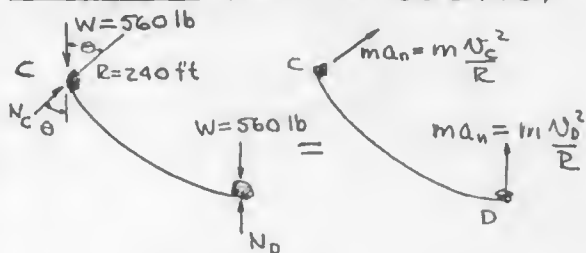
MAXIMUM,  $N_{\max}$   
AND MINIMUM,  
 $N_{\min}$  NORMAL  
FORCE ON  
THE CAR  
AS IT GOES  
FROM A TO D

NORMAL FORCE AT BSEE SOLUTION TO PROB. 13.41,  $N_B = 1670 \text{ lb}$ NEWTON'S LAWFROM B TO C (CAR MOVES IN A STRAIGHT LINE)

$$N_B' - W \cos 40^\circ = ma$$

$$N_B' = (560) \cos 40^\circ$$

$$N_B' = 429 \text{ lb}$$

AT C AND D (CAR IN THE CURVE AT C)

AT C

$$N_C - W \cos \theta = \frac{W}{g} \frac{v_c^2}{R}$$

$$N_C = 560 \left( \cos \theta + \frac{v_c^2}{gR} \right)$$

AT D

$$N_D - W = \frac{W}{g} \frac{v_D^2}{R}$$

$$N_D = 560 \left( 1 + \frac{v_D^2}{gR} \right)$$

SINCE  $v_D > v_C$  AND  $\cos \theta < 1$ ,  $N_D > N_C$   
WORK AND ENERGY FROM A TO D

$$v_A = 0, T_A = 0 \quad T_D = \frac{1}{2} W \quad v_D^2 = \frac{280}{g} v_0^2$$

$$U_{A-B} = W(90 + 60) = (560 \text{ lb})(150 \text{ ft})$$

$$U_{A-B} = 84000 \text{ lb}\cdot\text{ft}$$

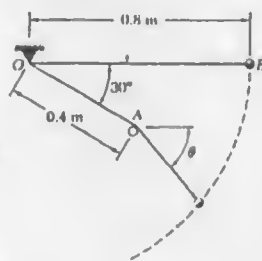
$$T_A + U_{A-B} = T_B \quad 0 + 84000 = 280 v_D^2$$

$$\frac{v_D^2}{g} = 300$$

$$N_D = 560 \left( 1 + \frac{v_D^2}{gR} \right) = 560 \left( 1 + \frac{300}{240} \right) = 1260 \text{ lb}$$

$$N_{\min} = N_B = 167.0 \text{ lb}; N_{\max} = N_D = 1260 \text{ lb}$$

13.43



GIVEN:

SPHERE RELEASED  
FROM REST AT B,  
( $v_B = 0$ )

FIND:

TENSION IN THE  
CORD,  
(a) JUST BEFORE  
IT COMES IN  
CONTACT WITH  
THE PEG  
(b) JUST AFTER  
CONTACT WITH PEG

VELOCITY OF THE SPHERE AS THE CORD CONTACTS A

$$v_B = 0 \quad T_B = 0$$

$$T_C = \frac{1}{2} m v_c^2$$

$$U_{B-C} = (mg)(0.4)$$

$$(0.8)(\sin 30^\circ) = 0.4$$

$$T_B + U_{B-C} = T_C$$

$$0 + 0.4 mg = \frac{1}{2} m v_c^2$$

$$v_c^2 = (0.8)(g)$$

NEWTON'S LAW

(a) CORD ROTATES ABOUT POINT O ( $R = L$ )

$$T - mg(\cos 60^\circ) = \frac{m v_c^2}{L}$$

$$T = mg(\cos 60^\circ) + \frac{m(0.8)g}{0.8}$$

$$T = \frac{3}{2} mg \quad T = 1.5 mg$$

(b) CORD ROTATES ABOUT A ( $R = L/2$ )

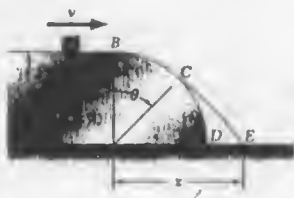
$$T - mg(\cos 60^\circ) = \frac{m v_c^2}{L/2}$$

$$T = mg/2 + m(0.8)g/0.4$$

$$T = \left( \frac{1}{2} + 2 \right) mg = \frac{5}{2} mg$$

$$T = 2.5 mg$$

13.44



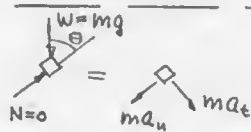
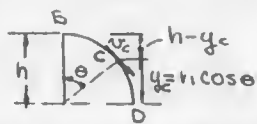
GIVEN:

$$v = 8 \text{ ft/s}$$

$$h = 3 \text{ ft}$$

FIND:

- (a)  $\theta$ , ANGLE AT WHICH BLOCK LEAVES THE SURFACE

(b)  $x$ 

BLOCK LEAVES SURFACE AT C WHEN THE NORMAL FORCE  $N = 0$

$$mg \cos \theta = m a_n$$

$$g \cos \theta = \frac{v_c^2}{h} \quad (1)$$

WORK-ENERGY PRINCIPLE

$$(a) T_B = \frac{1}{2} m v^2 = \frac{1}{2} m (8)^2 = 32 \text{ m}$$

$$T_C = \frac{1}{2} m v_c^2 \quad U_{B-C} = W(h-y) = mg(h-y_c)$$

$$T_B + U_{B-C} = T_C$$

USE EQ (1)

$$32 + mg(h-y) = \frac{1}{2} m v_c^2$$

$$32 + g(h-y) = \frac{1}{2} g y_c \quad (2)$$

$$32 + gh = \frac{3}{2} g y_c$$

$$y_c = (32 + gh) / (\frac{3}{2} g)$$

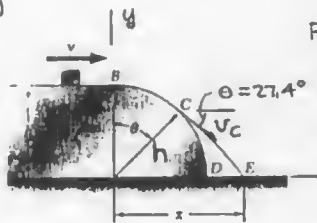
$$y_c = (32 + (32.2)(3)) / (\frac{3}{2}(32.2))$$

$$y_c = 2.6625 \text{ ft} \quad (3)$$

$$y_c = h \cos \theta \quad \cos \theta = \frac{y_c}{h} = \frac{2.6625}{3} = 0.8875$$

$$\theta = 27.4^\circ$$

(b)



FROM (1) AND (3)

$$v_c = \sqrt{g y_c}$$

$$v_c = \sqrt{(32.2)(2.6625)}$$

$$v_c = 9.259 \text{ ft/s}$$

$$\text{ATC: } (v_c)_x = v_c \cos \theta = (9.259)(\cos 27.4^\circ) = 8.220 \text{ ft/s}$$

$$(v_c)_y = -v_c \sin \theta = -(9.259)(\sin 27.4^\circ) = -4.261 \text{ ft/s}$$

$$y = y_c + (v_c)_y t - \frac{1}{2} g t^2 = 2.6625 - 4.261t - 16.1t^2$$

$$\text{ATE: } y = 0 \quad t^2 + 0.2647t - 0.1654 = 0$$

$$t = 0.2953 \text{ s}$$

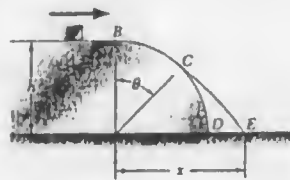
ATE:

$$x = h(\sin \theta) + (v_c)_x t = (3)(\sin 27.4^\circ) + (8.220)(0.2953)$$

$$x = 1.381 + 2.427 = 3.808 \text{ ft}$$

$$x = 3.81 \text{ ft}$$

13.45



GIVEN:

$$h = 2.5 \text{ m}$$

BLOCK LEAVES THE SURFACE WHEN  $\theta = 40^\circ$

FIND:

$$\text{INITIAL SPEED, } v$$

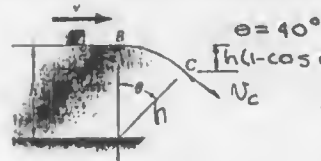
SEE LEFT; BLOCK LEAVES THE SURFACE WHEN  $N = 0$   $g \cos \theta = \frac{v_c^2}{h}$

$$h = 2.5 \text{ m}, \theta = 40^\circ$$

$$\text{THUS } v_c^2 = gh \cos \theta = (9.81)(2.5)(\cos 40^\circ)$$

$$v_c^2 = 18.79$$

WORK-ENERGY PRINCIPLE



$$\theta = 40^\circ$$

$$T_B = \frac{1}{2} m v^2$$

$$T_C = \frac{1}{2} m v_c^2 = \frac{1}{2} m (18.79)$$

$$T_C = 9.395 \text{ m}$$

$$U_{B-C} = mgh(1 - \cos \theta)$$

$$T_B + U_{B-C} = T_C$$

$$\frac{1}{2} m v^2 + mgh(1 - \cos \theta) = \frac{1}{2} m v_c^2$$

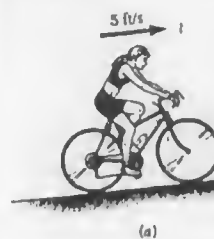
$$v^2 = -2gh(1 - \cos \theta) + 18.79$$

$$v^2 = -2(9.81 \text{ m/s}^2)(2.5 \text{ m})(1 - \cos 40^\circ) + 18.79$$

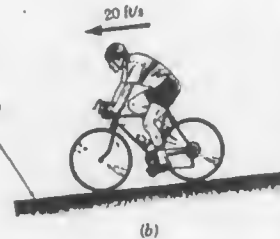
$$v^2 = 7.315$$

$$v_c = 2.70 \text{ m/s}$$

13.46



(a)



(b)

GIVEN:

- (a)  $v = 5 \text{ ft/s}$ , UP 3% SLOPE

BICYCLE WEIGHT,  $W_B = 15 \text{ lb}$ WOHANS WEIGHT  $W_W = 120 \text{ lb}$ 

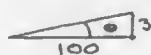
- (b)  $v = 20 \text{ ft/s}$ , DOWN 3% SLOPE, BRAKING

BICYCLE WEIGHT,  $W_B = 18 \text{ lb}$ HANS' WEIGHT,  $W_H = 180$ 

FIND:

- (a) POWER DEVELOPED BY THE WOHA,  $P_W$

- (b) POWER DISSIPATED BY THE BRAKES,  $P_B$

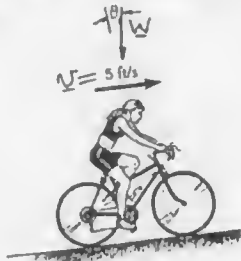


$$\tan \theta = \frac{3}{100} \quad \theta = 1.718^\circ$$

(CONTINUED)

### 13.46 continued

(a)



(a)

$$W = W_B + W_w = 15 + 120$$

$$W = 135 \text{ lb}$$

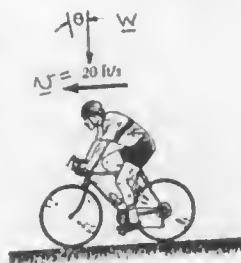
$$P_w = W \cdot v = (W \sin \theta)(v)$$

$$P_w = (135)(\sin 1.718^\circ)(5)$$

$$P_w = 20.24 \text{ ft}\cdot\text{lb/s}$$

$$P_w = 20.2 \text{ ft}\cdot\text{lb/s}$$

(b)



(b)

$$W = W_B + W_m = 18 + 180$$

$$W = 198 \text{ lb}$$

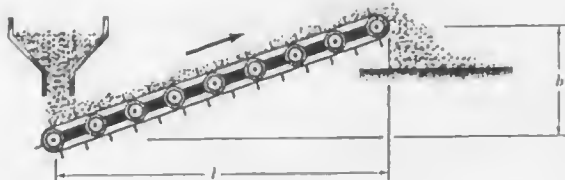
BRAKES MUST DISSIPATE THE POWER GENERATED BY THE BIKE AND THE MAN GOING DOWN THE SLOPE AT 20 ft/s

$$P_B = W \cdot v = (W \sin \theta)(v)$$

$$P_B = (198)(\sin 1.718^\circ)(20)$$

$$P_B = 118.7 \text{ ft}\cdot\text{lb/s}$$

### 13.47



GIVEN:

- (a) MASS FLOW RATE,  $m$  (kg/h),  $l$  (m),  $b$  (m)  
 (b) MASS FLOW RATE,  $W$  (tons/h),  $l$  (ft),  $b$  (ft)  
 MOTOR EFFICIENCY,  $\eta$

FIND:

- (a) Power  $P$  in kW  
 (b) Power in hp

(a) MATERIAL IS LIFTED TO A HEIGHT  $b$  AT A RATE,  $(m \text{ kg/h})(g \text{ m/s}^2) = (mg \text{ N/h})$

$$\text{THUS } \frac{\Delta U}{\Delta t} = \frac{(mg \text{ N/h})(b \text{ m})}{(3600 \text{ s/h})} = \left( \frac{mg b}{3600} \right) \frac{\text{N}\cdot\text{m}}{\text{s}}$$

$$\text{THUS, INCLUDING MOTOR EFFICIENCY } \eta$$

$$P(\text{kW}) = \frac{mg b (\text{N}\cdot\text{m/s})}{(3600) \left( \frac{1000 \text{ N}\cdot\text{m/s}}{\text{kW}} \right) (\eta)}$$

$$P(\text{kW}) = 0.278 \times 10^{-6} \frac{mg b}{\eta}$$

$$(b) \frac{\Delta U}{\Delta t} = \frac{[W(\text{tons/h})(2000 \text{ lb/ton})](b \text{ ft})}{3600 \text{ s/h}}$$

$$= \frac{W b}{1.8} \text{ ft}\cdot\text{lb/s}; 1 \text{ hp} = 550 \text{ ft}\cdot\text{lb/s}$$

$$\text{WITH } \eta, \text{ hp} = \left[ \frac{W \cdot b (\text{ft}\cdot\text{lb/s})}{1.8} \right] \left[ \frac{1 \text{ hp}}{550 \text{ ft}\cdot\text{lb/s}} \right] \left[ \frac{1}{\eta} \right] = \frac{0.101 \times 10^{-6} W b}{\eta}$$

### 13.48



GIVEN:

2000 lb CAR REAR WHEEL DRIVE, SKIDS FOR FIRST 60 ft WITH FRONT WHEELS OFF THE GROUND,  $\mu_k = 0.60$  ROLLS WITH SLIDING IMPENDING FOR REMAINING 1260 ft WITH 60% OF ITS WEIGHT ON REAR WHEELS,  $\mu_s = 0.85$

FIND:

- (a) HP DEVELOPED AT END OF 60 ft PORTION OF THE RACE  
 (b) HP DEVELOPED AT THE END OF THE RACE

(a) FIRST 60 ft (CALCULATE VELOCITY AT 60 ft)

FORCE GENERATED BY REAR WHEELS  $= \mu_k W$  SINCE CAR SKIDS. THUS  $F = (0.6)(2000 \text{ lb})$   
 $F = 1200 \text{ lb}$

$$\text{WORK AND ENERGY } T_1 = 0 \quad T_2 = \frac{1}{2} W v_{60}^2 = \frac{1000}{g} v_{60}^2$$

$$T_1 + U_{1-2} = T_2$$

$$U_{1-2} = (F)(60 \text{ ft}) = (1200 \text{ lb})(60 \text{ ft}) = 72,000 \text{ ft}\cdot\text{lb}$$

$$T_1 + U_{1-2} = T_2 \quad 0 + 72,000 = \frac{1000}{g} v_{60}^2$$

$$v_{60}^2 = (72)(32.2) = 2318.4$$

$$v_{60} = 48.15 \text{ ft/s}$$

$$\text{POWER} = F \cdot v_{60}$$

$$P = (1200 \text{ lb})(48.15 \text{ ft/s})$$

$$P = 57780 \text{ ft}\cdot\text{lb/s}$$

$$1 \text{ hp} = 550 \text{ ft}\cdot\text{lb/s}$$

$$\text{hp} = \frac{(57780 \text{ ft}\cdot\text{lb/s})}{(550 \text{ ft}\cdot\text{lb/s})} = 105.1$$

(b) END OF RACE (CALCULATE VELOCITY AT 1320 ft)

FOR FIRST 60 ft, FORCE GENERATED BY REAR WHEELS  $- F_s = 1200 \text{ lb}$  (SEE (a))  
 FOR REMAINING 1260 ft WITH 60% OF WEIGHT ON REAR WHEELS, THE FORCE GENERATED AT IMPENDING SLIDING IS  $\mu_s (0.60)(W) = (0.85)(0.60)(2000)$   
 $F_s = 1020 \text{ lb}$

$$T_1 + U_{1-2} = T_2 \quad T_1 = 0 \quad T_2 = \frac{1}{2} W v_{1320}^2 = \frac{1000}{g} v_{1320}^2$$

$$U_{1-2} = (F_s)(60 \text{ ft}) + (F_s)(1260 \text{ ft})$$

$$U_{1-2} = (1200 \text{ lb})(60 \text{ ft}) + (1020 \text{ lb})(1260 \text{ ft})$$

$$U_{1-2} = 1,357,200 \text{ ft}\cdot\text{lb}$$

$$0 + 1,357,200 = \frac{1000}{g} v_{1320}^2$$

$$v_{1320} = 209 \text{ ft/s}$$

$$\text{POWER} = F_s \cdot v_{1320}$$

$$P = (1020 \text{ lb})(209 \text{ ft/s}) = 213,230 \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$\text{hp} = \frac{(213,200 \text{ ft}\cdot\text{lb/s})}{(550 \text{ ft}\cdot\text{lb/s/hp})} = 388$$

13.49



GIVEN:

1000 kg CAR, REAR WHEEL DRIVE  
SKIDS FOR FIRST 20 M, WITH FRONT WHEELS  
OFF THE GROUND,  $\mu_k = 0.68$   
ROLLS WITH SLIDING IMPENDING FOR  
REMAINING 380 M WITH 80% OF ITS  
WEIGHT ON REAR WHEELS,  $\mu_s = 0.90$

FIND:

- (a) POWER DEVELOPED AT END OF 20 M (kW & hp)  
(b) POWER DEVELOPED AT END OF THE RACE (kW & hp)

(a) FIRST 20 M (CALCULATE VELOCITY AT 20 M)

FORCE GENERATED BY REAR WHEELS =  $\mu_k W$

SINCE CAR SKIDS, THUS  $F_s = (0.68)(1000)(g)$

$$F_s = (0.68)(1000 \text{ kg})(9.81 \text{ m/s}^2) = 6670.8 \text{ N}$$

WORK AND ENERGY  $T_1 = 0$ ,  $T_2 = \frac{1}{2} m v_{20}^2 = 500 v_{20}^2$

$$T_1 + U_{1-2} = T_2$$

$$U_{1-2} = (20 \text{ m})(F_s) = (20 \text{ m})(6670.8 \text{ N})$$

$$U_{1-2} = 133420 \text{ J}$$

$$0 + 133,420 = 500 v_{20}^2$$

$$v_{20}^2 = 133,420 / 500 = 266.83$$

$$v_{20} = 16.335 \text{ m/s}$$

$$\text{POWER} = (F_s)(v_{20}) = (6670.8 \text{ N})(16.335 \text{ m/s})$$

$$\text{POWER} = 108,970 \text{ J/s} = 108.97 \text{ kW/s}$$

$$1 \text{ kJ/s} = 1 \text{ kW}$$

$$1 \text{ hp} = 0.7457 \text{ kW} \quad \text{POWER} = 109.0 \text{ kJ/s} = 109.0 \text{ kW}$$

$$\text{POWER} = (109.0 \text{ kW}) / (0.7457 \text{ kW/hp}) = 146.2 \text{ hp}$$

(b) END OF RACE (CALCULATE VELOCITY AT 400 M)

FOR REMAINING 380 M, WITH

80% OF WEIGHT ON REAR WHEELS

THE FORCE GENERATED AT IMPENDING

SLIDING IS  $(\mu_s)(0.80)(mg)$

$$F_f = (0.90)(0.80)(1000 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_f = 7063.2 \text{ N}$$

WORK AND ENERGY, FROM 20 M (2) TO 28 M (3)

$$v_2 = 16.335 \text{ m/s} \quad (\text{FROM PART (a)})$$

$$T_2 = \frac{1}{2} (1000 \text{ kg})(16.335 \text{ m/s})^2$$

$$T_2 = 133420 \text{ J}$$

$$T_3 = \frac{1}{2} m v_{300}^2 = 500 v_{30}^2$$

$$U_{2-3} = (F_f)(380 \text{ m}) = (7063.2 \text{ N})(380 \text{ m})$$

$$U_{2-3} = 2,684,000 \text{ J}$$

$$T_2 + U_{2-3} = T_3$$

$$(133,420 \text{ J}) + (2,684,000 \text{ J}) = 500 v_{30}^2$$

$$v_{30} = 75.066 \text{ m/s}$$

$$\text{POWER} = (F_f)(v_{30}) = (7063.2 \text{ N})(75.066 \text{ m/s})$$

$$= 530,200 \text{ J}$$

kW

$$\text{POWER} = 530,200 \text{ J} = 530 \text{ kW}$$

hp

$$\text{POWER} = \frac{530 \text{ kW}}{(0.7457 \text{ kW/hp})} = 711 \text{ hp}$$

13.50



GIVEN:

CAR MASS,  $M_c = 1200 \text{ kg}$

LIFT MASS,  $M_L = 300 \text{ kg}$

SYSTEM RISES

2.8 M IN 15 S.

FIND:

- (a) AVERAGE POWER  
OUTPUT OF PUMP,  $(P_p)_A$   
(b) AVERAGE ELECTRIC  
POWER,  $(P_e)_A$ , WITH  $\eta = 82\%$

$$(a) (P_p)_A = (F)(v_A) = (m_c + m_L)(g)(v_A)$$

$$v_A = S/t = (2.8 \text{ m})/(15 \text{ s}) = 0.18667 \text{ m/s}$$

$$(P_p)_A = [(1200 \text{ kg}) + (300 \text{ kg})](9.81 \text{ m/s}^2)(0.18667 \text{ m/s})$$

$$(P_p)_A = 2747 \text{ J} = 2.75 \text{ kW}$$

$$(b) (P_e)_A = (P_p)/\eta = (2.75 \text{ kW})/(0.82)$$

$$(P_e)_A = 3.35 \text{ kW}$$

13.51



GIVEN:

CAR MASS,  $M_c = 1200 \text{ kg}$

LIFT MASS,  $M_L = 500 \text{ kg}$

PEAK VELOCITY AT

10 M HEIGHT IN

7.5 S INCREASING

UNIFORMLY. VELOCITY

DECREASES UNIFORMLY

TO 0, IN ANOTHER

7.5 S

PEAK DUMP POWER,

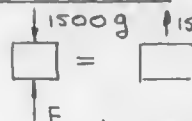
$P = 6 \text{ kW}$ , WHEN

VELOCITY IS MAXIMUM

FIND:

MAXIMUM LIFTING  
FORCE,  $F$

NEWTON'S LAW



$$Mg = (M_c + M_L)g = (1200 + 300)g$$

$$Mg = 1500g$$

$$+\Sigma F = F - 1500g = 1500a \quad (1)$$

SINCE MOTION IS UNIFORMLY ACCELERATED

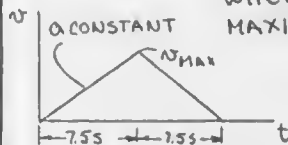
$a = \text{CONSTANT}$

THUS, FROM (1),  $F$  IS CONSTANT

AND PEAK POWER OCCURS

WHEN THE VELOCITY IS A

MAXIMUM AT 7.5 S.



$$a = \frac{v_{\text{MAX}}}{7.5 \text{ s}}$$

$$P = (6000 \text{ W}) = (F)(v_{\text{MAX}})$$

$$v_{\text{MAX}} = (6000)/F$$

$$\text{THUS } a = (6000)/(7.5)(F) \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$F - 1500g = (1500)(6000)/(7.5)(F)$$

$$F^2 - (1500 \text{ kg})(9.81 \text{ m/s}^2)F - \frac{(1500 \text{ kg})(6000 \text{ N} \cdot \text{m/s})}{(7.5 \text{ s})} = 0$$

$$F^2 - 14715F - 1.2 \times 10^6 = 0$$

$$F = 14,800 \text{ N}$$

$$F = 14.8 \text{ kN}$$

13.52

GIVEN:

$W = 100 \text{ TONS}$   
 $P = 400 \text{ hp}$   
 $U = 50 \text{ mi/h}$ , CONSTANT

FIND:

- (a)  $F_R$ , FORCE NEEDED TO OVERCOME AXLE FRICTION, ROLLING RESISTANCE AND AIR RESISTANCE  
 (b)  $\Delta P$ , ADDITIONAL HP TO MAINTAIN THE SAME SPEED UP A 1-PERCENT GRADE

$$(a) P = 400 \text{ hp} = \left(550 \frac{\text{ft} \cdot \text{lb}}{\text{s}} / \text{hp}\right) (400 \text{ hp}) = 220,000 \frac{\text{ft} \cdot \text{lb}}{\text{s}}$$

$$U = 50 \text{ mi/h} = 73.33 \text{ ft/s}$$

$$P = F_R \cdot U$$

$$F_R = P/U = \left(220,000 \frac{\text{ft} \cdot \text{lb}}{\text{s}}\right) / \left(73.33 \frac{\text{ft}}{\text{s}}\right)$$

$$F_R = 3000 \text{ lb}$$

$$(b) \theta = \tan^{-1} \frac{1}{100} = 0.573^\circ$$

$$W = (100 \text{ TONS}) (2000 \text{ lb/TON})$$

$$W = 200,000 \text{ lb}$$

$$\Delta P = W \sin \theta \cdot U$$

$$\Delta P = (200,000 \text{ lb}) (\sin 0.573^\circ) (73.33 \text{ ft/s})$$

$$\Delta P = 146,667 \text{ ft} \cdot \text{lb/s}$$

$$\Delta P = 267 \text{ hp}$$

13.53

GIVEN:

$W = 600 \text{ TONS}$   
 UNIFORM ACCELERATION FROM 0 TO 50 MI/H IN 40 S  
 CONSTANT 50 MI/H AFTER 40 S  
 HORIZONTAL TRACK  
 $F_R$ , FRICTION AND ROLLING RESISTANCE = 3000 lb  
 NEGLECT AIR RESISTANCE

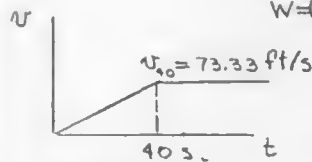
FIND:

$P$ , POWER REQUIRED AS A FUNCTION OF TIME  $t$ .

$$U = 50 \text{ mi/h} = 73.33 \text{ ft/s}$$

$$W = (600 \text{ TONS}) (2000 \text{ lb/TON})$$

$$W = 1,200,000 \text{ lb}$$



FOR UNIFORM MOTION

$$a = U/t_0 = (73.33 \text{ ft/s}) / (40 \text{ s})$$

$$a = 1.833 \text{ ft/s}^2$$

$$U = 1.833 t$$

$$\Sigma F = F - F_R = m a = \frac{W}{g} a$$

$$F = (3000 \text{ lb}) + \left( \frac{1,200,000 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (1.833 \text{ ft/s}^2)$$

$$F = 71,311 \text{ lb}$$

$$P = F \cdot U = (71,311) (1.833 t) = 130,710 t \frac{\text{lb} \cdot \text{ft}}{\text{s}}$$

$$P = 130,710 t / 550 = 238 t \text{ (hp)}$$

FOR  $t < 40 \text{ s}$ 

$$P = 238 t \text{ hp}$$

$$\text{FOR } t > 40 \text{ s } P = \frac{(3000)(73.3)}{(550)} = 400 \text{ hp}$$

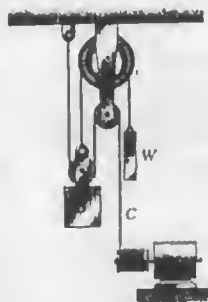
13.54

GIVEN:

$M_E = 3000 \text{ kg}$ , ELEVATOR MASS  
 $M_W = 1000 \text{ kg}$ , COUNTERWEIGHT

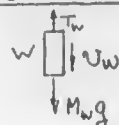
FIND:

- (a)  $P$  (kW), DELIVERED BY MOTOR WHEN VELOCITY OF E,  $U_E = 3 \text{ m/s}$  DOWN AND CONSTANT ( $a_E = 0$ )  
 (b)  $P$  (kW) WHEN  $U_E = 3 \text{ m/s}$  UPWARD  
 $a_E = 0.5 \text{ m/s}^2$  DOWN

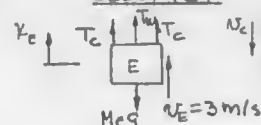


(a) ACCELERATION = 0

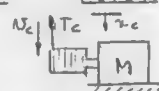
COUNTERWEIGHT



ELEVATOR



MOTOR



$$\Sigma F = T_W - M_W g = 0$$

$$T_W = (1000 \text{ kg}) (9.81 \text{ m/s}^2)$$

$$T_W = 9810 \text{ N}$$

$$+\Sigma F = 2T_E + T_W - M_E g = 0$$

$$2T_E = (9810 \text{ N}) + (3000 \text{ kg}) (9.81 \text{ m/s}^2)$$

$$T_E = 9810 \text{ N}$$

KINEMATICS

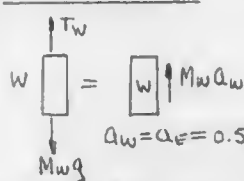
$$2x_E = x_C \quad 2\dot{x}_E = \dot{x}_C \quad U_C = 2U_E = 6 \text{ m/s}$$

$$P = T_C \cdot U_C = (9810 \text{ N}) (6 \text{ m/s}) = 58,860 \text{ J/s}$$

$$P \text{ (kW)} = 58.9$$

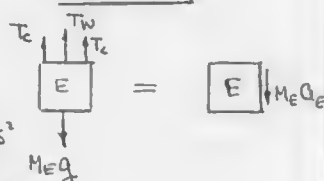
(b)  $a_E = 0.5 \text{ m/s}^2$  ↑,  $U_E = 3 \text{ m/s}$  ↓

COUNTERWEIGHT



$$a_W = a_E = 0.5 \text{ m/s}^2$$

ELEVATOR

COUNTERWEIGHT  $\Sigma F = M a$ 

$$\Sigma F = T_W - M_W g = M_W a_W$$

$$T_W = (1000 \text{ kg}) [(9.81 \text{ m/s}^2) + (0.5 \text{ m/s}^2)]$$

$$T_W = 10310 \text{ N}$$

ELEVATOR  $\Sigma F = M a$ 

$$+\Sigma F = 2T_E + T_W - M_E g = -M_E a_E$$

$$2T_E = (3000 \text{ kg}) [(9.81 \text{ m/s}^2) - (0.5 \text{ m/s}^2)] - 10310 \text{ N}$$

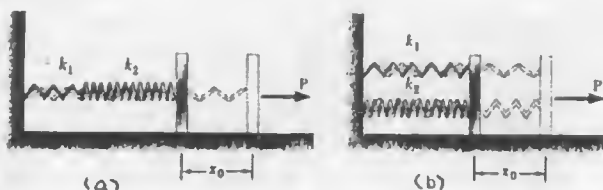
$$T_E = 8810 \text{ N} \quad U_C = 6 \text{ m/s} \text{ (SEE (a))}$$

$$P = T_C \cdot U_C = (8810 \text{ N}) (6 \text{ m/s})$$

$$P = 52,860 \text{ J} = 52.860 \text{ kJ} = 52.86 \text{ kW}$$

$$P \text{ (kW)} = 52.9$$

13.55



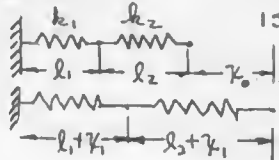
GIVEN:

P CAUSES DEFLECTION  $x_0$ , IS SLOWLY APPLIED(a) SPRINGS  $k_1$  AND  $k_2$  IN SERIES(b) SPRINGS  $k_1$  AND  $k_2$  IN PARALLEL

FIND:

SINGLE EQUIVALENT SPRING  $k_e$   
WHICH CAUSES THE SAME DEFLECTIONSYSTEM IS IN EQUILIBRIUM IN DEFLECTED  
 $x_0$  POSITION.

CASE (a)

FORCE IN BOTH SPRINGS  
IS THE SAME = P

$$x_0 = x_1 + x_2$$

$$x_0 = \frac{P}{k_e}$$

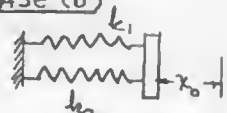
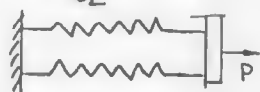
$$x_1 = \frac{P}{k_1} \quad x_2 = \frac{P}{k_2}$$

$$\text{THUS } \frac{P}{k_e} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

CASE (b)

DEFLECTION IN  
BOTH SPRINGS IS  
THE SAME =  $x_0$ 

$$P = k_1 x_0 + k_2 x_0$$

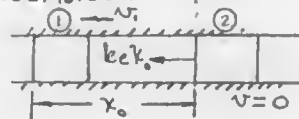
$$P = (k_1 + k_2) x_0$$

$$P = k_e x_0$$

EQUATING THE TWO EXPRESSIONS FOR  
 $P = (k_1 + k_2) x_0 = k_e x_0$ 

$$k_e = k_1 + k_2$$

13.56 continued

WE WILL USE AN EQUIVALENT SPRING CONSTANT  
 $k_e$  (SEE PROB. 13.55)

CHOOSE ① AT INITIAL UNDEFLECTED POSITION

$$V_1 = 0 \quad T_1 = \frac{1}{2} m V_1^2$$

CHOOSE ② AT  $x_0$  WHERE  $v = 0$ 

$$V_2 = \frac{1}{2} k_e x_0^2 \quad T_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{1}{2} m V_1^2 = \frac{1}{2} k_e x_0^2 + 0$$

$$\text{THUS } V_1 = V_{\text{MAX}} = x_0 \sqrt{\frac{k_e}{m}}$$

CASE (a)

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

$$V_{\text{MAX}} = x_0 \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

CASE (b)

$$k_e = k_1 + k_2$$

$$V_{\text{MAX}} = x_0 \sqrt{\frac{k_1 + k_2}{m}}$$

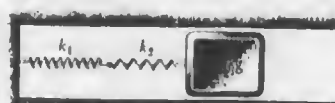
13.57

GIVEN:

$$k_1 = 12 \text{ kN/m}$$

$$k_2 = 8 \text{ kN/m}$$

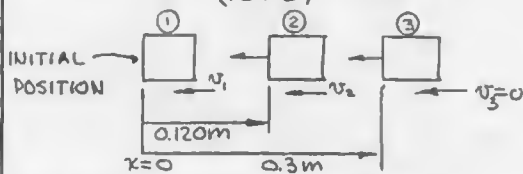
$$m = 16 \text{ kg}$$

INITIAL POSITION, 300mm  
TO RIGHT,  $v = 0$ 

FIND:

(a) MAXIMUM VELOCITY,  $V_{\text{MAX}}$ (b) VELOCITY 120 mm FROM INITIAL  
POSITIONUSE EQUIVALENT SPRING CONSTANT (SEE P 13.55)  
FOR SPRINGS IN SERIES,  $k_e = \frac{k_1 k_2}{k_1 + k_2}$ 

$$k_e = \frac{(12)(8)}{(12+8)} = 4.8 \text{ kN/m}$$



(a) AT ①, SPRING DEFLECTION, 0

$$V_1 = 0 \quad T_1 = \frac{1}{2} m V_1^2 = 8 V_1^2$$

$$\text{THUS } V_1 = V_{\text{MAX}}$$

AT ③,  $V_3 = 0 \quad T_3 = 0$ 

$$V_3 = \frac{1}{2} k_e x_3^2 = \left(\frac{4800}{2}\right)(0.3)^2 = 216 \text{ N}\cdot\text{m}$$

$$T_1 + V_1 = T_3 + V_3$$

$$8 V_{\text{MAX}}^2 + 0 = 0 + 216$$

$$V_{\text{MAX}}^2 = 27$$

$$V_{\text{MAX}} = 5.20 \text{ m/s}$$

(b)  $T_2 = \frac{1}{2} m V_2^2 = 8 V_2^2$ 

$$V_2 = \frac{1}{2} k_e x_2^2 = \left(\frac{4800}{2}\right)(0.120)^2 = 34.56 \text{ N}\cdot\text{m}$$

$$T_2 + V_2 = T_3 + V_3 \quad 8 V_2^2 + 34.56 = 0 + 216$$

$$V_2^2 = 22.68$$

$$V_2 = 4.76 \text{ m/s}$$

13.56



GIVEN:

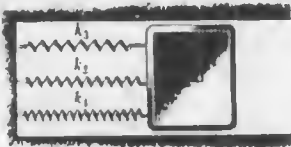
BLOCK OF MASS  $m$ BLOCK MOVED TO  $x_0$  AND RELEASED  
FROM REST.

FIND:

MAXIMUM VELOCITY,  $V_{\text{MAX}}$



13.58



GIVEN:

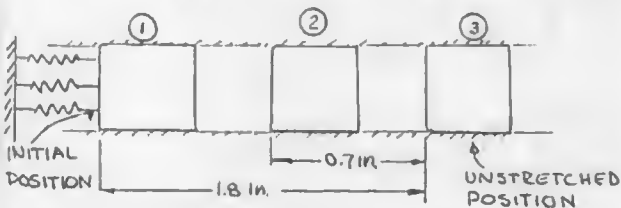
$W = 6 \text{ lb}$   
 $k_1 = 5 \text{ lb/in.}$   
 $k_2 = 10 \text{ lb/in.}$   
 $k_3 = 20 \text{ lb/in.}$   
 INITIAL DISPLACEMENT  
 $x_0 = 1.8 \text{ in. TO LEFT}$   
 FROM UNSTRETCHED  
 POSITION  
 $v_0 = 0$

FIND:

- (a) MAXIMUM VELOCITY,  $v_{\text{MAX}}$   
 (b) VELOCITY AT 0.7 in. FROM INITIAL POSITION

EQUIVALENT  $k_e = k_1 + k_2 + k_3$  (SEE P13.55 (b))

$$k_e = 5 + 10 + 20 = 35 \text{ lb/in} = 420 \text{ lb/ft}$$



- (a) MAXIMUM VELOCITY OCCURS AT ③ WHERE  
 THE SPRINGS ARE UNSTRETCHED

$$T_3 = \frac{1}{2} m v_{\text{MAX}}^2 = \frac{3}{g} v_{\text{MAX}}^2 \quad v_3 = 0$$

$$T_1 = 0 \quad v_1 = \frac{1}{2} k_e x_0^2 = \frac{(420 \text{ lb/ft})}{2} \left( \frac{1.8 \text{ in.}}{12 \text{ in./ft}} \right)^2$$

$$v_1 = 4.725 \text{ lb}\cdot\text{ft}$$

$$T_1 + v_1 = T_3 + v_3$$

$$0 + 4.725 = \frac{3}{g} v_{\text{MAX}}^2 + 0$$

$$v_{\text{MAX}}^2 = \frac{(32.2 \text{ ft/s}^2)(4.725 \text{ lb}\cdot\text{ft})}{3 \text{ lb}} = 50.715$$

$$v_{\text{MAX}} = 7.12 \text{ ft/s} \rightarrow$$

$$(b) \quad T_2 = \frac{1}{2} m v_2^2 = \frac{6}{2g} v_2^2 = \frac{3}{g} v_2^2$$

$$v_2 = \frac{1}{2} k_e x_2^2 = \frac{420 \text{ lb/ft}}{2} \left( \frac{0.7 \text{ in.}}{12 \text{ in./ft}} \right)^2$$

$$v_2 = 0.7146 \text{ lb}\cdot\text{ft}$$

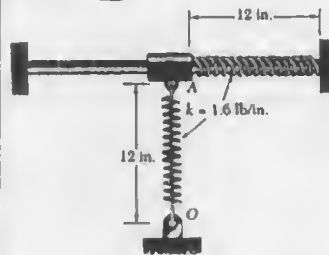
$$T_1 + v_1 = T_2 + v_2$$

$$0 + 4.725 = \frac{3}{g} v_2^2 + 0.7146$$

$$v_2^2 = \frac{(32.2 \text{ ft/s}^2)(4.010 \text{ lb}\cdot\text{ft})}{(3 \text{ lb})} = 43.05$$

$$v_2 = 6.56 \text{ ft/s} \rightarrow$$

13.59

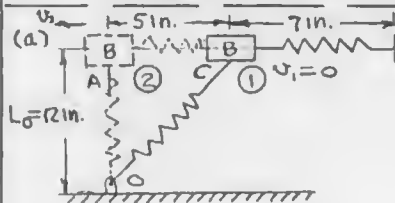


GIVEN:

$W_B = 10 \text{ lb}$   
 COLLAR B PUSHED  
 TO RIGHT,  $x_0 = 5 \text{ in.}$   
 AND RELEASED.  
 UNDEFORMED  
 LENGTH OF  
 EACH SPRING,  $L_F = 12 \text{ in.}$   
 $k = 1.6 \text{ lb/in. FOR}$   
 EACH SPRING

FIND:

- (a) MAXIMUM VELOCITY,  $v_{\text{MAX}}$   
 (b) MAXIMUM ACCELERATION,  $a_{\text{MAX}}$



MAXIMUM VELOCITY OCCURS AT A WHERE THE  
 COLLAR IS PASSING THROUGH ITS EQUILIBRIUM POSITION  
 POSITION ①

$$T_1 = 0 \quad k = (1.6 \text{ lb/in.})(12 \text{ in./ft}) = 19.2 \text{ lb/ft}$$

$$L_{OC} = \sqrt{5^2 + 12^2} = 13 \text{ in.}$$

$$\Delta L_{OC} = 13 \text{ in.} - 12 \text{ in.} = 1 \text{ in.} = \frac{1}{12} \text{ ft.}$$

$$\Delta L_{AC} = 5 \text{ in.} = \frac{5}{12} \text{ ft}$$

$$v_1 = \frac{1}{2} k (\Delta L_{OC})^2 + \frac{1}{2} k (\Delta L_{AC})^2 = \frac{(19.2 \text{ lb/ft})}{2} \left( \frac{1}{12} \text{ ft} \right)^2 + \left( \frac{5}{12} \text{ ft} \right)^2$$

$$v_1 = 1.733 \text{ lb}\cdot\text{ft}$$

POSITION ②

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} \left( \frac{10}{g} \right) v_{\text{MAX}}^2 = \frac{5}{g} v_{\text{MAX}}^2$$

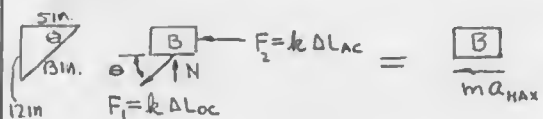
$$v_2 = 0 \quad (\text{BOTH SPRINGS ARE UNSTRETCHED})$$

$$T_1 + v_1 = T_2 + v_2 \quad 0 + 1.733 = \frac{5}{g} v_{\text{MAX}}^2 + 0$$

$$v_{\text{MAX}}^2 = \frac{(1.733 \text{ lb}\cdot\text{ft})(32.2 \text{ ft/s}^2)}{(5 \text{ lb})} = 11.16 \frac{\text{ft}^2}{\text{s}^2}$$

$$v_{\text{MAX}} = 3.34 \text{ ft/s} \rightarrow$$

(b) MAXIMUM ACCELERATION OCCURS AT C WHERE  
 THE HORIZONTAL FORCE ON THE COLLAR IS A  
 MAXIMUM



$$\Sigma F = ma \quad F_1 \cos \theta + F_2 = m a_{\text{MAX}}$$

$$k \Delta L_{OC} \cos \theta + k \Delta L_{AC} = m a_{\text{MAX}}$$

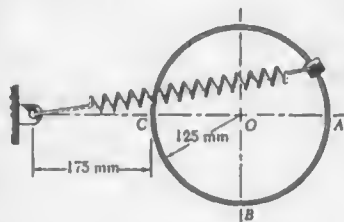
$$(19.2 \text{ lb/ft}) \left( \frac{1}{12} \text{ ft} \right) \left( \frac{5}{13} \right) + \left( \frac{5}{12} \text{ ft} \right) = \frac{10 \text{ lb}}{g} a_{\text{MAX}}$$

$$8.615 = \frac{10}{g} a$$

$$a_{\text{MAX}} = \frac{(8.615 \text{ lb})(32.2 \text{ ft/s}^2)}{(10 \text{ lb})}$$

$$a_{\text{MAX}} = 27.7 \text{ ft/s}^2 \rightarrow$$

13.60



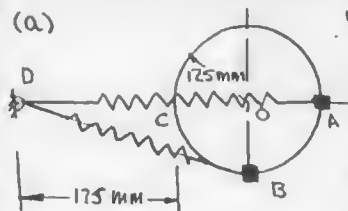
GIVEN:

MASS OF COLLAR  
 $m = 1.5 \text{ kg}$   
 $k = 400 \text{ N/m}$   
 UNDEFORMED  
 LENGTH OF  
 SPRING,  $L_0 = 150 \text{ mm}$   
 COLLAR RELEASED  
 FROM REST AT A

FIND:

- (a) VELOCITY OF THE COLLAR AT B,  $v_B$   
 (b) VELOCITY OF THE COLLAR AT C,  $v_C$

(a)



VELOCITY AT B

$$v_A = 0 \quad T_A = 0$$

$$\Delta L_{AB} = L_{AB} - L_0$$

$$\Delta L_{AB} = 425 \text{ mm} - 150 \text{ mm}$$

$$\Delta L_{AB} = 275 \text{ mm} = 0.275 \text{ m}$$

$$V_A = \frac{1}{2} k (\Delta L_{AB})^2$$

$$V_A = \frac{1}{2} (400 \text{ N/m}) (0.275 \text{ m})^2$$

$$V_A = 15.125 \text{ J}$$

$$T_B = \frac{1}{2} m v_B^2 = \left( \frac{1.5 \text{ kg}}{2} \right) (v_B^2) = (0.75) v_B^2$$

$$L_{BD} = (300^2 \text{ mm} + 125^2 \text{ mm})^{1/2} = 325 \text{ mm}$$

$$\Delta L_{BD} = L_{BD} - L_0 = (325 \text{ mm} - 150 \text{ mm}) = 175 \text{ mm} = 0.175 \text{ m}$$

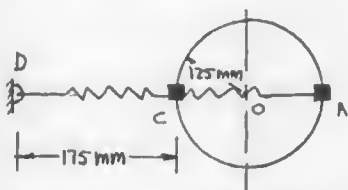
$$V_B = \frac{1}{2} k (\Delta L_{BD})^2 = \frac{1}{2} (400 \text{ N/m}) (0.175 \text{ m})^2 = 6.125 \text{ J}$$

$$T_A + V_A = T_B + V_B \quad 0 + 15.125 = 0.75 v_B^2 + 6.125$$

$$v_B^2 = \frac{(15.125 - 6.125)}{(0.75)} = 12.00 \frac{\text{m}^2}{\text{s}^2}$$

$$v_B = 3.46 \frac{\text{m}}{\text{s}}$$

(b) VELOCITY AT C



$$T_A = 0$$

$$V_A = 15.125 \text{ J (SEE (a))}$$

$$T_C = \frac{1}{2} m v_C^2 = \left( \frac{1.5 \text{ kg}}{2} \right) v_C^2 = 0.75 v_C^2$$

$$\Delta L_{OC} = L_0 - L_{OC} = (150 \text{ mm} - 175 \text{ mm}) = -25 \text{ mm}$$

$$V_C = \frac{1}{2} k (\Delta L_{OC})^2 = \frac{1}{2} (400 \text{ N/m}) (-0.025 \text{ m})^2$$

$$V_C = 0.125 \text{ J}$$

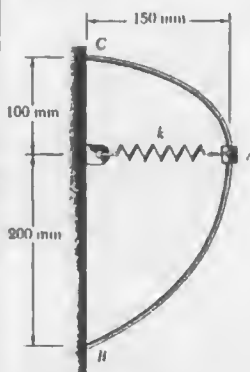
$$T_A + V_A = T_C + V_C$$

$$0 + 15.125 = 0.75 v_C^2 + 0.125$$

$$v_C^2 = 15 / 0.75 = 20$$

$$v_C = 4.47 \text{ m/s}$$

13.61



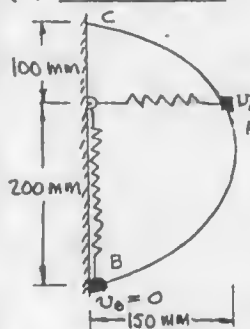
GIVEN:

HORIZONTAL PLANE  
 MASS OF COLLAR,  $m = 500 \text{ g}$   
 UNDEFORMED LENGTH  
 OF SPRING,  $L_0 = 80 \text{ mm}$   
 $k = 400 \text{ N/m}$

FIND:

- (a) VELOCITY AT A,  $v_A$   
 FOR VELOCITY AT B = 0  
 (b) VELOCITY AT C,  $v_C$

(a) VELOCITY AT A



$$T_A = \frac{1}{2} m v_A^2 = \left( \frac{0.5 \text{ kg}}{2} \right) v_A^2$$

$$T_A = (0.25) v_A^2$$

$$\Delta L_A = 0.150 \text{ m} - 0.080 \text{ m}$$

$$\Delta L_A = 0.070 \text{ m}$$

$$V_A = \frac{1}{2} k (\Delta L_A)^2$$

$$V_A = \frac{1}{2} (400 \times 10^3 \text{ N/m}) (0.070 \text{ m})^2$$

$$V_A = 980 \text{ J}$$

$$v_B = 0 \quad T_B = 0$$

$$\Delta L_B = 0.200 \text{ m} - 0.080 \text{ m} = 0.120 \text{ m}$$

$$V_B = \frac{1}{2} k (\Delta L_B)^2 = \frac{1}{2} (400 \times 10^3 \text{ N/m}) (0.120 \text{ m})^2$$

$$V_B = 2880 \text{ J}$$

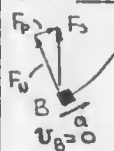
$$T_A + V_A = T_B + V_B \quad 0.25 v_A^2 + 980 = 0 + 2880$$

$$v_A^2 = (2880 - 980) / (0.25)$$

$$v_A^2 = 7600 \text{ m}^2/\text{s}^2$$

$$v_A = 87.2 \text{ m/s}$$

(b) VELOCITY AT C



SINCE SLOPE AT B IS POSITIVE THE COMPONENT OF THE SPRING FORCE  $F_P$ , PARALLEL TO THE ROD, CAUSES THE BLOCK TO MOVE BACK TOWARD A  
 $T_B = 0$ ,  $V_B = 2880 \text{ J (FROM PART (a))}$

$$T_C = \frac{1}{2} m v_C^2 = \left( \frac{0.5 \text{ kg}}{2} \right) v_C^2 = 0.25 v_C^2$$

$$\Delta L_C = 0.100 \text{ m} - 0.080 \text{ m} = 0.020 \text{ m}$$

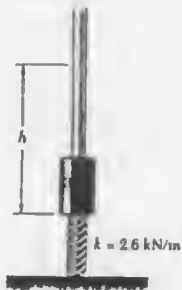
$$V_C = \frac{1}{2} k (\Delta L_C)^2 = \frac{1}{2} (400 \times 10^3 \text{ N/m}) (0.020 \text{ m})^2 = 80.0 \text{ J}$$

$$T_B + V_B = T_C + V_C \quad 0 + 2880 = 0.25 v_C^2 + 80.0$$

$$v_C^2 = 11200 \text{ m}^2/\text{s}^2$$

$$v_C = 105.8 \text{ m/s}$$

13.62

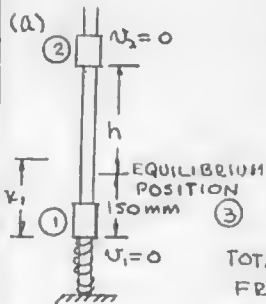


GIVEN:

MASS OF COLLAR,  $m = 3 \text{ kg}$   
 VERTICAL ROD,  $k = 2.6 \text{ kN/m}$   
 MASS IS PUSHED DOWN  
 150 mm FROM ITS  
 EQUILIBRIUM POSITION  
 AND RELEASED  
 SPRING IS UNATTACHED

FIND:

- (a) MAXIMUM HEIGHT  $h$   
 ABOVE EQUILIBRIUM  
 POSITION.  
 (b) MAXIMUM VELOCITY OF  
 THE COLLAR,  $V_{\text{MAX}}$



MAXIMUM HEIGHT IS REACHED  
 WHEN  $V_2 = 0$   
 THUS  $T_1 = T_2 = 0$

$$V = V_g + V_e$$

$$(V_g)_1 = 0 \quad \text{POSITION ①}$$

TOTAL SPRING DEFLECTION  
 FROM UNDEFLECTED SPRING  
 POSITION,  $x_1$   
 $x_1 = mg/k + 0.150$

$$x_1 = mg/k + 0.150 = \frac{(3 \text{ kg})(9.81 \text{ m/s}^2)}{(2.6 \times 10^3 \text{ N/m})} + 0.150 \text{ m}$$

$$x_1 = 0.01132 + 0.150 = 0.1613 \text{ m}$$

$$(V_e)_1 = \frac{1}{2} k x_1^2 = \frac{1}{2} (2.6 \times 10^3 \text{ N/m}) (0.1613 \text{ m})^2 = 33.83 \text{ J}$$

$$V_1 = 0 + 33.83 = 33.83 \text{ J}$$

$$\text{POSITION ②}$$

$$(V_g)_2 = mg(0.150 + h) = 3g(0.150 + h)$$

$$(V_e)_2 = 0 \quad (\text{SPRING IS NOT ATTACHED TO THE COLLAR})$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + (V_g)_1 + (V_e)_1 = 0 + (V_g)_2 + (V_e)_2$$

$$0 + 0 + 33.83 = 0 + 3g(0.150 + h) + 0$$

$$h = \frac{33.83 \text{ J}}{(3 \text{ kg})(9.81 \text{ m/s}^2)} - (0.150 \text{ m}) = 0.9995 \text{ m}$$

$$h = 1000 \text{ mm}$$

(b) MAXIMUM VELOCITY OCCURS

WHEN THE ACCELERATION = 0, I.E. AT EQUILIBRIUM  
 AT POSITION ③

$$T_3 = \frac{1}{2} m V_3^2 = \frac{1}{2} (3) V_{\text{MAX}}^2 = 1.5 V_{\text{MAX}}^2$$

$$V_3 = (V_g)_3 + (V_e)_3 = mg(0.150) + \frac{1}{2} k (x_1 - 0.150)^2$$

$$V_3 = (3 \text{ kg})(9.81 \text{ m/s}^2)(0.150 \text{ m}) + \frac{1}{2} (2.6 \times 10^3 \text{ N/m}) (0.1613 - 0.150 \text{ m})^2$$

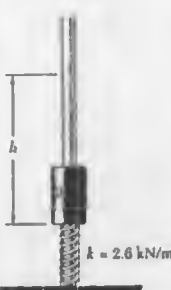
$$V_3 = 4.415 \text{ J} + 0.1660 \text{ J} = 4.581 \text{ J}$$

$$T_1 + V_1 = T_3 + V_3 \quad 0 + 33.83 = 1.5 V_{\text{MAX}}^2 + 4.581$$

$$V_{\text{MAX}}^2 = (29.249) / 1.5 = 19.50 \text{ m}^2/\text{s}^2$$

$$V_{\text{MAX}} = 4.42 \text{ m/s}$$

13.63



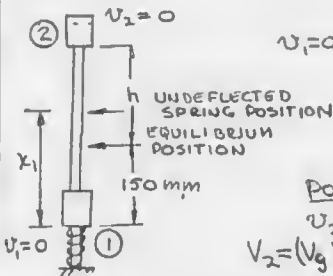
GIVEN:

SAME AS P13.62 AT LEFT  
 EXCEPT THAT THE SPRING  
 IS ATTACHED TO THE  
 COLLAR

FIND:

- (a) MAXIMUM HEIGHT  $h$  ABOVE  
 THE EQUILIBRIUM POSITION  
 (b) MAXIMUM VELOCITY OF  
 THE COLLAR,  $V_{\text{MAX}}$

(a)



POSITION ①

$$V_1 = 0, T_1 = 0$$

$$V_1 = 33.83 \text{ J}$$

(SAME AS P13.62  
 AT LEFT)

POSITION ②

$$V_2 = 0, T_2 = 0$$

$$V_2 = (V_g)_2 + (V_e)_2$$

$$(V_g)_2 = mg(h + 0.150) = 3g(h + 0.150)$$

$$(V_e)_2 = \frac{1}{2} k [h - (x_1 - 0.150)]^2 \quad x_1 = 0.1613 \text{ m}$$

$$(V_e)_2 = \frac{1}{2} (2.6 \times 10^3 \text{ N/m}) (h - 0.0113)^2 \quad (\text{P13.62 AT LEFT})$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 33.83 = 3g(h + 0.150) + \frac{1}{2} (2.6 \times 10^3) (h - 0.0113)^2$$

$$33.83 = 29.4h + 4.415 + 1.3 \times 10^3 h^2 - 29.4h + 0.166 \times 10^3$$

$$h^2 = (33.83 - 4.415 - 0.166 \times 10^3) / 1.3 \times 10^3$$

$$h^2 = 22.499$$

$$h = 0.1500 \text{ m}$$

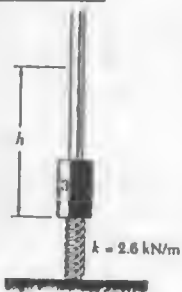
$$h = 150 \text{ mm}$$

(b) MAXIMUM VELOCITY

SEE (b) AT LEFT

$$V_{\text{MAX}} = 4.42 \text{ m/s}$$

13.64



GIVEN:

$$m = 3 \text{ kg}, k = 2.6 \text{ kN/m}$$

FIND:

- (a) COMPRESSION OF SPRING FROM UNDEFORMED POSITION IF COLLAR COMES TO EQUILIBRIUM  
(b) MAXIMUM COMPRESSION IF COLLAR IS SUDDENLY RELEASED

(a)

COLLAR IS IN EQUILIBRIUM



$$mg = 3g$$

$$+\uparrow \Sigma F = (2.6 \times 10^3 \text{ N/m})\delta - 3g = 0$$

$$\delta = \frac{(3 \text{ kg})(9.81 \text{ m/s}^2)}{(2.6 \times 10^3 \text{ N/m})}$$

$$\delta = 0.01132 \text{ m}$$

$$\delta = 11.32 \text{ mm}$$

(b)



MAXIMUM COMPRESSION OCCURS WHEN VELOCITY AT (2) IS ZERO  
 $T_1 = 0 \quad V_1 = 0$   
 $T_2 = 0 \quad V_2 = -mg\delta_{\max} + \frac{1}{2}k\delta_{\max}^2$

$$\delta_{\max} = \frac{(2)(3 \text{ kg})(9.81 \text{ m/s}^2)}{(2.6 \times 10^3 \text{ N/m})} = 0.02264 \text{ m}$$

$$\delta = 22.6 \text{ mm}$$

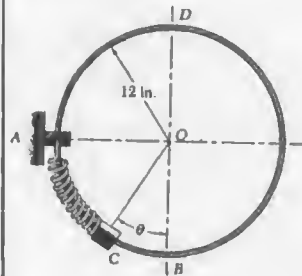
13.66

GIVEN:

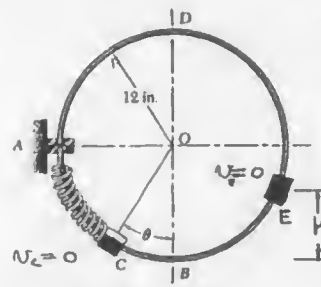
VERTICAL PLANE SPRING,  $k = 310 \text{ lb/ft}$ , UNDEFORMED LENGTH AB, IS UNATTACHED TO COLLAR,  $W = 80 \text{ lb}$ .  
 $\theta = 30^\circ$  ( $V = 0$ )  
 NO FRICTION

FIND:

- (a) MAXIMUM HEIGHT H ABOVE B REACHED BY THE COLLAR  
(b) MAXIMUM VELOCITY,  $V_{\max}$  OF THE COLLAR.



(a)



MAXIMUM HEIGHT ABOVE B IS REACHED WHEN THE VELOCITY AT E IS ZERO

$$T_C = 0$$

$$T_E = 0$$

$$V = V_C + V_g$$

POINT C

$$\Delta L_{BC} = (1 \text{ ft})\left(\frac{\pi}{6} \text{ RAD}\right)$$

$$\Delta L_{BC} = \frac{\pi}{6} \text{ ft}$$

$$\theta = 30^\circ = \pi/6 \text{ RAD}$$

$$R = 12 \text{ in.} = 1 \text{ ft}$$

$$(V_C)_E = \frac{1}{2}k(\Delta L_{BC})^2$$

$$(V_C)_E = \frac{1}{2}(310 \text{ lb/ft})\left(\frac{\pi}{6} \text{ ft}\right)^2 = 0.4112 \text{ lb}\cdot\text{ft}$$

$$(V_C)_g = WR(1 - \cos\theta) = \frac{(80 \text{ lb})}{(160 \text{ lb/ft})}(1 \text{ ft})(1 - \cos 30^\circ)$$

$$(V_C)_g = 0.06699 \text{ lb}\cdot\text{ft}$$

POINT E

$$(V_E)_C = 0 \text{ (SPRING IS UNATTACHED)}$$

$$(V_E)_g = WH = \left(\frac{8}{16}\right)(H) = \frac{H}{2} \text{ (lb}\cdot\text{ft)}$$

$$T_C + V_C = T_E + V_E$$

$$0 + 0.4112 + 0.06699 = 0 + 0 + \frac{H}{2}$$

$$H = 0.956 \text{ ft}$$

- (b) THE MAXIMUM VELOCITY IS AT B WHERE THE POTENTIAL ENERGY IS ZERO,  $V_B = V_{\max}$

$$T_C = 0 \quad V_C = 0.4112 + 0.06699 = 0.4782 \text{ lb}\cdot\text{ft}$$

$$T_B = \frac{1}{2}mV_B^2 = \frac{1}{2}\left(\frac{1}{2} \text{ lb}/32.2 \text{ ft/s}^2\right)V_{\max}^2$$

$$T_B = 0.07640 V_{\max}^2$$

$$V_B = 0$$

$$T_C + V_C = T_B + V_B \quad 0 + 0.4782 = (0.07640)V_{\max}^2$$

$$V_{\max}^2 = 61.59 \text{ ft}^2/\text{s}^2$$

$$V_{\max} = 7.85 \text{ ft/s}$$

13.65



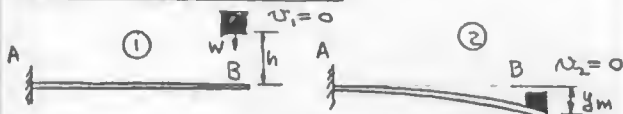
GIVEN:

STATIC DEFLECTION,  $y_{st}$  IS PROPORTIONAL TO  $W$ .

FIND:

$y_m$ , WHEN  $W$  IS DROPPED FROM  $h$

Denote by  $k$  an equivalent spring constant. Static deflection of beam is then  $y_{st} = \frac{W}{k}$  (1)

DROP  $W$  FROM HEIGHT  $h$ 

$$T_1 = 0 \quad V_1 = Wh \quad T_2 = 0 \quad V_2 = -Wy_m + \frac{1}{2}ky_m^2$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + Wh = 0 - Wy_m + \frac{1}{2}ky_m^2$$

FROM EQUATION (1),  $W = ky_{st}$   
 $ky_{st}(h + y_m) = \frac{1}{2}ky_m^2$

$$y_m^2 - 2y_{st}y_m - 2y_{st}h = 0 \quad y_m = y_{st}\left(1 + \sqrt{1 + \frac{2h}{y_{st}}}\right)$$

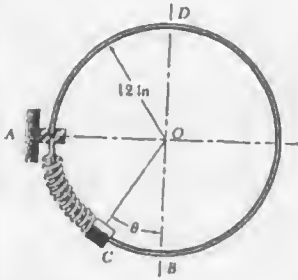
13.67

GIVEN:

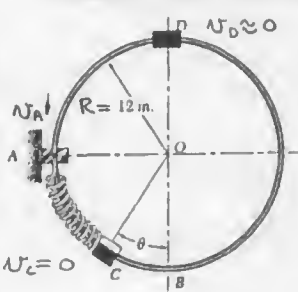
VERTICAL PLANE.  
SPRING,  $k = 316 \text{ lb/ft}$   
OF UNDEFORMED  
LENGTH  $AB$  IS  
UNATTACHED TO  
COLLAR  $C$   
OF WEIGHT  $W = 8.02 \text{ lb}$   
IS RELEASED  
FROM REST AT  
AN ANGLE  $\theta$   
NO FRICTION

FIND:

- (a) SMALLEST VALUE OF  $\theta$  FOR WHICH  
THE COLLAR WILL REACH POINT A  
(b) VALUE OF THE VELOCITY AT IT REACHES A.



- (a) SMALLEST ANGLE  $\theta$  OCCURS WHEN THE  
VELOCITY AT D IS CLOSE TO ZERO



$$v_c = 0 \quad v_D = 0$$

$$T_c = 0 \quad T_D = 0$$

$$V = V_e + V_g$$

POINT C

$$\Delta L_{BC} = (1 \text{ ft})(\theta) = \theta \text{ ft}$$

$$(V_c)_e = \frac{1}{2} k (\Delta L_{BC})^2$$

$$(V_c)_e = \frac{3}{2} \theta^2$$

$$(V_c)_g = WR(1 - \cos \theta)$$

$$(V_c)_g = \left( \frac{8.02}{16.02/16} \right) (1 \text{ ft}) (1 - \cos \theta)$$

$$(V_c)_g = \frac{1}{2} (1 - \cos \theta)$$

$$V_c = (V_c)_e + (V_c)_g = \frac{3}{2} \theta^2 + \frac{1}{2} (1 - \cos \theta)$$

POINT D

$$(V_D)_e = 0 \quad (\text{SPRING IS UNATTACHED})$$

$$(V_D)_g = W(2R) = (2)(0.516)(1 \text{ ft}) = 1.16 \text{ ft}$$

$$T_c + V_c = T_D + V_D \quad 0 + \frac{3}{2} \theta^2 + \frac{1}{2} (1 - \cos \theta) = 1$$

$$(1.5) \theta^2 - (0.5) \cos \theta = 0.5$$

$$\text{BY TRIAL } \theta = 0.7592 \text{ RAD}$$

$$\theta = 43.5^\circ$$

- (b) VELOCITY AT A

POINT D

$$v_D = 0, T_D = 0 \quad V_D = 1.16 \text{ ft} \quad (\text{SEE PART (a)})$$

POINT A

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} \left( \frac{0.516}{32.2 \text{ ft/s}^2} \right) v_A^2$$

$$T_A = 0.007640 v_A^2$$

$$V_A = (V_A)_g = W(2R) = (0.516)(1 \text{ ft}) = 0.516 \text{ ft}$$

$$T_A + V_A = T_D + V_D$$

$$0.007640 v_A^2 + 0.5 = 0 + 1$$

$$v_A^2 = 64.4 \text{ ft}^2/\text{s}^2$$

$$v_A = 8.02 \text{ ft/s}$$

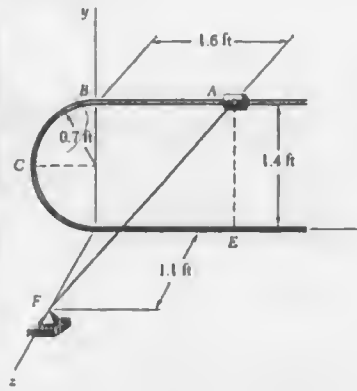
13.68

GIVEN:

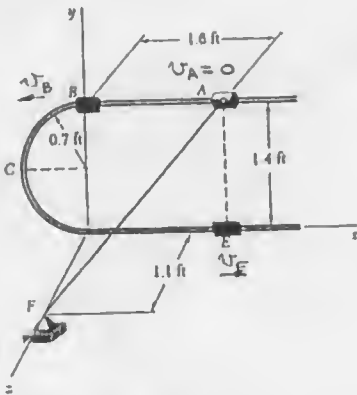
COLLAR,  $W = 2.71 \text{ lb}$   
UNDEFORMED  
LENGTH OF  
ELASTIC CORD  
 $L_0 = 0.9 \text{ ft}$   
 $k = 516 \text{ lb/ft}$   
 $v_A = 0$

FIND:

- SPEED OF COLLAR  
(a) AT B  
(b) AT E



- (a)



$$L_{AF} = \sqrt{(1.6)^2 + (1.4)^2 + (1.1)^2}$$

$$L_{AF} = 2.394 \text{ ft}$$

$$L_{BF} = \sqrt{(1.4)^2 + (1.1)^2}$$

$$L_{BF} = 1.780 \text{ ft}$$

$$L_{FE} = \sqrt{(1.6)^2 + (1.1)^2}$$

$$L_{FE} = 1.942 \text{ ft}$$

$$V = V_e + V_g$$

- (a) SPEED AT B

$$v_A = 0, T_A = 0$$

$$(v_A)_e = \frac{1}{2} k (\Delta L_{AF})^2$$

$$(v_A)_e = \frac{1}{2} (516 \text{ lb/ft}) (1.494 \text{ ft})^2$$

$$(v_A)_e = 5.580 \text{ lb} \cdot \text{ft}$$

$$(v_A)_g = (W)(1.4) = (2.71 \text{ lb})(1.4 \text{ ft}) = 3.78 \text{ lb} \cdot \text{ft}$$

$$v_A = (v_A)_e + (v_A)_g = 5.580 + 3.78 = 9.360 \text{ lb} \cdot \text{ft}$$

$$\text{POINT B} \quad T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} \left( \frac{2.71}{32.2 \text{ ft/s}^2} \right) v_B^2$$

$$T_B = 0.04193 v_B^2$$

$$(v_B)_e = \frac{1}{2} k (\Delta L_{BF})^2 \quad \Delta L_{BF} = L_{BF} - L_0 = 1.780 - 0.9$$

$$\Delta L_{BF} = 0.880 \text{ ft}$$

$$(v_B)_e = \frac{1}{2} (516 \text{ lb/ft}) (0.880 \text{ ft})^2 = 1.936 \text{ lb} \cdot \text{ft}$$

$$(v_B)_g = (W)(1.4) = (2.71 \text{ lb})(1.4 \text{ ft}) = 3.78 \text{ lb} \cdot \text{ft}$$

$$v_B = (v_B)_e + (v_B)_g = 1.936 + 3.78 = 5.716 \text{ lb} \cdot \text{ft}$$

$$T_A + v_A = T_B + v_B$$

$$0 + 9.360 = 0.04193 v_B^2 + 5.716$$

$$v_B^2 = (3.644) / (0.04193)$$

$$v_B^2 = 86.91 \text{ ft}^2/\text{s}^2$$

(CONTINUED)

$$v_B = 9.32 \text{ ft/s}$$

### 13.68 continued

(b) SPEED AT E

POINT A  $T_A = 0$   $V_A = 4.360 \text{ lb}\cdot\text{ft}$  (FROM PART (a))

POINT E

$$T_E = \frac{1}{2} m V_E^2 = \frac{1}{2} \left( \frac{2.7 \text{ lb}}{32.2 \text{ ft/s}^2} \right) V_E^2 = 0.04193 V_E^2$$

$$(V_E)_e = \frac{1}{2} k (\Delta L_{FE})^2 \quad \Delta L_{FE} = L_{FE} - L_0 = 1.942 - 0.900$$

$$\Delta L_{FE} = 1.042 \text{ ft}$$

$$(V_E)_e = \frac{1}{2} (5 \text{ lb/ft}) (1.042 \text{ ft})^2 = 2.714 \text{ lb}\cdot\text{ft}$$

$$(V_E)_g = 0 \quad V_E = 2.714 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_E + V_E \quad 0 + 4.360 = 0.04193 V_E^2 + 2.714$$

$$V_E^2 = 6.6456 / 0.04193$$

$$V_E^2 = 158.49 \text{ ft}^2/\text{s}^2$$

$$V_E = 12.59 \text{ ft/s}$$

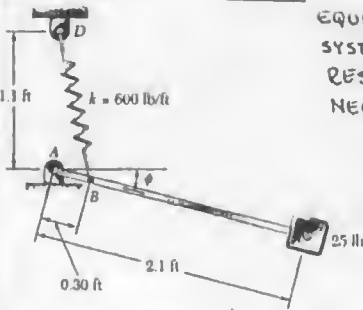
### 13.69

GIVEN:

EQUILIBRIUM FOR  $\phi = 0^\circ$   
SYSTEM RELEASED FROM  
REST WHEN  $\phi = 90^\circ$   
NEGLECT WEIGHT OF ROD

FIND:

VELOCITY OF  
BLOCK C AS  
IT PASSES  
THROUGH  
 $\phi = 0$



FIND THE UNSTRETCHED LENGTH OF THE SPRING



$\phi = 0$

$$\theta = \tan^{-1} \frac{1.1}{0.3} = 1.3045 \text{ RAD}$$

$$\theta = 74.745^\circ$$

$$L_{B0} = \sqrt{(1.1)^2 + 3^2}$$

$$L_{B0} = 1.140 \text{ ft}$$

$$\text{EQUILIBRIUM } \sum M_A = (0.3)(F_s \sin \theta) - (25)(2.1) = 0$$

$$F_s = \frac{(25 \text{ lb})(2.1 \text{ ft})}{(0.3 \text{ ft})(\sin 74.745^\circ)} = 181.39 \text{ lb}$$

$$F_s = k \Delta L_{B0}$$

$$181.39 \text{ lb} = (600 \text{ lb/ft})(\Delta L_{B0})$$

$$\Delta L_{B0} = 0.30232 \text{ ft}$$

$$\text{UNSTRETCHED LENGTH } L_0 = L_{B0} - \Delta L_{B0}$$

$$L_0 = 1.140 - 0.3023 = 0.83768 \text{ ft}$$

SPRING ELONGATION,  $\Delta L'_{B0}$ , WHEN  $\phi = 90^\circ$

$$\Delta L'_{B0} = (1.1 \text{ ft} + 0.3 \text{ ft}) - L_0$$

$$\Delta L'_{B0} = 1.4 \text{ ft} - 0.8377 \text{ ft} = 0.56232 \text{ ft}$$

### 13.69 continued

$\Delta L'_{B0}$  ( $\phi = 90^\circ$ )

$$V_1 = 0, T_1 = 0$$

$$V_1 = (V_1)_e + (V_1)_g$$

$$(V_1)_e = \frac{1}{2} k (\Delta L'_{B0})^2$$

$$(V_1)_e = \frac{1}{2} (600 \text{ lb/ft})(0.5623 \text{ ft})^2$$

$$(V_1)_e = 94.86 \text{ lb}\cdot\text{ft}$$

$$(V_1)_g = -(25 \text{ lb})(2.1 \text{ ft}) = -52.5 \text{ lb}\cdot\text{ft}$$

$$V_1 = 94.86 - 52.5 = 42.36 \text{ lb}\cdot\text{ft}$$

AT ② ( $\phi = 0^\circ$ )

$$(V_2)_e = \frac{1}{2} k (\Delta L_{B0})^2 = \frac{1}{2} (600 \text{ lb/ft})(0.3023 \text{ ft})^2$$

$$(V_2)_e = 27.42 \text{ lb}\cdot\text{ft}$$

$$(V_2)_g = 0 \quad V_2 = 27.42 \text{ lb}\cdot\text{ft}$$

$$T_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \left( \frac{25 \text{ lb}}{32.2 \text{ ft/s}^2} \right) V_2^2 = 0.3882 V_2^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 42.36 = 0.3882 V_2^2 + 27.42$$

$$V_2^2 = (14.94) / (0.3882)$$

$$V_2^2 = 38.48 \text{ ft}^2/\text{s}^2$$

$$V_2 = 6.20 \text{ ft/s}$$

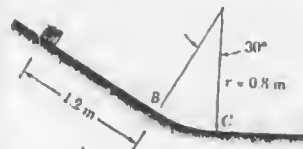
### 13.70

GIVEN:

300-g PELLETT  
RELEASED FROM  
REST. NO FRICTION

FIND:

FORCE ON PELLETT,  
(a) JUST BEFORE B  
(b) IMMEDIATELY  
AFTER B.



VELOCITY AT ②

$$V_1 = 0, T_1 = 0, V_1 = mg(1.2) \sin 30^\circ$$

$$V_1 = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(1.2 \text{ m})(\frac{1}{2})$$

$$V_1 = 1.766 \text{ J}$$

$$T_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} (0.3 \text{ kg})(V_2^2) = 0.15 V_2^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + 1.766 = 0.15 V_2^2 + 0 \quad V_2^2 = 11.77 \text{ m}^2/\text{s}^2$$

(a)  $\sum F = N - 3g \cos 30^\circ = 0$   
 $N = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(\frac{\sqrt{3}}{2}) = 2.55 \text{ N}$

(b)  $\sum F = N - 3g \cos 30^\circ = m v^2 / r = (0.3 \text{ kg})(11.77 \text{ m}^2/\text{s}^2) / (0.8 \text{ m})$   
 $N = 2.55 + 4.41 = 6.96 \text{ N}$

$$N = 6.96 \text{ N}$$



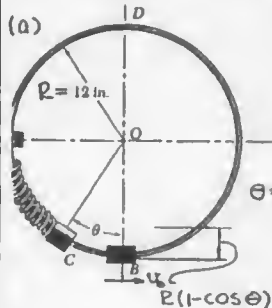
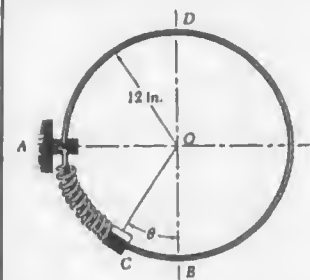
13.73

GIVEN:

VERTICAL PLANE  
SPRING,  $k = 31 \text{ lb/ft}$   
UNDEFORMED  
LENGTH = ARC AB,  
UNATTACHED TO  
COLLAR.  
COLLAR WEIGHT  
 $W = 8 \text{ oz}$ .  
 $\theta = 30^\circ$ .  
COLLAR RELEASED  
FROM REST AT C.

FIND:

- (a) VELOCITY AT B,  $V_c$   
(b) FORCE ON THE  
COLLAR FROM ROD AT B



$$V_c = 0, T_c = 0$$

$$T_B = \frac{1}{2} m V_B^2$$

$$T_B = \frac{1}{2} \left( \frac{8 \text{ oz}}{16 \text{ oz/lb}} \right) (32.2 \text{ ft/s}^2) V_B^2$$

$$T_B = 0.07764 V_B^2$$

$$V_c = (V_c)_e + (V_c)_g$$

$$\text{ARC } BC = \Delta L = R\theta$$

$$\Delta L_{BC} = (1 \text{ ft}) (30^\circ) \left( \frac{\pi}{180^\circ} \right)$$

$$\Delta L_{BC} = 0.5236 \text{ ft}$$

$$(V_c)_e = \frac{1}{2} k (\Delta L_{BC})^2$$

$$(V_c)_e = \frac{1}{2} (31 \text{ lb/ft}) (0.5236 \text{ ft})^2 = 0.4112 \text{ lb}\cdot\text{ft}$$

$$(V_c)_g = WR(1 - \cos \theta) = \frac{(8 \text{ oz})}{(16 \text{ oz/lb})} (1 \text{ ft}) (1 - \cos 30^\circ)$$

$$(V_c)_g = 0.06699 \text{ lb}\cdot\text{ft}$$

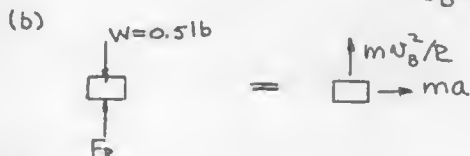
$$V_c = (V_c)_e + (V_c)_g = 0.4112 + 0.06699 = 0.4782 \text{ lb}\cdot\text{ft}$$

$$V_B = (V_B)_e + (V_B)_g = 0 + 0 = 0$$

$$T_c + V_c = T_B + V_B \quad 0 + 0.4782 = 0.07764 V_B^2$$

$$V_B^2 = 6.159 \text{ ft}^2/\text{s}^2$$

$$V_B = 7.85 \text{ ft/s}$$



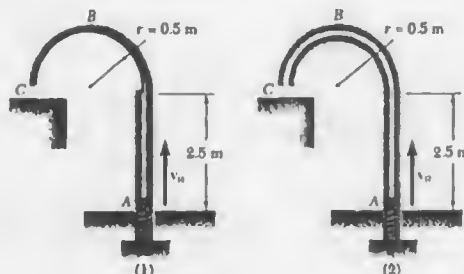
$$+\uparrow \Sigma F = F_R - W = m V_B^2 / R$$

$$F_R = 0.5 \text{ lb} + \frac{(0.5 \text{ lb})}{(32.2 \text{ ft/s}^2)} \left( \frac{6.159 \text{ ft}^2/\text{s}^2}{(1 \text{ ft})} \right)$$

$$F_R = 0.5 \text{ lb} + 0.09564 \text{ lb} = 1.456 \text{ lb}$$

$$F_R = 1.456 \text{ lb}$$

13.74

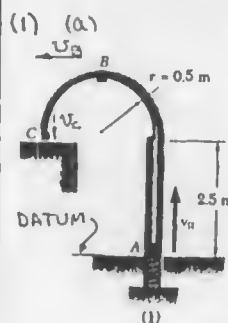


GIVEN:

PACKAGE, MASS  $m = 200\text{-g}$   
INITIAL VELOCITY,  $V_0$   
FRICTION LESS TUBE  
(1) TUBE IS OPEN ALONG CIRCULAR ARC  
(2) TUBE IS CLOSED THROUGHOUT

FIND:

- (a) SMALLEST VELOCITY  $V_0$  FOR PACKAGE  
TO REACH POINT C  
(b) FORCE EXERTED BY THE PACKAGE  
ON THE TUBE.



THE SMALLEST VELOCITY AT B  
WILL OCCUR WHEN THE FORCE  
EXERTED BY THE TUBE ON THE  
PACKAGE IS ZERO.

$$N = 0$$

$$mg = 0.2 \text{ g}$$

$$= \frac{m V_B^2}{r}$$

$$+\uparrow \Sigma F = 0 + mg = \frac{m V_B^2}{r}$$

$$V_B^2 = gr = (9.81 \text{ m/s}^2) (0.5 \text{ m})$$

$$V_B^2 = 4.905 \text{ m}^2/\text{s}^2$$

$$\text{AT A} \quad T_A = \frac{1}{2} m V_0^2 \quad V_A = 0$$

$$\text{AT B} \quad T_B = \frac{1}{2} m V_B^2 = \frac{1}{2} m (4.905) = 2.453 \text{ m}$$

$$V_B = mg(2.5 + 0.5) = 3mg$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m V_0^2 + 0 = 2.453 \text{ m} + 3mg$$

$$V_0^2 = 2[(2.453) + 3(9.81)] = 63.77$$

$$V_0 = 7.99 \text{ m/s}$$

$$\text{AT C} \quad T_C = \frac{1}{2} m V_C^2 \quad V_C = mg(2.5 \text{ m})$$

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2} m V_0^2 + 0 = \frac{1}{2} m V_C^2 + 2.5mg$$

$$V_C^2 = [63.77 - (5.0)(9.81)]$$

$$V_C^2 = 14.72 \text{ m}^2/\text{s}^2$$

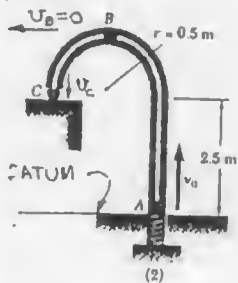
$$(b) \quad N_c = \frac{m V_C^2}{r} = \frac{(0.2 \text{ kg})(14.72 \text{ m}^2/\text{s}^2)}{(0.5 \text{ m})}$$

$$N_c = 5.89 \text{ N}$$



### 13.74 continued

(2) (a)



THE VELOCITY AT B CAN BE NEARLY EQUAL TO ZERO SINCE THE WEIGHT OF THE PACKAGE IS SUPPORTED BY THE TUBE.

$$\begin{aligned} \text{THUS, } v_B = 0 \quad T_B = 0 \\ v_B = mg(2.5 + 0.5) \\ v_B = 3mg \end{aligned}$$

$$T_A = \frac{1}{2} m v_0^2 \quad v_A = 0$$

$$T_B + v_B = T_A + v_A \quad 0 + 3mg = \frac{1}{2} m v_0^2 + 0$$

$$v_0^2 = 6g$$

$$v_0 = 7.67 \frac{m}{s}$$

(b)

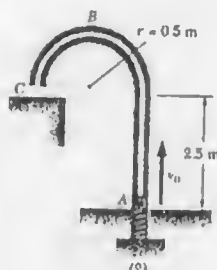
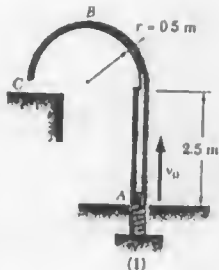
$$T_C = \frac{1}{2} m v_C^2 \quad v_C = mg(2.5 m)$$

$$T_A + v_A = T_C + v_C \quad \frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v_C^2 + 2.5 mg$$

$$v_C^2 = 6g - 5g = 9.81 \text{ m}^2/\text{s}^2$$

$$\begin{aligned} \left[ \begin{array}{c} \square \\ \downarrow mg \end{array} \right] &= \left[ \begin{array}{c} \square \\ \downarrow ma \end{array} \right] \rightarrow \frac{m v_C^2}{r} = N_C = m v_C^2 / r \\ N_C &= (0.2 \text{ kg})(9.81 \text{ m/s}^2) / (0.5 \text{ m}) \\ \text{PACKAGE ON TUBE, } N_C &= 3.92 \text{ N} \end{aligned}$$

### 13.75



GIVEN:

VELOCITY AT C,  $< 3.5 \text{ m/s}$  (REQUIRED)

FIND:

(a) LOOP (2) BUT NOT LOOP (1) CAN SATISFY REQUIREMENT THAT  $v_C < 3.5 \text{ m/s}$

(b) LARGEST ALLOWABLE VELOCITY  $v_0$  WHEN LOOP (2) IS USED AND  $v_C < 3.5 \text{ m/s}$ .

(a) LOOP (1), THE SMALLEST ALLOWABLE

VELOCITY AT B WILL OCCUR WHEN THE FORCE EXERTED BY THE TUBE ON THE PACKAGE IS ZERO

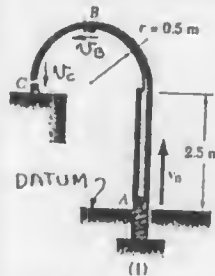
$$\begin{aligned} \left[ \begin{array}{c} \square \\ \downarrow N=0 \end{array} \right] &= \left[ \begin{array}{c} \square \\ \downarrow mg=0.2g \end{array} \right] \\ &= \left[ \begin{array}{c} \square \\ \downarrow m v_B^2 / r \end{array} \right] \end{aligned}$$

$$+\uparrow \Sigma F = 0 + mg = m v_B^2 / r$$

$$v_B^2 = gr = (9.81 \text{ m/s}^2)(0.5 \text{ m}) = 4.905 \text{ m}^2/\text{s}^2$$

$$v_B = 2.215 \text{ m/s}$$

### 13.75 continued



AT B

$$v_B = mg(2.5 + 0.5) = 3mg$$

THE VELOCITY AT B CANNOT BE LESS THAN  $2.215 \text{ m/s}$  IF THE PACKAGE IS TO MAINTAIN CONTACT WITH THE TUBE

FOR  $v_C$  TO BE AS SMALL AS POSSIBLE,  $v_B$  MUST BE AS SMALL AS POSSIBLE; THAT IS  $v_B = 2.215 \text{ m/s}$

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (2.215)^2$$

$$T_B = 2.453 \text{ m}$$

AT C

$$T_C = \frac{1}{2} m v_C^2$$

$$v_C = 2.5 mg$$

$$T_B + v_B = T_C + v_C$$

$$2.453 \text{ m} + 3mg = \frac{1}{2} m v_C^2 + 2.5 mg$$

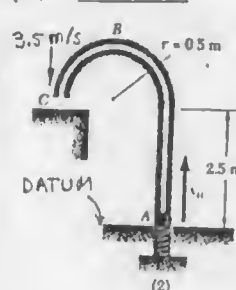
$$v_C^2 = 2 [2.453 + 0.5(9.81 \text{ m/s}^2)]$$

$$v_C^2 = 14.72 \text{ m}^2/\text{s}^2$$

$$v_C = 3.836 \text{ m/s} > 3.5 \text{ m/s}$$

THUS, LOOP (1) CANNOT MEET THE REQUIREMENT

(b) LOOP (2)



$$\text{AT A} \quad T_A = \frac{1}{2} m v_0^2$$

$$v_A = 0$$

AT C

$$v_C = 3.5 \text{ m/s} \quad T_C = \frac{1}{2} m (3.5)^2$$

$$T_C = 6.125 \text{ m}$$

$$v_C = 2.5 mg$$

$$T_A + v_A = T_C + v_C$$

$$\frac{1}{2} m v_0^2 + 0 = 6.125 \text{ m} + 2.5 mg$$

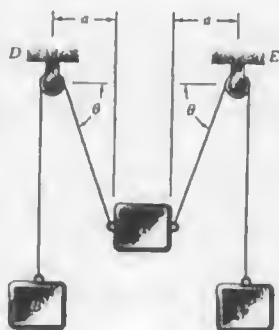
$$v_0^2 = 2 (6.125 + 2.5g) = 61.3 \text{ m}^2/\text{s}^2$$

$$v_0 = 7.83 \text{ m/s}$$

NOTE:

A LARGER VELOCITY AT A WOULD RESULT IN A VELOCITY AT C, GREATER THAN  $3.5 \text{ m/s}$

13.76

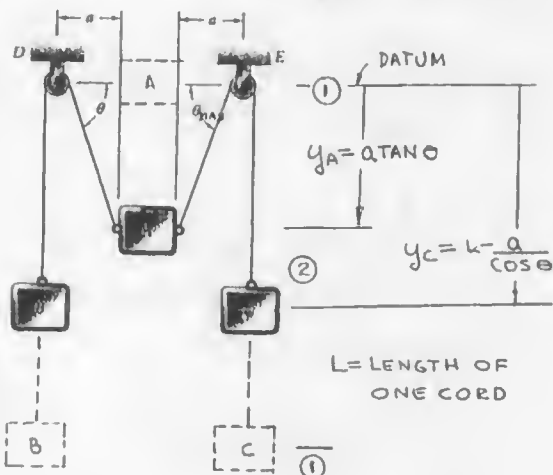


GIVEN:

ALL PACKAGES HAVE THE SAME WEIGHT AND  $v=0$  WHEN  $\theta=0$ .

FIND:

- (a) MAXIMUM  $\theta$   
(b) TENSION IN THE CORD,  $F$ , WHEN  $\theta = \theta_{\text{MAX}}$

(a)  $\theta_{\text{MAX}}$  WHEN  $v=0$ 

(a)

POSITION ①  $\theta=0$ 

$$v_A = v_B = v_C = 0 \quad T_1 = 0$$

$$V_1 = -2W(L-a)$$

POSITION ②  $v_A = v_B = v_C = 0 \quad T_2 = 0$  $\theta = \theta_{\text{MAX}}$ 

$$V_2 = -W a \tan \theta_{\text{MAX}} - (W_B + W_C) \left( L - \frac{a}{\cos \theta_{\text{MAX}}} \right)$$

$$V_2 = -W \left( a \tan \theta_{\text{MAX}} + 2L - \frac{2a}{\cos \theta_{\text{MAX}}} \right)$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 2W(L-a) = 0 - W \left( a \tan \theta_{\text{MAX}} + 2L - \frac{2a}{\cos \theta_{\text{MAX}}} \right)$$

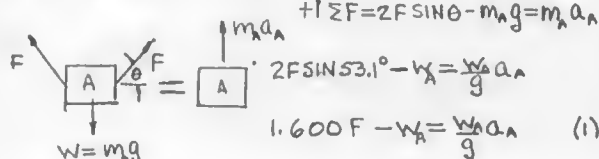
$$-2 \cos \theta_{\text{MAX}} = \sin \theta_{\text{MAX}} - 2$$

$$\sin \theta_{\text{MAX}} + 2 \cos \theta_{\text{MAX}} = 2$$

BY TRIAL

(b) AT  $\theta = 53.1^\circ$ 

$$\theta_{\text{MAX}} = 53.1^\circ$$

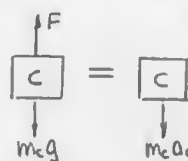


$$+\uparrow \Sigma F = 2F \sin \theta - m_A g = m_A a$$

$$2F \sin 53.1^\circ - W_A = \frac{W_A}{g} a_A$$

$$1.600F - W_A = \frac{W_A}{g} a_A \quad (1)$$

13.76 continued



$$+\uparrow \Sigma F = F - m_C g = -m_C a_C$$

$$F - W_C = -\frac{W_C}{g} a_C \quad (2)$$

KINEMATICS

$$a_A = \ddot{y}_A$$

$$a_C = \ddot{y}_C$$

$$\dot{y}_A = a \tan \theta$$

$$\dot{y}_C = L - \frac{a}{\cos \theta}$$

$$\text{LET } \dot{f}_A(\theta) = -a \sec^2 \theta \dot{\theta}$$

$$\dot{y}_C = a \tan \theta \dot{\theta}$$

$$\ddot{y}_A = \dot{f}_A(\theta) \dot{\theta} + f_A(\theta) \ddot{\theta} \quad \text{LET } \dot{f}_C = \frac{a \tan \theta}{\cos \theta} \dot{\theta}$$

$$\ddot{y}_C = \dot{f}_C \dot{\theta} + f_C(\theta) \ddot{\theta}$$

AT  $\theta_{\text{MAX}}, \dot{\theta} = 0, \theta_{\text{MAX}} = 53.1^\circ$ 

$$\text{THUS } \frac{\ddot{y}_A}{\ddot{y}_C} = \frac{f_A(53.1^\circ)}{f_C(53.1^\circ)} = \frac{-a \sec^2(53.1^\circ)}{a \tan(53.1^\circ) / \cos 53.1^\circ}$$

$$\frac{a_A}{a_C} = 1.250 \quad a_A = 1.250 a_C \quad (3)$$

REPLACE  $a_A$  IN (1) BY  $1.250 a_C$  FROM (3)

$$W_A = W_B = W_C = W$$

$$(1) \quad 1.600F - W = \frac{W}{g} (1.250 a_C)$$

$$(2) \quad F - W = -\frac{W}{g} a_C$$

$$1.600F - W = -1.250(F - W)$$

$$2.850F = 2.250W$$

$$F = 0.789W$$

13.77

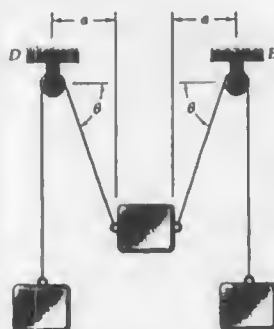
GIVEN:

$$W_A = 2 \text{ lb}$$

$$W_B = W_C = 3 \text{ lb}$$

$$v = 0, \text{ WHEN } \theta = 0$$

FIND:

(a) MAXIMUM  $\theta$ (b) TENSION  $F$  AT  $\theta_{\text{MAX}}$ 

REFER TO FIGURE IN P13.76 (a) AT LEFT

$$(a) \quad \theta = 0 \quad T_1 = 0 \quad V_1 = -(W_B + W_C)(L-a) = -6(L-a)$$

$$\theta = \theta_{\text{MAX}} \quad T_2 = 0 \quad V_2 = -2a \tan \theta_{\text{MAX}} - 6 \left( L - \frac{a}{\cos \theta_{\text{MAX}}} \right)$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 6(L-a) = 0 - 2a \tan \theta_{\text{MAX}} - 6 \left( L - \frac{a}{\cos \theta_{\text{MAX}}} \right)$$

$$-6 \cos \theta_{\text{MAX}} = 2 \sin \theta_{\text{MAX}} - 6$$

$$\text{BY TRIAL} \quad \theta_{\text{MAX}} = 36.9^\circ$$

$$(b) \text{ REFER TO (b) PROB 13.76}$$

$$2F \sin 36.9^\circ - W_A = \frac{W_A}{g} a_A \quad 1.201F - 2 = \frac{2}{g} a_A \quad (1)$$

$$F - W_C = -\frac{W_C}{g} a_C \quad F - 3 = -\frac{3}{g} a_C \quad (2)$$

$$\text{KINEMATICS} \quad \frac{a_A}{a_C} = \frac{\ddot{y}_A}{\ddot{y}_C} = \frac{\sec^2 36.9^\circ}{\tan 36.9^\circ / \cos 36.9^\circ} = 1.665 \quad (3)$$

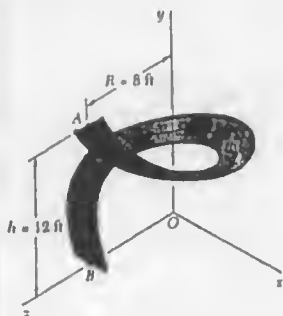
$$\text{SOLVE (1), (2), AND (3) FOR } F$$

$$F = 2.31 \text{ lb}$$

\*13.78

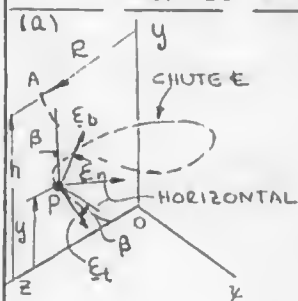
GIVEN:

PACKAGES RELEASED FROM REST AT A CHUTE IS BANKED SO THAT PACKAGES DO NOT TOUCH ITS EDGES. NO FRICTION. PACKAGE WEIGHT,  $W = 20 \text{ lb}$ . CHUTE IS A HELIX WITH PRINCIPAL NORMAL HORIZONTAL AND DIRECTED TOWARD  $y$  axis.



FIND:

- ANGLE  $\phi$  FORMED BY THE NORMAL TO THE SURFACE OF THE CHUTE AND THE PRINCIPAL NORMAL
- MAGNITUDE AND DIRECTION OF THE CHUTE ON THE PACKAGE AT B



AT POINT A

$$V_A = 0 \quad T_A = 0$$

$$V_A = mgh$$

AT ANY POINT P

$$T_P = \frac{1}{2} m v^2$$

$$V_P = Wy = mgy$$

$\underline{E}_n$ , ALONG PRINCIPAL NORMAL, HORIZONTAL AND DIRECTED TOWARD  $y$  axis

$$T_A + V_A = T_P + V_P$$

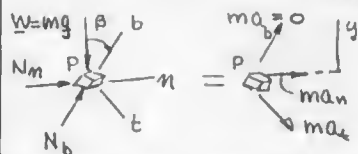
$$0 + mgh = \frac{1}{2} m v^2 + mgy$$

$$v^2 = 2g(h-y)$$

$\underline{E}_t$ , TANGENT TO CENTERLINE OF THE CHUTE

$\underline{E}_b$ , ALONG BINORMAL

$$\beta = \tan^{-1} \frac{h}{2\pi R} = \tan^{-1} \frac{(12 \text{ ft})}{2\pi(8 \text{ ft})} \quad \beta = 13.427^\circ$$



$$ma_b = 0 \quad \text{SINCE } a_b = 0$$

NOTE: FRICTION IS ZERO

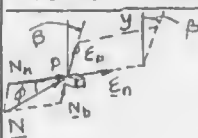
$$\sum F_t = ma_t \quad mgsin\beta = ma_t \quad a_t = gsin\beta$$

$$\sum F_b = ma_b \quad N_b - Wcos\beta = 0 \quad N_b = Wcos\beta$$

$$\sum F_n = ma_n \quad N_n = \frac{mv^2}{\rho} = \frac{m2g(h-y)}{\rho} = \frac{2W(h-y)}{\rho}$$

THE TOTAL NORMAL FORCE IS THE RESULTANT OF  $N_b$  AND  $N_n$ , LIES IN THE  $b-n$  PLANE AND FORMS ANGLE  $\phi$  WITH  $n$  AXIS.

\*13.78 continued



$$\tan\phi = N_b/N_n$$

$$\tan\phi = Wcos\beta / \frac{2(W(h-y))}{\rho}$$

$$\tan\phi = (\rho/2(h-y))cos\beta$$

$$\text{GIVEN: } \rho = R \left[ 1 + \left( \frac{h}{2\pi R} \right)^2 \right] = R(1 + \tan^2\beta) = \frac{R}{\cos^2\beta}$$

THUS:

$$\tan\phi = \frac{\rho}{2(h-y)} cos\beta = \frac{R}{2(h-y)cos\beta}$$

$$\tan\phi = \frac{8 \text{ ft}}{2(12-y)cos14.327^\circ} = \frac{4.113}{12-y}$$

$$\Leftrightarrow \cot\phi = 0.243(12-y)$$

(b) AT POINT B  $y=0$  FOR  $x, y, z$  AXES WE WRITE, WITH  $W = 20 \text{ lb}$

$$N_x = N_b sin\beta = Wcos\beta sin\beta = (20 \text{ lb})cos14.327^\circ sin14.327^\circ$$

$$N_x = 4.517 \text{ lb}$$

$$N_y = N_b cos\beta = Wcos^2\beta = (20 \text{ lb})cos^214.327^\circ$$

$$N_y = 18.922 \text{ lb}$$

$$N_z = -N_n = -2W \frac{h-y}{\rho} = -2 \frac{Wh-y}{R/cos^2\beta}$$

$$N_z = 2(20 \text{ lb}) \frac{(12 \text{ ft} - 0)}{8 \text{ ft}} cos^214.327^\circ \quad N_z = -56.765 \text{ lb}$$

$$N = \sqrt{(4.517)^2 + (18.922)^2 + (-56.765)^2} \quad N = 60.0 \text{ lb}$$

$$cos\theta_x = \frac{N_x}{N} = \frac{4.517}{60}$$

$$\theta_x = 85.7^\circ$$

$$cos\theta_y = \frac{N_y}{N} = \frac{18.922}{60}$$

$$\theta_y = 71.6^\circ$$

$$cos\theta_z = \frac{N_z}{N} = \frac{-56.765}{60}$$

$$\theta_z = 161.1^\circ$$

\*13.79

GIVEN:

$F(x, y, z)$  IS CONSERVATIVE

SHOW THAT:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$$

FOR A CONSERVATIVE FORCE, EQ (13.22) MUST BE SATISFIED

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

$$\text{WE NOW WRITE } \frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

$$\text{SINCE } \frac{\partial^2 V}{\partial x \partial y} = \frac{\partial^2 V}{\partial y \partial x}: \quad \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

WE OBTAIN IN A SIMILAR WAY

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$$

\*13.80

GIVEN:

$$\underline{F} = (yz\mathbf{i} + zx\mathbf{j} + xy\mathbf{k}) / xyz$$

SHOW:

(a)  $\underline{F}$  IS A CONSERVATIVE FORCE

FIND:

(b) THE POTENTIAL FUNCTION ASSOCIATED WITH  $\underline{F}$ 

$$(a) F_x = yz/xyz \quad F_y = zx/xyz$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial(1/x)}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x} = \frac{\partial(1/y)}{\partial x} = 0$$

$$\text{THUS } \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

THE OTHER TWO EQUATIONS DERIVED IN PROB. 13.80 ARE CHECKED IN A SIMILAR WAY

$$(b) \text{ RECALL THAT } F_x = -\frac{\partial V}{\partial x}, F_y = -\frac{\partial V}{\partial y}, F_z = -\frac{\partial V}{\partial z}$$

$$F_x = \frac{1}{x} = -\frac{\partial V}{\partial x} \quad V = -\ln x + f(y, z) \quad (1)$$

$$F_y = \frac{1}{y} = -\frac{\partial V}{\partial y} \quad V = -\ln y + g(z, x) \quad (2)$$

$$F_z = \frac{1}{z} = -\frac{\partial V}{\partial z} \quad V = -\ln z + h(x, y) \quad (3)$$

EQUATING (1) AND (2)

$$-\ln x + f(y, z) = -\ln y + g(z, x)$$

$$\text{THUS } f(y, z) = -\ln y + k(z) \quad (4)$$

$$g(z, x) = -\ln x + k(z) \quad (5)$$

EQUATING (2) AND (3)

$$-\ln z + h(x, y) = -\ln y + g(z, x)$$

$$\text{FROM (5)} \quad g(z, x) = -\ln z + l(x)$$

$$g(z, x) = -\ln x + k(z)$$

THUS

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

FROM (4)

$$f(y, z) = -\ln y - \ln z$$

SUBSTITUTE FOR  $f(y, z)$  IN (1)

$$V = -\ln x - \ln y - \ln z$$

$$V = -\ln xyz + C$$

\*13.81

GIVEN:

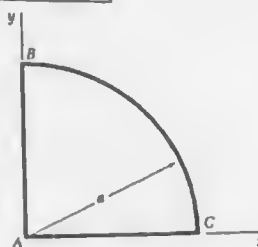
PARTICLE  $P(x, y)$   
ACTED UPON BY FORCE  $\underline{F}$

FIND:

WHETHER  $\underline{F}$  IS A  
CONSERVATIVE FORCE,  
AND COMPUTE THE  
WORK OF  $\underline{F}$  WHEN  
 $P(x, y)$  DESCRIBES A  
PATH ABCA, CLOCKWISE  
FOR,

$$(a) \underline{F} = ky\mathbf{i}$$

$$(b) \underline{F} = k(y\mathbf{i} + x\mathbf{j})$$



$$(a) F_x = ky \quad F_y = 0 \quad \frac{\partial F_x}{\partial y} = k \quad \frac{\partial F_y}{\partial x} = 0$$

THUS  $\frac{\partial F_x}{\partial y} \neq \frac{\partial F_y}{\partial x}$   $\underline{F}$  IS NOT CONSERVATIVE

$$U_{ABCA} = \oint_{ABCA} \underline{F} \cdot d\mathbf{r} = \int_A^B ky\mathbf{i} \cdot dy\mathbf{j} + \int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) + \int_C^A ky\mathbf{i} \cdot dx\mathbf{i}$$

$$\int_A^B = 0, \quad \underline{F} \text{ IS PERPENDICULAR TO THE PATH}$$

$$\int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) = \int_B^C ky dx$$

FROM B TO C THE PATH IS A QUARTER CIRCLE WITH ORIGIN AT A.

$$\text{THUS } x^2 + y^2 = a^2$$

$$y = \sqrt{a^2 - x^2}$$

$$\text{ALONG BC} \quad \int_B^C ky dx = \int_0^a k \sqrt{a^2 - x^2} dx$$

$$= \frac{\pi ka^2}{4}$$

$$\int_C^A ky\mathbf{i} \cdot dx\mathbf{i} = 0 \quad (y=0 \text{ ON CA})$$

$$U_{ABCA} = \int_A^B + \int_B^C + \int_C^A = 0 + \frac{\pi ka^2}{4} + 0$$

$$U_{ABCA} = \frac{\pi ka^2}{4}$$

$$(b) F_x = ky \quad F_y = kx \quad \frac{\partial F_x}{\partial y} = k, \quad \frac{\partial F_y}{\partial x} = k$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}, \quad \underline{F} \text{ IS CONSERVATIVE}$$

SINCE ABCA IS A CLOSED LOOP AND  $\underline{F}$  IS CONSERVATIVE,

$$U_{ABCA} = 0$$

\* 13.82

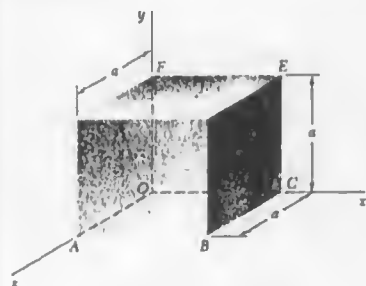
GIVEN:

POTENTIAL  
FUNCTION  
 $V(x, y, z) = -(x^2 + y^2 + z^2)^{1/2}$   
ASSOCIATED  
WITH FORCE  $\underline{P}$ .

FIND:

(a)  $x, y, z$   
COMPONENTS  
OF  $\underline{P}$

(b)  
WORK DONE  
BY  $\underline{P}$  FROM O TO D BY  
INTEGRATING ALONG  
THE PATH OABD,  $U_{OABD}$   
SHOW THAT  $U_{OABD} = \Delta V_{OD}$



$$(a) \quad P_x = -\frac{\partial V}{\partial x} = -\frac{\partial -(x^2 + y^2 + z^2)^{1/2}}{\partial x} = x(x^2 + y^2 + z^2)^{-1/2}$$

$$P_y = -\frac{\partial V}{\partial y} = -\frac{\partial -(x^2 + y^2 + z^2)^{1/2}}{\partial y} = y(x^2 + y^2 + z^2)^{-1/2}$$

$$P_z = -\frac{\partial V}{\partial z} = -\frac{\partial -(x^2 + y^2 + z^2)^{1/2}}{\partial z} = z(x^2 + y^2 + z^2)^{-1/2}$$

$$(b) \quad U_{OABD} = U_{OA} + U_{AB} + U_{BD}$$

O-A  $P_y$  AND  $P_x$  ARE PERPENDICULAR TO O-A  
AND DO NO WORK  
ALSO, ON O-A  $x=y=0$  AND  $P_z=1$

$$\text{THUS } U_{O-A} = \int_0^a P_z dz = \int_0^a dz = a$$

A-B  $P_z$  AND  $P_y$  ARE PERPENDICULAR TO A-B  
AND DO NO WORK  
ALSO ON A-B  $y=0, z=a$  AND  
 $P_x = x/(x^2 + a^2)^{1/2}$

$$\text{THUS } U_{A-B} = \int_0^a \frac{x dx}{(x^2 + a^2)^{1/2}} = a(\sqrt{2} - 1)$$

B-D  $P_x$  AND  $P_z$  ARE PERPENDICULAR TO  
B-D AND DO NO WORK  
ON B-D  $x=a, z=a$   $P_y = y/(y^2 + 2a^2)^{1/2}$

$$\text{THUS } U_{BD} = \int_0^a \frac{y}{(y^2 + 2a^2)^{1/2}} dy = (y^2 + 2a^2)^{1/2} \Big|_0^a$$

$$U_{BD} = (a^2 + 2a^2)^{1/2} - (2a^2)^{1/2} = a(\sqrt{3} - \sqrt{2})$$

$$U_{OABD} = U_{OA} + U_{AB} + U_{BD} = a + a(\sqrt{2} - 1) + a(\sqrt{3} - \sqrt{2})$$

$$U_{OABD} = a\sqrt{3}$$

$$\Delta V_{OD} = V(a, a, a) - V(0, 0, 0) = -(a^2 + a^2 + a^2)^{1/2} - 0$$

$$\Delta V_{OD} = -a\sqrt{3}$$

$$\text{THUS } U_{OABD} = -\Delta V_{OD}$$

\* 13.83

REFER TO FIG. P13.82 ON THE LEFT

GIVEN:

FROM SOLUTION TO (a) OF PROB. 13.82

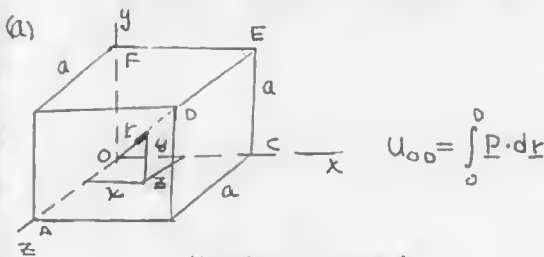
$$\underline{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

FIND:

(a) WORK DONE BY  $\underline{P}$  ALONG THE DIAGONAL  
OD

VERIFY:

(b) THAT WORK DONE AROUND THE  
CLOSED PATH OABDO IS ZERO.



$$\underline{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$d\underline{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$$

$$\underline{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

ALONG THE DIAGONAL  $x=y=z$

$$\text{THUS } \underline{P} \cdot d\underline{r} = \frac{3x}{(3x^2)^{1/2}} = \sqrt{3}$$

$$U_{OD} = \int_0^a \sqrt{3} dx = \sqrt{3}a$$

$$U_{OD} = \sqrt{3}a$$

(b)

$$U_{OABDO} = U_{OABD} + U_{DO}$$

FROM PROB 13.82

$$U_{OABD} = \sqrt{3}a \quad \text{AT LEFT}$$

THE WORK DONE FROM D TO O ALONG THE  
DIAGONAL IS THE NEGATIVE OF THE WORK  
DONE FROM O TO D

$$U_{DO} = -U_{OD} = -\sqrt{3}a \quad (\text{PART (a)})$$

THUS

$$U_{OABDO} = \sqrt{3}a - \sqrt{3}a = 0$$

\*13.84

GIVEN:

$$F = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) / (x^2 + y^2 + z^2)^{3/2}$$

PROVE:

(a)  $F$  IS CONSERVATIVE

FIND:

(b) THE POTENTIAL FUNCTION  $V(x, y, z)$  ASSOCIATED WITH  $F$

(a)  $F_x = x / (x^2 + y^2 + z^2)^{3/2}$   $F_y = y / (x^2 + y^2 + z^2)^{3/2}$

$$\frac{\partial F_x}{\partial y} = \frac{x(-\frac{3}{2})(2y)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \quad \frac{\partial F_y}{\partial x} = \frac{y(-\frac{3}{2})(2x)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}}$$

THUS  $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

THE OTHER TWO EQUATIONS DERIVED IN PROB. 13.79 ARE CHECKED IN A SIMILAR FASHION

(b) RECALLING THAT  $F_x = -\frac{\partial V}{\partial x}$ ,  $F_y = -\frac{\partial V}{\partial y}$ ,  $F_z = -\frac{\partial V}{\partial z}$

$(x = -\frac{\partial V}{\partial x} \quad V = -\int 1 / (x^2 + y^2 + z^2)^{3/2} dx$

$$V = (x^2 + y^2 + z^2)^{-\frac{1}{2}} + f(y, z)$$

SIMILARLY INTEGRATING  $\frac{\partial V}{\partial y}$  AND  $\frac{\partial V}{\partial z}$  SHOWS THAT

THE UNKNOWN FUNCTION  $f(x, y)$  IS A CONSTANT

$$V = \frac{1}{(x^2 + y^2 + z^2)^{1/2}}$$

13.85 continued

NEWTON'S SECOND LAW

$$F = ma_n: \frac{GMm}{r^2} = \frac{mv^2}{r} \quad v^2 = \frac{GM}{r}$$

$$T = \frac{1}{2} m v^2 = \frac{GMm}{2r} \quad V = -\frac{GMm}{r}$$

$$E = T + V = \frac{1}{2} \frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2} \frac{GMm}{r}$$

$$GM = g R_E^2 \quad E = -\frac{1}{2} \frac{g R_E^2 m}{r}$$

$$E = -\frac{1}{2} \frac{(9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})^2(3600 \text{ kg})}{r}$$

$$E = -\frac{716.15 \times 10^{15}}{r} \text{ (N}\cdot\text{m)}$$

FOR A GEOSYNCHRONOUS ORBIT ( $r_2 = 42,140 \times 10^6 \text{ m}$ )

$$E_{GS} = \frac{-716 \times 10^{15}}{42,140 \times 10^6} = -17.003 \times 10^9 \text{ J} = -17.003 \text{ GJ}$$

(a) AT 300 km ( $r_1 = 6.67 \times 10^6 \text{ m}$ )

$$E_{300} = \frac{-716 \times 10^{15}}{6.67 \times 10^6} = -107.42 \times 10^9 \text{ J} = -107.42 \text{ GJ}$$

ADDITIONAL ENERGY  $\Delta E_{300} = E_{GS} - E_{300}$

$$\Delta E_{300} = -17.003 + 107.42$$

$$\Delta E_{300} = 90.46 \text{ GJ}$$

(b) LAUNCH FROM THE EARTH ( $R_E = 6370 \text{ km}$ )

AT LAUNCH PAD  $E_E = V = -\frac{GMm}{R_E} = -\frac{g R_E^2 m}{R_E}$

$$E_E = -(9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})(3600 \text{ kg})$$

$$E_E = -224.96 \times 10^9 \text{ J} = -224.96 \text{ GJ}$$

ADDITIONAL ENERGY  $\Delta E_E = E_{GS} - E_E$

$$\Delta E_E = -17.003 + 224.96 = 208 \text{ GJ}$$

13.86

GIVEN:

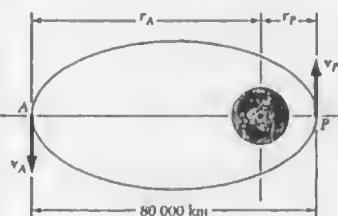
$$r_A / r_P = r_P / r_A$$

$$r_A + r_P = 80000 \text{ km}$$

ELLIPTIC ORBIT

FIND:

ENERGY PER UNIT MASS  $E/m$  REQUIRED TO PLACE THE SATELLITE IN ORBIT.



DETERMINE THE TOTAL ENERGY PER UNIT MASS FOR THE ELLIPTIC ORBIT AND SUBTRACT FROM IT THE ENERGY PER UNIT MASS ON THE EARTH TO GET THE ENERGY PER UNIT MASS NEEDED FOR PROPULSION. (EXCLUDING AIR RESISTANCE, THE WEIGHT OF THE BOOSTER ROCKET AND MANEUVERING.)

13.85

GIVEN:

3600-kg LAUNCHED FROM A CIRCULAR ORBIT AT 300 km ABOVE THE EARTH. ALTITUDE OF GEOSYNCHRONOUS (CIRCULAR) ORBIT = 35770 km

FIND:

(a) ENERGY NEEDED TO PLACE THE SATELLITE INTO GEOSYNCHRONOUS ORBIT FROM 300 km

(b) ENERGY NEEDED TO PLACE THE SATELLITE INTO A GEOSYNCHRONOUS ORBIT FROM THE EARTH (EXCLUDE AIR RESISTANCE)



$$r_2 = 6370 \text{ km} + 35770 \text{ km} = 42,140 \times 10^6 \text{ m}$$

ORBIT AT 300 km

$$r_1 = 6370 \text{ km} + 300 \text{ km} = 6.67 \times 10^6 \text{ m}$$

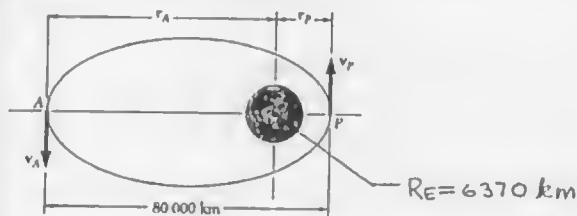
$$R_E = 6370 \text{ km}$$

FOR ANY CIRCULAR ORBIT OF RADIUS  $r$  THE TOTAL ENERGY  $E = T + V = \frac{1}{2} m v^2 - \frac{GMm}{r}$

$M$  = MASS OF THE EARTH

$m = 3600 \text{ kg}$  = SATELLITE MASS

# 13.86 continued



TOTAL ENERGY PER UNIT MASS FOR THE ORBIT

$$E_0 = T_A + V_A = T_P + V_P$$

$$E_0/m = \frac{v_A^2}{2} - \frac{GM}{r_A} = \frac{v_P^2}{2} - \frac{GM}{r_P} \quad (1)$$

$$v_A^2 \left(1 - \frac{r_P^2}{r_A^2}\right) = 2GM \left(\frac{1}{r_A} - \frac{1}{r_P}\right)$$

$$v_A/v_P = r_P/r_A \quad (\text{GIVEN})$$

$$v_A^2 \left(1 - \frac{r_A^2}{r_P^2}\right) = 2GM \left(\frac{r_P - r_A}{r_A r_P}\right)$$

$$v_A^2 \frac{(r_P - r_A)(r_P + r_A)}{r_P^2} = 2GM \frac{(r_P - r_A)}{r_A r_P}$$

$$v_A^2 = 2GM \frac{r_P}{r_A} \left(\frac{1}{r_P + r_A}\right) \quad (2)$$

SUBSTITUTING  $v_A$  IN (2) IN (1)

$$E_0/m = GM \frac{r_P}{r_A} \left(\frac{1}{r_P + r_A}\right) - \frac{GM}{r_A}$$

$$E_0/m = GM \frac{1}{r_A} \left[\frac{r_P - (r_P + r_A)}{r_P + r_A}\right] = -\frac{GM}{r_P + r_A}$$

$$GM = g R_E^2 = (9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})$$

$$r_P + r_A = 80,000 \times 10^3 \text{ m} \quad (\text{GIVEN})$$

$$E_0/m = \frac{-(9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})^2}{80,000 \times 10^3 \text{ m}}$$

$$E_0/m = 4.9765 \times 10^6 \frac{\text{N} \cdot \text{m}}{\text{kg}} = -4.9765 \frac{\text{MJ}}{\text{kg}}$$

TOTAL ENERGY PER UNIT MASS ON THE EARTH

$$E_E = T_E + V_E \quad v_E = 0 \quad T_E = 0 \quad V_E = -\frac{GM}{R_E}$$

$$E_E/m = -\frac{g R_E^2}{R_E} = -(9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})$$

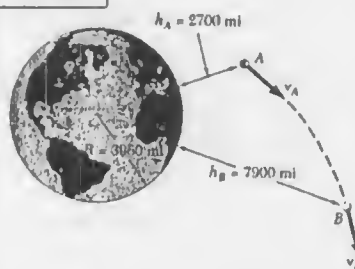
$$E_E/m = -62.490 \times 10^6 \frac{\text{N} \cdot \text{m}}{\text{kg}} = -62.49 \text{ MJ/kg}$$

ENERGY PER UNIT MASS NEEDED FOR PROPULSION,  $E_P/m = E_0/m - E_E/m$

$$E_P/m = -4.9765 \text{ MJ/kg} + 62.490 \text{ MJ/kg}$$

$$E_P/m = 57.5 \frac{\text{MJ}}{\text{kg}}$$

# 13.87

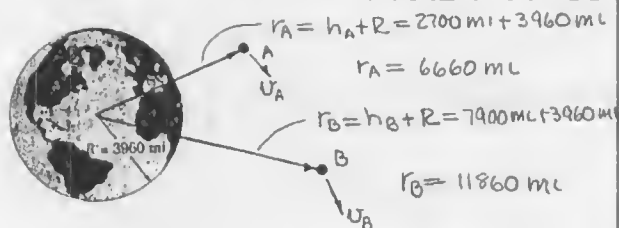


GIVEN:

$h_A$  AND  $h_B$   
AS SHOWN  
 $v_A = 20.2 \times 10^3 \frac{\text{mi}}{\text{h}}$

FIND:

$v_B$



$$\text{AT A, } v_A = 20.2 \times 10^3 \frac{\text{mi}}{\text{h}} = 29627 \text{ ft/s}$$

$$T_A = \frac{1}{2} m (29627 \text{ ft/s})^2 = 438.87 \times 10^6 \text{ m}$$

$$V_A = -\frac{GMm}{r_A} = -\frac{g R_E^2 m}{r_A}$$

$$r_A = 6660 \text{ mi} = 35.165 \times 10^6 \text{ ft}$$

$$R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$$

$$V_A = -\frac{(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2}{(35.165 \times 10^6 \text{ ft})} m = -400.3 \times 10^6 \text{ m}$$

AT B

$$T_B = \frac{1}{2} m v_B^2$$

$$V_B = -\frac{GMm}{r_B} = -\frac{g R_E^2 m}{r_B}$$

$$r_B = 11860 \text{ mi} = 62.621 \times 10^6 \text{ ft}$$

$$V_B = -\frac{(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2}{(62.621 \times 10^6 \text{ ft})} m$$

$$V_B = -224.8 \times 10^6 \text{ m}$$

$$T_A + V_A = T_B + V_B$$

$$438.87 \times 10^6 \text{ m} - 400.3 \times 10^6 \text{ m} = \frac{1}{2} m v_B^2 - 224.8 \times 10^6 \text{ m}$$

$$v_B^2 = 2[438.87 \times 10^6 - 400.3 \times 10^6 + 224.8 \times 10^6]$$

$$v_B^2 = 526.75 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$v_B = 22.95 \times 10^3 \text{ ft/s} = 15.65 \times 10^3 \text{ mi/h}$$

$$v_B = 15.65 \times 10^3 \frac{\text{mi}}{\text{h}}$$

13.88

GIVEN:

LUNAR EXCURSION MODULE (LEM)

FIND:

ENERGY PER POUND NEEDED TO  
ESCAPE MOON'S GRAVITATIONAL  
FIELD STARTING FROM

- (a) MOON'S SURFACE  
(b) CIRCULAR ORBIT 50 MI.  
ABOVE THE MOON'S SURFACE

NOTE:  $GM_{\text{MOON}} = 0.0123 GM_{\text{EARTH}}$ BY EQ. 12.30  $GM_{\text{MOON}} = 0.0123 g R_E^2$ 

AT  $\infty$  DISTANCE FROM MOON:  $r_2 = \infty$ , ASSUME  $v_2 = 0$   
 $E_2 = T_2 + V_2 = 0 - \frac{GMm}{\infty} = 0 - 0 = 0$

(a) ON SURFACE OF MOON  $R_M = 1081 \text{ mi} = 5.7077 \times 10^6 \text{ ft}$   
 $v_1 = 0$   $T_1 = 0$   $R_E = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$V_1 = -\frac{GM_M m}{R_M} \quad E_1 = T_1 + V_1 = 0 - \frac{0.0123 g R_E^2 m}{R_M}$$

$$E_1 = -\frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{(5.7077 \times 10^6 \text{ ft})}$$

WE = WEIGHT OF LEM ON THE EARTH

$$E_1 = (-30.336 \times 10^6 \frac{\text{ft}^2}{\text{s}^2}) m \quad m = \frac{W_E}{g}$$

$$E_1 = \frac{(-30.336 \times 10^6 \frac{\text{ft}^2}{\text{s}^2}) W_E}{32.2 \text{ ft/s}^2}$$

$$\Delta E = E_2 - E_1 = 0 + (942.1 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{lb}}) W_E$$

ENERGY PER POUND:  $\frac{\Delta E}{W_E} = 942 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{lb}}$

(b)



$$r_1 = R_M + 50 \text{ mi}$$

$$r_1 = (1081 \text{ mi} + 50 \text{ mi}) = 1131 \text{ mi} = 5.977 \times 10^6 \text{ ft}$$

NEWTON'S SECOND LAW:

$$F = ma_n: \frac{GM_M m}{r_1^2} = m \frac{v_1^2}{r_1}$$

$$v_1^2 = \frac{GM_M}{r_1} \quad T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m \frac{GM_M}{r_1}$$

$$V_1 = -\frac{GM_M m}{r_1}$$

$$E_1 = T_1 + V_1 = \frac{1}{2} \frac{GM_M m}{r_1} - \frac{GM_M m}{r_1}$$

$$E_1 = -\frac{1}{2} \frac{GM_M m}{r_1} = -\frac{1}{2} \frac{0.0123 g R_E^2 m}{r_1}$$

$$E_1 = -\frac{1}{2} \frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{5.977 \times 10^6 \text{ ft}}$$

$$E_1 = \frac{(14.498 \times 10^6 \frac{\text{ft}^2}{\text{s}^2}) W_E}{(32.2 \text{ ft/s}^2)} = 450.2 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{lb}} W_E$$

$$\Delta E = E_2 - E_1 = 0 + 450.2 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{lb}} W_E$$

ENERGY PER POUND

$$\frac{\Delta E}{W_E} = 450 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{lb}}$$

13.89

GIVEN:

SATELLITE OF MASS  $m$ CIRCULAR ORBIT OF RADIUS  $r$  ABOUT EARTH

FIND:

- (a) ITS POTENTIAL ENERGY  
(b) ITS KINETIC ENERGY  
(c) ITS TOTAL ENERGY

(a) POTENTIAL ENERGY  $V = -\frac{GMm}{r} = -\frac{gR^2 m}{r} + \text{CONSTANT}$

CHOOSING THE CONSTANT (CF EQ 13.17)

SO THAT  $V=0$  FOR  $r=R$ :

$$V = mgr(1 - \frac{R}{r})$$

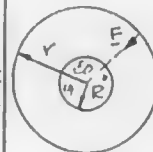
(b) KINETIC ENERGY

NEWTON'S SECOND LAW

$$F = ma_n: \frac{GMm}{r^2} = m \frac{v^2}{r}$$

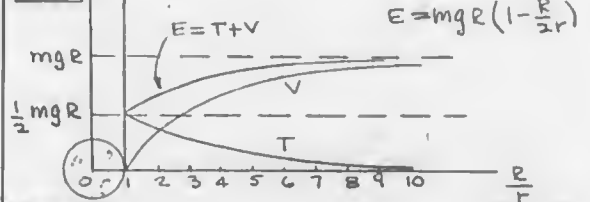
$$v^2 = \frac{GM}{r} = \frac{gR^2}{r}$$

$$T = \frac{1}{2} m v^2 \quad T = \frac{1}{2} m \frac{gR^2}{r}$$

(c) TOTAL ENERGY

$$E = T + V = \frac{1}{2} m \frac{gR^2}{r} + mgr(1 - \frac{R}{r})$$

ENERGY



13.90

GIVEN:

SATELLITE IN A CIRCULAR ORBIT

FIND:

ENERGY REQUIRED TO PLACE IT INTO  
ORBIT AT (a) 600 km, (b) 6000 km

BEFORE LAUNCHING:  $r_1 = R = 6.37 \times 10^6 \text{ m}$ ;  $v_1 = 0$ 

$$E_1 = T_1 + V_1 = 0 - \frac{GMm}{R} = -\frac{gR^2 m}{R} = -mgR$$

IN CIRCULAR ORBIT OF RADIUS  $r_2$ : [cf. EQ 12.30]

NEWTON'S SECOND LAW

$$F = ma_n: \frac{GMm}{r_2^2} = m \frac{v_2^2}{r_2}$$

$$v_2^2 = \frac{GM}{r_2} = \frac{gR^2}{r_2}$$

$$E_2 = T_2 + V_2 = \frac{1}{2} m v_2^2 - \frac{GMm}{r_2}$$

$$E_2 = \frac{1}{2} m \frac{gR^2}{r_2} - \frac{gR^2 m}{r_2} = -\frac{1}{2} \frac{gR^2 m}{r_2}$$

ENERGY IMPARTED IS

$$\Delta E = E_2 - E_1 = -\frac{1}{2} \frac{gR^2 m}{r_2} - (-mgR) = Rmg(1 - \frac{R}{2r_2})$$

ENERGY PER KG IS

$$\Delta E/m = Rg(1 - \frac{R}{2r_2})$$

(a)  $r_2 = 6370 + 600 = 6970 \text{ km}$

$$\Delta E/m = (6.37 \times 10^6)(9.81)(1 - \frac{6370}{2(6970)}) = 33.9 \frac{\text{MJ}}{\text{kg}}$$

(b)  $r_2 = 6370 + 6000 = 12370 \text{ km}$

$$\Delta E/m = (6.37 \times 10^6)(9.81)(1 - \frac{6370}{2(12370)}) = 46.4 \frac{\text{MJ}}{\text{kg}}$$



13.91

GIVEN:

EQ (13.17),  $V_g = -\frac{WR^2}{r}$   
 DISTANCE ABOVE EARTH'S SURFACE,  $y$

SHOW:

(a)  $V_g = Wy$  (FIRST ORDER APPROXIMATION)

DERIVE

(b) A SECOND ORDER APPROXIMATION

$$V_g = -\frac{WR^2}{r} \quad \text{SETTING } r = R + y: V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$$

$$V_g = -WR \left(1 + \frac{y}{R}\right)^{-1} = -WR \left[1 + \frac{(1)}{1} \frac{y}{R} + \frac{(1)(1)}{1 \cdot 2} \left(\frac{y}{R}\right)^2 + \dots\right]$$

WE ADD THE CONSTANT  $WR$ , WHICH IS EQUIVALENT TO CHANGING THE DATUM FROM  $r = \infty$  TO  $r = R$ :

$$V_g = WR \left[ \frac{y}{R} - \left(\frac{y}{R}\right)^2 + \dots \right]$$

(a) FIRST ORDER APPROXIMATION:

$$V_g = WR \left(\frac{y}{R}\right) = Wy \quad [\text{EQ 13.16}]$$

(b) SECOND ORDER APPROXIMATION:

$$V_g = WR \left[ \frac{y}{R} - \left(\frac{y}{R}\right)^2 \right]$$

$$V_g = Wy - Wy^2/R$$

13.92

GIVEN:

CELESTIAL BODY IN CIRCULAR ORBIT,  
 RADIUS  $r = 60$  LIGHT YEARS  
 VELOCITY  $U = 1.2 \times 10^6$  MI/H  
 ABOUT A POINT OF MASS,  $M_B$

FIND:

RATIO  $M_B/M_S$ , WHERE  $M_S$  IS THE  
 MASS OF THE SUN

$$U = 1.2 \times 10^6 \text{ MI/H} = 1.76 \times 10^6 \text{ FT/S}$$

$$r = 60 \text{ LIGHT YEARS}$$

1 LIGHT YEAR IS THE DISTANCE  
 TRAVELED BY LIGHT IN ONE YEAR  
 SPEED OF LIGHT = 186,300 MI/S

$$r = (60 \text{ YR}) \left( \frac{186,300 \text{ MI}}{\text{S}} \right) \left( \frac{5280 \text{ FT}}{\text{MI}} \right) \left( \frac{365 \text{ DAYS}}{\text{YR}} \right) \left( \frac{24 \text{ H}}{\text{DAY}} \right) \left( \frac{3600 \text{ S}}{\text{H}} \right)$$

$$r = 1.8612 \times 10^{18} \text{ FT}$$



NEWTON'S SECOND LAW

$$F = \frac{G M_B M}{r^2} = m \frac{U^2}{r}$$

$$M_B = \frac{r U^2}{G}$$

$$G M_{\text{EARTH}} = g R_{\text{EARTH}}^2 = (32.2 \frac{\text{FT}}{\text{S}^2}) (3960 \text{ MI} \times 5280 \frac{\text{FT}}{\text{MI}})^2 = 4.077 \times 10^{15} \frac{\text{FT}^3}{\text{S}^2}$$

$$M_{\text{SUN}} = 330,000 M_E: G M_{\text{SUN}} = 330,000 G M_{\text{EARTH}}$$

$$G M_{\text{SUN}} = (330,000 \times 4.077 \times 10^{15}) = 4.645 \times 10^{21} \frac{\text{FT}^3}{\text{S}^2}$$

$$G = 4.645 \times 10^{21} / M_{\text{SUN}}$$

$$M_B = \frac{r U^2}{G} = r U^2 M_{\text{SUN}} / 4.645 \times 10^{21}$$

$$M_B / M_{\text{SUN}} = \frac{(1.8612 \times 10^{18}) (1.76 \times 10^6)^2}{4.645 \times 10^{21}} = 1.24 \times 10^9$$

13.93

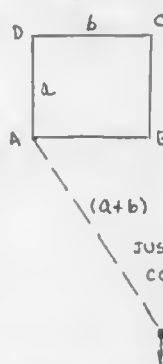
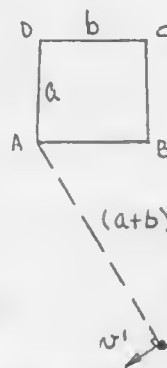
GIVEN:

FRICTIONLESS PLATE  
 FIRMLY ATTACHED TO  
 A HORIZONTAL PLANE  
 CORD ABC ATTACHED  
 TO THE PLATE AT A  
 AND TO A SPHERE AT C  
 $U_0$  = INITIAL VELOCITY  
 OF SPHERE CAUSES IT  
 TO MAKE A COMPLETE  
 CIRCUIT AND RETURN  
 TO C



FIND:

VELOCITY OF THE SPHERE AS IT STRIKES C IF

(a)  $U_0$  IS PARALLEL TO BC(b)  $U_0$  IS PERPENDICULAR TO BC.(a)  $U_0$  PARALLEL TO BCJUST BEFORE  
CORD IS TAUTJUST AFTER  
CORD IS  
TAUT

ANGULAR MOMENTUM IS CONSERVED ABOUT A

$$b U_0 = (a+b) U'$$

$$U' = \frac{b U_0}{(a+b)}$$

AS THE SPHERE CONTINUES ITS CIRCUIT TO POINT  
 C ITS VELOCITY IS ALWAYS PERPENDICULAR TO  
 THE CORD AND ENERGY IS CONSERVED

THUS  $U_c = U'$ 

$$U_c = \frac{b U_0}{(a+b)}$$

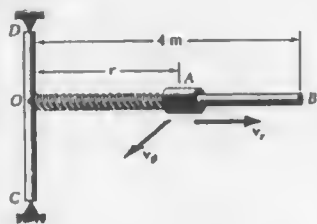
(b)  $U_0$  PERPENDICULAR TO BC

AS THE SPHERE MAKES A COMPLETE CIRCUIT  
 AROUND THE PLATE ITS VELOCITY IS ALWAYS  
 PERPENDICULAR TO THE CORD AND ENERGY  
 IS CONSERVED

THUS  $U_c = U_0$ 

$$U_c = U_0$$

13.94

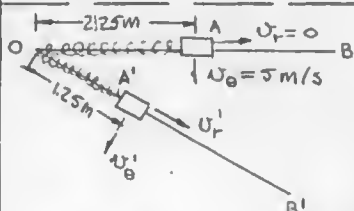


GIVEN:

$k = 750 \text{ N/m}$   
 UNDEFORMED SPRING  
 LENGTH,  $r_0 = 1.5 \text{ m}$   
 COLLAR MASS,  $M = 2.4 \text{ kg}$   
 INITIALLY,  
 $r = 2.25 \text{ m}$ ,  $v_\theta = 5 \text{ m/s}$   
 $v_r = 0$

FIND:

$v_r'$  AND  $v_\theta'$  WHEN  
 $r = 1.25 \text{ m}$



CONSERVATION OF ANGULAR MOMENTUM (ABOUT O)

$$(2.25 \text{ m})(m)(5 \text{ m/s}) = (1.25 \text{ m})(M)(v_\theta')$$

$$v_\theta' = (2.25)(5)/(1.25) = 9.00 \text{ m/s}$$

NO FRICTION

CONSERVATION OF ENERGY

$$T + V = T' + V'$$

$$T = \frac{1}{2} M (v_r^2 + v_\theta^2) = \frac{1}{2} (2.4 \text{ kg}) (0 + (5 \text{ m/s})^2)$$

$$T = 30.0 \text{ J}$$

$$V = \frac{1}{2} k (r - r_0)^2 = \frac{1}{2} (750 \text{ N/m}) (2.25 \text{ m} - 1.5 \text{ m})^2$$

$$V = 210.9 \text{ J}$$

$$v_\theta' = 9.00 \text{ m/s}, v_r'$$

$$T' = \frac{1}{2} M (v_r'^2 + v_\theta'^2) = \frac{1}{2} (2.4 \text{ kg}) (v_r'^2 + (9.00 \text{ m/s})^2)$$

$$T' = 1.2 v_r'^2 + 97.2$$

$$V' = \frac{1}{2} k (r' - r_0)^2 = \frac{1}{2} (750 \text{ N/m}) (1.25 \text{ m} - 1.5 \text{ m})^2$$

$$V' = 23.44 \text{ J}$$

$$T + V = T' + V'$$

$$30 + 210.9 = 1.2 v_r'^2 + 97.2 + 23.44$$

$$1.2 v_r'^2 = 120.26$$

$$v_r'^2 = 100.22$$

$$v_r' = 10.01 \text{ m/s}$$

$$v_r' = 10.01 \text{ m/s}$$

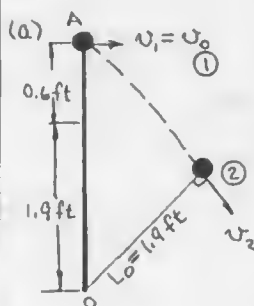
13.95

GIVEN:

ELASTIC CORD FIXED AT O  
 $k = 10 \text{ lb/ft}$   
 UNDEFORMED LENGTH,  $L_0 = 1.9 \text{ ft}$   
 WEIGHT OF BALL,  $W = 1.5 \text{ lb}$   
 HORIZONTAL FRICTIONLESS  
 PLANE  
 INITIAL VELOCITY  $v_0$   
 PERPENDICULAR TO OA

FIND:

- (a) SMALLEST ALLOWABLE  
 $v_0$  IF CORD DOES NOT  
 BECOME SLACK  
 (b) CLOSEST DISTANCE  $d$  FOR  
 $v_0'$  EQUAL TO HALF VALUE  
 FOR  $v_0$  FOUND IN (a)



THE CORD WILL NOT GO  
 SLACK IF  $v_2$  IS  
 PERPENDICULAR TO  
 THE UNDEFORMED CORD  
 LENGTH,  $L_0$ , AT ②

CONSERVATION OF ANGULAR MOMENTUM

$$2.5 v_1 = 1.9 v_2$$

$$v_2 = \frac{2.5}{1.9} v_1 = 1.3158 v_0$$

CONSERVATION OF ENERGY

POINT ①  $v_1 = v_0$   $T_1 = \frac{1}{2} \frac{W}{g} v_0^2 = 0.75 v_0^2$

$$V_1 = \frac{1}{2} k (L - L_0)^2 = \frac{1}{2} (10 \text{ lb/ft}) (2.5 \text{ ft} - 1.9 \text{ ft})^2$$

$$V_1 = 1.800 \text{ lb}\cdot\text{ft}$$

POINT ②  $T_2 = \frac{1}{2} \frac{W}{g} v_2^2 = 0.75 v_2^2$

$$\Delta L = 0 \quad V = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0.75 v_0^2 + 1.800 = 0.75 v_2^2 + 0$$

FROM CONS OF ANG MOM  $v_2 = 1.3158 v_0$

$$0.75 v_0^2 [(1.3158)^2 - 1] = 1.800$$

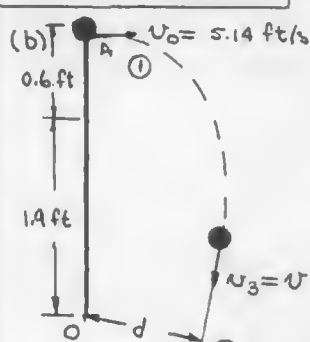
$$v_0^2 = \frac{(1.8 \text{ lb}\cdot\text{ft})(32.2 \text{ ft/s}^2)}{(0.75 \text{ lb})(0.7313)}$$

$$v_0^2 = 105.67 \frac{\text{ft}^2}{\text{s}^2}$$

$$v_0 = 10.28 \frac{\text{ft}}{\text{s}}$$

(CONTINUED)

### 13.95 continued



THE BALL TRAVELS  
IN A STRAIGHT LINE  
AFTER THE CORD  
GOES SLACK

CONS. OF ANG. MOMENTUM

$$(2.5)(5.14) = d v$$

$$d = \frac{12.85}{v}$$

CONS. OF ENERGY

$$v_1 = 5.14 \text{ ft/s}$$

$$T_1 = \frac{1}{2} \frac{W}{g} v_1^2 = \frac{1}{2} \frac{(1.5 \text{ lb})}{32.2 \text{ ft/s}^2} (5.14 \text{ ft/s})^2$$

POINT 1

$$T_1 = 0.6154 \text{ ft} \cdot \text{lb}$$

$$V_1 = \frac{1}{2} k (L - L_0)^2 = \frac{1}{2} (10 \text{ lb/ft}) (2.5 \text{ ft} - 1.9 \text{ ft})^2 = 1.800 \text{ lb} \cdot \text{ft}$$

$$\text{POINT 3} \quad T_3 = \frac{1}{2} \frac{W}{g} v_3^2 = \frac{0.75}{g} v^2$$

$$V_3 = 0$$

$$T_1 + V_1 = T_3 + V_3$$

$$0.6154 + 1.800 = \frac{0.75}{g} v^2 + 0$$

$$v = 10.18 \text{ ft/s}$$

FROM CONS. OF ANG. MOM.

$$d = \frac{12.85}{v} = \frac{12.85}{10.18} = 1.262 \text{ ft}$$

### 13.96

GIVEN:

ELASTIC CORD FIXED AT O  
 $k = 10 \text{ lb/ft}$   
UNDEFORMED LENGTH  $L_0 = 1.9 \text{ ft}$   
WEIGHT OF BALL,  $W = 1.5 \text{ lb}$   
HORIZONTAL FRICTIONLESS PLANE  
 $v_0$  PERPENDICULAR TO OA  
 $d = 0.8 \text{ ft}$  AFTER CORD BECOMES SLACK

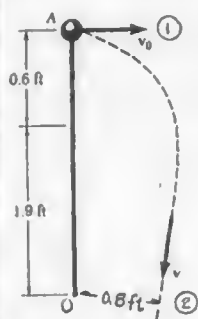
FIND:

(a) INITIAL SPEED,  $v_0$

(b) MAXIMUM SPEED,  $v_m$



(a)



CONSERVATION OF ANGULAR MOMENTUM ABOUT O

$$2.5 v_0 = 0.8 v$$

$$v = 3.125 v_0$$

CONSERVATION OF ENERGY

POINT 1

$$v_1 = v_0 \quad T_1 = \frac{1}{2} \frac{W}{g} v_0^2 = \frac{0.75}{g} v_0^2$$

$$V_1 = \frac{1}{2} k (L - L_0)^2 = \frac{1}{2} (10 \text{ lb/ft}) (2.5 \text{ ft} - 1.9 \text{ ft})^2$$

$$V_1 = 1.800 \text{ lb} \cdot \text{ft}$$

### 13.96 continued

$$\text{POINT 2} \quad v_2 = v \quad T_2 = \frac{1}{2} \frac{W}{g} v^2 = \frac{0.75}{g} v^2$$

$$V_2 = 0 \quad (\text{CORD IS SLACK})$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{0.75}{g} v_0^2 + 1.800 = \frac{0.75}{g} v^2 + 0$$

FROM CONS. OF ANG. MOM.,  $v = 3.125 v_0$

$$\frac{0.75}{g} v_0^2 [(3.125)^2 - 1] = 1.800$$

$$v_0^2 = \frac{(1.800 \text{ lb} \cdot \text{ft}) (32.2 \text{ ft/s}^2)}{(0.75 \text{ lb}) (8.7656)}$$

$$v_0^2 = 8.816 \text{ ft}^2/\text{s}^2$$

$$v_0 = 2.97 \text{ ft/s}$$

(b) MAXIMUM VELOCITY OCCURS WHEN THE BALL IS AT ITS MINIMUM DISTANCE FROM O (WHEN  $d = 0.8 \text{ ft}$ )

$$v_m = 3.125 v_0 = (3.125)(2.97) = 9.28 \text{ ft/s}$$

$$v_m = 9.28 \text{ ft/s}$$

### 13.97

GIVEN:

SPHERE OF MASS,  $m = 0.6 \text{ kg}$   
FORCE BETWEEN A AND O DIRECTED TOWARD O OF MAGNITUDE  $F = (80/r^2) \text{ N}$   
 $v_A = 20 \text{ m/s}$   
HORIZONTAL FRICTIONLESS PLANE

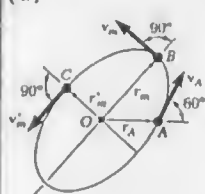


FIND:

(a) MAXIMUM AND MINIMUM DISTANCES FROM O

(b) CORRESPONDING VALUES OF THE SPEED

(a)



THE FORCE EXERTED ON THE SPHERE PASSES THROUGH O. ANGULAR MOMENTUM ABOUT O IS CONSERVED

MINIMUM VELOCITY IS AT B WHERE THE DISTANCE FROM O IS MAXIMUM  
MAXIMUM VELOCITY IS AT C WHERE DISTANCE FROM O IS MINIMUM

$$r_A m v_A \sin 60^\circ = r_m m v_m$$

$$(0.5 \text{ m}) (0.6 \text{ kg}) (20 \text{ m/s}) \sin 60^\circ = r_m (0.6 \text{ kg}) v_m$$

$$v_m = 8.66/r_m \quad (1)$$

CONSERVATION OF ENERGY

$$\text{AT POINT A} \quad T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} (0.6 \text{ kg}) (20 \text{ m/s})^2 = 120 \text{ J}$$

$$V = \int F dr = \int \frac{80}{r^2} dr = -\frac{80}{r} \quad V_A = -\frac{80}{0.5} = -160 \text{ J}$$

$$\text{AT POINT B} \quad T_B = \frac{1}{2} m v_m^2 = \frac{1}{2} (0.6 \text{ kg}) v_m^2 = 0.3 v_m^2$$

(AND POINT C)

$$V_B = -\frac{80}{r_m}$$

$$T_A + V_A = T_B + V_B$$

$$120 - 160 = 0.3 v_m^2 - \frac{80}{r_m} \quad (2)$$

SUBSTITUTE

$$(1) \text{ INTO } (2) \quad -40 = (0.3) \left( \frac{8.66}{r_m} \right)^2 - \frac{80}{r_m}$$

$$r_m^2 - 2 r_m + 0.5625 = 0 \quad (\text{CONTINUED})$$

### 13.97 continued

$$r'_m = 0.339 \text{ m AND } r_m = 1.661 \text{ m}$$

$$r_{\text{MAX}} = 1.661 \text{ m}$$

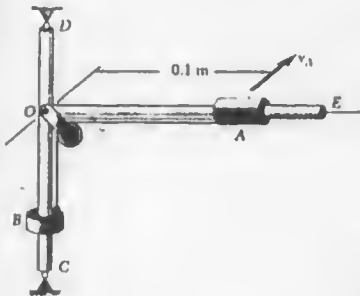
$$r_{\text{MIN}} = 0.339 \text{ m}$$

(b) SUBSTITUTE  $r'_m$  AND  $r_m$  FROM RESULTS OF PART (a) INTO (1) TO GET CORRESPONDING MAXIMUM AND MINIMUM VALUES OF THE SPEED

$$v'_m = \frac{8.66}{0.339} = 25.6 \frac{\text{m}}{\text{s}} \quad v_{\text{MAX}} = 25.6 \frac{\text{m}}{\text{s}}$$

$$v'_m = \frac{8.66}{1.661} = 5.21 \frac{\text{m}}{\text{s}} \quad v_{\text{MIN}} = 5.21 \frac{\text{m}}{\text{s}}$$

### 13.98



GIVEN:

$$m_A = 1.8 \text{ kg}$$

$$m_B = 0.7 \text{ kg}$$

$$\text{INITIALLY, } v_A = 2.1 \frac{\text{m}}{\text{s}}$$

$$\text{AND } v_B = 0$$

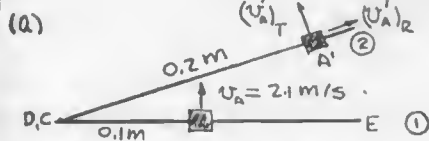
A STOP IS  
SUDDENLY  
REMOVE AT B

FIND:

$$(a) v'_A \text{ WHEN } m_A$$

$$\text{IS } 0.2 \text{ m FROM O}$$

$$(b) v'_A \text{ WHEN } v_B = 0$$



CONSERVATION OF ANGULAR MOMENTUM ABOUT DC

$$(0.1 \text{ m})(m_A)(v_A) = (0.2 \text{ m})(m_A)(v'_A)_T$$

$$(v'_A)_T = \left(\frac{0.1}{0.2}\right)(2.1 \frac{\text{m}}{\text{s}}) = 1.05 \text{ m/s}$$

CONSERVATION OF ENERGY

$$\textcircled{1} v_A = 2.1 \text{ m/s} \quad T_1 = \frac{1}{2} (1.8 \text{ kg}) (2.1 \text{ m/s})^2 = 3.969 \text{ J}$$

$$v_B = 0$$

CHOOSE DATUM FOR B AT ITS INITIAL POSITION  
AND NOTE THAT THE POTENTIAL ENERGY OF A  
DOES NOT CHANGE. THUS WE TAKE  $V_1 = 0$

$$\textcircled{2} (v'_A)_T = 1.050 \text{ m/s} \quad (v'_A)_R = v'_B \quad (\text{KINEMATICS})$$

$$T_2 = \frac{1}{2} m_A [(v'_A)_T^2 + (v'_A)_R^2] + \frac{1}{2} m_B (v'_B)^2$$

$$T_2 = \frac{1}{2} (1.8 \text{ kg}) [(1.050 \text{ m/s})^2 + (v'_A)_R^2] + \frac{1}{2} (0.7 \text{ kg}) (v'_A)_R^2$$

$$T_2 = 0.9923 + 1.25 (v'_A)_R^2$$

$$V_2 = m_B g (0.1 \text{ m}) = (0.7 \text{ kg}) (9.81 \text{ m/s}^2) (0.1 \text{ m}) = 0.6867 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2 \quad 3.969 + 0 = 0.9923 + 1.25 (v'_A)_R^2 + 0.6867$$

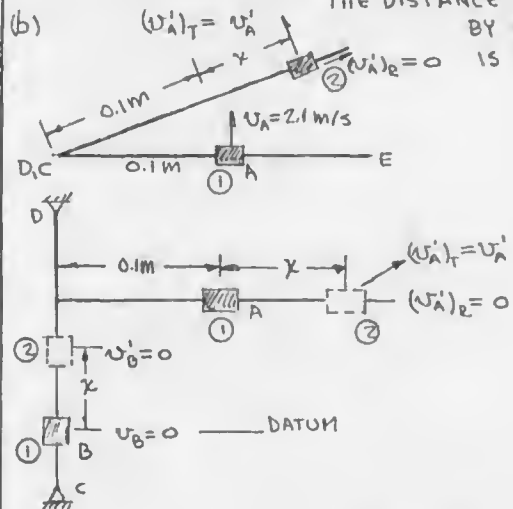
$$(v'_A)_R^2 = 1.832 \frac{\text{m}^2}{\text{s}^2}; \quad (v'_A)_R = 1.354 \frac{\text{m}}{\text{s}}$$

$$v'_A = \sqrt{(v'_A)_T^2 + (v'_A)_R^2} = \sqrt{(1.05)^2 + (1.354)^2} = 1.713 \frac{\text{m}}{\text{s}}$$

$$v'_A = 1.713 \quad \theta = \tan^{-1} \frac{(v'_A)_R}{(v'_A)_T} = \tan^{-1} \frac{1.354}{1.05} = 51.8^\circ$$

### 13.98 continued

WHEN B COMES TO REST  
THE DISTANCE  $x$  MOVED  
BY A AND B  
IS UNKNOWN



CONSERVATION OF ANGULAR MOMENTUM ABOUT DC

$$(0.1 \text{ m})(m_A)(v_A) = (0.1 \text{ m} + x \text{ m})(m_A)(v'_A)_T$$

$$\text{KINEMATICS } (v'_A)_R = (v'_B) = 0, \text{ THUS } (v'_A)_T = v'_A$$

$$v_A = 2.1 \text{ m/s}$$

$$(0.1)(2.1) = (0.1 + x) v'_A \quad x = \frac{0.21}{v'_A} - 0.1$$

CONSERVATION OF ENERGY

$$\text{AT } \textcircled{1} v_A = 2.1 \text{ m/s}$$

$$v_B = 0$$

$$T_1 = \frac{1}{2} m_A v_A^2 = \frac{1}{2} (1.8 \text{ kg}) (2.1)^2$$

$$T_1 = 3.969 \text{ J}$$

$$V_1 = 0$$

$$\text{AT } \textcircled{2} v_B = 0, (v'_A)_R = 0$$

$$T_2 = \frac{1}{2} m_A v_A'^2 = 0.9 v_A'^2$$

$$V_2 = m_B g x = (0.7 \text{ kg}) (9.81 \frac{\text{m}}{\text{s}^2}) x$$

$$V_2 = 6.867 x$$

$$T_1 + V_1 = T_2 + V_2$$

$$3.969 + 0 = 0.9 v_A'^2 + 6.867 x$$

$$\text{FROM CONSERVATION OF ANG. MOM. } x = \frac{0.21}{v'_A} - 0.1$$

$$\text{THUS } 3.969 = 0.9 v_A'^2 + (6.867) \left( \frac{0.21}{v'_A} - 0.1 \right)$$

$$3.969 v_A' = 0.9 v_A'^3 + 1.442 - 0.6867 v_A'$$

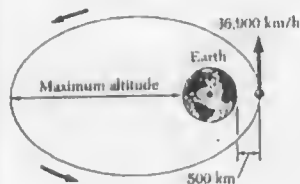
$$4.6557 v_A' = 0.9 v_A'^3 + 1.442$$

$$5.173 v_A' = v_A'^3 + 1.602$$

BY TRIAL

$$v_A' = 0.316 \text{ m/s}$$

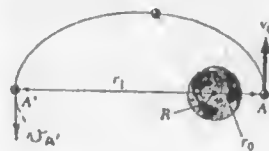
13.99



GIVEN:

SATELLITE LAUNCHED  
AS SHOWN

FIND:

MAXIMUM ALTITUDE,  
USING CONSERVATION  
OF ENERGY AND  
CONSERVATION OF  
MOMENTUM

$$\begin{aligned}
 R &= 6370 \text{ km} \\
 r_0 &= 500 \text{ km} + 6370 \text{ km} \\
 r_0 &= 6870 \text{ km} \\
 &= 6.87 \times 10^6 \text{ m} \\
 v_0 &= 36,900 \text{ km/h} \\
 &= \frac{36.9 \times 10^3 \text{ m}}{3.6 \times 10^3 \text{ s}} \\
 &= 10.25 \times 10^3 \text{ m/s}
 \end{aligned}$$

CONSERVATION OF ANGULAR MOMENTUM

$$\begin{aligned}
 r_0 m v_0 &= r_1 m v_A \quad r_0 = r_{\text{min}}, \quad r_1 = r_{\text{max}} \\
 v_A &= \left( \frac{r_0}{r_1} \right) v_0 = \left( \frac{6.87 \times 10^6}{r_1} \right) (10.25 \times 10^3) \\
 v_A &= \frac{70.418 \times 10^9}{r_1} \quad (1)
 \end{aligned}$$

CONSERVATION OF ENERGY

POINT A

$$v_0 = 10.25 \times 10^3 \text{ m/s}$$

$$T_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m (10.25 \times 10^3)^2$$

$$T_A = (m)(52.53 \times 10^6) \text{ (J)}$$

$$\begin{aligned}
 V_A &= -\frac{GMm}{r_0} \quad GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 \\
 & \quad GM = 398 \times 10^{12} \text{ m}^3/\text{s}^2 \\
 & \quad r_0 = 6.87 \times 10^6 \text{ m} \\
 V_A &= -\frac{(398 \times 10^{12} \text{ m}^3/\text{s}^2)m}{(6.87 \times 10^6 \text{ m})} = -57.93 \times 10^6 \text{ (J)}
 \end{aligned}$$

POINT A'

$$T_{A'} = \frac{1}{2} m v_{A'}^2$$

$$V_{A'} = -\frac{GMm}{r_1} = -\frac{398 \times 10^{12} \text{ m}^3/\text{s}^2}{r_1} \text{ (J)}$$

$$T_A + V_A = T_{A'} + V_{A'}$$

$$52.53 \times 10^6 \text{ m} - 57.93 \times 10^6 \text{ m} = \frac{1}{2} m v_{A'}^2 - \frac{398 \times 10^{12}}{r_1}$$

SUBSTITUTING FOR  $v_{A'}$  FROM (1)

$$-5.402 \times 10^6 = \frac{(70.418 \times 10^9)^2}{(21)(r_1)^2} - \frac{398 \times 10^{12}}{r_1}$$

$$-5.402 \times 10^6 = \frac{(2.4793 \times 10^{21})}{r_1^2} - \frac{398 \times 10^{12}}{r_1}$$

$$(5.402 \times 10^6) r_1^2 - (398 \times 10^{12}) r_1 + 2.4793 \times 10^{21} = 0$$

$$r_1 = 66.7 \times 10^6 \text{ m}, 6.87 \times 10^6 \text{ m}$$

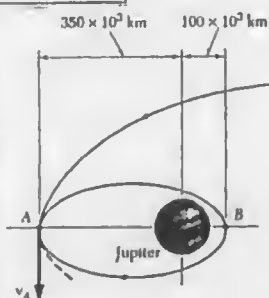
$$r_{\text{max}} = 66,700 \text{ km}$$

13.100

GIVEN:

 $v_A = 26.9 \text{ km/s}$   
 MASS OF JUPITER  
 $M_J = 319 M_E$ 

FIND:

 $\Delta v_A$  TO BRING THE  
 SPACE CRAFT TO  
 WITHIN  $100 \times 10^3 \text{ km}$   
 AT B


CONSERVATION OF ENERGY

POINT A

$$T_A = \frac{1}{2} m (v_A - \Delta v_A)^2$$

$$V_A = -\frac{GM_J m}{r_A}$$

$$\begin{aligned}
 GM_J &= 319 GM_E = 319 g R_E^2 \\
 R_E &= 6.37 \times 10^6 \text{ m} \\
 GM_J &= (319)(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 \\
 GM_J &= 126.98 \times 10^{15} \text{ m}^3/\text{s}^2
 \end{aligned}$$

$$r_A = 350 \times 10^3 \text{ m} \quad V_A = -\frac{(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(350 \times 10^3 \text{ m})}$$

$$V_A = -(362.8 \times 10^6) \text{ m}$$

POINT B

$$T_B = \frac{1}{2} m v_B^2$$

$$V_B = -\frac{GM_J m}{r_B} = -\frac{(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(100 \times 10^3 \text{ m})}$$

$$V_B = -(1269.8 \times 10^6) \text{ m}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m (v_A - \Delta v_A)^2 - 362.8 \times 10^6 \text{ m} = \frac{1}{2} m v_B^2 - 1269.8 \times 10^6 \text{ m}$$

$$(v_A - \Delta v_A)^2 - v_B^2 = -1814 \times 10^6 \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM

$$r_A = 350 \times 10^3 \text{ m} \quad r_B = 100 \times 10^3 \text{ m}$$

$$r_A m (v_A - \Delta v_A) = r_B m v_B$$

$$v_B = \left( \frac{r_A}{r_B} \right) (v_A - \Delta v_A) = \left( \frac{350}{100} \right) (v_A - \Delta v_A) \quad (2)$$

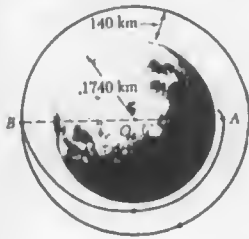
SUBSTITUTE  $v_B$  IN (2) INTO (1)

$$(v_A - \Delta v_A)^2 [1 - (3.5)^2] = -1814 \times 10^6$$

$$\begin{aligned}
 (v_A - \Delta v_A)^2 &= 1612.4 \times 10^6 \quad (v_A - \Delta v_A) = \pm 12.698 \times 10^3 \text{ m/s} \\
 \text{(TAKE + ROOT, - ROOT REVERSES FLIGHT DIRECTION)} \\
 v_A &= 26.9 \times 10^3 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \Delta v_A &= (26.9 \times 10^3 \text{ m/s} - 12.698 \times 10^3 \text{ m/s}) \\
 \Delta v_A &= 14.20 \text{ km/s}
 \end{aligned}$$

13.101



GIVEN:

AT ENGINE SHUTOFF AT A  
 $r_A = 1740 + 1748 \text{ km}$   
 AT B,  $r_B = 1740 + 140 = 1880 \text{ km}$   
 COMMAND MODULE IN A CIRCULAR ORBIT

FIND:

- (a) SPEED AT A AT ENGINE SHUTOFF.  
 (b) RELATIVE VELOCITY WHEN APPROACHES COMMAND MODULE AT A

CONSERVATION OF ANG. MOMENTUM

$$m r_A v_A = m r_B v_B$$

$$v_B = \frac{r_A}{r_B} v_A = \frac{1748}{1880} v_A$$

$$v_B = 0.9298 v_A \quad (1)$$

CONSERVATION OF ENERGY

$$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GM_{\text{MOON}} M}{r_A}$$

AT POINT A

$$M_{\text{MOON}} = 0.0123 M_{\text{EARTH}}$$

$$GM_{\text{MOON}} = 0.0123 GM_{\text{EARTH}} = 0.0123 g R_{\text{EARTH}}^2$$

$$GM_{\text{MOON}} = (0.0123) (9.81 \frac{\text{m}}{\text{s}^2}) (6.37 \times 10^6 \text{ m})^2$$

$$GM_{\text{MOON}} = 4.896 \times 10^{12} \frac{\text{m}^3}{\text{s}^2} \quad r_A = 1748 \times 10^3 \text{ m}$$

$$V_A = \frac{-4.896 \times 10^{12} \text{ m}^3/\text{s}^2}{(1748 \times 10^3 \text{ m})} = -2.801 \times 10^6 \text{ m/s}$$

$$\text{AT POINT B} \quad T_B = \frac{1}{2} m v_B^2 \quad r_B = 1880 \times 10^3 \text{ m}$$

$$V_B = -\frac{GM_{\text{MOON}} M}{r_B} = -\frac{(4.896 \times 10^{12} \text{ m}^3/\text{s}^2) M}{(1880 \times 10^3 \text{ m})} = -2.604 \times 10^6 \text{ m/s}$$

$$T_A + V_A = T_B + V_B; \quad \frac{1}{2} m v_A^2 - 2.801 \times 10^6 \text{ m} = \frac{1}{2} m v_B^2 - 2.604 \times 10^6 \text{ m}$$

$$v_A^2 = v_B^2 + 393.3 \times 10^3 \left( \frac{\text{m}^2}{\text{s}^2} \right) \quad (2)$$

(a) SPEED AT A

SUBSTITUTE  $v_B$  IN (1) INTO (2)

$$v_A^2 (1 - (0.9298)^2) = 393.3 \times 10^3$$

$$v_A^2 = 2.903 \times 10^6 \quad v_A = 1.704 \times 10^3 \frac{\text{m}}{\text{s}} \quad v_A = 1704 \frac{\text{m}}{\text{s}}$$

(b) AT POINT B

$$\text{FROM (1) AND RESULT IN (a)} \quad v_B = (0.9298)(1704)$$

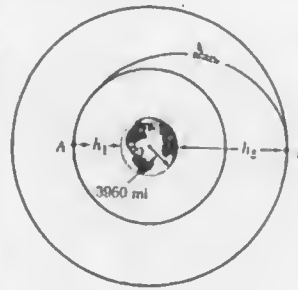
$$v_B = 1584.0 \frac{\text{m}}{\text{s}}$$

COMMAND MODULE IS IN CIRCULAR ORBIT,  $r_B = 1880 \times 10^3 \text{ m}$   
 (EQ 12.44)

$$v_{\text{circ}} = \sqrt{\frac{GM_{\text{MOON}}}{r_B}} = \sqrt{\frac{4.896 \times 10^{12}}{1.88 \times 10^6}} = 1613.8 \frac{\text{m}}{\text{s}}$$

$$\text{RELATIVE VELOCITY} = v_{\text{circ}} - v_B = 1613.8 - 1584.0 = 29.8 \frac{\text{m}}{\text{s}}$$

13.102



GIVEN:

$$h_1 = 200 \text{ mi} \\ h_2 = 500 \text{ mi}$$

FIND:

- FOR A SPACECRAFT TRANSFERRING FROM A CIRCULAR ORBIT TO A CIRCULAR ORBIT AT B  
 (a) INCREASES IN SPEED AT A AND B.  
 (b) TOTAL ENERGY PER UNIT MASS TO EXECUTE THE TRANSFER

ELLIPTICAL ORBIT BETWEEN A AND B

CONS. OF ANG. MOMENTUM

$$m r_A v_A = m r_B v_B$$

$$v_A = \frac{r_B}{r_A} v_B = \frac{23.549}{21.965} v_B$$

$$v_A = 1.0721 v_B \quad (1)$$

$$r_A = 3960 \text{ mi} + 200 \text{ mi} = 4160 \text{ mi}$$

$$r_A = 21.965 \times 10^6 \text{ ft}$$

$$r_B = 3960 \text{ mi} + 500 \text{ mi} = 4460 \text{ mi}$$

$$r_B = 23.549 \times 10^6 \text{ ft}$$

$$E = (3960)(5280) = 20,909 \times 10^6 \text{ ft}$$

$$\text{CONSERVATION OF ENERGY} \quad GM = g R^2 = (32.2 \frac{\text{ft}}{\text{s}^2}) (20,909 \times 10^6)^2$$

$$GM = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

POINT A:

$$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A} = -\frac{(14.077 \times 10^{15}) M}{(21.965 \times 10^6)}$$

POINT B:

$$T_B = \frac{1}{2} m v_B^2 \quad V_B = -\frac{GMm}{r_B} = -\frac{(14.077 \times 10^{15}) M}{(23.549 \times 10^6)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m v_A^2 - 640.89 \times 10^6 \text{ m} = \frac{1}{2} m v_B^2 - 597.79 \times 10^6 \text{ m}$$

$$v_A^2 - v_B^2 = 86.219 \times 10^6$$

$$\text{FROM (1)} \quad v_A = 1.0721 v_B \quad v_B^2 (1.0721^2 - 1) = 86.219 \times 10^6$$

$$v_B^2 = 576.98 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$v_B = 24,020.4 \text{ ft/s}$$

CIRCULAR ORBIT AT A AND B

$$\text{(EQ. 12.44)} \quad (v_A)_c = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077 \times 10^{15}}{21.965 \times 10^6}} = 25316 \text{ ft/s}$$

$$(v_B)_c = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.077 \times 10^{15}}{23.549 \times 10^6}} = 24450 \text{ ft/s}$$

(a) INCREASES IN SPEED AT A AND AT B

$$\Delta v_A = v_A - (v_A)_c = 25753 - 25316 = 437 \text{ ft/s}$$

$$\Delta v_B = (v_B)_c - v_B = 24449 - 24020 = 429 \text{ ft/s}$$

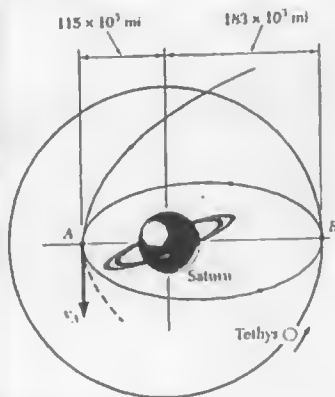
(b) TOTAL ENERGY PER UNIT MASS

$$E/m = \frac{1}{2} [v_A^2 - (v_A)_c^2 + (v_B)_c^2 - v_B^2]$$

$$E/m = \frac{1}{2} [(25753)^2 - (25313)^2 + (24450)^2 - (24020)^2]$$

$$E/m = 216 \times 10^6 \text{ ft}^2/\text{s}^2$$

13.103



GIVEN:

$$v_A = 68.8 \times 10^3 \text{ ft/s}$$

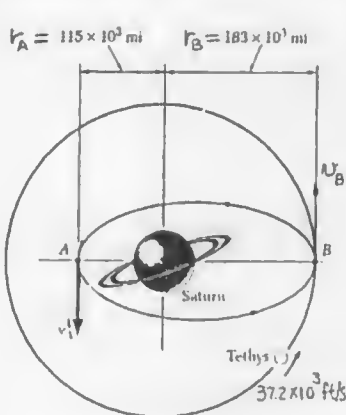
$$v_{\text{Tethys}} = 37.2 \times 10^3 \text{ ft/s}$$

IN CIRCULAR ORBIT

FIND:

- (a) DECREASE IN SPEED,  $\Delta v_A$  OF A SPACECRAFT AT A TO ACHIEVE AN ELLIPTICAL ORBIT THROUGH A AND B
- (b) THE SPEED  $v_B$  OF THE SPACECRAFT AS IT REACHES B

(a)



$$r_A = 115 \times 10^3 \text{ mi}$$

$$r_B = 183 \times 10^3 \text{ mi}$$

$$v_A = 607.2 \times 10^6 \text{ ft}$$

$$v_B = 966.2 \times 10^6 \text{ ft}$$

$v_A'$  = SPEED OF SPACECRAFT IN THE ELLIPTICAL ORBIT AFTER ITS SPEED HAS BEEN DECREASED

ELLIPTICAL ORBIT BETWEEN A AND B  
CONSERVATION OF ENERGY

POINT A  $T_A = \frac{1}{2} m v_A'^2$   $V_A = -\frac{G M_{\text{SAT}} M}{r_A}$

$M_{\text{SAT}}$  = MASS OF SATURN, DETERMINE  $G M_{\text{SAT}}$  FROM THE SPEED OF TETHYS IN ITS CIRCULAR ORBIT

(Eq 12.44)  $v_{\text{circ}} = \sqrt{\frac{G M_{\text{SAT}}}{r}}$   $G M_{\text{SAT}} = r_B v_{\text{circ}}^2$

$$G M_{\text{SAT}} = (966.2 \times 10^6 \text{ ft}) (37.2 \times 10^3 \text{ ft/s})^2 = 1.337 \times 10^{18} \text{ ft}^3/\text{s}^2$$

$$V_A = -\frac{(1.337 \times 10^{18} \text{ ft}^3/\text{s}^2) m}{(607.2 \times 10^6 \text{ ft})} = -2.202 \times 10^7 \text{ ft}^2/\text{s}^2$$

POINT B  $T_B = \frac{1}{2} m v_B'^2$   $V_B = -\frac{G M_{\text{SAT}} M}{r_B} = -\frac{(1.337 \times 10^{18} \text{ ft}^3/\text{s}^2) m}{(966.2 \times 10^6 \text{ ft})}$

$$T_A + V_A = T_B + V_B; \frac{1}{2} m v_A'^2 - 2.202 \times 10^7 m = \frac{1}{2} m v_B'^2 - 1.384 \times 10^9 m$$

$$v_A'^2 - v_B'^2 = 1.636 \times 10^9$$

CONSERVATION OF ANGULAR MOMENTUM

$$r_A m v_A' = r_B m v_B' \quad v_B' = \frac{r_A}{r_B} v_A' = \frac{607.2 \times 10^6}{966.2 \times 10^6} v_A' = 0.6284 v_A'$$

$$v_A'^2 [1 - (0.6284)^2] = 1.636 \times 10^9 \quad v_A' = 52005 \text{ ft/s}$$

$$\Delta v_A = v_A - v_A' = 68800 - 52005 = 16795 \text{ ft/s}$$

(b)  $v_B = \frac{r_A}{r_B} v_A' = (0.6284)(52005) = 32700 \text{ ft/s}$

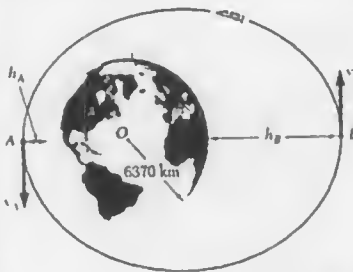
13.104

GIVEN:

$$h_A = 2400 \text{ km}$$

$$h_B = 9600 \text{ km}$$

FIND:

SPEED,  $v_A$ 

$$r_A = 6370 \text{ km} + 2400 \text{ km} = 8770 \text{ km}$$

$$r_B = 6370 \text{ km} + 9600 \text{ km} = 15970 \text{ km}$$

CONSERVATION OF MOMENTUM  $r_A m v_A = r_B m v_B$ 

$$v_B = \frac{r_A}{r_B} v_A = \frac{8770}{15970} v_A = 0.5492 v_A \quad (1)$$

CONSERVATION OF ENERGY

$$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{G M M}{r_A} \quad T_B = \frac{1}{2} m v_B^2 \quad V_B = -\frac{G M M}{r_B}$$

$$G M = g R^2 = (9.81 \text{ m/s}^2) (6370 \times 10^3 \text{ m})^2 = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$V_A = -\frac{(398.1 \times 10^{12} \text{ m}^3/\text{s}^2) m}{8770 \times 10^3} = -4.539 \times 10^6 \text{ m}^2/\text{s}^2$$

$$V_B = -\frac{(398.1 \times 10^{12} \text{ m}^3/\text{s}^2) m}{(15970 \times 10^3)} = -2.493 \text{ m}^2/\text{s}^2$$

$$T_A + V_A = T_B + V_B \quad \frac{1}{2} m v_A^2 - 4.539 \times 10^6 m = \frac{1}{2} m v_B^2 - 2.493 \times 10^6 m$$

SUBSTITUTE FOR  $v_B$  IN (2) FROM (1)

$$v_A^2 [1 - (0.5492)^2] = 40.92 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_A^2 = 58.59 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_A = 7.65 \times 10^3 \text{ m/s}$$

$$v_A = 27.6 \times 10^3 \text{ km/h}$$

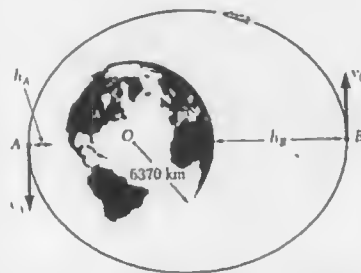
13.105

GIVEN:

$$v_A = 26.3 \times 10^3 \text{ km/h}$$

$$v_B = 18.5 \times 10^3 \text{ km/h}$$

FIND:

ALTITUDE,  $h_B$ 

$$r_A = 6370 \text{ km} + h_A$$

$$r_B = 6370 \text{ km} + h_B$$

$$v_A = 26.3 \times 10^3 \text{ km/h}$$

$$v_B = 18.5 \times 10^3 \text{ km/h} = 5.14 \times 10^3 \text{ m/s}$$

CONSERVATION OF MOMENTUM  $r_A m v_A = r_B m v_B$ 

$$r_A v_A = r_B v_B \quad r_A = \frac{v_B}{v_A} r_B = \frac{18.5}{26.3} r_B$$

$$r_A = 0.7034 r_B \quad (1)$$

### 13.105 : continued

#### CONSERVATION OF ENERGY

$$T_A = \frac{1}{2} m v_A^2 \quad T_A = \frac{1}{2} m (7.31 \times 10^3)^2 = 26.69 \times 10^6 \text{ m}$$

$$T_B = \frac{1}{2} m v_B^2 \quad T_B = \frac{1}{2} m (5.14 \times 10^3)^2 = 13.20 \times 10^6 \text{ m}$$

$$V_A = -\frac{GMm}{r_A} \quad GM = gR^2 = (9.81 \frac{\text{m}}{\text{s}^2})(6370 \times 10^3)^2$$

$$GM = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$V_A = -\frac{398.1 \times 10^{12}}{r_A}$$

$$V_B = -\frac{GMm}{r_B} = -\frac{398.1 \times 10^{12}}{r_B}$$

$$T_A + V_A = T_B + V_B$$

$$26.69 \times 10^6 \text{ m} - \frac{398.1 \times 10^{12}}{r_A} = 13.20 \times 10^6 \text{ m} - \frac{398.1 \times 10^{12}}{r_B}$$

SUBSTITUTE FOR  $r_A$  FROM (1)

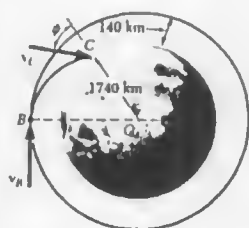
$$\frac{398.1 \times 10^{12}}{r_B} \left( \frac{1}{(0.7034)} - 1 \right) = 13.49 \times 10^6$$

$$\frac{1}{r_B} = 80.37 \times 10^{-9}$$

$$r_B = 12.442 \times 10^6 \text{ m} = 12442 \text{ km}$$

$$h_B = r_B - R = 12442 \text{ km} - 6370 \text{ km} = 6070 \text{ km}$$

### \* 13.106

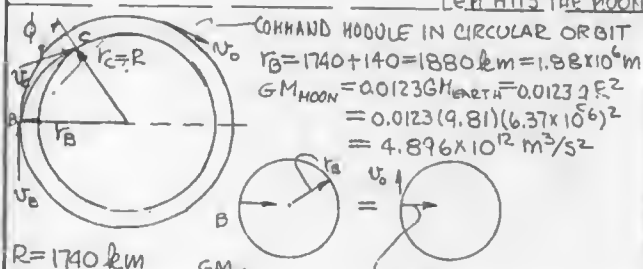


GIVEN:

COMMAND MODULE IN CIRCULAR ORBIT AT AN ALTITUDE OF 140 km. ATTACHED LEM CAST ALRIPT AT RELATIVE VELOCITY OF 200 m/s

FIND:

$v_c$  AND  $\phi$  AS THE LEM HITS THE MOON



$$R = 1740 \text{ km}$$

$$ZF = ma_n \quad \frac{GMm}{r_B^2} = m \frac{v_0^2}{r_B} \quad v_0 = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{4.896 \times 10^{12}}{1.88 \times 10^6}}$$

$$v_0 = 1614 \text{ m/s} \quad v_B = 1614 - 200 = 1414 \text{ m/s}$$

#### CONSERVATION OF ENERGY BETWEEN B AND C

$$\frac{1}{2} m v_B^2 - \frac{GMm}{r_B} = \frac{1}{2} m v_c^2 - \frac{GMm}{r_c} \quad r_c = R$$

$$v_c^2 = v_B^2 + 2 \frac{GM}{r_B} \left( \frac{r_B}{R} - 1 \right)$$

$$v_c^2 = (1414 \text{ m/s})^2 + 2 \left( \frac{4.896 \times 10^{12} \text{ m}^3/\text{s}^2}{(1.88 \times 10^6 \text{ m})} \right) \left( \frac{1.88 \times 10^6}{1.74 \times 10^6} - 1 \right)$$

$$v_c^2 = 1.999 \times 10^6 + 0.4191 \times 10^6 = 2.418 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_c = 1555 \frac{\text{m}}{\text{s}}$$

### \* 13.106 continued

#### CONSERVATION OF ANGULAR MOMENTUM

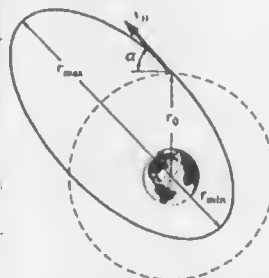
$$r_B m v_B = r_m v_c \sin \phi$$

$$\sin \phi = \frac{r_B v_B}{r_c v_c} = \frac{(1.88 \times 10^6 \text{ m})(1414 \frac{\text{m}}{\text{s}})}{(1.74 \times 10^6 \text{ m})(1555 \frac{\text{m}}{\text{s}})} = 0.98249$$

$$\phi = 79.26^\circ$$

$$\phi = 79.3^\circ$$

### 13.107

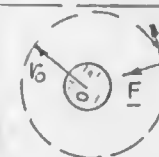


GIVEN:

SATELLITE PROJECTED AT VELOCITY  $v_0$  AT AN ANGLE  $\alpha$  WITH ITS INTENDED CIRCULAR ORBIT.

FIND:

$r_{\text{MAX}}$  AND  $r_{\text{MIN}}$



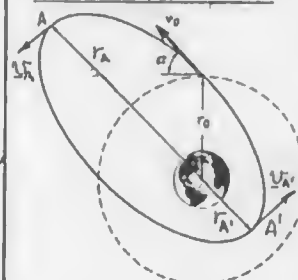
FOR CIRCULAR ORBIT OF RADIUS  $r_0$

$$F = ma_n \quad \frac{GMm}{r_0^2} = m \frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

BUT  $v_0$  FORMS AN ANGLE  $\alpha$  WITH THE INTENDED CIRCULAR PATH

FOR ELLIPTIC ORBIT



CONS OF ANG MOMENTUM

$$r_0 m v_0 \cos \alpha = r_A m v_A$$

$$v_A = \left( \frac{r_0}{r_A} \cos \alpha \right) v_0 \quad (1)$$

CONS OF ENERGY

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_0} =$$

$$\frac{1}{2} m v_A^2 - \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0} \left( 1 - \frac{r_0}{r_A} \right)$$

SUBSTITUTE FOR  $v_A$  FROM (1)

$$v_0^2 \left[ 1 - \left( \frac{r_0}{r_A} \right)^2 \cos^2 \alpha \right] = \frac{2GM}{r_0} \left( 1 - \frac{r_0}{r_A} \right)$$

$$\text{BUT } v_0^2 = \frac{GM}{r_0}, \text{ THUS } 1 - \left( \frac{r_0}{r_A} \right)^2 \cos^2 \alpha = 2 \left( 1 - \frac{r_0}{r_A} \right)$$

$$\cos^2 \alpha \left( \frac{r_0}{r_A} \right)^2 - 2 \left( \frac{r_0}{r_A} \right) + 1 = 0$$

SOLVING FOR  $\frac{r_0}{r_A}$

$$\frac{r_0}{r_A} = \frac{2 \pm \sqrt{4 - 4 \cos^2 \alpha}}{2 \cos^2 \alpha} = \frac{1 \pm \sin \alpha}{1 - \sin^2 \alpha}$$

$$r_A = \frac{(1 + \sin \alpha)(1 - \sin \alpha)}{1 \pm \sin \alpha} r_0 = (1 \mp \sin \alpha) r_0$$

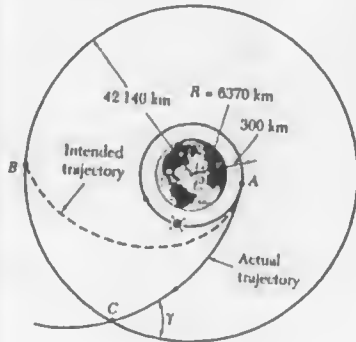
ALSO VALID FOR POINT A

THUS

$$r_{\text{MAX}} = (1 + \sin \alpha) r_0 \quad r_{\text{MIN}} = (1 - \sin \alpha) r_0$$



13.108

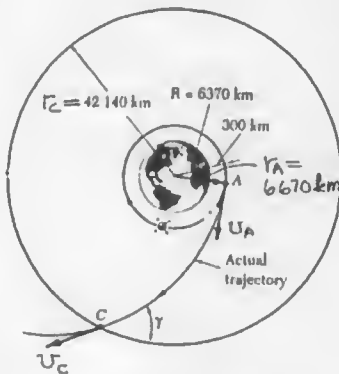


GIVEN:

COMMUNICATION  
SATELLITE AT A  
IS LAUNCHED  
WITH A VELOCITY  
RELATIVE TO A  
SPACE PLATFORM  
IN CIRCULAR  
ORBIT OF  
( $U_A$ )<sub>R</sub> = 3.44 km/s

FIND:

ANGLE  $\gamma$  AT  
WHICH THE  
SATELLITE  
CROSSES THE  
CIRCULAR  
ORBIT AT C.



$$R = 6370 \text{ km}$$

$$r_A = 6370 \text{ km} + 300 \text{ km}$$

$$r_A = 6.67 \times 10^6 \text{ m}$$

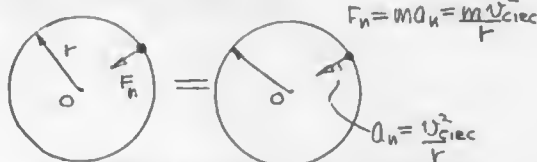
$$r_C = 42.14 \times 10^6 \text{ m}$$

$$GM = gR^2$$

$$GM = (9.81 \frac{\text{m}}{\text{s}^2})(6.37 \times 10^6 \text{ m})^2$$

$$GM = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

FOR ANY CIRCULAR ORBIT



$$F_n = \frac{GMm}{r^2} = m \frac{v_{circ}^2}{r}$$

$$v_{circ} = \sqrt{\frac{GM}{r}}$$

VELOCITY AT A

$$(v_A)_{circ} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.1 \times 10^{12} \text{ m}^3/\text{s}^2}{6.67 \times 10^6 \text{ m}}} = 7.726 \times 10^3 \text{ m/s}$$

$$v_A = (v_A)_{circ} + (v_A)_R = 7.726 \times 10^3 + 3.44 \times 10^3 = 11.165 \times 10^3 \text{ m/s}$$

VELOCITY AT C

CONSERVATION OF ENERGY  $T_A + V_A = T_C + V_C$ 

$$\frac{1}{2} m v_A^2 - \frac{GMm}{r_A} = \frac{1}{2} m v_C^2 - \frac{GMm}{r_C}$$

$$v_C^2 = v_A^2 + 2GM \left( \frac{1}{r_C} - \frac{1}{r_A} \right) = (11.165 \times 10^3)^2 + 2(398.1 \times 10^{12}) \left( \frac{1}{42.14 \times 10^6} - \frac{1}{6.67 \times 10^6} \right)$$

$$v_C^2 = 124.67 \times 10^6 - 100.48 \times 10^6 = 24.19 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_C = 4.919 \times 10^3 \text{ m/s}$$

CONSERVATION OF ANGULAR MOMENTUM

$$r_A m v_A = r_C m v_C \cos \gamma$$

$$\cos \gamma = \frac{r_A v_A}{r_C v_C} = \frac{(6.67 \times 10^6)(11.165 \times 10^3)}{(42.14 \times 10^6)(4.919 \times 10^3)}$$

$$\cos \gamma = 0.35926$$

$$\gamma = 68.9^\circ$$

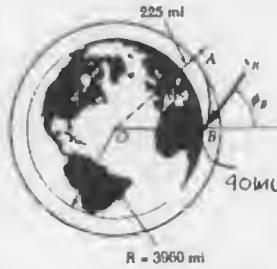
13.109

GIVEN:

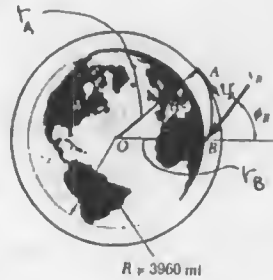
VEHICLE IN CIRCULAR  
ORBIT AT ALTITUDE  
OF 225 MI. SPEED  
DECREASED AT A  
SO THAT IT REACHES  
ALTITUDE AT B OF  
40 MI AT AN  
ANGLE  $\phi_B = 60^\circ$

FIND:

(a)  $U_A$ , AS VEHICLE  
LEAVES ITS CIRCULAR ORBIT  
(b)  $U_B$



(a)



$$r_A = 3960 \text{ mi} + 225 \text{ mi} = 4185 \text{ mi}$$

$$r_A = 4185 \text{ mi} \times 5280 \frac{\text{ft}}{\text{mi}} = 22097 \times 10^3 \text{ ft}$$

$$r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi}$$

$$r_B = 4000 \times 5280 = 21120 \times 10^3 \text{ ft}$$

$$R = 3960 \text{ mi} = 20909 \times 10^3 \text{ ft}$$

$$GM = gR^2 = (32.2 \frac{\text{ft}}{\text{s}^2})(20909 \times 10^3)^2$$

$$GM = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

CONSERVATION OF ENERGY

$$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A} = -\frac{14.077 \times 10^{15}}{22097 \times 10^3} m = -637.1 \times 10^6 m$$

$$T_B = \frac{1}{2} m v_B^2 \quad V_B = -\frac{GMm}{r_B} = -\frac{14.077 \times 10^{15}}{21120 \times 10^3} m = -666.5 \times 10^6 m$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m v_A^2 - 637.1 \times 10^6 m = \frac{1}{2} m v_B^2 - 666.5 \times 10^6 m$$

$$v_A^2 = v_B^2 - 58.94 \times 10^6 \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM

$$r_A m v_A = r_B m v_B \sin \phi_B$$

$$v_B = \frac{(r_A) v_A}{(r_B) (\sin \phi_B)} = \frac{4185}{4000 (\sin 60^\circ)} v_A$$

$$v_B = 1.208 v_A \quad (2)$$

SUBSTITUTE  $v_B$  FROM (2) IN (1)

$$v_A^2 = (1.208 v_A)^2 - 58.94 \times 10^6$$

$$v_A^2 [(1.208)^2 - 1] = 58.94 \times 10^6$$

$$v_A^2 = 128.27 \times 10^6 \text{ ft}^2/\text{s}^2$$

(a)

$$v_A = 11.32 \times 10^3 \text{ ft/s}$$

(b) FROM (2)

$$v_B = 1.208 v_A = 1.208 (11.32 \times 10^3) = 13.68 \times 10^3 \text{ ft/s}$$

$$v_B = 13.68 \times 10^3 \text{ ft/s}$$

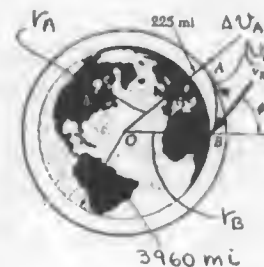
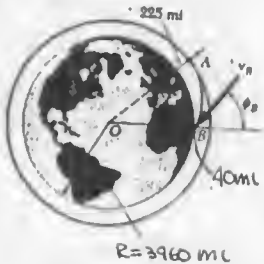
\*13.110

GIVEN:

VEHICLE AT A IN CIRCULAR ORBIT IS GIVEN AN INCREMENTAL VELOCITY  $\Delta V_A$  TOWARD O. ALTITUDES AS SHOWN. ENERGY EXPENDITURE IS 50% OF THAT USED IN PROB 13.109

FIND:

$v_B$  AND  $\phi_B$

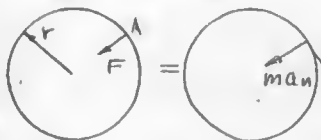


$$r_A = 3960 \text{ mi} + 225 \text{ mi} \\ r_A = 4185 \text{ mi} = 22.097 \times 10^6 \text{ ft} \\ r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi} \\ r_B = 21.120 \times 10^6 \text{ ft}$$

$$GM = gR^2 = (32.2 \frac{\text{ft}}{\text{s}^2}) (3960 \times 5280 \text{ ft})^2$$

$$GM = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

VELOCITY IN CIRCULAR ORBIT AT 225 MI ALTITUDE



$$F = \frac{GMm}{r^2} \\ a_n = \frac{v_{A, \text{circ}}^2}{r^2}$$

NEWTON'S SECOND LAW

$$F = ma_n \quad \frac{GMm}{r^2} = m \frac{v_{A, \text{circ}}^2}{r}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077 \times 10^{15}}{22.097 \times 10^6}} = 25.24 \times 10^3 \text{ ft/s}$$

ENERGY EXPENDITURE

FROM PROB. 13.109  $v_A = 11.32 \times 10^3 \text{ ft/s}$

$$\text{ENERGY, } \Delta E_{109} = \frac{1}{2} m (v_A)_{\text{circ}}^2 - \frac{1}{2} m v_A^2$$

$$\Delta E_{109} = \frac{1}{2} m (25.24 \times 10^3)^2 - \frac{1}{2} m (11.32 \times 10^3)^2$$

$$\Delta E_{109} = 254.46 \times 10^6 \text{ m ft} \cdot \text{lb} \quad (1)$$

$$\Delta E_{110} = (0.50) \Delta E_{109} = (254.46 \times 10^6 \text{ m ft} \cdot \text{lb}) / 2 \text{ ft} \cdot \text{lb}$$

THUS, ADDITIONAL KINETIC ENERGY AT A IS

$$\frac{1}{2} m (\Delta v_A)^2 = \Delta E_{110} = (254.46 \times 10^6 \text{ m ft} \cdot \text{lb}) / 2 \quad (1)$$

CONSERVATION OF ENERGY BETWEEN A AND B

$$T_A = \frac{1}{2} m (v_A)_{\text{circ}}^2 + (\Delta v_A)^2 \quad v_A = -\frac{GM}{r_A}$$

$$T_B = \frac{1}{2} m v_B^2 \quad v_B = -\frac{GM}{r_B}$$

$$T_A + v_A = T_B + v_B$$

$$\frac{1}{2} m (25.24 \times 10^3)^2 + 254.46 \times 10^6 \text{ m ft} \cdot \text{lb} - \frac{14.077 \times 10^{15} \text{ m}}{22.097 \times 10^6} = \frac{1}{2} m v_B^2 - \frac{14.077 \times 10^{15} \text{ m}}{21.120 \times 10^6}$$

$$v_B^2 = 637.4 \times 10^6 + 254.46 \times 10^6 - 1274.1 \times 10^6 + 1333 \times 10^6$$

$$v_B^2 = 950.4 \times 10^3$$

$$v_B = 30.8 \times 10^3 \text{ ft/s}$$

CONSERVATION OF ANGULAR MOMENTUM BETWEEN A AND B

$$r_A m (v_A)_{\text{circ}} = r_B m v_B \sin \phi_B$$

$$\sin \phi_B = \left( \frac{r_A}{r_B} \right) \left( \frac{v_A)_{\text{circ}}}{v_B} \right) = \left( \frac{4185}{4000} \right) \left( \frac{25.24 \times 10^3}{30.8 \times 10^3} \right) = 0.8565 \\ \phi_B = 58.9^\circ$$

13.111

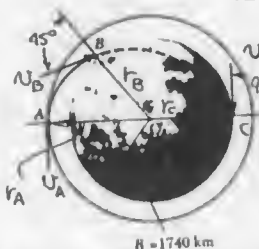
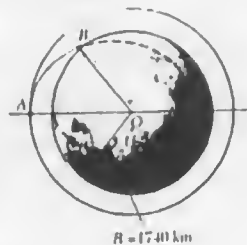
GIVEN:

LEM AT AN ALTITUDE OF 140 km IS SET ADRIFT FROM A CIRCULAR ORBIT AND ITS SPEED IS REDUCED

FIND:

(a) SMALLEST REDUCTION OF SPEED TO MAKE SURE THE LEM WILL HIT THE MOON

(b) THE REDUCTION IN SPEED WHICH WILL CAUSE THE LEM TO HIT THE MOON AT  $45^\circ$



$$r_A = 1740 \text{ km} + 140 \text{ km} = 1880 \times 10^3 \text{ m} \\ r_B = r_C = R = 1740 \text{ km} = 1740 \times 10^3 \text{ m}$$

$$GM_{\text{moon}} = 0.0123 GM_E \\ = 0.0123 g R_E^2 \\ = (0.0123) (9.81 \text{ m/s}^2) (6370 \text{ km})^2 \\ GM_{\text{moon}} = 4.896 \times 10^{12} \text{ m}^3/\text{s}^2$$

VELOCITY IN A CIRCULAR ORBIT AT 140 km ALTITUDE

$$v_{\text{circ}} \sqrt{\frac{GM_{\text{moon}}}{r_A}} = \sqrt{\frac{4.896 \times 10^{12} \text{ m}^3/\text{s}^2}{1880 \times 10^3 \text{ m}}} = 1.6138 \times 10^3 \text{ m/s}$$

(a) AN ELLIPTIC TRAJECTORY BETWEEN A AND C, WHERE THE LEM IS JUST TANGENT TO THE SURFACE OF THE MOON, WILL GIVE THE SMALLEST REDUCTION OF SPEED AT A WHICH WILL CAUSE IMPACT

CONSERVATION OF ENERGY (A AND C)

$$T_A = \frac{1}{2} m v_A^2 \quad v_A = -\frac{GM}{r_A} = -\frac{4.896 \times 10^{12} \text{ m}^3/\text{s}^2}{1880 \times 10^3 \text{ m}} = -2.604 \times 10^6 \text{ m}^2/\text{s}^2$$

$$T_C = \frac{1}{2} m v_C^2 \quad v_C = -\frac{GM}{r_C} = -\frac{4.896 \times 10^{12} \text{ m}^3/\text{s}^2}{1740 \times 10^3 \text{ m}} = -2.814 \times 10^6 \text{ m}^2/\text{s}^2$$

$$T_A + v_A = T_C + v_C \quad \frac{1}{2} m v_A^2 = 2.604 \times 10^6 \text{ m}^2/\text{s}^2 = \frac{1}{2} m v_C^2 - 2.814 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_A^2 = v_C^2 - 419.1 \times 10^3 \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM (A AND C)

$$r_A m v_A = r_C m v_C$$

$$v_C = \frac{r_A}{r_C} v_A = \frac{1880}{1740} v_A = 1.0805 v_A \quad (2)$$

REPLACE  $v_C$  IN (1) BY (2)

$$v_A^2 = (1.0805 v_A)^2 - 419.1 \times 10^3$$

$$v_A^2 [(1.0805)^2 - 1] = 419.1 \times 10^3 \quad v_A^2 = 2.502 \times 10^6$$

$$v_A = 1582 \text{ m}$$

$$\Delta v_A = (v_A)_{\text{circ}} - v_A = 1614 - 1582 = 31.5 \text{ m/s}$$

(b) CONSERVATION OF ENERGY (A AND B)

SINCE  $r_B = r_C$  CONSERVATION OF ENERGY IS THE SAME AS BETWEEN A AND C.

$$\text{THUS FROM (1)} \quad v_A^2 = v_B^2 - 419.1 \times 10^3 \quad (1')$$

CONSERVATION OF ANGULAR MOMENTUM (A AND B)

$$r_A m v_A = r_B m v_B \sin \phi \quad \phi = 45^\circ$$

$$v_B = \frac{r_A v_A}{r_B \sin \phi} = \frac{1880 v_A}{1740 \sin 45^\circ} = 1.528 v_A \quad (3)$$

REPLACE  $v_B$  IN (1') BY (3)

$$v_A^2 = (1.528 v_A)^2 - 419.1 \times 10^3$$

$$v_A^2 = 313.98 \times 10^3 \quad v_A = 560 \text{ m/s}$$

$$\Delta v_A = (v_A)_{\text{circ}} - v_A = 1614 - 560 = 1053 \text{ m/s}$$

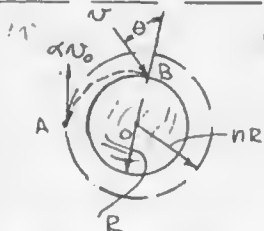
13.112

GIVEN:

SPACE PROBE IN CIRCULAR ORBIT  
OF RADIUS  $nR$ , WITH VELOCITY  $U_0$   
ABOUT A PLANET OF RADIUS  $R$ .

SHOW THAT:

- (a) PROBE WILL HIT THE PLANET AT AN ANGLE  $\theta$   
WITH THE VERTICAL, IF ITS VELOCITY IS  
REDUCED TO  $\alpha U_0$  WHERE  $\alpha = \sin \theta \sqrt{\frac{2(n-1)}{n^2 - \sin^2 \theta}}$   
(b) PROBE WILL MISS THE  
PLANET IF  $\alpha > \sqrt{\frac{2}{1+n}}$



(a) CONSERVATION OF ENERGY

$$\text{AT A } T_A = \frac{1}{2} m (\alpha U_0)^2$$

$$V_A = -\frac{GMm}{nR}$$

AT B

$$T_B = \frac{1}{2} m U^2$$

$$V_B = -\frac{GMm}{R}$$

M = MASS OF PLANET

m = MASS OF PROBE

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m (\alpha U_0)^2 - \frac{GMm}{nR} = \frac{1}{2} m U^2 - \frac{GMm}{R} \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM

$$nRm\alpha U_0 = RmU \sin \theta$$

$$U = \frac{n\alpha U_0}{\sin \theta} \quad (2)$$

REPLACE  $U$  IN (1) BY (2)

$$(\alpha U_0)^2 - \frac{2GM}{nR} = \left( \frac{n\alpha U_0}{\sin \theta} \right)^2 - \frac{2GM}{R} \quad (3)$$

FOR ANY CIRCULAR ORBIT

NEWTON'S SECOND LAW

$$-\frac{GMm}{r^2} = m \frac{U_{\text{circ}}^2}{r}$$

$$U_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

$$\text{FOR } r = nR \quad U_0 = U_{\text{circ}} = \sqrt{\frac{GM}{nR}}$$

SUBSTITUTE FOR  $U_0$  IN (3)

$$\alpha^2 \frac{GM}{nR} - \frac{2GM}{nR} = n^2 \alpha^2 \left( \frac{GM}{\sin^2 \theta (nR)} \right) - \frac{2GM}{R}$$

$$\alpha^2 \left[ 1 - \frac{n^2}{\sin^2 \theta} \right] = 2(1-n)$$

$$\alpha^2 = \frac{2(1-n)(\sin^2 \theta)}{(\sin^2 \theta - n^2)} = \frac{2(n-1)\sin^2 \theta}{(n^2 - \sin^2 \theta)}$$

$$\alpha = \sin \theta \sqrt{\frac{2(n-1)}{n^2 - \sin^2 \theta}} \quad (\text{QED})$$

(b) PROBE WILL JUST MISS THE PLANET IF  $\theta \geq 90^\circ$ ,

$$\alpha = \sin 90^\circ \sqrt{\frac{2(n-1)}{n^2 - \sin^2 90^\circ}} = \sqrt{\frac{2}{n+1}}$$

NOTE:  $n^2 - 1 =$ 

$$(n-1)(n+1)$$

13.113

GIVEN:

 $V_P$  AND  $V_A$  AS SHOWN

SHOW THAT:

$$U_A^2 = \frac{2GM}{r_A + r_P} \frac{r_P}{r_A}$$

$$U_P^2 = \frac{2GM}{r_A + r_P} \frac{r_A}{r_P}$$



CONSERVATION OF ANGULAR MOMENTUM

$$r_A m U_A = r_P m U_P$$

$$U_A = \frac{r_P}{r_A} U_P \quad (1)$$

CONSERVATION OF ENERGY

$$\frac{1}{2} m U_P^2 - \frac{GMm}{r_P} = \frac{1}{2} m U_A^2 - \frac{GMm}{r_A} \quad (2)$$

SUBSTITUTING FOR  $U_A$  FROM (1) INTO (2)

$$U_P^2 - \frac{2GM}{r_P} = \left( \frac{r_P}{r_A} \right)^2 U_P^2 - \frac{2GM}{r_A}$$

$$\left( 1 - \left( \frac{r_P}{r_A} \right)^2 \right) U_P^2 = 2GM \left( \frac{1}{r_P} - \frac{1}{r_A} \right)$$

$$\frac{r_A^2 - r_P^2}{r_A^2} U_P^2 = 2GM \frac{r_A - r_P}{r_A r_P}$$

$$\text{WITH } r_A^2 - r_P^2 = (r_A - r_P)(r_A + r_P)$$

$$U_P^2 = \frac{2GM}{r_A + r_P} \frac{r_A}{r_P} \quad (3)$$

EXCHANGING SUBSCRIPTS P AND A

$$U_A^2 = \frac{2GM}{r_A + r_P} \frac{r_P}{r_A} \quad (\text{QED})$$

13.114

GIVEN:

EARTH SATELLITE OF MASS  $m$   
DESCRIBING AN ELLIPTIC ORBIT  
 $r_A$  IS MAXIMUM AND  $r_P$  IS MINIMUM  
DISTANCES TO EARTH'S CENTER

SHOW THAT:

TOTAL ENERGY  $E = -\frac{GMm}{r_A + r_P}$ , WHERE  
 $M$  = MASS OF THE EARTH



SEE SOLUTION TO  
PROB 13.113 (ABOVE)  
FOR DERIVATION OF  
EQUATION (3)

$$U_P^2 = \frac{2GM}{r_A + r_P} \frac{r_A}{r_P}$$

TOTAL ENERGY AT POINT P IS

$$E = T_P + V_P = \frac{1}{2} m U_P^2 - \frac{GMm}{r_P}$$

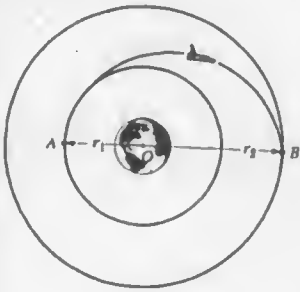
$$= \frac{1}{2} \left[ \frac{2GMm}{r_A + r_P} \frac{r_A}{r_P} \right] - \frac{GMm}{r_P}$$

$$= GMm \left[ \frac{r_A}{r_P(r_A + r_P)} - \frac{1}{r_P} \right] = GMm \frac{(r_A - r_P - r_P)}{r_P(r_A + r_P)}$$

$$E = -\frac{GMm}{r_A + r_P}$$

NOTE: RECALL THAT GRAVITATIONAL POTENTIAL  
OF A SATELLITE IS DEFINED AS BEING  
ZERO AT AN INFINITE DISTANCE FROM  
THE EARTH

13.115



GIVEN:

SPACECRAFT OF MASS  $m$  IN CIRCULAR ORBIT OF RADIUS  $r_1$  ABOUT THE EARTH

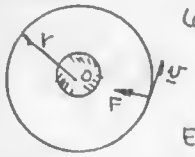
SHOW THAT:

(a) ADDITIONAL ENERGY  $\Delta E$  TO TRANSFER IT TO A CIRCULAR ORBIT OF LARGER RADIUS  $r_2$  IS  

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2}$$

(b) AMOUNTS OF ENERGY AT A AND B ARE

$$\Delta E_A = \frac{r_2}{r_1 + r_2} \Delta E, \Delta E_B = \frac{r_1}{r_1 + r_2} \Delta E$$



(a) FOR A CIRCULAR ORBIT OF RADIUS  $r$   
 $F = ma_n; \frac{GMm}{r^2} = m \frac{v^2}{r}$   
 $v^2 = \frac{GM}{r}$   
 $E = T + V = \frac{1}{2} m v^2 - \frac{GMm}{r} = -\frac{1}{2} \frac{GMm}{r} \quad (1)$

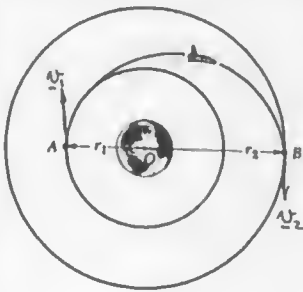
THUS  $\Delta E$  REQUIRED TO PASS FROM CIRCULAR ORBIT OF RADIUS  $r_1$  TO CIRCULAR ORBIT OF RADIUS  $r_2$  IS

$$\Delta E = E_2 - E_1 = -\frac{1}{2} \frac{GMm}{r_2} + \frac{1}{2} \frac{GMm}{r_1}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \quad (2) \text{ (Q.E.D.)}$$

(b) FOR AN ELLIPTIC ORBIT WE RECALL EQ (3) DERIVED IN PROBLEM 13.113 (WITH  $v_p = v_1$ )

$$v_1^2 = \frac{2GM}{r_1 + r_2} \frac{r_2}{r_1}$$



AT POINT A: INITIALLY SPACECRAFT IS IN A CIRCULAR ORBIT OF RADIUS  $r_1$

$$v_{circ}^2 = \frac{GM}{r_1}$$

$$T_{circ} = \frac{1}{2} m v_{circ}^2 = \frac{1}{2} m \frac{GM}{r_1}$$

AFTER THE SPACECRAFT ENGINES ARE FIRED AND IT IS PLACED ON A SEMI-ELLIPTIC PATH AB, WE RECALL

$$v_1^2 = \frac{2GM}{(r_1 + r_2)} \frac{r_2}{r_1}$$

AND

$$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m \frac{2GM r_2}{r_1(r_1 + r_2)}$$

AT POINT A, THE INCREASE IN ENERGY IS

$$\Delta E_A = T_1 - T_{circ} = \frac{1}{2} m \frac{2GM r_2}{r_1(r_1 + r_2)} - \frac{1}{2} m \frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1 + r_2} \left[ \frac{GMm(r_2 - r_1)}{2r_1 r_2} \right]$$

RECALL EQ (2):  $\Delta E_A = \frac{r_2}{(r_1 + r_2)} \Delta E \quad \text{(Q.E.D.)}$

A SIMILAR DERIVATION AT POINT B YIELDS,  $\Delta E_B = \frac{r_1}{(r_1 + r_2)} \Delta E \quad \text{(Q.E.D.)}$

13.116

GIVEN:

MISSILE FIRED FROM THE GROUND WITH VELOCITY  $v_0$  AT AN ANGLE  $\phi_0$  WITH THE VERTICAL, REACHES A MAXIMUM ALTITUDE  $\alpha R$  WHERE  $R$  IS THE RADIUS OF THE EARTH

SHOW THAT:

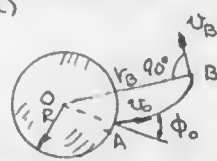
$$(a) \sin \phi_0 = (1 + \alpha) \sqrt{1 - \frac{\alpha}{1 + \alpha} \left( \frac{v_{esc}}{v_0} \right)^2}$$

WHERE  $v_{esc}$  = ESCAPE VELOCITY

FIND:

(b) RANGE OF ALLOWABLE VALUES OF  $v_0$

(a)



$$r_A = R$$

CONSERVATION OF ANG. MOM.

$$R m v_0 \sin \phi_0 = r_B m v_B$$

$$r_B = R + \alpha R = (1 + \alpha) R \quad (1)$$

$$v_B = \frac{R v_0 \sin \phi_0}{(1 + \alpha) R} = \frac{v_0 \sin \phi_0}{(1 + \alpha)}$$

CONSERVATION OF ENERGY

$$T_A + V_A = T_B + V_B \quad \frac{1}{2} m v_0^2 - \frac{GMm}{R} = \frac{1}{2} m v_B^2 - \frac{GMm}{(1 + \alpha) R}$$

$$v_0^2 - v_B^2 = \frac{2GM}{R} \left( 1 - \frac{1}{1 + \alpha} \right) = \frac{2GM}{R} \left( \frac{\alpha}{1 + \alpha} \right)$$

SUBSTITUTE FOR  $v_B$  FROM (1)

$$v_0^2 \left( 1 - \frac{\sin^2 \phi_0}{(1 + \alpha)^2} \right) = \frac{2GM}{R} \left( \frac{\alpha}{1 + \alpha} \right)$$

FROM EQ. (12.43):  $v_{esc}^2 = \frac{2GM}{R}$

$$v_0^2 \left[ 1 - \frac{\sin^2 \phi_0}{(1 + \alpha)^2} \right] = v_{esc}^2 \left( \frac{\alpha}{1 + \alpha} \right)$$

$$\frac{\sin^2 \phi_0}{(1 + \alpha)^2} = 1 - \left( \frac{v_{esc}}{v_0} \right)^2 \frac{\alpha}{1 + \alpha} \quad (2)$$

$$\sin \phi_0 = (1 + \alpha) \sqrt{1 - \frac{\alpha}{1 + \alpha} \left( \frac{v_{esc}}{v_0} \right)^2} \quad \text{Q.E.D.}$$

(b) ALLOWABLE VALUES OF  $v_0$  (FOR WHICH MAXIMUM ALTITUDE  $= \alpha R$ )

$$0 \leq \sin^2 \phi_0 \leq 1$$

FOR  $\sin \phi_0 = 0$ , FROM (2)

$$0 = 1 - \left( \frac{v_{esc}}{v_0} \right)^2 \frac{\alpha}{1 + \alpha}$$

$$v_0 = v_{esc} \sqrt{\frac{\alpha}{1 + \alpha}}$$

FOR  $\sin \phi_0 = 1$ , FROM 2

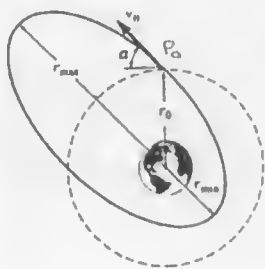
$$\frac{1}{(1 + \alpha)^2} = 1 - \left( \frac{v_{esc}}{v_0} \right)^2 \frac{\alpha}{1 + \alpha}$$

$$\left( \frac{v_{esc}}{v_0} \right)^2 = \frac{1}{\alpha} \left( 1 + \alpha - \frac{1}{1 + \alpha} \right) = \frac{1 + 2\alpha + \alpha^2 - 1}{\alpha(1 + \alpha)} = \frac{2 + \alpha}{1 + \alpha}$$

$$v_0 = v_{esc} \sqrt{\frac{1 + \alpha}{2 + \alpha}}$$

$$v_{esc} \sqrt{\frac{\alpha}{1 + \alpha}} \leq v_0 \leq v_{esc} \sqrt{\frac{1 + \alpha}{2 + \alpha}}$$

\*13.117

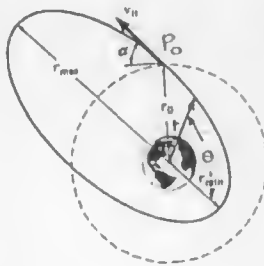


GIVEN:

FROM PROB. 13.107  
 $r_{\min} = r_0(1 - \sin \alpha)$   
 $r_{\max} = r_0(1 + \sin \alpha)$

SHOW THAT:

INTENDED CIRCULAR ORBIT AND RESULTING ELLIPTIC ORBIT INTERSECT AT THE ENDS OF THE MINOR AXIS OF THE ELLIPTIC ORBIT AT  $P_0$



IF THE POINT OF INTERSECTION  $P_0$  OF THE CIRCULAR AND ELLIPTIC ORBITS IS AT AN END OF THE MINOR AXIS, THEN  $U_0$  IS PARALLEL TO THE MAJOR AXIS. THIS WILL BE THE CASE ONLY IF  $\alpha + 90^\circ = \theta_0$ . THAT IS IF  $\cos \theta_0 = -\sin \alpha$ . WE MUST THEREFORE PROVE THAT  $\cos \theta_0 = -\sin \alpha$  (1)

WE RECALL FROM EQ (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C \cos \theta \quad (2)$$

WHEN  $\theta = 0$ ,  $r = r_{\min}$  AND  $r_{\min} = r_0(1 - \sin \alpha)$

$$\frac{1}{r_0(1 - \sin \alpha)} = \frac{GM}{h^2} + C \quad (3)$$

FOR  $\theta = 180^\circ$ ,  $r = r_{\max} = r_0(1 + \sin \alpha)$

$$\frac{1}{r_0(1 + \sin \alpha)} = \frac{GM}{h^2} - C \quad (4)$$

ADDING (3) AND (4) AND DIVIDING BY 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0} \left( \frac{1}{1 - \sin \alpha} + \frac{1}{1 + \sin \alpha} \right) = \frac{1}{r_0 \cos^2 \alpha}$$

SUBTRACTING (4) FROM (3) AND DIVIDING BY 2:

$$C = \frac{1}{2r_0} \left( \frac{1}{1 - \sin \alpha} - \frac{1}{1 + \sin \alpha} \right) = \frac{1}{2r_0} \frac{2 \sin \alpha}{1 - \sin^2 \alpha}$$

$$C = \frac{\sin \alpha}{r_0 \cos^2 \alpha}$$

SUBSTITUTE FOR  $\frac{GM}{h^2}$  AND  $C$  INTO EQ (2)

$$\frac{1}{r} = \frac{1}{r_0 \cos^2 \alpha} (1 + \sin \alpha \cos \theta) \quad (5)$$

LETTING  $r = r_0$  AND  $\theta = \theta_0$  IN EQ (5), WE HAVE

$$\cos^2 \alpha = 1 + \sin \alpha \cos \theta_0$$

$$\cos \theta_0 = \frac{\cos^2 \alpha - 1}{\sin \alpha} = -\frac{\sin^2 \alpha}{\sin \alpha} = -\sin \alpha$$

THIS PROVES THE VALIDITY OF EQ (1) AND THUS  $P_0$  IS AN END OF THE MINOR AXIS OF THE ELLIPTIC ORBIT

\*13.118

GIVEN:

SPACE VEHICLE UNDER GRAVITATIONAL ATTRACTION OF A PLANET OF MASS  $M$  (FIG. 13.15, SHOWN BELOW)

FIND:

(a) TOTAL ENERGY PER UNIT MASS,  $E/m$ , IN TERMS OF  $r_{\min}$  AND  $U_{\max}$  AND THE ANGULAR MOMENTUM PER UNIT MASS,  $h$ .

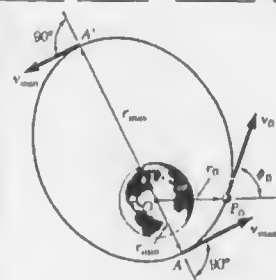
DERIVE:

$$(b) \frac{1}{r_{\min}} = \frac{GM}{h^2} \left[ 1 + \sqrt{1 + \frac{2E}{m} \left( \frac{h}{GM} \right)^2} \right]$$

SHOW THAT:

$$(c) \text{ECCENTRICITY } E = \sqrt{1 + \frac{2E}{m} \left( \frac{h}{GM} \right)^2}$$

(d) TRAJECTORY IS a,  
 HYPERBOLA IF  $E > 0$   
 ELLIPSE IF  $E = 0$   
 PARABOLA IF  $E < 0$



(a) POINT A

ANGULAR MOMENTUM PER UNIT MASS

$$h = \frac{h_0}{m} = \frac{r_{\min} m U_{\max}}{m}$$

$$h = r_{\min} U_{\max} \quad (1)$$

(b) ENERGY PER UNIT MASS

$$E/m = \frac{1}{m} (T + V)$$

$$E/m = \frac{1}{m} \left( \frac{1}{2} m U_{\max}^2 - \frac{GMm}{r_{\min}} \right) = \frac{1}{2} U_{\max}^2 - \frac{GM}{r_{\min}} \quad (2)$$

(b) FROM EQ. (1):  $U_{\max} = h/r_{\min}$ . SUBSTITUTING INTO (2)

$$E/m = \frac{1}{2} \frac{h^2}{r_{\min}^2} - \frac{GM}{r_{\min}}$$

$$\left( \frac{1}{r_{\min}} \right)^2 - \frac{2GM}{h^2} \cdot \frac{1}{r_{\min}} - \frac{2(E/m)}{h^2} = 0$$

SOLVING THE QUADRATIC:  $\frac{1}{r_{\min}} = \frac{GM}{h^2} + \sqrt{\left( \frac{GM}{h^2} \right)^2 + \frac{2(E/m)}{h^2}}$   
 REARRANGING

$$\frac{1}{r_{\min}} = \frac{GM}{h^2} \left[ 1 + \sqrt{1 + \frac{2E}{m} \left( \frac{h}{GM} \right)^2} \right] \quad (3)$$

(c) ECCENTRICITY OF THE TRAJECTORY

$$\text{EQ (12.39')} \quad \frac{1}{r} = \frac{GM}{h^2} (1 + E \cos \theta)$$

WHEN  $\theta = 0$ ,  $\cos \theta = 1$  AND  $r = r_{\min}$ , THUS

$$\frac{1}{r_{\min}} = \frac{GM}{h^2} (1 + E) \quad (4)$$

COMPARING (3) AND (4)  $E = \sqrt{1 + \frac{2E}{m} \left( \frac{h}{GM} \right)^2} \quad (5)$

(d) RECALLING DISCUSSION ON PAGES 708, 709 AND IN VIEW OF EQ. (5)

1. HYPERBOLA IF  $E > 1$ , THAT IS IF  $E > 0$

2. PARABOLA IF  $E = 1$ , THAT IS IF  $E = 0$

3. ELLIPSE IF  $E < 1$ , THAT IS IF  $E < 0$

NOTE: FOR CIRCULAR ORBIT  $E = 0$  AND

$$1 + \frac{2E}{m} \left( \frac{h}{GM} \right)^2 = 0 \text{ OR } E = -\left( \frac{GM}{h} \right)^2 \frac{m}{2}$$

BUT FOR CIRCULAR ORBIT  $U^2 = \frac{GM}{r}$  AND  $h^2 = U^2 r^2 = GM r$

THUS  $E = -\frac{1}{2} m \left( \frac{GM}{r} \right) = -\frac{1}{2} \frac{GMm}{r}$  (CHECKS WITH (1) FOUND IN 13.115)

13.119

GIVEN:

PARTICLE MASS  $m = 1.6 \text{ kg}$ , ACTED UPON BY A FORCE  $\mathbf{F} = (10 \sin 2t) \mathbf{i} + (12 \cos 2t) \mathbf{j}$  (F IN NEWTONS, t IN SECONDS) VELOCITY OF PARTICLE,  $\mathbf{v}_0 = 0$  AT  $t = 0$

FIND:

MAGNITUDE AND DIRECTION OF  $\mathbf{v}$  AT  $t = 4 \text{ s}$ 

$$m \mathbf{v}_0 + \int_0^t \mathbf{F} dt = m \mathbf{v}_4 \quad \text{for } t = 4 \text{ sec} \quad \mathbf{v}_0 = 0$$

$$0 + \int_0^4 [(10 \sin 2t) \mathbf{i} + (12 \cos 2t) \mathbf{j}] dt = 1.6 \mathbf{v}_4$$

$$0 + [-5 \cos 2t \mathbf{i} + 6 \sin 2t \mathbf{j}]_0^4 = 1.6 \mathbf{v}_4$$

$$(-5 \cos 8 + 1 \cos 0) \mathbf{i} + (6 \sin 8 - 0) \mathbf{j} = 1.6 \mathbf{v}_4$$

$$\mathbf{v}_4 = 3.58 \mathbf{i} + 3.71 \mathbf{j}$$

$$|\mathbf{v}_4| = 5.16 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{3.71}{3.58} = 46.0^\circ$$

13.121

GIVEN:

$$v_A = 30 \frac{\text{ft}}{\text{s}}, \mu_k = 0.30$$

FIND:

TIME FOR THE BLOCK TO REACH B WHERE  $v_B = 0$ , IF (a)  $\theta = 0$ , (b)  $\theta = 20^\circ$

(a)  $\theta = 0$ 

$$\begin{aligned} \rightarrow \square + \square &= \square \rightarrow m \mathbf{v}_B = 0 \\ m \mathbf{v}_A &+ \square \uparrow \mathbf{F} t = \mu_k W t \\ N t &= W t \end{aligned}$$

$$\frac{W}{g} v_A - \mu_k W t = 0 \quad t = \frac{v_A}{g \mu_k} = \frac{(30 \frac{\text{ft}}{\text{s}})}{(32.2 \frac{\text{ft}}{\text{s}^2})(0.30)} = 3.11 \text{ s}$$

(b)  $\theta = 20^\circ$ 

$$\begin{aligned} & \begin{aligned} & \text{Block A: } m \mathbf{v}_A \rightarrow \\ & \text{Block B: } \begin{aligned} & \text{Forces: } W \sin 20^\circ \downarrow, W \cos 20^\circ \rightarrow, F t = \mu_k N t \uparrow, N t = W \cos 20^\circ \end{aligned} \\ & \text{Block B: } m \mathbf{v}_B = 0 \end{aligned} \end{aligned}$$

IMPULSE-MOMENTUM IN X DIRECTION

$$+ \frac{W}{g} v_A - \mu_k W t \cos 20^\circ - W t \sin 20^\circ = 0$$

$$t = \frac{v_A}{g(\mu_k \cos 20^\circ + \sin 20^\circ)} = \frac{30 \text{ ft/s}}{(32.2 \text{ ft/s}^2)(0.30 \cos 20^\circ + \sin 20^\circ)} = 1.493 \text{ s}$$

13.120

GIVEN:

5-lb PARTICLE ACTED UPON BY A FORCE  $\mathbf{F} = -2t^2 \mathbf{i} + (3-t) \mathbf{j}$  (F IN POUNDS AND t IN SECONDS) VELOCITY OF THE PARTICLE IS  $\mathbf{v}_0 = (10 \text{ ft/s}) \mathbf{i}$  AT  $t = 0$ .

FIND:

(a) TIME AT WHICH VELOCITY IS PARALLEL TO THE y-AXIS

(b) THE CORRESPONDING VELOCITY

$$(a) \quad m \mathbf{v}_0 + \int_0^t \mathbf{F} dt = m(\mathbf{v}_x \mathbf{i} + \mathbf{v}_y \mathbf{j})$$

BUT  $v_x = 0$ , IF VELOCITY IS PARALLEL TO y-AXIS

$$\left(\frac{5}{g}\right) 10 \mathbf{i} + \int_0^t [-2t^2 \mathbf{i} + (3-t) \mathbf{j}] dt = \frac{5}{g} \mathbf{v}_y \mathbf{j}$$

$$\frac{50}{g} - \left(\frac{2t^3}{3}\right) \mathbf{i} + \left(3t - \frac{t^2}{2}\right) \mathbf{j} = \frac{5}{g} \mathbf{v}_y \mathbf{j} \quad (1)$$

$$\left(\frac{50}{g} - \frac{2t^3}{3}\right) \mathbf{i} + \left(3t - \frac{t^2}{2}\right) \mathbf{j} = \frac{5}{g} \mathbf{v}_y \mathbf{j}$$

SINCE THE X COMPONENT OF THE VELOCITY IS ZERO

$$\frac{50}{32.2} - \frac{2t^3}{3} = 0 \quad t^3 = 2.329$$

$$t = 1.326 \text{ s}$$

(b) SUBSTITUTE  $t = 1.326$  IN (1)

$$0 \mathbf{i} + \left[3(1.326) - \frac{(1.326)^2}{2}\right] \mathbf{j} = \frac{5}{32.2} \mathbf{v}_y \mathbf{j}$$

$$3.098 \mathbf{j} = 0.1553 \mathbf{v}_y \mathbf{j}$$

$$\mathbf{v}_y = 19.95 \text{ ft/s}$$

13.122

GIVEN:

$$v_A = 10 \text{ m/s}, \mu_k = 0.30$$

$$\theta = 30^\circ$$

FIND:

TIME FOR THE BLOCK TO REACH  $v = 10 \text{ m/s}$  DOWN AND TO THE LEFT



$$\begin{aligned} & \begin{aligned} & \text{Block A: } m \mathbf{v}_A \rightarrow \\ & \text{Block B: } \begin{aligned} & \text{Forces: } W \sin 30^\circ \downarrow, W \cos 30^\circ \rightarrow, F t = \mu_k N t \uparrow, N t = W \cos 30^\circ \end{aligned} \\ & \text{Block B: } m \mathbf{v}_B = 0 \end{aligned} \end{aligned}$$

UP THE PLANE TO B

$$+ \frac{W}{g} v_A - \mu_k W t \cos 30^\circ - W t \sin 30^\circ = 0$$

$$t_{AB} = \frac{v_A}{g(\mu_k \cos 30^\circ + \sin 30^\circ)} = \frac{10 \text{ m/s}}{(9.81 \frac{\text{m}}{\text{s}^2})(0.30 \cos 30^\circ + \sin 30^\circ)} = 1.342 \text{ s}$$

$$\begin{aligned} & \begin{aligned} & \text{Block C: } m \mathbf{v}_C \rightarrow \\ & \text{Block B: } \begin{aligned} & \text{Forces: } W \sin 30^\circ \downarrow, W \cos 30^\circ \rightarrow, F t = \mu_k N t \uparrow, N t = W \cos 30^\circ \end{aligned} \\ & \text{Block B: } m \mathbf{v}_B = 0 \end{aligned} \end{aligned}$$

DOWN THE PLANE TO C

$$+ \frac{W}{g} v_B + \mu_k W t \cos 30^\circ - W t \sin 30^\circ = -\frac{W}{g} v_C$$

$$t_{BC} = \frac{10 \text{ m/s}}{g(\sin 30^\circ - \mu_k \cos 30^\circ)} = \frac{10 \text{ m/s}}{(9.81 \frac{\text{m}}{\text{s}^2})(\frac{1}{2} - 0.30 \cos 30^\circ)} = 4.244 \text{ s}$$

$$t = t_{AB} + t_{BC} = 1.342 + 4.244 = 5.59 \text{ s}$$

13.123

GIVEN:

REAR (DRIVE) WHEELS OF A CAR SLIP FOR FIRST 60 ft WITH FRONT WHEELS JUST OFF THE GROUND.  $\mu_k = 0.60$   
 WHEELS ROLL WITHOUT SLIPPING FOR THE REMAINING 1260 ft WITH 60% OF THE WEIGHT ON THE REAR WHEELS.  $\mu_s = 0.85$   
 IGNORE AIR AND ROLLING RESISTANCE

FIND:

- (a) SHORTEST TIME FOR THE CAR TO TRAVEL THE FIRST 60 ft STARTING FROM REST  
 (b) MINIMUM TIME FOR THE CAR TO RUN THE WHOLE RACE

(a) FIRST 60 ft

VELOCITY AT 60 ft REAR WHEELS SKID TO GENERATE THE MAXIMUM FORCE RESULTING IN MAXIMUM VELOCITY AND MINIMUM TIME SINCE ALL THE WEIGHT IS ON THE REAR WHEELS THIS FORCE IS  $F = \mu_k N = 0.60W$

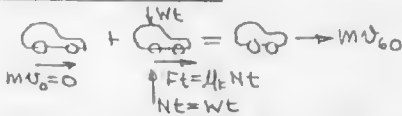
WORK AND ENERGY:  $T_0 + U_{0-60} = T_{60}$   
 $T_0 = 0$   $U_{0-60} = (F)(60)$   $T_{60} = \frac{1}{2} m v_{60}^2$

$$0 + (\mu_k W)(60) = \frac{1}{2} \frac{W}{g} v_{60}^2$$

$$v_{60}^2 = (2)(0.60)(60 \text{ ft})(32.2 \text{ ft/s}^2)$$

$$v_{60} = 48.15 \text{ ft/s}$$

IMPULSE - MOMENTUM



$$0 + \mu_k W t_{0-60} = \frac{W}{g} v_{60} \quad v_{60} = 48.15 \text{ ft/s}$$

$$t_{0-60} = \frac{48.15 \text{ ft/s}}{(0.60)(32.2 \text{ ft/s}^2)}$$

$$t_{0-60} = 2.49 \text{ s}$$

(b) FOR THE WHOLE RACE

THE MAXIMUM FORCE ON THE WHEELS FOR THE FIRST 60 ft IS  $F = \mu_k W = 0.60W$   
 FOR REMAINING 1260 ft THE MAXIMUM FORCE IF THERE IS NO SLIDING AND 60% OF THE WEIGHT IS ON THE REAR (DRIVE) WHEELS IS  $F = \mu_s (0.60)W = 0.85(0.60)W = 0.510W$

VELOCITY AT 1320 ft

WORK AND ENERGY  $T_0 + U_{0-60} + U_{60-1320} = T_{1320}$

$$T_0 = 0 \quad U_{0-60} = (0.60W)(60 \text{ ft}), \quad U_{60-1320} = (0.510W)(1260 \text{ ft})$$

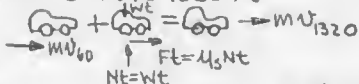
$$T_{1320} = \frac{1}{2} \frac{W}{g} v_{1320}^2$$

$$0 + 36W + (0.510)(1260)W = \frac{1}{2} \frac{W}{g} v_{1320}^2$$

$$v_{1320} = 209 \text{ ft/s}$$

IMPULSE - MOMENTUM

FROM 60 ft to 1320 ft



$$F = \mu_s N = 0.510W$$

$$v_{60} = 48.15 \text{ ft/s}$$

$$v_{1320} = 209 \text{ ft/s}$$

$$\left(\frac{W}{g}\right)(48.15) + 0.510W t_{60-1320} = \frac{W}{g}(209); \quad t_{60-1320} = 9.79 \text{ s}$$

$$t_{0-1320} = t_{0-60} + t_{60-1320} = 2.49 + 9.80 = 12.29 \text{ s}$$

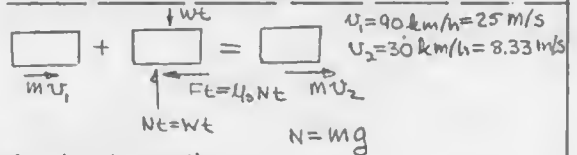
13.124

GIVEN:

TRUCK ON LEVEL ROAD TRAVELING AT 90 km/h  
 BRAKES ARE APPLIED TO SLOW IT TO 30 km/h  
 ANTISKID BRAKING SYSTEM LIMITS BRAKING FORCE SO THAT WHEELS ARE AT IMPENDING SLIDING.  $\mu_s = 0.65$

FIND:

SHORTEST TIME FOR TRUCK TO SLOW DOWN



$$m v_1 - \mu_s N t = m v_2$$

$$m(25 \text{ m/s}) - (0.65)m(9.81 \frac{\text{m}}{\text{s}^2})t = m(8.33 \text{ m/s})$$

$$t = \frac{25 - 8.33}{(0.65)(9.81)} = 2.61 \text{ s}$$

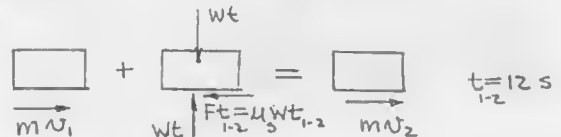
13.125

GIVEN:

TRAIN DECREASES SPEED FROM 200 km/h to 90 km/h AT A CONSTANT RATE IN 12 s.

FIND:

SMALLEST ALLOWABLE COEFFICIENT OF FRICTION IF A TRUNK IS NOT TO SLIDE



$$v_1 = 200 \text{ km/h} = 55.56 \text{ m/s}$$

$$v_2 = 90 \text{ km/h} = 25.0 \text{ m/s}$$

$$m v_1 - \mu_s m g t_{1-2} = m v_2$$

$$(55.56 \frac{\text{m}}{\text{s}}) - \mu_s (9.81 \frac{\text{m}}{\text{s}^2})(12 \text{ s}) = 25 \text{ m/s}$$

$$\mu_s = \frac{(55.56 - 25.0)}{(9.81)(12)} = 0.2596$$

$$\mu_s = 0.260$$

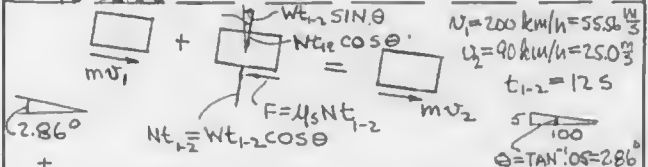
13.126

GIVEN:

TRAIN DECREASES SPEED FROM 200 km/h TO 90 km/h DOWN A 5% GRADE AT A CONSTANT RATE IN 12 s.

FIND:

SMALLEST COEFFICIENT OF FRICTION IF A TRUNK IS NOT TO SLIDE



$$m v_1 - \mu_s m g t_{1-2} \cos \theta + m g t_{1-2} \sin \theta = m v_2$$

$$(55.56 \frac{\text{m}}{\text{s}}) - \mu_s (9.81 \frac{\text{m}}{\text{s}^2})(12 \text{ s}) \cos 2.86^\circ + (9.81 \frac{\text{m}}{\text{s}^2})(12 \text{ s}) \sin 2.86^\circ = 25 \frac{\text{m}}{\text{s}}$$

$$\mu_s = \frac{55.56 - 25.0 + (9.81)(12) \sin 2.86^\circ}{(9.81)(12) \cos 2.86^\circ} = 0.310$$

13.127

GIVEN:

TRUCK SLOWS FROM 60 mi/h TO 20 mi/h  
DOWN A 4% GRADE WITH ITS WHEELS  
JUST ABOUT TO SLIDE  $\mu_s = 0.60$

FIND:

SHORTEST TIME FOR TRUCK TO SLOW DOWN

$\theta = \tan^{-1} \frac{4}{100} = 2.29^\circ$   
 $m\vec{v}_1 + \sum \text{IMP}_{1-2} = m\vec{v}_2$   
 $m\vec{v}_1 + W t \sin \theta - F t = m\vec{v}_2$   
 $v_1 = 60 \text{ mi/h} = 88 \text{ ft/s}$   
 $v_2 = 20 \text{ mi/h} = 29.33 \text{ ft/s}$   
 $N = W \cos \theta$   
 $F = \mu_s N = \mu_s W \cos \theta$   
 $(88 - 29.33) = (32.2 - 0.60 \cos 2.29^\circ) t$   
 $t = \frac{88 - 29.33}{32.2[0.60 \cos 2.29^\circ - \sin 2.29^\circ]} = 3.26 \text{ s}$

13.128

GIVEN:

INITIAL BOAT SPEED =  
 $v_1 = 8 \text{ mi/h}$   
 BOAT SPEED  
 10 SEC AFTER  
 SPINNAKER  
 IS RAISED =  
 $v_2 = 12 \text{ mi/h}$   
 $W = 980 \text{ lb}$

FIND:

NET FORCE PROVIDED BY THE SPINNAKER OVER  
 THE 10 SEC. INTERVAL

$$v_1 = 8 \text{ mi/h} = 11.73 \text{ ft/s}$$

$$t = 10 \text{ SEC}$$

$$v_2 = 12 \text{ mi/h} = 17.60 \text{ ft/s}$$

$$m\vec{v}_1 + F_N t_{1-2} = m\vec{v}_2$$

$$m \cdot v_1 + \text{IMP}_{1-2} = m v_2$$

$$m (11.73 \text{ ft/s}) + F_N (10 \text{ s}) = m (17.60 \text{ ft/s})$$

$$F_N = \frac{(1800 \text{ lb})(17.60 \text{ ft/s} - 11.73 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(10 \text{ s})} = 17.86 \text{ lb}$$

NOTE:

$F_N$  IS THE NET FORCE PROVIDED BY THE  
 SAILS. THE FORCE ON THE SAILS IS  
 ACTUALLY GREATER AND INCLUDES THE  
 FORCE NEEDED TO OVERCOME THE WATER  
 RESISTANCE ON THE HULL.

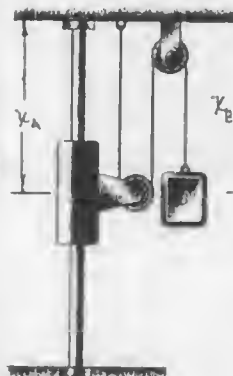
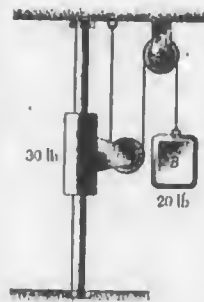
13.129

GIVEN:

SYSTEM RELEASED FROM  
 REST.

FIND:

TIME FOR A TO REACH A  
 VELOCITY OF 2 ft/s



KINEMATICS

LENGTH OF CABLE IS  
 CONSTANT

$$L = 2x_A + x_B$$

$$\frac{dL}{dt} = 2v_A + v_B = 0$$

$$v_B = -2v_A$$

$(v_A)_2 = 2 \text{ ft/s}$   
 COLLAR A  
 $m_A = \frac{W_A}{g} = \frac{30}{g}$   
 $(m_A v_A)_2 = 0$   
 $(2T) t_{1-2} - W_A t_{1-2} = m(v_A)_2$   
 $0 + (2T - 30) t_{1-2} = \left(\frac{30}{g}\right)(2)$   
 $(T - 15) t_{1-2} = \frac{30}{g} \quad (1)$   
 COLLAR B  
 $m_B = \frac{W_B}{g} = \frac{20}{g}$   
 $(v_B)_2 = 2(v_A)_2 = 4 \text{ ft/s}$   
 $(m_B v_B)_2 = 0$   
 $W_B t_{1-2} - T t_{1-2} + W_B t_{1-2} = m(v_B)_2$   
 $30 + (20 - T) t_{1-2} = \frac{20}{g}(4) \quad (2)$

ADD EQ (1) AND (2) (ELIMINATING T)

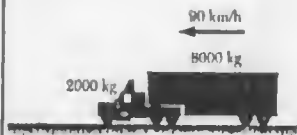
$$(20 - 15) t_{1-2} = \frac{(30 + 80)}{g} = \frac{110}{g}$$

$$t_{1-2} = \frac{22}{32.2} = 0.683 \text{ s}$$

$$t = 0.683 \text{ sec.}$$



13.130



GIVEN:

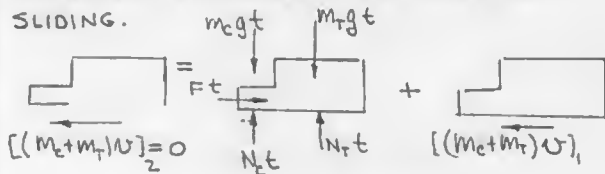
$m_c = 2000 \text{ kg}$   
 $m_T = 8000 \text{ kg}$   
 INITIAL  $v = 90 \text{ km/h}$   
 FINAL  $v = 0$   
 TRAILER BRAKES FAIL  
 $\mu_s = 0.65$

FIND:

- (a) SHORTEST TIME FOR RIG TO COME TO A STOP  
 (b) FORCE ON THE COUPLING DURING THIS TIME

$$v = 90 \text{ km/h} = 25 \text{ m/s}$$

(a) THE SHORTEST TIME FOR THE RIG TO COME TO A STOP WILL BE WHEN THE FRICTION FORCE ON THE WHEELS IS MAXIMUM. THE DOWNWARD FORCE EXERTED BY THE TRAILER ON THE CAB IS ASSUMED TO BE ZERO. SINCE THE TRAILER BRAKES FAIL ALL OF THE BRAKING FORCE IS SUPPLIED BY THE WHEELS OF THE CAB, WHICH IS MAXIMUM WHEN THE WHEELS OF THE CAB ARE AT IMPENDING SLIDING.



$$F_{t_2} = \mu_s N_{c_2} \quad N_c = m_c g = (2000)g$$

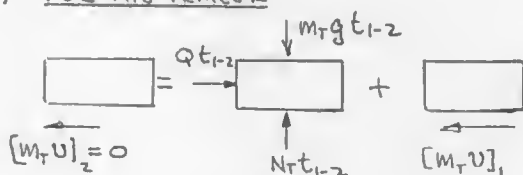
$$F_{t_2} = (0.65)(2000)g$$

$$+ \quad (m_c + m_T)v_2 = -F_{t_2} + (m_c + m_T)v_1$$

$$0 = -(0.65)(2000 \text{ kg})(9.81 \text{ m/s}^2)(t_{1-2}) = (10000 \text{ kg})(25 \text{ m/s})$$

$$t_{1-2} = 19.60 \text{ s}$$

(b) FOR THE TRAILER



$$+ \quad (m_T v)_2 = -Q t_{1-2} + (m_T v)_1$$

FROM (a)  $t_{1-2} = 19.60 \text{ s}$ 

$$0 = -Q(19.60 \text{ s}) + (8000 \text{ kg})(25 \text{ m/s})$$

$$Q = 10204 \text{ N}$$

$$Q = 10.20 \text{ kN (c)}$$

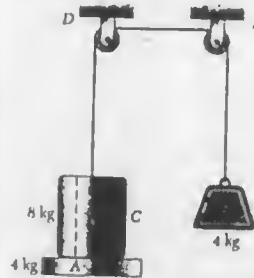
13.131

GIVEN:

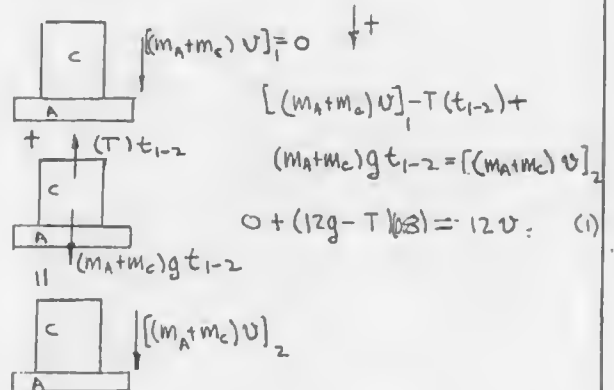
$m_A = 4 \text{ kg}$   
 $m_B = 4 \text{ kg}$   
 $m_C = 8 \text{ kg}$   
 SYSTEM IS RELEASED FROM REST

FIND:

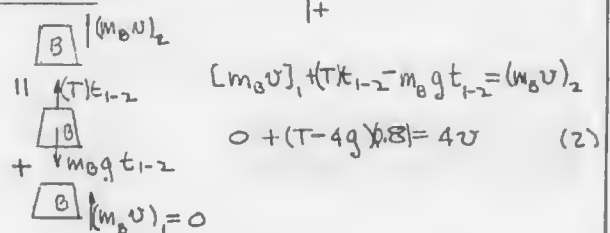
- (a) VELOCITY OF BLOCK B AFTER 0.8 SEC.  
 (b) FORCE EXERTED BY C ON A



(a) BLOCKS A AND C



BLOCK B

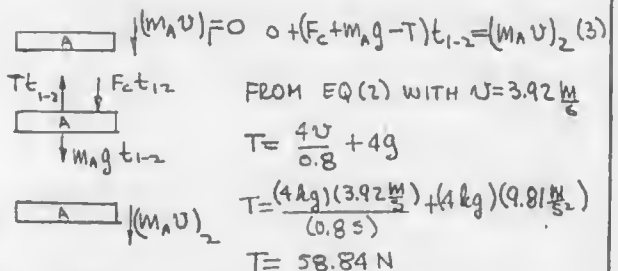


ADDING (1) AND (2), (ELIMINATING T)

$$(12g - 4g)(0.8) = (12 + 4)v$$

$$v = \frac{(8 \text{ kg})(9.81 \text{ m/s}^2)(0.8 \text{ s})}{16 \text{ kg}} = 3.92 \frac{\text{m}}{\text{s}} \quad v_B = 3.92 \frac{\text{m}}{\text{s}}$$

(b) COLLAR A

FROM EQ (2) WITH  $v = 3.92 \frac{\text{m}}{\text{s}}$ 

$$T = \frac{4v}{0.8} + 4g$$

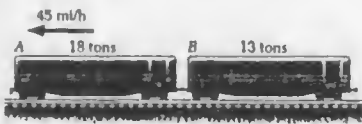
$$T = \frac{(4 \text{ kg})(3.92 \frac{\text{m}}{\text{s}})}{(0.8 \text{ s})} + (4 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})$$

$$T = 58.84 \text{ N}$$

SOLVING FOR  $F_c$  IN (3)

$$F_c = (4 \text{ kg})(3.92 \frac{\text{m}}{\text{s}}) - (4 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) + 58.84 \text{ N} = 39.2 \text{ N}$$

13.132



GIVEN:

$W_A = 18 \text{ TONS}$   
 $W_B = 13 \text{ TONS}$   
 INITIAL VELOCITY  
 $U = 45 \text{ mi/h}$   
 BRAKING FORCE  
 APPLIED TO EACH  
 CAR,  $F_B = 4300 \text{ lb}$

FIND:

- (a) TIME REQUIRED FOR THE TRAIN TO STOP  
 (b) THE FORCE IN THE COUPLING AS THE TRAIN SLOWS

(a) ENTIRE TRAIN  $U_i = 45 \text{ mi/h} = 66 \text{ ft/s}$ 

$$\begin{array}{c}
 \boxed{A} \quad \boxed{B} = \boxed{A} \quad \boxed{B} + \boxed{A} \quad \boxed{B} \\
 (M_A + M_B)U_2 = 0 \quad F_B t_{1-2} \quad F_B t_{1-2} \quad (M_A + M_B)U_1
 \end{array}$$

$$W_A + W_B = 18 + 13 = 31 \text{ TONS} = 62000 \text{ lb}$$

$$+ \quad 0 = -(4300 + 4300)t_{1-2} + \frac{62,000}{g}(66)$$

$$t_{1-2} = \frac{(62000 \text{ lb})(66 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(8600 \text{ lb})} = 14.78 \text{ s}$$

(b) CAR A  $W_A = 18 \text{ TONS} = 36000 \text{ lb}$ ,  $t_{1-2} = 14.78 \text{ s}$ 

$$\begin{array}{c}
 \boxed{A} = \boxed{A} + \boxed{A} \\
 M_A U_2 = 0 \quad F_B t_{1-2} \quad F_B t_{1-2} \quad M_A U_1 \\
 0 - [(4300 \text{ lb}) + F_c][14.78 \text{ s}] = \frac{(36,000 \text{ lb})}{(32.2 \text{ ft/s}^2)}(66 \text{ ft/s})
 \end{array}$$

$$F_c = 692.5 \text{ lb}$$

$$F_c = 693 \text{ lb T}$$

13.133



GIVEN:

$W_A = 18 \text{ T}$ ,  $W_B = 13 \text{ T}$   
 INITIAL VELOCITY  
 $U = 45 \text{ mi/h}$   
 BRAKING FORCE  
 $F_B = 4300 \text{ lb}$   
 APPLIED TO B  
 BUT NOT TO A.

FIND:

- (a) TIME REQUIRED FOR THE TRAIN TO STOP  
 (b) FORCE IN THE COUPLING AS THE TRAIN SLOWS

(a) ENTIRE TRAIN  $U_i = 45 \text{ mi/h} = 66 \text{ ft/s}$ 

$$\begin{array}{c}
 \boxed{A} \quad \boxed{B} = \boxed{A} \quad \boxed{B} + \boxed{A} \quad \boxed{B} \\
 (M_A + M_B)U_2 = 0 \quad F_B t_{1-2} \quad (M_A + M_B)U_1
 \end{array}$$

$$W_A + W_B = 18 + 13 = 31 \text{ TONS} = 62000 \text{ lb}$$

$$+ \quad 0 = -(4300 \text{ lb})t_{1-2} + \frac{(62000 \text{ lb})}{(32.2 \text{ ft/s}^2)}(66 \text{ ft/s})$$

$$t_{1-2} = 29.55 \text{ s}$$

$$t_{1-2} = 29.6 \text{ s}$$

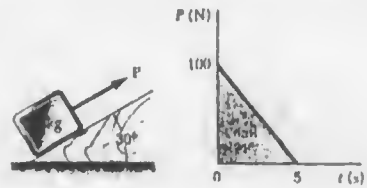
(b) CAR A

$$\begin{array}{c}
 \boxed{A} = \boxed{A} + \boxed{A} \\
 M_A U_2 = 0 \quad F_c t_{1-2} \quad M_A U_1
 \end{array}$$

$$+ \quad 0 = -F_c(t_{1-2}) + M_A U_1 \quad t_{1-2} = 29.55 \text{ s}$$

$$F_c = \frac{(36000 \text{ lb})(66 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(29.55 \text{ s})} = 2497 \text{ lb} \quad F_c = 2500 \text{ lb T}$$

13.134



GIVEN:

6-lb BLOCK ACTED UPON BY P AS SHOWN  
 IS INITIALLY AT REST. NO FRICTION

FIND:

(a) VELOCITY AT  $t = 5 \text{ s}$ 

(b) TIME AT WHICH THE VELOCITY IS ZERO

$$\begin{array}{c}
 \text{Block on } 30^\circ \text{ incline} \\
 \text{Forces: } mg \sin 30^\circ \text{ down, } mg \cos 30^\circ \text{ normal, } P \text{ up, } N \text{ normal} \\
 \text{Initial velocity } mU_1 = 0 \\
 \text{Graph: } P = 100 - 20t \\
 \text{(a) } t_{1-2} = 5 \text{ s} \quad (mU)_1 - mg \sin 30^\circ t_{1-2} + \int_0^{t_{1-2}} P dt = (mU)_2 \\
 0 - (6)(9.81)(0.5) + \int_0^5 (100 - 20t) dt = 6U_2 \\
 (-2.5)(9.81) + \frac{(100)(5) - (10)(5)^2}{6} = U_2 \\
 U_2 = 17.14 \text{ m/s}
 \end{array}$$

(b) AT  $t = 5 \text{ s}$ ,  $U_2 = 17.14 \text{ m/s}$  (FROM (a)).

AFTER  $t = 5 \text{ s}$ ,  $P = 0$ . DENOTE BY  $t'$  THE  
 TIME FOR THE BLOCK TO COME TO REST AFTER  
 $t = 5 \text{ s}$

AFTER  $t = 5 \text{ s}$ 

$$\begin{array}{c}
 \text{Block on } 30^\circ \text{ incline} \\
 \text{Forces: } mg \sin 30^\circ \text{ down, } mg \cos 30^\circ \text{ normal, } N \text{ normal} \\
 \text{Initial velocity } mU_2 = 17.14 \text{ m/s} \\
 \text{Final velocity } mU_3 = 0
 \end{array}$$

$$+ \quad 30^\circ$$

$$mU_2 - mg \sin 30^\circ t' = mU_3$$

$$(6)(17.14) - (6)(9.81)(0.5)t' = 0$$

$$t' = 3.49 \text{ s}$$

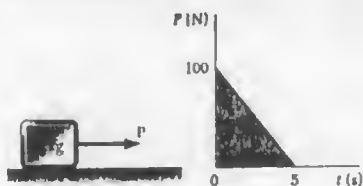
THE TOTAL TIME FOR THE BLOCK TO  
 COME TO REST IS

$$t = 5 + t'$$

$$t = 5 + 3.49 = 8.49 \text{ s}$$

$$t = 8.49 \text{ s}$$

13.135



GIVEN:

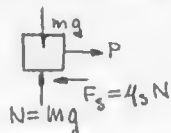
A 6-lb block is acted upon by the force  $P$  as shown and is initially at rest. Coefficients of friction,  $\mu_s = 0.60$ ,  $\mu_k = 0.45$ .

FIND:

- (a) Velocity of the block at  $t = 5$  s  
(b) Maximum velocity of the block

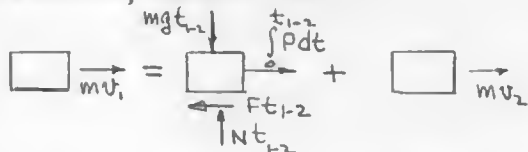
(a)

CHECK TO SEE IF THE BLOCK MOVES WHEN  $P$  IS APPLIED



$$\begin{aligned}\sum F &= 0 \quad P - \mu_s N = 0 \\ P &= \mu_s mg \\ P &= (0.60)(6 \text{ lb})(9.81 \frac{\text{m}}{\text{s}^2}) \\ P &= 35.3 \text{ N}\end{aligned}$$

SINCE 35.3 N IS LESS THAN THE INITIAL VALUE OF  $P = 100$  N, THE BLOCK MOVES.



$$P = 100 - 20t \quad t_{1-2} = 5 \text{ s} \quad F = \mu_k mg = (0.45)(6)(9.81)$$

$$mv_1 = \int_0^{t_{1-2}} P dt - F t_{1-2} + mv_2$$

$$0 = \int_0^5 (100 - 20t) dt - (0.45)(6)(9.81)(5) = 6v_2$$

$$0 = 500 - 250 - 132.4 + 6v_2$$

$$v_2 = 19.59 \text{ m/s}$$

(b) DETERMINE TIME AT WHICH THE VELOCITY IS A MAXIMUM, WHICH MUST OCCUR AT  $t < 5$  s

$$0 = \int_0^t (100 - 20t) dt - (0.45)(6)(9.81)t + 6v \quad (1)$$

$$\frac{dv}{dt} = 0; \quad 100 - 20t - 26.49 = 0$$

$$t = 3.68 \text{ s WHEN } v \text{ IS MAXIMUM}$$

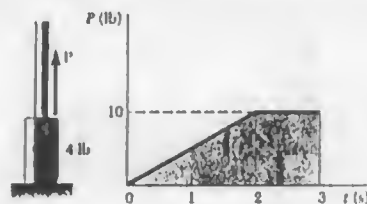
SUBSTITUTE  $t = 3.68$  s IN EQ (1)

$$0 = (100)(3.68) - 10(3.68)^2 - 97.47 + 6v_{\text{MAX}}$$

$$v_{\text{MAX}} = \frac{134.67}{6} = 22.45 \text{ m/s}$$

$$v_{\text{MAX}} = 22.5 \text{ m/s}$$

13.136



GIVEN:

A 4-lb block is acted upon by the force  $P$  as shown and is initially at rest. NO FRICTION.

FIND:

- (a) Velocity at  $t = 2$  s  
(b) Velocity at  $t = 3$  s

THE BLOCK DOES NOT MOVE UNTIL  $P = 4$  lb FROM  $t = 0$  TO  $t = 2$  s  $P = 5t$  THUS, THE BLOCK STARTS TO MOVE WHEN  $t = 4/5 = 0.8$  s

$$\begin{aligned}& \uparrow m v_2 \quad (a) \text{ FOR } 0 < t < 2 \text{ s} \\& \uparrow \int_{t_1}^{t_2} P dt \quad P = 5t \\& \uparrow t_1 = 0.8 \text{ s} \quad t_2 = 2 \text{ s}, v_1 = 0 \\& m v_1 + \int_{t_1}^{t_2} P dt - W(t_2 - t_1) = m v_2 \\& 0 + \int_{0.8}^2 5t dt - 4(2 - 0.8) = \frac{4}{g} v_2\end{aligned}$$

$$v_2 = \frac{(32.2 \frac{\text{ft}}{\text{s}^2})}{4(1 \text{ lb})} \left[ \left( \frac{5 \text{ lb}}{2} \right) (2 \text{ s})^2 - (0.8 \text{ s})^2 \right] - (4 \text{ lb})(2 \text{ s} - 0.8 \text{ s})$$

$$v_2 = 28.98 \text{ ft/s} \quad v_2 = 29.0 \text{ ft/s}$$

(b) FROM  $t = 2$  s TO  $t = 3$  s

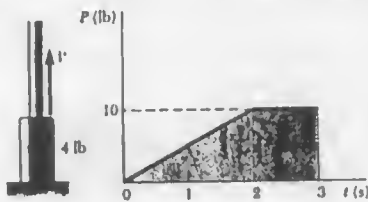
$$\begin{aligned}& \uparrow m v_3 \quad v_2 = 29.0 \text{ ft/s, FROM (a)} \\& \uparrow \int_{t_2}^{t_3} P dt \quad P = 10 \text{ lb} \quad 2 \leq t \leq 3 \text{ s} \\& \uparrow t_2 = 2 \text{ s} \quad t_3 = 3 \text{ s} \\& m v_2 + \int_{t_2}^{t_3} P dt - W(t_3 - t_2) = m v_3 \\& \left( \frac{4}{g} \right) (29.0) + \int_2^3 10 dt - 4(3 - 2) = \frac{4}{g} v_3\end{aligned}$$

$$v_3 = (29.0 \text{ ft/s}) + \frac{(32.2 \frac{\text{ft}}{\text{s}^2})}{4(1 \text{ lb})} [(6 \text{ lb})(1 \text{ s})] = v_3$$

$$v_3 = 29.0 + 48.3 = 77.3 \text{ ft/s}$$

$$v_3 = 77.3 \text{ ft/s}$$

13.137



GIVEN:

COLLAR INITIALLY AT REST IS ACTED UPON BY A FORCE  $P(t)$  AS SHOWN. NO FRICTION

FIND:

- (a) THE MAXIMUM VELOCITY OF THE COLLAR,  $v_{MAX}$   
 (b) THE TIME WHEN THE VELOCITY IS ZERO.

(1) DETERMINE TIME AT WHICH COLLAR STARTS TO MOVE

$$P = 5t, 0 < t < 2s$$

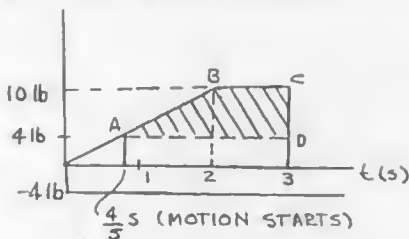
COLLAR MOVES WHEN  $P = 4 \text{ lb}$  OR  $t = \frac{P}{5} = \frac{4}{5} s$

$$m v_1 + \int_{t_1}^{t_2} P dt - \int_{t_1}^{t_2} W dt = m v_2$$

FOR  $t < 2s$   $P = 5t$  (lb)  
 $2s < t < 3s$   $P = 10$  lb  
 $t > 3s$   $P = 0$

FOR  $t < 3s$   $W = 4 \text{ lb}$

THE MAXIMUM VELOCITY OCCURS WHEN THE TOTAL IMPULSE IS MAXIMUM.



$$\text{AREA}_{ABCD} = \text{MAX IMPULSE} = \frac{1}{2}(6 \text{ lb})\left(\frac{6}{5} s\right) + (6 \text{ lb})(1 s)$$

$$\text{AREA}_{ABCD} = 9.6 \text{ lb}\cdot s$$

$$0 + 9.6 \text{ lb}\cdot s = \left(\frac{4 \text{ lb}}{32.2 \text{ ft/s}^2}\right) v_{MAX}$$

$$v_{MAX} = 77.3 \text{ ft/s}$$

(b) VELOCITY IS ZERO WHEN TOTAL IMPULSE IS ZERO AT  $t + \Delta t$

FOR  $\frac{4}{5} s < t < 3s$ , IMPULSE =  $9.6$  (lb·s), PART (a)

FOR  $\Delta t$  BEYOND  $3s$  IMPULSE =  $-4 \Delta t$  (lb·s)  
 THUS

$$\text{TOTAL IMPULSE} = 0 = 9.6 - 4 \Delta t$$

$$\Delta t = 2.4 s$$

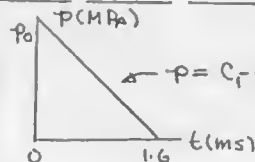
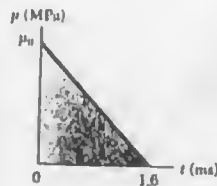
$$\text{TIME TO ZERO VELOCITY } t = 3s + 2.4s = 5.4s$$

13.138

GIVEN:

20-g BULLET  
 10 MM DIAMETER RIFLE  
 BARREL  
 EXIT VELOCITY OF THE  
 BULLET = 700 m/s  
 TIME BULLET TO EXIT  
 = 1.6 ms  
 VARIATION OF PRESSURE  
 AS SHOWN

FIND:



$$p = C_1 - C_2 t$$

AT  $t=0$   $p=p_0=C_1-C_2(0)$   
 $C_1=p_0$   
 AT  $t=1.6 \times 10^{-3} s$   $p=0$   
 $0=C_1-C_2(1.6 \times 10^{-3} s)$   
 $C_2=p_0/(1.6 \times 10^{-3} s)$

$$m = 20 \times 10^{-3} \text{ kg}$$

$$A = \frac{\pi (10^{-3})^2}{4}$$

$$A = 78.54 \times 10^{-6} \text{ m}^2$$

$$0 + A \int_0^{1.6 \times 10^{-3}} p dt = m v_2$$

$$0 + A \int_0^{1.6 \times 10^{-3}} (C_1 - C_2 t) dt = \frac{20 \times 10^{-3}}{g}$$

$$(78.54 \times 10^{-6} \text{ m}^2) \left[ C_1 (1.6 \times 10^{-3}) - \frac{C_2 (1.6 \times 10^{-3})^2}{2} \right] =$$

$$(20 \times 10^{-3} \text{ kg})(100 \text{ m/s})$$

$$1.6 \times 10^{-3} C_1 - 1.280 \times 10^{-6} C_2 = 178.25 \times 10^3$$

$$(1.6 \times 10^{-3} \text{ m}^2 \cdot s) p_0 - \frac{(1.280 \times 10^{-6} \text{ m}^2 \cdot s^2)}{(1.6 \times 10^{-3} s)} p_0 = 178.25 \times 10^3 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

$$p_0 = 222.8 \times 10^6 \text{ N/m}^2$$

$$p_0 = 223 \text{ MPa}$$

13.139

GIVEN:

25-g BULLET, 10 mm DIA. RIFLE BARREL  
 EXIT VELOCITY = 520 m/s  
 TIME FOR BULLET TO EXIT = 1.44 ms  
 PRESSURE MODEL  
 $p(t) = (950 \text{ MPa}) (e^{-t/(0.16 \text{ ms})})$

FIND:

% ERROR IF GIVEN EQUATION FOR  $p(t)$  IS USED  
 TO CALCULATE THE EXIT VELOCITY

$$A = \frac{\pi (10^{-3})^2}{4} = 78.54 \times 10^{-6} \text{ m}^2$$

$$0 + \int_0^{1.44 \times 10^{-3}} A p dt = m v_2$$

$$0 + (78.54 \times 10^{-6} \text{ m}^2) \int_0^{1.44 \times 10^{-3}} (950 \times 10^6 \frac{\text{N}}{\text{m}^2}) (e^{-t/(0.16 \times 10^{-3})}) dt = (25 \times 10^{-3} \text{ kg}) v_2$$

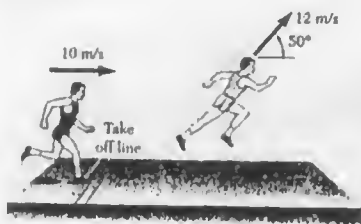
$$(78.54 \times 10^{-6}) (950 \times 10^6) (0.16 \times 10^{-3}) (e^{-1.44/0.16} - 1) = 25 \times 10^{-3} v_2$$

$$v_2 = 477.46 \text{ m/s}$$

$$\text{ERROR} = 477.46 - 520 = -42.54 \text{ m/s}$$

$$\% \text{ ERROR} = 100 (-42.54/520) = 8.18 \%$$

13.140



GIVEN:

INITIAL  
VELOCITY AT  
TAKE OFF  
= 10 m/s.  
VELOCITY  
AFTER  
TAKEOFF =  
12 m/s AT 50°  
IMPACT TIME  
= 0.18 s.

FIND:

VERTICAL COMPONENT OF THE AVERAGE  
IMPULSIVE FORCE ON ATHLETE'S FOOT  
FROM THE GROUND. (IN TERMS OF HIS WEIGHT  $W$ )

$$m\vec{v}_1 + \vec{W}\Delta t + \vec{P}_H\Delta t + \vec{P}_V\Delta t = m\vec{v}_2$$

$v_1 = 10 \text{ m/s}$        $v_2 = 12 \text{ m/s}$

$$m\vec{v}_1 + (\vec{P} - \vec{W})\Delta t = m\vec{v}_2 \quad \Delta t = 0.18 \text{ s}$$

VERTICAL COMPONENTS

$$0 + (P_V - W)(0.18) = (W/g)(12)(\sin 50^\circ)$$

$$P_V = W + \frac{(12)(\sin 50^\circ)}{(9.81)(0.18)} W$$

$$P_V = 6.21 W$$

13.141

GIVEN:

VELOCITY BEFORE  
LANDING = 30 ft/s  
AT 35°  
IMPACT TIME  
BEFORE COMING  
TO A STOP  
= 0.22 s  
WEIGHT = 185 lb

FIND:

HORIZONTAL COMPONENT OF THE AVERAGE  
IMPULSIVE FORCE ON THE ATHLETE'S FEET

$$m\vec{v}_1 + \vec{W}\Delta t + \vec{P}_H\Delta t + \vec{P}_V\Delta t = m\vec{v}_2$$

$v_1 = 30 \text{ ft/s}$        $v_2 = 0$

$$m\vec{v}_1 + (\vec{P} - \vec{W})\Delta t = m\vec{v}_2$$

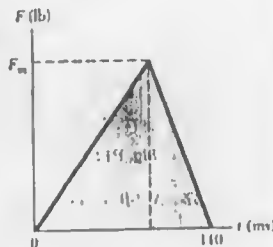
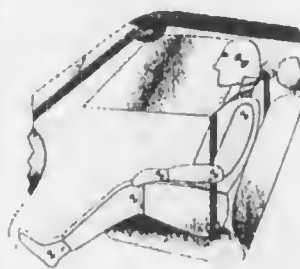
HORIZONTAL COMPONENTS

$$\frac{W}{g}(30)(\cos 35^\circ) - P_H(0.22) = 0$$

$$P_H = \frac{(185 \text{ lb})(30 \text{ ft/s})/\cos 35^\circ}{(32.2 \text{ ft/s}^2)(0.22 \text{ s})} = 641.7 \text{ lb}$$

$$P_H = 642 \text{ lb}$$

13.142



GIVEN:

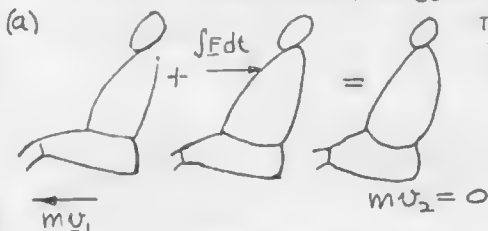
AUTOMOBILE TRAVELING AT 45 mi/h  
COMES TO A STOP IN 110 MS.  
FORCE ACTING ON MAN AS SHOWN  
MAN'S WEIGHT = 200 lb

FIND:

(a) AVERAGE IMPULSIVE FORCE EXERTED  
ON THE BELT AS SHOWN

(b) MAXIMUM FORCE  $F_m$  EXERTED ON THE BELT  
FORCE ON THE BELT IS OPPOSITE

TO THE  
DIRECTION  
SHOWN



$$v_1 = 45 \text{ mi/h} = 66 \text{ ft/s}, \quad W = 200 \text{ lb}$$

$$m\vec{v}_1 - \int F dt = m\vec{v}_2 \quad \int F dt = F_{AVE} \Delta t$$

$$\frac{(200 \text{ lb})(66 \text{ ft/s})}{(32.2 \text{ ft/s}^2)} - F_{AVE}(0.110 \text{ s}) = 0 \quad \Delta t = 0.110 \text{ s}$$

$$F_{AVE} = \frac{(200)(66)}{(32.2)(0.110)} = 3727 \text{ lb} \quad F_{AVE} = 3730 \text{ lb}$$

(b)

IMPULSE = AREA UNDER  
F-t DIAGRAM =  $\frac{1}{2} F_m(0.110 \text{ s})$

FROM (a), IMPULSE =  
 $F_{AVE} \Delta t = (3727 \text{ lb})(0.110 \text{ s})$

$$\frac{1}{2} F_m(0.110) = (3727)(0.110)$$

$$F_m = 7450 \text{ lb}$$

13.143

GIVEN:

1.60Z GOLF BALL HAS A VELOCITY OF  
125 ft/s AFTER IMPACT  
DURATION OF IMPACT =  $t_0 = 0.5 \text{ ms}$   
FORCE DURING IMPACT  $F = F_m \sin(\pi t/t_0)$

FIND:

MAXIMUM FORCE  $F_m$  ON THE BALL

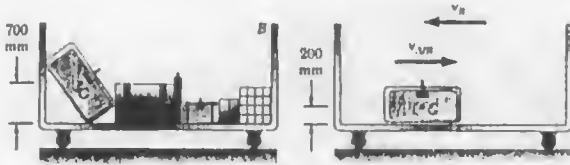
$$m\vec{v}_1 + \int F dt = m\vec{v}_2$$

$v_1 = 0$        $v_2 = 125 \text{ ft/s}$

$$0 + \int_0^{0.5 \times 10^{-3}} F_m \sin\left(\frac{\pi t}{0.5 \times 10^{-3}}\right) dt = \frac{(1.6/16)(125)}{(32.2)}$$

$$F_m = 1220 \text{ lb}$$

13.144



GIVEN:

15-kg SUITCASE A

40-kg LUGGAGE CARRIER B

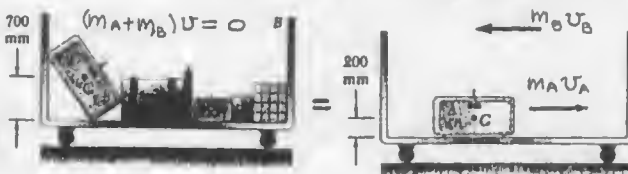
INITIAL VELOCITY OF CARRIER,  $v_B = 0.8 \text{ m/s}$ 

FIND:

(a)  $v_{A/B}$ (b)  $v_B'$  AFTER THE SUITCASE HITS THE RIGHT SIDE OF THE CARRIER WITHOUT PENETRATING

(c) ENERGY LOST BY THE IMPACT OF THE SUITCASE ON THE FLOOR OF THE CARRIER

(a) SINCE THERE ARE NO EXTERNAL FORCES ACTING ON THE SYSTEM OF THE SUITCASE A AND THE LUGGAGE CARRIER B, IN THE HORIZONTAL DIRECTION, LINEAR MOMENTUM IS CONSERVED



$$(m_A + m_B)U = m_A v_A + m_B v_B$$

$$0 = 0 \quad v_B = -0.8 \text{ m/s} \quad v_A = v_{A/B} + v_B$$

$$m_B = 40 \text{ kg} \quad m_A = 15 \text{ kg}$$

$$0 = (15 \text{ kg})(v_{A/B} - 0.8 \text{ m/s}) + 40 \text{ kg}(-0.8 \text{ m/s})$$

$$v_{A/B} = \frac{(40 \text{ kg})(0.8 \text{ m/s}) + 0.8 \text{ m/s}}{(15 \text{ kg})} = 2.93 \text{ m/s}$$

$$v_{A/B} = 2.93 \text{ m/s} \rightarrow$$

(b) MOMENTUM IS CONSERVED BEFORE AND AFTER THE SUITCASE HITS THE LUGGAGE CARRIER



$$m_A v_A + m_B v_B = (m_A + m_B)U'$$

$$U' = \frac{m_A v_A + m_B v_B}{(m_A + m_B)}$$

FROM (a)

$$U_A = v_{A/B} + v_B = 2.93 - 0.8 = 2.13 \text{ m/s}$$

$$U' = (15)(2.13) - (40)(0.8) = 0 \quad U = 0$$

(c) BEFORE SUITCASE FALLS,  $E_1 = m_A g (7 \text{ m})$   
AFTER SUITCASE HITS THE BOTTOM OF THE CARRIER  $E_2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 + m_A g (0.200 \text{ m})$

$$\text{ENERGY LOST, } \Delta E_L = E_1 - E_2 \quad E_1 = 15 \text{ g} (7)$$

$$\Delta E_L = (15)(9.81)(0.7) - \frac{1}{2}(15)(2.13)^2 - \frac{1}{2}(40)(0.8)^2 - (15)(9.81)(0.2)$$

$$\Delta E_L = 26.7 \text{ J}$$

13.145



GIVEN:

BEFORE COUPLING, 20-Mg CAR IS TRAVELING

AT 4 km/h AS SHOWN

40-Mg CAR HAS ITS WHEELS LOCKED

 $\mu_k = 0.30$ , 40-Mg CAR ONLY

FIND:

(a) VELOCITY OF BOTH CARS IMMEDIATELY AFTER COUPLING

(b) THE TIME FOR BOTH CARS TO COME TO REST

(a) THE MOMENTUM OF THE SYSTEM CONSISTING OF THE TWO CARS IS CONSERVED IMMEDIATELY BEFORE AND AFTER COUPLING.

$$\begin{array}{c} 40 \text{ Mg} \quad 20 \text{ Mg} \\ \hline 40 v = 0 \quad (20)(4) \end{array} = \begin{array}{c} 40 \text{ Mg} \quad 20 \text{ Mg} \\ \hline (20+40)U' \end{array}$$

BEFORE COUPLING                      AFTER COUPLING

$$\Sigma m v = \Sigma m v'$$

$$0 + (20 \text{ Mg})(4 \text{ km/h}) = (20 \text{ Mg} + 40 \text{ Mg})(v')$$

$$v' = \frac{(20)(4)}{(20+40)} = 1.333 \text{ km/h}$$

(b) AFTER COUPLING

$$\begin{array}{c} 60 \text{ Mg} \\ \hline 60 v_2 = 0 \end{array} = \begin{array}{c} 60 \text{ Mg} \\ \hline \int F_f dt \end{array} + \begin{array}{c} 60 \text{ Mg} \\ \hline 60 v_1 \end{array}$$

THE FRICTION FORCE ACTS ONLY ON THE 40 Mg CAR SINCE ITS WHEELS ARE LOCKED. THUS,

$$F_f = \mu_k N_{40} = (0.30)(40 \times 10^3 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})$$

$$F_f = 117.72 \times 10^3 \text{ N}$$

$$\text{FROM (a)} \quad v_i = v' = 1.333 \text{ km/h} = 0.3704 \text{ m/s}$$

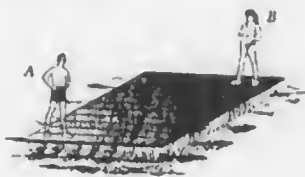
IMPULSE-MOMENTUM

$$\Sigma m v_i + \int_0^t F_f dt = \Sigma m v_2$$

$$(60 \times 10^3 \text{ kg})(0.3704 \text{ m/s}) - \int_0^t (117.72 \times 10^3 \text{ N}) dt = 0$$

$$t = \frac{(60 \times 10^3)(0.3704)}{(117.72 \times 10^3)} = 0.1888 \text{ s}$$

13.146



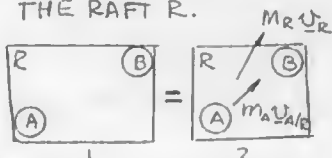
GIVEN:

$W_A = 190 \text{ lb}$   
 $W_B = 125 \text{ lb}$   
 $\text{RAFT } W_R = 300 \text{ lb}$   
 $V_{A/R} = 2 \text{ ft/s}$   
 TOWARD B, AFTER  
 THE RAFT BREAKS  
 LOOSE FROM ITS  
 ANCHOR.

FIND:

- (a) SPEED OF THE RAFT,  $V_R$ , IF B DOES NOT MOVE  
 (b) SPEED  $V_B$  OF B, IF THE RAFT IS NOT TO  
 MOVE

- (a) THE SYSTEM CONSISTS OF A AND B AND THE RAFT R.



MOMENTUM IS CONSERVED

$$(\sum m\mathbf{v})_1 = (\sum m\mathbf{v})_2$$

$$0 = m_A v_A + m_B v_B + m_R v_R \quad (1)$$

$$v_A = v_{A/R} + v_R \quad v_B = v_{B/R} + v_R \quad v_{B/R} = 0$$

$$v_A = 2 \text{ ft/s} + v_R \quad v_B = v_R$$

$$0 = m_A [2 + v_R] + m_B v_R + m_R v_R$$

$$v_R = \frac{-2 m_A}{(m_A + m_B + m_R)} = \frac{-(2 \text{ ft/s})(190 \text{ lb})}{(190 \text{ lb} + 125 \text{ lb} + 300 \text{ lb})}$$

$$v_R = 0.618 \text{ ft/s}$$

- (b) FROM EQ (1)

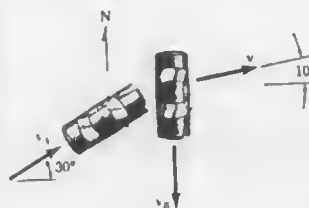
$$0 = m_A v_A + m_B v_B + 0 \quad (v_R = 0)$$

$$v_B = -\frac{m_A v_A}{m_B} \quad v_A = v_{A/R} + v_R = 2 \text{ ft/s}$$

$$v_B = -\frac{(2 \text{ ft/s})(190 \text{ lb})}{(125 \text{ lb})} = 3.04 \text{ ft/s}$$

$$v_B = 3.04 \text{ ft/s}$$

13.147



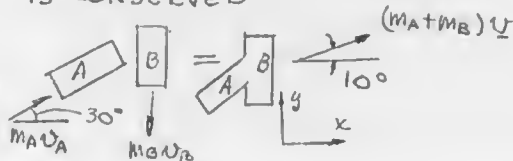
GIVEN:

$m_A = 1500 \text{ kg}$   
 $m_B = 1200 \text{ kg}$   
 BOTH CARS  
 TOGETHER, SKID  
 AT  $10^\circ$  NORTH OF  
 EAST AFTER  
 IMPACT

FIND:

- (a) WHO WAS  
 GOING FASTER  
 (b) SPEED OF  
 THE FASTER  
 CAR IF SLOWER  
 CAR WAS GOING  
 AT  $50 \text{ km/h}$

- (a) TOTAL MOMENTUM OF THE TWO CARS IS CONSERVED



$$\sum m\mathbf{v}_x: m_A v_A \cos 30^\circ = (m_A + m_B) v \cos 10^\circ \quad (1)$$

$$\sum m\mathbf{v}_y: m_A v_A \sin 30^\circ - m_B v_B = (m_A + m_B) v \sin 10^\circ \quad (2)$$

DIVIDING (1) INTO (2)

$$\frac{\sin 30^\circ}{\cos 30^\circ} - \frac{m_B v_B}{m_A v_A \cos 30^\circ} = \frac{\sin 10^\circ}{\cos 10^\circ}$$

$$\frac{v_B}{v_A} = \frac{(\tan 30^\circ - \tan 10^\circ) (m_A \cos 30^\circ)}{m_B}$$

$$\frac{v_B}{v_A} = (0.4010) \frac{(1500) (\cos 30^\circ)}{(1200)}$$

$$\frac{v_B}{v_A} = 0.434 \quad v_A = 2.30 v_B$$

THUS, A WAS GOING FASTER

- (b) SINCE  $v_B$  WAS THE SLOWER CAR  
 $v_B = 50 \text{ km/h}$

$$v_A = (2.30)(50) = 115.2 \text{ km/h}$$

MOTHER AND CHILD TRAVELING AT 2.2 km/h  
INITIALLY.  $m_m = 55 \text{ kg}$   $m_c = 20 \text{ kg}$   
CHILD'S SPEED DECREASES TO 3.6 km/h  
IN 3 s AS THE MOTHER PULLS ON THE ROPE

(a) MOTHERS SPEED AT THE END OF THE 3 s INTERVAL

(b) AVERAGE VALUE OF THE TENSION IN THE ROPE DURING THE 3 S INTERVAL

(a) CONSIDER MOTHER AND CHILD AS A SINGLE SYSTEM. ASSUMING THE FRICTION FORCE ON THE SKIS IS NEGLECTIBLE MOMENTUM IS CONSERVED



$$\cdot \overleftarrow{(m_c v_c)} \quad \overleftarrow{(m_H v_H)} \quad \overleftarrow{(m_c v'_c)} \quad \overleftarrow{(m_H v'_H)}$$

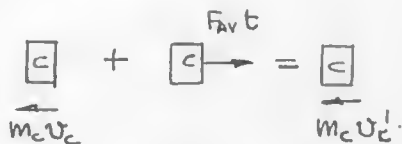
$$m_c v_c + m_H v_H = m_c v'_c + m_H v'_H$$

$$v_c = v_H = 7.2 \text{ km/h} \quad v_c' = 3.6 \text{ km/h}$$

$$(20)(7.2) + (55)(7.2) = 20(3.6) + (55)(V_M')$$

$$v_M' = 8.51 \text{ km/h}$$

(b) CHILD ALONE



$t = 35$

$$m_c u_c - F_{Av} t = m_c u_c'$$

$$v_c = 7.2 \text{ km/h} = 2 \text{ m/s} \quad v_c' = 3.6 \text{ km/h} = 1 \text{ m/s}$$

$$(20\text{ kg})(2\text{ m/s}) - F_{\text{AV}}(3\text{ s}) = (20\text{ kg})(1\text{ m/s})$$

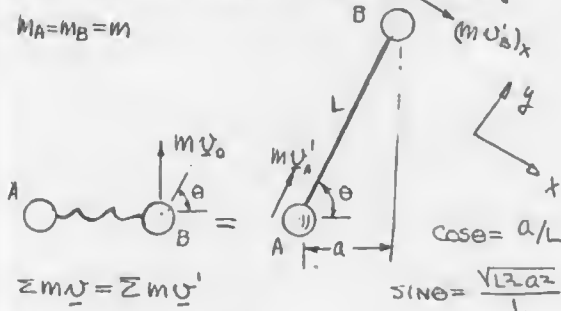
$$F_{AV} = \frac{(20 \text{ kg})(1 \text{ m/s})}{(3 \text{ s})} = 6.67 \text{ kg} \cdot \text{m/s}^2$$

$$F_{Av} = 6.67 \text{ N}$$

A AND B ON A HORIZONTAL  
FRICTIONLESS PLANE  
ARE ATTACHED BY AN  
INEXTENSIBLE CORD OF  
LENGTH L  
MASS OF A = MASS OF B  
 $u_B = v_0, u_A = 0$   
INITIALLY

(a)  $U_A'$  AND  $U_B'$  AFTER THE CORD BECOMES TAUT  
(b) THE ENERGY LOST AS THE CORD BECOMES TAUT

(a) FOR THE SYSTEM CONSISTING OF BOTH BALLS CONNECTED BY A CORD THE TOTAL MOMENTUM IS CONSERVED / (m.u.)



$$x: -mV_0 \cos \theta = m(u_a')_x \quad (1)$$

$$(v'_B)_x = -v_o \cos \theta = -v_o \frac{a}{L}$$

$$y: mv_0 \sin \theta = mv_A' + m_1(v_B')_y \quad (2)$$

SINCE THE CORD IS INEXTENSIBLE

$$v_A' = (v_B')_y \quad (3)$$

THUS FROM (2)  $v_o \sin \theta = 2v_A'$

$$v_A' = (v_0/2L)\sqrt{L^2 - a^2}$$

FROM (3)

$$(v_B')_y = v_A' = (v_0/2L) \sqrt{L^2 - a^2}$$

$$v_B' = \sqrt{(v_B')_x^2 + (v_B')_y^2} = v_c \sqrt{\frac{n^2}{L^2} + \frac{(L^2 - a^2)}{4L^2}}$$

$$V_B' = (V_0/2L) \sqrt{L^2 + 3a^2}$$

(b) INITIAL  $T = \frac{1}{2} m v_0^2$

$$T' = \frac{1}{2} m (v_A')^2 + \frac{1}{2} m (v_B')^2 = \frac{1}{2} m (v_0/2L)^2 [(L^2 a^2) + (L^2 3a^2)]$$

$$T' = \frac{1}{2} (m v_0^2 / 4L^2) (2L^2 + 2a^2) = (m v_0^2 / 4L^2) (L^2 + a^2)$$

$$\Delta T = T - T' = \frac{1}{2} m v_0^2 - (m v_0^2 / 4L^2)(L^2 + a^2)$$

$$\Delta T = (mv_0^2/4L^2)(L^2 - a^2)$$



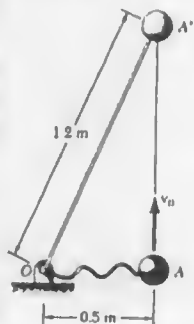
13.150

GIVEN:

2-kg SPHERE CONNECTED BY AN INEXTENSIBLE CORD OF LENGTH 1.2 m TO POINT O ON A HORIZONTAL FRICTIONLESS PLANE. INITIAL VELOCITY  $v_0$  PERPENDICULAR TO OA

FIND:

MAXIMUM ALLOWABLE  $v_0$  IF IMPULSE OF THE FORCE EXERTED ON THE CORD IS NOT TO EXCEED 3 N·S.



FOR THE SPHERE AT A IMMEDIATELY BEFORE AND AFTER THE CORD BECOMES TAUT

$$m\vec{v}_0 + \vec{F}\Delta t = m\vec{v}_A'$$

$$\Delta\theta = \cos^{-1}(.5/1.2) = 65.38^\circ$$

$$m\vec{v}_0 + \vec{F}\Delta t = m\vec{v}_A'$$

$$+ \Delta\theta \quad m v_0 \sin\theta - F\Delta t = 0 \quad F\Delta t = 3 \text{ N}\cdot\text{s}$$

$$v_0 = \frac{3 \text{ N}\cdot\text{s}}{(2 \text{ kg}) \sin(65.38^\circ)}$$

$$kg = \frac{\text{N}\cdot\text{s}^2}{\text{m}}$$

$$v_0 = 1.650 \text{ m/s}$$

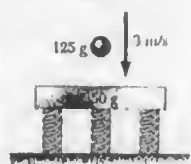
13.151

GIVEN:

ASSES ALL INITIAL VELOCITY OF THE BALL AS SHOWN. NO ENERGY LOST IN THE IMPACT

FIND:

(a) VELOCITY OF THE BALL IMMEDIATELY AFTER IMPACT  
(b) IMPULSE OF THE FORCE EXERTED BY THE PLATE ON THE BALL



(a) FOR THE SYSTEM WHICH IS THE BALL AND THE PLATE, MOMENTUM IS CONSERVED

$$(m\vec{v})_B = (m\vec{v}')_B$$

$$(m\vec{v})_P = 0 \quad (m\vec{v}')_P$$

ALL FORCES ARE NON-IMPULSIVE EXCEPT THE EQUAL AND OPPOSITE FORCES BETWEEN THE PLATE AND THE BALL

$$+ \downarrow (m\vec{v})_B = -(m\vec{v}')_B + (m\vec{v}')_P$$

$$(0.125 \text{ kg})(3 \text{ m/s}) = -(0.125 \text{ kg})(v_B') + (0.250 \text{ kg})v_P'$$

$$v_P' = 0.5v_B' + 1.5 \quad (1)$$

SINCE THERE IS NO ENERGY LOST THE KINETIC ENERGY OF THE SYSTEM IS CONSERVED

(CONTINUED)

13.151 continued

$$\text{BEFORE IMPACT, } T = \frac{1}{2} m_B v_B^2 = \frac{1}{2} (0.125 \text{ kg})(3 \text{ m/s})^2 = 0.5625 \text{ J}$$

$$\text{AFTER IMPACT } T' = \frac{1}{2} m_B (v_B')^2 + \frac{1}{2} m_P (v_P')^2$$

$$\text{SUBSTITUTE FOR } v_P' \text{ FROM (1)}$$

$$T' = \frac{1}{2} (0.125 \text{ kg})(v_B')^2 + \frac{1}{2} (0.250 \text{ kg})(0.5v_B' + 1.5)^2$$

$$T' = 0.09375(v_B')^2 + 0.1875v_B' + 0.2813$$

$$T = T' \quad 0.5625 = 0.09375(v_B')^2 + 0.1875v_B' + 0.2813$$

$$v_B'^2 + 2v_B' - 3 = 0$$

$$v_B' = \frac{-2 \pm \sqrt{4+12}}{2} = -1 \pm 2 = -3, +1$$

$$(v_B' = -3 \text{ m/s BEFORE IMPACT}) \quad v_B' = 1 \text{ m/s}$$

(b) BALL ALONE

$$(m\vec{v})_B + \vec{F}\Delta t = (m\vec{v}')_B$$

$$+ \uparrow (0.125 \text{ kg})(-3 \text{ m/s}) + F\Delta t = (0.125 \text{ kg})(1 \text{ m/s})$$

$$F\Delta t = 0.5 \text{ N}\cdot\text{s}$$

13.152

GIVEN:

BULLET FIRED INTO THE BLOCK AS SHOWN

FIND:

HORIZONTAL AND VERTICAL COMPONENTS OF THE IMPULSE ON THE BULLET.



FOR THE SYSTEM WHICH IS THE BULLET AND THE BLOCK, MOMENTUM IN THE HORIZONTAL DIRECTION IS CONSERVED

$$m\vec{v}_0 + \vec{F}\Delta t = (M+m)\vec{v}'$$

$$\vec{m\vec{v}} = 0$$

$$-m v_0 \cos\theta = (M+m)v' \quad v' = \frac{-m v_0 \cos\theta}{(M+m)}$$

BULLET ALONE

$$m\vec{v}_0 + \vec{R}_x \Delta t = m\vec{v}'$$

$$+ \rightarrow -m v_0 \cos\theta + R_x \Delta t = m v'$$

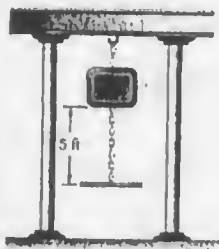
$$R_x \Delta t = m v_0 \cos\theta \left[ 1 - \frac{m}{M+m} \right]$$

$$R_x \Delta t = \frac{mM}{m+M} v_0 \cos\theta$$

$$+ \uparrow -m v_0 \sin\theta + P_y \Delta t = 0$$

$$P_y \Delta t = m v_0 \sin\theta$$

13.153



GIVEN:

RIGID BEAM WEIGHS 240 lb  
BLOCK WEIGHS 60 lb  
INITIAL VELOCITY OF THE  
BLOCK = 0 AND IT IS  
DROPPED FROM 5 ft.

FIND:

INITIAL IMPULSE EXERTED  
ON THE CHAIN AND THE  
ENERGY ABSORBED BY THE  
CHAIN IF THE SUPPORTING  
COLUMNS ARE,  
(a) RIGID, (b) EQUIVALENT TO  
TWO ELASTIC SPRINGS

VELOCITY OF THE BLOCK JUST BEFORE IMPACT

$$T_1 = 0 \quad V_1 = Wh = (60 \text{ lb})(5 \text{ ft}) = 300 \text{ lb} \cdot \text{ft}$$

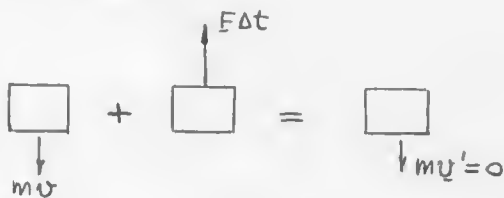
$$T_2 = \frac{1}{2} m v^2 \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300 = \frac{1}{2} \left( \frac{60}{g} \right) v^2$$

$$v = \sqrt{(600)(32.2)/60} = 17.94 \text{ ft/s}$$

(a) RIGID COLUMNS



$$+ \uparrow -mv + F\Delta t = 0 \quad \left( \frac{60}{g} \right) (17.94) = F\Delta t$$

$$F\Delta t = 33.43 \text{ lb} \cdot \text{s} \quad \uparrow \text{ ON THE BLOCK}$$

$$F\Delta t = 33.4 \text{ lb} \cdot \text{s}$$

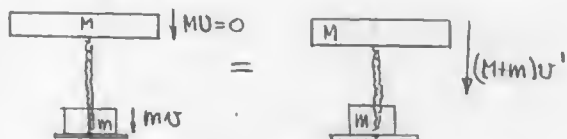
ALL OF THE KINETIC ENERGY OF THE BLOCK  
IS ABSORBED BY THE CHAIN

$$T = \frac{1}{2} \left( \frac{60}{g} \right) (17.94)^2 = 300 \text{ ft} \cdot \text{lb}$$

$$E = 300 \text{ ft} \cdot \text{lb}$$

(b) ELASTIC COLUMNS

MOENTUM OF SYSTEM OF BLOCK AND BEAM  
IS CONSERVED



$$mv = (M+m)v' \quad v' = \frac{m}{m+M} v = \frac{60}{300} (17.94 \text{ ft/s})$$

$$v' = 3.59 \text{ ft/s}$$

REFERRING TO FIGURE IN PART (a)

$$-mv + F\Delta t = -mv'$$

$$F\Delta t = m(v - v') = (60/g) (17.94 - 3.59) = 26.7 \text{ lb} \cdot \text{s}$$

$$E = \frac{1}{2} m v^2 - \frac{1}{2} m v'^2 - \frac{1}{2} M v'^2 = \frac{60}{2g} [(17.94)^2 - (3.59)^2] - \frac{240}{2g} (3.59)^2$$

$$E = 240 \text{ ft} \cdot \text{lb}$$

13.154



GIVEN:

$W_B = 5.02$   
INITIAL SPEED  
OF THE BALL  
= 90 mi/h  
AVERAGE  
SPEED OF  
THE GLOVE  
DURING IMPACT  
= 30 ft/s  
OVER A 6 in.  
DISTANCE

FIND:

AVERAGE IMPULSIVE FORCE EXERTED ON  
THE PLAYERS HAND

$$\begin{aligned} \text{Ball: } mv' &= 0 \quad \text{Player: } mv = 0 \quad \text{Player: } mv = 0 \\ \text{Ball: } mv' &= 0 \quad \text{Player: } mv = 0 \quad \text{Player: } mv = 0 \end{aligned}$$

$$t = \frac{d}{v_{av}} = \frac{(6/12)}{30} = (1/60) \text{ s}$$

$$+ \rightarrow 0 - F_{av} t + m v \quad F_{av} = \frac{W_B}{g t}$$

$$F_{av} = \frac{m v}{t} = \frac{(5/16 \text{ lb})(132 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(1/60 \text{ s})} = 76.9 \text{ lb}$$

13.155

GIVEN:

IDENTICAL COLLARS  
WITH VELOCITIES  
AS SHOWN.  
 $e = 0.65$ ,  $M = 1.2 \text{ kg}$   
NO FRICTION

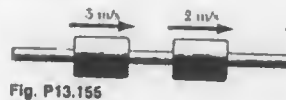


Fig. P13.155

FIND:

(a)  $v_A'$  AND  $v_B'$   
AFTER IMPACT  
(b) ENERGY LOST  
DURING IMPACT

(a) TOTAL MOMENTUM IS CONSERVED

$$v_A = 3 \text{ m/s} \quad v_B = 2 \text{ m/s} \quad \begin{matrix} v_A' & v_B' \\ \text{A} & \text{B} \end{matrix} = \begin{matrix} v_A' & v_B' \\ \text{A} & \text{B} \end{matrix} \quad m_A = m_B = m \quad M = 1.2 \text{ kg}$$

$$m v_A + m v_B = m v_A' + m v_B'$$

$$+ \rightarrow (3 \text{ m/s}) + (2 \text{ m/s}) = v_A' + v_B'$$

$$7 \text{ m/s} = v_A' + v_B' \quad (1)$$

RELATIVE VELOCITIES ALONG LINE OF IMPACT

$$v_B' - v_A' = e(v_A - v_B) \quad e = 0.65$$

$$v_B' - v_A' = (0.65)(3 \text{ m/s} - 2 \text{ m/s}) = 1.95 \text{ m/s} \quad (2)$$

ADDING (1) AND (2)

$$2 v_B' = 8.95 \quad v_B' = 4.48 \text{ m/s}$$

FROM (1) WITH  $v_B' = 4.48 \text{ m/s}$ 

$$v_A' = 7 \text{ m/s} - 4.48 \text{ m/s} = 2.53 \text{ m/s} \rightarrow$$

(b) ENERGY LOST DURING IMPACT

$$E_L = T_A + T_B - T_A' - T_B'$$

$$E_L = \frac{1}{2} (1.2 \text{ kg}) [5^2 + 2^2 - (4.475)^2 - (2.525)^2]$$

$$E_L = 1.559 \text{ N} \cdot \text{m}$$

13.156



GIVEN:

IDENTICAL COLLARS  
MOVE TOWARD EACH  
OTHER WITH  
VELOCITIES SHOWN  
 $e = 0$

SHOW THAT:

(a) AFTER IMPACT THE COMMON VELOCITY  
 $U' = (1/2)(v_A - v_B)$

(b) THE ENERGY LOSS IS  $\frac{1}{4}m(v_A + v_B)^2$

$$(a) \quad \begin{array}{c} v_A \quad v_B \quad v' \\ \boxed{A} \quad \boxed{B} = \boxed{A+B} \end{array} \quad \begin{array}{l} m_A = m_B = m \\ e = 0 \end{array}$$

CONSERVATION OF TOTAL MOMENTUM

$$\pm m v_A - m v_B = 2m U' \quad U' = \frac{1}{2}(v_A - v_B)$$

(b) ENERGY LOSS

$$E_L = T_A + T_B - (T_A' + T_B')$$

$$E_L = \frac{1}{2}m(v_A^2 + v_B^2) - \frac{1}{2}m(U'^2 + U'^2)$$

FROM (a)

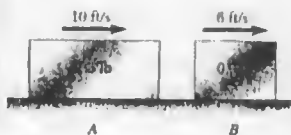
$$U' = \frac{1}{2}(v_A - v_B)$$

$$E_L = \frac{1}{2}m(v_A^2 + v_B^2) - \frac{1}{2}m\left[\frac{1}{2}(v_A - v_B)^2\right]$$

$$E_L = \frac{1}{2}m(v_A^2 + v_B^2) - \frac{1}{4}m(v_A^2 - 2v_A v_B + v_B^2)$$

$$E_L = \frac{1}{4}m[v_A^2 + 2v_A v_B + v_B^2] = \frac{1}{4}m(v_A + v_B)^2$$

13.157



GIVEN:

INITIAL VELOCITIES  
AS SHOWN  
 $w_A = 1.5 \text{ lb}$ ,  $w_B = 0.9 \text{ lb}$   
AFTER IMPACT  
 $U_B' = 10.5 \text{ ft/s}$   
NO FRICTION

FIND:

$e$ , THE COEFFICIENT  
OF RESTITUTION

THE TOTAL MOMENTUM IS CONSERVED

$$U_A = 10 \text{ ft/s} \quad U_B = 6 \text{ ft/s} \quad U_A' \quad U_B' = 10.5 \text{ ft/s}$$

$$\begin{array}{c} 1.5 \text{ lb} \quad 0.9 \text{ lb} = 1.5 \text{ lb} \quad 0.9 \text{ lb} \\ A \quad B \quad A' \quad B \end{array}$$

$$\pm m_A U_A + m_B U_B = m_A U_A' + m_B U_B'$$

$$\frac{(1.5 \text{ lb})(10 \text{ ft/s}) + (0.9 \text{ lb})(6 \text{ ft/s})}{g} = \frac{(1.5 \text{ lb})(U_A') + (0.9 \text{ lb})(10.5 \text{ ft/s})}{g}$$

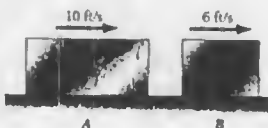
$$U_A' = \frac{15 + 5.4 - 9.45}{1.5} = 7.30 \text{ ft/s}$$

COEFFICIENT OF RESTITUTION

$$e = \frac{U_B' - U_A'}{U_A - U_B} = \frac{10.5 - 7.30}{10 - 6} = 0.800$$

$$e = 0.800$$

13.158



GIVEN:

INITIAL VELOCITIES  
AS SHOWN  
 $w_A = 1.5 \text{ lb}$ ,  $w_B = 0.9 \text{ lb}$   
 $e = 0.75$   
NO FRICTION

FIND:

(a) AFTER IMPACT

$U_A'$  AND  $U_B'$

(b) ENERGY LOSS DUE  
TO THE IMPACT

(a) THE TOTAL MOMENTUM IS CONSERVED

$$U_A = 10 \text{ ft/s} \quad U_B = 6 \text{ ft/s} \quad U_A' \quad U_B'$$

$$\begin{array}{c} 1.5 \text{ lb} \quad 0.9 \text{ lb} = 1.5 \text{ lb} \quad 0.9 \text{ lb} \\ A \quad B \quad A \quad B \end{array}$$

$$\pm m_A U_A + m_B U_B = m_A U_A' + m_B U_B'$$

$$\frac{(1.5 \text{ lb})(10 \text{ ft/s}) + (0.9 \text{ lb})(6 \text{ ft/s})}{g} = \frac{(1.5 \text{ lb})U_A' + (0.9 \text{ lb})U_B'}{g}$$

$$15 + 5.4 = 20.4 = 1.5 U_A' + 0.9 U_B' \quad (1)$$

RELATIVE VELOCITIES

$$(U_A - U_B)e = (U_B' - U_A')$$

$$(10 - 6)(0.75) = U_B' - U_A'$$

$$U_B' - U_A' = 3 \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$U_B' = 10.38 \text{ ft/s}$$

$$U_A' = 7.38 \text{ ft/s}$$

(b) ENERGY LOSS

$$E_L = \frac{1}{2}m_A U_A^2 + \frac{1}{2}m_B U_B^2 - \frac{1}{2}m_A U_A'^2 - \frac{1}{2}m_B U_B'^2$$

$$E_L = \frac{1}{2g} [(1.5)(10)^2 + (0.9)(6)^2 - (1.5)(7.375)^2 - (0.9)(10.375)^2]$$

$$E_L = \frac{1}{(2)(32.2 \text{ ft/s}^2)} (150 + 32.4 - 81.585 - 96.876) (1 \text{ lb} \cdot \text{ft/s}^2)$$

$$E_L = \frac{3.937}{(2)(32.2)} = 0.06113$$

$$E_L = 0.0611 \text{ ft} \cdot \text{lb}$$

13.159



GIVEN:

INITIALLY  $U_A = U_B = 0$ ,  $U_C = 1.5 \text{ m/s}$

ALL CARS HAVE THE SAME WEIGHT

$e_{BC} = 0.8$ ,  $e_{AB} = 0.5$

FIND:

$U_A'$ ,  $U_B'$ ,  $U_C'$  AFTER ALL COLLISIONS

$m_A = m_B = m_C = m$

COLLISION BETWEEN B AND C

THE TOTAL MOMENTUM IS CONSERVED

$$\begin{array}{c} U_B' \quad U_C' \quad U_B = 0 \quad U_C = 1.5 \text{ m/s} \\ \boxed{B} \quad \boxed{C} = \boxed{B} \quad \boxed{C} \end{array}$$

$$\pm m U_B' + m U_C' = m U_B + m U_C$$

$$U_B' + U_C' = 0 + 1.5 \quad (1)$$

(CONTINUED)

# 13.159 continued

## RELATIVE VELOCITIES

$$(v_B - v_C)(e_{BC}) = (v'_C - v'_B)$$

$$(-1.5)(0.8) = (v'_C - v'_B)$$

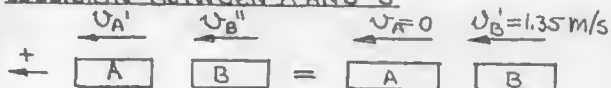
$$-1.2 = v'_C - v'_B \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v'_B = 1.35 \text{ m/s}$$

$$v'_C = 0.15 \text{ m/s}$$

SINCE  $v'_B > v'_C$ , CAR B COLLIDES WITH CAR A  
COLLISION BETWEEN A AND B



$$m v_A' + m v_B'' = m v_A + m v_B'$$

$$v_A' + v_B'' = 0 + 1.35 \quad (3)$$

## RELATIVE VELOCITIES

$$(v_A - v_B)e_{AB} = (v_B'' - v_A')$$

$$(0 - 1.35)(0.5) = v_B'' - v_A'$$

$$v_A' - v_B'' = 0.675 \quad (4)$$

SOLVING (3) AND (4) SIMULTANEOUSLY

$$2v_A' = 1.35 + 0.675$$

$$v_A' = 1.013 \text{ m/s}$$

$$v_B'' = 0.338 \text{ m/s}$$

SINCE  $v'_C < v_B'' < v_A'$  THERE ARE NO FURTHER COLLISIONS

# 13.160 continued

## RELATIVE VELOCITIES

$$(v_A - v_B)e = (v'_B - v'_A)$$

$$v_0 e = v'_B - v'_A \quad (2)$$

SOLVING EQUATIONS (1) AND (2) SIMULTANEOUSLY

$$v'_A = v_0(1-e)/2$$

$$v'_B = v_0(1+e)/2$$

(b) SECOND COLLISION (BETWEEN B AND C)

THE TOTAL MOMENTUM IS CONSERVED

$$m v_B' + m v_C = m v_B'' + m v_C'$$

$$v_B' + v_C = v_B'' + v_C'$$

USING THE RESULT FROM (a) FOR  $v_B'$

$$v_0(1+e)/2 + 0 = v_B'' + v_C' \quad (3)$$

## RELATIVE VELOCITIES

$$(v_B' - v_C)e = v_C' - v_B''$$

SUBSTITUTING AGAIN FOR  $v_B'$  FROM (a)

$$v_0(1+e)(e) = v_C' - v_B'' \quad (4)$$

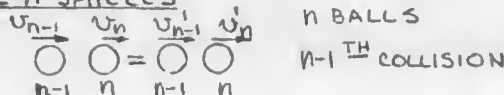
SOLVING EQUATIONS (3) AND (4) SIMULTANEOUSLY

$$v_C' = \frac{1}{2} [v_0(1+e)/2 + v_0(1+e)(e)/2]$$

$$v_C' = v_0(1+e)^2/4$$

$$v_B'' = v_0(1-e^2)/4$$

(c) FOR n SPHERES



WE NOTE FROM THE ANSWER TO PART (b), WITH  $n=3$

$$v'_n = v'_3 = v'_C = v_0(1+e)^2/4$$

$$\text{OR } v'_3 = v_0(1+e)^{(3-1)}/2^{(3-1)}$$

THUS FOR n BALLS

$$v'_n = v_0(1+e)^{(n-1)}/2^{(n-1)}$$

(d) FOR  $n=6$  AND  $e=0.95$

FROM THE ANSWER TO PART (c) WITH  $n=6$

$$v'_6 = v_0(1+0.95)^{(6-1)}/2^{(6-1)}$$

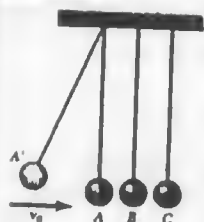
$$v'_6 = 0.881 v_0$$

$$v'_6 = 0.881 v_0$$

# 13.160

## GIVEN:

SPHERES A, B, C OF EQUAL WEIGHT  
INITIAL VELOCITY OF A IS  $v_0$  AND B AND C ARE AT REST.  $e$  IS THE SAME FOR ALL SPHERES



## FIND:

(a)  $v'_A$  AND  $v'_B$  AFTER THE FIRST COLLISION

(b)  $v''_B$  AND  $v'_C$  AFTER THE SECOND COLLISION

(c) FOR n SPHERES, THE VELOCITY  $v'_n$  AFTER IT IS HIT FOR THE FIRST TIME

(d) USING THE RESULT FROM PART (c) THE VELOCITY OF THE LAST SPHERE FOR  $n=6$  AND  $e=0.95$

(a) FIRST COLLISION (BETWEEN A AND B)

THE TOTAL MOMENTUM IS CONSERVED

$$m v_A + m v_B = m v'_A + m v'_B$$

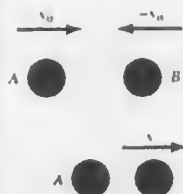
$$v_A + v_B = v'_A + v'_B$$

$$v_0 = v'_A + v'_B \quad (1)$$

13.161

GIVEN:

$m_A = 3 \text{ kg}$   
 INITIAL VELOCITIES OF  
 DISKS A AND B ARE  
 EQUAL AND OPPOSITE  
 OF MAGNITUDE  $U_0$   
 AFTER IMPACT  $U_A' = 0$   
 $e = 0.5$ . NO FRICTION



FIND:

- (a)  $m_B$   
 (b) RANGE OF VALUES FOR  
 $m_B$  IF  $e$  IS UNKNOWN

(a) TOTAL MOMENTUM CONSERVED

$$U_A = U_0 \quad U_B = -U_0 \quad U_A' = 0 \quad U_B' = U'$$

$$\begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \text{A} \\ \text{B} \end{array} = \begin{array}{c} \text{A} \\ \text{B} \end{array}$$

$$m_A U_A + m_B U_B = m_A U_A' + m_B U_B'$$

$$(3 \text{ kg})(U_0) + m_B(-U_0) = 0 + m_B U'$$

$$U' = 3U_0/m_B - U_0$$

$$U' = U_0(3/m_B - 1) \quad (1)$$

RELATIVE VELOCITIES

$$\begin{array}{c} \rightarrow \\ + \end{array} \quad (U_A - U_B)e = (U_B' - U_A')$$

$$2U_0 e = U' - 0$$

$$U' = 2U_0 e \quad (2)$$

SUBSTITUTE FOR  $U'$  IN EQUATION (1) FROM (2)

$$2U_0 e = U_0(3/m_B - 1) \quad (3)$$

$$e = 0.5 \quad (2)(5) = 3/m_B - 1$$

$$m_B = 3/2 \text{ kg}$$

(b) FROM EQ. (3)

$$2e + 1 = 3/m_B$$

$$m_B = \frac{3}{(2e+1)}$$

$$e = 0 \quad m_B = 3 \text{ kg}$$

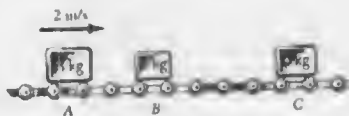
$$e = 1 \quad m_B = 1 \text{ kg}$$

$$1 \text{ kg} < m_B < 3 \text{ kg}$$

13.162

GIVEN:

INITIALLY B  
 AND C ARE  
 AT REST,  
 $U_A = 2 \text{ m/s}$   
 $e = 0.3$   
 MASSES AS  
 SHOWN



FIND:

- (a)  $U_C'$  AFTER A HITS B  
 AND B HITS C  
 (b)  $U_A''$  AFTER A HITS B  
 THE SECOND TIME

(a) PACKAGES A AND B

$$U_A = 2 \text{ m/s} \quad U_B = 0 \quad U_A' = \quad U_B' =$$

$$\begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \text{A} \\ \text{B} \end{array} = \begin{array}{c} \text{A} \\ \text{B} \end{array}$$

TOTAL MOMENTUM CONSERVED

$$m_A U_A + m_B U_B = m_A U_A' + m_B U_B'$$

$$(8 \text{ kg})(2 \text{ m/s}) + 0 = (8 \text{ kg})U_A' + (4 \text{ kg})U_B'$$

$$4 = 2U_A' + U_B' \quad (1)$$

RELATIVE VELOCITIES

$$(U_A - U_B)e = (U_B' - U_A')$$

$$(2)(0.3) = U_B' - U_A' \quad (2)$$

SOLVING EQUATIONS (1) AND (2) SIMULTANEOUSLY

$$U_A' = 1.133 \text{ m/s} \rightarrow$$

$$U_B' = 1.733 \text{ m/s} \rightarrow$$

PACKAGES B AND C

$$U_B' = 1.733 \text{ m/s} \quad U_C = 0 \quad U_B'' = \quad U_C' =$$

$$\begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \text{B} \\ \text{C} \end{array} = \begin{array}{c} \text{B} \\ \text{C} \end{array}$$

$$m_B U_B' + m_C U_C = m_B U_B'' + m_C U_C'$$

$$(4 \text{ kg})(1.733 \text{ m/s}) + 0 = 4 U_B'' + 6 U_C'$$

$$6.932 = 4U_B'' + 6U_C' \quad (3)$$

RELATIVE VELOCITIES

$$(U_B' - U_C)e = U_C' - U_B''$$

$$(1.733)(0.3) = 0.5199 = U_C' - U_B'' \quad (4)$$

SOLVING EQUATIONS (3) AND (4) SIMULTANEOUSLY

$$U_C' = 0.901 \text{ m/s} \rightarrow$$

$$U_B'' = 0.381 \text{ m/s} \rightarrow$$

(b) PACKAGES A AND B (SECOND TIME)

$$U_A' = 1.133 \text{ m/s} \quad U_B'' = 0.381 \text{ m/s} \quad U_A'' = \quad U_B''' =$$

$$\begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \text{A} \\ \text{B} \end{array} = \begin{array}{c} \text{A} \\ \text{B} \end{array}$$

TOTAL MOMENTUM CONSERVED

$$(8)(1.133) + (4)(0.381) = 8U_A'' + 4U_B'''$$

$$10.588 = 8U_A'' + 4U_B''' \quad (5)$$

RELATIVE VELOCITIES

$$(U_A' - U_B'')e = U_B''' - U_A''$$

$$(1.133 - 0.381)(0.3) = 0.2256 = U_B''' - U_A'' \quad (6)$$

SOLVING (5) AND (6) SIMULTANEOUSLY

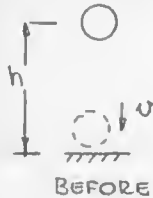
$$U_A'' = 0.807 \text{ m/s} \rightarrow$$

13.163

GIVEN:

BALL DROPPED FROM A HEIGHT OF 100-IN. ONTO A RIGID SURFACE MUST REBOUND TO A HEIGHT  $53 \text{ IN.} \leq h' \leq 58 \text{ IN}$

FIND:

RANGE OF ALLOWABLE VALUES OF  $e$ 

UNIFORM ACCELERATED MOTION

$$v = \sqrt{2gh}$$

$$v' = \sqrt{2gh'}$$

$$e = \frac{v'}{v}$$

$$e = \sqrt{\frac{h'}{h}}$$

COEFFICIENT OF RESTITUTION

HEIGHT OF DROP  $h = 100 \text{ IN}$ HEIGHT OF BOUNCE  $53 \text{ IN.} \leq h' \leq 58 \text{ IN}$ 

THUS

$$\sqrt{\frac{53}{100}} \leq e \leq \sqrt{\frac{58}{100}}$$

$$0.728 \leq e \leq 0.762$$

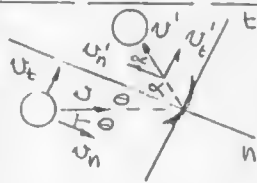
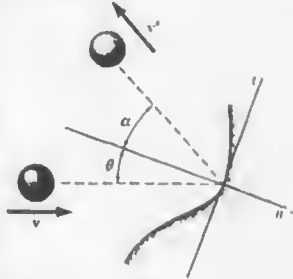
13.164

GIVEN:

BALLS HITS SURFACE AT AN ANGLE  $\theta$  AND REBOUNDS AT AN ANGLE  $\alpha$

SHOW:

$\alpha > \theta$  AND THAT % LOSS IN KINETIC ENERGY IS  $100(1-e^2)\cos^2\theta$



MOMENTUM IN  $t$  DIRECTION IS CONSERVED (NO FRICTION)

$$m v_t = m v'_t$$

$$v \sin \theta = v' \sin \alpha \quad (1)$$

COEFFICIENT OF RESTITUTION ( $n$ -DIRECTION)

$$v_n e = v'_n \quad v (\cos \theta) (e) = v' \cos \alpha \quad (2)$$

DIVIDE EQ (2) INTO EQ.(1)

$$\frac{\tan \alpha}{\tan \theta} = e$$

THUS

FOR  $0 < e < 1$   $\tan \alpha > \tan \theta$  AND  $\alpha > \theta$ 

% LOSS IN KINETIC ENERGY

SQUARING BOTH SIDES OF (1) AND (2) AND ADDING

$$v^2 (\sin^2 \theta + e^2 \cos^2 \theta) = (v')^2$$

$$\Delta T = \frac{1}{2} m [v^2 - (v')^2] = \frac{1}{2} m v^2 [1 - (\sin^2 \theta + e^2 \cos^2 \theta)]$$

$$\Delta T = \frac{1}{2} m v^2 \cos^2 \theta (1 - e^2)$$

$$\% \text{ LOSS} = 100 \frac{\Delta T}{\frac{1}{2} m v^2} = 100 (1 - e^2) \cos^2 \theta$$

13.165

GIVEN:

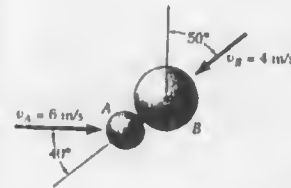
INITIAL VELOCITIES AS SHOWN

$$m_A = 600 \text{ g}$$

$$m_B = 1 \text{ kg}$$

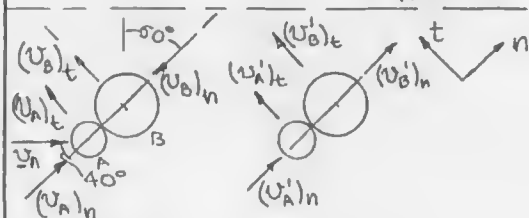
$$e = 0.8$$

NO FRICTION



FIND:

$v'_A$  AND  $v'_B$  AFTER IMPACT



BEFORE

AFTER

$$v_A = 6 \text{ m/s}$$

$$(v_A)_n = (6)(\cos 40^\circ) = 4.596 \text{ m/s}$$

$$(v_A)_t = -6(\sin 40^\circ) = -3.857 \text{ m/s}$$

$$v_B = (v_B)_n = -4 \text{ m/s}$$

$$(v_B)_t = 0$$

 $t$ -DIRECTIONTOTAL MOMENTUM CONSERVED

$$m_A (v_A)_t + m_B (v_B)_t = m_A (v'_A)_t + m_B (v'_B)_t$$

$$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)_t$$

$$-2.314 \text{ N} = 0.6 (v'_A)_t + (v'_B)_t \quad (1)$$

BALL A ALONE MOMENTUM CONSERVED

$$m_A (v_A)_t = m_A (v'_A)_t \quad -3.857 = (v'_A)_t$$

$$(v'_A)_t = -3.857 \text{ m/s} \quad (2)$$

REPLACE  $(v'_A)_t$  IN (2) IN EQUATION (1)

$$-2.314 = 0.6(-3.857) + (v'_B)_t$$

$$-2.314 = -2.314 + (v'_B)_t$$

$$(v'_B)_t = 0$$

 $n$ -DIRECTIONRELATIVE VELOCITIES

$$[(v_A)_n - (v_B)_n] e = (v'_B)_n - (v'_A)_n$$

$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$

$$6.877 = (v'_B)_n - (v'_A)_n \quad (3)$$

TOTAL MOMENTUM CONSERVED

$$m_A (v_A)_n + m_B (v_B)_n = m_A (v'_A)_n + m_B (v'_B)_n$$

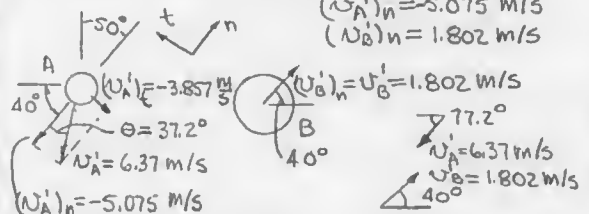
$$(0.6 \text{ kg})(4.596 \text{ m/s}) + (1 \text{ kg})(-4 \text{ m/s}) = (0.6 \text{ kg})(v'_A)_n + (1 \text{ kg})(v'_B)_n$$

$$-1.2424 = (v'_B)_n + 0.6 (v'_A)_n \quad (4)$$

SOLVING EQ. (4) AND (3) SIMULTANEOUSLY

$$(v'_A)_n = -5.075 \text{ m/s}$$

$$(v'_B)_n = 1.802 \text{ m/s}$$





13.168



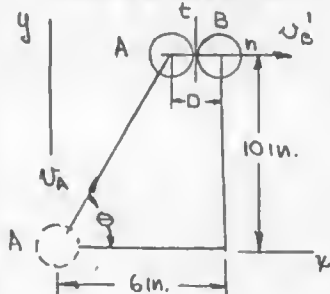
GIVEN:

DIAMETER OF  
BALLS,  $D = 2.37$   
 $u_A = 3$  ft/s  
AFTER  
IMPACT,  $u_B'$   
IN THE  $x$   
DIRECTION  
 $e = 0.9$

FIND:

- (a)  $\theta$   
(b)  $u_B'$

(a) SINCE  $u_B'$  IS IN THE  $x$ -DIRECTION AND (ASSUMING NO FRICTION) THE COMMON TANGENT BETWEEN A AND B AT IMPACT MUST BE PARALLEL TO THE  $y$ -AXIS



THUS  
 $\tan \theta = \frac{10}{6-D}$   
 $\theta = \tan^{-1} \frac{10}{6-2.37} = 70.04^\circ$   
 $\theta = 70.0^\circ$

(b) CONSERVATION OF MOMENTUM IN  $x$  (H) DIRECTION

$$m u_A \cos \theta + m (u_B)_n = m (u_A')_n + m u_B'$$

$$(3)(\cos 70.04) + 0 = (u_A')_n + u_B'$$

$$1.0241 = (u_A')_n + (u_B')$$

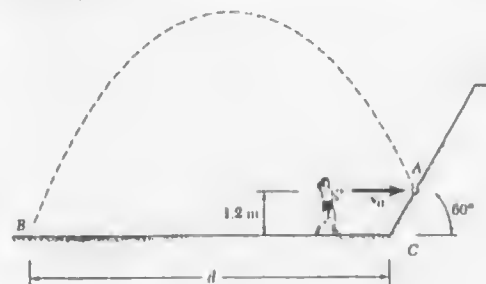
RELATIVE VELOCITIES IN THE  $n$  DIRECTION

$$e = 0.9 \quad (u_A \cos \theta - (u_B)_n) e = u_B' - (u_A')_n$$

$$(1.0241 - 0)(0.9) = u_B' - (u_A')_n$$

$$(1) + (2) \quad 2u_B' = 1.0241(1.9), \quad u_B' = 0.972 \text{ ft/s}$$

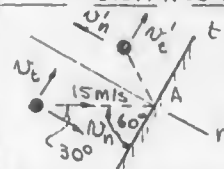
13.170



GIVEN:

INITIAL VELOCITY,  $u_0 = 15$  m/s  
 $e = 0.9$

FIND:

DISTANCE  $d$ 


MOMENTUM IN  $t$  DIRECTION  
IS CONSERVED

$$m u_0 \sin 30^\circ = m u_t'$$

$$(15)(\sin 30^\circ) = u_t'$$

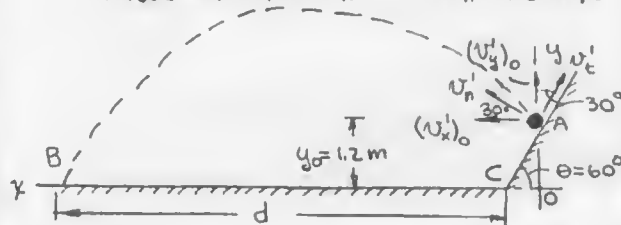
$$u_t' = 7.5 \text{ m/s}$$

COEFF OF RESTITUTION IN  $n$ -DIRECTION

$$(u_0 \cos 30^\circ) e = u_n'$$

$$(15)(\cos 30^\circ)(0.9) = u_n'$$

$$u_n' = 11.69 \text{ m/s}$$



WRITE  $u'$  IN TERMS OF  $x$  AND  $y$  COMPONENTS

$$(u_x')_0 = u_n' \cos 30^\circ - u_t' \sin 30^\circ$$

$$(u_x')_0 = (11.69)(\cos 30^\circ) - (7.5)(\sin 30^\circ) = 6.374 \text{ m/s}$$

$$(u_y')_0 = u_n' \sin 30^\circ + u_t' \cos 30^\circ$$

$$(u_y')_0 = (11.69)(\sin 30^\circ) + (7.5)(\cos 30^\circ) = 12.340 \text{ m/s}$$

MOTION OF A PROJECTILE (ORIGIN AT O)

$$y = y_0 + (u_y')_0 t - (g t^2)/2$$

$$y = 1.2 + (12.340 \text{ m/s}) t - (9.81 \text{ m/s}^2) t^2/2$$

TIME TO REACH POINT B ( $y_B = 0$ )

$$0 = 1.2 + 12.340 t_B - (4.81/2) t_B^2$$

$$t_B = 2.610 \text{ s}$$

$$x = x_0 + (u_x')_0 t$$

$$x = 0 + 6.374 t$$

$$x_B = (6.374)(t_B) = (6.374 \text{ m/s})(2.610 \text{ s})$$

$$x_B = 16.63 \text{ m}$$

$$d = x_B - 1.2 \cot 60^\circ = 15.94 \text{ m}$$

$$d = 15.94 \text{ m}$$

13.169



GIVEN:

BALL THROWN WITH  
VELOCITY  $u$  AS  
SHOWN. BALL  
REBOUNDS IN A  
DIRECTION  
PARALLEL TO  $OA$

FIND:

COEFFICIENT OF  
RESTITUTION  $e$   
BETWEEN THE BALL  
AND THE WALL

LAW OF SINES

$$\frac{\sin \theta}{R/2} = \frac{\sin 135^\circ}{R}$$

$$\theta = 20.705^\circ$$

$$\alpha = 45^\circ - 20.705^\circ = 24.295^\circ$$

CONS. OF MOM FOR WALL IN  $t$  DIRECTION,  $-u \sin \theta = -u' \sin \alpha$

COEFF. OF RESTITUTION IN  $n$ ;  $-u(\cos \theta) e = u' \cos \alpha$

$$\text{DIVIDING, } \frac{\tan \theta}{e} = \tan \alpha \quad e = \frac{\tan 20.705^\circ}{\tan 24.295^\circ} = 0.871$$



13.171

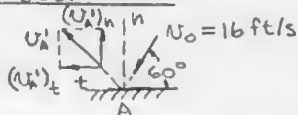
GIVEN:

INITIAL VELOCITY  
OF BALL AS  
SHOWN AT A  
 $e = 0.6$   
 $U_B$  IS  
HORIZONTAL

FIND:

- (a)  $h$  AND  $d$   
(b)  $U_B$

(a) REBOUND AT A

CONSERVATION OF MOMENTUM IN THE  $t$ -DIRECTION

$$m U_0 \cos 60^\circ = m (U'_A)_t$$

$$(U'_A)_t = (16 \text{ ft/s}) (\cos 60^\circ) = 8.00 \text{ ft/s}$$

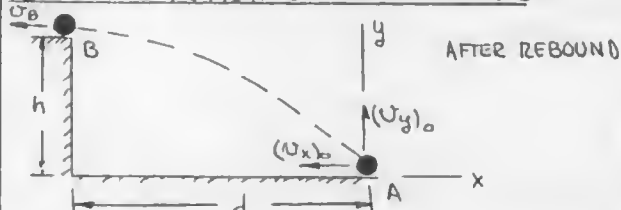
COEFF. OF RESTITUTION IN THE  $n$ -DIRECTION

$$-(U'_A)_n - 0 = 0 - (U'_A)_n$$

$$(16 \text{ ft/s}) (\sin 60^\circ) (0.6) = (U'_A)_n$$

$$(U'_A)_n = 8.314 \text{ ft/s}$$

PROJECTILE MOTION BETWEEN A AND B



$$(U_x)_0 = -(U'_A)_t = -8.00 \text{ ft/s}$$

$$(U_y)_0 = (U'_A)_n = 8.314 \text{ ft/s}$$

X-DIRECTION

$$x = (U_x)_0 t = -8t,$$

$$U_x = -8 \text{ ft/s}$$

y-DIRECTION

$$y = (U_y)_0 t - \frac{1}{2} g t^2 = 8.314 t - (16.1) t^2$$

$$U_y = (U_y)_0 - g t = 8.314 - 32.2 t$$

AT B,  $(U_B)_y = 0$ 

$$(U_B)_y = 0 = 8.314 - 32.2 t_{A-B}$$

$$t_{A-B} = 0.2582 \text{ s}$$

 $y_B = h$ 

$$h = (8.314) t_{A-B} - (16.1) t_{A-B}^2$$

$$h = (8.314) (0.2582) - (16.1) (0.2582)^2$$

$$h = 1.073 \text{ ft}$$

$$x_B = -d = -8 t_{A-B}$$

$$d = (8) (0.2582) = 2.065 \text{ ft}$$

$$d = 2.07 \text{ ft}$$

$$(b) U_B = (U_x)_0 = -8.00 \text{ ft/s}$$

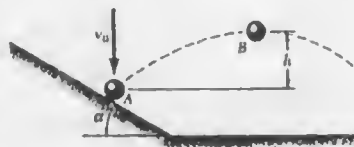
$$U_B = -8.00 \text{ ft/s}$$

13.172

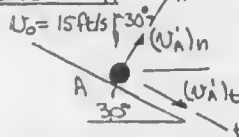
GIVEN:

$U_0 = 15 \text{ ft/s}$   
 $\alpha = 30^\circ$   
 $e = 0.8$

FIND:

 $h$ 

REBOUND AT A

CONSERVATION OF MOMENTUM IN THE  $t$ -DIRECTION

$$m U_0 \sin 30^\circ = m (U'_A)_t$$

$$(U'_A)_t = (15 \text{ ft/s}) (\sin 30^\circ) = 7.5 \text{ ft/s}$$

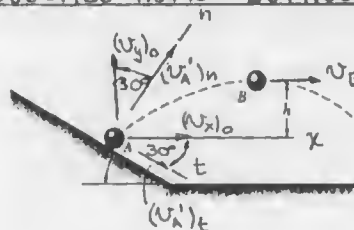
RELATIVE VELOCITIES IN THE  $n$ -DIRECTION

$$(-U_0 \cos 30^\circ - 0) e = 0 - (U'_A)_n$$

$$(U'_A)_n = (0.8) (15 \text{ ft/s}) (\cos 30^\circ)$$

$$(U'_A)_n = 10.392 \text{ ft/s}$$

PROJECTILE MOTION BETWEEN A AND B

AFTER  
REBOUND

$$(U_x)_0 = (U'_A)_t \cos 30^\circ + (U'_A)_n \sin 30^\circ$$

$$(U_x)_0 = (7.5) (\cos 30^\circ) + (10.392) \sin 30^\circ$$

$$(U_x)_0 = 11.691 \text{ ft/s}$$

$$(U_y)_0 = -(U'_A)_t \sin 30^\circ + (U'_A)_n \cos 30^\circ$$

$$(U_y)_0 = -(7.5) (\sin 30^\circ) + (10.392) \cos 30^\circ$$

$$(U_y)_0 = 5.2497 \text{ ft/s}$$

X-DIRECTION

$$x = (U_x)_0 t \quad U_x = (U_x)_0$$

$$x = 11.69 t \quad U_x = 11.69 \text{ ft/s} = U_B$$

y-DIRECTION

$$y = (U_y)_0 t - \frac{1}{2} g t^2$$

$$U_y = (U_y)_0 - g t$$

$$\text{AT A } U_y = 0 = (U_y)_0 - g t_{A-B}$$

$$t_{A-B} = U_y / g = \frac{5.2497 \text{ ft/s}}{32.2 \text{ ft/s}^2}$$

$$t_{A-B} = 0.1630 \text{ s}$$

AT B

$$y = h = (U_y)_0 t_{A-B} - \frac{g t_{A-B}^2}{2}$$

$$h = (5.2497) (0.1630) - \frac{(32.2) (0.1630)^2}{2} = 0.428 \text{ ft}$$

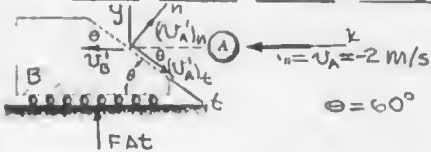
13.173



GIVEN:

$$m_A = 1.2 \text{ kg} \\ m_B = 4.8 \text{ kg} \\ v_0 = 2 \text{ m/s} \\ e = 1, \theta = 60^\circ$$

FIND:

 $v_B'$  AFTER IMPACT

A ALONE MOMENTUM CONSERVED IN t-DIRECTION

$$m_A v_A \cos 60^\circ = m_A (v_A')_t \\ (v_A')_t = -(2 \text{ m/s})(.5) = -1 \text{ m/s}$$

A AND B TOTAL MOMENTUM CONSERVED ALONG THE X-AXIS

$$m_A v_A + m_B v_B = m_A [(v_A')_t \cos \theta + (v_A')_n \sin \theta] + m_B v_B' \\ (1.2 \text{ kg})(-2 \text{ m/s}) + 0 = (1.2 \text{ kg})[(-1 \text{ m/s})(\cos 60^\circ) + (v_A')_n (\sin 60^\circ)] - 4.8 \text{ kg}(v_B')$$

$$-1.8 = 1.0392(v_A')_n - 4.8 v_B' \quad (1)$$

RELATIVE VELOCITIES IN THE n DIRECTION

$$[v_A \sin \theta - (v_B)_n] e = (v_B')_n - (v_A')_n \\ (v_B)_n = 0 \\ e = 1 \\ v_A = 2 \text{ m/s} \\ \theta = 60^\circ \\ (-2)(\sin 60^\circ)(1) = -v_B' \sin 60^\circ - (v_A')_n \\ -1.732 = -0.866 v_B' - (v_A')_n \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$(v_A')_n = 1.184 \text{ m/s} \quad v_B' = 0.632 \text{ m/s}$$

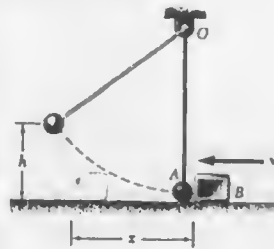
13.174

GIVEN:

$$m_B = 1 \text{ kg} \\ m_A = 0.5 \text{ kg} \\ v_0 = 2 \text{ m/s} \\ \mu_k = 0.6 \\ e = 0.8$$

FIND:

- (a) MAX. HEIGHT  $h$   
(b) DISTANCE  $x$  TRAVELED BY THE BLOCK



$$v_A = 0 \quad v_B = 2 \text{ m/s} \quad v_A' = v_B' \\ \text{BEFORE} \quad \text{AFTER}$$

VELOCITIES JUST AFTER IMPACT

TOTAL MOMENTUM IN THE HORIZONTAL DIRECTION IS CONSERVED

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \\ 0 + (1 \text{ kg})(2 \text{ m/s}) = (0.5 \text{ kg})(v_A') + (1 \text{ kg})(v_B')$$

$$4 = v_A' + 2 v_B' \quad (1)$$

RELATIVE VELOCITIES

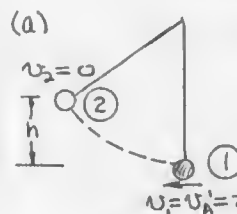
$$(v_A - v_B) e = (v_B' - v_A')$$

$$(0 - 2)(0.8) = v_B' - v_A'$$

$$-1.6 = v_B' - v_A' \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v_B' = 0.8 \text{ m/s} \quad v_A' = 2.4 \text{ m/s}$$



CONSERVATION OF ENERGY

$$T_1 = \frac{1}{2} m_A v_1^2 \quad v_1 = 0$$

$$T_1 = \frac{1}{2} m_A (2.4 \text{ m/s})^2 = 2.88 m_A$$

$$T_2 = 0$$

$$V_2 = m_A g h$$

$$T_1 + V_1 = T_2 + V_2 \quad 2.88 m_A + 0 = 0 + m_A (9.81)$$

$$h = 0.294 \text{ m}$$

(b) WORK AND ENERGY

$$v_2 = 0 \quad v_B = v_B' = 0.8 \text{ m/s} \\ v_1 = v_B' = 0.8 \text{ m/s} \\ N = W_B = m_B g \\ F_f = \mu_k N$$

$$T_1 = \frac{1}{2} m_B v_1^2 = \frac{1}{2} m_B (0.8 \text{ m/s})^2 = 0.32 m_B \quad T_2 = 0$$

$$U_{1-2} = -F_f x = -\mu_k N x = -\mu_k m_B g x = -(0.6)(m_B)(9.81)x$$

$$U_{1-2} = -5.886 m_B x$$

$$T_1 + U_{1-2} = T_2 \quad 0.32 m_B - 5.886 m_B x = 0 \\ x = 0.0544 \text{ m}$$

13.175

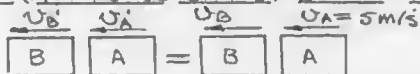
GIVEN:

$m_A = m_B = 1.5 \text{ kg}$   
INITIALLY,  
 $U_A = 5 \text{ m/s}, U_B = 0$   
NO FRICTION  
(1)  $e = 1$   
(2)  $e = 0$



FIND:

- (a) MAXIMUM DEFLECTION OF THE SPRING  
(b) FINAL VELOCITY OF BLOCK A



PHASE I IMPACT

(CONSERVATION OF TOTAL MOMENTUM)

$$+ \quad m_A U_A + m_B U_B = m_A U_A' + m_B U_B'$$

$$m_A = m_B$$

$$5 + 0 = U_A' + U_B' \quad (1)$$

RELATIVE VELOCITIES

$$(U_A - U_B)e = (U_B' - U_A')$$

$$(5 - 0)e = U_B' - U_A' \quad (2)$$

ADDING (1) AND (2)

$$\frac{5(1+e)}{2} = U_B'$$

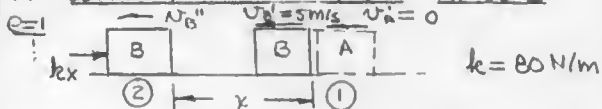
SUBTRACTING (2) FROM (1)

$$\frac{5(1-e)}{2} = U_A'$$

$$e = 1 \quad U_B' = 5 \text{ m/s} \quad U_A' = 0$$

$$e = 0 \quad U_B' = 2.5 \text{ m/s} \quad U_A' = 2.5 \text{ m/s}$$

(a) CONSERVATION OF ENERGY PHASE II



$$T_1 = \frac{1}{2} m_B (U_B)^2 = \frac{1}{2} (1.5 \text{ kg}) (5 \text{ m/s})^2 = 18.75 \text{ J}$$

$$V_1 = 0$$

$$\text{AT } X = X_{\text{MAX}}, T_2 = 0; V_2 = \frac{1}{2} k (X_{\text{MAX}})^2 = (40) (X_{\text{MAX}})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$18.75 + 0 = 0 + 40 X_{\text{MAX}}^2$$

$$e = 1 \quad X_{\text{MAX}} = 0.685 \text{ m}$$

$e = 2$  BOTH A AND B HAVE THE SAME VELOCITY  
INITIALLY AT ① OF 2.5 m/s

$$\text{THUS } T_1 = \frac{1}{2} (m_A + m_B) (U_A)^2 = \frac{1}{2} (3 \text{ kg}) (2.5 \text{ m/s})^2$$

$$T_1 = 9.375 \text{ J} \quad V_1 = 0$$

$$\text{AT } X = X_{\text{MAX}} \quad T_2 = 0 \quad V_2 = \frac{1}{2} k X_{\text{MAX}}^2 = 40 X_{\text{MAX}}^2$$

$$T_1 + V_1 = T_2 + V_2 \quad 9.375 + 0 = 40 X_{\text{MAX}}^2$$

$$e = 0 \quad X_{\text{MAX}} = 0.484 \text{ m}$$

13.175 continued

(b)  $e = 1$ , BLOCK B IS RETURNED TO POSITION ① WITH A VELOCITY OF 5 m/s  $\rightarrow$  SINCE ENERGY IS CONSERVED, AND IMPACTS BLOCK A WHICH IS AT REST, IN THE IMPACT TOTAL MOMENTUM IS CONSERVED AND PHASE I IS REPEATED WITH THE VELOCITIES OF A AND B INTERCHANGED. THUS  $U_A'' = 5 \text{ m/s} \rightarrow$  AND  $U_B'' = 0$ . SINCE THERE IS NO FRICTION THESE VELOCITIES ARE THE FINAL VELOCITIES OF A AND B

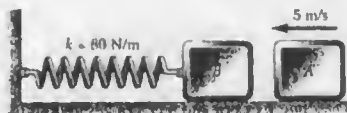
$$e = 1 \quad U_A'' = 5 \text{ m/s} \rightarrow$$

$e = 0$ , BLOCKS A AND B ARE RETURNED TO POSITION ① WITH THE SAME VELOCITY OF 2.5 m/s  $\rightarrow$  SINCE ENERGY IS CONSERVED. THERE IS NO ADDITIONAL IMPACT AND THE SPRING SLOWS BLOCK B DOWN AND A AND B SEPARATE WITH A CONTINUING WITH A VELOCITY OF 2.5 m/s TO THE RIGHT

$$e = 0 \quad U_A'' = 2.5 \text{ m/s} \rightarrow$$

13.176

GIVEN:



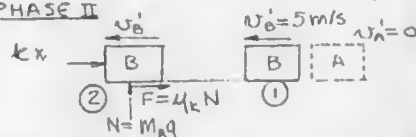
$m_A = m_B = 1.5 \text{ kg}$   
INITIALLY  
 $U_A = 5 \text{ m/s}, U_B = 0$   
 $\mu_k = 0.3, \mu_s = 0.5$   
 $e = 1$

FIND:

FINAL POSITION OF (a) BLOCK A (b) BLOCK B

IMPACT SEE PHASE I OF PROB 13.175,  $e = 1$   
AFTER IMPACT  $U_A' = 0, U_B' = 5 \text{ m/s}$

PHASE II

MAXIMUM DEFLECTION  $X_{\text{MAX}}$  OF THE SPRING

WORK AND ENERGY

$$T_1 = \frac{1}{2} m_B (U_B)^2 = \frac{1}{2} (1.5 \text{ kg}) (5 \text{ m/s})^2 = 18.75 \text{ J}$$

$$T_2 = 0 \quad U_{1-2} = \int -kx dx - \int \mu_k m_B g dx$$

$$U_{1-2} = -\frac{1}{2} (80) (X_{\text{MAX}})^2 - (0.3) (1.5 \text{ kg}) (9.81 \text{ m/s}^2) X_{\text{MAX}}$$

$$T_1 + U_{1-2} = T_2$$

$$18.75 - 40 X_{\text{MAX}}^2 - 4.4145 X_{\text{MAX}} = 0$$

$$X_{\text{MAX}} = 0.632 \text{ m}$$

PHASE III RETURN OF B TO POSITION ① BEFORE IMPACT WITH A

$$T_2 = 0, U_{2-1} = \frac{1}{2} k (X_{\text{MAX}})^2 - \mu_k m_B g X_{\text{MAX}}$$

$$T_1 = \frac{1}{2} m_B (U_B')^2 = (0.75) (U_B')^2 \quad U_{2-1} = (40) (0.632)^2 - (4.4145) (0.632)$$

$$0 + 13.173 = (0.75) (U_B')^2 \quad U_{2-1} = 15.977 - 2.790 = 13.173$$

$$U_B' = 4.191 \text{ m/s} \rightarrow$$

AFTER IMPACT WITH A AT POSITION ①

RELATIVE VELOCITIES

$$+ \quad (U_A'' - U_B'')(e) = (U_A' - U_B') \\ (4.191 - 0)(1) = U_A'' - U_B'' \quad (1)$$

(CONTINUED)

### 13.176 continued

#### CONSERVATION OF TOTAL MOMENTUM AT ①

$$\begin{aligned} \rightarrow m_A u_A' + m_B u_B' &= m_A u_A'' + m_B u_B'' \quad m_A = m_B \\ 0 + 4.191 &= u_A'' + u_B'' \quad (2) \end{aligned}$$

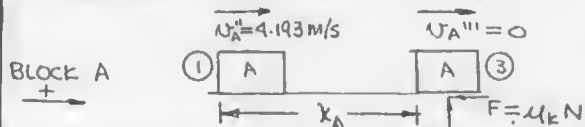
ADDING EQUATIONS (1) AND (2)

$$2(4.191) = 2u_A''$$

$$u_A'' = 4.191 \text{ m/s}$$

$$\text{FROM EQ (2)} \quad u_B'' = 4.191 - 4.191 = 0$$

(a) PHASE IV (VELOCITY OF B = 0 AT ①)



$$\begin{aligned} T_1 &= \frac{1}{2} m_A (u_A'')^2 = (0.75 \text{ kg})(4.191 \text{ m/s})^2 \quad N = m_A g \\ T_1 &= 13.173 \text{ J} \quad U_{1-3} = -\mu_k m_B g x_A = -(0.3)(1.5 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) x_A \end{aligned}$$

$$U_{1-3} = -4.415 x_A$$

$$T_3 = 0$$

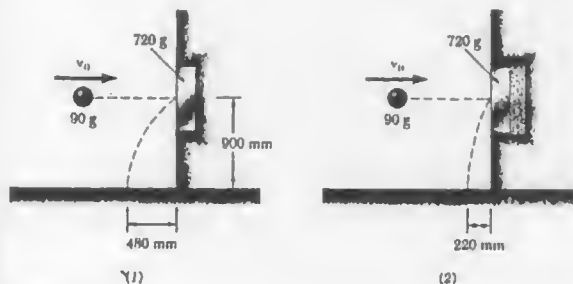
$$T_1 + U_{1-3} = T_3 \quad 13.173 - 4.415 x_A = 0$$

$$\text{FINAL POSITION OF A} \quad x_A = 2.98 \text{ m}$$

(b)  $u_B''' = 0$  AT IMPACT POINT AND THE SPRING IS UNDEFLECTED AT THIS POINT.

$$x_B = 0$$

### 13.177



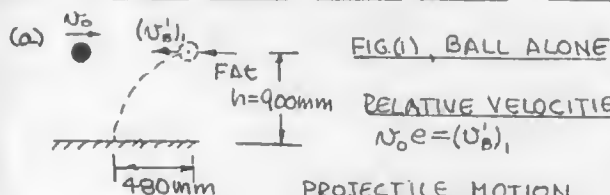
GIVEN:

BALL REBOUNDS AS SHOWN IN FIGURES (1) AND (2). FOAM RUBBER BEHIND PLATE IN (2)

FIND:

(a) COEFFICIENT OF RESTITUTION  $e$  BETWEEN THE BALL AND THE PLATE

(b) THE INITIAL VELOCITY  $u_0$



RELATIVE VELOCITIES

$$u_0 e = (u_B')_1$$

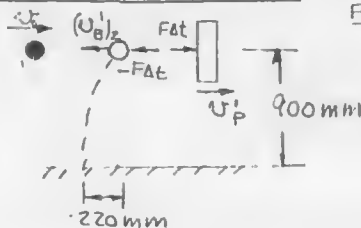
PROJECTILE MOTION

$t$  = TIME FOR THE BALL TO HIT THE GROUND

$$0.480 \text{ m} = u_0 e t \quad (1)$$

### 13.177 continued

FIG (2) BALL AND PLATE



RELATIVE VELOCITIES

$$\rightarrow (u_B - u_P) e = u_P' + (u_B')_2$$

$$u_B = u_0 \quad u_P = 0$$

$$u_0 e = u_P' + (u_B')_2 \quad (2)$$

CONSERVATION OF MOMENTUM

$$\rightarrow m_B u_B + m_P u_P = m_B (-u_B')_2 + m_P (u_P')$$

$$(0.09 \text{ kg})(u_0) + 0 = (0.09 \text{ kg})(-u_B')_2 + (0.720 \text{ kg})u_P'$$

$$u_0 = (-u_B')_2 + 8 u_P' \quad (3)$$

SOLVING (2) AND (3) SIMULTANEOUSLY FOR  $(u_B')_2$

$$(u_B')_2 = u_0 \frac{(8e-1)}{9}$$

PROJECTILE MOTION

$$0.220 \text{ m} = u_0 \frac{(8e-1)}{9} t \quad (4)$$

DIVIDING EQ (4) BY EQ. (3)

$$\frac{0.220}{0.480} = \frac{8e-1}{9e}$$

$$4.125e = 8e-1$$

$$e = 0.258$$

(b) FROM FIG (1)

PROJECTILE MOTION

$$h = \frac{1}{2} g t^2$$

$$0.900 = \frac{(9.81)}{2} t^2, \quad 1.80 = 9.81 t^2 \quad (5)$$

EQUATION (1)

$$0.480 = u_0 e t$$

$$t = \frac{(0.480)}{(0.258) u_0} = \frac{1.860}{u_0}$$

SUBSTITUTING FOR  $t$  IN (5)

$$1.800 = (9.81) \left( \frac{1.860}{u_0} \right)^2$$

$$u_0^2 = 18.855$$

$$u_0 = 4.34 \text{ m/s}$$

13.178

GIVEN:

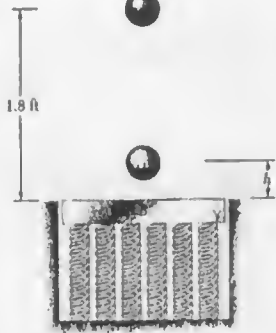
$$W_A = 1.3 \text{ lb}$$

$$W_B = 2.6 \text{ lb}$$

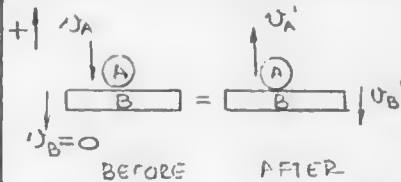
$$e = 0.8$$

FIND:

- (a) REBOUND HEIGHT  $h$  OF THE SPHERE A  
(b) EQUIVALENT SPRING CONSTANT  $k$  IF THE MAXIMUM DEFLECTION OF THE PLATE IS  $3h$



(a) VELOCITY OF A AND B AFTER IMPACT



INITIAL VELOCITY OF A (BEFORE IMPACT)

$$v_A = 0 \text{ (A)} \quad T_1 = 0 \quad V_1 = (W_A/g)(1.8 \text{ ft})$$

$$T_2 = \frac{1}{2} W_A v_A^2 \quad V_2 = 0$$

1.8 ft

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1.8 W_A g = \frac{1}{2} W_A v_A^2$$

$$v_A^2 = (2)(1.8)(32.2 \text{ ft/s}^2) = 115.92 \text{ ft}^2/\text{s}^2$$

$$v_A = 10.77 \text{ ft/s} \downarrow$$

VELOCITIES AFTER IMPACT

CONSERVATION OF MOMENTUM

$$W_A(-v_A) + W_B(v_B) = W_A v_A' + W_B(-v_B')$$

$$\frac{1.3}{g}(-10.77) + 0 = \frac{1.3}{g} v_A' + \frac{2.6}{g}(-v_B')$$

$$-10.77 = v_A' - 2v_B' \quad (1)$$

RELATIVE VELOCITIES

$$-v_A - v_B = e(-v_B' - v_A')$$

$$(10.77)(0.8) = v_B' + v_A' \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v_A' = 2.15 \text{ ft/s} \quad v_B' = 6.46 \text{ ft/s}$$

REBOUND HEIGHT OF A (CONSERVATION OF ENERGY)

$$v_A = 0 \text{ (A)} \quad T_2 = \frac{1}{2} W_A v_A^2 \quad V_2 = 0$$

$$T_3 = 0 \quad V_3 = m_B g h$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2} W_A v_A^2 + 0 = 0 + m_B g h$$

$$h = \frac{1}{2} (2.15 \text{ ft/s})^2 / (32.2 \text{ ft/s}^2) = 0.0720 \text{ ft}$$

$$(b) \downarrow \text{B} \quad T_2 = \frac{1}{2} W_B v_B^2 \quad V_2 = 0$$

$$v_B = 6.46 \text{ ft/s} \quad 3h = 0.216 \text{ ft} \quad T_4 = 0 \quad V_4 =$$

$$V_4 = \frac{1}{2} k (3h)^2$$

$$T_2 + V_2 = T_4 + V_4$$

$$\frac{1}{2} (2.6 \text{ lb}) (6.46 \text{ ft/s})^2 = \frac{1}{2} k (0.216)^2$$

$$k = 72.2 \text{ lb/ft}$$

13.179

GIVEN:

FIGURE AS SHOW IN 13.178 (LEFT)

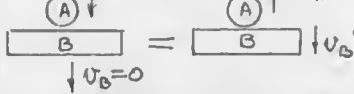
$$W_A = 1.3 \text{ lb}, W_B = 2.6 \text{ lb}$$

$$\text{EQUIV SINGLE SPRING } k = 5 \frac{\text{lb}}{\text{in}}$$

FIND:

- (a) VALUE OF  $e$  FOR WHICH  $h$  IS A MAXIMUM  
(b) CORRESPONDING VALUE OF  $h$   
(c) CORRESPONDING MAXIMUM DEFLECTION OF B  
(d) INITIAL VELOCITY OF A (BEFORE IMPACT)

FROM SOLUTION TO PROB 13.178,  $v_A = 10.77 \text{ ft/s} \downarrow$   
 $v_A = 10.77 \text{ ft/s}$



CONSERVATION OF MOMENTUM

$$W_A(-v_A) + W_B v_B = W_A v_A' + W_B(-v_B')$$

$$\frac{1.3}{g}(-10.77) + 0 = \frac{1.3}{g} v_A' + \frac{2.6}{g}(-v_B')$$

$$-10.77 = v_A' - 2v_B' \quad (1)$$

RELATIVE VELOCITIES

$$-v_A - v_B = e(-v_B' - v_A')$$

$$10.77 e = v_B' + v_A' \quad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY FOR  $v_A'$ 

$$3v_A' = (10.77)(2e - 1)$$

$h$  IS MAXIMUM WHEN  $v_A'$  IS MAXIMUM, THAT IS WHEN  $e = 1$

 $e = 1$ (b) FOR  $e = 1$ 

$$v_A' = 10.77/3 = 3.59 \text{ ft/s}$$

FOR A ALONE

CONSERVATION OF ENERGY

$$v_A = 0 \text{ (A)} \quad T_1 = \frac{1}{2} W_A (v_A')^2 \quad V_1 = 0$$

$$T_2 = 0 \quad V_2 = W_A h$$

$$v_A = 3.59 \text{ ft/s} \quad T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} W_A (v_A')^2 + 0 = 0 + W_A h$$

$$h = \frac{1}{2} \frac{(3.59 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)} = 0.200 \text{ ft}$$

(c) FOR B ALONE

CONSERVATION OF ENERGY

$$v_B = 0 \text{ (B)} \quad T_1 = \frac{1}{2} W_B (v_B')^2$$

$$T_2 = 0 \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} W_B (v_B')^2 = 0$$

$$v_B' = 0$$

$$T_1 = \frac{1}{2} W_B (v_B')^2 = \frac{1}{2} (2.6 \text{ lb}) (7.18 \text{ ft/s})^2$$

$$T_1 = 2.08 \text{ ft} \cdot \text{lb} \quad V_1 = 0$$

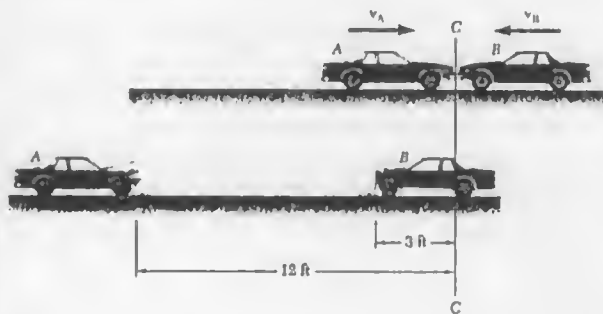
$$T_3 = 0 \quad V_3 = \frac{1}{2} k x^2 \quad k = 5 \text{ lb/in} = 60 \text{ lb/ft}$$

$$V_3 = \frac{1}{2} (60)(x)^2 = 30x^2$$

$$T_1 + V_1 = T_3 + V_3$$

$$2.08 + 0 = 0 + 30x^2 \quad x = 0.263 \text{ ft}$$

13.180

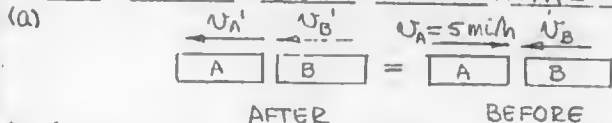


GIVEN:

CARS A AND B OF THE SAME MASS  
BEFORE COLLISION  $v_A = 5 \text{ mi/h}$   
BRAKES LOCKED,  $\mu_k = 0.30$   
CARS AT REST IN POSITION SHOWN

FIND:

- (a) THE SPEED OF B,  $v_B$ , BEFORE IMPACT  
(b) EFFECTIVE COEFF. OF RESTITUTION,  $e$



AT C

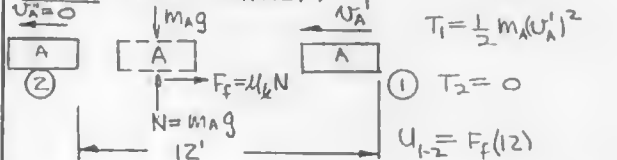
CONSERVATION OF TOTAL MOMENTUM

$$+ \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$-7.33 + v_B = v_A' + v_B' \quad (1)$$

WORK AND ENERGY

CAR A (AFTER IMPACT)



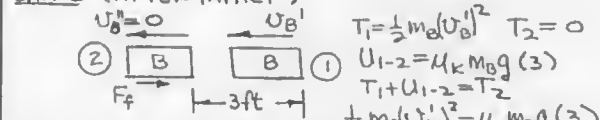
$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m_A (v_A')^2 - \mu_k m_A g (12) = 0$$

$$(v_A')^2 = (2)(12 \text{ ft})(0.3)(32.2 \text{ ft/s}^2) = 231.84 \text{ ft/s}^2$$

$$v_A' = 15.226 \text{ ft/s}$$

CAR B (AFTER IMPACT)



$$v_B' = (2)(3 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$(v_B')^2 = 57.96 \text{ ft/s}^2 \quad v_B' = 7.613 \text{ ft/s}$$

$$\text{FROM (1)} \quad v_B = 7.333 + v_A' + v_B' = 7.333 + 15.226 + 7.613$$

$$v_B = 30.2 \text{ ft/s} = 20.6 \text{ mi/h}$$

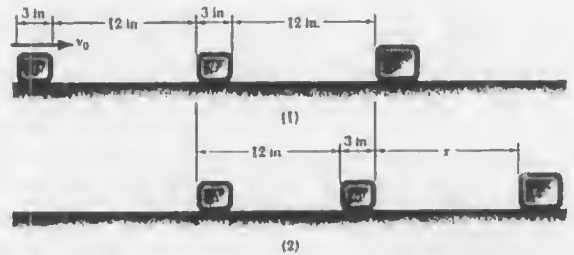
(b) RELATIVE VELOCITIES

$$(-v_A - v_B)e = v_B' - v_A'$$

$$(-7.333 - 30.2)e = 7.613 - 15.226$$

$$e = \frac{-(-7.613)}{-(-37.53)} = 0.2028 \quad e = 0.203$$

13.181



GIVEN:

$w_A = w_B = 0.8 \text{ lb}$ ,  $w_C = 2.4 \text{ lb}$ ,  $\mu_k = 0.30$   
INITIALLY  $v_A = v_0 = 15 \text{ ft/s}$ ,  $v_B = v_C = 0$   
AFTER A STRIKES B AND B STRIKES C ALL  
BLOCKS COME TO REST AS SHOWN IN (2)

FIND:

- (a) COEFF. OF RESTITUTION BETWEEN A AND B  
AND BETWEEN B AND C.  
(b) THE DISPLACEMENT  $x$  OF BLOCK C

(a) WORK AND ENERGY

VELOCITY OF A JUST BEFORE IMPACT WITH B

$$v_0 = 15 \text{ ft/s} \quad v_A = 15 \text{ ft/s} \quad T_1 = \frac{1}{2} w_A v_0^2 \quad T_2 = \frac{1}{2} w_A (v_A')^2$$

$$T_1 = \frac{1}{2} w_A v_0^2 \quad T_2 = \frac{1}{2} w_A (v_A')^2$$

$$U_{1-2} = -\mu_k w_A (1 \text{ ft})$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} w_A v_0^2 - \mu_k w_A (1 \text{ ft}) = \frac{1}{2} w_A (v_A')^2$$

$$(v_A')^2 = v_0^2 - 2\mu_k g = (15 \text{ ft/s})^2 - 2(0.3)(32.2 \text{ ft/s}^2)(1 \text{ ft})$$

$$(v_A')^2 = 205.68 \text{ ft/s}^2, (v_A') = 14.342 \text{ ft/s}$$

VELOCITY OF A AFTER IMPACT WITH B  $(v_A')_2$ 

$$(v_A')_2 = v_A = 0 \quad T_2 = \frac{1}{2} w_A (v_A')^2 \quad T_3 = 0$$

$$U_{2-3} = -\mu_k w_A (3/12)$$

$$T_2 + U_{2-3} = T_3, \frac{1}{2} w_A (v_A')^2 - (\mu_k)(w_A)(1 \text{ ft}) = 0$$

$$(v_A')^2 = 2(0.3)(32.2 \text{ ft/s}^2)(\frac{1}{4} \text{ ft}) = 4.83 \text{ ft/s}^2$$

$$(v_A')_2 = 2.198 \text{ ft/s}$$

CONSERVATION OF MOMENTUM AS A HITS B

$$(v_A)_2 = v_0 = 0 \quad (v_A)_2 = v_B' \quad (v_A)_2 = 14.342 \text{ ft/s}$$

$$(v_A)_2 = 2.198 \text{ ft/s}$$

$$m_A (v_A)_2 + m_B v_B = m_A (v_A')_2 + m_B v_B'$$

$$14.342 + 0 = 2.198 + v_B' \quad v_B' = 12.144 \text{ ft/s}$$

RELATIVE VELOCITIES (A AND B)

$$+ \quad ((v_A)_2 - v_B) e_{AB} = v_B' - (v_A')_2$$

$$(14.342 - 0) e_{AB} = 12.144 - 2.198$$

$$e_{AB} = 0.694$$

WORK AND ENERGY

VELOCITY OF B JUST BEFORE IMPACT WITH C

$$(v_B)_2 = v_0 = 0 \quad (v_B)_2 = v_C' \quad T_2 = \frac{1}{2} w_B (v_B')^2 = \frac{w_B}{2g} (12.144)^2$$

$$T_4 = \frac{1}{2} w_B (v_B')^2 = \frac{w_B}{2g} (v_B')^2$$

$$U_{2-4} = -\mu_k w_B (1 \text{ ft}) = (\mu_k) w_B$$

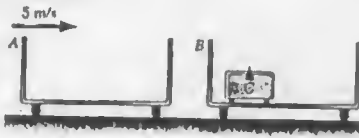
$$T_2 + U_{2-4} = T_4, \frac{(12.144)^2}{2g} - 0.3 = \frac{(v_B')^2}{2g}$$

$$(v_B')_4 = 11.321 \text{ ft/s}$$

(CONTINUED)



13.183



GIVEN:

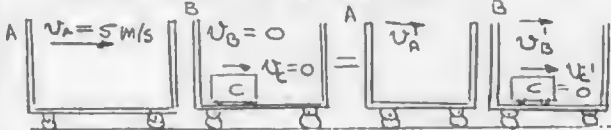
MASS OF CARRIERS  
 $m_A = m_B = 40 \text{ kg}$   
 MASS OF LUGGAGE  $m_C = 15 \text{ kg}$   
 $e_{BC} = 0.30$   
 $e_{AB} = 0.80$

FIND:

- (a) VELOCITY OF CARRIER B AFTER C HITS THE WALL OF B THE FIRST TIME  
 (b) THE TOTAL ENERGY LOST IN THE IMPACT BETWEEN B AND C.

(a) IMPACT BETWEEN A AND B

TOTAL MOMENTUM CONSERVED



$$m_A u_A + m_B u_B = m_A u_A' + m_B u_B' \quad m_A = m_B = 40 \text{ kg}$$

$$5 \text{ m/s} + 0 = u_A' + u_B' \quad (1)$$

RELATIVE VELOCITIES

$$(u_A - u_B) e_{AB} = u_B' - u_A'$$

$$(5 - 0)(0.80) = u_B' - u_A' \quad (2)$$

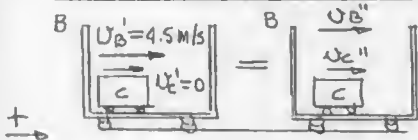
ADDING (1) AND (2)

$$(5 \text{ m/s})(1 + 0.80) = 2 u_B'$$

$$u_B' = 4.5 \text{ m/s} \rightarrow$$

IMPACT BETWEEN B AND C (AFTER A HITS B)

TOTAL MOMENTUM CONSERVED



$$m_B u_B' + m_C u_C' = m_B u_B'' + m_C u_C''$$

$$(40 \text{ kg})(4.5 \text{ m/s}) + 0 = (40 \text{ kg}) u_B'' + (15 \text{ kg}) u_C''$$

$$4.5 = u_B'' + 0.375 u_C'' \quad (3)$$

RELATIVE VELOCITIES

$$(u_B' - u_C') e_{BC} = u_C'' - u_B''$$

$$(4.5 - 0)(0.30) = u_C'' - u_B'' \quad (4)$$

ADDING (3) AND (4)

$$(4.5)(1 + 0.3) = (1.375) u_C''$$

$$u_C'' = 4.2545 \text{ m/s}$$

$$(3) \quad u_B'' = 4.5 - 0.375(4.2545) = 2.90 \text{ m/s}$$

(b)

$$\Delta T_L = (T_B' + T_C') - (T_B'' + T_C'')$$

$$T_B' = \frac{1}{2} m_B (u_B')^2 = \frac{1}{2} (40 \text{ kg}) (4.5 \text{ m/s})^2 = 405 \text{ J}$$

$$T_C' = 0 \quad T_B'' = \frac{1}{2} m_B (u_B'')^2 = \frac{1}{2} (40 \text{ kg}) (2.90)^2 = 168.72 \text{ J}$$

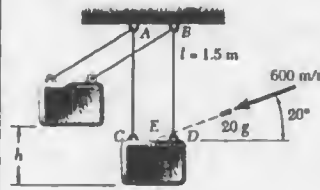
$$T_C'' = \frac{1}{2} m_C (u_C'')^2 = \frac{1}{2} (15 \text{ kg}) (4.2545 \text{ m/s})^2 = 135.76 \text{ J}$$

$$\Delta T_L = (405 + 0) - (168.72 + 135.76) = 100.5 \text{ J} \quad \Delta T_L = 100.5 \text{ J}$$

13.184

GIVEN:

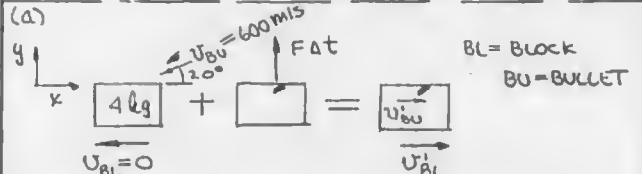
INITIAL VELOCITY OF 20g BULLET = 600 m/s  
 MASS OF BLOCK = 4 kg



FIND:

- (a) MAXIMUM HEIGHT  $h$   
 (b) TOTAL IMPULSE BY THE CORDS ON THE BLOCK

(a)



TOTAL MOMENTUM IN X IS CONSERVED

$$m_{BL} u_{BL} + m_{BU} u_{BU} \cos 20^\circ = m_{BL} u_{BL}' + m_{BU} u_{BU}' \quad (u_{BL}' = u_{BU}')$$

$$0 + (0.02 \text{ kg})(-600 \text{ m/s})(\cos 20^\circ) = (4.02 \text{ kg})(u_{BL}') \quad (u_{BL}' = u_{BU}')$$

$$u_{BL}' = 2.805 \text{ m/s}$$

CONSERVATION OF ENERGY

$$T_1 = \frac{1}{2} (m_{BL} + m_{BU}) (u_{BL}')^2$$

$$T_1 = \frac{1}{2} (4.02 \text{ kg}) (2.805 \text{ m/s})^2$$

$$T_1 = 15.815 \text{ J}$$

$$V_1 = 0$$

$$T_2 = 0 \quad V_2 = (m_{BL} + m_{BU}) g h$$

$$V_2 = (4.02 \text{ kg})(9.81 \text{ m/s}^2)(h) = 39.44 h$$

$$T_1 + V_1 = T_2 + V_2$$

$$15.815 + 0 = 0 + 39.44 h$$

$$h = 0.401 \text{ m}$$

(b) REFER TO FIGURE IN PART (a)

IMPULSE-MOMENTUM IN y DIRECTION

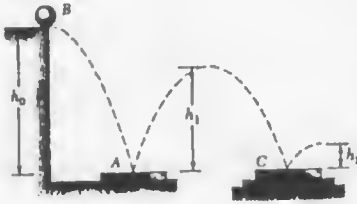
$$m_{BU} u_{BU} \sin 20^\circ + F \Delta t = (m_{BL} + m_{BU}) (u_{BL})_y$$

$$(0.02 \text{ kg})(-600 \text{ m/s})(\sin 20^\circ) + F \Delta t = 0 \quad (u_{BL})_y = 0$$

$$F \Delta t = 4.10 \text{ N} \cdot \text{s}$$



13.185



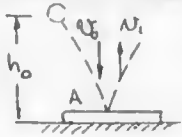
GIVEN:

MASS OF BALL  
 $m_B = 70g$   
 $h_0 = 210g$   
 BALL DROPS FREELY FROM B  
 $h_2 = 0.25m$   
 $m_A = m_B = 210g$   
 FOAM RUBBER SUPPORT AT C

FIND:

- (a) COEFFICIENT OF RESTITUTION BETWEEN THE BALL AND THE PLATES  
 (b) THE HEIGHT  $h_1$  OF THE BALL'S FIRST BOUNCE

(a) PLATE ON HARD GROUND (FIRST REBOUND)

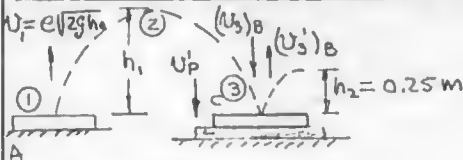


CONS OF ENERGY  
 $\frac{1}{2} m_B v_0^2 = m_B g h_0$   
 $v_0 = \sqrt{2g h_0}$

RELATIVE VELOCITIES

$$v_0 e = v_1 \quad v_1 = e \sqrt{2g h_0}$$

PLATE ON FOAM RUBBER SUPPORT AT C



CONSERVATION OF ENERGY

POINTS ① AND ③  $v_1 = v_3 = 0$ 

$$\frac{1}{2} m_B v_1^2 = \frac{1}{2} m_B (v_3)_B^2$$

$$(v_3)_B = e \sqrt{2g h_0}$$

CONSERVATION OF MOMENTUM

$$\uparrow \text{ AT } \textcircled{3} \quad m_B (-v_3)_B + m_P v_P = m_B (v_3)_B - m_P v_P'$$

$$\frac{m_P}{m_B} = \frac{210}{70} = 3$$

$$-e \sqrt{2g h_0} = (v_3)_B - 3 v_P' \quad (1)$$

RELATIVE VELOCITIES

$$[-(v_3)_B - (v_P)'] e = -v_P' - (v_3)_B$$

$$e \sqrt{2g h_0} + 0 = v_P' + (v_3)_B \quad (2)$$

MULTIPLY (2) BY 3 AND ADD TO (1)

$$4(v_3)_B = \sqrt{2g h_0} (3e^2 - e)$$

CONSERVATION OF ENERGY AT ③,  $(v_3)_B = \sqrt{2g h_2}$ 

$$\text{THUS } 4 \sqrt{2g h_2} = \sqrt{2g h_0} (3e^2 - e)$$

$$4 \sqrt{\frac{h_2}{h_0}} = 4 \sqrt{\frac{0.25}{1.5}} = 3e^2 - e$$

$$3e^2 - e - 1.633 = 0 \quad e = 0.923$$

(b) FROM (a),  $v_1 = e \sqrt{2g h_0}$ POINTS ① AND ② CONS OF ENERGY  $\frac{1}{2} m_B v_1^2 = m_B g h_1$ 

$$\frac{1}{2} e^2 2g h_0 = g h_1$$

$$h_1 = (0.923)^2 (1.5) = 1.278m$$

13.186

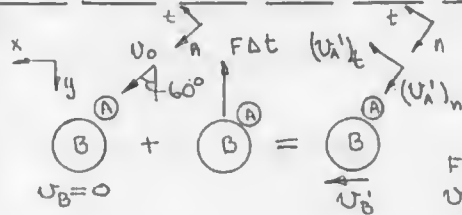
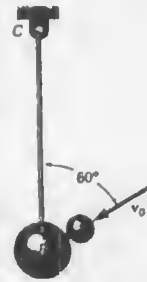
GIVEN:

$m_B = 700g$ ,  $m_A = 350g$   
 $e = 1$

A STRIKES B WITH VELOCITY  $v_0$  AT  $60^\circ$  AS SHOWN  
 CORD BC ATTACHED TO B IS INEXTENSIBLE  
 NO FRICTION

FIND:

VELOCITY OF EACH BALL AFTER IMPACT, CHECK THAT NO ENERGY IS LOST IN THE IMPACT



FROM KINEMATICS  
 $v_B$  IS IN THE X DIRECTION

BALL A ALONE

MOMENTUM IN t DIRECTION IS CONSERVED

$$m_A (v_A)_t = m_A (v_A')_t, (v_A)_t = 0$$

$$\text{THUS } (v_A')_t = 0 \text{ AND } (v_A')_n = v_A' \text{ AT } 60^\circ$$

BALLS A AND B

TOTAL MOMENTUM IN X DIRECTION CONSERVED

$$m_A v_0 \sin 60^\circ = m_A (v_A')_n \sin 60^\circ + m_B v_B'$$

$$(0.350) \sqrt{2} v_0 = (0.350) \sqrt{2} v_A' + 0.700 v_B'$$

$$v_0 = v_A' + 2.309 v_B' \quad (1)$$

RELATIVE VELOCITIES (n-DIRECTION)

$$[(v_A)_n - (v_B)_n] e = (v_B')_n - (v_A')_n$$

$$(v_0 - 0)(1) = v_B' \sin 60^\circ - v_A'$$

$$v_0 = 0.866 v_B' - v_A' \quad (2)$$

ADDING (1) AND (2)

$$2v_0 = (2.309 + 0.866) v_B'$$

$$v_B' = 0.630 v_0$$

$$\text{FROM (1)} \quad v_A' = v_0 - (2.309)(0.630 v_0) = -0.455 v_0$$

$$v_A' = 0.455 v_0$$

ENERGY

$$\Delta T = \frac{1}{2} m_A v_0^2 - \frac{1}{2} m_A v_A'^2 - \frac{1}{2} m_B v_B'^2$$

$$\Delta T = \frac{1}{2} [(0.350)(v_0^2 - (0.455 v_0)^2) - (0.700)(0.630 v_0)^2]$$

$$\Delta T = \frac{1}{2} [0.350(1 - 0.2065) - 0.700(0.3969)] v_0^2$$

$$\Delta T = \frac{1}{2} [0.278 - 0.278] v_0^2 = 0 \text{ (CHECK)}$$

13.187



GIVEN:

$$m_A = 700 \text{ g}$$

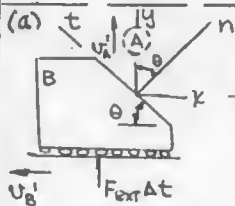
$$m_B = 2.1 \text{ kg}$$

$$e = 0.6$$

SPHERE A REBOUNDS UP

FIND:

- (a) ANGLE  $\theta$   
(b) ENERGY LOST



MOMENTUM OF SPHERE A ALONE IS CONSERVED IN THE t-DIRECTION

$$m_A v_0 \cos \theta = m_A v_A' \sin \theta$$

$$v_0 = v_A' \tan \theta \quad (1)$$

TOTAL MOMENTUM IS CONSERVED IN THE x-DIRECTION

$$m_B v_B + m_A v_0 = m_B v_B' + m_A (v_A')_x \quad v_B = 0, (v_A')_x = 0$$

$$0 + 0.700 v_0 = 2.1 v_B' + 0$$

$$v_B' = v_0 / 3 \quad (2)$$

RELATIVE VELOCITIES IN THE n-DIRECTION

$$(-v_0 \sin \theta - 0)e = -v_B' \sin \theta - v_A' \cos \theta$$

$$(v_0)(0.6) = v_B' + v_A' \cot \theta \quad (3)$$

SUBSTITUTING  $v_B'$  FROM (2) INTO (3)

$$0.6 v_0 = 0.333 v_0 + v_A' \cot \theta$$

$$0.267 v_0 = v_A' \cot \theta \quad (4)$$

DIVIDE (4) INTO (1)

$$\frac{1}{0.267} = \frac{\tan \theta}{\cot \theta} = \tan^2 \theta$$

$$\tan \theta = 1.935 \quad \theta = 62.7^\circ$$

(b) FROM (1)  $v_0 = v_A' \tan \theta = v_A' (1.935)$

$$v_A' = 0.5168 v_0, \quad v_B' = v_0 / 3 \quad (2)$$

$$T_{\text{LOST}} = \frac{1}{2} m_A v_A^2 - \frac{1}{2} (m_A (v_A')^2 + m_B v_B'^2)$$

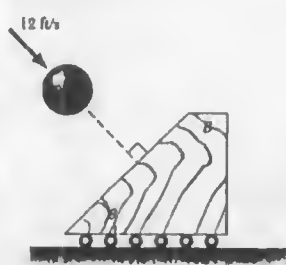
$$T_{\text{LOST}} = \frac{1}{2} (0.7)(v_0)^2 - \frac{1}{2} [(0.7)(0.5168 v_0)^2 + (2.1)(v_0/3)^2]$$

$$T_{\text{LOST}} = \frac{1}{2} [0.7 - 0.1870 - 0.2333] v_0^2$$

$$T_{\text{LOST}} = 0.1400 v_0^2 \text{ J}$$

$$T_{\text{LOST}} = 0.1400 v_0^2$$

13.188



GIVEN:

$$m_A = 3 \text{ lb}$$

$$m_B = 9 \text{ lb}$$

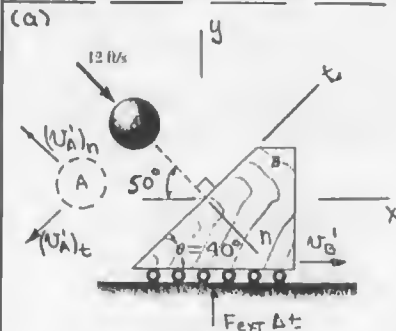
$$e = 0.50$$

$$\theta = 40^\circ$$

$$v_A = 12 \text{ ft/s}$$

FIND:

- (a) VELOCITIES AFTER IMPACT,  $v_A'$  AND  $v_B'$   
(b) ENERGY LOST



MOMENTUM OF THE SPHERE A ALONE IS CONSERVED IN THE t-DIRECTION

$$m_A (v_A)_t = m_A (v_A')_t \quad (v_A)_t = 0$$

$$(v_A')_t = 0 \quad (v_A')_n = v_A' \sin 50^\circ$$

TOTAL MOMENTUM IS CONSERVED IN THE x-DIRECTION

$$m_A v_A \cos 50^\circ + m_B v_B = m_A (-v_A') \cos 50^\circ + m_B v_B'$$

$$v_B = 0 \quad v_A = 12 \text{ ft/s}$$

$$\left(\frac{3}{9}\right)(12)(\cos 50^\circ) + 0 = \left(\frac{3}{9}\right)(-v_A')(\cos 50^\circ) + \left(\frac{9}{9}\right)v_B'$$

(1)  $23.140 = -1.9284 v_A' + 9 v_B'$   
RELATIVE VELOCITIES IN THE n-DIRECTION

$$(v_A - v_B)e = (v_B' \cos 50^\circ + v_A') \quad v_B = 0$$

$$(12 - 0)(0.5) = 0.6428 v_B' + v_A' \quad v_A = 12 \text{ ft/s}$$

$$(2) \quad 6 = 0.6428 v_B' + v_A' \quad e = 0.50$$

SOLVING EQ. (1) AND (2) SIMULTANEOUSLY

$$v_B' = 3.39 \text{ ft/s} \rightarrow$$

$$v_A' = 3.82 \text{ ft/s} \nearrow 50^\circ$$

$$(b) T_{\text{LOST}} = \frac{1}{2} m_A v_A^2 - \frac{1}{2} (m_A (v_A')^2 + m_B (v_B')^2)$$

$$T_{\text{LOST}} = \frac{1}{2} \left[ \left( \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (12 \text{ ft/s})^2 - \left( \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (3.82 \text{ ft/s})^2 - \left( \frac{9 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (3.39 \text{ ft/s})^2 \right]$$

$$T_{\text{LOST}} = \frac{1}{2} [12.064 - 3.212] = 4.42 \text{ ft} \cdot \text{lb}$$

$$T_{\text{LOST}} = 4.42 \text{ ft} \cdot \text{lb}$$

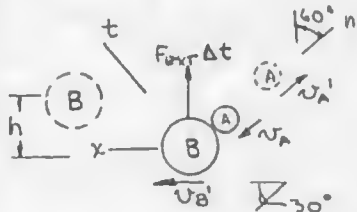
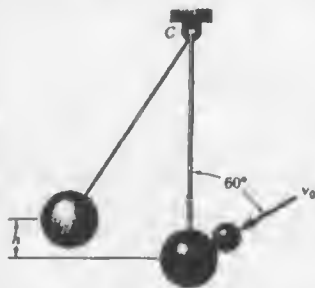
13.189

GIVEN:

$W_B = 12 \text{ oz}$   
 $W_A = 6 \text{ oz}$   
 $U_0 = 4.8 \text{ ft/s}$  AT  
 $60^\circ$  AS SHOWN  
 $e = 1$

FIND:

HEIGHT  $h$  REACHED  
 BY BALL B



TOTAL MOMENTUM IN THE  
 X-DIRECTION IS CONSERVED

$$m_A U_A \sin 60^\circ + m_B (U_B)_x = m_A (-U_A') \sin 60^\circ + m_B U_B'$$

$$U_A = U_0 = 4.8 \text{ ft/s} \quad (U_B)_x = 0$$

$$\left(\frac{6}{16}\right)(4.8)(\sin 60^\circ) + 0 = -\left(\frac{6}{16}\right)(U_A') \sin 60^\circ + \left(\frac{12}{16}\right)U_B'$$

$$4.1568 = -0.866 U_A' + 2 U_B' \quad (1)$$

RELATIVE VELOCITY IN THE  $n$ -DIRECTION

$$[-U_A - (U_B)_n]e = -U_B' \cos 30^\circ - U_A'$$

$$(-4.8 - 0)(1) = -0.866 U_B' - U_A' \quad (2)$$

SOLVE EQ (1) AND (2) SIMULTANEOUSLY

$$U_B' = 3.0232 \text{ ft/s} \quad U_A' = 2.18 \text{ ft/s}$$

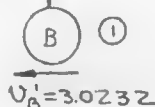
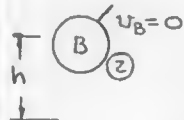
CONSERVATION OF ENERGY

BALL B

$$T_1 = \frac{1}{2} m_B (U_B')^2$$

$$T_1 = \frac{1}{2} \frac{W_B}{g} (3.0232)^2$$

$$V_1 = 0$$



$$T_2 = 0 \quad V_2 = W_B h$$

$$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2} \frac{W_B}{g} (3.0232)^2 = 0 + W_B h$$

$$h = \frac{(3.0232)^2}{(2)(32.2)} = 0.1419 \text{ ft}$$

$$h = 0.1419 \text{ ft}$$

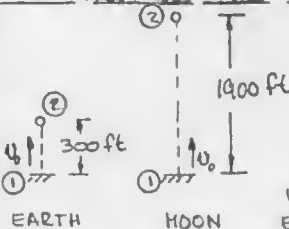
13.190

GIVEN:

PELLET OF WEIGHT  $W = 2 \text{ oz}$ ,  
 SHOT VERTICALLY RISES TO  
 $h_e = 300 \text{ ft}$  ON EARTH  
 $h_m = 1900 \text{ ft}$  ON THE MOON  
 ACCELERATION OF GRAVITY ON  
 THE MOON,  $g_m = 0.165 g_e$

FIND:

ENERGY LOSS DUE TO DRAG FOR PELLET ON THE EARTH



SINCE THE PELLET IS  
 SHOT FROM THE SAME  
 PISTOL THE INITIAL  
 VELOCITY  $U_0$  IS THE SAME  
 ON THE MOON AND ON  
 THE EARTH

WORK AND ENERGY

$$\text{EARTH: } T_1 = \frac{1}{2} m U_0^2$$

$$U_{1-2} = -m g_e (300 \text{ ft}) - E_L$$

( $E_L$  = LOSS OF ENERGY DUE TO  
 DRAG)

$$T_2 = 0$$

$$T_1 - 300 m g_e - E_L = 0 \quad (1)$$

MOON:

$$T_1 = \frac{1}{2} m U_0^2$$

$$U_{1-2} = -m g_m (1900)$$

$$T_2 = 0$$

$$T_1 - 1900 m g_m = 0 \quad (2)$$

SUBTRACT (1) FROM (2)

$$-1900 m g_m + 300 m g_e + E_L = 0$$

$$g_m = 0.165 g_e$$

$$m = \frac{(2/16)}{g_e}$$

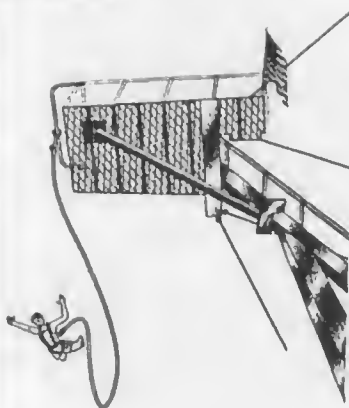
$$E_L = (1900) \left(\frac{2}{16}\right) (0.165 g_e) - 300 \left(\frac{2}{16}\right) g_e$$

$$E_L = 1.688 \text{ ft} \cdot \text{lb}$$

13.191

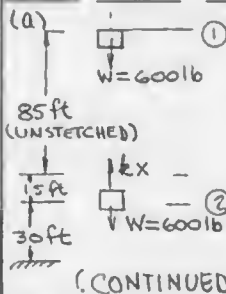
GIVEN:

130 FT TOWER  
 ELASTIC CABLE  
 $L = 85 \text{ ft}$   
 UNSTRETCHED  
 CABLE IS TO  
 STRETCH TO  
 100 FT WHEN  
 A 600 LB WEIGHT  
 ATTACHED TO IT  
 IS DROPPED  
 FROM THE  
 TOWER



FIND:

(a)  $k$  FOR THE CABLE  
 (b) DISTANCE FROM  
 THE GROUND WHEN  
 A 186 LB MAN JUMPS



CONSERVATION OF ENERGY

$$U_1 = 0 \quad T_1 = 0 \quad V_1 = 100 \text{ W}$$

$$\text{DATUM AT } (2) \quad V_1 = (100 \text{ ft})(600 \text{ lb}) = 6 \times 10^4 \text{ ft} \cdot \text{lb}$$

$$U_2 = 0 \quad T_2 = 0$$

$$V_2 = V_g + V_e = 0 + \frac{1}{2} k (15 \text{ ft})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6 \times 10^4 = 0 + (1/2) k$$

$$k = 533 \text{ lb/ft}$$

(CONTINUED)

# 13.191 continued

FROM (a),  $k = 533 \text{ lb/ft}$

(b)

85 ft (UNSTRETCHED)

130 ft

(STRETCH) d

①

② DATUM

$W = 186 \text{ lb}$

$T_1 = 0$

$V_1 = (186)(130 - d)$

$T_2 = 0$

$V_2 = V_g + V_e = 0 + \frac{1}{2}(533)(30 - 85d)^2$

$V_2 = (266.67)(45 - d)^2$

$T_1 + V_1 = T_2 + V_2$

$0 + (186)(130 - d) = 0 + (266.67)(45 - d)^2$

$266.7d^2 - 23815d + 516018 = 0$

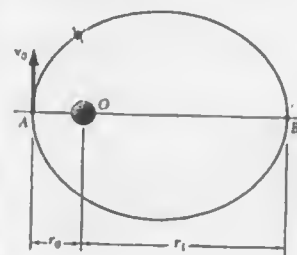
$d = 23815 \pm \sqrt{(23815)^2 - 4(266.7)(516018)} = 36.99 \text{ ft}$

$52.3 \text{ ft}$

DISCARD 52.3 ft (ASSUMES CORD ACTS IN COMPRESSION WHEN REBOUND OCCURS)

$d = 37.0 \text{ ft}$

# 13.193

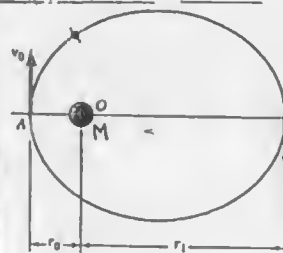


GIVEN:

PLANET OF MASS  $M$  AT  
SATELLITE IN AN  
ELLIPTICAL ORBIT

DERIVE:

$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$   
WHERE  $h$  IS THE  
ANGULAR MOMENTUM  
USE CONSERVATION  
OF ENERGY AND  
CONSERVATION OF  
ANGULAR MOMENTUM



ANGULAR MOMENTUM

$$h = r_0 v_0 = r_1 v_1$$

$$v_0 = \frac{h}{r_0} \quad v_1 = \frac{h}{r_1} \quad (1)$$

CONSERVATION OF ENERGY

$$T_A + V_A = T_B + V_B$$

$$T_A = \frac{1}{2} M v_0^2$$

$$V_A = -\frac{GMm}{r_0}$$

$$T_B = \frac{1}{2} M v_1^2 \quad V_B = -\frac{GMm}{r_1}$$

$$T_A + V_A = T_B + V_B \quad \frac{1}{2} M v_0^2 - \frac{GMm}{r_0} = \frac{1}{2} M v_1^2 - \frac{GMm}{r_1}$$

$$v_0^2 - v_1^2 = 2GM \left[ \frac{1}{r_0} - \frac{1}{r_1} \right] = 2GM \left[ \frac{r_1 - r_0}{r_1 r_0} \right]$$

SUBSTITUTE FOR  $v_0$  AND  $v_1$  FROM (1)

$$h^2 \left[ \frac{1}{r_0^2} - \frac{1}{r_1^2} \right] = 2GM \left[ \frac{r_1 - r_0}{r_1 r_0} \right]$$

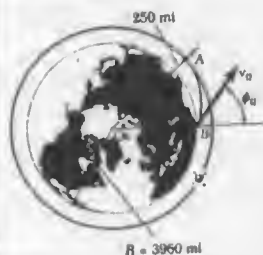
$$h^2 \left[ \frac{r_1^2 - r_0^2}{r_1^2 r_0^2} \right] = \frac{h^2}{r_1^2 r_0^2} (r_1 + r_0)(r_1 - r_0) = 2GM \left[ \frac{r_1 - r_0}{r_1 r_0} \right]$$

$$h^2 \left( \frac{1}{r_0} + \frac{1}{r_1} \right) = 2GM \quad \left( \frac{1}{r_0} + \frac{1}{r_1} \right) = \frac{2GM}{h^2} \quad (\text{QED})$$

# 13.194

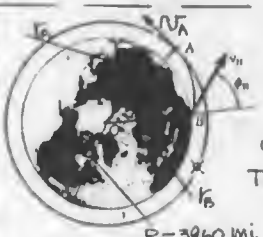
GIVEN:

SHUTTLE ALTITUDE AT  
 $B = 40 \text{ MI}$ ,  $\phi_0 = 55^\circ$   
MUST BE TANGENT  
TO ORBIT AT POINT  
A AT AN ALTITUDE  
OF 250 MI. ENGINE  
TURNED OFF AT B



FIND:

$v_0$



CONSERVATION OF ENERGY

$$T_B + V_B = T_A + V_A$$

$$T_B = \frac{1}{2} M v_0^2 \quad V_B = -\frac{GMm}{r_B}$$

$$T_A = \frac{1}{2} M v_A^2 \quad V_A = -\frac{GMm}{r_A}$$

$$GM = gR^2 \quad (\text{EQ 12.30})$$

$$T_A + V_A = T_B + V_B \quad \frac{1}{2} M v_0^2 - gR^2 \frac{1}{r_B} = \frac{1}{2} M v_A^2 - gR^2 \frac{1}{r_A}$$

$$v_A^2 = v_0^2 - \frac{2gR^2}{r_B} \left( 1 - \frac{r_B}{r_A} \right)$$

$$r_A = 3960 + 250 = 4210 \text{ MI} \quad r_B = 3960 + 40 = 4000 \text{ MI}$$

$$v_A^2 = v_0^2 - \frac{2(32.2)(3960)^2}{4000} \left( 1 - \frac{4000}{4210} \right)$$

$$v_A^2 = v_0^2 - 66.495 \times 10^6 \quad (1)$$

$$v_A v_A = r_B v_0 \sin \phi; \quad v_A = (4000/4210) v_0 \sin 55^\circ = 0.77829 v_0 \quad (2)$$

$$(2) \text{ AND } (1) \quad [1 - (0.77829)^2] v_0^2 = 66.495 \times 10^6 \quad v_0 = 12,910 \text{ ft/s}$$

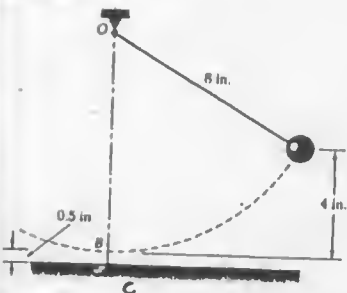
# 13.192

GIVEN:

2-OZ SPHERE A.  
MAGNET AT B  
EXERTS A FORCE  
 $F = 0.1/r^2 \text{ (lb.m)}$   
SPHERE RELEASED  
FROM POSITION  
SHOWN

FIND:

SPEED OF A  
AS IT PASSES  
THROUGH B



POTENTIAL ENERGY  
OF THE MAGNET

$$F = 0.1/r^2 = -dV/dr$$

$$V_M = -0.1/r$$

$$v_1 = 0 \quad T_1 = 0$$

$$v_1 = (v_M)_1 + (v_g)_1$$

$$(v_M)_1 = -0.1/r_1$$

$$r_1^2 = (4 + 0.5)^2 + [8^2 - (8 - 4)^2] = 68.25$$

$$r_1 = 8.2614 \text{ in.}$$

$$(v_M)_1 = (-0.1)/(8.2614) = -0.012105 \text{ lb.m}$$

$$(v_g)_1 = W(4.5 \text{ in.}) = (2/16 \text{ lb})(4.5 \text{ in.})$$

$$(v_g)_1 = 0.5625 \text{ lb.m}$$

$$v_1 = (v_g)_1 + (v_M)_1 = 0.5625 - 0.012105 = 0.5504 \text{ lb.m}$$

$$v_1 = 0.045866 \text{ lb.ft}$$

$$T_2 = \frac{1}{2} M v_0^2 = \frac{1}{2} \left( \frac{2}{16} \text{ lb} \right) v_0^2 \quad v_0^2 = 0.001941 \quad v_0^2$$

$$v_2 = (v_g)_2 + (v_M)_2 = \left( \frac{2}{16} \text{ lb} \right) (0.5 \text{ in.}) - \left( \frac{0.1}{r_2} \right) = -0.1375 \text{ lb.m}$$

$$v_2 = -0.1375 \text{ lb.m} = -0.011458 \text{ lb.ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0.045866 = 0.001941 v_0^2 - 0.011458$$

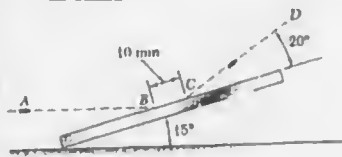
$$v_0^2 = 29.53 \text{ ft}^2/\text{s}^2$$

$$v_0 = 5.43 \text{ ft/s}$$

13.195

GIVEN:

25-g BULLET  
INITIAL VELOCITY  
 $U_1 = 600 \text{ m/s}$ ,  
HORIZONTAL  
RICOCHET  
VELOCITY  $U_2$   
 $= 400 \text{ m/s}$   
AT  $20^\circ$

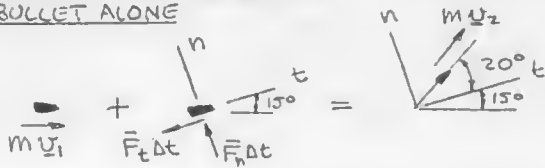


BULLET LEAVES A 10-MM SCRATCH ON THE  
PLATE AT AN AVERAGE SPEED OF  $500 \text{ m/s}$

FIND:

THE MAGNITUDE AND DIRECTION OF THE  
AVERAGE IMPULSIVE FORCE EXERTED BY  
THE BULLET ON THE PLATE

IMPULSE AND MOMENTUM  
BULLET ALONE



$$mU_1 + \bar{F}_c \Delta t = mU_2$$

t DIRECTION  $mU_1 \cos 15^\circ - \bar{F}_c \Delta t = mU_2 \cos 20^\circ$

$$\bar{F}_c \Delta t = (0.025 \text{ kg}) [600 \text{ m/s} \cos 15^\circ - 400 \text{ m/s} \cos 20^\circ]$$

$$\bar{F}_c \Delta t = 5.092 \text{ kg} \cdot \text{m/s}$$

$$\Delta t = \frac{S_{\text{Scr}}}{U_{\text{Av}}} = \frac{0.010 \text{ m}}{500 \text{ m/s}} = 20 \times 10^{-6} \text{ s}$$

$$\bar{F}_c = (5.092 \text{ kg} \cdot \text{m/s}) / (20 \times 10^{-6} \text{ s}) = 254.6 \times 10^3 \text{ kg} \cdot \text{m/s}^2$$

$$\bar{F}_c = 254.6 \text{ kN}$$

n DIRECTION

$$-mU_1 \sin 15^\circ + \bar{F}_n \Delta t = mU_2 \sin 20^\circ$$

$$\bar{F}_n \Delta t = (0.025 \text{ kg}) [600 \text{ m/s} \sin 15^\circ + 400 \text{ m/s} \sin 20^\circ]$$

$$\bar{F}_n \Delta t = 7.3025 \text{ kg} \cdot \text{m/s} \quad \Delta t = 20 \times 10^{-6} \text{ s}$$

$$\bar{F}_n = (7.3025 \text{ kg} \cdot \text{m/s}) / (20 \times 10^{-6} \text{ s}) = 365.1 \times 10^3 \text{ kg} \cdot \text{m/s}^2$$

$$\bar{F}_n = 365.1 \text{ kN}$$

$$F = \sqrt{254.6^2 + 365.1^2} = 445.1 \text{ kN}$$

$$\theta = \tan^{-1} \frac{\bar{F}_c}{\bar{F}_n} = \tan^{-1} \frac{254.6}{365.1}$$

$$\theta = 34.9^\circ$$

$$\alpha = 34.9 + 15^\circ = 49.9^\circ$$

FORCE OF THE BULLET  
ON THE PLATE

$$\bar{F} = 445 \text{ kN}$$



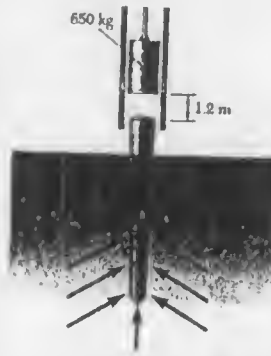
13.196

GIVEN:

650 kg HAMMER  
DROPS 1.2 m AND  
DRIVES A 140 kg  
PILE 110 mm INTO  
THE GROUND  
 $e = 0$

FIND:

AVERAGE RESISTANCE  
OF THE GROUND TO  
PENETRATION



VELOCITY OF THE HAMMER AT IMPACT  
CONSERVATION OF ENERGY

$$\begin{aligned} \textcircled{1} \quad m \quad U_1 = 0 \quad \textcircled{2} \quad T_1 = 0 \quad V_H = m g (1.2 \text{ m}) \\ V_H = (0.650 \text{ kg}) (9.81 \frac{\text{m}}{\text{s}^2}) (1.2 \text{ m}) \\ V_1 = 7.652 \text{ J} \\ T_2 = \frac{1}{2} m U_H^2 = 0.650 \frac{U_H^2}{2} = 0.325 U_H^2 \\ m = 0.650 \text{ kg} \quad V_2 = 0 \end{aligned}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 7.652 = 0.325 U^2$$

$$U^2 = 23.54 \text{ m}^2/\text{s}^2$$

$$U = 4.852 \text{ m/s}$$

VELOCITY OF PILE AFTER IMPACT

SINCE THE IMPACT IS PLASTIC ( $e = 0$ ), THE VELOCITY  
OF THE PILE AND HAMMER ARE THE SAME AFTER  
IMPACT

CONSERVATION OF MOMENTUM

$$U_H = 4.852 \text{ m/s}$$

$$\begin{aligned} \text{H} \downarrow \quad \text{P} \downarrow \quad U_H &= \text{H} \downarrow \quad \text{P} \downarrow \quad U' = U_H' = U_P' \\ U_P' &= U_H' = U_P' \end{aligned}$$

THE GROUND REACTION AND THE WEIGHTS ARE  
NON-IMPULSIVE

$$\text{THUS} \quad m_H U_H = (m_H + m_P) U'$$

$$U' = \frac{m_H U_H}{(m_H + m_P)} = \frac{(650)}{(650 + 140)} (4.852 \text{ m/s}) = 3.992 \text{ m/s}$$

WORK AND ENERGY  $d = 0.110 \text{ m}$

$$\begin{aligned} \text{H} \downarrow \quad \text{P} \downarrow \quad T_2 + m_H g d &= \text{H} \downarrow \quad \text{P} \downarrow \quad T_3 = 0 \\ T_2 &= T_3 = 0 \end{aligned}$$

$$T_2 + U_2 - 3 = T_3 \quad T_2 = \frac{1}{2} (m_H + m_P) (U')^2$$

$$T_3 = 0$$

$$T_2 = \frac{1}{2} (650 + 140) (3.992)^2$$

$$T_2 = 6.295 \times 10^3 \text{ J}$$

$$U_2 - 3 = (m_H + m_P) g d - F_{\text{Av}} d = (650 + 140) (9.81) (0.110) - F_{\text{Av}} (0.110)$$

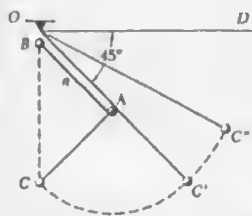
$$U_2 - 3 = 852.49 - (0.110) F_{\text{Av}}$$

$$T_2 + U_2 - 3 = T_3$$

$$6.295 \times 10^3 + 852.49 - (0.110) F_{\text{Av}} = 0$$

$$F_{\text{Av}} = (7147.5) / (0.110) = 64.98 \times 10^3 \text{ N} \quad F_{\text{Av}} = 65 \text{ kN}$$

13.197



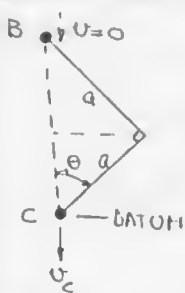
GIVEN:

SPHERE RELEASED FROM REST AT B. CORD OF LENGTH  $2a$  BECOMES TAUT AT C

FIND:

VERTICAL DISTANCE FROM OD TO THE HIGHEST POINT C' REACHED BY THE SPHERE

VELOCITY AT POINT C (BEFORE THE CORD IS TAUT)  
CONSERVATION OF ENERGY FROM B TO C.



$$T_B = 0$$

$$V_B = mg(2)(\frac{\sqrt{2}}{2})a = mga\sqrt{2}$$

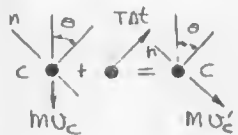
$$T_C = \frac{1}{2}mv_c^2 \quad V_c = 0$$

$$T_B + V_B = T_C + V_c$$

$$0 + mga\sqrt{2} = \frac{1}{2}mv_c^2 + 0$$

$$v_c = \sqrt{2\sqrt{2}ga}$$

VELOCITY AT C (AFTER THE CORD BECOMES TAUT)  
LINEAR MOMENTUM PERPENDICULAR TO THE CORD IS CONSERVED



$$\theta = 45^\circ$$

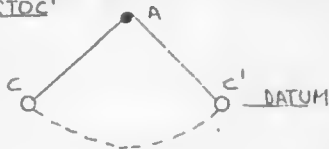
$$-mv_c \sin \theta = mv_c'$$

$$v_c' = (\sqrt{2\sqrt{2}})(\frac{\sqrt{2}}{2})\sqrt{ga}$$

$$v_c' = \sqrt{2}ga = \sqrt[4]{2}ga$$

NOTE: THE WEIGHT OF THE SPHERE IS A NON-IMPULSIVE FORCE

VELOCITY AT C' (CONSERVATION OF ENERGY)  
C TO C'



$$T_C = \frac{1}{2}m(v_c')^2 \quad V_c = 0$$

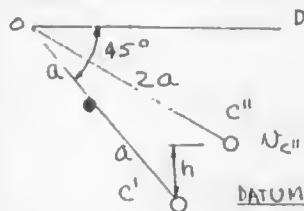
$$T_{C'} = \frac{1}{2}m(v_{C'}')^2 \quad V_{C'} = 0$$

$$T_C + V_C = T_{C'} + V_{C'}$$

$$\frac{1}{2}m(v_c')^2 + 0 = \frac{1}{2}m(v_{C'}')^2 + 0$$

$$v_{C'}' = v_c'$$

C' TO C'' (CONSERVATION OF ENERGY)



$$T_{C'} = \frac{1}{2}m(v_{C'}')^2$$

$$T_{C''} = \frac{1}{2}m(\sqrt[4]{2}ga)^2$$

$$T_{C''} = \frac{\sqrt{2}}{2}mga$$

$$V_{C''} = 0$$

$$T_{C''} = 0$$

$$V_{C''} = mgh$$

$$T_{C'} + V_{C'} = T_{C''} + V_{C''}$$

$$\frac{\sqrt{2}}{2}mga + 0 = 0 + mgh$$

$$h = \frac{\sqrt{2}}{2}a$$

$$h = 0.707a$$

13.198

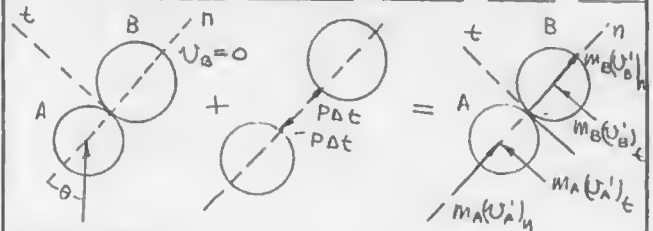
GIVEN:

$m_A$  AND  $m_B$  SLIDING ON A FRICTIONLESS SURFACE INITIALLY,  $v_B = 0$   
 $v_A = v_0$  AT ANGLE  $\theta$   
COEFFICIENT OF RESTITUTION,  $e$

SHOW:

THAT A COMPONENT OF THE VELOCITY OF A AFTER IMPACT IS,

- (a) POSITIVE IF  $m_A > em_B$   
(b) NEGATIVE IF  $m_A < em_B$   
(c) ZERO IF  $m_A = em_B$



$$m_A v_A = m_A v_0$$

DISKS A AND B (TOTAL MOMENTUM CONSERVED)

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

NORMAL DIRECTION:

$$m_A v_0 \cos \theta + 0 = m_A (v_A')_n + m_B (v_B')_n \quad (1)$$

RELATIVE VELOCITIES

$$[v_A \cos \theta - (v_B)_n]e = (v_B')_n - (v_A')_n$$

$$v_0 (\cos \theta)e = (v_B')_n - (v_A')_n \quad (2)$$

MULTIPLY (2) BY  $m_B$  AND SUBTRACT IT FROM (1)

$$v_0 \cos \theta (m_A - em_B) = (m_A + m_B)(v_A')_n$$

$$(v_A')_n = (v_0 \cos \theta) \frac{(m_A - em_B)}{(m_A + m_B)} \quad (3)$$

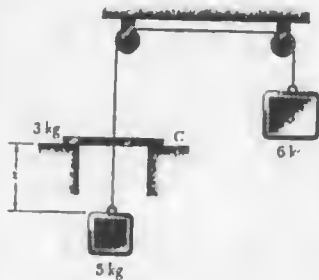
FROM EQUATION (3)

(a)  $m_A > em_B$   $(v_A')_n$  POSITIVE

(b)  $m_A < em_B$   $(v_A')_n$  NEGATIVE

(c)  $m_A = em_B$   $(v_A')_n = 0$

13.199



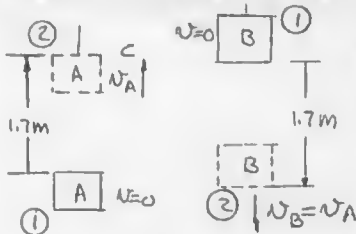
GIVEN:

MASSSES A, B  
AND C AS SHOWN  
INITIAL  
 $U_A = 0, U_C = 1.7 \text{ m/s}$   
IMPACT BETWEEN  
A AND C IS  
PLASTIC,  $e = 0$

FIND:

- (a)  $U'_A, U'_B, U'_C$   
IMMEDIATELY  
AFTER A HITS C  
(b) DISTANCE A AND  
C MOVE BEFORE  
STOPPING  
(c)  $\lambda$  AFTER ONE  
COMPLETE CYCLE

(a) VELOCITY OF A JUST BEFORE IT HITS C

CONS. OF ENERGY  
DATUM AT ①

POSITION ①  
 $(U_A)_1 = (U_B)_1 = 0$   
 $T_1 = 0$   
 $V_1 = 0$

POSITION ②  $T_2 = \frac{1}{2} m_A (U_A')^2 + \frac{1}{2} m_B U_B'^2$

$U_A = U_B$  (KINEMATICS)

$T_2 = \frac{1}{2} (5+6) U_A'^2 = \frac{11}{2} U_A'^2$

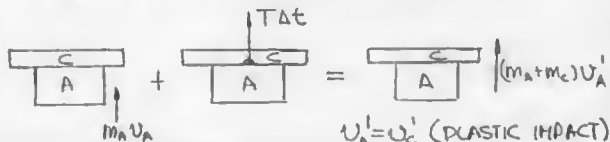
$V_2 = m_A g (1.7) - m_B g (1.7) = (5-6) (g) (1.7)$

$V_2 = -1.7g$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 0 = \frac{11}{2} U_A'^2 - 1.7g$

$U_A'^2 = \left(\frac{3.4}{11}\right) (9.81) = 3.032 \frac{\text{m}^2}{\text{s}^2} \quad U_A' = 1.741 \text{ m/s}$

VELOCITY OF A AND C AFTER A HITS C



IMPULSE-MOMENTUM A AND C

$+ \uparrow m_A U_A + T \Delta t = (m_A + m_C) U_A'$

$(5)(1.741) + T \Delta t = 8 U_A' \quad (1)$

$U_B = U_A; U_B' = U_A'$  (CORD REMAINS TIGHT)

B ALONE

$+ \downarrow m_B U_B + T \Delta t = m_B U_B'$

$m_B U_B = m_B U_A \quad m_B U_A - T \Delta t = m_B U_A'$

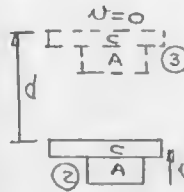
$(6)(1.741) - T \Delta t = 6 U_A' \quad (2)$

13.199 continued

ADDING EQUATIONS (1) AND (2),  $11(1.741) = 4 U_A'$   
 $U_A' = 1.3679 \text{ m/s}$

$U_A' = U_B' = U_C' = 1.368 \text{ m/s}$

DISTANCE A AND C MOVE BEFORE STOPPING

CONS. OF ENERGY  
DATUM AT ②

POSITION ②

$T_2 = \frac{1}{2} (m_A + m_B + m_C) (U_A')^2$

$T_2 = \left(\frac{14}{2}\right) (1.3681)^2$

$T_2 = 13.103 \text{ J}$   
 $V_2 = 0$

POSITION ③

$T_3 = 0 \quad V_3 = (m_A + m_C) g d - m_B g d$

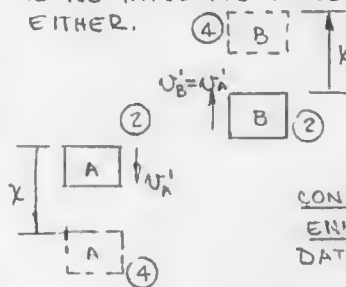
$V_3 = (8-6) g d = 2 g d$

$T_2 + V_2 = T_3 + V_3$

$13.103 + 0 = 0 + 2 g d$

$d = (13.103) / (2)(9.81) = 0.6679 \text{ m}$   
 $d = 0.668 \text{ m}$

(b) AS THE SYSTEM RETURNS TO POSITION ②, AFTER STOPPING IN POSITION ③, ENERGY IS CONSERVED AND THE VELOCITIES OF A, B AND C BEFORE THE COLLAR AT C, IS REMOVED, ARE THE SAME AS THEY WERE IN (a) ABOVE WITH THE DIRECTIONS REVERSED. THUS,  $U_A' = U_C' = U_B' = 1.3679 \text{ m/s}$ . AFTER THE COLLAR C IS REMOVED THE VELOCITIES OF A AND B REMAIN THE SAME SINCE THERE IS NO IMPULSIVE FORCE ACTING ON EITHER.

CONSERVATION OF  
ENERGY  
DATUM AT ②

$T_2 = \frac{1}{2} (m_A + m_B) (U_A')^2$

$T_2 = \frac{1}{2} (5+6) (1.3679)^2$

$T_2 = 10.291 \text{ J}$

$V_2 = 0$

$T_4 = 0 \quad V_4 = m_B g x - m_A g x$

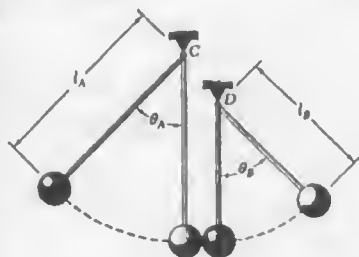
$V_4 = (6-5) g x$

$T_2 + V_2 = T_4 + V_4$

$10.291 + 0 = (1)(9.81) x$

$x = 1.049$

13.200

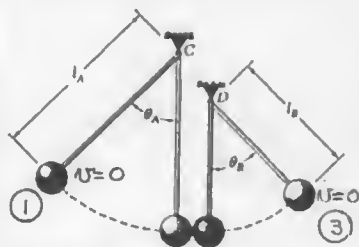


GIVEN:

SPHERE A IS RELEASED FROM REST AT AN ANGLE  $\theta_A$ . SPHERE B IS AT REST, IS HIT BY A, AND RISES TO A MAXIMUM ANGLE  $\theta_B = \theta_A$ .

FIND:

$\theta_B$  IN TERMS OF  $l_B/l_A$  AND  $e$ .



$$m_A = m_B = m \quad (2)$$

$$\theta_A = \theta_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgl_A(1 - \cos\theta_A) = \frac{1}{2} m v_A^2 + 0$$

$$v_A^2 = 2gl_A(1 - \cos\theta_A) \quad (1)$$

CONSERVATION OF MOMENTUM AT (2)

$$\frac{(A)(B)}{m v_A \quad v_B = 0} = \frac{(A)(B)}{m v_A' \quad v_B'}$$

$$m v_A + m v_B = m v_A' + m v_B'$$

$$v_A + 0 = v_A' + v_B' \quad (2)$$

RELATIVE VELOCITIES AT (2)

$$(v_A - v_B)e = v_B' - v_A' \quad v_A e = v_B' - v_A' \quad (3)$$

ADDING EQUATIONS (2) AND (3) AND SOLVING FOR  $v_B'$ ,

$$v_B' = \frac{1}{2}(1+e)v_A \quad (4)$$

CONSERVATION OF ENERGY (2)  $\rightarrow$  (3)

SPHERE B

$$\text{POSITION (2)} \quad T_2 = \frac{1}{2} m (v_B')^2 \quad V_2 = 0$$

$$T_3 = 0$$

$$V_3 = mgl_B(1 - \cos\theta_B)$$

$$T_2 + V_2 = T_3 + V_3 \quad \frac{1}{2} m (v_B')^2 + 0 = 0 + mgl_B(1 - \cos\theta_B)$$

$$(v_B')^2 = 2gl_B(1 - \cos\theta_B) \quad (5)$$

SUBSTITUTE  $v_B'$  FROM EQ. (4) INTO EQ. (5)

$$\frac{1}{4}(1+e)^2 v_A^2 = 2gl_B(1 - \cos\theta_B) \quad (6)$$

DIVIDE (1) INTO (6) AND SET  $\theta_A = \theta_B$ 

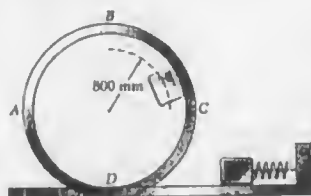
$$\frac{1}{4}(1+e)^2 v_A^2 = \frac{2gl_B(1 - \cos\theta_B)}{2gl_A(1 - \cos\theta_B)}$$

$$l_B/l_A = (1+e)^2/4$$

13.201

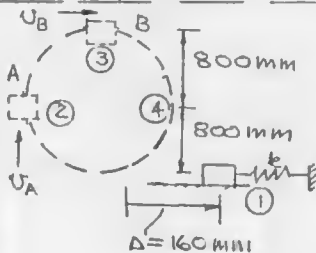
GIVEN:

300-g BLOCK  
SPRING OF CONSTANT  
 $k = 600 \text{ N/m}$  IS  
INITIALLY COMPRESSED  
160 mm WHEN  
THE BLOCK IS  
RELEASED.  
NO FRICTION



FIND:

FORCE EXERTED BY THE LOOP ABCD ON THE BLOCK AS IT PASSES THROUGH,  
(a) POINT A  
(b) POINT B, (c) POINT C



VELOCITIES AT A AND B

CONSERVATION OF ENERGY, DATUM AT (1)

POSITION (1)

$$V = 0 \quad T_1 = 0$$

$$V_1 = \frac{1}{2} k \Delta^2$$

$$V_1 = \frac{1}{2} (600 \text{ N/m}) (0.160 \text{ m})^2$$

$$V_1 = 7.68 \text{ J}$$

$$\text{POSITION (2)} \quad T_2 = \frac{1}{2} m v_A^2 = \frac{1}{2} (0.3) v_A^2 = 0.15 v_A^2$$

$$V_2 = mgl(0.800 \text{ m}) = (0.3 \text{ kg})(9)(0.8 \text{ m}) = 0.24 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + 7.68 = 0.15 v_A^2 + 0.24 \text{ J}$$

$$v_A^2 = \frac{7.68 - (0.24)(9.81)}{0.15}$$

$$v_A^2 = 35.50 \text{ m}^2/\text{s}^2$$

$$\text{POSITION (3)} \quad T_3 = \frac{1}{2} m v_B^2 = \frac{1}{2} (0.3) v_B^2 = 0.15 v_B^2$$

$$V_3 = mgl(1.6 \text{ m}) = (0.3)(9)(1.6) = 0.48 \text{ J}$$

$$T_1 + V_1 = T_3 + V_3 \quad 0 + 7.68 = 0.15 v_B^2 + 0.48 \text{ J}$$

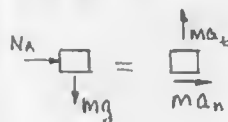
$$v_B^2 = \frac{7.68 - (0.48)(9.81)}{0.15} = 19.81 \text{ m}^2/\text{s}^2$$

$$\text{POSITION (4)} \quad \text{SINCE } V_4 = V_2 \text{ THE VELOCITY } v_A = v_C$$

$$v_C^2 = 35.50 \text{ m}^2/\text{s}^2$$

NEWTON'S SECOND LAW

(a) AT A



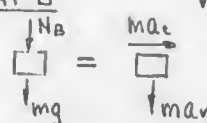
$$\Sigma F_n = N_A = m a_n$$

$$a_n = \frac{v_A^2}{R} = \frac{(35.50 \text{ m}^2/\text{s}^2)}{(0.8 \text{ m})}$$

$$N_A = (0.3 \text{ kg})(35.50 \text{ m}^2/\text{s}^2)/(0.8 \text{ m})$$

$$N_A = 13.31 \text{ N} \rightarrow$$

(b) AT B



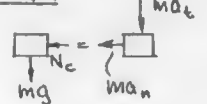
$$\Sigma F_n = N_B + mg = m a_n$$

$$a_n = \frac{v_B^2}{R} = \frac{(19.81 \text{ m}^2/\text{s}^2)}{(0.8 \text{ m})}$$

$$N_B = \frac{(0.3 \text{ kg})(19.81 \text{ m}^2/\text{s}^2)}{(0.8 \text{ m})} - (0.3 \text{ kg})(9.81 \text{ m/s}^2)$$

$$N_B = 4.49 \text{ N} \downarrow$$

(c) AT (C)



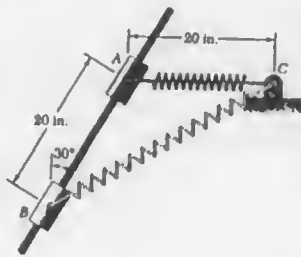
$$\Sigma F_n = N_C = m a_n \quad a_n = \frac{v_C^2}{R}$$

$$N_C = (0.3 \text{ kg})(35.50 \text{ m}^2/\text{s}^2)/(0.8 \text{ m})$$

$$N_C = 13.31 \text{ N}$$



## 13.C1



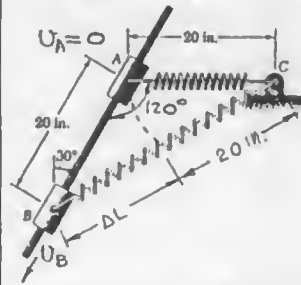
## GIVEN:

COLLAR  $W_A = 12 \text{ lb}$   
 SPRING IS  
 UNSTRETCHED WHEN  
 COLLAR IS AT A.  
 COLLAR RELEASED  
 FROM REST AT A

## FIND:

VELOCITY AT B  
 FOR  $k = 0.1 \text{ lb/in}$   
 TO  $2.0 \text{ lb/in}$  IN  
 $0.1 \text{ lb/in}$  INCREMENTS

WRITE EQUATION FOR  $U_B$  IN TERMS OF  $k$



## ANALYSIS

$$(20 + \Delta L)^2 = 20^2 + 20^2 - 2(20)(20)\cos 120^\circ$$

$$(20 + \Delta L)^2 = 800 + 400 = 1200$$

$$\Delta L = 14.64 \text{ in} = 1.220 \text{ ft}$$

## CONSERVATION OF ENERGY

$$U_A = 0 \quad T_A = \frac{1}{2} m U_A^2 = 0$$

$$T_B = \frac{1}{2} m U_B^2 = \frac{6}{g} U_B^2$$

$$V_A = 0 \quad (\text{DATUM AT A})$$

$$V_B = V_g + V_e$$

$$V_B = -(12 \text{ lb}) \left( \frac{20 \text{ ft}}{12} \right) (\cos 30^\circ) + \frac{1}{2} (k (16 \text{ in}) (12 \text{ in/ft}) (1.220 \text{ ft})^2)$$

$$V_B = (-17.32 + 8.932k) (16 \text{ ft}), (\text{INPUT } k \text{ IN lb/in.})$$

$$T_A + V_A = T_B + V_B \quad 0 + 0 = \left( \frac{6 \text{ lb}}{32.2 \text{ W/s}^2} \right) U_B^2 - 17.32 + 8.932k$$

$$U_B = [92.95 - 47.933k]^{1/2} (\text{ft/s}) \quad (1)$$

## OUTLINE OF PROGRAM

INPUT  $k$  IN (1) IN lb/in IN 0.1 lb/in INCREMENTS  
 AND STOP WHEN  $k = 2.0 \text{ lb/in}$

PRINT VALUES OF  $U_B$  (IN ft/s)

NOTE: COLLAR NEVER REACHES B FOR  
 $k > (92.95/47.933) = 1.939 \text{ lb/in}$

## PROGRAM OUTPUT

13.C1

K (LB/IN)	VELOCITY (FT/S)
0.10	9.39
0.20	9.13
0.30	8.86
0.40	8.59
0.50	8.31
0.60	8.01
0.70	7.71
0.80	7.39
0.90	7.06
1.00	6.71
1.10	6.34
1.20	5.95
1.30	5.54
1.40	5.08
1.50	4.59
1.60	4.03
1.70	3.39
1.80	2.58
1.90	1.37

## 13.C2

## GIVEN:

CAR WEIGHT,  $W = 2000 \text{ lb}$   
 FOR FIRST 60 FT ALL WEIGHT IS ON  
 THE REAR WHEELS WHICH ARE SLIPPING  
 FOR REMAINING 1260 FT, 60 % OF THE  
 WEIGHT IS ON THE REAR WHEELS  
 WITH SLIPPING IMPENDING.

$$\mu_s = 0.60 \quad \mu_k = 0.85$$

$$\text{AERODYNAMIC DRAG } F_d = 0.0098 U^2$$

$$\text{WITH } U \text{ IN FT/S AND } F_d \text{ IN LB.}$$

## FIND:

VELOCITY AND ELAPSED TIME WITH AND WITHOUT DRAG.  
 EVERY 5 FT FOR THE FIRST 60 FT AND  
 EVERY 90 FT FOR THE REMAINING 1260 FT.

ANALYSIS USE WORK AND ENERGY IN INCREMENTS  
 OF  $\Delta x_i = 0.1 \text{ ft}$ , BETWEEN  $i$ TH AND  $i+1$ TH INTERVAL

$$\boxed{\quad} + \boxed{\quad} = \boxed{\quad} \quad \text{TO GET } U_{i+1}$$

$N=W$

$(U_i = 0 \text{ FOR } i=0)$

$$T_i + (U_i \rightarrow U_{i+1}) = T_{i+1} \quad \frac{1}{2} m U_i^2 - (F_d + F_f) \Delta x_i = \frac{1}{2} m U_{i+1}^2$$

$$(1) \quad U_{i+1} = [U_i^2 + \frac{2g}{W} (F_f - F_d) \Delta x_i]^{1/2} \quad F_d = 0.0098 U_i^2$$

$$(2) \quad \Delta t_i = \frac{2 \Delta x_i}{(U_i + U_{i+1})} \quad \text{FIRST 60 FT } F_f = \mu_k W = (0.85) W$$

FOR REMAINING 1260 FT

$$F_f = (0.60) \mu_s W = 0.36 W$$

$$\Delta x_i = 0.1 \text{ ft}$$

$$g = 32.2 \text{ ft/s}^2$$

$$W = 2000 \text{ lb}$$

## OUTLINE OF PROGRAM

IDENTIFY  $U_i$  AND  $U_{i+1}$  AS THE VELOCITIES IN THE  $i$ TH  
 INTERVAL WITHOUT AND WITH DRAG, WITH  $U_i = 0$

AND  $F_f = 0.85 W$  USE A LOOP TO SOLVE FOR  $U_{i+1}$

AND TO SOLVE FOR  $t_i$ , SUM  $\Delta x_i$  TO FIND  $x_i$  AND

SUM  $\Delta t_i$  TO FIND  $t_i$ . PRINT  $U_i, t_i, x_i$  AT 5 FT INTERVALS

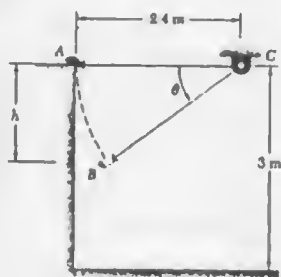
REPEAT FOR REMAINING 1260 FT WITH  $F_f = 0.36 W$ .

PRINT  $x_i, U_i, t_i$  AT 90 FT INTERVALS

13.C2

DISTANCE (FT)	V (FT/S) NO DRAG	T (S)	V (FT/S) DRAG	T (S)
5.	13.90	0.719	13.89	0.720
10.	19.66	1.017	19.64	1.018
15.	24.07	1.246	24.05	1.247
20.	27.80	1.439	27.76	1.440
25.	31.08	1.609	31.02	1.610
30.	34.05	1.762	33.97	1.764
35.	36.78	1.903	36.67	1.905
40.	39.31	2.035	39.19	2.037
45.	41.70	2.158	41.55	2.161
50.	43.95	2.275	43.78	2.278
55.	46.10	2.386	45.90	2.390
60.	48.15	2.492	47.92	2.496
60 FT. TO 1320 FT AT 90 FT. INTERVALS				
150.	72.65	3.984	71.76	4.000
240.	90.74	5.086	89.00	5.119
330.	105.78	6.002	103.02	6.057
420.	118.93	6.803	115.02	6.882
510.	130.77	7.524	125.60	7.630
600.	141.62	8.184	135.09	8.320
690.	151.70	8.798	143.71	8.966
780.	161.15	9.373	151.63	9.575
870.	170.08	9.917	158.95	10.155
960.	178.56	10.433	165.75	10.709
1050.	186.66	10.926	172.10	11.242
1140.	194.42	11.398	178.06	11.756
1230.	201.88	11.852	183.67	12.253
1320.	209.07	12.290	188.96	12.736

## 13.C3



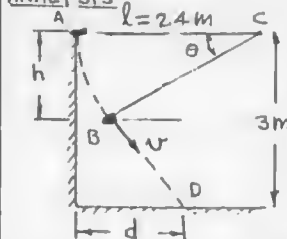
## GIVEN:

5-kg BAG  
ROPE = 2.4 m LONG  
INITIAL VELOCITY ZERO

## FIND:

FOR VALUES OF MAXIMUM TENSION  $F_m$  FROM 40 TO 140 N IN 5-N INCREMENTS, THE  
(a) DISTANCE  $h$   
(b) DISTANCE  $d$  FROM THE WALL TO THE POINT WHERE THE BAG HITS THE FLOOR.

## ANALYSIS



BAG MOVES ALONG A CIRCULAR ARC AB UNTIL THE ROPE BREAKS (RADIUS,  $l$ )

## NEWTON'S LAW

$$\begin{aligned} \sum F_r &= m a_r \\ F_m - mg \cos \theta &= \frac{mv^2}{l} \\ F_m &= \frac{mv^2}{l} + mg \cos \theta \end{aligned}$$

## CONSERVATION OF ENERGY

$$v = \sqrt{2gh} \quad (1)$$

$$F_m = \frac{2mg}{l} h + mg \cos \theta = \frac{3mg}{l} h$$

$$\sin \theta = \frac{h}{l} \quad (2) \quad h = \frac{F_m l}{3mg} \quad (3)$$

## FROM B TO D (PROJECTILE TRAJECTORY)

$$v_H = v \sin \theta \rightarrow d = (l - l \cos \theta) + v_H t_D \quad (4)$$

$$v_V = v \cos \theta \quad (3-h) = v_V t_D + g t_D^2 / 2$$

$$t_D = \frac{-v_V + \sqrt{v_V^2 + 2(3-h)g}}{g} \quad (5)$$

## OUTLINE OF PROGRAM

WITH  $l = 2.4$  m,  $m = 5$  kg,  $g = 9.81$  m/s<sup>2</sup> IN EQUATION (3), AND FOR  $F_m$  IN 5-N INCREMENTS FROM 40 TO 140 N, SOLVE FOR  $h$ . FOR EACH  $h$ , SOLVE FOR  $v$  (EQ 1), AND  $\theta$  (EQ 2). SOLVE FOR  $v_H$  AND  $v_V$  (EQ 6) AND WITH  $v_V$  AND  $h$ , SOLVE FOR  $t_D$  (EQ 5) AND WITH  $\theta$ ,  $h$ ,  $t_D$  SOLVE FOR  $d$  IN (EQ 4). PRINT  $h$  AND  $d$  FOR EACH  $F_m$ .

## PROGRAM OUTPUT

FORCE (NEWTONS)	H (METERS)	d (METERS)
40	0.652	0.503
45	0.734	0.585
50	0.815	0.668
55	0.897	0.752
60	0.979	0.839
65	1.060	0.927
70	1.142	1.017
75	1.223	1.109
80	1.305	1.203
85	1.386	1.300
90	1.468	1.401
95	1.549	1.505
100	1.631	1.615
105	1.713	1.731
110	1.794	1.854
115	1.876	1.989
120	1.957	2.137
125	2.039	2.306
130	2.120	2.504
135	2.202	2.753
140	2.283	3.101

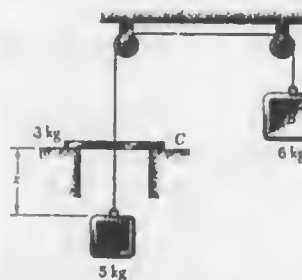
## 13.C4

## GIVEN:

INITIALLY  
 $v_A = 0$   $v = 1.7$  m  
PLASTIC IMPACT  
BETWEEN A AND C  
 $e = 0$

## FIND:

(a) TIME TO COMPLETE 10 COMPLETE CYCLES  
(b) VALUE OF  $k$  AFTER THE 10<sup>TH</sup> CYCLE

ANALYSIS (FOR THE  $i^{th}$  CYCLE)

REFER TO FIGURES IN THE SOLUTION TO PROB. 13.199 FROM (1) TO (2) CONSERVATION OF ENERGY (BEFORE IMPACT)

$$T_1 = 0, v_1 = 0, T_2 = \frac{1}{2} (5+6) (v_A^2) = \frac{11}{2} (v_A^2)$$

$$v_2 = (5-6)g(x_i) = -gx_i$$

$$T_1 + v_1 = T_2 + v_2 \quad 0 + 0 = \frac{11}{2} (v_A^2) - gx_i \quad (v_A^2) = \frac{2}{11} gx_i \quad (1)$$

TIME  $(t_{1-2})_i$

ACCELERATION FROM (1) TO (2) IS CONSTANT, THUS AVERAGE VELOCITY IS  $(\bar{v}_A)_i = \frac{0 + (v_A)_i}{2}$

$$\text{AND } (\bar{v}_A)_i = \frac{x_i}{(t_{1-2})_i} \quad (t_{1-2})_i = \frac{2x_i}{(v_A)_i} = \frac{2x_i}{\sqrt{\frac{2}{11}gx_i}} \quad (t_{1-2})_i = 1.498 \sqrt{x_i} \quad (2)$$

(AFTER IMPACT) AT (2)

## IMPULSE-MOMENTUM FOR A AND C

$$5(v_A)_i + 6(v_C)_i = (5+6)(v_A')_i$$

## IMPULSE-MOMENTUM FOR B

$$6(v_A')_i - 6(v_C')_i = 6(v_A')_i$$

$$\text{ADDING } 11(v_A')_i = 14(v_A')_i \quad (v_A')_i = \frac{14}{11} (v_A)_i = \sqrt{\frac{11}{98}gx_i} \quad (3)$$

FROM (2) TO (4), (SEE (b) IN SOLUTION TO PROB. 13.199)

## CONSERVATION OF ENERGY DATUM AT (2)

$$T_2 = \frac{1}{2} (5+6) (v_A')^2 = \frac{11}{2} \left( \frac{14}{11} (v_A)_i \right)^2 = \frac{121}{196} gx_i \quad v_2 = 0$$

$$T_4 = 0 \quad v_4 = m_B g x_{i+1} - m_A g x_{i+1} = (6-5)g x_{i+1} = g x_{i+1}$$

$$T_2 + v_2 = T_4 + v_4 \quad \frac{121}{196} gx_i + 0 = 0 + g x_{i+1} \quad x_{i+1} = \frac{121}{196} x_i \quad (4)$$

TIME (2) TO (4)  $(t_{2-4})_i$

$$(t_{2-4})_i = \frac{2x_{i+1}}{(v_A')_i} = \frac{2 \left( \frac{121}{196} x_i \right)}{\left( \frac{14}{11} \sqrt{\frac{11}{98}gx_i} \right)} = 1.1766 \sqrt{x_i} \quad (5)$$

TIME FROM (2) TO (3) AND FROM (3) TO (2)

$$T_2 = \frac{1}{2} (m_A + m_B + m_C) (v_A')^2 = 7 \left( \frac{11}{98} gx_i \right) = \frac{77}{98} gx_i$$

$$T_3 = 0 \quad v_3 = (m_A + m_C)g d_i - m_B g d_i = (6-6)g d_i = 0$$

$$T_2 + v_2 = T_3 + v_3 \quad \frac{77}{98} gx_i + 0 = 0 + 0 \quad d_i = \frac{77}{196} x_i$$

$$(t_{2-3})_i = \frac{2d_i}{(v_A')_i}$$

$$(t_{2-3})_i = 2 \left( \frac{77}{196} x_i \right) / \left( \frac{14}{11} \sqrt{\frac{11}{98}gx_i} \right) = 0.7488 \sqrt{x_i} \quad (6)$$

$$(t_{3-2})_i = (t_{2-3})_i = 0.7488 \sqrt{x_i} \quad (7)$$

(CONTINUED)

### 13.C4 continued

TOTAL TIME TO COMPLETE THE  $L^{\text{TH}}$  CYCLE

Eqs. (2) + (6) + (7) + (5)

$$t_L = (t_{1-2})_L + (t_{2-3})_L + (t_{3-2})_L + (t_{2-1})_L$$

$$t_L = (1.498 + 0.7488 + 0.7488 + 1.1766) \sqrt{x_L}$$

$$t_L = 4.172 \sqrt{x_L} \quad (8)$$

OUTLINE OF PROGRAM

SET  $x_L = 1.7 \text{ M}$  ( $L=1$ )

(a) CALCULATE  $x_{L+1}$  FROM EQUATION (4) FOR  $L=1$  TO  $L=10$ . FOR EACH VALUE OF  $x$  USE EQUATION (8) TO DETERMINE  $t$  FOR THE  $L^{\text{TH}}$  CYCLE. SUM  $t$ 'S TO OBTAIN THE TOTAL TIME THROUGH THE  $10^{\text{TH}}$  CYCLE.

(b) FOR  $L=10$  OBTAIN  $x$  FOR THE TENTH CYCLE

PRINT TOTAL TIME AND  $x$  FOR THE  $10^{\text{TH}}$  CYCLE.

PROGRAM OUTPUT

TOTAL TIME=23.1 SECONDS

$x$  FOR THE TENTH CYCLE=0.01367 METERS

### 13.C5

GIVEN:

$m_B = 700 \text{ g}$ ,  $m_A = 350 \text{ g}$   
 $v_0 = 6 \text{ m/s}$   $v_B = 0$   
 $\theta_0 = 20^\circ$  TO  $150^\circ$  IN  
 $10^\circ$  INCREMENTS

FIND:

$u_A'$  AND  $u_B'$  AFTER IMPACT

AND ENERGY LOST FOR,

(a)  $e=1$

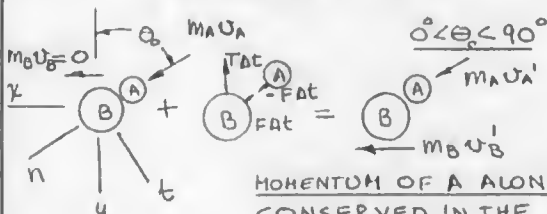
(b)  $e=0.75$

(c)  $e=0$



ANALYSIS:

DEVELOP FORMULAS FOR  $u_A'$  AND  $u_B'$  IN TERMS OF  $\theta$  AND  $e$



MOENTUM OF A ALONE IS CONSERVED IN THE  $t$  DIRECTION

$m_A(u_A)_t = m u_A't$   
 $(u_A)_t = 0$  THUS  $u_A't = 0$   
 AND  $u_A'$  IS ALONG THE  $n$  AXIS

KINEMATICS

$$(u_B')_x = u_B'$$

### 13.C5 continued

CONSERVATION OF MOMENTUM IN THE  $x$  DIRECTION FOR A AND B TOGETHER

$$m_A u_A \sin \theta_0 = m_A u_A' \sin \theta_0 + m_B u_B'$$

$$m_A = 0.350 \text{ kg} \quad m_B = 0.700 \text{ kg} \quad u_A = u_B = 6 \text{ m/s}$$

$$6 \sin \theta_0 = u_A' \sin \theta_0 + 2 u_B' \quad (1)$$

RELATIVE VELOCITIES IN THE  $n$  DIRECTION

$$(u_A - 0)e = u_B' \sin \theta_0 - u_A' \quad u_A = u_B = 6 \text{ m/s}$$

$$6e = u_B' \sin \theta_0 - u_A' \quad (2)$$

MULTIPLY (2) BY  $\sin \theta_0$  AND ADD TO (1) TO GET  $u_B'$

$$u_B' = \frac{6 \sin \theta_0 (1+e)}{(2 + \sin^2 \theta_0)} \quad (3)$$

SUBSTITUTE (3) IN (2) FOR  $u_A'$

$$u_A' = \frac{6 \sin^2 \theta_0 - 12e}{(2 + \sin^2 \theta_0)} \quad (4)$$

FOR  $\theta_0 \geq 90^\circ$ ,  $\Delta t = 0$ , AND BALL A AT A VELOCITY OF  $6 \text{ m/s}$  HITS BALL B WHICH IS AT  $0$  VELOCITY AND IS NOT CONSTRAINED BY THE CORD. THUS IF ONLY MAGNITUDES ARE CONSIDERED  $u_A'$  AND  $u_B'$  HAVE VALUES FOR  $110^\circ < \theta < 90^\circ$  WHICH ARE THE AS FOR  $\theta = 90^\circ$

ENERGY LOST

$$\Delta E = \frac{1}{2} m_A u_A^2 - \frac{1}{2} (m_A u_A'^2 + m_B u_B'^2)$$

$$\Delta E = \frac{1}{2} (0.350) [u_A^2 - u_A'^2] - \frac{1}{2} (0.700) u_B'^2 \quad (5)$$

OUTLINE OF PROGRAM

INPUT  $\theta_0$  INTO EQUATIONS (3) AND (4) FROM  $20^\circ$  TO  $90^\circ$  IN INCREMENTS OF  $5^\circ$  FOR  $e=1$ ,  $e=0.75$  AND  $e=0$  TO OBTAIN  $u_A'$  AND  $u_B'$ . SUBSTITUTE  $u_A'$  AND  $u_B'$  IN (5) TO OBTAIN  $\Delta E$ . PRINT  $e$ ,  $\theta_0$ ,  $u_A'$ ,  $u_B'$ ,  $\Delta E$

PROGRAM OUTPUT

#### 13.C5

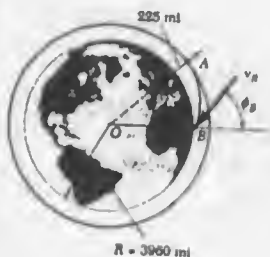
e	THETA (DEG)	VEL A (M/S)	VEL B (M/S)	% E LOST
1.00	20.	-5.337	1.939	0.0
1.00	30.	-4.667	2.667	0.0
1.00	40.	-3.945	3.196	0.0
1.00	50.	-3.278	3.554	0.0
1.00	60.	-2.727	3.779	0.0
1.00	70.	-2.325	3.911	0.0
1.00	80.	-2.081	3.979	0.0
1.00	90.	-2.000	4.000	0.0
0.75	20.	-3.920	1.696	41.3
0.75	30.	-3.333	2.333	38.9
0.75	40.	-2.702	2.797	36.3
0.75	50.	-2.118	3.109	33.8
0.75	60.	-1.636	3.307	31.8
0.75	70.	-1.284	3.422	30.4
0.75	80.	-1.071	3.482	29.5
0.75	90.	-1.000	3.500	29.2
0.00	20.	0.332	0.969	94.5
0.00	30.	0.667	1.333	88.9
0.00	40.	1.027	1.598	82.9
0.00	50.	1.361	1.777	77.3
0.00	60.	1.636	1.890	72.7
0.00	70.	1.838	1.956	69.4
0.00	80.	1.959	1.990	67.3
0.00	90.	2.000	2.000	66.7

VALUES FOR ANGLES OF  $90$  TO  $150$  DEGREES ARE THE SAME AS THOSE FOR  $90$  DEGREES

## 13.C6

## GIVEN:

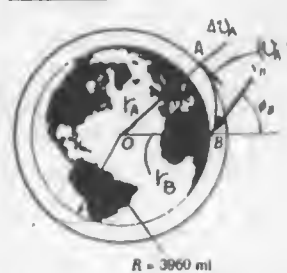
INITIAL CIRCULAR ORBIT  
OF 225 MI ABOVE THE  
SURFACE OF THE EARTH  
INCREMENTAL VELOCITY  
 $\Delta U_A$  TOWARD THE  
CENTER OF THE EARTH



## FIND:

$U_B$  AND  $\phi_B$  AT AN ALTITUDE  
OF 40 MI FOR ENERGY  
EXPENDITURE OF 5  
TO 100 % OF THAT  
USED IN PROB. 13.109  
IN 5% INCREMENTS

## ANALYSIS



## CONSERVATION OF ENERGY

## AT POINT A

$$T_A = \frac{1}{2} m [(U_A)_{\text{circ}}^2 + (\Delta U)^2]$$

$$V_A = -\frac{GM}{r_A}$$

## AT POINT B

$$T_B = \frac{1}{2} m U_B^2$$

$$V_B = -\frac{GM}{r_B}$$

$$r_A = 3960 + 225 = 4185 \text{ mi}$$

$$r_B = 3960 + 40 = 4000 \text{ mi}$$

$$T_A + V_A = T_B + V_B$$

$$(U_A)_{\text{circ}}^2 = \frac{gR^2}{r_A}$$

$$\frac{1}{2} m [(U_A)_{\text{circ}}^2 + (\Delta U)^2] - \frac{GMm}{r_A} = \frac{1}{2} m U_B^2 - \frac{GMm}{r_B}$$

$$U_B^2 = (U_A)_{\text{circ}}^2 + (\Delta U)^2 + 2GM \left[ \frac{1}{r_B} - \frac{1}{r_A} \right] \quad (1)$$

## ENERGY EXPENDITURE IN PROB. 13.109

LET  $U_A$  = VELOCITY AT A IN PROB. 13.109 TO  
BRING THE VEHICLE TO B AT  $\phi = 60^\circ$

FROM 13.109,  $U_A = 11.32 \times 10^3 \text{ ft/s}$

$$\text{ENERGY EXPENDITURE, } E = \frac{1}{2} m [(U_A)_{\text{circ}}^2 - (U_A)^2] \quad (2)$$

## ENERGY EXPENDITURE IN THIS PROBLEM

$$KE = \frac{1}{2} m (\Delta U)^2, \text{ WHERE } K \text{ IS THE \% ENERGY}$$

USED IN PROB 13.109.

SOLVING FOR  $(\Delta U)^2$  AND REPLACING E BY  
EQUATION (2)

$$(\Delta U)^2 = \frac{K}{100} [(U_A)_{\text{circ}}^2 - (U_A)^2] \quad (3)$$

EQUATION FOR  $U_B$  (SUBSTITUTE (3) INTO (1))

$$U_B = \left\{ (U_A)_{\text{circ}}^2 + \frac{K}{100} [(U_A)_{\text{circ}}^2 - (U_A)^2] + 2GM \left[ \frac{1}{r_B} - \frac{1}{r_A} \right] \right\}^{\frac{1}{2}} \quad (4)$$

## CONSTANTS:

$$(U_A)_{\text{circ}}^2 = \frac{gR^2}{r_A} = \frac{(32.2)(3960)(5280)^2}{(4185)(5280)}$$

$$(U_A)_{\text{circ}}^2 = 637.07 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$\text{FROM 13.109, } (U_A)^2 = (11.32 \times 10^3)^2 = 128.14 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$2GM \left[ \frac{1}{r_B} - \frac{1}{r_A} \right] = 2gR^2 \left[ \frac{1}{r_B} - \frac{1}{r_A} \right] = \frac{2(32.2)(3960)(5280)^2}{5280} \left[ \frac{1}{4000} - \frac{1}{4185} \right]$$

(CONTINUED)

## 13.C6 continued

$$2GM \left[ \frac{1}{r_B} - \frac{1}{r_A} \right] = 58.93 \times 10^6 \text{ ft}^2/\text{s}^2$$

EQUATION FOR  $\phi_B$ 

## CONSERVATION OF ANGULAR MOMENTUM

$$r_A (U_A)_{\text{circ}} = r_B U_B \sin \phi_B$$

$$\phi_B = \sin^{-1} [r_A (U_A)_{\text{circ}} / r_B U_B] \quad (5)$$

## OUTLINE OF PROGRAM

INPUT CONSTANTS INTO EQUATION (4) AND  
SOLVE FOR  $U_B$  FOR VALUES OF K OF 5%  
TO 100% AT INTERVALS OF 5% FOR EACH  
VALUE OF  $U_B$  AND USING THE GIVEN CONSTANT  
VALUES OF  $(U_A)_{\text{circ}}$ ,  $r_A$  AND  $r_B$ , USE EQUATION (5)  
TO SOLVE FOR  $\phi_B$ . PRINT K,  $U_B$  AND  $\phi_B$ .

## PROGRAM OUTPUT

13.C6

K (%)	VB (FT/S)	PHI (DEGREES)
5.	26860.	79.5
10.	27329.	75.1
15.	27791.	71.8
20.	28245.	69.2
25.	28692.	67.0
30.	29132.	65.0
35.	29566.	63.3
40.	29993.	61.7
45.	30414.	60.3
50.	30830.	58.9
55.	31240.	57.7
60.	31644.	56.6
65.	32044.	55.5
70.	32438.	54.5
75.	32828.	53.6
80.	33214.	52.7
85.	33595.	51.8
90.	33971.	51.0
95.	34344.	50.3
100.	34712.	49.5

14.1



GIVEN:

- (1) 15-kg SUITCASE A TOSSED WITH VELOCITY OF 3 m/s  $\rightarrow$ .
- (2) 20-kg SUITCASE B TOSSED WITH VELOCITY OF 2 m/s  $\rightarrow$ .
- (3) 25-kg CARRIER INITIALLY AT REST

FIND: FINAL VELOCITY OF CARRIER

- (a) IF 15-kg SUITCASE IS TOSSED FIRST.
- (b) IF 20-kg SUITCASE IS TOSSED FIRST.

(a) 15-kg SUITCASE TOSSED ON CARRIER FIRST:

CONSERVATION OF MOMENTUM:

$$(15 \text{ kg})(3 \text{ m/s}) + (15 \text{ kg} + 25 \text{ kg})v_1$$

$$(15)(3) = (40)v_1$$

$$v_1 = 1.125 \text{ m/s}$$

20-kg SUITCASE TOSSED NEXT:

$$(20)(2) + (40)(1.125) = (20+40)v_2$$

$$(20)(2) + (40)(1.125) = 60v_2$$

$$v_2 = 1.417 \text{ m/s} \rightarrow$$

(b) 20-kg SUITCASE TOSSED ON CARRIER FIRST:

CONSERVATION OF MOMENTUM:

$$(20)(2) + (20 \text{ kg} + 25 \text{ kg})v_1$$

$$(20)(2) = (45)v_1$$

$$v_1 = 0.8889 \text{ m/s}$$

15-kg SUITCASE TOSSED NEXT:

$$(15)(3) + (45)(0.8889) = (15+45)v_2$$

$$(15)(3) + (45)(0.8889) = 60v_2$$

$$v_2 = 1.417 \text{ m/s} \rightarrow$$

14.2



GIVEN:

EMPLOYEE TOSSES TWO SUITCASES A AND B ON CARRIER WITH  $v_0 = 2.4 \text{ m/s} \rightarrow$ .

MASS OF SUITCASE A IS 15 kg  
MASS OF CARRIER IS 25 kg

(a) FIND  $m_B$  KNOWING THAT $v_{\text{FINAL}} = 1.2 \text{ m/s} \rightarrow$ .(b) FIND  $v_{\text{FINAL}}$  IF B IS TOSSED FIRST

(a) CONSERVATION OF MOMENTUM:

$$m_B(2.4 \text{ m/s}) + (15 \text{ kg})(2.4 \text{ m/s}) = (m_B + 15 + 25)v_{\text{FINAL}}$$

$$2.4(m_B + 15) = (m_B + 40)v_{\text{FINAL}} \quad (1)$$

LET  $v_{\text{FINAL}} = 1.2 \text{ m/s}$ :

$$2.4m_B + 36 = 1.2m_B + 48$$

$$m_B = 10.00 \text{ kg}$$

(b) F.B.I. EQUATION IS STILL VALID

LET  $m_B = 10.00 \text{ kg}$  IN EQ. (1):

$$2.4(10 + 15) = (10 + 40)v_{\text{FINAL}}$$

$$v_{\text{FINAL}} = 1.200 \text{ m/s}$$

14.3



GIVEN:

180-lb MAN  
120-lb WOMAN  
300-lb BOAT  
MAN AND WOMAN DIVE WITH  
16 ft/s VELOCITY W/R BOAT.

FIND: FINAL VELOCITY OF BOAT IF

(a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST.

(a) WOMAN DIVES FIRST

CONSERVATION OF MOMENTUM:

$$(300+180)v_1 + 120(16-v_1)$$

$$0 =$$

$$\pm x\text{-COMP: } -(300+180)v_1 + 120(16-v_1) = 0 \quad (1)$$

$$-600v_1 + 1920 = 0 \quad v_1 = 3.20 \text{ ft/s}$$

MAN DIVES NEXT:

$$(300+180)v_1 + 300v_2 + 180(16-v_2)$$

$$\pm x\text{-COMP: } -(300+180)v_1 = -300v_2 + 180(16-v_2) \quad (2)$$

$$-480(3.20) = -480v_2 + 2880, \quad v_2 = 9.20 \text{ ft/s} \leftarrow$$

(b) MAN DIVES FIRST.

SIMILAR ANALYSIS YIELDS THE FOLLOWING ERS.:

$$-(300+120)v_1 + 180(16-v_1) = 0 \quad v_1 = 4.80 \text{ ft/s}$$

$$-(300+120)v_1 = -300v_2 + 120(16-v_2), \quad v_2 = 9.37 \text{ ft/s} \leftarrow$$

14.4



GIVEN:

180-lb MAN  
120-lb WOMAN  
300-lb BOAT  
MAN AND WOMAN DIVE WITH  
16 ft/s VELOCITY W/R BOAT  
IN OPPOSITE DIRECTIONS

FIND: FINAL VELOCITY OF BOAT IF

(a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST

(a) WOMAN DIVES FIRST

CONSERVATION OF MOMENTUM:

$$120(16-v_1) + (300+180)v_1$$

$$0 =$$

$$\pm x\text{ COMP: } -120(16-v_1) + (300+180)v_1 = 0 \quad (1)$$

$$600v_1 - 1920 = 0 \quad v_1 = 3.20 \text{ ft/s}$$

MAN DIVES NEXT:

$$(300+180)v_1 + 300v_2 + 180(v_2+16)$$

$$\pm x\text{ COMP: } 480(3.20) = 300v_2 + 180(v_2+16) \quad (2)$$

$$480v_2 = -1344, \quad v_2 = -2.80 \quad v_2 = 2.80 \text{ ft/s} \leftarrow$$

(b) MAN DIVES FIRST

SIMILAR ANALYSIS YIELDS THE FOLLOWING EQS.:

$$+180(16+v_1) + (300+120)v_1 = 0 \quad v_1 = -4.80 \text{ ft/s}$$

$$420(-4.80) = 300v_2 + 120(v_2-16)$$

$$420v_2 = -96$$

$$v_2 = -0.229 \text{ ft/s}$$

$$v_2' = 0.229 \text{ ft/s} \leftarrow$$

14.5



GIVEN: IDENTICAL CARS.  $v_0 = 1,920 \text{ m/s}$   
 AFTER A HITS B:  $(v_B)_1 = 1,680 \text{ m/s}$   
 AFTER B HITS C:  $(v_B)_2 = 0,210 \text{ m/s}$   
 AFTER A HITS B AGAIN:  $(v_B)_3 = 0,23625 \text{ m/s}$   
 FIND: (a) FINAL VELOCITIES OF A AND C, (b) COEFF. e

CONSERVATION OF MOMENTUM:

$$m(1,920) = m(v_A)_1 + m(1,680) \quad \text{A HITS B}$$

$$(v_A)_1 = 1,920 - 1,680 = 0,240 \text{ m/s}$$

$$m(1,680) = m(0,210) + m(v_C)_F \quad \text{B HITS C}$$

$$(v_C)_F = 1,680 - 0,210 = 1,470 \text{ m/s}$$

$$m(0,240) + m(0,210) = m(v_A)_F + m(0,23625) \quad \text{A HITS B AGAIN}$$

$$(v_A)_F = 0,21375 \text{ m/s}$$

$$0,240 + 0,210 = (v_A)_F + 0,23625 \quad (v_A)_F = 0,21375 \text{ m/s}$$

$$(a) \quad v_A = 0,214 \text{ m/s} \rightarrow ; v_C = 1,470 \text{ m/s} \rightarrow$$

$$(b) \text{ FIRST COLLISION: } e = \frac{1,680 - 0,240}{1,920} = 0,750$$

$$\text{SECOND COLLISION: } e = \frac{1,470 - 0,210}{1,680} = 0,750$$

$$\text{THIRD COLLISION: } e = \frac{0,23625 - 0,21375}{0,240 - 0,210} = 0,750$$

14.6



GIVEN: IDENTICAL CARS.  $v_0 = 2,00 \text{ m/s}$   
 AFTER A HITS B:  $(v_A)_1 = 0,400 \text{ m/s}$   
 AFTER B HITS C:  $v_C = 1,280 \text{ m/s}$   
 AFTER A HITS B AGAIN:  $(v_A)_2 = 0,336 \text{ m/s}$   
 FIND: (a)  $v_B$  AFTER EACH COLLISION, (b) COEFF. e

CONSERVATION OF MOMENTUM:

$$m(2,00) = m(0,400) + m(v_B)_1 \quad \text{A HITS B}$$

$$(v_B)_1 = 1,600 \text{ m/s} \rightarrow$$

$$m(1,600) = m(v_B)_2 + m(1,280) \quad \text{B HITS C}$$

$$(v_B)_2 = 0,320 \text{ m/s} \rightarrow$$

$$m(0,400) + m(0,320) = m(0,336) + m(v_B)_3 \quad \text{A HITS B AGAIN}$$

$$(v_B)_3 = 0,384 \text{ m/s} \rightarrow$$

$$0,400 + 0,320 = 0,336 + (v_B)_3 \quad (v_B)_3 = 0,384 \text{ m/s}$$

$$(b) \text{ FIRST COLLISION: } e = \frac{1,600 - 0,400}{2,00} = 0,600$$

$$\text{SECOND COLLISION: } e = \frac{1,280 - 0,320}{1,600} = 0,600$$

$$\text{THIRD COLLISION: } e = \frac{0,384 - 0,336}{0,400 - 0,320} = 0,600$$

14.7



GIVEN:

BULLET FIRED  
 THROUGH A AND,  
 BECOMES EMBEDDED  
 IN B. BLOCKS MOVE  
 WITH  $v = 5 \text{ ft/s}$   
 AND  $v_B = 9 \text{ ft/s}$

FIND: (a) WEIGHT  $w$  OF BULLET  
 (b) VELOCITY OF BULLET BETWEEN A AND B.

CONSERVATION OF MOMENTUM:

(a) BULLET GOES THROUGH BLOCK A INTO BLOCK B  
 $(w)(1500) = (5)(5) + (4,95 + w)(9)$

$$1500w = 30 + 44,55 + 9w$$

$$1491w = 74,55$$

$$w = 0,0500 \text{ lb}, \quad w = 0,800 \text{ oz}$$

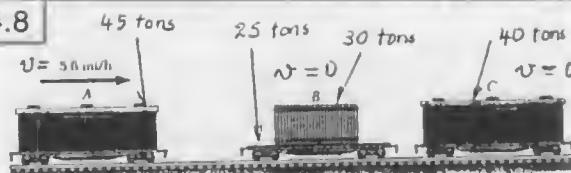
(b) BULLET GOES THROUGH BLOCK A

$$(0,05)(1500) = (5)(5) + (0,05)v$$

$$75 = 30 + 0,05v$$

$$v = 900 \text{ ft/s}$$

14.8



GIVEN: CARS AND CONTAINER SHOWN.

CARS GET COUPLED AS THEY HIT EACH OTHER.

FIND:

VELOCITY OF CAR A AFTER EACH COUPLING,  
 ASSUMING THAT CONTAINER

(a) DOES NOT SLIDE ON FLATCAR

(b) SLIDES AFTER FIRST COUPLING BUT HITS STOP  
 BEFORE SECOND COUPLING

(c) SLIDES AFTER BOTH COUPLINGS

CONSERVATION OF MOMENTUM:

(a) CONTAINER DOES NOT SLIDE

$$(45)(5,6) = (100)v_1 = (140)v_2$$

$$252 = 100v_1 = 140v_2$$

$$v_1 = 2,52 \text{ m/s}$$

$$v_2 = 1,800 \text{ m/s}$$

(b) CONTAINER SLIDES AFTER 1<sup>st</sup> COUPLING, STOPS BEFORE 2<sup>nd</sup>

$$(45)(5,6) = (70)v_1 = (140)v_2$$

$$252 = 70v_1 = 140v_2$$

$$v_1 = 3,60 \text{ m/s}$$

$$v_2 = 1,800 \text{ m/s}$$

(c) CONTAINER SLIDES AND STOPS ONLY AFTER 2<sup>nd</sup> COUPLING

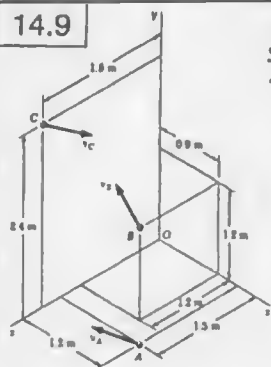
$$(45)(5,6) = (70)v_1 = (110)v_2$$

$$252 = 70v_1 = 110v_2$$

$$v_1 = 3,60 \text{ m/s}$$

$$v_2 = 2,29 \text{ m/s}$$

14.9



GIVEN:

SYSTEM OF PARTICLES WITH  
 $m_A = 3 \text{ kg}$ ,  $m_B = 4 \text{ kg}$ ,  
 $m_C = 5 \text{ kg}$

AND VELOCITIES (m/s)

$$\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$$

$$\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$$

FIND:

ANGULAR MOMENTUM  $\mathbf{H}_O$ 

$$\mathbf{H}_O = \mathbf{r}_A \times m_A \mathbf{v}_A + \mathbf{r}_B \times m_B \mathbf{v}_B + \mathbf{r}_C \times m_C \mathbf{v}_C$$

USING DETERMINANT FORM FOR VECTOR PRODUCTS  
 AND FACTORING MASSES:

$$\mathbf{H}_O = (3 \text{ kg}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -4 & 4 & 6 \end{vmatrix} + 4 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -6 & 8 & 4 \end{vmatrix} + 5 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k} - 19.2\mathbf{i} - 43.2\mathbf{j} + 57.6\mathbf{k} + 6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k}$$

$$\mathbf{H}_O = -(31.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (64.8 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (48.0 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

14.10

GIVEN:

SYSTEM OF PARTICLES OF PROB. 14.9.

FIND:

(a) POSITION VECTOR  $\mathbf{z}$  OF MASS CENTER G.

(b) LINEAR MOMENTUM OF SYSTEM.

(c) ANGULAR MOMENTUM  $\mathbf{H}_G$  OF SYSTEM.

ALSO: VERIFY THAT ANSWERS TO PROBS. 14.9  
 AND 14.10 SATISFY EQUATION

$$\mathbf{H}_O = \mathbf{z} \times m \mathbf{v} + \mathbf{H}_G$$

(a) EQ. (14.12):

$$m \mathbf{z} = \sum m_i \mathbf{z}_i$$

$$(3+4+5)\mathbf{z} = 3(1.2\mathbf{i} + 1.5\mathbf{k}) + 4(0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}) + 5(2.4\mathbf{j} + 1.8\mathbf{k})$$

$$12\mathbf{z} = 7.2\mathbf{i} + 16.8\mathbf{j} + 18.3\mathbf{k}$$

$$\mathbf{z} = (0.600\text{ m})\mathbf{i} + (1.400\text{ m})\mathbf{j} + (1.525\text{ m})\mathbf{k}$$

$$(b) \mathbf{L} = \sum m_i \mathbf{v}_i = 3(-4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) + 4(-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}) + 5(2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{L} = (-26.0 \text{ kg}\cdot\text{m/s})\mathbf{i} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{k}$$

$$(c) \mathbf{H}_G = \mathbf{r}_{A/G} \times m_A \mathbf{v}_A + \mathbf{r}_{B/G} \times m_B \mathbf{v}_B + \mathbf{r}_{C/G} \times m_C \mathbf{v}_C$$

$$\text{WHERE } \mathbf{r}_{A/G} = \mathbf{r}_A - \mathbf{z} = 1.2\mathbf{i} + 1.5\mathbf{k} - (0.6\mathbf{i} + 1.4\mathbf{j} + 1.525\mathbf{k})$$

$$= 0.6\mathbf{i} - 1.4\mathbf{j} - 0.025\mathbf{k}$$

$$\mathbf{r}_{B/G} = \mathbf{r}_B - \mathbf{z} = 0.9\mathbf{i} - 0.2\mathbf{j} - 0.325\mathbf{k}$$

$$\mathbf{r}_{C/G} = \mathbf{r}_C - \mathbf{z} = -0.6\mathbf{i} + \mathbf{j} + 0.275\mathbf{k}$$

$$\mathbf{H}_G = (3 \text{ kg}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & -1.4 & -0.025 \\ -4 & 4 & 6 \end{vmatrix} + 4 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & -0.2 & -0.325 \\ -6 & 8 & 4 \end{vmatrix} + 5 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.6 & 1 & 0.275 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -24.9\mathbf{i} - 10.5\mathbf{j} - 3.6\mathbf{k} + 7.2\mathbf{i} + 3\mathbf{j} + 7.0\mathbf{k} - 11.75\mathbf{i} - 22.5\mathbf{j} + 8\mathbf{k}$$

$$\mathbf{H}_G = -(29.45 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (16.75 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (3.20 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

(CONTINUED)

14.10 continued

WE COMPUTE  $\mathbf{z} \times m \mathbf{v}$ :

$$\mathbf{z} \times m \mathbf{v} = \mathbf{z} \times \mathbf{L} = (0.6\mathbf{i} + 1.4\mathbf{j} + 1.525\mathbf{k}) \times (-26\mathbf{i} + 14\mathbf{j} + 14\mathbf{k})$$

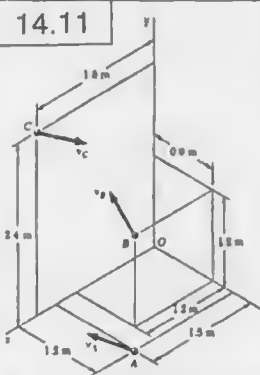
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix} = -1.75\mathbf{i} - 48.05\mathbf{j} + 44.8\mathbf{k}$$

$$\text{THUS: } \mathbf{z} \times m \mathbf{v} + \mathbf{H}_G = -1.75\mathbf{i} - 48.05\mathbf{j} + 44.8\mathbf{k} - 29.45\mathbf{i} - 16.75\mathbf{j} + 3.20\mathbf{k}$$

$$= -31.2\mathbf{i} - 64.8\mathbf{j} + 48.0\mathbf{k}$$

WHICH IS THE EXPRESSION OBTAINED FOR  $\mathbf{H}_O$  IN PROB. 14.9.

14.11



GIVEN: SYSTEM OF PARTICLES

WITH  $m_A = 3 \text{ kg}$ ,  $m_B = 4 \text{ kg}$ ,  $m_C = 5 \text{ kg}$ 

AND VELOCITIES (m/s)

$$\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$$

$$\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$$

FIND:

(a)  $v_x$  AND  $v_y$  FOR WHICH $\mathbf{H}_O$  IS PARALLEL TO  $z$  AXIS(b) CORRESPONDING  $\mathbf{H}_O$ .

$$\mathbf{H}_O = \mathbf{r}_A \times m_A \mathbf{v}_A + \mathbf{r}_B \times m_B \mathbf{v}_B + \mathbf{r}_C \times m_C \mathbf{v}_C$$

$$= (3 \text{ kg}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -4 & 4 & 6 \end{vmatrix} + 4 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -6 & 8 & 4 \end{vmatrix} + 5 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k} + (19.2 - 4.8v_y)\mathbf{i} + (4.8v_x - 14.4)\mathbf{j} + (3.6v_y - 4.8v_x)\mathbf{k} + 6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k}$$

$$\mathbf{H}_O = (7.2 - 4.8v_y)\mathbf{i} + (-36 + 4.8v_x)\mathbf{j} + (-9.6 + 3.6v_y - 4.8v_x)\mathbf{k}$$

(a) FOR  $\mathbf{H}_O$  TO BE  $\parallel z$  AXIS:

$$H_x = 7.2 - 4.8v_y = 0 \quad H_y = -36 + 4.8v_x = 0$$

$$v_x = 7.50 \text{ m/s}, \quad v_y = 1.500 \text{ m/s}$$

(b)  $\mathbf{H}_O = H_z \mathbf{k} = (-9.6 + 3.6 \times 1.500 - 4.8 \times 7.50)\mathbf{k}$ 

$$\mathbf{H}_O = -(40.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

14.12

GIVEN: SAME SYSTEM OF PARTICLES

WITH SAME VELOCITY DATA AS IN PROB. 14.11

FIND:

(a)  $v_x$  AND  $v_y$  FOR WHICH  $\mathbf{H}_O$  IS PARALLEL TO  $y$  AXIS.(b) CORRESPONDING  $\mathbf{H}_O$ .

SEE SOLUTION OF PROB. 14.11 FOR DERIVATION  
 OF EQ. (1):

$$\mathbf{H}_O = (7.2 - 4.8v_y)\mathbf{i} + (-36 + 4.8v_x)\mathbf{j} + (-9.6 + 3.6v_y - 4.8v_x)\mathbf{k}$$

(a) FOR  $\mathbf{H}_O$  TO BE  $\parallel y$  AXIS:

$$H_x = 7.2 - 4.8v_y = 0 \quad H_z = -9.6 + 3.6v_y - 4.8v_x = 0$$

$$v_y = 1.500 \text{ m/s} \quad -7.6 + 3.6(1.500) - 4.8v_x = 0$$

$$v_x = -0.875 \text{ m/s}$$

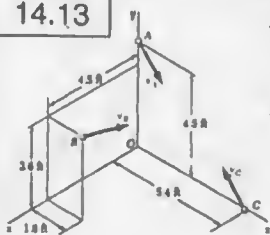
$$v_x = -0.875 \text{ m/s}, \quad v_y = 1.500 \text{ m/s}$$

(b)  $\mathbf{H}_O = H_y \mathbf{j} = [-36 + 4.8 \times (-0.875)]\mathbf{j}$ 

$$\mathbf{H}_O = -(40.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j}$$



14.13



GIVEN: SYSTEM OF PARTICLES  
WITH  $W_A = 9.66 \text{ lb}$ ,  $W_B = 6.44 \text{ lb}$ ,  
 $W_C = 12.88 \text{ lb}$   
AND VELOCITIES (ft/s)

$$\vec{v}_A = 4\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{v}_B = 4\hat{i} + 3\hat{j}$$

$$\vec{v}_C = -2\hat{i} + 4\hat{j} + 2\hat{k}$$

FIND: ANG. MOMENTUM  $\vec{H}_O$ .

$$\vec{H}_O = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B + \vec{r}_C \times m_C \vec{v}_C$$

USING DETERMINANT FORM FOR VECTOR PRODUCTS AND FACTORING MASSES:

$$\vec{H}_O = \frac{9.66}{32.2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4.5 & 0 \\ 4 & 2 & 2 \end{vmatrix} + \frac{6.44}{32.2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.8 & 3.6 & 4.5 \\ 4 & 3 & 0 \end{vmatrix} + \frac{12.88}{32.2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5.4 & 0 & 0 \\ -2 & 4 & 2 \end{vmatrix}$$

$$= 0.3(9\hat{i} - 18\hat{k}) + 0.2(-13.5\hat{i} + 18\hat{j} - 9\hat{k}) + 0.4(-10.8\hat{j} + 21.6\hat{k})$$

$$\vec{H}_O = -(0.720 \text{ ft} \cdot \text{lb} \cdot \text{s})\hat{i} + (1.440 \text{ ft} \cdot \text{lb} \cdot \text{s})\hat{k}$$

14.14

GIVEN: SYSTEM OF PARTICLES OF PROB. 14.13.

FIND: (a) POSITION VECTOR  $\vec{r}$  OF MASS CENTER G.

(b) LINEAR MOMENTUM OF SYSTEM.

(c) ANGULAR MOMENTUM  $\vec{H}_G$  OF SYSTEM.

ALSO: VERIFY THAT ANSWERS TO PROBS. 14.13 AND 14.14 SATISFY EQUATION

$$\vec{H}_O = \vec{r} \times m \vec{v} + \vec{H}_G$$

(a) EQ. (14.12):  $m\vec{r} = \sum m_i \vec{r}_i$

WHERE  $m_A = 0.3$ ,  $m_B = 0.2$ ,  $m_C = 0.4$ ,  $m = 0.9$

$$0.9\vec{r} = 0.3(4.5\hat{j}) + 0.2(1.8\hat{i} + 3.6\hat{j} + 4.5\hat{k}) + 0.4(5.4\hat{k})$$

$$\vec{r} = (2.80 \text{ ft})\hat{i} + (2.30 \text{ ft})\hat{j} + (1.00 \text{ ft})\hat{k}$$

$$(b) \vec{L} = \sum m_i \vec{r}_i \times \vec{v}_i = 0.3(4\hat{i} + 2\hat{j} + 2\hat{k}) \times (4\hat{i} + 2\hat{j} + 2\hat{k}) + 0.2(-2\hat{i} + 4\hat{j} + 2\hat{k}) \times (4\hat{i} + 3\hat{j})$$

$$\vec{L} = (1.200 \text{ lb} \cdot \text{s})\hat{i} + (2.80 \text{ lb} \cdot \text{s})\hat{j} + (1.400 \text{ lb} \cdot \text{s})\hat{k}$$

$$(c) \vec{H}_G = \vec{r}_{A/G} \times m_A \vec{v}_A + \vec{r}_{B/G} \times m_B \vec{v}_B + \vec{r}_{C/G} \times m_C \vec{v}_C$$

WHERE

$$\vec{r}_{A/G} = \vec{r}_A - \vec{r} = 4.5\hat{j} - (2.8\hat{i} + 2.3\hat{j} + \hat{k}) = -2.8\hat{i} + 2.2\hat{j} - \hat{k}$$

$$\vec{r}_{B/G} = \vec{r}_B - \vec{r} = 1.8\hat{i} + 3.6\hat{j} + 4.5\hat{k} - (2.8\hat{i} + 2.3\hat{j} + \hat{k}) = -1\hat{i} + 1.3\hat{j} + 3.5\hat{k}$$

$$\vec{r}_{C/G} = \vec{r}_C - \vec{r} = 5.4\hat{k} - (2.8\hat{i} + 2.3\hat{j} + \hat{k}) = -2.8\hat{i} - 2.3\hat{j} - \hat{k}$$

$$\vec{H}_G = 0.3 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2.8 & 2.2 & -1 \\ 4 & 2 & 2 \end{vmatrix} + 0.2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1.3 & 3.5 \\ 4 & 3 & 0 \end{vmatrix} + 0.4 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2.8 & -2.3 & -1 \\ -2 & 4 & 2 \end{vmatrix}$$

$$= 0.3(6.4\hat{i} + 1.6\hat{j} - 14.4\hat{k}) + 0.2(-10.5\hat{i} + 14\hat{j} - 8.2\hat{k}) + 0.4(-0.6\hat{i} - 3.2\hat{j} + 5.8\hat{k})$$

$$\vec{H}_G = -(0.420 \text{ ft} \cdot \text{lb} \cdot \text{s})\hat{i} + (2.00 \text{ ft} \cdot \text{lb} \cdot \text{s})\hat{j} - (3.64 \text{ ft} \cdot \text{lb} \cdot \text{s})\hat{k}$$

COMPUTE  $\vec{r} \times m \vec{v}$ :

$$\vec{r} \times m \vec{v} = \vec{r} \times \vec{L} = (2.8\hat{i} + 2.3\hat{j} + \hat{k}) \times (1.2\hat{i} + 2.8\hat{j} + 1.4\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.8 & 2.3 & 1 \\ 1.2 & 2.8 & 1.4 \end{vmatrix} = 0.420\hat{i} - 2.72\hat{j} + 5.08\hat{k}$$

$$\text{THUS: } \vec{r} \times m \vec{v} + \vec{H}_G = 0.420\hat{i} - 2.72\hat{j} + 5.08\hat{k} + (-0.420\hat{i} + 2\hat{j} - 3.64\hat{k})$$

$$= -0.720\hat{j} + 1.440\hat{k}$$

WHICH IS THE EXPRESSION OBTAINED FOR  $\vec{H}_O$  IN PROB. 14.13.

14.15

GIVEN:

900-LB SPACE VEHICLE WITH  $\vec{v}_0 = (1200 \text{ ft/s})\hat{i}$   
AS IT PASSES THROUGH O AT  $t=0$ , IT EXPLODES INTO  
A (450 lb), B (300 lb), C (150 lb)

AT  $t=4 \text{ s}$ , POSITIONS OF A AND B ARE

A (3840 ft, -960 ft, -1920 ft)

B (6480 ft, 1200 ft, 2640 ft)

FIND: POSITION OF C AT THAT TIME

MOTION OF MASS CENTER:

SINCE THERE IS NO EXTERNAL FORCE,

$$\vec{r} = \vec{v}_0 t = (1200 \text{ ft/s})\hat{i} (4 \text{ s}) = (4800 \text{ ft})\hat{i}$$

EQUATION (14.12)

$$m\vec{r} = \sum m_i \vec{r}_i$$

$$(900 \text{ lb})(4800\hat{i}) = (450 \text{ lb})(3840\hat{i} - 960\hat{j} - 1920\hat{k}) + (300 \text{ lb})(6480\hat{i} + 1200\hat{j} + 2640\hat{k}) + (150 \text{ lb})\vec{r}_C$$

$$150\vec{r}_C = (900 \times 4800 - 450 \times 3840 - 300 \times 6480)\hat{i} +$$

$$(450 \times 960 - 300 \times 1200)\hat{j} + (450 \times 1920 - 300 \times 2640)\hat{k}$$

$$= 648,000\hat{i} + 72,000\hat{j} + 72,000\hat{k}$$

$$\vec{r}_C = (4320 \text{ ft})\hat{i} + (480 \text{ ft})\hat{j} + (480 \text{ ft})\hat{k}$$

14.16

GIVEN:

30-LB PASSES THROUGH O WITH  
VELOCITY  $\vec{v}_0 = (120 \text{ ft/s})\hat{i}$  WHEN IT EXPLODES  
INTO FRAGMENTS A (12 lb) AND B (18 lb).  
AT  $t=3 \text{ s}$ , POSITION OF A IS A (300 ft, 24 ft, -48 ft).

FIND: POSITION OF B AT THAT TIME

ASSUME:  $a_y = -g = -32.2 \text{ ft/s}^2$

MOTION OF MASS CENTER:

IT MOVES AS IF PROJECTILE HAD NOT EXPLODED.

$$\vec{r} = \vec{v}_0 t - \frac{1}{2} g t^2 \hat{j}$$

$$= (120 \text{ ft/s})(3 \text{ s})\hat{i} - \frac{1}{2} (32.2 \text{ ft/s}^2)(3 \text{ s})^2 \hat{j}$$

$$= (360 \text{ ft})\hat{i} - (144.9 \text{ ft})\hat{j}$$

EQUATION (14.12):

$$m\vec{r} = \sum m_i \vec{r}_i$$

$$m\vec{r} = m_A \vec{r}_A + m_B \vec{r}_B$$

$$\frac{30}{g} (360\hat{i} - 144.9\hat{j}) = \frac{12}{g} (300\hat{i} + 24\hat{j} - 48\hat{k}) + \frac{18}{g} \vec{r}_B$$

$$18\vec{r}_B = (30 \times 360 - 12 \times 300)\hat{i} +$$

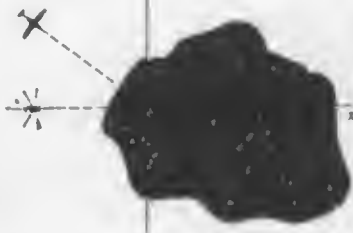
$$(-30 \times 144.9 - 12 \times 24)\hat{j} + (12 \times 48)\hat{k}$$

$$= 7200\hat{i} - 4635\hat{j} + 576\hat{k}$$

$$\vec{r}_B = (400 \text{ ft})\hat{i} - (258 \text{ ft})\hat{j} + (32.0 \text{ ft})\hat{k}$$



14.17



GIVEN:

AIRPLANE:  $m_A = 1500 \text{ kg}$   
 HELICOPTER:  $m_H = 3000 \text{ kg}$   
 COLLIDE AT 1200 m  
 ABOVE O.  
 4 MIN. BEFORE:  
 HELICOPTER WAS  
 8.4 km WEST OF O;  
 PLANE WAS 16 km WEST  
 AND 12 km NORTH OF O.

AFTER COLLISION, HELICOPTER BREAKS INTO  
 $H_1$  (1000 kg) AND  $H_2$  (2000 kg)

FIND: POINT A WHERE WRECKAGE OF PLANE  
 WILL BE FOUND, KNOWING THAT FRAGMENTS OF HELI-  
 COPTER WERE AT  $H_1$  (500 m, -100 m) AND  $H_2$  (600 m, -500 m).

MOTION OF MASS CENTER G:

$$\text{AT COLLISION: } \vec{v}_H = \frac{(5400 \text{ m})}{4(60 \text{ s})} \hat{i} = (35.00 \text{ m/s}) \hat{i}$$

$$\vec{v}_A = \frac{(16000 \text{ m}) \hat{i} - (12000 \text{ m}) \hat{j}}{4(60 \text{ s})} = (66.67 \text{ m/s}) \hat{i} - (50 \text{ m/s}) \hat{j}$$

VELOCITY OF MASS CENTER:

$$(m_H + m_A) \vec{v} = m_H \vec{v}_H + m_A \vec{v}_A$$

$$4500 \vec{v} = 3000 (35.00 \hat{i}) + 1500 (66.67 \hat{i} - 50 \hat{j})$$

$$\vec{v} = (45.556 \text{ m/s}) \hat{i} - (16.667 \text{ m/s}) \hat{j}$$

VERTICAL MOTION OF G:

$$h = \frac{1}{2} g t^2 \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1200 \text{ m})}{9.81 \text{ m/s}^2}} = 15.641 \text{ s}$$

POSITION OF G AT TIME OF GROUND IMPACT:

$$\vec{r} = \vec{v} t = (45.556 \hat{i} - 16.667 \hat{j})(15.641)$$

$$\vec{r} = (712.55 \text{ m}) \hat{i} - (260.69 \text{ m}) \hat{j} \quad (1)$$

FROM EQ. (14.12):

$$(m_H + m_A) \vec{r} = m_{H_1} \vec{r}_{H_1} + m_{H_2} \vec{r}_{H_2} + m_A \vec{r}_A \quad (2)$$

$$4500 (712.55 \hat{i} - 260.69 \hat{j}) =$$

$$1000 (500 \hat{i} - 100 \hat{j}) + 2000 (600 \hat{i} - 500 \hat{j}) + 1500 \vec{r}_A$$

$$1.5 \vec{r}_A = (4.5 \times 712.55 - 500 - 2 \times 600) \hat{i} + (-4.5 \times 260.69 + 100 + 2 \times 500) \hat{j}$$

$$\vec{r}_A = (1004 \text{ m}) \hat{i} - (467 \text{ m}) \hat{j}$$

14.18

GIVEN: SAME AS FOR PROB. 14.17.

FIND: POINT WHERE FRAGMENT  $H_2$  WILL BE  
 FOUND, KNOWING THAT WRECKAGE OF PLANE  
 WAS FOUND AT A (1200 m, 80 m) AND FRAGMENT  
 $H_1$  AT  $H_1$  (400 m, -200 m).

SEE SOLUTION OF PROB. 14.17 FOR DERIVATION OF

$$\vec{r} = (712.55 \text{ m}) \hat{i} - (260.69 \text{ m}) \hat{j} \quad (1)$$

$$(m_H + m_A) \vec{r} = m_{H_1} \vec{r}_{H_1} + m_{H_2} \vec{r}_{H_2} + m_A \vec{r}_A \quad (2)$$

SUBSTITUTING DATA:

$$4500 (712.55 \hat{i} - 260.69 \hat{j}) =$$

$$1000 (400 \hat{i} - 200 \hat{j}) + 2000 \vec{r}_{H_2} + 1500 (1200 \hat{i} + 80 \hat{j})$$

$$2 \vec{r}_{H_2} = (4.5 \times 712.55 - 400 - 1.5 \times 1200) \hat{i} +$$

$$(-4.5 \times 260.69 + 200 - 1.5 \times 80) \hat{j}$$

$$\vec{r}_{H_2} = (303 \text{ m}) \hat{i} - (547 \text{ m}) \hat{j}$$

14.19



GIVEN: CARS A (1500 kg), B (1300 kg), AND C (1200 kg)  
 WERE TRAVELING AS SHOWN WHEN CAR A HITS CAR B.

AT THAT INSTANT CAR C IS AT  $x_C = 10 \text{ m}$ ,  $y_C = 3 \text{ m}$ .  
 CAR C HITS A AND B, AND ALL CARS HIT P ( $x_P, y_P$ ).

FIND: (a) TIME  $t$  FROM FIRST COLLISION TO STOP AT P.  
 (b) SPEED  $v_A$  OF CAR A

KNOWING THAT  $x_P = 18 \text{ m}$ ,  $y_P = 13.9 \text{ m}$ 

MOTION OF MASS CENTER

FINAL POSITION OF MASS CENTER OF SYSTEM  
 IS THE SAME AS IF THE CARS HAD NOT  
 COLLIDED AND HAD KEPT MOVING WITH THEIR  
 ORIGINAL VELOCITIES.

$$(m_A + m_B + m_C) \vec{r}_P = m_A (v_A t) \hat{i} + m_B (v_B t) \hat{j} +$$

$$m_C (x_C \hat{i} + y_C \hat{j} - v_C t \hat{i})$$

WHERE  $v_B = 72 \text{ km/h} = 20 \text{ m/s}$ ,  $v_C = 90 \text{ km/h} = 25 \text{ m/s}$

$$4000 \vec{r}_P = 1500 v_A t \hat{i} + 1300 (20 t) \hat{j} + 1200 (10 \hat{i} + 3 \hat{j} - 25 t \hat{i})$$

$$\vec{r}_P = (0.375 v_A - 7.5) t \hat{i} + 3 \hat{i} + 6.5 t \hat{j} + 0.9 \hat{j}$$

$$\text{THUS: } x_P = (0.375 v_A - 7.5) t + 3, \quad y_P = 6.5 t + 0.9 \quad (1)$$

$$(a) \text{ MAKING } y_P = 13.9 \text{ m: } 13.9 = 6.5 t + 0.9$$

$$t = 2.00 \text{ s}$$

$$(b) \text{ MAKING } x_P = 18 \text{ m AND } t = 2 \text{ s:}$$

$$18 = (0.375 v_A - 7.5) 2 + 3 \quad v_A = 40 \text{ m/s} = 144 \text{ km/h}$$

14.20

GIVEN: SAME AS FOR PROB. 14.19.

FIND: COORDINATES OF POLE P, KNOWING THAT  
 $v_A = 129.6 \text{ km/h}$  AND THAT TIME FROM FIRST  
 COLLISION TO STOP AT P IS  $t = 2.4 \text{ s}$ .

SEE SOLUTION OF PROB. 14.19 FOR DERIVATION OF

$$\vec{r}_P = (0.375 v_A - 7.5) t + 3, \quad y_P = 6.5 t + 0.9 \quad (1)$$

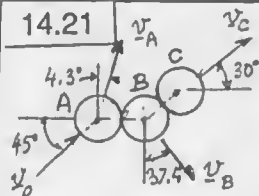
MAKING  $v_A = 129.6 \text{ km/h} = 36 \text{ m/s}$  AND  $t = 2.4 \text{ s}$   
 IN Eqs. (1):

$$x_P = (0.375 \times 36 - 7.5)(2.4) + 3 = 17.40 \text{ m}$$

$$y_P = 6.5 (2.4) + 0.9 = 16.50 \text{ m}$$

$$x_P = 17.40 \text{ m}, y_P = 16.50 \text{ m}$$

14.21



GIVEN: 3 BALLS OF SAME MASS  
BALL A STRIKES B AND C  
WHICH ARE AT REST.

BEFORE IMPACT,  $v_0 = 12 \text{ ft/s}$   
AFTER IMPACT,  $v_C = 6.29 \text{ ft/s}$

FIND:

(a)  $v_A$ , (b)  $v_B$  AFTER IMPACT

### CONSERVATION OF LINEAR MOMENTUM

IN X DIRECTION:

$$m(12 \text{ ft/s}) \cos 45^\circ = m v_A \sin 4.3^\circ + m v_B \sin 37.4^\circ + m(6.29) \cos 30^\circ$$

$$0.07498 v_A + 0.60738 v_B = 3.0380 \quad (1)$$

IN Y DIRECTION:

$$m(12 \text{ ft/s}) \sin 45^\circ = m v_A \cos 4.3^\circ - m v_B \cos 37.4^\circ + m(6.29) \sin 30^\circ$$

$$0.99719 v_A - 0.79441 v_B = 5.3403 \quad (2)$$

(a) MULTIPLY (1) BY 0.79441, (2) BY 0.60738, AND ADD:

$$0.66524 v_A = 5.6570 \quad v_A = 8.50 \text{ ft/s}$$

(b) MULTIPLY (1) BY 0.99719, (2) BY -0.07498, AND ADD:

$$0.66524 v_B = 2.6290 \quad v_B = 3.95 \text{ ft/s}$$

14.23 continued

$$t^2 - 0.20053t - 3.0581 = 0$$

$$t = \frac{0.20053 + \sqrt{(0.20053)^2 + 4(3.0581)}}{2} = 1.8519 \text{ s}$$

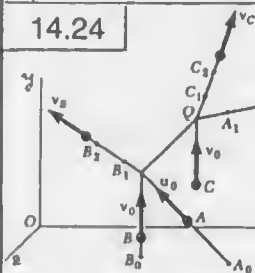
HORIZONTAL MOTION:

$$\vec{r}_P = (v_x t) \hat{i} + (v_y t) \hat{j}$$

$$= (9.8361)(1.8519) \hat{i} + (1.3115)(1.8519) \hat{j}$$

$$\vec{r}_P = (18.22 \text{ m}) \hat{i} + (2.43 \text{ m}) \hat{j}$$

14.24



GIVEN:

BEFORE COLLISIONS ALPHA

PARTICLE A MOVED WITH

$\vec{u}_0 = -(480 \text{ m/s}) \hat{i} + 600 \hat{j} - 640 \hat{k}$

NUCLEI B AND C MOVED

WITH  $\vec{u}_0 = (480 \text{ m/s}) \hat{j}$

AFTER COLLISIONS, THEY

MOVED ALONG PATHS WHERE

$A_1(240, 220, 160)$ ,  $A_2(320, 300, 200)$

$B_1(107, 200, 170)$ ,  $B_2(74, 270, 160)$

$C_1(200, 212, 130)$ ,  $C_2(200, 260, 115)$

(DIMENSIONS IN mm)

FIND: SPEED OF EACH PARTICLE AFTER COLLISIONS.

MASS OF OXYGEN NUCLEUS =  $m$ , MASS OF  $\alpha$  PARTICLE =  $\frac{1}{4}m$

BEFORE COLLISIONS:

$\alpha$  PARTICLE:  $\vec{u}_0 = -480 \hat{i} + 600 \hat{j} - 640 \hat{k}$

NUCLEI B AND C:  $\vec{u}_0 = 480 \hat{j}$

AFTER COLLISIONS:

$$\vec{v}_A = \vec{v}_A \frac{A_1 A_2}{A_1 A_2} = \frac{80\hat{i} + 80\hat{j} + 40\hat{k}}{120} \quad v_A = (0.6667 \hat{i} + 0.6667 \hat{j} + 0.3333 \hat{k}) v_A$$

$$\vec{v}_B = \vec{v}_B \frac{B_1 B_2}{B_1 B_2} = \frac{-33\hat{i} + 70\hat{j} - 10\hat{k}}{78.03} \quad v_B = (-0.4229 \hat{i} + 0.8971 \hat{j} - 0.12816 \hat{k}) v_B$$

$$\vec{v}_C = \vec{v}_C \frac{C_1 C_2}{C_1 C_2} = \frac{18\hat{i} - 15\hat{k}}{50.29} \quad v_C = (0.9545 \hat{j} - 0.2983 \hat{k}) v_C$$

CONSERVATION OF MOMENTUM:

$$\frac{1}{4} m u_0 + 2m v_0 = \frac{1}{4} m v_A + m v_B + m v_C$$

$$-120 \hat{i} + 150 \hat{j} - 160 \hat{k} + 960 \hat{j} = (0.1667 \hat{i} + 0.1667 \hat{j} + 0.08333 \hat{k}) v_A$$

$$(-0.4229 \hat{i} + 0.8971 \hat{j} - 0.12816 \hat{k}) v_B + (0.9545 \hat{j} - 0.2983 \hat{k}) v_C$$

EQUATING THE COEFFICIENTS OF THE UNIT VECTORS:

$$0.1667 v_A - 0.4229 v_B = -120 \quad (1)$$

$$0.1667 v_A + 0.8971 v_B + 0.9545 v_C = 1110 \quad (2)$$

$$0.08333 v_A - 0.12816 v_B - 0.2983 v_C = -160 \quad (3)$$

$$\text{MULTIPLY (2) BY } 0.2983, \text{ (3) BY } 0.9545 \text{ AND ADD:}$$

$$0.12926 v_A + 0.14528 v_B = 178.39 \quad (4)$$

$$\text{MULTIPLY (1) BY } 0.14528, \text{ (4) BY } 0.4229 \text{ AND ADD:}$$

$$0.7888 v_A = 58.01 \quad v_A = 735.4 \text{ m/s}$$

$$v_A = 735 \text{ m/s}$$

$$\text{FROM (1):}$$

$$0.1667(735.4) - 0.4229 v_B = -120 \quad v_B = 573.6 \text{ m/s}$$

$$v_B = 574 \text{ m/s}$$

$$\text{FROM (3):}$$

$$0.08333(735.4) - 0.12816(573.6) - 0.2983 v_C = -160$$

$$v_C = 495.4 \text{ m/s} \quad v_C = 495 \text{ m/s}$$

$$\text{FROM (3):}$$

$$0.08333(735.4) - 0.12816(573.6) - 0.2983 v_C = -160$$

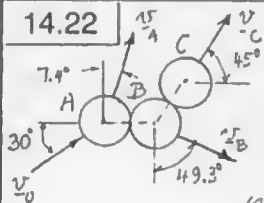
$$v_C = 495.4 \text{ m/s} \quad v_C = 495 \text{ m/s}$$

$$\text{FROM (3):}$$

$$0.08333(735.4) - 0.12816(573.6) - 0.2983 v_C = -160$$

$$v_C = 495.4 \text{ m/s} \quad v_C = 495 \text{ m/s}$$

14.22



GIVEN: 3 BALLS OF SAME MASS  
BALL A STRIKES B AND C  
WHICH ARE AT REST.

BEFORE IMPACT,  $v_0 = 12 \text{ ft/s}$   
AFTER IMPACT,  $v_C = 6.29 \text{ ft/s}$

FIND:

(a)  $v_A$ , (b)  $v_B$  AFTER IMPACT.

### CONSERVATION OF LINEAR MOMENTUM

IN X DIRECTION:

$$m(12 \text{ ft/s}) \cos 30^\circ = m v_A \sin 7.4^\circ + m v_B \sin 49.3^\circ + m(6.29) \cos 45^\circ$$

$$0.12880 v_A + 0.75813 v_B = 5.9446 \quad (1)$$

IN Y DIRECTION:

$$m(12 \text{ ft/s}) \sin 30^\circ = m v_A \cos 7.4^\circ - m v_B \cos 49.3^\circ + m(6.29) \sin 45^\circ$$

$$0.99167 v_A - 0.65210 v_B = 1.5523 \quad (2)$$

(a) MULTIPLY (1) BY 0.65210, (2) BY 0.75813, AND ADD:

$$0.83581 v_A = 5.0533 \quad v_A = 6.05 \text{ ft/s}$$

(b) MULTIPLY (1) BY 0.99167, (2) BY -0.12880, AND ADD:

$$0.83581 v_B = 5.6951 \quad v_B = 6.81 \text{ ft/s}$$

14.23

GIVEN: 3-kg BIRD FLYING 15m ABOVE  
GROUND WITH  $\vec{v}_B = (10 \text{ m/s}) \hat{i}$  IS HIT BY 50-g ARROW  
WITH  $\vec{v}_A = (60 \text{ m/s}) \hat{j} + (80 \text{ m/s}) \hat{k}$ .

FIND: DISTANCE FROM O UNDER POINT OF IMPACT TO  
P WHERE BIRD HITS THE GROUND.

### CONSERVATION OF MOMENTUM:

$$(3000 \text{ g})(10 \text{ m/s}) \hat{i} + (50 \text{ g})(60 \hat{j} + 80 \hat{k}) = (3050 \text{ g}) \vec{v}$$

VELOCITY OF BIRD AND ARROW AFTER IMPACT:

$$\vec{v} = (9.8361 \text{ m/s}) \hat{i} + (0.98361 \text{ m/s}) \hat{j} + (1.3115 \text{ m/s}) \hat{k}$$

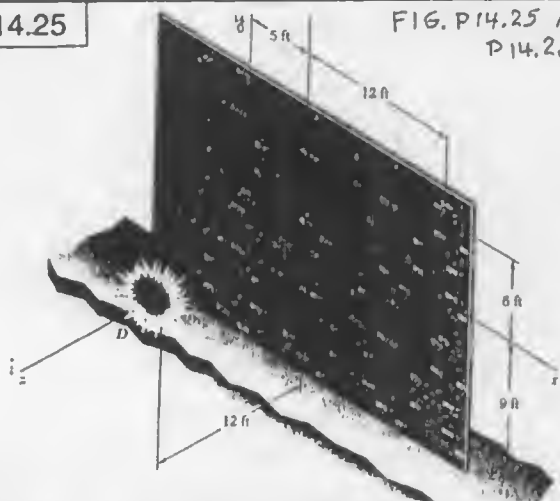
VERTICAL MOTION:

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2 \quad \text{MAKE } y = 0:$$

$$0 = 15 \text{ m} + (0.98361 \text{ m/s}) t - \frac{1}{2} (9.81 \text{ m/s}^2) t^2$$

(CONTINUED)

14.25

FIG. P14.25 AND  
P14.26

**GIVEN:** 12-lb SHELL EXPLODES AT D INTO FRAGMENTS, A (5 lb), B (4 lb), AND C (3 lb), WHICH HIT WALL AS SHOWN. VELOCITY OF SHELL WAS  $\mathbf{v}_0 = (40 \text{ ft/s})\mathbf{i} - (30 \text{ ft/s})\mathbf{j} - (1200 \text{ ft/s})\mathbf{k}$ . FIND: SPEED OF EACH FRAGMENT

**CONSERVATION OF MOMENTUM:**

$$(12/8)\mathbf{v}_0 = (5/8)\mathbf{v}_A + (4/8)\mathbf{v}_B + (3/8)\mathbf{v}_C$$

$$12(40\mathbf{i} - 30\mathbf{j} - 1200\mathbf{k}) = 5(-\frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{k})v_A +$$

$$4(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k})v_B + 3(-\frac{2}{5}\mathbf{j} - \frac{4}{5}\mathbf{k})v_C$$

EQUATE COEFFICIENTS OF UNIT VECTORS:

$$\textcircled{1} -\frac{25}{13}v_A + \frac{4}{3}v_B = 480 \quad (1)$$

$$\textcircled{2} \frac{4}{3}v_B - \frac{9}{5}v_C = -360 \quad (2)$$

$$\textcircled{3} -\frac{60}{13}v_A - \frac{4}{3}v_B - \frac{12}{5}v_C = -14,400 \quad (3)$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$$v_A = 1677.64, \quad v_B = 1389.84, \quad v_C = 1229.51$$

$$v_A = 1678 \text{ ft/s}; \quad v_B = 1390 \text{ ft/s}; \quad v_C = 1230 \text{ ft/s}$$

14.26

SEE FIGURE AT TOP OF PAGE

**GIVEN:** 12-lb SHELL EXPLODES AT D INTO FRAGMENTS A (4 lb), B (3 lb), AND C (5 lb), WHICH HIT WALL AS SHOWN. VELOCITY OF SHELL WAS  $\mathbf{v}_0 = (40 \text{ ft/s})\mathbf{i} - (30 \text{ ft/s})\mathbf{j} - (1200 \text{ ft/s})\mathbf{k}$ .

**FIND:** SPEED OF EACH FRAGMENT

**CONSERVATION OF MOMENTUM:**

$$(12/8)\mathbf{v}_0 = (4/8)\mathbf{v}_A + (3/8)\mathbf{v}_B + (5/8)\mathbf{v}_C$$

$$12(40\mathbf{i} - 30\mathbf{j} - 1200\mathbf{k}) = 4(-\frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{k})v_A +$$

$$3(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k})v_B + 5(-\frac{2}{5}\mathbf{j} - \frac{4}{5}\mathbf{k})v_C$$

EQUATE COEFFICIENTS OF UNIT VECTORS:

$$\textcircled{1} -\frac{20}{13}v_A + 2v_B = 480 \quad (1)$$

$$\textcircled{2} v_B - 3v_C = -360 \quad (2)$$

$$\textcircled{3} -\frac{48}{13}v_A - 2v_B - 4v_C = -14,400 \quad (3)$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$$v_A = 2097.05, \quad v_B = 1853.11, \quad v_C = 737.705$$

$$v_A = 2097 \text{ ft/s}; \quad v_B = 1853 \text{ ft/s}; \quad v_C = 738 \text{ ft/s}$$

14.27

DERIVE  $\mathbf{H}_0 = \mathbf{\bar{r}} \times m \mathbf{\bar{v}} + \mathbf{H}_G$ , WHERE

$$\mathbf{H}_0 = \sum (\mathbf{r}_i \times m_i \mathbf{v}_i) \quad (14.7)$$

$$\mathbf{H}_G = \sum (\mathbf{r}_i' \times m_i \mathbf{v}_i') \quad (14.24)$$

AND  $m$  = TOTAL MASS OF SYSTEM,

$\mathbf{\bar{r}}$  = POSITION VECTOR OF G;  $\mathbf{\bar{v}}$  = VELOCITY OF G.

MAKING  $\mathbf{r}_i = \mathbf{\bar{r}} + \mathbf{r}_i'$  IN EQ. (14.7):

$$\mathbf{H}_0 = \sum (\mathbf{\bar{r}} + \mathbf{r}_i') \times m_i \mathbf{v}_i$$

$$= \mathbf{\bar{r}} \times \sum m_i \mathbf{v}_i + \sum \mathbf{r}_i' \times m_i \mathbf{v}_i$$

$$\text{BUT } \sum m_i \mathbf{v}_i = m \mathbf{\bar{v}}$$

$$\text{AND, BY (14.24): } \sum \mathbf{r}_i' \times m_i \mathbf{v}_i' = \mathbf{H}_G$$

THEREFORE:

$$\mathbf{H}_0 = \mathbf{\bar{r}} \times m \mathbf{\bar{v}} + \mathbf{H}_G \quad (\text{Q.E.D.})$$

14.28

DERIVE  $\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$  (14.23)

DIRECTLY FROM  $\sum \mathbf{M}_0 = \dot{\mathbf{H}}_0$  (14.11)

BY USING EQUATION DERIVED IN PROB. 14.27.

WE REDUCE THE FORCES TO THE VECTORS SHOWN. IT FOLLOWS THAT

$$\sum \mathbf{M}_0 = \mathbf{\bar{r}} \times \sum \mathbf{F} + \sum \mathbf{M}_G \quad (1)$$

FROM PROB. 14.27:  $\mathbf{H}_0 = \mathbf{\bar{r}} \times m \mathbf{\bar{v}} + \mathbf{H}_G$

DIFFERENTIATE:  $\dot{\mathbf{H}}_0 = \dot{\mathbf{r}} \times m \mathbf{\bar{v}} + \mathbf{\bar{r}} \times m \dot{\mathbf{v}} + \dot{\mathbf{H}}_G$

$$= \mathbf{\bar{v}} \times m \mathbf{\bar{v}} + \mathbf{\bar{r}} \times m \mathbf{\bar{a}} + \dot{\mathbf{H}}_G$$

BUT  $\mathbf{\bar{v}} \times m \mathbf{\bar{v}} = 0$  AND  $m \mathbf{\bar{a}} = \sum \mathbf{F}$ . THUS

$$\dot{\mathbf{H}}_0 = \mathbf{\bar{r}} \times \sum \mathbf{F} + \dot{\mathbf{H}}_G \quad (2)$$

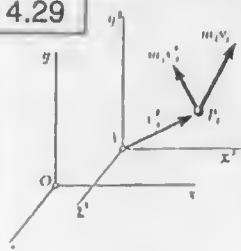
SUBSTITUTE FOR  $\sum \mathbf{M}_0$  FROM (1) AND  $\dot{\mathbf{H}}_0$  FROM (2) INTO (14.11):

$$\mathbf{\bar{r}} \times \sum \mathbf{F} + \sum \mathbf{M}_G = \mathbf{\bar{r}} \times \sum \mathbf{F} + \dot{\mathbf{H}}_G$$

OR

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G \quad (\text{Q.E.D.})$$

14.29



**GIVEN:**

NEWTONIAN FRAME  $Oxyz$

AND FRAME  $A'x'y'z'$  IN TRANSLATION W/R TO  $Oxyz$ .

LET  $\mathbf{H}_A' = \sum \mathbf{r}_i' \times m_i \mathbf{v}_i'$  (1)

AND  $\mathbf{H}_A = \sum \mathbf{r}_i \times m_i \mathbf{v}_i$  (2)

WHERE  $\mathbf{v}_i'$  AND  $\mathbf{v}_i$  DENOTE VELOCITIES W/R  $A'x'y'z'$

AND  $Oxyz$ , RESPECTIVELY

SHOW THAT  $\mathbf{H}_A = \mathbf{H}_A'$  AT GIVEN INSTANT

IF, AND ONLY IF, ONE OF THE FOLLOWING CONDITIONS IS SATISFIED AT THAT INSTANT:

(a)  $\mathbf{v}_A = 0$  WITH RESPECT TO  $Oxyz$ ,

(b) A COINCIDES WITH MASS CENTER G OF SYSTEM OF PARTICLES,

(c)  $\mathbf{v}_A$  IS DIRECTED ALONG  $AG$ .

(CONTINUED)

## 14.29 continued

WE RECALL:

$$\vec{H}_A' = \sum \vec{r}_i' \times m_i \vec{v}_i' \quad (1)$$

$$\vec{H}_A = \sum \vec{r}_i \times m_i \vec{v}_i \quad (2)$$

LET  $\vec{v}_i' = \vec{v}_A + \vec{v}_i'$  IN EQ. (2):

$$\vec{H}_A = \sum \vec{r}_i \times m_i (\vec{v}_A + \vec{v}_i') = (\sum m_i \vec{r}_i) \times \vec{v}_A + \sum \vec{r}_i \times m_i \vec{v}_i'$$

BUT, BY (14.12):  $\sum m_i \vec{r}_i' = m \vec{r}' = m \vec{AG}$ 

RECALLING EQ. (1), WE WRITE

$$\vec{H}_A = m \vec{AG} \times \vec{v}_A + \vec{H}_A'$$

THIS EQUATION REDUCES TO  $\vec{H}_A = \vec{H}_A'$  IF(a)  $\vec{v}_A = 0$ , (b)  $A \equiv G$ , (c)  $\vec{v}_A \parallel \vec{AG}$  (Q.E.D.)

## 14.30

GIVEN:

FRAME  $Ax'y'z'$  IN TRANSLATION WITH RESPECT TO NEWTONIAN FRAME  $Oxyz$ .

$$\text{LET } \vec{H}_A' = \sum \vec{r}_i' \times m_i \vec{v}_i' \quad (1)$$

WHERE  $\vec{r}_i'$  AND  $\vec{v}_i'$  ARE DEFINED W/R FRAME  $Ax'y'z'$ AND LET  $\sum \vec{M}_A$  BE THE SUM OF THE MOMENTS OF THE EXTERNAL FORCES ABOUT A.SHOW THAT THE RELATION  $\sum \vec{M}_A = \dot{\vec{H}}_A$ 

IS VALID IF, AND ONLY IF, ONE OF THE FOLLOWING CONDITIONS IS SATISFIED:

(a)  $Ax'y'z'$  IS A NEWTONIAN FRAME OF REFERENCE

(b) A COINCIDES WITH MASS CENTER G OF SYSTEM OF PARTICLES.

(c)  $\vec{a}_A$  IS DIRECTED ALONG AG

DIFFERENTIATE EQ. (1):

$$\begin{aligned} \dot{\vec{H}}_A' &= \sum \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum \vec{r}_i' \times m_i \dot{\vec{v}}_i' \\ &= \sum \vec{v}_i' \times m_i \vec{v}_i' + \sum \vec{r}_i' \times m_i \vec{a}_i' \end{aligned}$$

BUT  $\vec{v}_i' \times \vec{v}_i' = 0$  AND  $\vec{a}_i' = \vec{a}_i - \vec{a}_A$ 

$$\text{THUS: } \dot{\vec{H}}_A' = \sum (\vec{r}_i' \times m_i \vec{a}_i) - (\sum m_i \vec{r}_i') \times \vec{a}_A$$

BUT, BY (14.12):  $\sum m_i \vec{r}_i' = m \vec{r}' = m \vec{AG}$ AND, SINCE  $\vec{a}_i$  IS ACCELERATION W/R NEWTONIAN FRAME, WE HAVE, BY EQ. (14.5),

$$\sum (\vec{r}_i' \times m_i \vec{a}_i) = \sum (\vec{r}_i' \times \vec{F}_i) = \sum \vec{M}_A$$

THEREFORE

$$\dot{\vec{H}}_A' = \sum \vec{M}_A - m \vec{AG} \times \vec{a}_A$$

THIS EQUATION REDUCES TO  $\dot{\vec{H}}_A' = \sum \vec{M}_A$  IF(a)  $\vec{a}_A = 0$  FRAME  $Ax'y'z'$  IS IN UNIFORM TRANSLATION W/R NEWTONIAN FRAME  $Oxyz$  AND IS ITSELF A NEWTONIAN FRAME,(b)  $\vec{AG} = 0$ ; A COINCIDES WITH G,(c)  $\vec{AG} \times \vec{a}_A = 0$ ;  $\vec{a}_A$  IS DIRECTED ALONG AG

(Q.E.D.)

## 14.31

GIVEN: REFERRING TO PROB. 14.1,

ASSUME THAT

(1) 15-kg SUITCASE FIRST TOSSED WITH  $\vec{v} = 3 \text{ m/s}$ (2) 20-kg SUITCASE THEN TOSSED WITH  $\vec{v} = 2 \text{ m/s}$ 

(3) 25-kg CARRIER INITIALLY AT REST.

FIND: ENERGY LOST AS

(a) FIRST SUITCASE HITS CARRIER

(b) SECOND SUITCASE HITS CARRIER

(a) BEFORE FIRST SUITCASE HITS CARRIER:

$$T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} (15 \text{ kg}) (3 \text{ m/s})^2 = 67.50 \text{ J}$$

FIRST IMPACT: CONSERVATION OF MOMENTUM

$$(15 \text{ kg})(3 \text{ m/s}) = (25 + 15) v_1 \quad v_1 = 1.125 \text{ m/s}$$

$$T_1 = \frac{1}{2} (25 \text{ kg} + 15 \text{ kg}) (1.125 \text{ m/s})^2 = 25.313 \text{ J}$$

$$\text{EN. LOST} = T_0 - T_1 = 67.50 \text{ J} - 25.313 \text{ J} = 42.2 \text{ J}$$

(b) JUST BEFORE SECOND SUITCASE HITS:

$$\begin{aligned} T_1' &= T_1 + \frac{1}{2} (20 \text{ kg}) (2 \text{ m/s})^2 = 25.313 \text{ J} + 40 \text{ J} \\ &= 65.313 \text{ J} \end{aligned}$$

SECOND IMPACT: CONSERVATION OF MOMENTUM

$$(25 \text{ kg} + 15 \text{ kg}) (1.125 \text{ m/s}) + (20 \text{ kg}) (2 \text{ m/s}) = (60 \text{ kg}) v_2$$

$$v_2 = 1.4167 \text{ m/s}$$

$$T_2 = \frac{1}{2} (60 \text{ kg}) (1.4167 \text{ m/s})^2 = 60.208 \text{ J}$$

$$\text{EN. LOST} = T_1' - T_2 = 65.313 \text{ J} - 60.208 \text{ J} = 5.10 \text{ J}$$

## 14.32

GIVEN: COLLISIONS DESCRIBED

IN PROB. 14.5. WE RECALL THAT

INITIAL VELOCITY OF CAR A WAS  $v_A = 1.920 \text{ m/s}$ AFTER A HITS B:  $(v_B)_1 = 1.680 \text{ m/s}$ AFTER B HITS C:  $(v_B)_2 = 0.210 \text{ m/s}$ AFTER A AGAIN HITS B:  $(v_B)_3 = 0.23625 \text{ m/s}$ 

MASS OF EACH CAR = 1500 kg

FIND: ENERGY LOST AFTER ALL COLLISIONS HAVE TAKEN PLACE.

FROM SOLUTION OF PROB. 14.5 WE HAVE THE FOLLOWING FINAL VELOCITIES:

$$v_A = 0.21375 \text{ m/s}, \quad v_B = 0.23625 \text{ m/s},$$

$$v_C = 1.470 \text{ m/s}$$

INITIAL ENERGY:

$$T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} (1500 \text{ kg}) (1.920 \text{ m/s})^2 = 2764.8 \text{ J}$$

FINAL ENERGY:

$$\begin{aligned} T_f &= \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 = \frac{1}{2} m (v_A^2 + v_B^2 + v_C^2) \\ &= \frac{1}{2} (1500 \text{ kg}) [(0.21375 \text{ m/s})^2 + (0.23625 \text{ m/s})^2 + (1.470 \text{ m/s})^2] \\ &= 1696.8 \text{ J} \end{aligned}$$

ENERGY LOST:

$$= T_0 - T_f = 2764.8 \text{ J} - 1696.8 \text{ J} = 1068 \text{ J}$$

### 14.33 GIVEN:

180-lb MAN AND 120-lb WOMAN OF PROB. 14.3 JUMP FROM SAME END OF 300-lb BOAT WITH VELOCITY OF 16 ft/s WITH RESPECT TO BOAT.

FIND:

WORK DONE BY WOMAN AND BY MAN IF WOMAN DIVES FIRST.

TOTAL K.E. AFTER WOMAN DIVES

FROM PART a OF SOLUTION OF PROB. 14.3:

VEL. OF BOAT =  $(v_B)_1 = 3.20 \text{ ft/s}$

THUS, VEL. OF WOMAN =  $(v_W)_1 = 16 - 3.20 = 12.8 \text{ ft/s}$

K.E. =  $T_1 = \frac{1}{2} m_W (v_W)_1^2 + \frac{1}{2} (m_B + m_M) (v_B)_1^2$

$$T_1 = \frac{1}{2} \frac{120}{32.2} (12.8)^2 + \frac{1}{2} \frac{420}{32.2} (3.20)^2 = 381.61 \text{ ft} \cdot \text{lb}$$

WORK OF WOMAN =  $T_1 = 381.61 \text{ ft} \cdot \text{lb}$

TOTAL K.E. AFTER MAN DIVE

FROM ANSWER TO PART a OF PROB. 14.3:

VEL. OF BOAT =  $(v_B)_2 = 9.20 \text{ ft/s}$

THUS, VEL. OF MAN =  $(v_M)_2 = 16 - 9.20 = 6.80 \text{ ft/s}$

K.E. =  $T_2 = \frac{1}{2} m_W (v_W)_2^2 + \frac{1}{2} m_M (v_M)_2^2 + \frac{1}{2} m_B (v_B)_2^2$

$$= \frac{1}{2} \frac{120}{32.2} (12.8)^2 + \frac{1}{2} \frac{180}{32.2} (6.80)^2 + \frac{1}{2} \frac{300}{32.2} (9.20)^2$$

$$T_2 = 828.82 \text{ ft} \cdot \text{lb}$$

WORK OF MAN =  $T_2 - T_1 = 828.82 - 381.61 = 447 \text{ ft} \cdot \text{lb}$

### 14.34 GIVEN:

BULLET OF PROB. 14.7 FIRED WITH  $v_0 = 1500 \text{ ft/s}$  THROUGH 6-lb BLOCK A BECOMES EMBEDDED IN 4.95-lb BLOCK B. BLOCKS MOVE WITH  $v_A = 5 \text{ ft/s}$  AND  $v_B = 9 \text{ ft/s}$ .

FIND:

ENERGY LOST AS BULLET

(a) PASSES THROUGH BLOCK A

(b) BECOMES EMBEDDED IN BLOCK B

FROM ANSWER TO PROB. 14.7:

WEIGHT OF BULLET =  $W = 0.800 \text{ oz} = 0.0500 \text{ lb}$

VEL. OF BULLET BETWEEN BLOCKS =  $v_1 = 900 \text{ ft/s}$

(a) ENERGY LOST AS BULLET PASSES THROUGH A

INITIAL K.E. =  $T_0 = \frac{1}{2} \frac{W}{g} v_0^2 = \frac{1}{2} \frac{0.0500 \text{ lb}}{32.2 \text{ ft/s}^2} (1500 \text{ ft/s})^2$

$$T_0 = 1746.89 \text{ ft} \cdot \text{lb}$$

K.E. OF SYSTEM AFTER BULLET PASSES THROUGH A:

$$= T_1 = \frac{1}{2} \frac{W}{g} v_1^2 + \frac{1}{2} \frac{W_A}{g} v_A^2 = \frac{1}{2} \frac{0.0500}{32.2} (900)^2 + \frac{1}{2} \frac{6}{32.2} (5)^2$$

$$T_1 = 628.80 + 2.33 = 631.21 \text{ ft} \cdot \text{lb}$$

EN. LOST =  $T_0 - T_1 = 1746.89 - 631.21 = 1116 \text{ ft} \cdot \text{lb}$

(b) ENERGY LOST AS BULLET LEAVES EMBEDDED IN B

FINAL K.E. =  $T_2 = \frac{1}{2} \frac{W_A}{g} v_A^2 + \frac{1}{2} \frac{(W_B + W)}{g} v_B^2$

$$T_2 = \frac{1}{2} \frac{6}{32.2} (5)^2 + \frac{1}{2} \frac{4.95 + 0.05}{32.2} (9)^2 = 8.616 \text{ ft} \cdot \text{lb}$$

EN. LOST =  $T_1 - T_2 = 631.21 - 8.616 = 623 \text{ ft} \cdot \text{lb}$

### 14.35

GIVEN: AUTOMOBILE A, OF MASS  $m_A$ , COLLIDES WITH AUTOMOBILE B OF MASS  $m_B$ . WE ASSUME PLASTIC IMPACT AND THAT ENERGY ABSORBED BY EACH AUTOMOBILE EQUALS ITS K.E. WITH RESPECT TO MOVING FRAME ATTACHED TO MASS CENTER OF SYSTEM.



(a) SHOW THAT  $E_A/E_B = m_B/m_A$ , WHERE  $E_A$  AND  $E_B$  ARE ENERGIES ABSORBED BY A AND B.

(b) FIND  $E_A$  AND  $E_B$  IF  $m_A = 1600 \text{ kg}$ ,  $m_B = 900 \text{ kg}$ ,  $v_A = 90 \text{ km/h}$ ,  $v_B = 60 \text{ km/h}$ .

BEFORE COLLISION: VELOCITY  $\bar{v}$  OF MASS CENTER G:

$$(m_A + m_B) \bar{v} = m_A v_A + m_B v_B \quad \bar{v} = \frac{m_A v_A + m_B v_B}{m_A + m_B}$$

NOTION OF AUTOS RELATIVE TO G:

$$v_{A/G} = v_A - \bar{v} = v_A - \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m_B (v_A - v_B)}{m_A + m_B}$$

$$v_{B/G} = \frac{m_A (v_B - v_A)}{m_A + m_B} \quad \text{Similarly: } v_{B/G} = -\frac{m_A}{m_A + m_B} (v_A + v_B)$$

$$T_{A/G} = \frac{1}{2} m_A v_{A/G}^2 = \frac{1}{2} \frac{m_A m_B^2}{(m_A + m_B)^2} (v_A + v_B)^2 \quad (1)$$

$$T_{B/G} = \frac{1}{2} m_B v_{B/G}^2 = \frac{1}{2} \frac{m_A^2 m_B}{(m_A + m_B)^2} (v_A + v_B)^2 \quad (2)$$

AFTER COLLISION:

SINCE THERE IS NO EXTERNAL FORCE, G KEEPS MOVING WITH VELOCITY  $\bar{v}$ .

SINCE IMPACT IS PLASTIC:  $v_A' = v_B' = \bar{v}$

AND  $v_{A/G}' = v_{B/G}' = 0$ . THUS:  $T_{A/G}' = T_{B/G}' = 0$

IT FOLLOWS THAT  $E_A = T_{A/G}$  AND  $E_B = T_{B/G}$

(a) DIVIDING (1) BY (2):

$$\frac{E_A}{E_B} = \frac{T_{A/G}}{T_{B/G}} = \frac{m_B}{m_A} \quad (\text{Q.E.D.})$$

(b) SUBSTITUTING IN (1) AND (2) THE GIVEN DATA,  $m_A = 1600 \text{ kg}$ ,  $m_B = 900 \text{ kg}$ ,  $v_A = 90 \text{ km/h} = 25 \text{ m/s}$ ,  $v_B = 60 \text{ km/h} = 16.67 \text{ m/s}$ . WE FIND  $E_A = 160.0 \text{ kJ}$ ,  $E_B = 320 \text{ kJ}$

### 14.36 GIVEN: CAR COLLISION OF PROB. 14.35

DEFINE: SEVERITY OF A COLLISION =  $E/E_0$

WHERE  $E$  = ENERGY ABSORBED BY CAR IN COLLISION, AND  $E_0$  = EN. ABSORBED BY SAME CAR IN A TEST

WHERE IT HITS AN IMMOVABLE WALL WITH VELOC.  $v_0$

SHOW THAT COLLISION OF PROB. 14.35 IS  $(m_A/m_B)^2$  TIMES MORE SEVERE FOR CAR B THAN FOR CAR A.

ENERGIES ABSORBED IN TESTS OF A AND B:

$$(E_A)_0 = \frac{1}{2} m_A v_0^2 \quad (E_B)_0 = \frac{1}{2} m_B v_0^2 \quad (3)$$

SEVERITY OF COLLISION FOR CAR A =  $E_A/(E_A)_0$

SEVERITY OF COLLISION FOR CAR B =  $E_B/(E_B)_0$

RECALLING EQS. (3) AND FROM PROB. 14.35 THAT

$E_A/E_B = m_B/m_A$ , WE HAVE

$$\text{SEVERITY OF COLL. FOR B} = \frac{E_B (E_A)_0}{E_A (E_B)_0} =$$

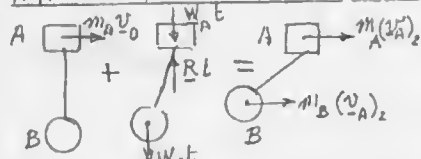
$$= \frac{m_A}{m_B} \frac{\frac{1}{2} m_A v_0^2}{\frac{1}{2} m_B v_0^2} = \left( \frac{m_A}{m_B} \right)^2 \quad (\text{Q.E.D.})$$

14.37

SOLVE SAMPLE PROB. 14.4, ASSUMING THAT CART A IS GIVEN VELOCITY  $v_0 \rightarrow$  AND THAT BALL B IS AT REST.

(a) VELOCITY OF B AT MAXIMUM ELEVATION

(IMPULSE-MOMENTUM METHOD):



WE NOTE THAT WHEN B REACHES MAX. HEIGHT  $(v_B)_2 = (v_B)_1$

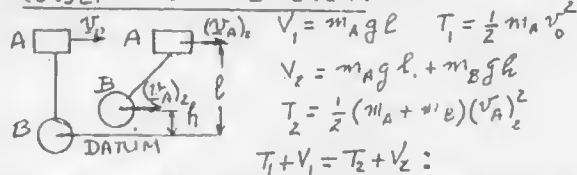
$$\sum m v_1 + \sum \text{Ext Imp}_{1 \rightarrow 2} = \sum m v_2$$

$$\text{X COMP.: } m_A v_0 = (m_A + m_B)(v_A)_2$$

$$(v_B)_2 = (v_A)_2 = \frac{m_A}{m_A + m_B} v_0 \rightarrow (1)$$

(b) MAXIMUM HEIGHT REACHED BY B

CONSERVATION OF ENERGY:



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m_A v_0^2 + m_A g \ell = \frac{1}{2} (m_A + m_B) (v_A)_2^2 + m_A g \ell + m_B g \ell$$

SUBSTITUTING FOR  $(v_A)_2$  FROM (1):

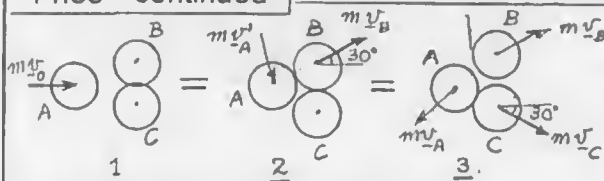
$$\frac{1}{2} m_A v_0^2 = \frac{1}{2} \frac{m_A^2}{m_A + m_B} v_0^2 + m_B g \ell$$

$$h = \frac{v_0^2}{2g} \frac{m_A^2}{(m_A + m_B)^2}, \quad h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g}$$

(SAME ANSWER AS FOR PART b OF SP14.4)

14.38 continued

(b) A HITS B BEFORE C



CONS. OF MOMENTUM FROM 1 TO 2:

$$\text{X COMP.: } m v_0 = m (v_A)_2 + m v_B \cos 30^\circ \quad (v_A)_2 = v_0 - v_B \cos 30^\circ \quad (4)$$

$$\text{Y COMP.: } 0 = m (v_A)_2 \sin 30^\circ + m v_B \sin 30^\circ \quad (v_A)_2 = -v_B \sin 30^\circ \quad (5)$$

SOLVE BOTH MEMBERS OF (4) AND (5) AND ADD:

$$v_A^2 = v_0^2 - 2 v_0 v_B \cos 30^\circ + v_B^2 \quad (6)$$

CONS. OF ENERGY FROM 1 TO 2:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 \quad v_0^2 = v_A^2 + v_B^2 \quad (7)$$

CARRYING INTO (6) AND SOLVING FOR  $v_B$ :  $v_B = v_0 \cos 30^\circ$  (7)

CONS. OF MOMENTUM FROM 1 TO 3:

$$\text{X COMP.: } m v_0 = m (v_A)_3 + m v_B \cos 30^\circ + m v_C \cos 30^\circ$$

$$\text{Y COMP.: } 0 = m (v_A)_3 \sin 30^\circ + m v_B \sin 30^\circ + m v_C \sin 30^\circ$$

SUBSTITUTE FOR  $v_B$  FROM (7) AND SOLVE FOR  $(v_A)_3$  AND  $(v_C)_3$ 

$$(v_A)_3 = v_0 \sin 30^\circ - v_C \cos 30^\circ \quad (8)$$

$$(v_A)_3 = -v_0 \sin 30^\circ \cos 30^\circ + v_C \sin 30^\circ \quad (9)$$

$$\text{SQUARING AND ADDING: } v_A^2 = 0.25 v_0^2 - 0.866 v_0 v_C + v_C^2 \quad (10)$$

CONS. OF ENERGY FROM 1 TO 3:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \quad v_0^2 = v_A^2 + v_B^2 + v_C^2 \quad (11)$$

SUBSTITUTE FOR  $v_A$  AND  $v_B$  FROM (10) AND (7) INTO (11):

$$v_0^2 = 0.25 v_0^2 - 0.866 v_0 v_C + v_C^2 + 0.25 v_0^2 + v_C^2, \quad v_C = 0.433 v_0$$

CARRYING INTO (8) AND (9):

$$(v_A)_3 = 0.25 v_0 - 0.433 v_0 \cos 30^\circ = -0.125 v_0$$

$$(v_A)_3 = -0.433 v_0 + 0.433 v_0 \sin 30^\circ = -0.2165 v_0$$

THUS:  $v_A = 0.25 v_0 \angle 60^\circ$ ;  $v_B = 0.866 v_0 \angle 30^\circ$ ;  $v_C = 0.433 v_0 \angle 30^\circ$ 

14.38

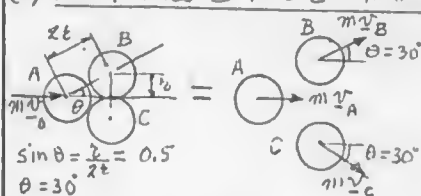


GIVEN:

BALL A HITS WITH  $v_0$   
BALLS B AND C WHICH  
ARE AT REST. ASSUME  
CONSERVATION OF  
ENERGY.

FIND: FINAL VELOCITY OF EACH BALL, IF  
(a) A STRIKES B AND C SIMULTANEOUSLY,  
(b) A HITS B BEFORE IT HITS C

(a) A STRIKES B AND C SIMULTANEOUSLY



CONSERVATION OF MOMENTUM

X COMP:

$$0 = m v_B \sin 30^\circ - m v_C \sin 30^\circ$$

$$v_B = v_C$$

$$v_0 - v_A = v_B \sqrt{3} \quad (1)$$

$$v_0^2 - v_A^2 = 2 v_B^2 \quad (2)$$

$$v_0 + v_A = \frac{2}{\sqrt{3}} v_B \quad (3)$$

$$v_0 + v_A = \frac{2}{\sqrt{3}} v_B$$

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$$v_0 + v_A = \frac{2}{\sqrt{3}} v_B$$

14.39



GIVEN:

A HITS B WITH  
 $v_0 = 15 \text{ ft/s}$ . ASSUME  
CONS. OF ENERGY  
FIND:  
MAGNITUDES OF  $v_A$ ,  
 $v_B$ , AND  $v_C$ .

CONS. OF MOMENTUM:

$$\text{X COMP.: } m v_0 \cos 45^\circ = m v_B \sin 30^\circ + m v_C \cos 30^\circ \quad (1)$$

$$\text{Y COMP.: } m v_0 \sin 45^\circ = m v_A - m v_B \cos 30^\circ + m v_C \sin 30^\circ \quad (2)$$

MULTIPLY (1) BY  $\sin 30^\circ$ , (2) BY  $\cos 30^\circ$ , SUBTRACT, ANDSOLVE FOR  $v_B$ :  $v_B = 0.8660 v_A - 0.2588 v_0$  (3)CARRY INTO (1) AND SOLVE FOR  $v_C$ :

$$v_C = 0.8165 v_0 - 0.57735 (0.8660 v_A - 0.2588 v_0)$$

$$v_C = -0.5 v_A + 0.9659 v_0 \quad (4)$$

CONS. OF ENERGY:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2, \quad v_0^2 = v_A^2 + v_B^2 + v_C^2$$

SUBSTITUTE FOR  $v_B$  AND  $v_C$  FROM (3) AND (4):

$$v_0^2 = v_A^2 + (0.8660 v_A - 0.2588 v_0)^2 + (-0.5 v_A + 0.9659 v_0)^2$$

$$2 v_A^2 - 1.414 v_0 v_A = 0 \quad v_A = 0.7071 v_0$$

$$\text{FROM (3) AND (4): } v_B = 0.3536 v_0, \quad v_C = 0.6124 v_0$$

GIVEN DATA:  $v_0 = 15 \text{ ft/s}$ . THEREFORE:

$$v_A = 10.61 \text{ ft/s}; \quad v_B = 5.30 \text{ ft/s}; \quad v_C = 9.19 \text{ ft/s}$$



14.40



GIVEN:

A HITS B WITH  $v_0 = 15 \text{ ft/s}$   
 ASSUME CONSERVATION  
 OF ENERGY.

FIND:

MAGNITUDES OF  $v_A$ ,  
 $v_B$ , AND  $v_C$ .

CONS. OF MOMENTUM:

$$\pm x \text{ COMP: } m v_0 \cos 30^\circ = m v_B \sin 45^\circ + m v_C \cos 45^\circ \quad (1)$$

$$\pm y \text{ COMP: } m v_0 \sin 30^\circ = m v_A - m v_B \cos 45^\circ + m v_C \sin 45^\circ \quad (2)$$

SUBTRACT (2) FROM (1) AND DIVIDE BY  $m$ :

$$0.3660 v_0 = -v_A + 1.4142 v_B \quad v_B = 0.7071 v_A + 0.2588 v_0 \quad (3)$$

ADD (1) AND (2) AND DIVIDE BY  $m$ :

$$1.3660 v_0 = v_A + 1.4142 v_C \quad v_C = -0.7071 v_A + 0.9659 v_0 \quad (4)$$

CONS. OF ENERGY:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \quad v_0^2 = v_A^2 + v_B^2 + v_C^2$$

SUBSTITUTE FOR  $v_B$  AND  $v_C$  FROM (3) AND (4):

$$v_0^2 = v_A^2 + (0.7071 v_A + 0.2588 v_0)^2 + (-0.7071 v_A + 0.9659 v_0)^2$$

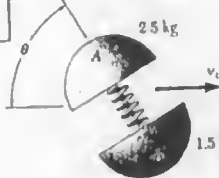
$$2 v_0^2 - v_0 v_A = 0 \quad v_A = 0.5 v_0$$

$$\text{FROM (2) AND (4): } v_B = 0.6124 v_0, \quad v_C = 0.6124 v_0$$

GIVEN DATA:  $v_0 = 15 \text{ ft/s}$ . THEREFORE:

$$v_A = 7.50 \text{ ft/s}; \quad v_B = 9.19 \text{ ft/s}; \quad v_C = 9.19 \text{ ft/s}$$

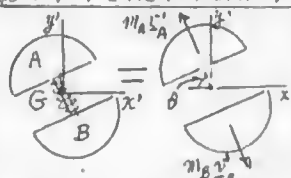
14.41



GIVEN:  $v_0 = 8 \text{ m/s}$ .  
 POTENTIAL ENERGY OF  
 SPRING = 120 J.  
 CORD CUT WHEN  $\theta = 30^\circ$ .

FIND:

$v_A$  AND  $v_B$  AFTER  
 CORD IS CUT.

CONS. OF LINEAR MOM. W/R FRAME  $Gx'y'z'$  $\Delta$  CRAMER'S:

$$0 = m_A v_A' - m_B v_B'$$

$$v_B' = \frac{m_A}{m_B} v_A' = \frac{2.5}{1.5} v_A'$$

$$v_B' = \frac{5}{3} v_A' \quad (1)$$

CONS. OF ENERGY W/R FRAME  $Gx'y'z'$ :

$$120 \text{ J} = \frac{1}{2} (2.5) v_A'^2 + \frac{1}{2} (1.5) v_B'^2 \quad 5 v_A'^2 + 3 v_B'^2 = 480 \quad (2)$$

SUBSTITUTE FOR  $v_B'$  FROM (1) INTO (2):

$$5 v_A'^2 + 3 \left( \frac{5}{3} v_A' \right)^2 = 480 \quad v_A'^2 = 36 \quad v_A' = 6 \text{ m/s} \rightarrow \theta$$

$$\text{FROM (1): } v_B' = \frac{5}{3} (6 \text{ m/s}) \quad v_B' = 10 \text{ m/s} \rightarrow \theta$$

WITH RESPECT TO FIXED FRAME  $Ox'y'z'$ :

$$v_A = v_A' + v_A'' = 8 \text{ m/s} \rightarrow + 6 \text{ m/s} \rightarrow \theta \quad (3)$$

$$v_B = v_B' + v_B'' = 8 \text{ m/s} \rightarrow + 10 \text{ m/s} \rightarrow \theta \quad (4)$$

FOR  $\theta = 30^\circ$ : EQ. (3):

$$\pm x \text{ COMP: } (v_A)_x = 8 - 6 \cos 30^\circ = 2.804 \text{ m/s}$$

$$\pm y \text{ COMP: } (v_A)_y = 6 \sin 30^\circ = 3 \text{ m/s}$$

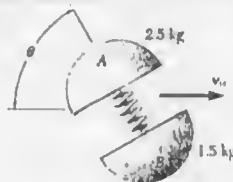
$$\text{EQ. (4): } v_A = 4.11 \text{ m/s} \angle 46.9^\circ$$

$$\pm x \text{ COMP: } (v_B)_x = 8 + 10 \cos 30^\circ = 16.660 \text{ m/s}$$

$$\pm y \text{ COMP: } (v_B)_y = -10 \sin 30^\circ = -5 \text{ m/s}$$

$$v_B = 17.39 \text{ m/s} \angle 16.7^\circ$$

14.42

GIVEN:  $v_0 = 8 \text{ m/s}$ .

POTENTIAL ENERGY OF  
 SPRING = 120 J.

CORD CUT WHEN  $\theta = 120^\circ$ .

FIND:

$v_A$  AND  $v_B$  AFTER  
 THE CORD IS CUT.

SEE SOLUTION OF PROB. 14.41 FOR DERIVATION OF

EQ. (3) AND (4). WITH  $\theta = 120^\circ$ , WE HAVE

$$v_A = v_0 + v_A' = 8 \text{ m/s} + 6 \text{ m/s} \angle 60^\circ \quad (3')$$

$$v_B = v_0 + v_B' = 8 \text{ m/s} + 10 \text{ m/s} \angle 60^\circ \quad (4')$$

EQ. (3'):

$$\pm x \text{ COMP: } (v_A)_x = 8 + 6 \cos 60^\circ = 8 + 3 = 11 \text{ m/s}$$

$$\pm y \text{ COMP: } (v_A)_y = 6 \sin 60^\circ = 5.196 \text{ m/s}$$

$$v_A = 12.17 \text{ m/s} \angle 25.3^\circ$$

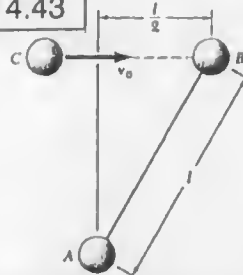
EQ. (4'):

$$\pm x \text{ COMP: } (v_B)_x = 8 - 10 \cos 60^\circ = 8 - 5 = 3 \text{ m/s}$$

$$\pm y \text{ COMP: } (v_B)_y = -10 \sin 60^\circ = -8.660 \text{ m/s}$$

$$v_B = 9.17 \text{ m/s} \angle 70.9^\circ$$

14.43



GIVEN:

THREE SPHERES, EACH OF MASS  $m$ .

A AND B ARE CONNECTED

BY TAUT, INEXTENSIBLE CORD.

C STRIKES B AS SHOWN.

ASSUME CONS. OF ENERGY.

FIND:

VELOCITY OF EACH SPHERE  
 AFTER IMPACT.

EFFECT ON CONSTRAINTS ON FINAL VELOCITIES

$$\begin{aligned} \vec{I}_{AB} \Delta \vec{v} &= m \vec{v}_A' \\ m \vec{v}_0 &= m \vec{v}_C + m \vec{v}_C' = m \vec{v}_C' \\ v_A &= v_A' \angle 60^\circ \\ v_C &= v_C' \end{aligned}$$

BECAUSE CORD AB IS INEXTENSIBLE,

COMPONENT OF  $v_B$  ALONG ABMUST BE EQUAL TO  $v_A$ .

$$v_B = v_A \angle 60^\circ + v_{B/A} \angle 30^\circ \quad (1)$$

CONS. OF MOMENTUM FOR SYSTEM:

$$\begin{aligned} m v_0 &= m v_C + 2 m v_A + m v_{B/A} \\ \pm y \text{ COMP: } 0 &= 2 m v_A \sin 60^\circ - m v_{B/A} \sin 30^\circ \\ v_{B/A} &= 2 \sqrt{3} v_A \quad (2) \end{aligned}$$

$$\pm x \text{ COMP: } m v_0 = m v_C + 2 m v_A \cos 60^\circ + m v_{B/A} \cos 30^\circ$$

$$\text{DIVIDING BY } m \text{ AND SUBSTITUTING FOR } v_{B/A} \text{ FROM (2): } v_0 = v_C + v_A + \frac{\sqrt{3}}{2} (2 \sqrt{3} v_A) \quad v_C = v_0 - 4 v_A \quad (3)$$

CONS. OF ENERGY:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \quad v_0^2 = 2 v_A^2 + v_{B/A}^2 + v_C^2 \quad (4)$$

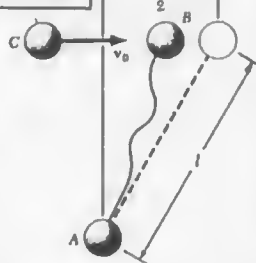
SUBSTITUTE FOR  $v_{B/A}$  AND  $v_C$  FROM (2) AND (3) INTO (4):

$$v_0^2 = 2 v_A^2 + 12 v_A^2 + v_C^2 - 8 v_0 v_A + 16 v_A^2 \quad v_A = \frac{4}{15} v_0 \angle 60^\circ$$

$$\text{FROM (3): } v_C = v_0 - \frac{16}{15} v_0 = -\frac{1}{15} v_0 \quad v_C = \frac{1}{15} v_0$$

$$\text{FROM (1) AND (2): } v_B = \frac{4}{15} v_0 \angle 60^\circ + \frac{8 \sqrt{3}}{15} v_0 \angle 30^\circ, \quad v_B = 0.961 v_0 \angle 13.9^\circ$$

14.44



GIVEN:

THREE SPHERES, EACH OF MASS  $m$ . A AND B ARE CONNECTED BY INEXTENSIBLE CORD WHICH IS SLACK. C STRIKES B AS SHOWN WITH PERFECTLY ELASTIC IMPACT.

FIND:

- (a) VELOCITY OF EACH SPHERE AFTER CORD BECOMES TAUT.  
(b) FRACTION OF INITIAL K.E. LOST WHEN CORD BECOMES TAUT.

(a) DETERMINATION OF VELOCITIES

IMPACT OF C AND B



CONS. OF MOMENTUM:

$$m v_0 = m v_C + m v_B$$

$$v_C + v_B = v_0 \quad (1)$$

CONS. OF ENERGY (PERFECTLY ELASTIC IMPACT):

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_C^2 + \frac{1}{2} m v_B^2 \quad v_C^2 + v_B^2 = v_0^2 \quad (2)$$

$$\text{SQUARE (1): } v_C^2 + 2 v_C v_B + v_B^2 = v_0^2$$

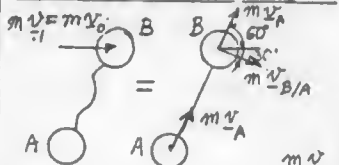
$$\text{SUBTRACT (2): } 2 v_C v_B = 0$$

$v_B = 0$  CORRESPONDS TO INITIAL CONDITIONS AND SHOULD BE ELIMINATED. THEREFORE

$$v_C = 0$$

$$\text{FROM (1): } v_B = v_0$$

CORD AB BECOMES TAUT



BECAUSE CORD IS INEXTENSIBLE, COMPONENT OF  $\vec{v}_B$  ALONG AB MUST BE EQUAL TO  $\vec{v}_A$ .

CONS. OF MOMENTUM:

$$m v_0 = 2 m v_A + m v_{B/A}$$

$$\uparrow y \text{ COMP: } 0 = 2 m v_A \sin 60^\circ - m v_{B/A} \sin 30^\circ$$

$$v_{B/A} = 2\sqrt{3} v_A \quad (3)$$

$$\rightarrow x \text{ COMP: } m v_0 = 2 m v_A \cos 60^\circ + m v_{B/A} \cos 30^\circ$$

DIVIDING BY  $m$  AND SUBSTITUTING FOR  $v_{B/A}$  FROM (3):

$$v_0 = 2 v_A (0.5) + (2\sqrt{3} v_A)(\sqrt{3}/2)$$

$$v_0 = 4 v_A \quad v_A = 0.25 v_0 \quad v_{B/A} = 0.25 v_0 \sqrt{3}$$

CARRYING INTO (3):  $v_{B/A} = 2\sqrt{3} (0.25 v_0) = 0.866 v_0$ 

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = (0.25 v_0 \cos 60^\circ + 0.866 v_0 \cos 30^\circ) \hat{i} + (0.25 v_0 \sin 60^\circ - 0.866 v_0 \sin 30^\circ) \hat{j}$$

$$\vec{v}_B = 0.875 v_0 \hat{i} - 0.2125 v_0 \hat{j}$$

$$\angle_B = 0.4125 v_0 \approx 13.9^\circ$$

$$v_B = 0.901 v_0 \approx 13.9^\circ$$

(b) FRACTION OF K.E. LOST

$$T_0 = \frac{1}{2} m v_0^2$$

$$T_{\text{FINAL}} = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2$$

$$= \frac{1}{2} m (0.25 v_0)^2 + \frac{1}{2} m (0.90139 v_0)^2 + \frac{1}{2} m (0)$$

$$= \frac{1}{2} m (0.875) v_0^2$$

$$\text{K.E. LOST} = T_0 - T_{\text{FINAL}} = \frac{1}{2} m (1 - 0.875) v_0^2 = \frac{1}{2} \cdot \frac{1}{8} m v_0^2$$

$$\text{FRACTION OF K.E. LOST} = \frac{1}{8}$$

14.45

GIVEN:

360-kg SPACE VEHICLE WITH  $\vec{v}_0 = (450 \text{ m/s}) \hat{i}$ . AS IT PASSES THROUGH O, EXPLOSIVE CHARGES SEPARATE IT INTO 3 PARTS: A (60 kg), B (120 kg), AND C (180 kg). SHORTLY AFTER, THE POSITIONS OF THE 3 PARTS ARE A (72 m, 72 m, 648 m), B (180 m, 396 m, 972 m), C (-144 m, -288 m, 576 m). VELOCITY OF B IS  $\vec{v}_B = (150 \text{ m/s}) \hat{i} + (330 \text{ m/s}) \hat{j} + (660 \text{ m/s}) \hat{k}$ . X-COMP OF VELOCITY OF C IS  $(v_C)_x = -120 \text{ m/s}$ .

FIND: VELOCITY OF A.

CONSERVATION OF ANGULAR MOMENTUM ABOUT O

SINCE VEHICLE PASSES THROUGH O,  $H_0 = 0$ , OR

$$H_0 = \sum \vec{r} \times m \vec{v} = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B + \vec{r}_C \times m_C \vec{v}_C = 0$$

USING DETERMINANT FORM:

$$H_0 = 60 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 72 & 72 & 648 \\ (v_A)_x & (v_A)_y & (v_A)_z \end{vmatrix} + 120 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 180 & 396 & 972 \\ 150 & 330 & 660 \end{vmatrix} + 180 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -144 & -288 & 576 \\ (v_C)_x & (v_C)_y & (v_C)_z \end{vmatrix} = 0$$

EQUATING TO ZERO THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ , AND DIVIDING BY 60:

$$(1) \quad 72(v_A)_z - 648(v_A)_y - 1188 \times 10^3 - 864(v_C)_z - 1728(v_C)_y = 0$$

$$(2) \quad 648(v_A)_z - 72(v_A)_x + 540 \times 10^3 - 207.36 \times 10^3 + 432(v_C)_z = 0$$

$$(3) \quad 72(v_A)_y - 72(v_A)_x + 0 - 432(v_C)_y - 103.68 \times 10^3 = 0$$

OR, AFTER REDUCTIONS:

$$(v_A)_z - 9(v_A)_y - 12(v_C)_z - 24(v_C)_y = 1650 \quad (1)$$

$$-(v_A)_z + 9(v_A)_x + 6(v_C)_z = 2130 \quad (2)$$

$$(v_A)_y - (v_A)_x - 6(v_C)_y = 1440 \quad (3)$$

CONSERVATION OF LINEAR MOMENTUM

$$m \vec{v}_0 = m_A \vec{v}_A + m_B \vec{v}_B + m_C \vec{v}_C$$

$$360(450 \hat{i}) = 60[(v_A)_x \hat{i} + (v_A)_y \hat{j} + (v_A)_z \hat{k}] + 120[150 \hat{i} + 330 \hat{j} + 660 \hat{k}] + 180[-120 \hat{i} + (v_C)_y \hat{j} + (v_C)_z \hat{k}]$$

EQUATING THE COEFF. OF THE UNIT VECTORS AND DIVIDING BY 60:

$$(1) \quad (v_A)_x + 300 - 360 = 0 \quad (v_A)_x = 60 \text{ m/s} \quad (4)$$

$$(2) \quad (v_A)_y + 660 + 3(v_C)_z = 0 \quad (v_A)_y = -660 - 3(v_C)_z \quad (5)$$

$$(3) \quad (v_A)_z + 1320 + 3(v_C)_z = 2700 \quad (v_A)_z = 1380 - 3(v_C)_z \quad (6)$$

SUBSTITUTING FROM (4), (5), (6) INTO (2) AND (3):

$$-1380 + 3(v_C)_z + 9(60) + 6(v_C)_z = 2130 \quad (v_C)_z = 930 \text{ m/s}$$

$$-660 - 3(v_C)_z - 60 - 6(v_C)_y = 1440 \quad (v_C)_y = -270 \text{ m/s}$$

SUBSTITUTING FOR  $(v_C)_y$  AND  $(v_C)_z$  INTO (5) AND (6):

$$(v_A)_y = -660 - 3(-270) = 60 \text{ m/s}$$

$$(v_A)_z = 1380 - 3(930) = 390 \text{ m/s}$$

RECALLING FROM (4) THAT  $(v_A)_x = 60 \text{ m/s}$ , WE HAVE

$$\vec{v}_A = (60.0 \text{ m/s}) \hat{i} + (60.0 \text{ m/s}) \hat{j} + (390 \text{ m/s}) \hat{k}$$

CHECK

SINCE EQ. (1) WAS NOT USED IN OUR SOLUTION, WE CAN USE IT TO CHECK THE ANSWER.

SUBSTITUTING THE VALUES OBTAINED FOR

$(v_A)_x$ ,  $(v_A)_y$ ,  $(v_C)_z$ , AND  $(v_C)_y$  INTO THE LEFT-HAND MEMBER OF EQ. (1), WE OBTAIN

$$390 - 9(60) - 12(330) - 24(-270) =$$

$$390 - 540 - 3960 + 5760 = 1650 \quad \text{O.K.}$$



14.46

GIVEN:

IN SCATTERING EXPERIMENT OF PROB. 14.24

IT IS KNOWN THAT PARTICLE A IS PROJECTED FROM  $A_0(260, -20, 340)$  AND COLLIDES WITH C AT  $Q(200, 180, 140)$ . FIND:

COORDINATES OF  $B_0$  WHERE PATH OF B INTERSECTS  $xy$  PLANE.

CONS. OF ANGULAR MOMENTUM ABOUT Q:

SINCE PATHS OF A AFTER COLLISIONS AND OF C BEFORE AND AFTER COLLISION PASS THROUGH Q THE CORRESPONDING ANG. MOMENTA ARE ZERO (FIG. P14.24). CONS. OF ANGULAR MOMENTUM OF ALL PARTICLES ABOUT Q IS EXPRESSED AS

$$\vec{Q}A_0 \times m_A \vec{u}_0 + \vec{Q}B_0 \times m_B \vec{v}_0 = \vec{Q}B_1 \times m_B \vec{v}_B \quad (1)$$

$$\text{WHERE } \vec{Q}A_0 = \vec{r}_{A_0} - \vec{r}_Q = (260\hat{i} - 20\hat{j} + 340\hat{k}) - (200\hat{i} + 180\hat{j} + 140\hat{k})$$

$$= 60\hat{i} - 200\hat{j} + 200\hat{k}$$

$$\vec{Q}B_0 = (\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}$$

$$\vec{Q}B_1 = \vec{r}_{B_1} - \vec{r}_Q = (107\hat{i} + 200\hat{j} + 170\hat{k}) - (200\hat{i} + 180\hat{j} + 140\hat{k})$$

$$= -93\hat{i} + 20\hat{j} + 30\hat{k}$$

$$\vec{u}_0 = -480\hat{i} + 600\hat{j} - 640\hat{k} \quad \vec{v}_0 = 480\hat{j}$$

AND, FROM SOLUTION OF PROB. 14.24:

$$\vec{v}_B = \vec{v}_B \hat{z}_B = 573.6(-0.4229\hat{i} + 0.8971\hat{j} - 0.12816\hat{k})$$

$$= -242.6\hat{i} + 514.6\hat{j} - 73.51\hat{k}$$

SUBSTITUTING INTO (1) AND USING DETERMINANTS:

$$\frac{m}{4} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 60 & -200 & 200 \\ -480 & 600 & -640 \end{vmatrix} + m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \Delta x & \Delta y & \Delta z \\ 0 & 480 & 0 \end{vmatrix} = m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -93 & 20 & 30 \\ -242.6 & 514.6 & -73.51 \end{vmatrix}$$

EQUATING THE COEFFICIENTS OF  $\hat{i}$  AND  $\hat{k}$ :

$$\left(\frac{\hat{i}}{4}\right) \frac{1}{4}(8000) - 480\Delta z = -16908 \quad \Delta z = 39.39 \text{ mm}$$

$$\left(\frac{\hat{k}}{4}\right) \frac{1}{4}(-60000) + 480\Delta x = -43006 \quad \Delta x = -58.35 \text{ mm}$$

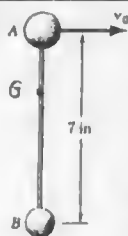
$$x_{B_0} = x_Q + \Delta x = 200 - 58.4$$

$$z_{B_0} = 141.6 \text{ mm}$$

$$z_{B_0} = z_Q + \Delta z = 140 + 39.4$$

$$z_{B_0} = 179.4 \text{ mm}$$

14.47



GIVEN:

5-lb SPHERE A AND 2-lb SPHERE B CONNECTED BY RIGID ROD REST ON HORIZONTAL, FRICTIONLESS SURFACE WHEN A IS GIVEN VELOCITY  $\vec{v}_0 = (10.5 \text{ ft/s})\hat{i}$ .

FIND:

(a) LINEAR MOM. AND ANG. MOM.  $H_G$ (b)  $\vec{v}_A$  AND  $\vec{v}_B$  AFTER  $180^\circ$  ROTATION.POSITION OF MASS CENTER  $AG + BG = 7 \text{ in.}$ 

$$AG(5 \text{ lb}) = BG(2 \text{ lb}), \quad BG = 2.5AG, \quad 3.5AG = 7, \quad AG = 2 \text{ in.}$$

(a) LINEAR AND ANG. MOMENTUM.

$$L = m_A \vec{v}_0 = \frac{5 \text{ lb}}{32.2 \text{ ft/s}^2} (10.5 \text{ ft/s})\hat{i} = (1.6304 \text{ lb}\cdot\text{s})\hat{i}$$

$$L = (1.630 \text{ lb}\cdot\text{s})\hat{i}$$

$$H_G = \vec{G}A \times m_A \vec{v}_0 = (2 \text{ in.})\hat{j} \times (1.6304 \text{ lb}\cdot\text{s})\hat{i}$$

$$= -(3.2608 \text{ in.}\cdot\text{lb}\cdot\text{s})\hat{k}$$

$$= -(0.27174 \text{ ft}\cdot\text{lb}\cdot\text{s})\hat{k}$$

$$H_G = -(0.272 \text{ ft}\cdot\text{lb}\cdot\text{s})\hat{k}$$

(CONTINUED)

14.47 continued

(b) VELOCITIES OF A AND B AFTER  $180^\circ$  ROTATION

CONS. OF LINEAR MOMENTUM:

$$m_A \vec{v}_0 = m_A \vec{v}_A' + m_B \vec{v}_B'$$

$$(5/32)(10.5) = (5/32)\vec{v}_A' + (2/32)\vec{v}_B'$$

$$5\vec{v}_A' + 2\vec{v}_B' = 52.5 \quad (1)$$

CONS. OF ANG. MOM. ABOUT G:

$$I_G \vec{\omega}_0 = I_G \vec{\omega}' = I_A \vec{\omega}_A' + I_B \vec{\omega}_B'$$

$$(2 \text{ in.})(5/32)(10.5) = -(2 \text{ in.})(5/32)\vec{v}_A' + (5 \text{ in.})(2/32)\vec{v}_B'$$

$$\text{MULTIPLY BY 32 AND DIVIDE BY 2: } -5\vec{v}_A' + 5\vec{v}_B' = 52.5 \quad (2)$$

$$\text{ADD (1) AND (2): } 7\vec{v}_B' = 105 \quad \vec{v}_B' = +15.00 \text{ ft/s.}$$

$$\text{FROM (1): } 5\vec{v}_A' + 2(15) = 52.5 \quad \vec{v}_A' = +4.50 \text{ ft/s}$$

$$\vec{v}_A' = (4.50 \text{ ft/s})\hat{i} ; \vec{v}_B' = (15.00 \text{ ft/s})\hat{i}$$

14.48



GIVEN:

5-lb SPHERE A AND 2-lb SPHERE B CONNECTED BY RIGID ROD REST ON HORIZONTAL, FRICTIONLESS SURFACE WHEN B IS GIVEN VELOCITY  $\vec{v}_0 = (10.5 \text{ ft/s})\hat{i}$ .

FIND:

(a) LINEAR MOM. AND ANG. MOM.  $H_G$ (b)  $\vec{v}_A$  AND  $\vec{v}_B$  AFTER  $180^\circ$  ROTATION.POSITION OF MASS CENTER  $AG + BG = 7 \text{ in.}$ 

$$AG(5 \text{ lb}) = BG(2 \text{ lb}), \quad BG = 2.5AG, \quad 3.5AG = 7, \quad AG = 2 \text{ in.}$$

(a) LINEAR AND ANG. MOMENTUM

$$L = m_B \vec{v}_0 = \frac{2 \text{ lb}}{32.2 \text{ ft/s}^2} (10.5 \text{ ft/s})\hat{i} = (0.6522 \text{ lb}\cdot\text{s})\hat{i}$$

$$L = (0.652 \text{ lb}\cdot\text{s})\hat{i}$$

$$H_G = \vec{G}B \times m_B \vec{v}_0 = -(5 \text{ in.})\hat{j} \times (0.6522 \text{ lb}\cdot\text{s})\hat{i}$$

$$= +(3.261 \text{ in.}\cdot\text{lb}\cdot\text{s})\hat{k}$$

$$H_G = +(0.272 \text{ ft}\cdot\text{lb}\cdot\text{s})\hat{k}$$

(b) VELOCITIES OF A AND B AFTER  $180^\circ$  ROTATION.

CONS. OF LINEAR MOMENTUM:

$$m_B \vec{v}_0 = m_A \vec{v}_A' + m_B \vec{v}_B'$$

$$(2/32)(10.5) = (5/32)\vec{v}_A' + (2/32)\vec{v}_B'$$

$$5\vec{v}_A' + 2\vec{v}_B' = 21 \quad (1)$$

CONS. OF ANG. MOM. ABOUT G:

$$I_G \vec{\omega}_0 = I_G \vec{\omega}' = I_A \vec{\omega}_A' + I_B \vec{\omega}_B'$$

$$(5 \text{ in.})(2/32)(10.5) = (2 \text{ in.})(5/32)\vec{v}_A' - (5 \text{ in.})(2/32)\vec{v}_B'$$

MULTIPLY BY 32 AND DIVIDE BY 2:

$$5\vec{v}_A' - 5\vec{v}_B' = 52.5 \quad (2)$$

SUBTRACT (2) FROM (1):

$$7\vec{v}_B' = -31.5$$

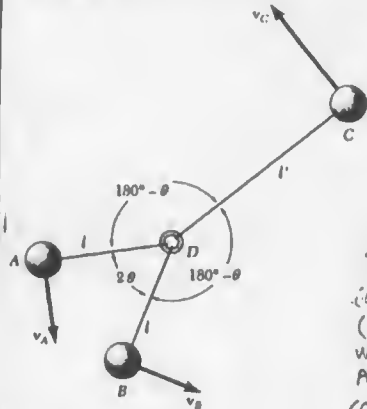
$$\vec{v}_B' = -4.50 \text{ ft/s}$$

FROM (1):  $5\vec{v}_A' + 2(-4.50) = 21$ 

$$\vec{v}_A' = +6.00 \text{ ft/s}$$

$$\vec{v}_A' = (6.00 \text{ ft/s})\hat{i} ; \vec{v}_B' = -(4.50 \text{ ft/s})\hat{i}$$

14.49 and 14.50



**GIVEN:**  
THREE IDENTICAL SPHERES  
CONNECTED TO RING D AT  
THEIR MASS CENTER  
SLIDE ON HORIZONTAL,  
FRICTIONLESS SURFACE  
( $l = 2l \cos \theta$ ).  
 $v_A = v_B = v_0$  WHEN  
CORD CD BREAKS.  
**FIND AFTER CORDS**  
AD AND BD BECOME TAUT  
(a) SPEED OF RING D  
(b) RELATIVE SPEED AT  
WHICH A AND B ROTATE  
ABOUT D  
(c) PERCENT ENERGY OF

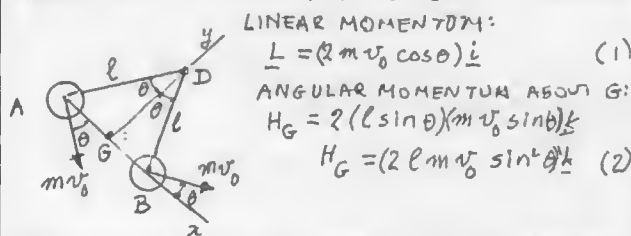
ORIGINAL SYSTEM LOST WHEN AD AND BD BECOME TAUT.

PROB. 14.49: ASSUME  $\theta = 30^\circ$ .

PROB. 14.50: ASSUME  $\theta = 45^\circ$ .

WE CONSIDER THE FOLLOWING TWO POSITIONS OF  
THE SPHERES A AND B AND THE RING D.

**POSITION 1:** IMMEDIATELY AFTER CORD CD BREAKS



LINEAR MOMENTUM:

$$\underline{L} = (2m v_0 \cos \theta) \underline{i} \quad (1)$$

ANGULAR MOMENTUM ABOUT G:

$$H_G = (2l \sin \theta)(m v_0 \sin \theta) l$$

$$H_G = (2l m v_0 \sin^2 \theta) l \quad (2)$$

**POSITION 2:** AFTER CORDS AD AND BD BECOME TAUT:

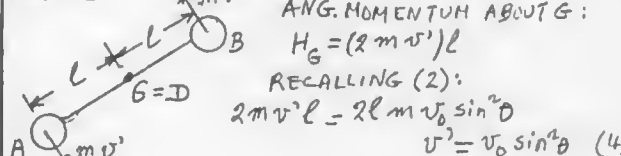
(a) SPEED OF MASS CENTER (NOW LOCATED AT D).

RECALLING (1):

$$\underline{L} = (2m) \underline{v} = (2m v_0 \cos \theta) \underline{i} \quad \underline{v} = (v_0 \cos \theta) \underline{i}$$

$$v_D = \underline{v} = v_0 \cos \theta \quad (3)$$

(b) RELATIVE SPEED  $v'$  AT WHICH A AND B ROTATE  
ABOUT D



ANG. MOMENTUM ABOUT G:

$$H_G = (2m v') l$$

RECALLING (2):

$$2m v' l = 2l m v_0 \sin^2 \theta$$

$$v' = v_0 \sin^2 \theta \quad (4)$$

(c) ENERGY LOST:

CONSIDERING SYSTEM OF 3 SPHERES:

INITIALLY,  $v_C = (l/l) v_A = (2 \cos \theta) v_0$ . THEREFORE

$$T_0 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 = m v_0^2 (1 + 2 \cos^2 \theta)$$

$$T_f = \frac{1}{2} (2m) v_D^2 + 2 \left( \frac{1}{2} m v'^2 \right) + \frac{1}{2} m v_C^2$$

$$= m [v_0^2 \cos^2 \theta + m (v_0 \sin^2 \theta)^2 + 2 v_0^2 \cos^2 \theta]$$

$$= m v_0^2 (3 \cos^2 \theta + \sin^4 \theta)$$

$$\% \text{ LOSS} = 100 \frac{T_0 - T_f}{T_0} = 100 \frac{1 + 2 \cos^2 \theta - 3 \cos^2 \theta - \sin^4 \theta}{1 + 2 \cos^2 \theta} = 100 \frac{\sin^2 \theta \cos^2 \theta}{1 + 2 \cos^2 \theta} \quad (5)$$

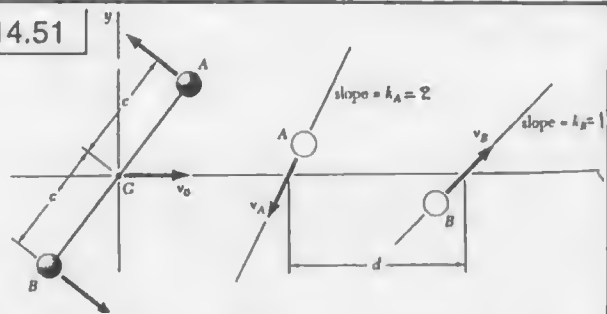
PROB. 14.49: MAKING  $\theta = 30^\circ$  IN EQS. (3), (4), AND (5)

(a)  $0.866 v_0$ . (b)  $0.250 v_0$ . (c)  $7.50\%$

PROB. 14.50: MAKING  $\theta = 45^\circ$  IN EQS. (3), (4), AND (5):

(a)  $0.707 v_0$ . (b)  $0.500 v_0$ . (c)  $12.50\%$

14.51



**GIVEN:** TWO SMALL IDENTICAL SPHERES A AND B,  
CONNECTED BY A CORD SLIDE ON A HORIZONTAL, FRICTION-  
LESS SURFACE. INITIALLY THEY ROTATE WITH  $\dot{\theta} = 8 \text{ rad/s}$   
ABOUT G, AND G HAS VELOCITY  $\underline{v}_0 = v_0 \underline{i}$ .  
AFTER CORD BREAKS, SPHERES MOVE ALONG PATHS  
WITH  $k_A = 2$ ,  $k_B = 1$ , AND  $d = 625 \text{ mm}$ .

**FIND:**

(a) SPEEDS  $v_0$ ,  $v_A$ , AND  $v_B$ , (b) length  $2c$  of cord

CONSERVATION OF LINEAR MOMENTUM

$$\text{BEFORE BREAK: } \underline{L}_0 = (2m) \underline{v} \quad \underline{L}_0 = 2m v_0 \underline{i}$$

AFTER BREAK:

$$\begin{aligned} \underline{L} &= m \underline{v}_A + m \underline{v}_B = m \left( -\frac{1}{\sqrt{5}} v_A \underline{i} + \frac{2}{\sqrt{5}} v_A \underline{j} \right) + m \left( -\frac{2}{\sqrt{2}} v_B \underline{i} + \frac{1}{\sqrt{2}} v_B \underline{j} \right) \\ \underline{L} &= m \underline{v}_A + m \underline{v}_B = m \left( -\frac{1}{\sqrt{5}} v_A \underline{i} + \frac{2}{\sqrt{5}} v_A \underline{j} \right) + m \left( -\frac{2}{\sqrt{2}} v_B \underline{i} + \frac{1}{\sqrt{2}} v_B \underline{j} \right) \end{aligned}$$

SETTING  $\underline{L} = \underline{L}_0$  AND EQUATING COEFF. OF UNIT VECTORS:

$$\textcircled{1} \quad -\frac{1}{\sqrt{5}} v_A + \frac{2}{\sqrt{2}} v_B = 2 v_0 \quad (1)$$

$$\textcircled{2} \quad \frac{2}{\sqrt{5}} v_A + \frac{1}{\sqrt{2}} v_B = 0 \quad (2)$$

$$\text{SUBTRACTING (2) FROM (1): } \frac{1}{\sqrt{5}} v_A = 2 v_0 \quad v_A = 2\sqrt{5} v_0 \quad (3)$$

$$\text{SUBSTITUTING FOR } v_A \text{ INTO (2): } v_B = \frac{2\sqrt{2}}{\sqrt{5}} (2\sqrt{5} v_0) = 4\sqrt{2} v_0 \quad (4)$$

CONSERVATION OF ANGULAR MOMENTUM

$$\text{BEFORE BREAK: } (H_G)_0 = 2m c^2 \dot{\theta} = 2m c^2 (8 \text{ rad/s}) = 16 m c^2$$

$$\text{AFTER BREAK: } H_G = H_A + m (v_B) d = m \left( \frac{1}{12} (4\sqrt{2} v_0) (0.625 \text{ m}) \right) -$$

$$= 2.5 m v_0$$

$$\text{SETTING } H_G = (H_G)_0: 2.5 m v_0 = 16 m c^2 \quad v_0 = 6.40 c^2 \quad (5)$$

CONSERVATION OF ENERGY

$$\text{BEFORE BREAK: } T_0 = \frac{1}{2} (2m) v_0^2 + \frac{1}{2} (2m) (c \dot{\theta})^2 = m (v_0^2 + c^2 \dot{\theta}^2)$$

$$\text{LETTING } \dot{\theta} = 8 \text{ rad/s AND USING (5): } T_0 = m (40.96 c^4 + 64 c^4)$$

$$\text{AFTER BREAK: } T = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

RECALLING (3), (4), AND (5):

$$T = \frac{1}{2} m (20 v_0^2 + 32 v_0^2) = 26 m v_0^2 = 1064.96 m c^4$$

$$\text{SETTING } T = T_0: 1064.96 c^4 = 40.96 c^4 + 64 c^4$$

$$1024 c^4 = 64 \quad c^4 = 0.0625 \quad c = 0.250 \text{ m}$$

$$\text{FROM (5): } v_0 = 6.40 (0.0625) = 0.400 \text{ m/s}$$

$$\text{FROM (3): } v_A = 2\sqrt{5} (0.4) = 1.789 \text{ m/s}$$

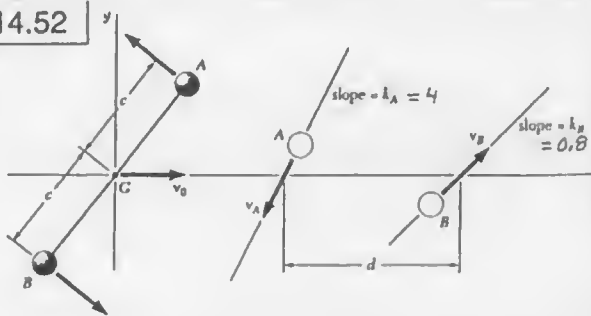
$$\text{FROM (4): } v_B = 4\sqrt{2} (0.4) = 2.26 \text{ m/s}$$

**ANSWERS:**

$$(a) \quad v_0 = 0.400 \text{ m/s}; v_A = 1.789 \text{ m/s}; v_B = 2.26 \text{ m/s}$$

$$(b) \quad \text{LENGTH OF CORD} = 2c = 500 \text{ mm}$$

14.52



**GIVEN:** TWO SMALL IDENTICAL SPHERES A AND B, CONNECTED BY A CORD OF LENGTH  $2c = 600 \text{ mm}$  SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE. INITIALLY THEY ROTATE WITH  $\dot{\theta} = 12 \text{ rad/s}$  ABOUT G, AND G MOVES WITH  $\underline{v}_G = v_0 \underline{i}$ . AFTER CORD BREAKS, SPHERES MOVE ALONG PATHS WITH  $k_A = 4$  AND  $k_B = 0.8$ .

**FIND:** SPEEDS  $v_0$ ,  $v_A$ , AND  $v_B$ , (b) DISTANCE  $d$

#### CONSERVATION OF LINEAR MOMENTUM

BEFORE BREAK:  $\underline{L}_0 = (2m)\underline{v}_G$   $\underline{L}_0 = 2m\dot{\theta} \underline{i}$

AFTER BREAK:

$$\underline{L} = m\underline{v}_A + m\underline{v}_B = m\left(-\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{41}}v_B\right)\underline{i} + m\left(\frac{4}{\sqrt{17}}v_A + \frac{4}{\sqrt{41}}v_B\right)\underline{j}$$

SETTING  $\underline{L} = \underline{L}_0$  AND EQUATING COEFF. OF UNIT VECTORS.

$$\textcircled{1} -\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{41}}v_B = 2v_0 \quad (1)$$

$$\textcircled{2} -\frac{4}{\sqrt{17}}v_A + \frac{4}{\sqrt{41}}v_B = 0, \quad v_B = \frac{\sqrt{41}}{\sqrt{17}}v_A \quad (2)$$

SUBSTITUTE FOR  $v_B$  INTO (1):

$$-\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{17}}v_A = 2v_0 \quad v_A = \frac{\sqrt{17}}{2}v_0 \quad (3)$$

$$\text{FROM (2): } v_B = \frac{\sqrt{41}}{\sqrt{17}} \cdot \frac{\sqrt{17}}{2}v_0 = \frac{\sqrt{41}}{2}v_0 \quad (4)$$

#### CONSERVATION OF ANGULAR MOMENTUM

BEFORE BREAK:  $(H_G)_0 = 2m\dot{\theta}c = 2m(0.3 \text{ m})(12 \text{ rad/s}) = m(2.16)$

AFTER BREAK:  $H_G = H_A = m(v_B)_y d = m d \frac{4}{\sqrt{41}} \frac{\sqrt{41}}{2}v_0 = 2m d v_0$

SETTING  $H_G = (H_G)_0$ :  $2m d v_0 = m(2.16)$   $v_0 d = 1.08$  (5)

#### CONSERVATION OF ENERGY

BEFORE BREAK:  $T_0 = \frac{1}{2}(2m)v_0^2 + \frac{1}{2}(2m)(c\dot{\theta})^2$   
 $= m v_0^2 + m(0.3 \times 12)^2 = m(v_0^2 + 12.96)$

AFTER BREAK:  $T = \frac{1}{2}m v_A^2 + \frac{1}{2}m v_B^2$

RECALLING (3) AND (4):  $T = \frac{1}{2}m v_0^2 \left(\frac{17}{4} + \frac{41}{4}\right) = 7.25m v_0^2$

SETTING  $T = T_0$ :  $7.25v_0^2 = v_0^2 + 12.96$   $v_0^2 = 1.440 \text{ m}^2/\text{s}^2$

FROM (3):  $v_A = \frac{\sqrt{17}}{2}(1.440) = 2.969 \text{ m/s}$

FROM (4):  $v_B = \frac{\sqrt{41}}{2}(1.440) = 4.610 \text{ m/s}$

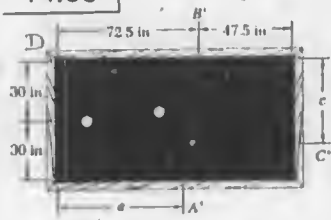
FROM (5):  $d = \frac{1.08}{1.440} = 0.750 \text{ m}$

#### ANSWERS:

(a)  $v_0 = 1.440 \text{ m/s}$ ;  $v_A = 2.97 \text{ m/s}$ ;  $v_B = 4.61 \text{ m/s}$

(b) DISTANCE  $d = 0.750 \text{ m} = 750 \text{ mm}$

14.53



#### GIVEN:

BALL A HITS BALL B WITH  $\underline{v}_0 = (12 \text{ ft/s})\underline{i}$ , THEN C, THEN SIDE OF TABLE AT A' (WHERE  $a = 66 \text{ in.}$ ) WITH  $\underline{v}_A = -(5.76 \text{ ft/s})\underline{j}$

#### FIND:

(a) VELOCITIES OF B AND C

(b) DISTANCE  $c$  WHERE

BALL C HITS SIDE

#### CONSERVATION OF LINEAR MOMENTUM

$$m\underline{v}_0 \underline{i} = -m\underline{v}_A \underline{j} + m(\underline{v}_B)_x \underline{i} + m(\underline{v}_B)_y \underline{j} + m\underline{v}_C \underline{i}$$

EQUATING COEFF. OF UNIT VECTORS:

$$\textcircled{1} m\underline{v}_0 = m(\underline{v}_B)_x + m\underline{v}_C \quad (\underline{v}_B)_x + \underline{v}_C = \underline{v}_0 = 12 \text{ ft/s} \quad (1)$$

$$\textcircled{2} 0 = -m\underline{v}_A + (\underline{v}_B)_y \quad (\underline{v}_B)_y = \underline{v}_A = 5.76 \text{ ft/s} \quad (2)$$

#### CONSERVATION OF ANG. MOMENTUM ABOUT CORNER D

$$+(30 \text{ in.})\underline{v}_0 = -(66 \text{ in.})\underline{v}_A + (72.5 \text{ in.})(\underline{v}_B)_y + c\underline{v}_C$$

$$30(12) = -66(5.76) + (72.5)(5.76) + c\underline{v}_C$$

$$c\underline{v}_C = 322.56 \quad (3)$$

#### CONSERVATION OF ENERGY

$$\frac{1}{2}m\underline{v}_0^2 = \frac{1}{2}m\underline{v}_A^2 + \frac{1}{2}m[(\underline{v}_B)_x^2 + (\underline{v}_B)_y^2] + \frac{1}{2}m\underline{v}_C^2$$

DIVIDING BY  $m$ , MULTIPLYING BY 2, AND SUBSTITUTING FOR  $\underline{v}_0$ ,  $\underline{v}_A$ ,  $(\underline{v}_B)_y$  THEIR VALUES AND  $(\underline{v}_B)_x = 12 - \underline{v}_C$  FROM (1):

$$(12)^2 = (5.76)^2 + (12 - \underline{v}_C)^2 + (5.76)^2 + \underline{v}_C^2$$

DIVIDING BY 2:  $\underline{v}_C^2 = 12\underline{v}_C + (5.76)^2 = 0$ ,  $\underline{v}_C = 6 \pm 1.68$

WITH  $\underline{v}_C = 6 - 1.68 = 4.32$ , EQ. (3) YIELDS  $c = 74.7 \text{ (IMPOSSIBLE)}$

THEREFORE:  $\underline{v}_C = 6 + 1.68 = 7.68$   $\underline{v}_C = 7.68 \text{ ft/s}$

FROM (1):  $(\underline{v}_B)_x = 12 - 7.68 = 4.32$

$\underline{v}_B = (4.32 \text{ ft/s})\underline{i} + (5.76 \text{ ft/s})\underline{j}$  OR  $\underline{v}_B = 7.20 \text{ ft/s} \angle 53.1^\circ$

FROM (3):  $c(7.68) = 322.56$   $c = 42.0 \text{ in.}$

14.54

(SEE FIGURE OF PROB. 14.53)

**GIVEN:** BALL A HITS B WITH  $\underline{v}_A = (15 \text{ ft/s})\underline{i}$ ,

THEN C; BALL C HITS SIDE AT C = 48 in. WITH  $\underline{v}_C = (9.6 \text{ ft/s})\underline{i}$

**FIND:** (a)  $\underline{v}_A$  AND  $\underline{v}_B$ , (b) DISTANCE  $a$ .

#### CONSERVATION OF LINEAR MOMENTUM

$$m\underline{v}_0 \underline{i} = -m\underline{v}_A \underline{j} + m(\underline{v}_B)_x \underline{i} + m(\underline{v}_B)_y \underline{j} + m\underline{v}_C \underline{i}$$

$$\textcircled{1} m\underline{v}_0 = m(\underline{v}_B)_x + m\underline{v}_C \quad (\underline{v}_B)_x = 15 - 9.6 = 5.40 \text{ ft/s} \quad (1)$$

$$\textcircled{2} 0 = -m\underline{v}_A + (\underline{v}_B)_y \quad (\underline{v}_B)_y = \underline{v}_A \quad (2)$$

#### CONSERVATION OF ANGULAR MOMENTUM ABOUT CORNER D

$$+(30 \text{ in.})\underline{v}_0 = -a\underline{v}_A + (72.5 \text{ in.})(\underline{v}_B)_y + c\underline{v}_C$$

SUBSTITUTING GIVEN DATA AND USING EQ. (2):

$$30(15 \text{ ft/s}) = -a\underline{v}_A + 72.5\underline{v}_A + 48(9.6 \text{ ft/s})$$

$$(a - 72.5)\underline{v}_A = 10.8 \quad (3)$$

#### CONSERVATION OF ENERGY

$$\frac{1}{2}m\underline{v}_0^2 = \frac{1}{2}m\underline{v}_A^2 + \frac{1}{2}m[(\underline{v}_B)_x^2 + (\underline{v}_B)_y^2] + \frac{1}{2}m\underline{v}_C^2$$

DIVIDING BY  $m$ , MULTIPLYING BY 2, AND SUBSTITUTING:

$$(15)^2 = \underline{v}_A^2 + (5.40)^2 + \underline{v}_A^2 + (9.6)^2$$

$$\underline{v}_A^2 = 51.84 \quad \underline{v}_A = 7.20 \text{ ft/s} \quad \underline{v}_A = 7.20 \text{ ft/s} \angle 53.1^\circ$$

FROM (1) AND (2):

$$\underline{v}_B = (\underline{v}_B)_x \underline{i} + (\underline{v}_B)_y \underline{j} = (5.40 \text{ ft/s})\underline{i} + (7.20 \text{ ft/s})\underline{j}$$

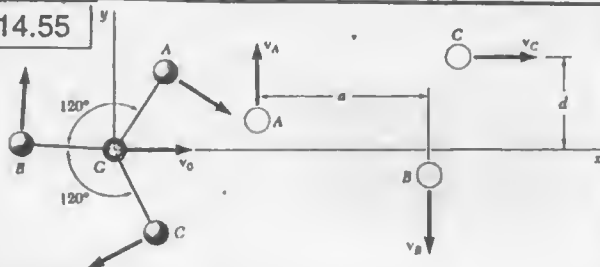
$$\text{OR } \underline{v}_B = 9.00 \text{ ft/s} \angle 53.1^\circ$$

FROM (3):  $(a - 72.5)7.20 = 10.8$

$$a = 72.5 + 1.5$$

$$a = 74.0 \text{ in.}$$

14.55



GIVEN:

THREE SMALL IDENTICAL SPHERES CONNECTED BY 200-MM-LONG STRINGS TO RING G SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE.

INITIALLY, SPHERES ROTATE ABOUT G WITH 0.8 m/s RELATIVE VELOCITY AND RING MOVES WITH  $\vec{v}_0 = (0.4 \text{ m/s})\hat{i}$ . SUDDENLY RING BREAKS AND SPHERES MOVE FREELY AS SHOWN WITH  $a = 346 \text{ mm}$ .

FIND:

(a) VELOCITY OF EACH SPHERE, (b) DISTANCE  $d$ .

CONSERVATION OF LINEAR MOMENTUM

BEFORE BREAK:  $\vec{L}_0 = (3m)\vec{v} = 3m(0.4\hat{i}) = m(1.2\text{ m/s})\hat{i}$

AFTER BREAK:  $\vec{L} = m\vec{v}_A\hat{i} - m\vec{v}_B\hat{j} + m\vec{v}_C\hat{i}$

$\vec{L} = \vec{L}_0: m\vec{v}_C\hat{i} + m(\vec{v}_A - \vec{v}_B)\hat{j} = m(1.2 \text{ m/s})\hat{i}$

THEREFORE:

$$\vec{v}_A = \vec{v}_B \quad (1)$$

$$\vec{v}_C = 1.200 \text{ m/s} \quad \vec{v}_C = 1.200 \text{ m/s} \rightarrow (2)$$

CONSERVATION OF ANGULAR MOMENTUM

BEFORE BREAK:  $\frac{1}{2}(H_0)_0 = 3m\ell v^2 = 3m(0.2\text{ m})(0.8\text{ m/s}) = 0.480 \text{ m}$

AFTER BREAK:

$$\frac{1}{2}H_0 = -m\vec{v}_A \times \vec{r}_A + m\vec{v}_B \times \vec{r}_B + m\vec{v}_C \times \vec{r}_C$$

$$H_0 = (H_0)_0:$$

$$0.346 m \vec{v}_A + m \vec{v}_C d = 0.480 \text{ m}$$

RECALLING (2):

$$0.346 \vec{v}_A + 1.200 d = 0.480$$

$$d = 0.400 - 0.28833 \vec{v}_A \quad (3)$$

CONSERVATION OF ENERGY

BEFORE BREAK:

$$T_0 = \frac{1}{2}(3m)\vec{v}^2 + 3\left(\frac{1}{2}m\vec{v}^2\right)$$

$$= \frac{3}{2}m(\vec{v}_0^2 + v^2) = \frac{3}{2}m[(0.4)^2 + (0.8)^2]m = 1.200 \text{ m}$$

AFTER BREAK:

$$T = \frac{1}{2}m\vec{v}_A^2 + \frac{1}{2}m\vec{v}_B^2 + \frac{1}{2}m\vec{v}_C^2$$

$T = T_0$ : SUBSTITUTING FOR  $\vec{v}_B$  FROM (1) AND  $\vec{v}_C$  FROM (2):

$$\frac{1}{2}[v_A^2 + v_A^2 + (1.200)^2] = 1.200$$

$$v_A^2 = 0.480$$

$$\vec{v}_A = \vec{v}_B = 0.69282 \text{ m/s}$$

(a) VELOCITIES:

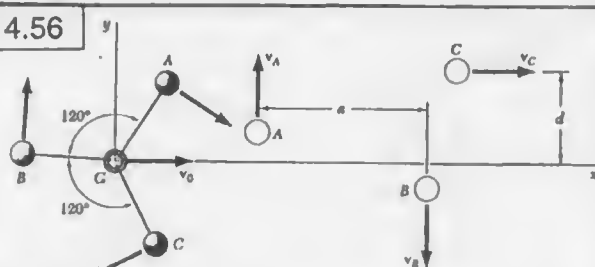
$$\vec{v}_A = 0.693 \text{ m/s} \uparrow; \vec{v}_B = 0.693 \text{ m/s} \downarrow; \vec{v}_C = 1.200 \text{ m/s} \rightarrow$$

(b) DISTANCE  $d$ :

$$\text{FROM (3): } d = 0.400 - 0.28833(0.69282) = 0.20024 \text{ m}$$

$$d = 200 \text{ mm}$$

14.56



GIVEN:

THREE SMALL IDENTICAL SPHERES CONNECTED BY STRINGS OF LENGTH  $\ell$  TO RING G SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE.

INITIALLY, SPHERES ROTATE ABOUT G AND RING MOVES AS SHOWN. SUDDENLY RING BREAKS AND SPHERES MOVE FREELY IN  $xy$  PLANE. WE KNOW THAT  $\vec{v}_A = (1.039 \text{ m/s})\hat{i}$ ,  $\vec{v}_C = (1.800 \text{ m/s})\hat{i}$ ,  $a = 416 \text{ mm}$ ,  $d = 240 \text{ mm}$ .

FIND:

(a) VEL.  $\vec{v}_0$  OF RING, (b) LENGTH  $\ell$  OF STRINGS (c) RATE IN rad/s AT WHICH SPHERES WERE ROTATING

CONSERVATION OF LINEAR MOMENTUM

$$(3m)\vec{v} = m\vec{v}_A + m\vec{v}_B + m\vec{v}_C$$

$$3m\vec{v}_0\hat{i} = m(1.039 \text{ m/s})\hat{i} - m\vec{v}_B\hat{j} + m(1.800 \text{ m/s})\hat{i}$$

EQUATING COEFF. OF UNIT VECTORS:

$$\textcircled{1} 3\vec{v}_0 = 1.800 \text{ m/s}$$

$$\textcircled{a} \vec{v}_0 = 0.600 \text{ m/s} \rightarrow$$

$$\textcircled{2} 0 = 1.039 \text{ m/s} - \vec{v}_B$$

$$\vec{v}_B = 1.039 \text{ m/s} \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUMBEFORE BREAK:  $\frac{1}{2}(H_0)_0 = 3m\ell^2\dot{\theta}$ 

AFTER BREAK:

$$\frac{1}{2}H_0 = -m\vec{v}_A \times \vec{r}_A + m\vec{v}_B \times \vec{r}_B + m\vec{v}_C \times \vec{r}_C$$

$$+ m\vec{v}_A (x_A + 0.416)$$

$$+ m\vec{v}_C (0.240)$$

$$= m(1.039)(0.416)$$

$$+ m(1.800)(0.240)$$

$$= m(0.864224)$$

$$(H_0)_0 = H_0: 3m\ell^2\dot{\theta} = m(0.864224) \quad \ell^2\dot{\theta} = 0.28807 \quad (2)$$

CONSERVATION OF ENERGY

BEFORE BREAK:

$$T_0 = \frac{1}{2}(3m)\vec{v}^2 + 3\left(\frac{1}{2}m\vec{v}^2\right) = \frac{3}{2}m\vec{v}_0^2 + \frac{3}{2}m(\ell\dot{\theta})^2$$

$$= \frac{3}{2}m(0.600)^2 + \frac{3}{2}m\ell^2\dot{\theta}^2$$

AFTER BREAK:

$$T = \frac{1}{2}m\vec{v}_A^2 + \frac{1}{2}m\vec{v}_B^2 + \frac{1}{2}m\vec{v}_C^2$$

$$= \frac{1}{2}m[(1.039)^2 + (1.039)^2 + (1.800)^2] = \frac{1}{2}m(5.399)$$

$$T = T_0: \frac{1}{2}m(5.399) = \frac{3}{2}m(0.600)^2 + \frac{3}{2}m\ell^2\dot{\theta}^2$$

$$\ell^2\dot{\theta}^2 = 1.4397 \quad (3)$$

$$\text{DIVIDING (3) BY (2): } \dot{\theta} = \frac{1.4397}{0.28807} = 4.9976$$

$$\textcircled{b} \text{ FROM (2): } \ell^2 = \frac{0.28807}{4.9976} \quad \ell = 0.2401 \text{ m}$$

$$\ell = 240 \text{ mm}$$

$$\textcircled{c} \text{ RATE OF ROTATION} = \dot{\theta} = 5.00 \text{ rad/s}$$

14.57



GIVEN:

VEL. OF STREAM = 25 m/s  $A = 300 \text{ mm}^2$ 

FIND: FORCE EXERTED BY STREAM ON MAILBOX.

$$(\Delta m) \vec{v}_0 + \boxed{\text{MAILBOX}} = 0 \quad (\Delta m) \vec{v}_0 - P \Delta t = 0$$

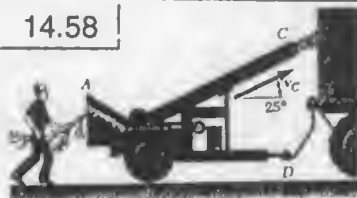
$$P = \frac{\Delta m}{\Delta t} v_0 = (\rho A v_0) v_0 = \rho A v_0^2$$

$$P = (1000 \text{ kg/m}^3)(300 \times 10^{-6} \text{ m}^2)(25 \text{ m/s})^2$$

$$P = 187.5 \text{ N}$$

NOTE: FORCE  $P$  SHOWN ON SKETCH IS FORCE APPLIED BY MAILBOX ON STREAM. FORCE EXERTED BY STREAM ON MAILBOX IS 187.5 N  $\rightarrow$

14.58



GIVEN:

TREE LIMBS ARE FED INTO SHREDDER AT RATE OF 5 kg/s AND CHIPS ARE SPEWED WITH  $v_c = 20 \text{ m/s}$ .

FIND: HORIZ. COMP. OF FORCE EXERTED ON HITCH AT D.

$$v_c = 20 \text{ m/s}$$

$$(\Delta m) \vec{v}_A + \sum \vec{F} \Delta t = (\Delta m) \vec{v}_C$$

$$\sum \vec{F} = \frac{\Delta m}{\Delta t} \vec{v}_C = (5 \text{ kg/s})(20 \text{ m/s}) \angle 25^\circ$$

FORCE EXERTED ON CHIPS =  $\sum \vec{F} = 100 \text{ N} \angle 25^\circ$

$$\text{FREE BODY: SHREDDER}$$

$$\sum F_x = 0: F_x - (100 \text{ N}) \cos 25^\circ = 0$$

$$F_x = 90.6 \text{ N} \rightarrow$$

$$\text{ON HITCH: } F_x = 90.6 \text{ N} \leftarrow$$

14.59



GIVEN:

WATER DISCHARGED AT RATE OF 2000 gal/min WITH VEL. OF 150 ft/s.

FIND: THRUST OF ENGINE TO KEEP BOAT STATIONARY.

$$\text{EQ. (14.38): } (\Delta m) \vec{v}_A + \sum \vec{F} \Delta t = (\Delta m) \vec{v}_B$$

WHERE  $\vec{v}_A = 0$ ,  $\vec{v}_B = 150 \text{ ft/s} \angle 35^\circ$

$$\text{FORCE EXERTED ON STREAM:}$$

$$\sum \vec{F} = \frac{\Delta m}{\Delta t} \vec{v}_B = \left( \frac{2000 \text{ gal}}{\text{min}} \times \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{62.4 \text{ lb}}{\text{ft}^3} \right) \left( \frac{1}{32.2 \text{ ft/s}^2} \right) (150 \text{ ft/s})$$

$$\sum \vec{F} = 1295.4 \text{ lb} \angle 35^\circ$$

THRUST OF ENGINE =  $(\sum \vec{F})_x = (1295.4 \text{ lb}) \cos 35^\circ$

THRUST = 1061 lb  $\leftarrow$

14.60



GIVEN:

ENGINE PROPELS PLOW AT SPEED OF 12 mi/h. PLOW PROJECTS 180 TONS OF SNOW PER MINUTE WITH VELOCITY OF 40 ft/s WR TO CAR.

FIND:

(a) FORCE EXERTED BY ENGINE ON CAR  
(b) LATERAL FORCE EXERTED BY TRACK.

WE MEASURE ALL VELOCITIES W/R PLOW CAR AND APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PLOW CAR, THE SNOW IT CONTAINS, AND THE SNOW ENTERING IN THE TIME INTERVAL  $\Delta t$ .

$$-(\Delta m) \vec{u}_1 + \boxed{\text{PLOW CAR}} = \boxed{\text{PLOW CAR + SNOW}}$$

$$-(\Delta m) \vec{u}_1 + (P \Delta t) \vec{k} + (R \Delta t) \vec{j} + (L \Delta t) \vec{i} = (\Delta m) \vec{u}_2$$

$$-(\Delta m) \vec{u}_1 + (P \Delta t) \vec{k} + (R \Delta t) \vec{j} - (W \Delta t) \vec{j} = (\Delta m) \vec{u}_2 (\cos 30^\circ \vec{i} + \sin 30^\circ \vec{j})$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$\text{(k)} \quad -(\Delta m) u_1 + P \Delta t = 0 \quad P = \frac{\Delta m}{\Delta t} u_1 \quad (1)$$

$$\text{(i)} \quad L \Delta t = (\Delta m) u_2 \cos 30^\circ \quad L = \frac{\Delta m}{\Delta t} u_2 \cos 30^\circ \quad (2)$$

WITH GIVEN DATA:

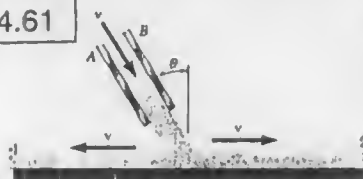
$$u_1 = 12 \text{ mi/h} = 17.60 \text{ ft/s}, \quad u_2 = 40 \text{ ft/s}$$

$$\frac{\Delta m}{\Delta t} = (180 \text{ tons/min}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{2000 \text{ lb}}{1 \text{ ton}} \right) \left( \frac{1}{32.2 \text{ ft/s}^2} \right) = 186.34 \text{ lb} \cdot \text{s/ft}^2$$

$$\text{(a) EQ. (1): } P = (186.34 \text{ lb} \cdot \text{s/ft}^2)(17.60 \text{ ft/s}) \quad P = (3280 \text{ lb}) \vec{k}$$

$$\text{(b) EQ. (2): } L = (186.34 \text{ lb} \cdot \text{s/ft}^2)(40 \text{ ft/s}) \cos 30^\circ \quad L = (6450 \text{ lb}) \vec{i}$$

14.61



GIVEN:

$v = 30 \text{ m/s}$   
STREAM SEPARATED INTO TWO STREAMS WITH  $Q_1 = 100 \text{ L/min}$  AND  $Q_2 = 500 \text{ L/min}$

FIND: (a)  $\theta$ , (b) TOTAL FORCE EXERTED BY STREAM ON PLATE.

WE NOTE THAT  $Q = Q_1 + Q_2 \quad (1)$

$$(\Delta m) \vec{v} + \boxed{\text{PLATE}} = \boxed{\text{STREAM 1}} + \boxed{\text{STREAM 2}}$$

$$(\Delta m) \vec{v} + P \Delta t = (\Delta m_1) \vec{v}_1 + (\Delta m_2) \vec{v}_2$$

IMPULSE-MOMENTUM PRINCIPLE:

$$\pm x \text{ COMP: } (\Delta m) v \sin \theta = (\Delta m_2) v - (\Delta m_1) v$$

$$(P \Delta t) v \sin \theta = (P Q_2 \Delta t) v - (P Q_1 \Delta t) v$$

$$Q \sin \theta = Q_2 - Q_1 \quad (2)$$

$$\pm y \text{ COMP: } -(\Delta m) v \cos \theta + P \Delta t = 0$$

$$P = \frac{\Delta m}{\Delta t} v \cos \theta \quad P = \rho Q v \cos \theta \quad (3)$$

(a) FROM (1):  $Q = 100 + 500 = 600 \text{ L/min}$

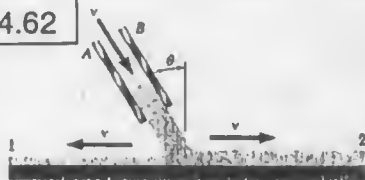
$$\text{FROM (2): } \sin \theta = \frac{Q_2 - Q_1}{Q} = \frac{500 - 100}{600} = \frac{2}{3} \quad \theta = 41.8^\circ$$

$$\text{(b) FROM (3): } P = (1 \text{ kg/L})(600 \text{ L/min}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) (30 \text{ m/s}) \cos 41.8^\circ$$

$$P = 224 \text{ N} \uparrow$$

FORCE EXERTED BY STREAM ON PLATE = 224 N  $\uparrow$

14.62



GIVEN:

$U = 40 \text{ m/s}$ ,  $\theta = 30^\circ$   
 TOTAL FORCE  
 EXERTED BY STREAM  
 ON BLADE =  $500 \text{ N}$

FIND:  $Q_1$  AND  $Q_2$   
 OF RESULTING  
 STREAMS.

WE NOTE THAT  $Q = Q_1 + Q_2$  (1)

SEE SOLUTION OF PROB. 14.61 FOR DERIVATION OF

$$Q \sin \theta = Q_2 - Q_1 \quad (2) \quad P = \rho Q U \cos \theta \quad (3)$$

$$\text{FROM (3): } Q = \frac{P}{\rho U \cos \theta} = \frac{500 \text{ N}}{(1 \text{ kg/L})(40 \text{ m/s}) \cos 30^\circ} = 14.434 \text{ L/s}$$

$$= 866.03 \text{ L/min}$$

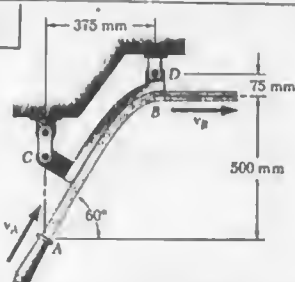
$$\text{ADDING (1) AND (2): } Q(1 + \sin \theta) = 2Q_2$$

$$Q_2 = \frac{1 + \sin \theta}{2} Q = \frac{1 + \sin 30^\circ}{2} (866.03 \text{ L/min}) = 649.52 \text{ L/min}$$

$$\text{FROM (1): } Q_1 = Q - Q_2 = 866.03 - 649.52 = 216.51 \text{ L/min}$$

$$Q_1 = 217 \text{ L/min}; Q_2 = 650 \text{ L/min}$$

14.63

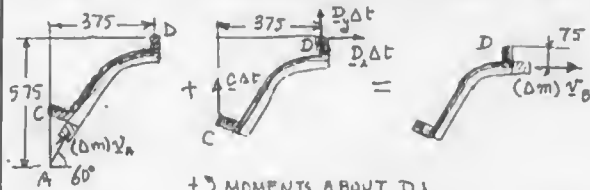


GIVEN:

WATER DISCHARGED  
 AT RATE  $Q = 1.2 \text{ m}^3/\text{min}$   
 WITH  $V_A = V_B = 25 \text{ m/s}$

FIND:  
 COMPONENTS OF  
 REACTIONS AT C AND D.  
 (NEGLECT WEIGHT  
 OF VANE).

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO  
 THE BLADE, WATER IN CONTACT WITH THE BLADE, AND  
 WATER STRIKING THE BLADE IN INTERVAL  $\Delta t$ .



+ MOMENTS ABOUT D:

$$575(\Delta m)V_A \cos 60^\circ - 375(\Delta m)V_B \sin 60^\circ - 375 C \Delta t = 75(\Delta m)V_B$$

$$375 C = \frac{\Delta m}{\Delta t} (25 \text{ m/s}) (575 \cos 60^\circ - 375 \sin 60^\circ - 75)$$

$$C = -7.484 \frac{\Delta m}{\Delta t}$$

$$\text{BUT } \frac{\Delta m}{\Delta t} = \rho Q = (1000 \text{ kg/m}^3) \left( \frac{1.2 \text{ m}^3}{60 \text{ s}} \right) = 20 \text{ kg/s}$$

$$\text{THUS: } C = -7.484 (20) = -149.68 \text{ N}$$

$$C_x = 0, C_y = 149.7 \text{ N} \uparrow$$

$$\pm x \text{ COMP: } (\Delta m)V_A \cos 60^\circ + D_x \Delta t = (\Delta m)V_B$$

$$D_x = \frac{\Delta m}{\Delta t} (25 \text{ m/s}) (1 - \cos 60^\circ) = (20 \text{ kg/s}) (25 \text{ m/s}) (1 - \cos 60^\circ)$$

$$D_x = 250 \text{ N} \rightarrow$$

$$\pm y \text{ COMP: } (\Delta m)V_A \sin 60^\circ + C \Delta t + D_y \Delta t = 0$$

$$D_y = -\frac{\Delta m}{\Delta t} (25 \text{ m/s}) \sin 60^\circ - (-149.7 \text{ N})$$

$$= -(20 \text{ kg/s}) (25 \text{ m/s}) \sin 60^\circ + 149.7 \text{ N}$$

$$= -433.0 \text{ N} + 149.7 \text{ N} = -283.3 \text{ N}$$

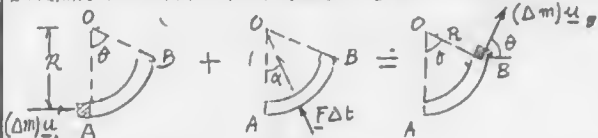
$$\frac{D_y}{y} = 283 \text{ N} \downarrow$$

14.64

ASSUME THAT BLADE AB OF SAMPLE PROB. 14.7  
 IS IN THE SHAPE OF AN ARC OF CIRCLE.

SHOW THAT RESULTANT FORCE  $F$  EXERTED BY THE BLADE  
 ON THE STREAM IS APPLIED AT MIDPOINT C OF ARC AB.

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE  
 PORTION OF STREAM IN CONTACT WITH THE BLADE AND  
 ENTERING IN CONTACT IN INTERVAL  $\Delta t$ .



WE RECALL THAT  $U_A = U_B = U$

$$\pm \text{ MOMENTS ABOUT O: } R(\Delta m)U + \text{MOM. OF } F = R(\Delta m)U$$

THUS: MOM. OF  $F$  ABOUT O = 0; LINE OF ACTION OF  $F$  PASSES  
 THROUGH O.

$$\pm x \text{ COMP: } (\Delta m)U - (F \Delta t) \sin \alpha = (\Delta m)U \cos \theta$$

$$\text{OR: } F(\Delta t) \sin \alpha = (\Delta m)U(1 - \cos \theta) \quad (1)$$

$$\pm y \text{ COMP: } 0 + F(\Delta t) \cos \alpha = (\Delta m)U \sin \theta \quad (2)$$

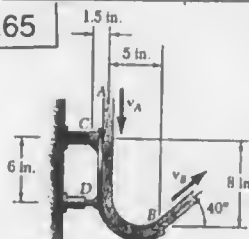
DIVIDE (1) BY (2):

$$\tan \alpha = \frac{1 - \cos \theta}{\sin \theta} = \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \tan \frac{\theta}{2} \quad \alpha = \frac{\theta}{2}$$

THUS: LINE OF ACTION OF  $F$  BISSECTS  $\angle AOB$ ,

$F$  IS APPLIED AT MIDPOINT C OF ARC AB.  
 (Q.E.D.)

14.65



GIVEN:

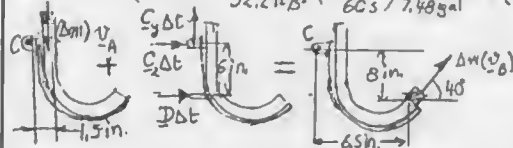
STREAM OF WATER WITH  
 $Q = 150 \text{ gal/min}$   
 $V_A = V_B = 60 \text{ ft/s}$   
 REACTION AT D HORIZONTAL

FIND:

COMPONENTS OF REACTIONS  
 AT C AND D  
 (NEGLECT WEIGHT OF VANE.)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE VANE, THE  
 WATER IN CONTACT WITH IT, AND THE MASS  $\Delta m$  OF WATER ENTERING  
 AND LEAVING THE SYSTEM IN THE INTERVAL  $\Delta t$ . WE NOTE

$$\text{THAT } \Delta m = \rho Q \Delta t = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \left( \frac{150 \text{ gal}}{60 \text{ s}} \right) \left( \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right) \Delta t = (0.6477 \frac{\text{lb}}{\text{s}}) \Delta t$$



$$\pm \text{ MOM. about C: } -(\Delta m)V_A(1.5 \text{ in.}) + D \Delta t(6 \text{ in.}) =$$

$$= (\Delta m)V_B \cos 40^\circ(8 \text{ in.}) + (\Delta m)V_B \sin 40^\circ(6.5 \text{ in.})$$

$$D \Delta t(6 \text{ in.}) = (0.6477 \frac{\text{lb}}{\text{s}}) \Delta t (60 \text{ ft/s}) (11.806 \text{ in.}) \quad D = 76.47 \text{ lb}$$

$$\pm x \text{ COMP: } C_x \Delta t + D \Delta t = (\Delta m)V_B \cos 40^\circ$$

$$C_x = (0.6477 \frac{\text{lb}}{\text{s}}) (60 \text{ ft/s}) \cos 40^\circ - 76.47 \text{ lb} = -46.7 \text{ lb}$$

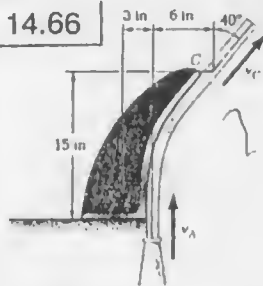
$$\pm y \text{ COMP: } -(\Delta m)V_A + C_y \Delta t = (\Delta m)V_B \sin 40^\circ$$

$$C_y = (0.6477 \frac{\text{lb}}{\text{s}}) (60 \text{ ft/s}) (\sin 40^\circ + 1) = 63.8 \text{ lb}$$

$$C_x = 46.7 \text{ lb} \leftarrow, C_y = 63.8 \text{ lb} \uparrow$$

$$D_x = 76.5 \text{ lb} \rightarrow, D_y = 0$$

14.66



GIVEN:

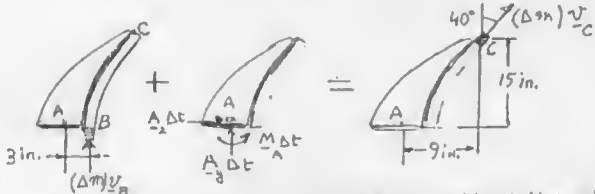
STREAM OF WATER WITH  
 $Q = 200 \text{ gal/min}$   
 AND  $V_B = V_C = 100 \text{ ft/s}$

FIND:

FORCE-COUPLE SYSTEM  
 APPLIED TO VANE AT A.  
 (NEGLECT WEIGHT OF VANE)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE VANE. THE WATER ENTERS AT A, LEAVES AT B, AND THE MASS  $\Delta m$  OF WATER ENTERS LEAVING THE SYSTEM IN  $\Delta t$ . WE NOTE THAT

$$\Delta m = \rho Q \Delta t = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \left( \frac{200 \text{ gal}}{60 \text{ s}} \right) \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \Delta t = (0.8636 \frac{\text{lb}}{\text{s}}) \Delta t$$



$$\pm x \text{ COMP: } A_x \Delta t = (\Delta m) V_C \sin 40^\circ = (0.8636 \Delta t)(100 \text{ ft/s}) \sin 40^\circ$$

$$A_x = 55.51 \text{ lb} \quad A_x = 55.5 \text{ lb} \rightarrow$$

$$\pm y \text{ COMP: } (\Delta m) V_B + A_y \Delta t = (\Delta m) V_C \cos 40^\circ$$

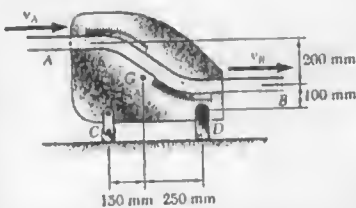
$$A_y = (0.8636)(100 \text{ ft/s})(\cos 40^\circ - 1) = -20.2 \text{ lb}, \quad A_y = 20.2 \text{ lb} \uparrow$$

$$\pm \text{ MOMENTS ABOUT A: } (\Delta m) V_B (3 \text{ in.}) + M_A \Delta t = -(\Delta m) V_C \sin 40^\circ (15 \text{ in.}) + (\Delta m) V_C \cos 40^\circ (9 \text{ in.})$$

$$M_A = (0.8636 \text{ lb/s}) (100 \text{ ft/s}) [-(15 \text{ in.}) \sin 40^\circ + (9 \text{ in.}) \cos 40^\circ - 3 \text{ in.}]$$

$$M_A = -496.3 \text{ lb}\cdot\text{in.} \quad M_A = 496 \text{ lb}\cdot\text{in.} \curvearrowright$$

14.67



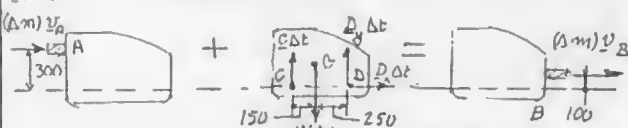
GIVEN:

STREAM OF WATER WITH  
 CROSS-SECTION  $A = 600 \text{ mm}^2$   
 AND  $V_A = V_B = 20 \text{ m/s}$ .  
 COMBINED MASS OF  
 PLATE AND VANES  
 IS  $5 \text{ kg}$ .

FIND:

REACTIONS AT C AND D.

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO PLATE, VANES  
 WATER IN CONTACT WITH PLATE, AND MASS  $\Delta m$  OF WATER  
 ENTERING AND LEAVING SYSTEM IN INTERVAL  $\Delta t$ .



$$\Delta m = \rho Q \Delta t = \rho A V \Delta t = (1000 \text{ kg/m}^3)(600 \times 10^{-6} \text{ m}^2)(20 \text{ m/s}) \Delta t = (12 \text{ kg/s}) \Delta t$$

$$\pm x \text{ COMP: } (\Delta m) V_A + D_x \Delta t = (\Delta m) V_B, \quad D_x = (1200)(20 - 20) = 0$$

$$\pm \text{ MOM. ABOUT D: } (\Delta m) V_A (300) + C \Delta t (400) - W \Delta t (250) = (\Delta m) V_B (100)$$

$$1100 C = (12 \text{ kg/s})(20 \text{ m/s})(100 - 300) + (5 \times 9.81 \text{ N})(250) = -35,738$$

$$C = -89.34 \text{ N} \quad C = 89.3 \text{ N} \downarrow$$

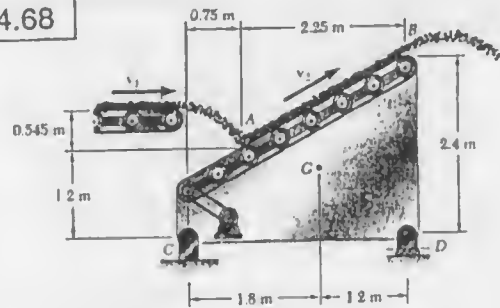
$$\pm y \text{ COMP: } (-89.34 - 5 \times 9.81 + D_y) \Delta t = 0$$

$$D_y = 138.39$$

RECALLING THAT  $D_x = 0$ .

$$D = 138.4 \text{ N} \uparrow$$

14.68

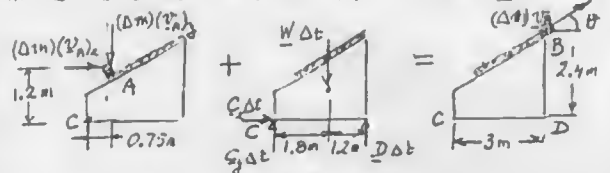


GIVEN: COAL DISCHARGED FROM FIRST TO SECOND  
 CONVEYOR BELT AT RATE OF  $120 \text{ kg/s}$  WITH  
 $V_1 = 3 \text{ m/s}$  AND  $V_2 = 4.25 \text{ m/s}$ . MASS OF SECOND  
 BELT ASSEMBLY AND COAL IT SUPPORTS IS  $472 \text{ kg}$ .  
 FIND: COMPONENTS OF REACTIONS AT C AND D.

MASS OF COAL ENTERING AND LEAVING SYSTEM IN  $\Delta t$ :  
 $\Delta m = (120 \text{ kg/s}) \Delta t$  (1)

VELOCITY  $V_A$  WITH WHICH COAL  
 HITS SECOND BELT:  
 $(V_A)_x = V_1 = 3 \text{ m/s} \rightarrow$  (2)  
 $(V_A)_y = \sqrt{2gh} = \sqrt{2(9.81)(0.545)}$   
 $(V_A)_y = 3.27 \text{ m/s} \downarrow$  (3)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE  
 SECOND BELT ASSEMBLY, THE COAL IT SUPPORTS, AND THE  
 MASS  $\Delta m$  OF COAL HITTING IT IN INTERVAL  $\Delta t$ :



WE NOTE THAT  $\tan \theta = \frac{1.2 \text{ m}}{2.25 \text{ m}} \quad \theta = 28.07^\circ$

$\pm \text{ MOM. ABOUT C:}$

$$(\Delta m)(V_A)_x (1.2 \text{ m}) + (\Delta m)(V_A)_y (0.75 \text{ m}) + (W \Delta t)(1.8 \text{ m}) - (D \Delta t)(3 \text{ m})$$

$$= (\Delta m)(V_B \cos \theta)(2.4 \text{ m}) - (\Delta m)(V_B \sin \theta)(3 \text{ m})$$

$$D(3 \text{ m}) = (472 \text{ kg})(9.81 \text{ m/s}^2)(1.8 \text{ m})$$

$$+ (120 \text{ kg/s}) [(3 \text{ m/s})(1.2 \text{ m}) + (3.27 \text{ m/s})(0.75 \text{ m})]$$

$$- (120 \text{ kg/s})(4.25 \text{ m/s}) [(2.4 \text{ m}) \cos 28.07^\circ - (3 \text{ m}) \sin 28.07^\circ]$$

$$D(3 \text{ m}) = 8334.6 \text{ N}\cdot\text{m} + 726.30 \text{ N}\cdot\text{m} - 260.09 \text{ N}\cdot\text{m} = 8700 \text{ N}\cdot\text{m}$$

$$D = 2900 \text{ N} \quad D_x = 0, \quad D_y = 2900 \text{ N} \uparrow$$

$$\pm x \text{ COMP: } (\Delta m)(V_A)_x + C_x \Delta t = (\Delta m) V_B \cos \theta$$

$$C_x = (120 \text{ kg/s})(3 \text{ m/s}) \cos 28.07^\circ - (120 \text{ kg/s})(3 \text{ m/s})$$

$$= 90.0 \text{ N} \quad C_x = 90.0 \text{ N} \rightarrow$$

$$\pm y \text{ COMP: } -(\Delta m)(V_A)_y + C_y \Delta t + D \Delta t - W \Delta t = (\Delta m) V_B \sin \theta$$

$$C_y = -2900 \text{ N} + (472 \text{ kg})(9.81 \text{ m/s}^2) + (120 \text{ kg/s})(3.27 + 4.25 \sin 28.07^\circ) \text{ m/s}$$

$$= 2362.7 \text{ N} \quad C_y = 2360 \text{ N} \uparrow$$

NOTE: W

WHEN BELT IS AT REST:

$$\sum \mathcal{M}_C = 0 \quad D(3 \text{ m}) - W(1.8 \text{ m}) = 0$$

$$3D - (472 \times 9.81)(1.8) = 0$$

$$D = 2778 \text{ N} \quad D = 2780 \text{ N} \uparrow$$

$$C = 472 \times 9.81 - 2778 \quad C = 1852 \text{ N} \uparrow$$





14.69

**GIVEN:**

PLANE CRUISES AT 900 km/h.  
SCOOPS AIR AT RATE OF 90 kg/s AND  
DISCHARGES IT AT 660 m/s RELATIVE TO PLANE.

**FIND:** TOTAL DRAG DUE TO AIR FRICTION

WE APPLY EQ. (14.39):  $\Sigma F = \frac{dm}{dt} (v_B - v_A)$   
WITH RESPECT TO PLANE.

WE HAVE:  $\Sigma F = D = \text{TOTAL DRAG}$ ,

$$v_B = 660 \text{ m/s}, \quad v_A = 900 \text{ km/h} = 900 \frac{1000 \text{ m}}{3600 \text{ s}} = 250 \text{ m/s}$$

$$\text{EQ. (14.39): } D = (90 \text{ kg/s}) (660 \text{ m/s} - 250 \text{ m/s})$$

$$D = 36.9 \text{ kN}$$

14.70

**GIVEN:**

PLANE IN LEVEL FLIGHT AT 570 mi/h.  
DRAG DUE TO AIR FRICTION = 7500 lb  
EXHAUST VEL. = 1800 ft/s RELATIVE TO PLANE

**FIND:** RATE IN lb/s AT WHICH AIR PASSES THRU ENGINE

WE APPLY EQ. (14.39):  $\Sigma F = \frac{dm}{dt} (v_B - v_A)$   
WITH RESPECT TO PLANE.

WE HAVE  $\Sigma F = \text{DRAG} = 7500 \text{ lb}$

$$v_B = 1800 \text{ ft/s}, \quad v_A = 570 \text{ mi/h} = 836 \text{ ft/s}$$

$$\text{EQ. (14.39): } 7500 \text{ lb} = \frac{dm}{dt} (1800 \text{ ft/s} - 836 \text{ ft/s})$$

$$\frac{dm}{dt} = (7500 \text{ lb} \cdot \text{s}) (32.2 \text{ ft/s}^2) = 251 \text{ lb/s}$$

14.71

**GIVEN:**

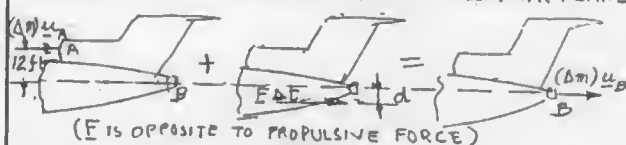
ENGINE SCOOPS IN AIR  
AT A RATE OF 200 lb/s  
AND DISCHARGES IT AT B  
AT 2000 ft/s W/R PLANE

**FIND:**

THRUST OF ENGINE

WHEN AIRPLANE SPEED IS (a) 300 mi/h, (b) 600 mi/h.

WE APPLY IMP.-MOM. PRINCIPLE USING VELOC. W/R PLANE



$$\pm \Sigma \text{COMP.}: (\Delta m) u_A + F \Delta t = (\Delta m) u_B$$

$$F = \frac{\Delta m}{\Delta t} (u_B - u_A) = \frac{200 \text{ lb/s}}{32.2 \text{ ft/s}^2} (2000 \text{ ft/s} - v)$$

$$\pm \Sigma \text{MOM. ABOUT B: } -(\Delta m) u_A (12 \text{ ft}) + (F \Delta t) d = 0$$

$$F d = \frac{\Delta m}{\Delta t} (12 \text{ ft}) u_A = \frac{200 \text{ lb/s}}{32.2 \text{ ft/s}^2} (12 \text{ ft}) v$$

$$(a) \quad v = 300 \text{ mi/h} = 440 \text{ ft/s}$$

$$\text{EQ. (1): } F = \frac{200}{32.2} (2000 - 440) = 9,689 \text{ lb}$$

$$\text{EQ. (2): } F d = \frac{200}{32.2} (12) (440) = 32,795 \text{ lb} \cdot \text{ft}$$

$$\text{DIVIDE (2) BY (1): } d = 3.38 \text{ ft}$$

ANSWER: 9690 lb, 3.38 ft BELOW B

$$(b) \quad v = 600 \text{ mi/h} = 880 \text{ ft/s}$$

$$\text{EQ. (1): } F = \frac{200}{32.2} (2000 - 880) = 6,956 \text{ lb}$$

$$\text{EQ. (2): } F d = \frac{200}{32.2} (12) (880) = 65,590 \text{ lb} \cdot \text{ft}$$

$$\text{DIVIDE (2) BY (1): } d = 9.43 \text{ ft}$$

ANSWER: 6960 lb, 9.43 ft BELOW B

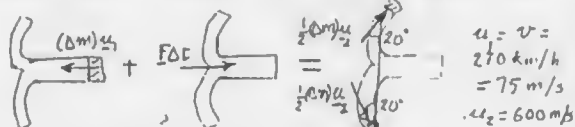
14.72

**GIVEN:**

IN REVERSE THRUST,  
ENGINE SCOOPS AIR AT  
RATE OF 120 kg/s AND  
DISCHARGES IT AS SHOWN  
WITH VELOCITY OF 600 m/s  
RELATIVE TO ENGINE.

**FIND:** REVERSE THRUST  
WHEN PLANE SPEED IS 270 km/h.

WE APPLY IMPULSE-MOMENTUM PRINCIPLE W/R PLANE



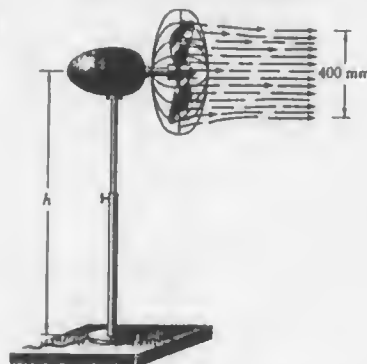
(F IS OPPOSITE TO REVERSE THRUST OF ENGINE)

$$\pm \Sigma \text{COMP.}: -(\Delta m) u_1 + F \Delta t = 2 \left[ \frac{1}{2} (\Delta m) u_2 \sin 20^\circ \right]$$

$$F = \frac{\Delta m}{\Delta t} (u_1 + u_2 \sin 20^\circ) = (120 \text{ kg/s}) (75 + 600 \sin 20^\circ) \text{ m/s}$$

$$F = 33.6 \text{ kN}$$

14.73



**GIVEN:** FLOOR FAN DELIVERS AIR WITH SPEED OF 6 m/s.  
IT IS SUPPORTED BY A 200-mm-DIAMETER CIRCULAR BASE  
AND ITS TOTAL WEIGHT IS 60 N.

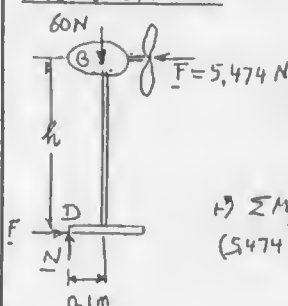
**FIND:** MAX. HEIGHT  $h$  IF FAN IS NOT TO TIP OVER.

(USE  $\rho = 1.21 \text{ kg/m}^3$  FOR AIR AND ASSUME  $v_A = v_B$ )

**THRUST:**

$$\text{FROM EQ. (14.39): } F = \frac{dm}{dt} (v_B - v_A) = \rho Q (v - 0) = \rho v A v$$

$$F = (1.21 \text{ kg/m}^3) \frac{\pi}{4} (0.400 \text{ m})^2 (6 \text{ m/s})^2 = 5,474 \text{ N}$$

**FREE BODY: FAN**

FORCE EXERTED ON FAN BY  
AIR STREAM IS EQUAL AND  
OPPOSITE TO THRUST.

WHEN FAN IS ABOUT TO  
TIP OVER, NORMAL FORCE  $N$   
IS APPLIED AT D.

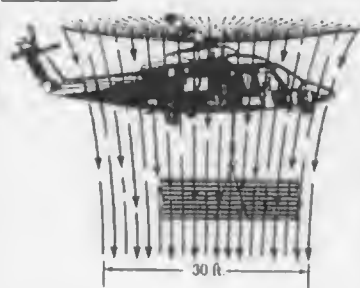
$$\pm \Sigma M_D = 0:$$

$$(5474 \text{ N}) h - (60 \text{ N}) (0.1 \text{ m}) = 0$$

$$h = 1.096 \text{ m}$$



14.74



GIVEN:

MAX. DOWNWARD AIR SPEED PRODUCED BY HELICOPTER IS 80 ft/s. WEIGHT OF HELICOPTER AND CREW IS 3500 lb.

FIND:

MAX. LOAD THAT HELICOPTER CAN LIFT WHILE HOVERING, (ASSUME  $\gamma = 0.076 \text{ lb/ft}^3$  FOR AIR.)

WE USE EQ. (14.39) TO DETERMINE THE THRUST  $F$ :

$$F = \frac{dm}{dt} (v_B - v_A) = \rho Q (v - 0) = \rho A v^2 = \frac{\gamma}{g} A v^2$$

$$F = \frac{0.076 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \frac{\pi (30 \text{ ft})^2 (80 \text{ ft/s})^2}{4} = 10,678 \text{ lb}$$

THE LIFT PROVIDED BY THE BLADE IS EQUAL AND OPPOSITE, THAT IS 10,678 lb. WE WRITE

$$+\uparrow \Sigma F_y = 0: 10,678 \text{ lb} - W - 3500 \text{ lb} = 0$$

$$W = 7178 \text{ lb}$$

$$W = 7180 \text{ lb}$$

14.75



GIVEN:

AIRLINER CRUISES AT 600 mi/h WITH EACH OF ITS THREE ENGINES DISCHARGING AIR AT 2000 ft/s RELATIVE TO PLANE.

FIND:

SPEED OF PLANE AFTER IT HAS LOST THE USE OF

(a) ONE ENGINE, (b) TWO ENGINES

(ASSUME THAT DRAG IS PROPORTIONAL TO  $v^2$ .)

WE USE EQ. (14.39) TO DETERMINE THE TOTAL THRUST OF THE ENGINES:

$$F = \frac{dm}{dt} (v_B - v_A) \quad \text{WHERE } v_B = 2000 \text{ ft/s}$$

$v$  = SPEED OF PLANE

$$\text{THUS: } F = \frac{dm}{dt} (2000 - v)$$

THE DRAG IS  $D = kv^2$

$$\text{EQUATING THRUST AND DRAG: } \frac{dm}{dt} (2000 - v) = kv^2 \quad (1)$$

WITH THREE ENGINES,  $v = 600 \text{ mi/h} = 880 \text{ ft/s}$

SUBSTITUTING IN EQ. (1):

$$\left( \frac{dm}{dt} \right)_3 (2000 - 880) = k (880)^2$$

$$\left( \frac{dm}{dt} \right)_3 = 691.43 k$$

(a) WITH TWO ENGINES:

$$\left( \frac{dm}{dt} \right)_2 = \frac{2}{3} \left( \frac{dm}{dt} \right)_3 = \frac{2}{3} (691.43 k) = 460.95 k$$

SUBSTITUTING IN EQ. (1):

$$460.95 k (2000 - v) = kv^2$$

$$v^2 + 460.95 v - 921.9 \times 10^3 = 0$$

$$v = \frac{-460.95 + \sqrt{(460.95)^2 + 4(921.9 \times 10^3)}}{2} = 756.96 \text{ ft/s}$$

$$v = 516 \text{ mi/h}$$

(b) WITH ONE ENGINE:

$$\left( \frac{dm}{dt} \right)_1 = \frac{1}{3} \left( \frac{dm}{dt} \right)_3 = \frac{1}{3} (691.43 k) = 230.48 k$$

SUBSTITUTING IN EQ. (1):

$$230.48 k (2000 - v) = kv^2$$

$$v^2 + 230.48 v - 460.95 \times 10^3 = 0$$

$$v = \frac{-230.48 + \sqrt{(230.48)^2 + 4(460.95 \times 10^3)}}{2} = 573.41 \text{ ft/s}$$

$$v = 391 \text{ mi/h}$$

14.76



GIVEN:

16-Mg PLANE MAINTAINS  $v = 774 \text{ km/h}$  WITH  $\alpha = 18^\circ$ . IT SCOOPS AIR AT RATE OF 300 kg/s AND DISCHARGES IT AT 665 m/s RELATIVE TO PLANE.

FIND: (a) INITIAL ACCELERATION IF PILOT CHANGES TO HORIZONTAL FLIGHT WITH SAME ENGINE SETTING  
(b) MAX. HORIZONTAL SPEED THAT WILL BE ATTAINED. (ASSUME THAT DRAG IS PROPORTIONAL TO  $v^2$ .)

DETERMINATION OF THRUST

SINCE AIRPLANE IS ACCELERATED IN HORIZONTAL FLIGHT, WE USE A REFERENCE FRAME AT REST WITH RESPECT TO THE ATMOSPHERE WHEN USING EQ. (14.39) TO DETERMINE THE THRUST  $F$  (CF. FOOTNOTE, PAGE 860).

$$F = \frac{dm}{dt} (v_B - v_A)$$

WHERE  $v_A = 0$ ,  $v_B = v_{\text{disch.}} - v_{\text{plane}}$

$$= 665 \text{ m/s} - 774 \text{ km/h} \left( \frac{1000 \text{ m}}{3600 \text{ s}} \right)$$

$$= 665 \text{ m/s} - 215 \text{ m/s} = 450 \text{ m/s}$$

$$F = (300 \text{ kg/s}) (450 \text{ m/s} - 0) = 135.0 \text{ kN}$$

AIRPLANE CLIMBING (NO ACCELERATION)

$$\Sigma F \Delta 18^\circ = 0$$

$$135.0 \text{ kN} - D - W \sin 18^\circ = 0$$

$$D = 135.0 \text{ kN} - (16 \text{ Mg}) (9.81 \text{ m/s}^2) \sin 18^\circ$$

$$= 135.0 \text{ kN} - 48.50 \text{ kN} = 86.50 \text{ kN}$$

(a) AT START OF HORIZONTAL FLIGHT

THRUST AND DRAG ARE STILL THE SAME

$$\Sigma F = ma$$

$$F - D = ma$$

$$(135.0 - 86.5) \times 10^3 \text{ N} = (16 \times 10^3 \text{ kg}) a$$

$$a = 3.03 \text{ m/s}^2$$

(b) AT MAX. SPEED IN HORIZONTAL FLIGHT

WE HAVE  $a = 0$

$$F_m - D_m = 0$$

(1)

WHERE

$$F_m = \frac{dm}{dt} (u - v_m) = (300 \text{ kg/s}) (665 \text{ m/s} - v_m) \quad (2)$$

ON THE OTHER HAND

$$D_m = kv_m^2$$

(3)

BUT, INITIALLY, WE HAD  $D = 86.50 \text{ kN}$

AND  $v = 774 \text{ km/h} = 215 \text{ m/s}$  AND, THEREFORE

$$D = kv^2$$

$$86.50 \times 10^3 \text{ N} = k (215 \text{ m/s})^2$$

(4)

DIVIDING (3) AND (4) MEMBER BY MEMBER:

$$\frac{D_m}{86.50 \times 10^3} = \frac{v_m^2}{(215)^2} \quad D_m = 1.8713 v_m^2 \quad (5)$$

SUBSTITUTING FOR  $F_m$  FROM (2) AND FOR  $D_m$  FROM (5) INTO (1):

$$300(665 - v_m) - 1.8713 v_m^2 = 0$$

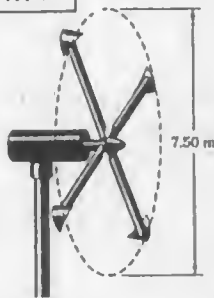
$$v_m^2 + 160.32 v_m - 106.61 \times 10^3 = 0$$

$$v_m = \frac{-160.32 + \sqrt{(160.32)^2 + 4(106.61 \times 10^3)}}{2} = 256.05 \text{ m/s}$$

$$= (256.05 \text{ m/s}) \left( \frac{3600 \text{ s}}{1000 \text{ m}} \right) = 921.78 \text{ km/h}$$

$$v_m = 922 \text{ km/h}$$

14.77



GIVEN:

WIND TURBINE-GENERATOR'S OUTPUT-POWER RATING IS 5 kW FOR 30 km/h WIND SPEED.

FIND FOR THAT WIND SPEED (a) KINETIC ENERGY OF AIR PARTICLES ENTERING CIRCLE PER SECOND.

(b) EFFICIENCY OF THIS ENERGY-CONVERSION SYSTEM. (ASSUME  $\rho = 1.21 \text{ kg/m}^3$  FOR AIR)

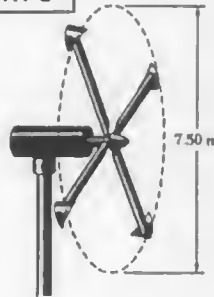
(a) KINETIC ENERGY PER SECOND

$$= \frac{1}{2} \frac{\Delta m}{\Delta t} v^2 = \frac{1}{2} \rho Q v^2 = \frac{1}{2} \rho (Av) v^2 = \frac{1}{2} \rho A v^3$$

$$= \frac{1}{2} (1.21 \text{ kg/m}^3) \frac{\pi}{4} (7.50 \text{ m})^2 \left( \frac{30 \times 10^3 \text{ m}}{3.6 \times 10^3 \text{ s}} \right)^3 = 15.47 \text{ kJ/s}$$

(b) EFFICIENCY =  $\frac{5 \text{ kW}}{15.47 \text{ kJ/s}} = 0.323$

14.78



GIVEN:

WIND TURBINE GENERATOR PRODUCES 28 kW OF ELECTRIC POWER WITH AN EFFICIENCY OF 0.35 AS AN ENERGY-CONVERSION SYSTEM

FIND:

(a) KINETIC ENERGY OF AIR PARTICLES ENTERING CIRCLE PER SECOND

(b) WIND SPEED (ASSUME  $\rho = 1.21 \text{ kg/m}^3$  FOR AIR.)

(a) KINETIC ENERGY PER SECOND

$$= \text{INPUT POWER} = \frac{\text{OUTPUT POWER}}{\text{EFFICIENCY}} = \frac{28 \text{ kW}}{0.35} = 80 \text{ kJ/s}$$

(b) WIND SPEED

$$\text{K.E. PER SECON} = \frac{1}{2} \frac{\Delta m}{\Delta t} v^2 = \frac{1}{2} \rho Q v^2 = \frac{1}{2} \rho (Av) v^2 = \frac{1}{2} \rho A v^3$$

$$\text{THEREFORE: } 80 \text{ kJ/s} = \frac{1}{2} (1.21 \text{ kg/m}^3) \frac{\pi}{4} (7.50 \text{ m})^2 v^3$$

$$v^3 = 2793.1 \quad v = 14.411 \text{ m/s} \quad v = 51.9 \text{ km/h}$$

14.79

GIVEN:

PLANE CRUISING IN LEVEL FLIGHT AT 600 mi/h SCOOPS IN AIR AT RATE OF 200 lb/s AND DISCHARGES IT AT 2200 ft/s RELATIVE TO PLANE.

FIND: (a) POWER USED TO PROPEL PLANE, (b) TOTAL ENGINE POWER (c) EFFICIENCY OF PLANE

(a) FROM EQ. (14.39):

$$\text{THRUST} = F = \frac{dm}{dt} (v_B - v_A), \quad \text{WHERE } v_B = 2200 \text{ ft/s}, \quad v_A = 600 \text{ mi/h} = 880 \text{ ft/s}$$

$$F = \frac{200 \text{ lb/s}}{32.2 \text{ ft/s}^2} (2200 - 880) \text{ ft/s} = 8,198.8 \text{ lb}$$

$$\text{PROPULSIVE POWER} = F v = (8,198.8 \text{ lb})(880 \text{ ft/s}) = 7,214,944 \text{ ft-lb/s} = 13,120 \text{ hp}$$

$$(b) \text{ POWER LOST IN EXHAUST} = \frac{1}{2} \frac{dm}{dt} v_{exh}^2 = \frac{1}{2} \frac{200}{32.2} (2200 - 880)^2$$

$$= 5.4112 \times 10^6 \text{ ft-lb/s} = 9,838 \text{ hp}$$

$$\text{TOTAL POWER} = 13,120 \text{ hp} + 9,838 \text{ hp} = 22,960 \text{ hp}$$

$$(c) \text{ EFFICIENCY} = \frac{13,120 \text{ hp}}{22,960 \text{ hp}} = 0.571$$

14.80

GIVEN:

PROPELLER OF SMALL PLANE HAS 6-ft-DIAMETER SLIPSTREAM AND PRODUCES 800-lb THRUST WHEN PLANE IS AT REST ON GROUND.

FIND: (a) SPEED OF THE AIR IN THE SLIPSTREAM, (b) VOLUME OF AIR PASSING THROUGH PROPELLER PER SECOND, (c) KINETIC ENERGY IMPARTED TO THE AIR PER SECOND (ASSUME  $\rho = 0.076 \text{ lb/ft}^3$  FOR AIR.)

(a) SPEED  $v$  OF AIR

APPLY EQ. (14.39), ASSUMING AIR ENTERS SLIPSTREAM WITH ZERO VELOCITY:

$$\text{THRUST} = F = \frac{dm}{dt} v = \rho Q v = \frac{\rho}{g} (Av) v = \frac{\rho}{g} A v^2$$

$$800 \text{ lb} = \frac{0.076 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \frac{\pi}{4} (6 \text{ ft})^2 v^2$$

$$800 \text{ lb} = (0.066734 \text{ lb} \cdot \text{s}^2/\text{ft}^4) v^2 \quad v^2 = 11,988 \text{ ft}^2/\text{s}^2$$

$$v = 109.49 \text{ ft/s} \quad v = 109.5 \text{ ft/s}$$

(b) VOLUME OF AIR PER SECOND

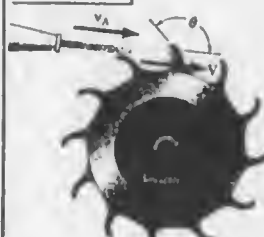
$$Q = Av = \frac{\pi}{4} (6 \text{ ft})^2 (109.49 \text{ ft/s}) \quad Q = 3100 \text{ ft}^3/\text{s}$$

(c) KINETIC ENERGY IMPARTED TO AIR PER SECOND

$$\frac{\Delta T}{\Delta t} = \frac{1}{2} \frac{dm}{dt} v^2 = \frac{1}{2} (\rho Av) v^2 = \frac{1}{2} \left( \frac{\rho}{g} A v^2 \right) v = \frac{1}{2} F v$$

$$= \frac{1}{2} (800 \text{ lb})(109.49 \text{ ft/s}) = 43,800 \text{ ft-lb/s}$$

14.81



GIVEN: PELTON-WHEEL TURBINE.

RATE AT WHICH WATER IS DEFLECTED BY BLADES EQUALS RATE AT WHICH WATER ISSUES FROM NOZZLE:  $\Delta m/\Delta t = \rho A v_A$ .

FIND: (a) VELOCITY  $v$  OF BLADES FOR MAXIMUM POWER,

(b) MAXIMUM POWER,

(c) MECHANICAL EFFICIENCY. (USE NOTATION OF SP 14.7)

IMPULSE-MOMENTUM PRINCIPLE

AS IN SAMPLE PROB. 14.7:

$$\Sigma F \text{ COMP: } (\Delta m) u - F_x \Delta t = (\Delta m) u \cos \theta$$

$$\text{BUT NOW } \Delta m = \rho A v_A \Delta t \quad u = v_A - v$$

$$\text{THUS: } F_x = \rho A v_A (v_A - v)(1 - \cos \theta)$$

$$\text{OUTPUT POWER} = F_x v = \rho A v_A (v_A - v)(1 - \cos \theta) v \quad (1)$$

$$\text{OR: OUTPUT POWER} = \rho A v_A (v_A v - v^2)(1 - \cos \theta)$$

(a) FOR MAX. POWER:  $d(\text{POWER})/dv = 0$ :

$$\rho A v_A (v_A - 2v)(1 - \cos \theta) = 0 \quad v = \frac{1}{2} v_A$$

(b) MAX. POWER: MAKE  $v = \frac{1}{2} v_A$  IN EQ. (1):

$$\text{MAX. POWER} = \rho A v_A \left( v_A - \frac{1}{2} v_A \right) \left( \frac{1}{2} v_A \right) (1 - \cos \theta) = \frac{1}{4} \rho A v_A^2 (1 - \cos \theta)$$

(c) EFFICIENCY

$$\text{INPUT POWER} = \frac{1}{2} \frac{\Delta m}{\Delta t} v_A^2 = \frac{1}{2} (\rho A v_A) v_A^2 = \frac{1}{2} \rho A v_A^3$$

$$= \frac{1}{2} \rho A v_A^3 \quad (2)$$

DIVIDE (1) BY (2):

$$\eta = \frac{\text{OUTPUT POWER}}{\text{INPUT POWER}} = \frac{\rho A v_A (v_A - v)(1 - \cos \theta) v}{\frac{1}{2} \rho A v_A^3}$$

$$\eta = 2 \frac{v}{v_A} \left( 1 - \frac{v}{v_A} \right) (1 - \cos \theta)$$

NOTE: MAXIMUM EFFICIENCY IS OBTAINED WHEN  $v = \frac{1}{2} v_A$  AND  $\theta = 180^\circ$ :

$$\eta_{\text{MAX}} = 2 \left( \frac{1}{2} \times \frac{1}{2} \right) (2) = 1$$

14.82



GIVEN:

CIRCULAR REENTRANT ORIFICE (BORDA'S MOUTHPIECE)

$$v_1 = 0, v_2 = v = \sqrt{2gh}$$

SHOW THAT:

$$d = D/\sqrt{2}$$

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO SECTION OF WATER INDICATED BY DASHED LINE AND TO MASS OF WATER  $\Delta m$  ENTERING AND LEAVING IN  $\Delta t$ .



$$\pm x \text{ COMP.}: 0 + P\Delta t = (\Delta m)v = (\rho Q \Delta t)v = (\rho A_2 v \Delta t)v$$

$$\text{TIPUS: } P = \rho A_2 v^2 = \rho \frac{\pi}{4} d^2 v^2 \quad (1)$$

BUT, RECALLING THAT THE PRESSURE AT A DEPTH  $h$  IS  $p = \rho gh$ , WE HAVE

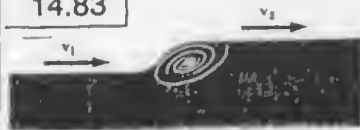
$$P = pA_1 = \rho gh A_1 = \rho gh \frac{\pi}{4} D^2$$

SUBSTITUTING THIS EXPRESSION IN (1) AND THE EXPRESSION GIVEN FOR  $v$ :

$$\rho gh \frac{\pi}{4} D^2 = \rho \frac{\pi}{4} d^2 (2gh)$$

$$D^2 = 2d^2 \quad d = \frac{D}{\sqrt{2}} \quad (\text{Q.E.D.})$$

14.83

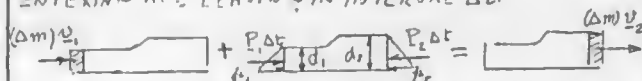


GIVEN:

HYDRAULIC JUMP.

CHANNEL WIDTH =  $b$ .EXPRESS RATE OF FLOW  $Q$  IN TERMS OF  $b, d_1, d_2$ .

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE WATER SECTION SHOWN AND TO THE MASS OF WATER  $\Delta m$  ENTERING AND LEAVING IN INTERVAL  $\Delta t$ .



$$\pm x \text{ COMP.}: (\Delta m)v_1 + P_1 \Delta t - P_2 \Delta t = (\Delta m)v_2$$

$$(\rho Q \Delta t) v_1 + P_1 \Delta t - P_2 \Delta t = (\rho Q \Delta t) v_2$$

$$\rho Q (v_1 - v_2) = P_2 - P_1 \quad (1)$$

$$\text{BUT } Q = A_1 v_1 = b d_1 v_1 \quad v_1 = Q/bd_1 \quad (2)$$

$$\text{AND } Q = A_2 v_2 = b d_2 v_2 \quad v_2 = Q/bd_2 \quad (3)$$

$$\text{ALSO: } P_1 = \frac{1}{2} \rho b d_1^3 \quad A_1 = \frac{1}{2} (\delta d_1) (b d_1) = \frac{1}{2} \delta b d_1^2 \quad (4)$$

$$\text{SIMILARLY: } P_2 = \frac{1}{2} \rho b d_2^3 \quad A_2 = \frac{1}{2} \delta b d_2^2 \quad (5)$$

SUBSTITUTE FROM (2), (3), (4), (5) INTO (1):

$$\rho Q \left( \frac{Q}{b d_1} - \frac{Q}{b d_2} \right) = \frac{1}{2} \delta b (d_2^3 - d_1^3)$$

$$\rho Q^2 \frac{d_2 - d_1}{b d_1 d_2} = \frac{1}{2} \delta b (d_2 + d_1)(d_2 - d_1)$$

DIVIDING THROUGH BY  $d_2 - d_1$  AND RECALLING THAT  $\delta = \rho g$ :

$$\frac{Q^2}{b d_1 d_2} = \frac{1}{2} g b (d_1 + d_2)$$

$$Q = b \sqrt{\frac{1}{2} g d_1 d_2 (d_1 + d_2)}$$

\* 14.84

GIVEN: FOR CHANNEL OF PROB. 14.83:

$$b = 12 \text{ ft}, d_1 = 4 \text{ ft}, d_2 = 5 \text{ ft}$$

FIND: RATE OF FLOW.

SEE SOLUTION OF PROB. 14.83 FOR DERIVATION OF

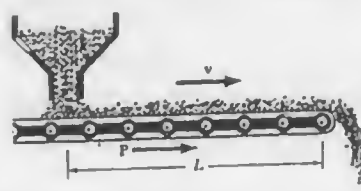
$$Q = b \sqrt{\frac{1}{2} g d_1 d_2 (d_1 + d_2)}$$

SUBSTITUTING THE GIVEN DATA:

$$Q = (12 \text{ ft}) \sqrt{\frac{1}{2} (32.2 \text{ ft/s}^2) (4 \text{ ft}) (5 \text{ ft}) (9 \text{ ft})}$$

$$Q = 646 \text{ ft}^3/\text{s}$$

14.85



GIVEN:

GRAVEL FALLS ON CONVEYOR BELT WITH NO VELOCITY AND AT THE CONSTANT RATE  $q = \Delta m/\Delta t$

(a) FIND MAGNITUDE OF FORCE  $P$  REQUIRED TO MAINTAIN A CONSTANT BELT SPEED.

(b) SHOW THAT K.E. REQUIRED BY GRAVEL IN GIVEN TIME INTERVAL IS HALF THE WORK DONE BY  $P$ . WHAT HAPPENS TO THE OTHER HALF OF WORK OF  $P$ ?

(a) WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE GRAVEL ON THE BELT AND TO THE MASS  $\Delta m$  OF GRAVEL HITTING AND LEAVING BELT IN INTERVAL  $\Delta t$ .



$$\pm x \text{ COMP.}: m v + P \Delta t = m v + (\Delta m) v$$

$$P = \frac{\Delta m}{\Delta t} v = q v \quad P = q v$$

(b) KINETIC ENERGY ACQUIRED PER UNIT TIME:

$$\frac{\Delta T}{\Delta t} = \frac{1}{2} \frac{\Delta m}{\Delta t} v^2 = \frac{1}{2} q v^2 \quad (1)$$

WORK DONE PER UNIT TIME:

$$\frac{\Delta U}{\Delta t} = \frac{P \Delta x}{\Delta t} = P v$$

RECALLING THE RESULT OF PART (a):

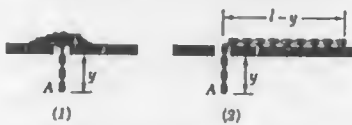
$$\frac{\Delta U}{\Delta t} = (q v) v = q v^2 \quad (2)$$

COMPARING ERS. (1) AND (2), WE CONCLUDE THAT

$$\frac{\Delta T}{\Delta t} = \frac{1}{2} \frac{\Delta U}{\Delta t} \quad (\text{Q.E.D.})$$

THE OTHER HALF OF THE WORK OF  $P$  IS DISSIPATED INTO HEAT BY FRICTION AS THE GRAVEL SLIPS ON THE BELT BEFORE REACHING THE SPEED  $v$ .

14.86



**GIVEN:** CHAIN OF LENGTH  $l$  AND MASS  $m$  FALLS THROUGH SMALL HOLE IN PLATE. CHAIN IS AT REST WHEN  $y$  IS VERY SMALL.

**FIND IN EACH CASE SHOWN:**

(a) ACCELERATION OF FIRST LINK A AS FUNCTION OF  $y$ .  
(b) VELOCITY OF CHAIN AS LAST LINK PASSES THRU HOLE.  
IN CASE 1, ASSUME THAT EACH LINK IS AT REST UNTIL IT FALLS THRU HOLE.

IN CASE 2, ASSUME THAT ALL LINKS HAVE THE SAME SPEED AT ANY GIVEN INSTANT.

**CASE 1:** WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PORTION OF CHAIN WHICH HAS ALREADY PASSED THROUGH THE HOLE AT TIME  $t$  AND TO THE PORTION WHICH WILL PASS IN INTERVAL  $\Delta t$ .

$\Delta m_2$  (NO MOM)

$$\downarrow \frac{m y}{l} v + \downarrow m g \frac{y}{l} \Delta t = \downarrow m (y + \Delta y) (v + \Delta v)$$

$$\downarrow \frac{m y}{l} v + \frac{m g y}{l} \Delta t = \frac{m}{l} (y + \Delta y) (v + \Delta v)$$

$$\text{DIVIDE BY } \Delta t \text{ AND LET } \Delta t \rightarrow 0: g y = y \frac{dv}{dt} + v \frac{dy}{dt} = \frac{d}{dt} (y v)$$

MULTIPLY BOTH SIDES BY  $y v dt$  AND NOTE THAT  $v dt = dy$ :

$$g y^2 dy = y v d(y v)$$

SET  $y v = u$  AND INTEGRATE:

$$\int_0^y g y^2 dy = \int_0^u u du$$

$$\frac{1}{3} g y^3 = \frac{1}{2} (y v)^2 \quad v^2 = \frac{2}{3} g y \quad (1)$$

(a) DIFFERENTIATE (1) WITH RESPECT TO  $t$ :

$$2 v \frac{dv}{dt} = \frac{2}{3} g \frac{dy}{dt} \quad \text{OR } 2 v a = \frac{2}{3} g v$$

$$a = \frac{1}{3} g$$

(b) AS LAST LINK PASSES THROUGH HOLE,  $y = l$  AND EQ (1)

$$\text{YIELDS } v^2 = \frac{2}{3} g l \quad v = \sqrt{\frac{2}{3} g l}$$

**CASE 2:** (a) AT TIME  $t$ , THE FORCE CAUSING THE ACCELERATION OF THE ENTIRE CHAIN IS THE WEIGHT OF THE LENGTH  $y$  OF CHAIN WHICH HAS PASSED THROUGH:

$$\Sigma F = m a:$$

$$m g \left( \frac{y}{l} \right) = m a \quad a = \frac{g y}{l}$$

(b) SETTING  $a = v \frac{dv}{dy}$ , WE HAVE

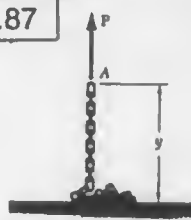
$$v dv = \frac{g}{l} y dy$$

INTEGRATING IN  $v$  FROM 0 TO  $v$  AND IN  $y$  FROM 0 TO  $l$ :

$$\frac{1}{2} v^2 = \frac{1}{2} \frac{g}{l} \frac{l^2}{2}$$

$$v = \sqrt{g l}$$

14.87



**GIVEN:**

CHAIN OF LENGTH  $l$  AND MASS  $m$  IS LYING IN A PILE ON FLOOR. IT IS RAISED AT A CONSTANT  $v$ .

**FIND FOR ANY  $y$ :**

(a) MAGNITUDE OF FORCE  $P$ ,  
(b) REACTION OF FLOOR.

(a) WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE LENGTH  $y$  OF CHAIN WHICH IS OFF THE FLOOR AND TO THE LENGTH  $\Delta y$  WHICH WILL BE SET IN MOTION DURING THE TIME INTERVAL  $\Delta t$ .

$$\downarrow P \Delta t + \downarrow m g \frac{y}{l} \Delta t = \uparrow m \frac{y + \Delta y}{l} v$$

$$\text{+ly COMP: } m \frac{y}{l} v + P \Delta t - m g \frac{y}{l} \Delta t = m \frac{y + \Delta y}{l} v$$

$$P \Delta t = \frac{m}{l} (g y \Delta t - y v + y v + v \Delta y)$$

DIVIDING BY  $\Delta t$ :

$$P = \frac{m}{l} (g y + v \frac{\Delta y}{\Delta t})$$

NOTING THAT  $\Delta y / \Delta t = v$ ,

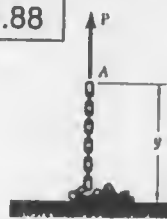
$$P = \frac{m}{l} (g y + v^2)$$

(b) THE REACTION OF THE FLOOR IS EQUAL TO THE WEIGHT OF CHAIN STILL ON THE FLOOR:

$$R = m g - m g \frac{y}{l}$$

$$R = m g \left( 1 - \frac{y}{l} \right)$$

14.88



**GIVEN:**

CHAIN OF LENGTH  $l$  AND MASS  $m$  IS LOWERED INTO A PILE ON THE FLOOR AT CONSTANT  $v$ .  
**FIND FOR ANY  $y$ :**

(a) MAGNITUDE OF FORCE  $P$ ,  
(b) REACTION OF THE FLOOR.

(a)  $P$  IS EQUAL TO THE WEIGHT OF CHAIN STILL OFF THE FLOOR:  $P = m g y / l$

(b) WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE LENGTH  $l - y$  OF CHAIN ON THE FLOOR AND TO THE LENGTH  $\Delta y$  WHICH HITS THE FLOOR IN  $\Delta t$ :

$$\downarrow m \frac{\Delta y}{l} v + \downarrow m g \frac{(l - y + \Delta y) \Delta t}{l} = \uparrow R \Delta t$$

$$\text{+ly COMP: } -m \frac{\Delta y}{l} v - \frac{m g}{l} (l - y + \Delta y) \Delta t + R \Delta t = 0$$

SOLVING FOR  $R$ :

$$R = \frac{m}{l} [g (l - y + \Delta y) + v \frac{\Delta y}{\Delta t}]$$

BUT  $\frac{\Delta y}{\Delta t} = v$  AND  $\Delta y \rightarrow 0$  WHEN  $\Delta t \rightarrow 0$ .

THEREFORE:

$$R = \frac{m}{l} [g (l - y) + v^2]$$

14.89



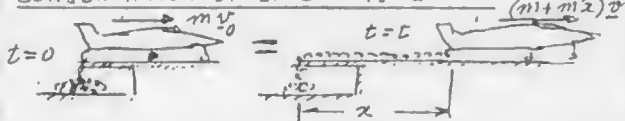
GIVEN:

AS PLANE OF MASS  $m$  LANDS WITH  $v_0$  ON CARRIER, ITS TAIL HOOKS INTO END OF CHAIN OF LENGTH  $\ell$ .

FIND: (a) MASS OF CHAIN REQUIRED TO REDUCE PLANE SPEED TO  $\beta v_0$  (WHERE  $\beta < 1$ ), (b) MAX. FORCE EXERTED BY CHAIN ON PLANE

LET  $m' =$  MASS OF CHAIN PER UNIT LENGTH  
 $x =$  DISTANCE TRAVELED AT TIME  $t$

CONSERVATION OF LINEAR MOMENTUM



$$\pm \sum X_{CMR}: m v_0 = (m + m'x) v \quad (1)$$

(a) WE WANT  $v = \beta v_0$  FOR  $x = \ell$ . SUBSTITUTE:

$$m v_0 = (m + m'\ell) \beta v_0$$

$$m v_0 (1 - \beta) = m' \ell \beta v_0$$

$$\text{MASS OF CHAIN} = m' \ell = \frac{1 - \beta}{\beta} m \quad (2)$$

(b) SOLVE EQ. (1) FOR  $v$ :

$$v = \frac{m v_0}{(m + m'x)} \quad (3)$$

$$a = \frac{dv}{dt} = - \frac{m v_0}{(m + m'x)^2} m' \frac{dx}{dt} = - \frac{m m' v_0 v}{(m + m'x)^2}$$

$$\text{OR, RECALLING (3): } a = - \frac{m^2 m' v_0^2}{(m + m'x)^3} \quad (4)$$

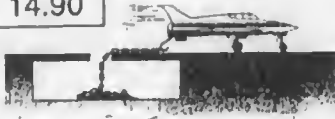
DECELERATION IS A MAXIMUM FOR  $x = 0$ . WE HAVE

$$(-a)_{\max} = \frac{m^2 m' v_0^2}{m^3} = \frac{m'}{m} v_0^2 \quad (5)$$

WRITING  $|F|_{\max} = m |a|_{\max}$  AND RECALLING (2):

$$|F|_{\max} = m' v_0^2 \quad |F|_{\max} = \frac{1 - \beta}{\beta} \frac{m v_0^2}{\ell}$$

14.90



GIVEN:

AS 6000-kg PLANE LANDS AT 180 km/h ON CARRIER, ITS TAIL HOOKS INTO END OF

80-m-LONG CHAIN OF MASS OF 50 kg/m.

FIND: (a) MIN. DECELERATION OF PLANE,

(b) VELOCITY WHEN ENTIRE CHAIN IS PULLED OUT

SEE SOLUTION OF PROB. 14.89 FOR DERIVATION OF ERS. (3) AND (5).

(a) FROM EQ. (5):

$$\text{MAX. DECEL.} = (-a)_{\max} = \frac{m'}{m} v_0^2 = \frac{50 \text{ kg/m}}{6000 \text{ kg}} \left( \frac{180 \text{ m/s}}{3.6} \right)^2$$

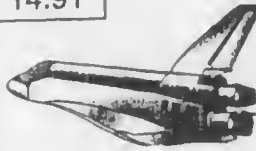
$$\text{MAX. DECEL.} = 20.8 \text{ m/s}^2$$

(b) FROM EQ. (3), FOR  $x = \ell = 80 \text{ m}$ :

$$v_{\max} = \frac{m v_0}{(m + m' \ell)} = \frac{(6000 \text{ kg})(180 \text{ km/h})}{6000 \text{ kg} + (50 \text{ kg/m})(80 \text{ m})}$$

$$v_{\max} = 108.0 \text{ km/h}$$

14.91



GIVEN:

EACH OF THE THREE ENGINES OF SPACE SHUTTLE BURNS PROPELLANT AT RATE OF 340 kg/s AND EJECTS IT WITH A RELATIVE VELOCITY OF 3750 m/s

FIND: TOTAL THRUST PROVIDED BY THE THREE ENGINES

FROM EQ. (14.44) FOR EACH ENGINE

$$P = \frac{dm}{dt} u = (340 \text{ kg/s})(3750 \text{ m/s}) = 1.275 \times 10^6 \text{ N}$$

FOR THE 3 ENGINES:

$$\text{TOTAL THRUST} = 3(1.275 \times 10^6 \text{ N}) = 3.83 \text{ MN}$$

14.92



GIVEN:

THE THREE ENGINES OF SPACE SHUTTLE PROVIDE A TOTAL THRUST OF 6 MN. PROPELLANT IS EJECTED WITH A RELATIVE VEL. OF 3750 m/s.

FIND: RATE AT WHICH PROPELLANT IS BURNED BY EACH OF THE THREE ENGINES.

$$\text{THRUST OF EACH ENGINE: } P = \frac{1}{3} (6 \text{ MN}) = 2 \times 10^6 \text{ N}$$

$$\text{EQ. (14.44): } P = \frac{dm}{dt} u$$

$$2 \times 10^6 \text{ N} = \frac{dm}{dt} (3750 \text{ m/s}) \quad \frac{dm}{dt} = \frac{2 \times 10^6 \text{ N}}{3750 \text{ m/s}} = 533 \text{ kg/s}$$

14.93

GIVEN:

ROCKET FIRED VERTICALLY FROM GROUND  
 WEIGHT OF ROCKET (INCLUDING FUEL) = 2400 lb

WEIGHT OF FUEL = 2000 lb

FUEL EJECTED AT RATE OF 25 lb/s WITH RELATIVE VELOCITY OF 12,000 ft/s.

FIND: ACCELERATION OF ROCKET

(a) AS IT IS FIRED,

(b) AS LAST PARTICLE OF FUEL IS BEING CONSUMED

EQ. (14.44):

$$P = \frac{dm}{dt} u = \frac{25 \text{ lb/s}}{g} (12,000 \text{ ft/s}) = \frac{300 \times 10^3}{g}$$

$$\begin{aligned} \uparrow \sum F &= ma: \\ P - W &= ma \\ a &= \frac{P}{m} - \frac{W}{m} = \frac{(300 \times 10^3)/g}{W/g} - g \\ a &= \frac{300 \times 10^3}{W} - g \quad (1) \end{aligned}$$

(a) AS ROCKET IS FIRED:

$$W = 2400 \text{ lb}$$

$$\text{FROM (1): } a = \frac{300 \times 10^3}{2400} - 32.2 = 125.0 - 32.2 = 92.8$$

$$a = 92.8 \text{ ft/s}^2 \uparrow$$

(b) AS LAST PARTICLE OF FUEL IS BEING CONSUMED:

$$W = 2400 - 2000 = 400 \text{ lb}$$

$$\text{FROM (1): } a = \frac{300 \times 10^3}{400} - 32.2 = 750 - 32.2 = 717.8$$

$$a = 718 \text{ ft/s}^2 \uparrow$$

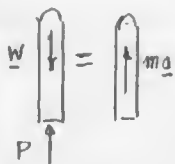
14.94

GIVEN:

ROCKET FIRED VERTICALLY FROM GROUND.  
WEIGHT OF ROCKET (INCLUDING FUEL) = 3000 lb  
WEIGHT OF FUEL = 2500 lb.  
FUEL CONSUMED AT RATE OF 30 lb/s.  
ACCELERATION INCREASES BY 750 ft/s<sup>2</sup> FROM TIME  
ROCKET IS FIRED TO TIME WHEN LAST PARTICLE OF FUEL  
IS CONSUMED.

FIND:

RELATIVE VELOCITY WITH WHICH IS EJECTED.



$$+\Sigma F = ma:$$

$$P - W = ma$$

$$a = \frac{P - W}{m} = \frac{P}{W/g} - g$$

$$a = \frac{P}{W}g - g \quad (1)$$

AS ROCKET IS FIRED; EQ. (1) YIELD.

$$a_0 = \frac{Pg}{3000\text{ lb}} - g \quad (2)$$

WHEN LAST PARTICLE IS FIRED:

$$a_0 + 750 \text{ ft/s}^2 = \frac{Pg}{500\text{ lb}} - g \quad (3)$$

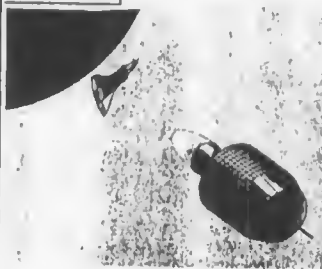
SUBTRACT (2) FROM (3):  $750 = Pg \left( \frac{1}{500} - \frac{1}{3000} \right)$   
 $750 = (1.6667 \times 10^{-3}) Pg$   
 $P = \frac{450 \times 10^3}{g}$

BUT, FROM EQ. (14.44):

$$P = \frac{dm}{dt} u: \frac{450 \times 10^3}{g} = \frac{30 \text{ lb/s}}{g} u$$

$$u = 15,000 \text{ ft/s}$$

14.95



GIVEN:

ENGINE OF COMMUNICATION  
SATELLITE IS FIRED TO  
INCREASE ITS VELOCITY BY  
8000 ft/s.  
WEIGHT OF SATELLITE  
(INCLUDING FUEL) = 10,000 lb.  
FUEL EJECTED WITH  
RELATIVE VEL. OF 13,750 ft/s.

FIND:

WEIGHT OF FUEL CONSUMED

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO  
SATELLITE AND FUEL EJECTED IN INTERVAL  $\Delta t$ .

$$mv + 0 = (m - \Delta m)(v + \Delta v) + \Delta m(u - v - \Delta v)$$

$$\begin{aligned} \Rightarrow mv &= (m - \Delta m)(v + \Delta v) + \Delta m(u - v - \Delta v) \\ mv &= mv + m\Delta v - \Delta m v - \Delta m \Delta v + \Delta m u - \Delta m v - \Delta m \Delta v \\ m\Delta v &= u\Delta m \end{aligned}$$

BUT  $\Delta m = q \Delta t$  AND  $m = m_0 - qt$ THUS  $(m_0 - qt)\Delta v = uq \Delta t$ 

$$\text{AS } \Delta t \rightarrow 0: \frac{dv}{dt} = \frac{uq}{m_0 - qt}$$

$$v = \int_0^t \frac{uq dt}{m_0 - qt} = -u [\ln(m_0 - qt)]_0^t$$

$$v = u \ln \frac{m_0}{m_0 - qt} \quad (1)$$

(CONTINUED)

14.95 continued

EXPRESSING EQ. (1) IN EXPONENTIAL FORM:

$$\frac{m_0}{m_0 - qt} = e^{v/u} \quad (2)$$

SETTING  $m_0 = (10,000 \text{ lb})/g$ ,  $v = 8000 \text{ ft/s}$ ,  $u = 13,750 \text{ ft/s}$ ,  
AND EXPRESSING  $q$  IN lb/s, WE HAVE

$$\frac{10,000/g}{(10,000 - qt)/g} = e^{8000/13,750} = e^{0.58182} = 1.7893$$

$$10,000 - qt = \frac{10,000}{1.7893} = 5,588.8 \quad qt = 4,411.2 \text{ lb}$$

$$\text{WEIGHT OF FUEL EXPENDED} = qt = 4410 \text{ lb}$$

14.96

GIVEN: COMMUNICATION SATELLITE OF PROB 14.95.

FIND: INCREASE IN VELOCITY AFTER 2500 lb HAS BEEN  
CONSUMED.

SEE SOLUTION OF PROB 14.95 FOR DERIVATION OF EQ. (1).

$$v = u \ln \frac{m_0}{m_0 - qt} \quad (1)$$

FROM DATA OF PROBS. 14.95 AND 14.96:

$$u = 13,750 \text{ ft/s}, \quad m_0 = (10,000 \text{ lb})/g, \quad qt = (2,500 \text{ lb})/g$$

SUBSTITUTE IN (1):

$$v = (13,750 \text{ ft/s}) \ln \frac{10,000/g}{10,000/g - 2,500/g} = (13,750 \text{ ft/s}) \ln(1.3333)$$

$$v = 3960 \text{ ft/s}$$

14.97

GIVEN:

A 540-kg SPACECRAFT IS MOUNTED ON  
TOP OF ROCKET OF MASS OF 19 Mg,  
INCLUDING 17.8 Mg OF FUEL.  
FUEL IS CONSUMED AT THE RATE  
OF 225 kg/s AND EJECTED WITH  
A RELATIVE VELOCITY OF 3600 m/s.

FIND:

MAXIMUM SPEED OF SPACECRAFT IF  
ROCKET IS FIRED VERTICALLY FROM  
THE GROUND.

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \quad (1)$$

DATA:

$$u = 3600 \text{ m/s}, \quad q = 225 \text{ kg/s}, \quad m_{\text{fuel}} = 17,800 \text{ kg}$$

$$m_0 = 19,000 \text{ kg} + 540 \text{ kg} = 19,540 \text{ kg}$$

$$\text{WE HAVE } m_{\text{fuel}} = qt, \quad 17,800 \text{ kg} = (225 \text{ kg/s})t$$

$$t = \frac{17,800 \text{ kg}}{225 \text{ kg/s}} = 79.111 \text{ s}$$

MAX. VELOCITY IS REACHED WHEN ALL FUEL HAS  
BEEN CONSUMED, THAT IS, WHEN  $qt = m_{\text{fuel}}$ . EQ. (1) YIELDS

$$v_m = u \ln \frac{m_0}{m_0 - m_{\text{fuel}}} - gt$$

$$= (3600 \text{ m/s}) \ln \frac{19,540}{19,540 - 17,800} - (9.81 \text{ m/s}^2)(79.111 \text{ s})$$

$$= (3600 \text{ m/s}) \ln 11.230 - 776.1 \text{ m/s} = 7430.8 \text{ m/s}$$

$$v_m = 7430 \text{ m/s}$$

14.98

GIVEN:

A 540-kg SPACECRAFT IS MOUNTED ON A TWO-STAGE ROCKET.  
 EACH STAGE HAS A MASS OF 9.5 Mg, INCLUDING 8.9 Mg OF FUEL.  
 FUEL IS CONSUMED AT A RATE OF 225 kg/s AND EJECTED WITH A RELATIVE VELOCITY OF 3600 m/s.  
 AS STAGE A EXPELS ITS LAST PARTICLE OF FUEL, ITS CASING IS JETTISONED.

FIND:

- (a) SPEED OF ROCKET AT THAT INSTANT.  
 (b) MAXIMUM SPEED OF SPACECRAFT

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - q t} - g t \quad (1)$$

(a) FIRST STAGE

$u = 3600 \text{ m/s}$ ,  $q = 225 \text{ kg/s}$ , MASS OF FUEL =  $m_f = 8900 \text{ kg}$   
 $m_0 = 2(9500 \text{ kg}) + 540 \text{ kg} = 19540 \text{ kg}$   
 WE HAVE  $m_f = q t_1$ ,  $t_1 = \frac{m_f}{q} = \frac{8900 \text{ kg}}{225 \text{ kg/s}}$ ,  $t_1 = 39.556 \text{ s}$

SUBSTITUTE INTO (1):

$$v_1 = (3600 \text{ m/s}) \ln \frac{19540}{19540 - 8900} - (9.81 \text{ m/s}^2)(39.556 \text{ s})$$

$$= (3600 \text{ m/s}) \ln 1.8365 - 388.04 \text{ m/s} = 1800.3 \text{ m/s}$$

$$v_1 = 1800 \text{ m/s}$$

(b) SECOND STAGE

$u = 3600 \text{ m/s}$ ,  $q = 225 \text{ kg/s}$ , MASS OF FUEL =  $m_f = 8900 \text{ kg}$   
 $m_1 = 9500 \text{ kg} + 540 \text{ kg} = 10040 \text{ kg}$   
 $m_f = q t_2$ ,  $t_2 = \frac{m_f}{q} = \frac{8900 \text{ kg}}{225 \text{ kg/s}}$ ,  $t_2 = 39.556 \text{ s}$

REPLACING  $v$  BY  $v_2 - v_1$  AND  $m_0$  BY  $m_1$  IN EQ. (1):

$$v_2 - v_1 = u \ln \frac{m_1}{m_1 - q t_2} - g t_2$$

$$= (3600 \text{ m/s}) \ln \frac{10040}{10040 - 8900} - (9.81 \text{ m/s}^2)(39.556 \text{ s})$$

$$= (3600 \text{ m/s}) \ln 1.8070 - 388.04 \text{ m/s} = 744.4 \text{ m/s}$$

$$v_2 = v_1 + 744.4 = 1800 + 744.4 = 2544.4 \text{ m/s}$$

$$v_2 = 2544 \text{ m/s}$$

14.99

GIVEN: SPACECRAFT OF PROB. 14.97.

FIND: ALTITUDE REACHED WHEN ALL THE FUEL OF THE LAUNCHING ROCKET IS CONSUMED.

WE RECALL DATA FROM PROB. 14.97 AND EQ. (1). SETTING  $v = dy/dt$ , WE HAVE

$$dy = (u \ln \frac{m_0}{m_0 - q t} - g t) dt \quad (1')$$

$$\int_0^h dy = \int_0^t (u \ln \frac{m_0}{m_0 - q t} - g t) dt = u \int_0^t \ln \frac{m_0}{m_0 - q t} dt - \frac{1}{2} g t^2$$

SETTING  $\frac{m_0 - q t}{m_0} = z$ , WE FIND THAT

$$u \int_0^t \ln \frac{m_0}{m_0 - q t} dt = u \int_1^{\frac{m_0}{m_0 - q t}} \ln z \left( -\frac{m_0}{q z} dz \right) = u \frac{m_0}{q} [z \ln z - z + 1]$$

$$\text{THUS: } h = \int_0^h dy = \frac{u m_0}{q} (z \ln z - z + 1) - \frac{1}{2} g t^2$$

$$h = u \left[ t + \frac{m_0 - q t}{q} \ln \frac{m_0 - q t}{m_0} \right] - \frac{1}{2} g t^2 \quad (2)$$

GIVEN DATA:  $u = 3600 \text{ m/s}$ ,  $q = 225 \text{ kg/s}$ ,  $m_0 = 19540 \text{ kg}$   
 $t = 79.111 \text{ s}$ ,  $q t = m_f = 17800 \text{ kg}$ ,  $m_1 - q t = 19540 - 17800 = 1740 \text{ kg}$   
 $h = 3600 \left[ 79.111 + \frac{1740}{225} \ln \frac{1740}{19540} \right] - \frac{1}{2} (9.81) (79.111)^2$   
 $= 3600 (79.111 - 18.704) - 30698 = 186770 \text{ m}$   
 $h = 186.8 \text{ km}$

[NOTE THAT  $g$  WAS ASSUMED CONSTANT]

14.100

GIVEN: SPACECRAFT AND TWO-STAGE

LAUNCHING ROCKET OF PROB. 14.98.

FIND ALTITUDE AT WHICH

(a) STAGE A IS RELEASED.

(b) FUEL OF BOTH STAGES HAS BEEN CONSUMED.

SEE SOLUTIONS OF SAMPLE PROB. 14.8 AND PROB. 14.99 FOR DERIVATION OF EQ. (2):

$$h = u \left[ t + \frac{m_0 - q t}{q} \ln \frac{m_0 - q t}{m_0} \right] - \frac{1}{2} g t^2 \quad (2)$$

(a) FIRST STAGE

FROM PROB. 14.98 WE HAVE

$u = 3600 \text{ m/s}$ ,  $q = 225 \text{ kg/s}$ ,  $m_0 = 19540 \text{ kg}$ ,  $t_1 = 39.556 \text{ s}$   
 $q t_1 = m_f = 8900 \text{ kg}$ ,  $m_0 - q t_1 = 19540 - 8900 = 10640 \text{ kg}$

EQ. (2) YIELDS

$$h_1 = (3600) \left[ 39.556 + \frac{10640}{225} \ln \frac{10640}{19540} \right] - \frac{1}{2} (9.81) (39.556)^2$$

$$= (3600) (39.556 - 28.744) - 76747 = 31248 \text{ m}$$

$$h_1 = 31.2 \text{ km}$$

(b) SECOND STAGE

USING AGAIN EQ. (2) AND ADDING  $h_1$  AND  $v_1 t_2$  TO IT,

$$h_2 = h_1 + v_1 t_2 + u \left[ t_2 + \frac{m_1 - q t_2}{q} \ln \frac{m_1 - q t_2}{m_1} \right] - \frac{1}{2} g t_2^2 \quad (3)$$

FROM PROB. 14.98, WE HAVE

$v_1 = 1800.3 \text{ m/s}$ ,  $u = 3600 \text{ m/s}$ ,  $q = 225 \text{ kg/s}$ ,  $t_2 = 39.556 \text{ s}$   
 $m_1 = 10040 \text{ kg}$ ,  $q t_2 = m_f = 8900 \text{ kg}$ ,  $m_1 - q t_2 = 1140 \text{ kg}$

EQ. (3) YIELDS

$$h_2 = 31248 + (1800.3)(39.556) + 3600 \left[ 39.556 + \frac{1140}{225} \ln \frac{1140}{10040} \right] - \frac{1}{2} (9.81) (39.556)^2$$

$$h_2 = 31248 + 71213 + 3600 (39.556 - 11.023) - 7675$$

$$= 197500 \text{ m}$$

$$h_2 = 197.5 \text{ km}$$

14.101

GIVEN:

COMMUNICATION SATELLITE OF PROB. 14.95

FUEL CONSUMED AT RATE OF 37.5 lb/s.

FIND: DISTANCE FROM SATELLITE TO SHUTTLE AT  $t = 60 \text{ s}$ .

SEE SOLUTION OF PROB. 14.95 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - q t} \quad (1)$$

SETTING  $v = dx/dt$ , WE HAVE

$$dx = (u \ln \frac{m_0}{m_0 - q t}) dt \quad (1')$$

$$x = \int_0^t (u \ln \frac{m_0}{m_0 - q t}) dt = -u \int_1^{\frac{m_0}{m_0 - q t}} \ln z \frac{m_0}{q z} dz$$

SETTING  $\frac{m_0 - q t}{m_0} = z$  WE HAVE  $dt = -\frac{m_0}{q} dz$  AND

$$x = \frac{m_0 u}{q} \int_1^{\frac{m_0}{m_0 - q t}} \ln z dz = \frac{m_0 u}{q} [z \ln z - z]_1^{\frac{m_0}{m_0 - q t}} = \frac{m_0 u}{q} (z \ln z - z + 1)$$

$$= \frac{m_0 u}{q} \left( \frac{m_0 - q t}{m_0} \ln \frac{m_0 - q t}{m_0} - 1 + \frac{q t}{m_0} + 1 \right)$$

$$x = u \left( t + \frac{m_0 - q t}{q} \ln \frac{m_0 - q t}{m_0} \right) \quad (2)$$

GIVEN DATA:  $q = (37.5 \text{ lb/s})/g$ ,  $t = 60 \text{ s}$ ,

AND FROM PROB. 14.95:

 $u = 13,750 \text{ ft/s}$ ,  $m_0 = (10,000 \text{ lb})/g$ THUS:  $m_0 - q t = (10,000/g) - (37.5/g)(60) = (7750 \text{ lb})/g$ 

AFTER SUBSTITUTION, EQ. (2) YIELDS

$$x = (13,750 \text{ ft/s}) \left( 60 + \frac{7750}{37.5} \ln \frac{7750}{10000} \right) \text{ s}$$

$$= (13,750) (60 - 52.678) = 100,680 \text{ ft}$$

$$= (100,680 \text{ ft}) \frac{1 \text{ mi}}{5280 \text{ ft}} = 19.068 \text{ mi}$$

 $x = 19.07 \text{ mi}$



14.102

GIVEN:

ROCKET OF PROB. 14.93.

FIND: (a) ALTITUDE AT WHICH ALL FUEL IS CONSUMED.  
(b) VELOCITY OF ROCKET AT THAT TIME.

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - q t} - g t \quad (1)$$

AND SOLUTION OF PROB. 14.99 FOR DERIVATION OF

$$h = u \left[ t + \frac{m_0 - q t}{q} \ln \frac{m_0}{m_0 - q t} \right] - \frac{1}{2} g t^2 \quad (2)$$

FROM STATEMENT OF PROB. 14.93, WE RECALL

$$u = 12,000 \text{ ft/s}, m_0 = (2400 \text{ lb})/g, q t = m_f = (2000 \text{ lb})/g$$

$$q = (25 \text{ lb/s})/g \quad t = \frac{m_f}{q} = \frac{2000 \text{ lb}}{25 \text{ lb/s}} = 80 \text{ s}$$

(a) ALTITUDE AT WHICH ALL FUEL IS CONSUMED

SUBSTITUTING DATA IN EQ. (2):

$$h = (12,000 \text{ ft/s}) \left[ 80 + \frac{2400 - 2000}{25} \ln \frac{2400}{2400 - 2000} \right] - \frac{1}{2} (32.2 \text{ ft/s}^2) (80 \text{ s})^2$$

$$h = (12,000) (80 - 20.668) - 103,040 = 512,944 \text{ ft}$$

$$h = \frac{512,944}{5280} = 97.148 \text{ mi} \quad h = 97.1 \text{ mi}$$

(b) VELOCITY OF ROCKET AT THAT TIME

SUBSTITUTING DATA IN EQ. (1):

$$v = (12,000 \text{ ft/s}) \ln \frac{2400}{2400 - 2000} - (32.2 \text{ ft/s}^2) (80 \text{ s})$$

$$= 12,000 \ln 6 - 2576 = 18,925 \text{ ft/s}$$

$$v = 18,930 \text{ ft/s}$$

14.103

GIVEN:

JET AIRPLANE WITH

 $v$  = SPEED OF AIRPLANE $u$  = RELATIVE SPEED OF EXPELLED GASESSHOW THAT MECHANICAL EFFICIENCY IS  $\eta = \frac{2v}{u+v}$ EXPLAIN WHY  $\eta = 1$  WHEN  $u = v$ .THRUST  $P$  IS OBTAINED FROM EQ. (14.39):

$$\Sigma F = \frac{dm}{dt} (v - v_A) \quad \text{WHERE } v_A = v = \text{AIRPLANE SPEED}$$

$$v_2 = u = \text{EXHAUST VEL. REL. TO PLANE}$$

$$\text{THUS: } F = \frac{dm}{dt} (u - v)$$

$$\text{USEFUL POWER} = Fv = \frac{dm}{dt} (u - v) v$$

WASTED POWER = K.E. IMPARTED PER SECOND TO EXHAUST GASES WHOSE ABSOLUTE VEL. IS  $u - v$ .

$$= \frac{1}{2} \frac{dm}{dt} (u - v)^2$$

TOTAL POWER = USEFUL POWER + WASTED POWER

$$= \frac{dm}{dt} [(u - v) v + \frac{1}{2} (u - v)^2] = \frac{dm}{dt} (u^2 - v^2 + \frac{1}{2} u^2 - \frac{1}{2} v^2 - uv)$$

$$= \frac{1}{2} \frac{dm}{dt} (u^2 - v^2) = \frac{1}{2} \frac{dm}{dt} (u + v)(u - v)$$

$$\text{EFFICIENCY} = \eta = \frac{\text{USEFUL POWER}}{\text{TOTAL POWER}} = \frac{(u - v) v}{\frac{1}{2} (u + v)(u - v)}$$

$$\eta = \frac{2v}{u + v} \quad (\text{Q.E.D.})$$

WHEN  $u = v$ , THE ABSOLUTE VELOCITY  $u - v$  OF THE EXPELLED GASES IS ZERO. THUS, NO ENERGY IS IMPARTED TO THE EXPELLED GASES AND NO POWER IS WASTED.

14.104

GIVEN:

ROCKET WITH SPEED  $v$ , EXPELLING FUEL WITH RELATIVE SPEED  $u$ .

SHOW THAT MECHANICAL EFFICIENCY IS  $\eta = 2uv/(u^2 + v^2)$ .  
EXPLAIN WHY  $\eta = 1$  WHEN  $u = v$ .

WE RECALL EQ. (14.44) FOR THRUST  $P$  OF ROCKET:

$$P = \frac{dm}{dt} u$$

$$\text{USEFUL POWER} = P v = \frac{dm}{dt} u v$$

WASTED POWER = K.E. ENERGY IMPARTED PER SECOND TO EXPELLED FUEL WHOSE ABSOLUTE VELOCITY IS  $u - v$ .

$$= \frac{1}{2} \frac{dm}{dt} (u - v)^2$$

TOTAL POWER = USEFUL POWER + WASTED POWER

$$= \frac{dm}{dt} u v + \frac{1}{2} \frac{dm}{dt} (u - v)^2$$

$$= \frac{1}{2} \frac{dm}{dt} (2uv + u^2 + v^2 - 2uv)$$

$$= \frac{1}{2} \frac{dm}{dt} (u^2 + v^2)$$

$$\text{EFFICIENCY} = \eta = \frac{\text{USEFUL POWER}}{\text{TOTAL POWER}} = \frac{\frac{dm}{dt} u v}{\frac{1}{2} \frac{dm}{dt} (u^2 + v^2)}$$

$$\eta = \frac{2uv}{u^2 + v^2} \quad (\text{Q.E.D.})$$

WHEN  $u = v$ , THE ABSOLUTE VELOCITY  $u - v$  OF THE EXPELLED FUEL IS ZERO. THUS, NO ENERGY IS IMPARTED TO THE EXPELLED FUEL AND NO POWER IS WASTED.

14.105

GIVEN:

30-g BULLET FIRED WITH  $v_0 = 480 \text{ m/s}$  INTO 5-kg BLOCK A, WHICH RESTS ON 4-kg CART C.  $\mu_k = 0.50$  BETWEEN BLOCK A AND CART C.

FIND (a) FINAL VELOCITY  $v_f$  OF CART AND BLOCK,  
(b) FINAL POSITION OF BLOCK ON CART.

CONSERVATION OF LINEAR MOMENTUM

$$m_0 v_0 = (m_0 + m_A) v' = (m_0 + m_A + m_C) v_f$$

BULLET FIRED      JUST AFTER IMPACT      BLOCK HAS STRIPPED SLIDING

$$m_0 v_0 = (m_0 + m_A) v' = (m_0 + m_A + m_C) v_f$$

$$(0.030 \text{ kg})(480 \text{ m/s}) = (5.030 \text{ kg}) v' = (9.030 \text{ kg}) v_f$$

$$v' = \frac{0.030}{5.030} (480 \text{ m/s}) = 2.863 \text{ m/s}$$

$$v_f = \frac{0.030}{9.030} (480 \text{ m/s}) = 1.5947 \text{ m/s}$$

$$(a) \text{ ANSWER IS } v_f = 1.595 \text{ m/s}$$

(b) WORK-ENERGY PRINCIPLE

JUST AFTER IMPACT:

$$T' = \frac{1}{2} (m_0 + m_A) v'^2 = \frac{1}{2} (5.030 \text{ kg})(2.863 \text{ m/s})^2 = 20.615 \text{ J}$$

FINAL KINETIC ENERGY:

$$T_f = \frac{1}{2} (m_0 + m_A + m_C) v_f^2 = \frac{1}{2} (9.030 \text{ kg})(1.5947 \text{ m/s})^2 = 11.482 \text{ J}$$

WORK OF FRICTION FORCE:

$$F = \mu_k N = \mu_k (m_0 + m_A) g = 0.50 (5.030)(9.81) = 24.672 \text{ N}$$

$$\text{WORK} = U = -F x = -24.672 x$$

$$T_f + U = T': 11.482 - 24.672 x = 20.615 \quad x = 0.370 \text{ m}$$



14.106



GIVEN: 80-Mg ENGINE A WITH  $v_0 = 6.5 \text{ km/h}$  STRIKES 20-Mg FLATCAR C WHICH IS AT REST AND CARRIES 30-Mg LOAD B. A AND C ARE COUPLED UPON IMPACT.

B CAN SLIDE ON C WITH  $\mu_k = 0.25$ .

FIND: VELOCITY OF CAR C

(a) IMMEDIATELY AFTER IMPACT

(b) AFTER B HAS SLID TO A STOP RELATIVE TO C.

CONSERVATION OF LINEAR MOMENTUM

FIRST NOTE THAT  $\epsilon$  WILL NOT HAVE DURING COUPLING OF A AND C, SINCE THE FRICTION FORCE EXERTED ON B BY C IS NONIMPULSIVE:  $F \Delta t = \mu_k N \Delta t \approx 0$ .

$$m_A v_0 = (m_A + m_C) v' = (m_A + m_C + m_B) v_f$$

(v\_B = 0)

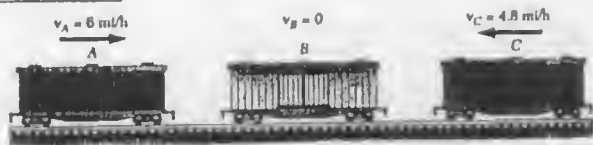
$$m_A v_0 = (m_A + m_C) v' = (m_A + m_C + m_B) v_f$$

$$(80 \text{ Mg})(6.5 \text{ km/h}) = (100 \text{ Mg}) v' = (130 \text{ Mg}) v_f$$

$$(a) v' = \frac{80}{100} (6.5 \text{ km/h}) \quad v' = 5.20 \text{ km/h}$$

$$(b) v_f = \frac{80}{130} (6.5 \text{ km/h}) \quad v_f = 4.00 \text{ km/h}$$

14.107



GIVEN: THREE IDENTICAL CARS WITH VELOCITIES SHOWN. CAR B IS FIRST HIT BY CAR A.

FIND: FINAL VELOCITY OF EACH CAR IF

(a) ALL CARS GET AUTOMATICALLY COUPLED,

(b) A AND B GET COUPLED, BUT B AND C BOUNCE OFF EACH OTHER WITH  $\epsilon = 1$  (I.E. NO ENERGY LOSS).

(a) ALL CARS AUTOMATICALLY COUPLED

CONSERVATION OF LINEAR MOMENTUM:

$$m_A v_A + m_B v_B + m_C v_C = (m_A + m_B + m_C) v_f$$

$$m(6 \text{ mi/h}) + 0 - m(4.8 \text{ mi/h}) = (3m) v_f$$

$$v_f = \frac{6 - 4.8}{3} = +0.4 \quad v_f = 0.400 \text{ mi/h}$$

(b) CARS A AND B ONLY GET COUPLED

CONSERVATION OF LINEAR MOMENTUM FOR A AND B:

$$m(6 \text{ mi/h}) + m(0) = (2m) v'$$

$$v' = 3 \text{ mi/h}$$

$$v' = 3 \text{ mi/h}$$

$$v' = 3 \text{ mi/h}$$

$$v' = 3 \text{ mi/h}$$

$$v' = 3 \text{ mi/h}$$

CONS. OF LINEAR MOMENTUM:

$$2m(3) - m(4.8) = 2m v'' + m v_C'$$

$$2v'' + v_C' = 1.2 \text{ mi/h} \quad (1)$$

(CONTINUED)

14.107 continued

CONSERVATION OF ENERGY ( $\epsilon = 1$ ):

RELATIVE VELOCITY AFTER AND BEFORE IMPACT ARE EQUAL:

$$v_C' - v'' = (3 + 4.8) \text{ mi/h} \quad (2)$$

SUBTRACTING (2) FROM (1):

$$3v'' = 1.2 - 7.8 \quad v'' = -2.20 \text{ mi/h}$$

THUS:

$$v_A' = v_B' = 2.20 \text{ mi/h}$$

SUBSTITUTING  $v'' = -2.20 \text{ mi/h}$  IN (1):

$$2(-2.20 \text{ mi/h}) + v_C' = 1.2 \text{ mi/h}$$

$$v_C' = +5.60 \text{ mi/h}$$

$$v_C' = 5.60 \text{ mi/h}$$

14.108

GIVEN:

9000-lb HELICOPTER A IS TRAVELING DUE EAST AT 75 mi/h AT ALTITUDE OF 2500 ft WHEN IT IS HIT BY 12,000-lb HELICOPTER B. THEIR ENTANGLED WRECKAGE FALLS TO THE GROUND IN 12 s AT POINT LOCATED 1500 ft EAST AND 384 ft SOUTH OF POINT OF IMPACT.

FIND: VELOCITY COMPONENTS OF HELICOPTER B JUST BEFORE COLLISION. (NEGLECT AIR RESISTANCE.)

VELOCITY OF WRECKAGE IMMEDIATELY AFTER COLLISION

$$v' = v_x' \hat{i} + v_y' \hat{j} + v_z' \hat{k}$$

BUT:

$$x = v_x' t \quad v_x' = \frac{x}{t} = \frac{1500 \text{ ft}}{12 \text{ s}} = 125 \text{ ft/s}$$

$$z = v_z' t \quad v_z' = \frac{z}{t} = \frac{384 \text{ ft}}{12 \text{ s}} = 32 \text{ ft/s}$$

$$-h = v_y' t - \frac{1}{2} g t^2 \quad v_y' = -\frac{h}{t} + \frac{1}{2} g t$$

$$= -\frac{2500 \text{ ft}}{12 \text{ s}} + \frac{1}{2} (32.2 \text{ ft/s}^2)(12 \text{ s})$$

$$v_y' = -15.133 \text{ ft/s}$$

$$\text{THUS: } v' = (125 \text{ ft/s}) \hat{i} - (15.133 \text{ ft/s}) \hat{j} + (32 \text{ ft/s}) \hat{k}$$

IMPACT: CONSERVATION OF LINEAR MOMENTUM

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

AFTER SUBSTITUTING DATA AND EXPRESSION FOUND FOR  $v'$ , AND NOTING THAT  $v_A = 75 \text{ mi/h} = 110 \text{ ft/s}$ ,

$$\frac{9000 \text{ lb}}{g} (110 \text{ ft/s}) \hat{i} + \frac{12,000 \text{ lb}}{g} v_B = \frac{21,000 \text{ lb}}{g} [(125 \text{ ft/s}) \hat{i} - (15.133 \text{ ft/s}) \hat{j} + (32 \text{ ft/s}) \hat{k}]$$

SOLVING FOR  $v_B$ :

$$v_B = 1.75 [(125 \text{ ft/s}) \hat{i} - (15.133 \text{ ft/s}) \hat{j} + (32 \text{ ft/s}) \hat{k}] - (82.5 \text{ ft/s}) \hat{i}$$

IT FOLLOWS THAT

$$(v_B)_x = 1.75(125 \text{ ft/s}) - 82.5 \text{ ft/s} = 136.25 \text{ ft/s} = 92.90 \text{ mi/h}$$

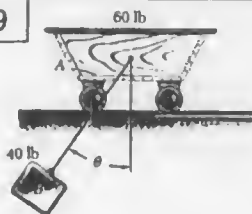
$$(v_B)_y = -1.75(15.133 \text{ ft/s}) = -26.48 \text{ ft/s}$$

$$(v_B)_z = 1.75(32 \text{ ft/s}) = 56.0 \text{ ft/s} = 38.18 \text{ mi/h}$$

ANSWER:

$$92.9 \text{ mi/h EAST, } 38.2 \text{ mi/h SOUTH, } 26.5 \text{ ft/s DOWN}$$

14.109



**GIVEN:**  
BLOCK B IS SUSPENDED FROM 6-FT CORD ATTACHED TO CART A. SYSTEM IS RELEASED FROM REST WHEN  $\theta = 35^\circ$ .  
**FIND:**  
VELOCITIES OF A AND B WHEN  $\theta = 0$ .

## CONSERVATION OF LINEAR MOMENTUM

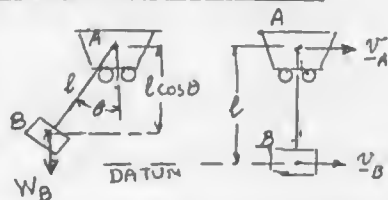
$$0 = m_A \vec{v}_A + m_B \vec{v}_B \quad \text{or} \quad 0 = m_A \vec{v}_A + m_B \vec{v}_B$$

$\pm x \text{ comp:}$

$$m_A \vec{v}_A + m_B \vec{v}_B = 0$$

$$\vec{v}_A = -\frac{m_B}{m_A} \vec{v}_B \quad (1)$$

## CONSERVATION OF ENERGY



INITIALLY:  $T_0 = 0$   $V_0 = W_B l(1 - \cos \theta) = m_B g l(1 - \cos \theta)$

AS B PASSES UNDER A:

$$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \quad V = 0$$

$$T_0 + V_0 = T + V$$

$$m_B g l(1 - \cos \theta) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1 - \cos \theta)$$

SUBSTITUTING FOR  $v_A$  FROM (1):

$$m_A \left(\frac{m_B}{m_A}\right)^2 v_B^2 + m_B v_B^2 = 2 m_B g l(1 - \cos \theta)$$

$$\frac{m_B}{m_A} (m_A + m_B) v_B^2 = 2 m_B g l(1 - \cos \theta)$$

$$\frac{m_A + m_B}{m_A} v_B^2 = 2 g l(1 - \cos \theta)$$

$$v_B = \sqrt{\frac{2 m_A}{m_A + m_B} g l(1 - \cos \theta)}$$

GIVEN DATA:

$$\frac{m_A}{m_A + m_B} = \frac{W_A}{W_A + W_B} = \frac{60 \text{ lb}}{60 \text{ lb} + 40 \text{ lb}} = 0.6$$

$$l = 6 \text{ ft}, \quad \theta = 35^\circ$$

$$v_B = \sqrt{2(0.6)(32.2 \text{ ft/s}^2)(6 \text{ ft})(1 - \cos 35^\circ)} = 6.4752 \text{ ft/s}$$

CARRYING THIS VALUE INTO (1):

$$v_A = -\frac{m_B}{m_A} v_B = -\frac{W_B}{W_A} v_B = -\frac{40 \text{ lb}}{60 \text{ lb}} (6.4752 \text{ ft/s})$$

$$= -4.3168 \text{ ft/s}$$

ANSWER:

$$\vec{v}_A = 4.32 \text{ ft/s} \leftarrow; \quad \vec{v}_B = 6.48 \text{ ft/s} \rightarrow$$

14.110



GIVEN:

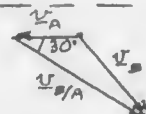
9-KG BLOCK B STARTS FROM REST AND SLIDES DOWN 15-KG WEDGE A.

FIND:

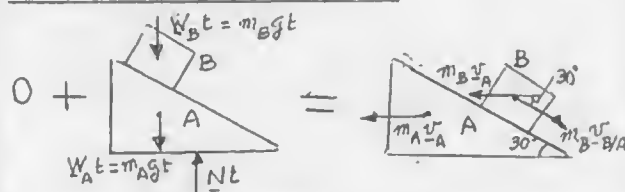
(a) VELOCITY OF B RELATIVE TO A AFTER IT HAS SLID 0.6 m

(b) CORRESPONDING VELOCITY OF WEDGE A. (NEGLECT FRICTION.)

WE RESOLVE  $\vec{v}_B$  INTO ITS COMPONENTS:  $\vec{v}_A$  AND  $\vec{v}_{B/A}$   $\angle 30^\circ$



## IMPULSE-MOMENTUM PRINCIPLE



$$\Sigma m \vec{v}_0 + \Sigma \vec{F} t = \Sigma m \vec{v}$$

$\pm x \text{ comp:}$

$$0 + 0 = m_B v_{B/A} \cos 30^\circ - m_A v_A - m_B v_A$$

$$v_A = \frac{m_B \cos 30^\circ}{m_A + m_B} v_{B/A} = \frac{(9 \text{ kg}) \cos 30^\circ}{15 \text{ kg} + 9 \text{ kg}} v_{B/A}$$

$$v_A = 0.32476 v_{B/A} \quad (1)$$

## CONSERVATION OF ENERGY

$$T_0 = 0$$

$$V_0 = m_B g h = (9 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) \sin 30^\circ = 26.487 \text{ J}$$

$$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \quad V = 0$$

REFERRING TO VELOCITY TRIANGLE SHOWN ABOVE AND USING THE LAW OF COSINES:

$$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B (v_A^2 + v_{B/A}^2 - 2 v_A v_{B/A} \cos 30^\circ)$$

RECALLING (1) AND SUBSTITUTING THE GIVEN VALUES:

$$T = \frac{1}{2} (15) (0.32476)^2 v_{B/A}^2 + \frac{1}{2} (9) [(0.32476)^2 + 1 - 2(0.32476) \cos 30^\circ] v_{B/A}^2$$

$$= 0.79102 v_{B/A}^2 + 2.44336 v_{B/A}^2 = 3.2344 v_{B/A}^2$$

$$T + V = T_0 + V_0$$

$$3.2344 v_{B/A}^2 = 26.487 \text{ J}$$

$$v_{B/A} = 2.8617 \text{ m/s}$$

(a)  $\vec{v}_{B/A} = 2.86 \text{ m/s} \angle 30^\circ$

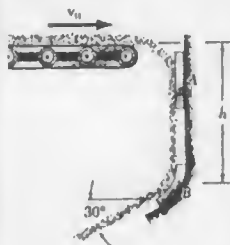
(b) FROM EQ. (1):

$$v_A = 0.32476 (2.8617 \text{ m/s})$$

$$= 0.92936 \text{ m/s}$$

$$\vec{v}_A = 0.929 \text{ m/s} \leftarrow$$

14.111



GIVEN:

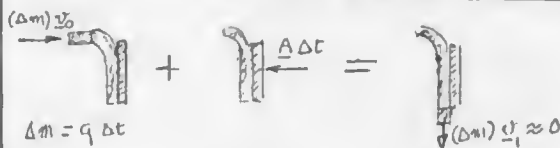
MASS  $q$  OF SAND DISCHARGED PER UNIT TIME FROM CONVEYOR BELT AND DEFLECTED BY PLATE AT A SO THAT IT FALLS IN A VERTICAL STREAM UNTIL IT IS DEFLECTED BY PLATE AT B. FIND FORCE REQUIRED TO HOLD

(A) PLATE A,

(B) PLATE B.

(NEGLECT FRICTION BETWEEN SAND AND PLATES.)

(a) IMPULSE-MOMENTUM PRINCIPLE FOR PLATE A AND SAND



$$\Delta m = q \Delta t$$

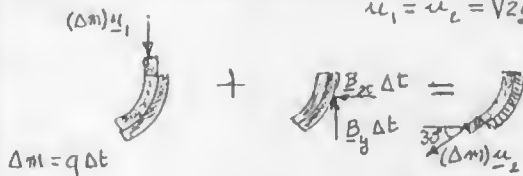
$$\pm x \text{ COMP.}: (\Delta m)v_0 - A\Delta t = 0$$

$$A = \frac{\Delta m}{\Delta t} v_0 = q v_0$$

$$A = q v_0 \leftarrow$$

(b) IMPULSE-MOMENTUM PRINCIPLE FOR PLATE B AND SAND

$$u_1 = u_2 = \sqrt{2gh}$$



$$\Delta m = q \Delta t$$

$$\pm x \text{ COMP.}: 0 - B_x \Delta t = -(\Delta m)u_2 \cos 30^\circ$$

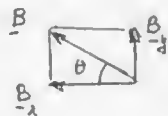
$$B_x = \frac{\Delta m}{\Delta t} u_2 \cos 30^\circ = q \sqrt{2gh} \frac{\sqrt{3}}{2}$$

$$B_x = \frac{1}{2} q \sqrt{6gh}$$

$$\pm y \text{ COMP.}: (\Delta m)u_1 - B_y \Delta t = (\Delta m)u_2 \sin 30^\circ$$

$$B_y = \frac{\Delta m}{\Delta t} (u_1 - u_2 \sin 30^\circ) = q \sqrt{2gh} \left(1 - \frac{1}{2}\right)$$

$$B_y = \frac{1}{2} q \sqrt{2gh}$$



$$B^2 = B_x^2 + B_y^2$$

$$= \left(\frac{q}{2}\right)^2 (6gh + 2gh)$$

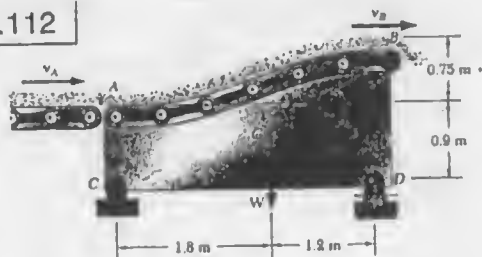
$$= 2q^2 gh$$

$$B = q \sqrt{2gh}$$

$$\tan \theta = \frac{B_y}{B_x} = \frac{\sqrt{2gh}}{\sqrt{6gh}} = \frac{1}{\sqrt{3}}, \quad \theta = 30^\circ$$

$$\underline{B} = q \sqrt{2gh} \nearrow 30^\circ \leftarrow$$

14.112



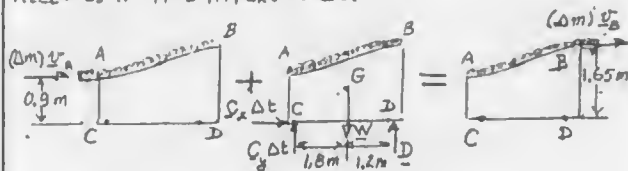
GIVEN:

SAND RECEIVED AT A AND DISCHARGED AT B AT A RATE OF  $100 \text{ kg/s}$  AND WITH  $v_A = v_B = 4.5 \text{ m/s}$ . COMBINED WEIGHT OF COMPONENT AND SAND IT SUPPORTS IS  $W = 4 \text{ kN}$ .

FIND:

REACTIONS AT C AND D.

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE COMPONENT, THE SAND IT SUPPORTS AND THE SAND IT RECEIVES IN THE INTERVAL  $\Delta t$ .



$$\pm x \text{ COMP.}: (\Delta m)v_A + C_x \Delta t = (\Delta m)v_B$$

$$C_x = \frac{\Delta m}{\Delta t} (v_B - v_A) = (100 \text{ kg/s}) (4.5 \text{ m/s} - 4.5 \text{ m/s}) = 0$$

+ MOMENTS ABOUT C:

$$-(\Delta m)v_A(0.9 \text{ m}) - (W\Delta t)(1.8 \text{ m}) + (D\Delta t)(3 \text{ m}) = -(\Delta m)v_B(1.65 \text{ m})$$

$$3D = 1.8W + \frac{\Delta m}{\Delta t} (0.9v_A) - \frac{\Delta m}{\Delta t} (1.65v_B)$$

$$= 1.8(4000 \text{ N}) + 0.9(100 \text{ kg/s})(4.5 \text{ m/s}) - 1.65(100 \text{ kg/s})(4.5 \text{ m/s})$$

$$= 6862.5 \text{ N}$$

$$D = 2287.5 \text{ N}$$

$$\underline{D} = 2.29 \text{ kN} \uparrow \leftarrow$$

+ y COMP.:

$$C_y \Delta t - W \Delta t + D \Delta t = 0$$

$$C_y = W - D = 4000 \text{ N} - 2287.5 \text{ N} = 1712.5 \text{ N}$$

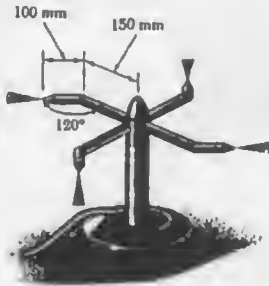
RECALLING THAT  $C_x = 0$ :

$$\underline{C} = 1.712 \text{ kN} \uparrow \leftarrow$$

NOTE. IF COMPONENT WAS STOPPED AND THE SAND WAS NOT MOVING, WE WOULD HAVE

$$\underline{C} = 1.600 \text{ kN} \uparrow, \quad \underline{D} = 2.40 \text{ kN} \uparrow$$

14.113



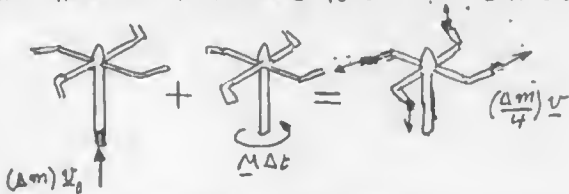
GIVEN:

EACH OF THE FOUR ROTATING ARMS OF SPRINKLER CONSISTS OF TWO STRAIGHT PORTIONS OF PIPE FORMING  $120^\circ$  ANGLE. EACH ARM DISCHARGES WATER AT THE RATE OF 20 L/min WITH RELATIVE VELOCITY OF 18 m/s. FRICTION IS EQUIVALENT TO COUPLE  $M = 0.375 \text{ N}\cdot\text{m}$ .

FIND:

CONSTANT RATE AT WHICH SPRINKLER ROTATES.

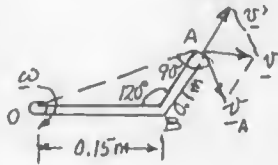
WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE SPRINKLER, THE WATER IT CONTAINS, AND THE MASS  $\Delta m$  OF WATER ENTERING IN INTERVAL  $\Delta t$ .



EQUATING MOMENTS ABOUT AXIS OF ROTATION  $O$ :

$$0 + M\Delta t = 4 [\text{MOMENT OF } (\frac{\Delta m}{4})v]$$

$M\Delta t = \text{MOMENT OF } (\Delta m)v \quad (1)$



THE VELOCITY  $v$  OF THE WATER LEAVING AN ARM IS THE RESULTANT OF THE VELOCITY  $v'$  RELATIVE TO THE ARM AND OF THE VELOCITY  $v_A$  OF NOZZLE:

$$v = v' + v_A$$

WHERE  $v' = 18 \text{ m/s}$  AND  $v_A = (OA)\omega$

BUT APPLYING THE LAW OF COSINES TO TRIANGLE  $OAB$ :

$$\begin{aligned} (OA)^2 &= (OB)^2 + (BA)^2 - 2(OB)(BA)\cos 120^\circ \\ &= (0.15\text{m})^2 + (0.10\text{m})^2 - 2(0.15\text{m})(0.10\text{m})\cos 120^\circ \\ (OA)^2 &= 0.0475 \text{ m}^2 \end{aligned}$$

THEREFORE:

$$\begin{aligned} + \text{J MOM. OF } v \text{ ABOUT } O &= \text{MOM. OF } v' + \text{MOM. OF } v_A \\ &= (0.15\text{m})v'\cos 30^\circ - (OA)(OA)\omega \\ &= (0.15\text{m})(18\text{m/s})\cos 30^\circ - (OA)^2\omega \\ &= 2.3383 \text{ m}^2/\text{s} - (0.0475 \text{ m}^2)\omega \end{aligned}$$

SUBSTITUTING INTO EQ. (1) AND RECALLING THAT  $M = 0.375 \text{ N}\cdot\text{m}$ :

$$(0.375 \text{ N}\cdot\text{m})\Delta t = (\Delta m)[2.3383 \text{ m}^2/\text{s} - (0.0475 \text{ m}^2)\omega]$$

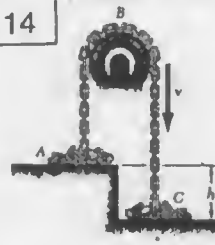
DIVIDING BY  $\Delta t$ , AND NOTING THAT

$$\frac{\Delta m}{\Delta t} = \rho Q = (1 \text{ kg/L})(20 \text{ L/min}) \frac{1 \text{ min}}{60 \text{ s}} = \frac{4}{3} \text{ kg/s}$$

WE HAVE

$$\begin{aligned} 0.375 \text{ N}\cdot\text{m} &= (\frac{4}{3} \text{ kg/s})[2.3383 \text{ m}^2/\text{s} - (0.0475 \text{ m}^2)\omega] \\ 2.3383 \text{ m}^2/\text{s} - (0.0475 \text{ m}^2)\omega &= 0.28125 \text{ m}^2/\text{s} \\ \omega &= 43.306 \text{ rad/s} \quad \omega = 414 \text{ rpm} \end{aligned}$$

14.114



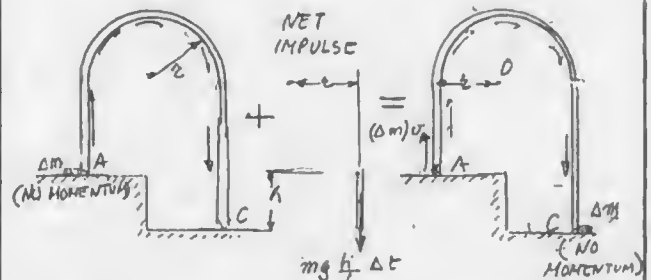
GIVEN:

WHEN GIVEN AN INITIAL SPEED  $v$ , THE CHAIN KEEPS MOVING OVER THE PULLEY.

FIND:

HEIGHT  $h$ , (NEGLECT FRICTION.)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PORTION OF CHAIN OF MASS  $m$  AND LENGTH  $L$  IN MOTION AT TIME  $t$  AND TO THE ELEMENT OF LENGTH  $\Delta z$  AND MASS  $\Delta m = \frac{m}{L}\Delta z$  WHICH WILL BE SET IN MOTION IN THE TIME  $\Delta t$  INTERVAL  $\Delta t$ .



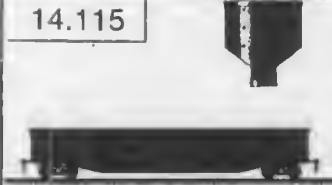
WE NOTE THAT THE ELEMENT AT A ACQUIRES A LINEAR MOMENTUM  $(\Delta m)v$  WHICH IS ADDED TO THE SYSTEM, WHILE THE MOMENTUM OF THE ELEMENT AT C IS LOST TO THE SYSTEM.

EQUATING MOMENTS ABOUT  $O$ :

$$+ \Delta O + (mg \frac{L}{2} \Delta t) \ell = (\Delta m)v \ell$$

$$h = \frac{\Delta z}{\Delta t} \frac{v}{g} = v \frac{v}{g} \quad h = \frac{v^2}{g}$$

14.115



GIVEN:

RAILROAD CAR OF MASS  $m_0$  AND LENGTH  $L$  APPROACHES CHUTE AT SPEED  $v_0$  TO BE LOADED WITH SAND AT RATE  $dm/dt = q$ .

FIND: (a) MASS OF CAR AND LOAD AFTER CAR HAS PASSED, (b) SPEED OF CAR AT THAT TIME.

CONSERVATION OF MOMENTUM IN HORIZONTAL DIRECTION

WE CONSIDER THE CAR AND THE MASS OF SAND  $qt$  WHICH FALLS INTO THE CAR IN THE TIME  $t$ .

$$(qt)v_1$$

$$m_0 v_0 = (m_0 + qt)v$$

$$\pm \text{2 COMP.}: m_0 v_0 = (m_0 + qt)v \quad v = \frac{m_0 v_0}{m_0 + qt} \quad (1)$$

LETTING  $v = \frac{dx}{dt}$  IN (1):

$$dx = \frac{m_0 v_0 dt}{m_0 + qt} \quad x = m_0 v_0 \int_0^t \frac{dt}{m_0 + qt}$$

(CONTINUED)

### 14.115 continued

$$x = \frac{m_0 v_0}{g} [\ln(m_0 + gL)]_0^L = \frac{m_0 v_0}{g} \ln \frac{m_0 + gL}{m_0} \quad (2)$$

USING THE EXPONENTIAL FORM:  $m_0 + gL = m_0 e^{gx/m_0 v_0}$

WHERE  $m_0 + gL$  REPRESENTS THE MASS AT TIME  $L$ , AFTER THE CAR HAS MOVED THROUGH  $x$ .

(a) MAKING  $x = L$  IN (2), WE OBTAIN THE FINAL MASS:

$$m_f = m_0 + gL = m_0 e^{gL/m_0 v_0}$$

(b) MAKING  $L = L_f$  IN (1), WE OBTAIN THE FINAL SPEED:

$$v = \frac{m_0 v_0}{m_0 + gL_f} = \frac{m_0}{m_f} v_0 = v_0 e^{-gL_f/m_0 v_0}$$

### 14.116



GIVEN:

SPACE VEHICLE DESCRIBING CIRCULAR ORBIT ABOUT THE EARTH AT SPEED OF 15,000 mi/h RELEASES AT ITS FRONT END A CAPSULE WITH A GROSS WEIGHT OF 1200 lb, INCLUDING 800 lb OF FUEL, WHICH IS CONSUMED AT THE RATE OF 40 lb/s AND EJECTED WITH RELATIVE VELOCITY OF 9000 ft/s.

FIND:

(a) TANGENTIAL ACCELERATION OF CAPSULE AS IT IS FIRED.

(b) MAX. SPEED ATTAINED BY THE CAPSULE.

FROM EQ. (14.44):

$$\text{THRUST} = P = \frac{dm}{dt} u = \frac{40 \text{ lb/s}}{32.2 \text{ ft/s}^2} (9000 \text{ ft/s}) = \frac{360 \times 10^3 \text{ lb}}{32.2}$$

$$(a) P = m_0 a_t: \frac{360 \times 10^3 \text{ lb}}{32.2} = \frac{1200 \text{ lb}}{32.2 \text{ ft/s}^2} a_t$$

$$a_t = \frac{360 \times 10^3 \text{ lb}}{1.2 \times 10^3} \text{ ft/s}^2 \quad a_t = 300 \text{ ft/s}^2$$

(b) MAX. SPEED OF CAPSULE RELATIVE TO SPACE VEHICLE IS OBTAINED FROM EXPRESSION DERIVED IN PROB. 14.95, OR FROM EXPRESSION OBTAINED IN SAMPLE PROB. 14.8 BY OMITTING THE TERM DUE TO GRAVITY.

$$v_{c/v} = u \ln \frac{m_0}{m_0 - qL}$$

WHERE  $u = (9000 \text{ ft/s})$

$$m_0 = \frac{1200 \text{ lb}}{g}, \quad m_0 - qL = \frac{1200 \text{ lb} - 800 \text{ lb}}{g} = \frac{400 \text{ lb}}{g}$$

$$\frac{m_0}{m_0 - qL} = \frac{1200}{400} = 3$$

THUS:

$$v_{c/v} = (9000 \text{ ft/s}) 2.197 = (9000 \text{ ft/s})(1.0986) = 9887.5 \text{ ft/s} = 6741 \text{ mi/h}$$

$$v_c = v_v + v_{c/v} = 15,000 \text{ mi/h} + 6741 \text{ mi/h} = 21,741.5 \text{ mi/h}$$

$$v_c = 21,700 \text{ mi/h}$$

### 14.C1



GIVEN:

WOMAN OF WEIGHT  $W_w$  STANDS READY TO DIVE WITH VELOCITY  $v_w$  RELATIVE TO BOAT OF WEIGHT  $W_b$ .

MAN OF WEIGHT  $W_m$  READY TO DIVE FROM OTHER END OF BOAT WITH RELATIVE VELOCITY  $v_m$ .

FIND:

VELOCITY OF BOAT AFTER BOTH SWIMMERS HAVE DIVED

IF (a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST

USE  $W_w = 120 \text{ lb}$ ,  $W_m = 180 \text{ lb}$ ,  $W_b = 300 \text{ lb}$ , AND

(PROB. 14.4):  $v_w = v_m = 16 \text{ ft/s}$

(i)  $v_w = 14 \text{ ft/s}$ ,  $v_m = 18 \text{ ft/s}$

(ii)  $v_w = 18 \text{ ft/s}$ ,  $v_m = 14 \text{ ft/s}$

ANALYSIS

(a) WOMAN DIVES FIRST:

$v'_b$  = VEL. OF BOAT AFTER WOMAN DIVES

$v_b$  = VEL. OF BOAT AFTER BOTH SWIMMERS HAVE DIVED

CONSERVATION OF MOMENTUM:

$$0 = \frac{W_w(v_w - v'_b)}{g} + \frac{(W_b + W_m)v'_b}{g} \quad v'_b = \frac{W_w v_w}{W_w + W_b + W_m} \quad (1)$$

$$\frac{(W_b + W_m)v'_b}{g} = \frac{W_b v_b}{g} + \frac{W_m(v_m + v_b)}{g}$$

$$(W_b + W_m)v'_b = W_b v_b + W_m(v_m + v_b) \quad v_b = v'_b - \frac{W_m v_m}{W_b + W_m}$$

SUBSTITUTING FOR  $v'_b$  FROM (1):

$$v_b = \frac{W_w v_w}{W_w + W_b + W_m} - \frac{W_m v_m}{W_b + W_m} \quad (2)$$

(b) MAN DIVES FIRST:

INTERCHANGE SUB  $w$  AND SUB  $m$  IN (2) AND CHANGE ALL SIGNS

$$v_b = -\frac{W_m v_m}{W_m + W_b + W_w} + \frac{W_w v_w}{W_b + W_w} \quad (3)$$

OUTLINE OF PROGRAM

INPUT  $W_w$ ,  $W_m$ ,  $W_b$ ,  $v_w$ ,  $v_m$ , AND EQS. (2) AND (3).

PROGRAM OUTPUT

PROB. 14.1

(a) Woman dives first  
Velocity of boat = -2.800  
(b) Man dives first  
Velocity of boat = -0.229

(i)

(a) Woman dives first  
Velocity of boat = -3.950  
(b) Man dives first  
Velocity of boat = -1.400

(ii)

(a) Woman dives first  
Velocity of boat = -1.650  
(b) Man dives first  
Velocity of boat = 0.943

## 14.C2

## GIVEN:

SYSTEM OF  $n$  PARTICLES  $A_i$  OF MASS  $m_i$ ,  
COORDINATES  $x_i, y_i, z_i$ , WITH VELOCITIES OF COMPONENTS  
( $v_x$ )<sub>i</sub>, ( $v_y$ )<sub>i</sub>, ( $v_z$ )<sub>i</sub>.

## FIND:

COMPONENTS OF ANGULAR MOMENTUM OF SYSTEM ABOUT  
ORIGIN O. USE PROGRAM TO SOLVE PROBS. 14.9  
AND 14.13.

## ANALYSIS

$$H_o = \sum_{i=1}^n r_i \times m_i v_i = \sum_{i=1}^n m_i \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_i & y_i & z_i \\ (v_x)_i & (v_y)_i & (v_z)_i \end{vmatrix}$$

$$H_x = \sum_{i=1}^n m_i [y_i(v_z)_i - z_i(v_y)_i] \quad (1)$$

$$H_y = \sum_{i=1}^n m_i [z_i(v_x)_i - x_i(v_z)_i] \quad (2)$$

$$H_z = \sum_{i=1}^n m_i [x_i(v_y)_i - y_i(v_x)_i] \quad (3)$$

## OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER AND SYSTEM OF UNITS USED  
IF SI UNITS, ENTER FOR  $i=1$  TO  $i=n$ :

$m_i$  (kg);  $x_i, y_i, z_i$  (m); ( $v_x$ )<sub>i</sub>, ( $v_y$ )<sub>i</sub>, ( $v_z$ )<sub>i</sub> (m/s)

IF U.S. CUSTOMARY UNITS, ENTER FOR  $i=1$  TO  $i=n$ :

$W_i$  (lb);  $x_i, y_i, z_i$  (ft); ( $v_x$ )<sub>i</sub>, ( $v_y$ )<sub>i</sub>, ( $v_z$ )<sub>i</sub> (ft/s)

AND COMPUTE  $m_i = W_i/32.2$

COMPUTE THE SUMS (1), (2), AND (3).

PRINT PROBLEM NUMBER

PRINT VALUES OBTAINED FOR  $H_x, H_y, H_z$ .

IF SI UNITS, RESULTS ARE EXPRESSED IN kg·m<sup>2</sup>/s.

IF U.S. CUSTOMARY UNITS, RESULTS ARE  
EXPRESSED IN ft·lb·s.

## PROGRAM OUTPUT

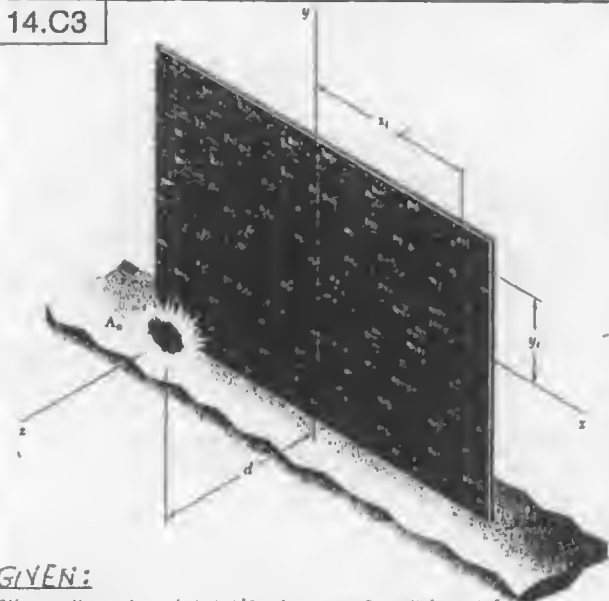
## Problem 14.09

Hx = -31.2 kg·m<sup>2</sup>/s  
Hy = -64.8 kg·m<sup>2</sup>/s  
Hz = 48.0 kg·m<sup>2</sup>/s

## Problem 14.13

Hx = 0.000 ft·lb·s  
Hy = -0.720 ft·lb·s  
Hz = 1.440 ft·lb·s

## 14.C3



## GIVEN:

SHELL MOVING WITH VELOCITY OF COMPONENTS

$v_x, v_y, v_z$  EXPLODES IN THREE FRAGMENTS OF WEIGHTS  
 $W_1, W_2, W_3$  AT POINT  $A_0$  AT DISTANCE  $d$  FROM WALL.  
FRAGMENTS HIT THE WALL AT POINTS  $A_i$  ( $i=1,2,3$ )  
OF COORDINATES  $x_i$  AND  $y_i$ .

FIND: SPEED OF EACH FRAGMENT AFTER EXPLOSION  
USE PROGRAM TO SOLVE (a) PROB. 14.25, (b) PROB. 14.26.

## ANALYSIS

DETERMINE DIRECTION COSINES OF PATH  $A_0A_i$  ( $i=1,2,3$ )

FIRST COMPUTE  $\ell_i = \sqrt{x_i^2 + y_i^2 + d^2}$  (1)

THEN  $(\lambda_x)_i = x_i/\ell_i$ ,  $(\lambda_y)_i = y_i/\ell_i$ ,  $(\lambda_z)_i = -d/\ell_i$  (2)

CONSERVATION OF LINEAR MOMENTUM:

$$\frac{1}{g}(W_1 + W_2 + W_3)(v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) = \frac{W_1}{g} v_1 \lambda_1 + \frac{W_2}{g} v_2 \lambda_2 + \frac{W_3}{g} v_3 \lambda_3$$

$$x\text{-COMP: } W_1(\lambda_x)_1 v_1 + W_2(\lambda_x)_2 v_2 + W_3(\lambda_x)_3 v_3 = (W_1 + W_2 + W_3) v_x \quad (3)$$

$$y\text{-COMP: } W_1(\lambda_y)_1 v_1 + W_2(\lambda_y)_2 v_2 + W_3(\lambda_y)_3 v_3 = (W_1 + W_2 + W_3) v_y \quad (4)$$

$$z\text{-COMP: } W_1(\lambda_z)_1 v_1 + W_2(\lambda_z)_2 v_2 + W_3(\lambda_z)_3 v_3 = (W_1 + W_2 + W_3) v_z \quad (5)$$

THESE 3 EQS. ARE SOLVED SIMULTANEOUSLY FOR  $v_1, v_2, v_3$ .

## OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER

ENTER VALUES OF  $v_x, v_y, v_z$ , AND  $d$

ENTER VALUES OF  $W_i, x_i, y_i$  FOR  $i=1,2,3$

COMPUTE DIRECTION COSINES FROM EQS. (1) AND (2)

COMPUTE COEFF. IN EQS. (3), (4), (5) AND SOLVE FOR  $v_1, v_2, v_3$   
BY COMPUTING

$$D = \begin{vmatrix} W_1(\lambda_x)_1 & W_2(\lambda_x)_2 & W_3(\lambda_x)_3 \\ W_1(\lambda_y)_1 & W_2(\lambda_y)_2 & W_3(\lambda_y)_3 \\ W_1(\lambda_z)_1 & W_2(\lambda_z)_2 & W_3(\lambda_z)_3 \end{vmatrix}, \quad D_i = \begin{vmatrix} \sum W_j v_j & W_i(\lambda_x)_i & W_i(\lambda_z)_i \\ \sum W_j v_j & W_i(\lambda_y)_i & W_i(\lambda_z)_i \\ \sum W_j v_j & W_i(\lambda_x)_i & W_i(\lambda_y)_i \end{vmatrix}, \text{ etc.}$$

AND  $v_1 = D_1/D$ ,  $v_2 = D_2/D$ ,  $v_3 = D_3/D$

## PROGRAM OUTPUT

## (a) Problem 14.25

VA = 1678 ft/s  
VB = 1390 ft/s  
VC = 1230 ft/s

## (b) Problem 14.26

VA = 2097 ft/s  
VB = 1853 ft/s  
VC = 738 ft/s

## 14.C4



GIVEN: AS 6000-KG PLANE LANDS ON CARRIER AT 180 km/h, ITS TAIL HOOKS INTO END OF 80-m LONG CHAIN OF ANCHORS. PER UNIT LENGTH  $m' = 50 \text{ kg/m}$  LYING BELOW DECK.

TASK: WRITE, USING 5-m INCREMENTS, THE DISTANCE TRAVELED BY THE PLANE AND THE CORRESPONDING VALUES OF THE TIME, THE VELOCITY, AND THE ACCELERATION OF THE PLANE.  
(ASSUME NO OTHER RESISTANCE.)

DIAGRAM

CONSERVATION OF LINEAR MOMENTUM:



$$m v_0 = (m + m'x) v \quad (1)$$

LETTING  $v = dx/dt$ :

$$m v_0 dt = (m + m'x) dx$$

$$t = \int_0^x \frac{m + m'x}{m v_0} dx = \left[ \frac{(m + m'x)^2}{2 m m' v_0} \right]_0^x = \frac{(m + m'x)^2 - m^2}{2 m m' v_0} \quad (2)$$

SOLVING (1) FOR  $v$ :  $v = \frac{m v_0}{m + m'x}$

DIFFERENTIATING (1) WITH RESPECT TO  $t$ :

$$0 = m' \frac{dx}{dt} v + (m + m'x) \frac{dv}{dt}$$

NOTING THAT  $dx/dt = v$  AND  $dv/dt = a$ :

$$0 = m' v^2 + (m + m'x) a \quad a = - \frac{m' v^2}{m + m'x} \quad (4)$$

OUTLINE OF PROGRAM

ENTER  $m = 6000 \text{ kg}$ ,  $m' = 50 \text{ kg/m}$ ,  $v_0 = 180 \text{ km/h} = 50 \text{ m/s}$   
FOR  $x = 0$  TO  $x = 80 \text{ m}$  AND USING 5-m INCREMENTS  
CALCULATE  $t$ ,  $v$ , AND  $a$  FROM EQS. (2), (3), (4) AND TABULATE

PROGRAM OUTPUT

Distance (m)	Time (s)	Velocity (km/h)	Acceleration (m/s <sup>2</sup> )
0.000	0.000	180.000	-20.833
5.000	0.102	172.800	-18.432
10.000	0.208	166.154	-16.386
15.000	0.319	160.000	-14.632
20.000	0.433	154.286	-13.120
25.000	0.552	148.966	-11.809
30.000	0.675	144.000	-10.667
35.000	0.802	139.355	-9.667
40.000	0.933	135.000	-8.789
45.000	1.069	130.909	-8.014
50.000	1.208	127.059	-7.327
55.000	1.352	123.429	-6.717
60.000	1.500	120.000	-6.173
65.000	1.652	116.757	-5.686
70.000	1.808	113.684	-5.249
75.000	1.969	110.769	-4.855
80.000	2.133	108.000	-4.500

## 14.C5



GIVEN:

A 16-Mg PLANE MAINTAINS A CONSTANT SPEED OF 774 km/h WHILE CLIMBING AT AN ANGLE  $\alpha = 18^\circ$ .

PLANE SCOOPS IN AIR AT RATE OF 300 kg/s AND DISCHARGES IT AT A RELATIVE SPEED OF 665 m/s. PILOT THEN CHANGES ANGLE OF CLIMB  $\alpha$  WHILE MAINTAINING THE SAME ENGINE SETTING.

FIND FOR VALUES OF  $\alpha$  FROM 0 TO 20° USING 1° INCREMENTS:

- (a) INITIAL ACCELERATION OF PLANE,  
(b) MAXIMUM SPEED THAT WILL BE ATTAINED.  
(ASSUME DRAG TO BE PROPORTIONAL TO  $v^2$ .)

ANALYSIS

FROM EQ. (14.39): THRUST =  $P = \frac{dm}{dt}(u - v)$

DENOTING RATE  $dm/dt$  BY  $R$ :

$$P = R(u - v) \quad (1)$$

WHILE CLIMBING AT  $v_0$  AND  $\alpha_0$ :

$$P_0 = R(u - v_0) \quad (2)$$

SINCE PLANE IS IN EQUILIBRIUM:

$$D_0 = P_0 - mg \sin \alpha_0 \quad (3)$$

- (a) PLANE CLIMBING AT ANGLE  $\alpha$  AND SPEED  $v_0$ :

$$\sum F_x = ma$$

$$P_0 - D_0 - mg \sin \alpha = ma$$

$$a = (P_0 - D_0 - mg \sin \alpha) / m \quad (4)$$

- (b) MAX. SPEED WHILE CLIMBING AT ANGLE  $\alpha$ :

ACCEL. WILL THEN BE ZERO AND PLANE IN EQUILIBRIUM.

$$\sum F_x = 0: P - D - mg \sin \alpha = 0 \quad (5)$$

BUT, SINCE  $D \propto v^2$ , WE HAVE

$$D = D_0 (v/v_0)^2$$

SUBSTITUTING FOR  $D$  INTO (5) AND FOR  $P$  FROM (1):

$$R(u - v) - (D_0/v_0^2) v^2 - mg \sin \alpha = 0$$

$$v^2 + \frac{R v_0^2}{D_0} v + (v_0^2/D_0)(mg \sin \alpha - Ru) = 0$$

$$\text{SET } B = R v_0^2 / D_0, \quad C = (v_0^2/D_0)(mg \sin \alpha - Ru) \quad (6)$$

MAX. SPEED:

$$v_{\max} = v = \frac{1}{2} (-B + \sqrt{B^2 - 4C}) \quad (7)$$

OUTLINE OF PROGRAM

ENTER  $m = 16 \times 10^3 \text{ kg}$ ,  $v_0 = 774 \text{ km/h} = 215 \text{ m/s}$ ,  $\alpha_0 = 18^\circ$ ,

$R = 300 \text{ kg/s}$ ,  $u = 665 \text{ m/s}$ ,  $g = 9.81 \text{ m/s}^2$ .

USE EQS. (2) AND (3) TO CALCULATE  $P_0$  AND  $D_0$ .

CALCULATE  $B = R v_0^2 / D_0$ .

FOR  $\alpha$  FROM 0 TO 20°, WITH 1° INCREMENTS

- (a) USE EQ. (4) TO CALCULATE  $a$

- (b) USE EQS. (6) AND (7) TO CALCULATE  $v_{\max}$

(CONTINUED)

## 14.C5 continued

### PROGRAM OUTPUT

alpha degrees	acceleration m/s <sup>2</sup>	max v km/h
0.000	3.031	921.796
1.000	2.860	913.933
2.000	2.689	906.020
3.000	2.518	898.060
4.000	2.347	890.053
5.000	2.176	882.002
6.000	2.006	873.907
7.000	1.836	865.770
8.000	1.666	857.594
9.000	1.497	849.378
10.000	1.328	841.126
11.000	1.160	832.839
12.000	0.992	824.518
13.000	0.825	816.166
14.000	0.658	807.785
15.000	0.492	799.375
16.000	0.327	790.940
17.000	0.163	782.481
18.000	0.000	774.000
19.000	-0.162	765.499
20.000	-0.324	756.981

## 14.C6 continued

### OUTLINE OF PROGRAM

ENTER  $g = 31.2 \text{ ft/s}^2$ ,  $m_0 = 2400/g$ ,  $m_s = 2000/g$ .

$q = 25/g$ ,  $u = 12,000 \text{ ft/s}$

COMPUTE FINAL TIME  $t_s = m_s/q = 2000/25 = 80s$

FOR  $t$  FROM 0 TO 80s AT 4-s INTERVALS,

COMPUTE

(a) ACCELERATION  $a$  FROM EQ. (4)

(b) VELOCITY  $v$  FROM EQ. (1)

(c) ELEVATION  $h$  FROM EQ. (3), DIVIDING RESULT BY 5280 TO OBTAIN  $h$  IN MILES.

### PROGRAM OUTPUT

t s	a ft/s <sup>2</sup>	v 10 <sup>3</sup> ft/s	h mi
0.000	92.800	0.000	0.000
4.000	98.235	0.382	0.143
8.000	104.164	0.787	0.584
12.000	110.657	1.216	1.341
16.000	117.800	1.673	2.434
20.000	125.695	2.159	3.883
24.000	134.467	2.679	5.714
28.000	144.271	3.236	7.952
32.000	155.300	3.835	10.628
36.000	167.800	4.481	13.775
40.000	182.086	5.180	17.431
44.000	198.569	5.940	21.639
48.000	217.800	6.772	26.449
52.000	240.527	7.688	31.921
56.000	267.800	8.702	38.122
60.000	301.133	9.838	45.137
64.000	342.800	11.123	53.066
68.000	396.371	12.596	62.037
72.000	467.800	14.317	72.213
76.000	567.800	16.376	83.814
80.000	717.800	18.925	97.148

## 14.C6 GIVEN:

ROCKET OF WEIGHT 2400 lb, INCLUDING 2000 lb OF FUEL, IS FIRED VERTICALLY FROM GROUND. IT CONSUMES FUEL AT RATE OF 25 lb/s AND EJECTS IT WITH RELATIVE VELOCITY OF 12,000 ft/s.

FIND FROM TIME OF IGNITION TO TIME WHEN LAST PARTICLE OF FUEL IS CONSUMED, AND AT 4-s TIME INTERVALS:

(a) ACCELERATION  $a$  OF ROCKET IN  $\text{ft/s}^2$ ,

(b) ITS VELOCITY  $v$  IN  $\text{ft/s}$ ,

(c) ITS ELEVATION  $h$  ABOVE GROUND IN MILES.

### ANALYSIS

WE RECALL FROM SAMPLE PROB. 14.B THAT

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \quad (1)$$

WHERE:  $v$  = VELOCITY OF ROCKET

$m_0$  = INITIAL WEIGHT OF ROCKET AND FUEL

$q$  = RATE AT WHICH FUEL IS CONSUMED

$u$  = RELATIVE VELOCITY AT WHICH FUEL IS EJECTED

LETTING  $y = dy/dt$  AND INTEGRATING  $y$  FROM 0 TO  $h$ :

$$h = \int_0^h dy = u \int_0^t \ln \frac{m_0}{m_0 - qt} dt - \frac{1}{2} gt^2 \quad (2)$$

TO CALCULATE THE INTEGRAL, WE SET  $\frac{m_0 - qt}{m_0} = z$

AND OBTAIN  $dt = -\frac{m_0}{q} dz$ . THEREFORE:

$$\begin{aligned} \int_0^t \ln \frac{m_0}{m_0 - qt} dt &= \int_1^z (-\ln z) \left(-\frac{m_0}{q}\right) dz \\ &= \frac{m_0}{q} \int_1^z \ln z dz = \frac{m_0}{q} [z \ln z - z]_1^z = \frac{m_0}{q} (z \ln z - z + 1) \end{aligned}$$

THUS, EQ. (2) YIELDS

$$\begin{aligned} h &= \frac{m_0 u}{q} \left( \frac{m_0 - qt}{m_0} \ln \frac{m_0 - qt}{m_0} - 1 + \frac{qt}{m_0} + 1 \right) - \frac{1}{2} gt^2 \\ h &= u \left( t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0} \right) - \frac{1}{2} gt^2 \quad (3) \end{aligned}$$

REWRITING EQ. (1) AS

$$v = u \ln m_0 - u \ln (m_0 - qt) - gt$$

AND DIFFERENTIATING WITH RESPECT TO  $t$ ,

$$a = \frac{dv}{dt} = -u \frac{-q}{m_0 - qt} - g \quad a = \frac{uq}{m_0 - qt} - g \quad (4)$$

(CONTINUED)



15.1 GIVEN:  $\Theta = 1.5t^3 - 4.5t^2 + 10$   
FIND:  $\Theta$ ,  $\omega$ , AND  $\alpha$   
 WHEN (a)  $t = 0$ , (b)  $t = 4s$ .

$$\omega = \frac{d\Theta}{dt} = 4.5t^2 - 9t$$

$$\alpha = \frac{d\omega}{dt} = 9t - 9$$

(a)  $t = 0$ :  $\Theta = 10 \text{ rad}$

$$\omega = 0$$

$$\alpha = -9 \text{ rad/s}^2$$

(b)  $t = 4s$ :  $\Theta = 1.5(4)^3 - 4.5(4)^2 + 10$

$$\Theta = 34 \text{ rad}$$

$$\omega = 4.5(4) - 9(4)$$

$$\omega = 36 \text{ rad/s}$$

$$\alpha = 9(4) - 9; \quad \alpha = 27 \text{ rad/s}^2$$

15.2 GIVEN:  $\Theta = 1.5t^3 - 4.5t^2 + 10$   
FIND:  $t$ ,  $\Theta$ , AND  $\alpha$  WHEN  $\omega = 0$

$$\omega = \frac{d\Theta}{dt} = 4.5t^2 - 9t$$

$$\alpha = \frac{d\omega}{dt} = 9t - 9$$

For  $\omega = 0$ :  $4.5t^2 - 9t = 0$   
 $t = 0$  AND  $t = 2$ .

$t = 0$ :  $\Theta = 10 \text{ rad}$ ,  $\alpha = -9 \text{ rad/s}^2$

$t = 2s$ :  $\Theta = 1.5(2)^3 - 4.5(2)^2 + 10$ ,  $\Theta = 4 \text{ rad}$   
 $\alpha = 9(2) - 9$ ,  $\alpha = 9 \text{ rad/s}^2$

15.3 GIVEN:  $\Theta = \Theta_0(1 - e^{-t/4})$  WITH  $\Theta_0 = 0.40 \text{ rad}$   
FIND:  $\Theta$ ,  $\omega$ , AND  $\alpha$   
 WHEN (a)  $t = 0$ , (b)  $t = 3s$ , (c)  $t = \infty$

$$\Theta = 0.40(1 - e^{-t/4})$$

$$\omega = \frac{d\Theta}{dt} = \frac{1}{4}(0.40)e^{-t/4} = 0.10 e^{-t/4}$$

$$\alpha = \frac{d\omega}{dt} = -\frac{1}{4}(0.10)e^{-t/4} = -0.025e^{-t/4}$$

(a)  $t = 0$ :  $\Theta = 0.40(1 - e^0)$   $\Theta = 0$   
 $\omega = 0.10 e^0$   $\omega = 0.1 \text{ rad/s}$   
 $\alpha = -0.025 e^0$   $\alpha = -0.025 \text{ rad/s}^2$

(b)  $t = 3s$ :  $\Theta = 0.40(1 - e^{-3/4})$   
 $= 0.40(1 - 0.4724)$ ,  $\Theta = 0.211 \text{ rad}$   
 $\omega = 0.10 e^{-3/4}$   
 $= 0.10(0.4724)$ ,  $\omega = 0.0472 \text{ rad/s}$   
 $\alpha = -0.025 e^{-3/4}$   
 $= -0.025(0.4724)$ ,  $\alpha = -0.0118 \text{ rad/s}^2$

(c)  $t = \infty$ :  $\Theta = 0.40(1 - e^{-\infty})$   
 $= 0.40(1 - 0)$   $\Theta = 0.4 \text{ rad}$   
 $\omega = 0.10 e^{-\infty}$   $\omega = 0$   
 $\alpha = -0.025 e^{-\infty}$   $\alpha = 0$

15.4 GIVEN:  $\Theta = \Theta_0 \sin\left(\frac{\pi t}{T}\right) - 0.5\Theta_0 \sin\left(\frac{2\pi t}{T}\right)$   
 WHERE  $\Theta_0 = 6 \text{ rad}$ ,  $T = 4s$ .  
FIND:  $\Theta$ ,  $\omega$ , AND  $\alpha$  WHEN (a)  $t = 0$ , (b)  $t = 2s$ .

$$\omega = \frac{d\Theta}{dt} = \Theta_0 \frac{\pi}{T} \cos\left(\frac{\pi t}{T}\right) - 0.5\Theta_0 \frac{2\pi}{T} \cos\left(\frac{2\pi t}{T}\right)$$

$$\alpha = \frac{d\omega}{dt} = -\Theta_0 \left(\frac{\pi}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right) + 0.5\Theta_0 \left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi t}{T}\right)$$

(a)  $t = 0$ :  $\Theta = 0$

$$\omega = 6 \frac{\pi}{4} - 0.5(6) \frac{2\pi}{4}$$

$$\omega = 0$$

$$\alpha = 0$$

(b)  $t = 2s$ :

$$\Theta = 6 \sin\left(\frac{2\pi}{4}\right) - 0.5(6) \sin\left(\frac{4\pi}{4}\right) = 6 - 0$$

$$\Theta = 6 \text{ rad}$$

$$\omega = 6 \left(\frac{\pi}{4}\right) \cos\left(\frac{2\pi}{4}\right) - 0.5(6) \frac{2\pi}{4} \cos\left(\frac{4\pi}{4}\right)$$

$$= 6 \frac{\pi}{4} (0) - 0.5(6) \frac{2\pi}{4} (-1) = \frac{6\pi}{4}$$

$$\omega = 4.71 \text{ rad/s}$$

$$\alpha = -6 \left(\frac{\pi}{4}\right)^2 \sin\left(\frac{2\pi}{4}\right) + 0.5(6) \left(\frac{2\pi}{4}\right)^2 \sin\left(\frac{4\pi}{4}\right)$$

$$= -6 \left(\frac{\pi}{4}\right)^2 (1) + 3 \left(\frac{2\pi}{4}\right)^2 (0) = -\frac{3}{8}\pi^2$$

$$\alpha = -3.70 \text{ rad/s}^2$$

15.5 GIVEN:  $\Theta = \Theta_0 \sin\left(\frac{\pi t}{T}\right) - 0.5\Theta_0 \sin\left(\frac{2\pi t}{T}\right)$

WHERE  $\Theta_0 = 6 \text{ rad}$ ,  $T = 4s$

FIND:  $\Theta$ ,  $\omega$ , AND  $\alpha$  WHEN  $t = 1s$

$$\omega = \frac{d\Theta}{dt} = \Theta_0 \frac{\pi}{T} \cos\left(\frac{\pi t}{T}\right) - 0.5\Theta_0 \frac{2\pi}{T} \cos\left(\frac{2\pi t}{T}\right)$$

$$\alpha = \frac{d\omega}{dt} = -\Theta_0 \left(\frac{\pi}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right) + 0.5\Theta_0 \left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi t}{T}\right)$$

$t = 1s$ :  $\Theta = 6 \sin\left(\frac{\pi}{4}\right) - 0.5(6) \sin\left(\frac{2\pi}{4}\right)$

$$= 6 \frac{\sqrt{2}}{2} - 0.5(6)(1)$$

$$\Theta = 1.243 \text{ rad}$$

$$\omega = 6 \left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right) - 0.5(6) \frac{2\pi}{4} \cos\left(\frac{2\pi}{4}\right)$$

$$= 6 \left(\frac{\pi}{4}\right) \frac{\sqrt{2}}{2} - 0.5(6) \left(\frac{\pi}{2}\right)(0)$$

$$\omega = 3.33 \text{ rad/s}$$

$$\alpha = -6 \left(\frac{\pi}{4}\right)^2 \sin\left(\frac{\pi}{4}\right) + 0.5(6) \left(\frac{2\pi}{4}\right)^2 \sin\left(\frac{2\pi}{4}\right)$$

$$= -6 \left(\frac{\pi}{4}\right)^2 \frac{\sqrt{2}}{2} + 0.5(6) \left(\frac{\pi}{2}\right)^2 (1)$$

$$\alpha = 4.79 \text{ rad/s}^2$$

15.6



GIVEN:  $t = 0$ ,  $\omega = 0$   
 $t = 6s$ ,  $\omega_1 = 3300 \text{ rpm} = 110\pi \text{ rad/s}$ .  
 THEN COASTS TO REST IN 80s.  
FIND: NUMBER OF REVOLUTIONS  
 (a) TO REACH SPEED OF 3300 rpm,  
 (b) TO COAST TO REST.

UNIFORMLY ACCELERATED MOTION:  $\omega_0 = 0$ ,  $t = 6s$ .

(a)  $\omega = \omega_0 + \alpha t$ ;  $110\pi = 0 + \alpha(6)$ ,  $\alpha = \frac{110}{6}\pi \text{ rad/s}^2$

$$\Theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \left(\frac{110}{6}\pi\right) (6s)^2 = 330\pi \text{ rad}$$

$$\Theta = (330\pi \text{ rad}) \frac{1 \text{ rev}}{2\pi \text{ rad}} \quad \Theta = 165 \text{ rev}$$

(b)  $\omega_1 = 110\pi \text{ rad/s}$ ,  $\omega_2 = 0$  WHEN  $t = 80s$

$$\omega_2 = \omega_1 + \alpha t$$

$$0 = 110\pi + \alpha(80s)$$

$$\alpha = -\frac{110}{80}\pi \text{ rad/s}^2$$

$$\Theta = \omega_1 t + \frac{1}{2} \alpha t^2 = (110\pi)(80) - \frac{1}{2} \left(\frac{110}{80}\pi\right) (80s)^2$$

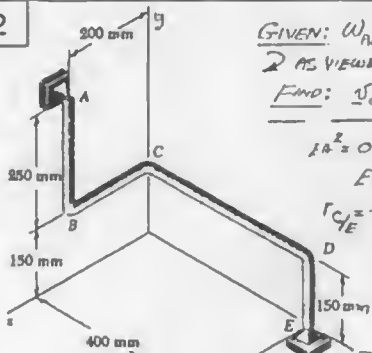
$$= 8800\pi - 4400\pi = 4400\pi \text{ rad}$$

$$\Theta = (4400\pi) \frac{1 \text{ rev}}{2\pi \text{ rad}}$$

$$\Theta = 2200 \text{ rev}$$



15.12



GIVEN:  $\omega_{AE} = 9 \text{ rad/s}$ ,  $\alpha_{AE} = 0$   
 2 AS VIEWED FROM E.

FIND:  $\underline{v}_C$  AND  $\underline{a}_C$

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\underline{r}_{CE} = (-0.4 \text{ m})\underline{i} + (0.15 \text{ m})\underline{j}$$

$$\underline{EA} = (-0.4 \text{ m})\underline{i} + (0.4 \text{ m})\underline{j} + (0.2 \text{ m})\underline{k}$$

$$\underline{\hat{EA}} = \frac{\underline{EA}}{EA} = \frac{1}{0.6}(-0.4 \underline{i} + 0.4 \underline{j} + 0.2 \underline{k}) = \frac{1}{3}(-2 \underline{i} + 2 \underline{j} + \underline{k})$$

$$\underline{\omega} = \omega_{AE} \underline{\hat{EA}} = (9 \text{ rad/s}) \frac{1}{3}(-2 \underline{i} + 2 \underline{j} + \underline{k})$$

$$\underline{\omega} = (-6 \text{ rad/s})\underline{i} + (6 \text{ rad/s})\underline{j} + (3 \text{ rad/s})\underline{k}$$

$$\underline{v}_C = \underline{\omega} \times \underline{r}_{CE} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ -0.4 & 0.15 & 0 \end{vmatrix} = -0.45 \underline{i} - 1.2 \underline{j} + (-0.9 + 2.4) \underline{k}$$

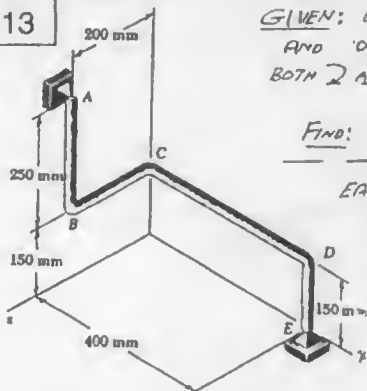
$$\underline{v}_C = (-0.45 \text{ m/s})\underline{i} - (1.2 \text{ m/s})\underline{j} + (1.5 \text{ m/s})\underline{k}$$

$$\underline{a}_C = \underline{\alpha} \times \underline{r}_{CE} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{CE}) = \underline{\alpha}_C \times \underline{r}_{CE} + \underline{\omega} \times \underline{v}_C$$

$$\underline{a}_C = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ -0.45 & -1.2 & 1.5 \end{vmatrix} = (9 + 3.6)\underline{i} + (-1.35 + 9)\underline{j} + (7.2 + 2.7)\underline{k}$$

$$\underline{a}_C = (12.60 \text{ m/s}^2)\underline{i} + (7.65 \text{ m/s}^2)\underline{j} + (9.90 \text{ m/s}^2)\underline{k}$$

15.13



GIVEN:  $\omega_{AE} = 9 \text{ rad/s}$   
 AND  $\alpha_{AE} = 45 \text{ rad/s}^2$ ,  
 BOTH 2 AS VIEWED FROM E.

FIND:  $\underline{v}_B$  AND  $\underline{a}_B$

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\underline{r}_{BA} = (-0.25 \text{ m})\underline{j}$$

$$\underline{EA} = (-0.4 \text{ m})\underline{i} + (0.4 \text{ m})\underline{j} + (0.2 \text{ m})\underline{k}$$

$$\underline{\hat{EA}} = \underline{EA}/EA = (-0.4 \underline{i} + 0.4 \underline{j} + 0.2 \underline{k})/0.6 = \frac{1}{3}(-2 \underline{i} + 2 \underline{j} + \underline{k})$$

$$\underline{\omega} = \omega_{AE} \underline{\hat{EA}} = (9 \text{ rad/s}) \frac{1}{3}(-2 \underline{i} + 2 \underline{j} + \underline{k})$$

$$\underline{\omega} = (-6 \text{ rad/s})\underline{i} + (6 \text{ rad/s})\underline{j} + (3 \text{ rad/s})\underline{k}$$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_{BE} = (-6 \underline{i} + 6 \underline{j} + 3 \underline{k}) \times (-0.25 \underline{j})$$

$$= 1.5 \underline{k} + 0.75 \underline{i}; \quad \underline{v}_B = (0.75 \text{ m/s})\underline{i} + (1.5 \text{ m/s})\underline{k}$$

$$\underline{\alpha} = \alpha_{AE} \underline{\hat{EA}} = (45 \text{ rad/s}^2) \frac{1}{3}(-2 \underline{i} + 2 \underline{j} + \underline{k})$$

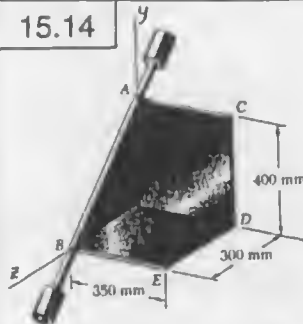
$$\underline{\alpha} = (-30 \text{ rad/s}^2)\underline{i} + (30 \text{ rad/s}^2)\underline{j} + (15 \text{ rad/s}^2)\underline{k}$$

$$\underline{a}_B = \underline{\alpha} \times \underline{r}_{BA} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{BA}) = \underline{\alpha} \times \underline{r}_{BA} + \underline{\omega} \times \underline{v}_B$$

$$\underline{a}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -30 & 30 & 15 \\ -0.25 & 0 & 1.5 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ 0.75 & 0 & 1.5 \end{vmatrix} = 3.75 \underline{i} + 25 \underline{k} + 9 \underline{i} + (2.25 + 9) \underline{j} - 4.5 \underline{k}$$

$$\underline{a}_B = (12.75 \text{ m/s}^2)\underline{i} + (11.25 \text{ m/s}^2)\underline{j} + (3 \text{ m/s}^2)\underline{k}$$

15.14



GIVEN:  $\omega_{AB} = 5 \text{ rad/s}$ ,  
 $\alpha_{AB} = 0$  AND VELOCITY  
 OF E IS DOWNWARD

FIND:  $\underline{v}_D$  AND  $\underline{a}_D$

SINCE  $\underline{v}_E$  IS DOWNWARD,  
 $\omega_{AB}$  IS  $\curvearrowright$  WHEN  
 VIEWED FROM A.

$$AB^2 = 0.4^2 + 0.3^2; AB = 0.5 \text{ m}$$

$$\underline{r}_{DB} = (0.35 \text{ m})\underline{i} - (0.3 \text{ m})\underline{k}$$

$$\underline{BA} = (0.4 \text{ m})\underline{j} - (0.3 \text{ m})\underline{k}$$

$$\underline{\hat{BA}} = \frac{\underline{BA}}{BA} = \frac{1}{0.5}(0.4 \underline{j} - 0.3 \underline{k}) = 0.8 \underline{j} - 0.6 \underline{k}$$

$$\underline{\omega} = \omega_{AB} \underline{\hat{BA}} = (5 \text{ rad/s})(0.8 \underline{j} - 0.6 \underline{k}) = (4 \text{ rad/s})\underline{j} - (3 \text{ rad/s})\underline{k}$$

$$\underline{v}_D = \underline{\omega} \times \underline{r}_{DB} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ 0.35 & 0 & -0.3 \end{vmatrix} = -1.2 \underline{i} - 1.05 \underline{j} - 1.4 \underline{k}$$

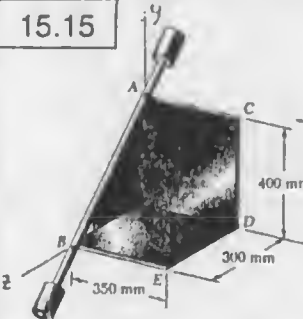
$$\underline{v}_D = (-1.2 \text{ m/s})\underline{i} - (1.05 \text{ m/s})\underline{j} - (1.4 \text{ m/s})\underline{k}$$

$$\underline{a}_D = \underline{\alpha} \times \underline{r}_{DB} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{DB}) = \underline{\alpha} \times \underline{r}_{DB} + \underline{\omega} \times \underline{v}_D$$

$$\underline{a}_D = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ -1.2 & -1.05 & -1.4 \end{vmatrix} = (-5.6 - 3.15)\underline{i} + 3.6 \underline{j} + 4.8 \underline{k}$$

$$\underline{a}_D = (-8.75 \text{ m/s}^2)\underline{i} + (3.6 \text{ m/s}^2)\underline{j} + (4.8 \text{ m/s}^2)\underline{k}$$

15.15



GIVEN:  $\omega_{AB} = 5 \text{ rad/s}$ ,  
 $\alpha_{AB} = -20 \text{ rad/s}^2$   
 AND VELOCITY OF E  
 IS DOWNWARD,  
 FIND:  $\underline{v}_D$  AND  $\underline{a}_D$ .

SINCE  $\underline{v}_E$  IS DOWNWARD,  
 $\omega_{AB}$  IS  $\curvearrowright$  WHEN  
 VIEWED FROM A.

$$AB^2 = 0.4^2 + 0.3^2; AB = 0.5 \text{ m}$$

$$\underline{r}_{DB} = (0.35 \text{ m})\underline{i} - (0.3 \text{ m})\underline{k}$$

$$\underline{BA} = (0.4 \text{ m})\underline{j} - (0.3 \text{ m})\underline{k}$$

$$\underline{\hat{BA}} = \frac{\underline{BA}}{BA} = \frac{1}{0.5}(0.4 \underline{j} - 0.3 \underline{k}) = 0.8 \underline{j} - 0.6 \underline{k}$$

$$\underline{\omega} = \omega_{AB} \underline{\hat{BA}} = (5 \text{ rad/s})(0.8 \underline{j} - 0.6 \underline{k}) = (4 \text{ rad/s})\underline{j} - (3 \text{ rad/s})\underline{k}$$

$$\underline{\alpha} = \alpha_{AB} \underline{\hat{BA}} = (-20 \text{ rad/s}^2)(0.8 \underline{j} - 0.6 \underline{k}) = (-16 \text{ rad/s}^2)\underline{j} + (12 \text{ rad/s}^2)\underline{k}$$

$$\underline{v}_D = \underline{\omega} \times \underline{r}_{DB} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ 0.35 & 0 & -0.3 \end{vmatrix} = -1.2 \underline{i} - 1.05 \underline{j} - 1.4 \underline{k}$$

$$\underline{v}_D = (-1.2 \text{ m/s})\underline{i} - (1.05 \text{ m/s})\underline{j} - (1.4 \text{ m/s})\underline{k}$$

$$\underline{a}_D = \underline{\alpha} \times \underline{r}_{DB} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{DB}) = \underline{\alpha} \times \underline{r}_{DB} + \underline{\omega} \times \underline{v}_D$$

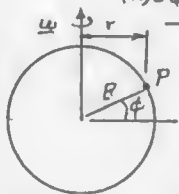
$$\underline{a}_D = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -16 & 12 \\ 0.35 & 0 & -0.3 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ -1.2 & -1.05 & -1.4 \end{vmatrix} = 4.8 \underline{i} + 4.2 \underline{j} + 5.6 \underline{k} + (-5.6 - 3.15)\underline{i} + 3.6 \underline{j} + 4.8 \underline{k}$$

$$\underline{a}_D = (-3.95 \text{ m/s}^2)\underline{i} + (7.8 \text{ m/s}^2)\underline{j} + (10.40 \text{ m/s}^2)\underline{k}$$

15.16

GIVEN: EARTH ROTATES  $2\pi$  radians in 23 h 56 m. RADIUS OF EARTH = 3960 mi

FIND: VELOCITY AND ACCELERATION OF POINT AT (a) EQUATOR, (b) PHILA, LATITUDE  $40^\circ$ , (c) NORTH POLE



$$24 \text{ h } 56 \text{ m} = 23.933 \text{ h}$$

$$\omega = \frac{2\pi \text{ rad}}{(23.933 \text{ h}) \left( \frac{3600 \text{ s}}{\text{h}} \right)} = 72.925 \times 10^{-6} \text{ rad/s}$$

$$R = (3960 \text{ mi}) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) = 20.91 \times 10^6 \text{ ft}$$

$$r = \text{RADIUS OF PATH} = R \cos \phi$$

(a) EQUATOR: LATITUDE =  $\phi = 0$

$$v = r\omega = R(\cos 0)\omega = (20.91 \times 10^6 \text{ ft})(72.925 \times 10^{-6} \text{ rad/s})$$

$$v = 1525 \text{ ft/s}$$

$$a = r\omega^2 = R(\cos 0)\omega^2 = (20.91 \times 10^6 \text{ ft})(72.925 \times 10^{-6} \text{ rad/s})^2$$

$$a = 0.118 \text{ ft/s}^2$$

(b) PHILADELPHIA: LATITUDE =  $\phi = 40^\circ$

$$v = r\omega = R(\cos 40^\circ)\omega = (20.91 \times 10^6 \text{ ft})(\cos 40^\circ)(72.925 \times 10^{-6} \text{ rad/s})$$

$$v = 1168 \text{ ft/s}$$

$$a = r\omega^2 = R(\cos 40^\circ)\omega^2 = (20.91 \times 10^6 \text{ ft})(\cos 40^\circ)(72.925 \times 10^{-6} \text{ rad/s})^2$$

$$a = 0.0852 \text{ ft/s}^2$$

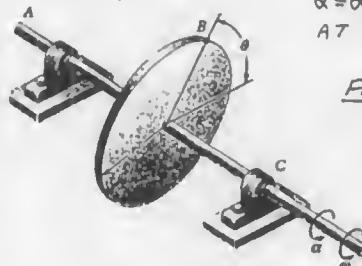
(c) NORTH POLE: LATITUDE =  $\phi = 0$

$$r = R \cos 0 = 0$$

$$v = a = 0$$

15.19

GIVEN:  $v = 600 \text{ mm}$   
 $\alpha = \alpha_0 e^{-t}$  WHERE  $\alpha_0 = 10 \text{ rad/s}^2$   
 AT  $t = 0$ ,  $\omega = 0$



FIND:  $a_B$  WHEN

(a)  $t = 0$ ,

(b)  $t = 0.5 \text{ s}$ ,

(c)  $t = \infty$ .

$$\alpha = \frac{d\omega}{dt} = \alpha_0 e^{-t}; \quad \int d\omega = \int \alpha_0 e^{-t} dt$$

$$\omega = \alpha_0 \left| -e^{-t} \right|_0^t \quad \omega = \alpha_0 (1 - e^{-t})$$

$$a_t = r\alpha = r\alpha_0 e^{-t} = (0.6 \text{ m})(10 \text{ rad/s}^2)e^{-t} = 6e^{-t}$$

$$a_n = r\omega^2 = r\alpha_0^2 (1 - e^{-t})^2 = (0.6 \times 10^2)(1 - e^{-t})^2 = 60(1 - e^{-t})^2$$

(a)  $t = 0$ :  $a_t = 6e^0 = 6 \text{ m/s}^2$   $a_n = 60(1 - e^0)^2 = 0$

$$a_B^2 = a_t^2 + a_n^2 = 6^2 \quad a_B = 6 \text{ m/s}^2$$

(b)  $t = 0.5 \text{ s}$ :

$$a_t = 6e^{-0.5} = 6(0.6065) = 3.639 \text{ m/s}^2$$

$$a_n = 60(1 - e^{-0.5})^2 = 60(1 - 0.6065)^2 = 9.789 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = (3.639)^2 + (9.789)^2 \quad a_B = 9.78 \text{ m/s}^2$$

(c)  $t = \infty$ :

$$a_t = 6e^{-\infty} = 0 \quad a_n = 60(1 - e^{-\infty})^2 = 60 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 0 + 60^2 \quad a_B = 60 \text{ m/s}^2$$

15.20

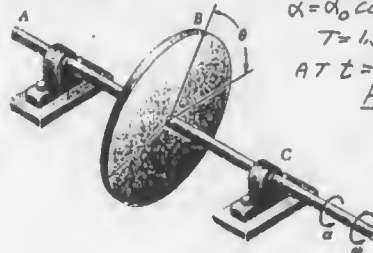
GIVEN:  $v = 250 \text{ mm}$   
 $\alpha = \alpha_0 \cos(\pi t/T)$  WHERE  
 $T = 1.5 \text{ s}$  AND  $\alpha_0 = 10 \text{ rad/s}^2$   
 AT  $t = 0$ ,  $\omega = 0$

FIND:  $a_B$  WHEN

(a)  $t = 0$ ,

(b)  $t = 0.5 \text{ s}$ ,

(c)  $t = 0.75 \text{ s}$ .



$$\alpha = \frac{d\omega}{dt} = \alpha_0 \cos\left(\frac{\pi t}{T}\right); \quad \int d\omega = \int \alpha_0 \cos\left(\frac{\pi t}{T}\right) dt$$

$$\omega = \alpha_0 \frac{T}{\pi} \left| \sin\left(\frac{\pi t}{T}\right) \right|_0^t \quad \omega = \alpha_0 \frac{T}{\pi} \sin\left(\frac{\pi t}{T}\right)$$

$$a_t = r\alpha = r\alpha_0 \cos\left(\frac{\pi t}{T}\right) = (0.25 \text{ m})(10 \text{ rad/s}^2) \cos\left(\frac{\pi t}{1.5}\right) = 2.5 \cos\left(\frac{\pi t}{1.5}\right)$$

$$a_n = r\omega^2 = r\alpha_0^2 \frac{T^2}{\pi^2} \sin^2\left(\frac{\pi t}{T}\right) = (0.25)(10^2) \frac{1.5^2}{\pi^2} \sin^2\left(\frac{\pi t}{1.5}\right) = 5.70 \sin^2\left(\frac{\pi t}{1.5}\right)$$

(a)  $t = 0$ :  $a_t = 2.5 \cos(0) = 2.5 \text{ m/s}^2$

$$a_n = 5.70 \sin^2(0) = 0$$

$$a_B^2 = a_t^2 + a_n^2 = 2.5^2 + 0 \quad a_B = 2.5 \text{ m/s}^2$$

(b)  $t = 0.5 \text{ s}$ :

$$a_t = 2.5 \cos\left(\frac{0.5\pi}{1.5}\right) = 2.5 \cos\left(\frac{\pi}{3}\right) = 1.25 \text{ m/s}^2$$

$$a_n = 5.70 \sin^2\left(\frac{0.5\pi}{1.5}\right) = 5.70 \sin^2\left(\frac{\pi}{3}\right) = 4.275 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 1.25^2 + 4.275^2 \quad a_B = 4.45 \text{ m/s}^2$$

(c)  $t = 0.75 \text{ s}$ :

$$a_t = 2.5 \cos\left(\frac{0.75\pi}{1.5}\right) = 2.5 \cos\left(\frac{\pi}{2}\right) = 0$$

$$a_n = 5.70 \sin^2\left(\frac{0.75\pi}{1.5}\right) = 5.70 \sin^2\left(\frac{\pi}{2}\right) = 5.70 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 0 + 5.70^2 \quad a_B = 5.70 \text{ m/s}^2$$

15.17

GIVEN: ONE YEAR = 365.24 DAYS AND  
 RADIUS OF ORBIT OF EARTH =  $93 \times 10^6 \text{ mi}$ .

FIND: FOR THE EARTH,  $v$  AND  $a$ .

$$\omega = \frac{2\pi \text{ rad}}{(365.24 \text{ DAYS}) \left( \frac{24 \text{ h}}{\text{DAY}} \right) \left( \frac{3600 \text{ s}}{\text{h}} \right)} = 199.11 \times 10^{-9} \text{ rad/s}$$

$$v = r\omega = (93 \times 10^6 \text{ mi}) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) (199.11 \times 10^{-9} \text{ rad/s})$$

$$v = 97,770 \text{ ft/s}$$

$$v = 66,700 \text{ mi/h}$$

$$a = r\omega^2 = (93 \times 10^6) (5280) (199.11 \times 10^{-9})^2$$

$$a = 19.47 \times 10^{-3} \text{ ft/s}^2$$

15.18

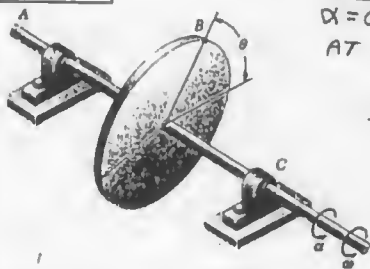
GIVEN:  $r = 200 \text{ mm}$   
 $\alpha = 0.3 \text{ rad/s}^2$  (CONSTANT)  
 AT  $t = 0$ ,  $\omega_0 = 0$

FIND:  $a_B$  WHEN

(a)  $t = 0$ ,

(b)  $t = 2 \text{ s}$ ,

(c)  $t = 4 \text{ s}$ .



UNIFORMLY ACCELERATED MOTION

$$\omega = \omega_0 + \alpha t = 0 + \alpha t \quad \omega = \alpha t$$

$$a_t = r\alpha$$

$$a_n = r\omega^2 = r\alpha^2 t^2$$

$$a_B^2 = a_t^2 + a_n^2 = r^2 \alpha^2 + r^2 \alpha^4 t^4 = r^2 \alpha^2 (1 + \alpha^2 t^4)$$

$$a_B = r\alpha (1 + \alpha^2 t^4)^{1/2}$$

$$r = 0.2 \text{ m}, \alpha = 0.3 \text{ rad/s}^2$$

$$a_B = (0.2)(0.3)(1 + (0.3)^2 t^4)^{1/2} = 0.06 (1 + 0.09 t^4)^{1/2}$$

(a)  $t = 0$ :  $a_B = 0.06 (1 + 0) = 0.06 \text{ m/s}^2$

$$a_B = 0.06 \text{ m/s}^2$$

(b)  $t = 2 \text{ s}$ :  $a_B = 0.06 (1 + 0.09 \times 2^4)^{1/2}$

$$a_B = 0.0937 \text{ m/s}^2$$

(c)  $t = 4 \text{ s}$ :  $a_B = 0.06 (1 + 0.09 \times 4^4)^{1/2}$

$$a_B = 0.294 \text{ m/s}^2$$

15.21



GIVEN:  $v_A = 15 \text{ in./s}$ ,  $a_A = 9 \text{ in./s}^2$   
 FIND: (a)  $\omega$  AND  $\alpha$  OF PULLEY, (b)  $a_B$

(a)  $v = 15 \text{ in./s}$ ,  $a = 9 \text{ in./s}^2$ ,  $v = r\omega$   
 $15 \text{ in./s} = (6 \text{ in.})\omega$ ;  $\omega = 2.5 \text{ rad/s}$   
 $a = r\alpha$ ;  $9 \text{ in./s}^2 = (6 \text{ in.})\alpha$   
 $\alpha = 1.5 \text{ rad/s}^2$

(b)  $a_B = 9 \text{ in./s}^2$   
 $a_n = r\omega^2 = (6 \text{ in.})(2.5 \text{ rad/s})^2$   
 $a_n = 37.5 \text{ in./s}^2$   
 $a_B = 37.5 \text{ in./s}^2$   
 $a_B = 38.6 \text{ in./s}^2 \angle 76.5^\circ$

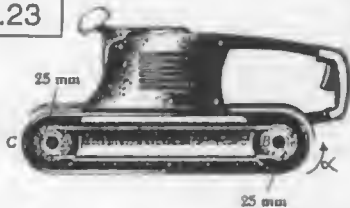
15.22



GIVEN:  $\omega = 4 \text{ rad/s}$   
 FIND:  $\alpha$  FOR WHICH  $a_B = 120 \text{ in./s}^2$

$a_B = r\alpha = (6 \text{ in.})\alpha$   
 $a_n = r\omega^2 = (6 \text{ in.})(4 \text{ rad/s})^2 = 96 \text{ in./s}^2$   
 $a_B^2 = a_n^2 + a_t^2$   
 $(120 \text{ in./s}^2)^2 = (96 \text{ in./s}^2)^2 + (6\alpha)^2$   
 $\alpha^2 = 144$ ,  $\alpha = \pm 12$ ,  $\alpha = 12 \text{ rad/s}^2$

15.23



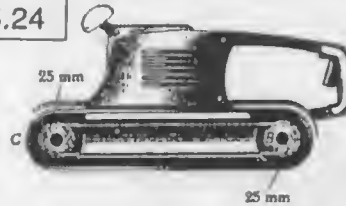
GIVEN:  
 $\alpha = 120 \text{ rad/s}^2$   
 WHEN  $t = 0$ ,  $\omega = 0$

FIND:  $a_C$  WHEN  
 (a)  $t = 0.5 \text{ s}$ ,  
 (b)  $t = 2 \text{ s}$ .

$\alpha = 120 \text{ rad/s}^2$ ,  $a_C = r\alpha = (0.025 \text{ m})(120 \text{ rad/s}^2)$   
 $a_C = 3 \text{ m/s}^2$   
 (a)  $t = 0.5 \text{ s}$ :  
 $\omega = \alpha t = (120 \text{ rad/s}^2)(0.5 \text{ s}) = 60 \text{ rad/s}$   
 $a_n = r\omega^2 = (0.025 \text{ m})(60 \text{ rad/s})^2$   
 $a_n = 90 \text{ m/s}^2$   
 $a_B^2 = a_n^2 + a_t^2 = 3^2 + 90^2$   
 $a_B = 90.05 \text{ m/s}^2$

(b)  $t = 2 \text{ s}$ :  $\omega = \alpha t = (120 \text{ rad/s}^2)(2 \text{ s}) = 240 \text{ rad/s}$   
 $a_n = r\omega^2 = (0.025 \text{ m})(240 \text{ rad/s})^2$   
 $a_n = 1440 \text{ m/s}^2$   
 $a_B^2 = a_n^2 + a_t^2 = 3^2 + 1440^2$   
 $a_B = 1440 \text{ m/s}^2$

15.24

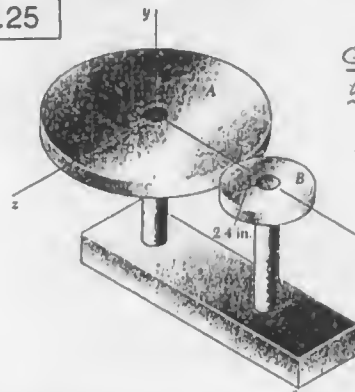


GIVEN: RATED  
 SPEED OF DRUMS  
 IS 2400 rpm  
 SANDER COMES TO  
 REST IN 10 s.  
 FIND:  $v_C$  AND  $a_C$   
 (a) BEFORE POWER

IS CUT OFF, (b) 9 s LATER.

$\omega_0 = 2400 \text{ rpm} = 251.3 \text{ rad/s}$ ,  $r = 0.025 \text{ m}$   
 (a)  $v_C = r\omega = (0.025 \text{ m})(251.3 \text{ rad/s})$ ;  $v_C = 6.28 \text{ m/s}$   
 $a_C = r\alpha = (0.025 \text{ m})(251.3 \text{ rad/s}^2)$ ;  $a_C = 15.79 \text{ m/s}^2$   
 (b) WHEN  $t = 10 \text{ s}$ ,  $\omega = 0$ .  
 $\omega = \omega_0 + \alpha t$ ;  $0 = 251.3 \text{ rad/s} + \alpha(10 \text{ s})$ ;  $\alpha = -25.13 \text{ rad/s}^2$   
 WHEN  $t = 9 \text{ s}$ :  
 $\omega = \omega_0 + \alpha t$ ;  $\omega = 251.3 \text{ rad/s} - (25.13 \text{ rad/s}^2)(9 \text{ s}) = 25.13 \text{ rad/s}$   
 $v_C = r\omega = (0.025 \text{ m})(25.13 \text{ rad/s})$ ;  $v_C = 0.628 \text{ m/s}$   
 $(a_C)_t = r\alpha = (0.025 \text{ m})(-25.13 \text{ rad/s}^2)$ ;  $(a_C)_t = -0.628 \text{ m/s}^2$   
 $(a_C)_n = r\omega^2 = (0.025 \text{ m})(25.13 \text{ rad/s})^2$ ;  $(a_C)_n = 15.79 \text{ m/s}^2$   
 $a_C^2 = (a_C)_t^2 + (a_C)_n^2 = (0.628 \text{ m/s}^2)^2 + (15.79 \text{ m/s}^2)^2$   
 $a_C = 15.80 \text{ m/s}^2$

15.25



GIVEN:  
 $\omega_B = (30 \text{ rad/s})\hat{j}$

IF NO SLIPPING OCCURS,  
 FIND: (a)  $\omega_A$   
 (b) ACCELERATIONS  
 OF POINTS IN  
 CONTACT.

(a) VELOCITIES:  $\omega_B = 30 \text{ rad/s}$  FOR NO SLIPPING:  
 $r_A \omega_A = r_B \omega_B$   
 $(6 \text{ in.})\omega_A = (2.4 \text{ in.})(30 \text{ rad/s})$   
 $\omega_A = 12 \text{ rad/s}$   
 $\omega_A = -(12 \text{ rad/s})\hat{j}$

(b) ACCELERATIONS:  
 $\omega_A = 12 \text{ rad/s}$ ,  $\omega_B = 30 \text{ rad/s}$   
 $r_A = 6 \text{ in.}$ ,  $r_B = 2.4 \text{ in.}$   
 $a_A = r_A \omega_A^2 = (6 \text{ in.})(12 \text{ rad/s})^2 = 864 \text{ in./s}^2 = 72 \text{ ft/s}^2$   
 $a_A = -(72 \text{ ft/s}^2)\hat{j}$   
 $a_B = r_B \omega_B^2 = (2.4 \text{ in.})(30 \text{ rad/s})^2 = 2160 \text{ in./s}^2 = 180 \text{ ft/s}^2$   
 $a_B = (180 \text{ ft/s}^2)\hat{j}$

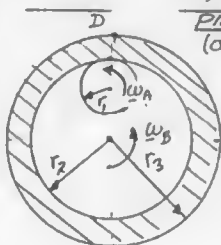
## 15.26 and 15.27



GIVEN: CONSTANT ANG. VELOCITY OF SHAFT:  $\omega_A = \omega_A \hat{k}$   
 FIND: (a) ANG. VELOCITY OF RING  $\omega_B$   
 (b) ACCELERATIONS OF POINTS SHAFT AND RING WHICH ARE IN CONTACT.

PROB. 15.26: IN TERMS OF  $\omega_A$ ,  $r_1$ ,  $r_2$ , AND  $r_3$ .

PROB. 15.27: WHEN  $\omega_A = 25 \text{ rad/s}$ ,  $r_1 = 12 \text{ mm}$ ,  $r_2 = 30 \text{ mm}$ , AND  $r_3 = 40 \text{ mm}$   
 ALSO, FIND ACCEL. OF POINT ON OUTSIDE OF B.



PROB. 15.26

(a) AT POINT OF CONTACT

$$r_1 \omega_A = r_2 \omega_B \quad \omega_B = \frac{r_1}{r_2} \omega_A$$

(b) ACCEL. OF POINTS OF CONTACT

$$\text{SHAFT A: } a_A = r_1 \omega_A^2$$

$$\text{RING B: } a_B = r_2 \omega_B^2 = r_2 \left( \frac{r_1}{r_2} \omega_A \right)^2$$

$$a_B = \frac{r_1^2}{r_2} \omega_A^2$$

ACCEL. OF POINT D ON OUTSIDE OF RING

$$a_D = r_3 \omega_B^2 = r_3 \left( \frac{r_1}{r_2} \omega_A \right)^2; \quad a_D = r_3 \left( \frac{r_1}{r_2} \right)^2 \omega_A^2$$

PROB. 15.27  $\omega_A = 25 \text{ rad/s}$ ,  $r_1 = 12 \text{ mm}$ ,  $r_2 = 30 \text{ mm}$ ,  $r_3 = 40 \text{ mm}$

$$(a) \omega_B = \frac{r_1}{r_2} \omega_A = \frac{12 \text{ mm}}{30 \text{ mm}} (25 \text{ rad/s}); \quad \omega_B = 10 \text{ rad/s}$$

$$(b) a_A = r_1 \omega_A^2 = (12 \text{ mm}) (25 \text{ rad/s})^2 = 7.5 \times 10^3 \text{ mm/s}^2$$

$$a_A = 7.5 \text{ m/s}^2$$

$$a_B = \frac{r_1^2}{r_2} \omega_A^2 = \frac{(12 \text{ mm})^2}{(30 \text{ mm})} (25 \text{ rad/s})^2 = 3 \times 10^3 \text{ mm/s}^2$$

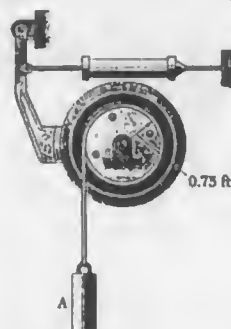
$$a_B = 3 \text{ m/s}^2$$

$$a_D = r_3 \left( \frac{r_1}{r_2} \right)^2 \omega_A^2 = (40 \text{ mm}) \left( \frac{12 \text{ mm}}{30 \text{ mm}} \right)^2 (25 \text{ rad/s})^2$$

$$a_D = 4 \times 10^3 \text{ mm/s}^2$$

$$a_D = 4 \text{ m/s}^2$$

## 15.28



GIVEN:

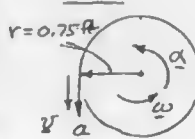
WHEN  $t = 0$ ,  $v_A = 9 \text{ ft/s} \downarrow$   
 BRAKE IS APPLIED  
 AND BLOCK COMES  
 TO REST AFTER  
 MOVING 18 ft.  
 ASSUMING UNIFORM  
 MOTION, FIND:  
 (a)  $\alpha$  OF DRUM  
 (b) TIME TO  
 COME TO REST

$$\text{BLOCK A: } v^2 - v_0^2 = 2as$$

$$0 - (9 \text{ ft/s})^2 = 2a(18 \text{ ft})$$

$$a = -2.25 \text{ ft/s}^2; \quad a = 2.25 \text{ ft/s}^2 \uparrow$$

DRUM:



$$v_A = r \omega$$

$$9 \text{ ft/s} = (0.75 \text{ ft}) \omega$$

$$\omega = 12 \text{ rad/s}$$

$$a = r \alpha$$

$$-(2.25 \text{ ft/s}^2) = (0.75 \text{ ft}) \alpha$$

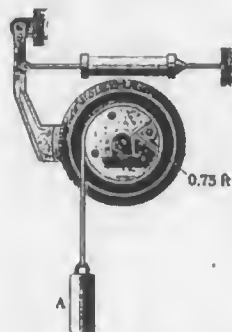
$$\alpha = -3 \text{ rad/s}^2; \quad \alpha = 3 \text{ rad/s}^2 \downarrow$$

UNIFORM MOTION  $\omega = 0$  WHEN  $t = t_f$

$$\omega = \omega_0 + \alpha t; \quad 0 = (12 \text{ rad/s}) - (3 \text{ rad/s}^2) t_f$$

$$t_f = 4 \text{ s}$$

## 15.29



GIVEN:

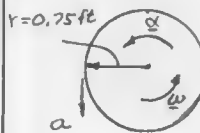
WHEN  $t = 0$ ,  $v = 0$ .  
 WHEN  $t = 5$ , BLOCK  
 HAS MOVED 16 ft.  
 ASSUMING UNIFORM  
 MOTION, FIND:  
 (a)  $\alpha$  OF DRUM  
 (b)  $\omega$  OF DRUM  
 WHEN  $t = 4 \text{ s}$

$$\text{BLOCK A: } s = v_0 t + \frac{1}{2} a t^2$$

$$16 \text{ ft} = 0 + \frac{1}{2} a (5 \text{ s})^2$$

$$a = +1.28 \text{ ft/s}^2 \quad a = 1.28 \text{ ft/s}^2 \downarrow$$

DRUM:



$$a = r \alpha$$

$$(1.28 \text{ ft/s}^2) = (0.75 \text{ ft}) \alpha$$

$$\alpha = 1.707 \text{ rad/s}^2$$

$$\alpha = 1.707 \text{ rad/s}^2 \downarrow$$

UNIFORM MOTION  $\omega_0 = 0$  WHEN  $t = 0$

$$\omega = \omega_0 + \alpha t$$

$$\text{WHEN } t = 4 \text{ s: } \omega = 0 + (1.707 \text{ rad/s}^2)(4 \text{ s})$$

$$\omega = 6.83 \text{ rad/s}$$

$$\omega = 6.83 \text{ rad/s} \downarrow$$

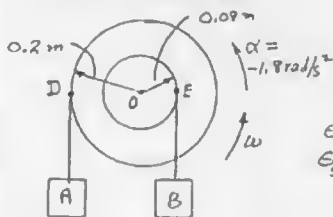
15.30



GIVEN: FOR PULLEY,  
 $\omega_0 = 0.8 \text{ rad/s}$ ,  
 $\alpha = 1.8 \text{ rad/s}^2$ .

FIND: WHEN  $t = 5\text{ s}$ ,  
 THE VELOCITY AND  
 POSITION OF  
 (a) BLOCK A,  
 (b) BLOCK B.

MOTION OF PULLEY



UNIF. ACCEL. MOTION

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ \omega_5 &= (0.8 \text{ rad/s}) + (1.8 \text{ rad/s}^2)(5 \text{ s}) \\ \omega_5 &= 9.8 \text{ rad/s} \\ \omega_5 &= 8.2 \text{ rad/s} \\ \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ \theta_5 &= 0 + (0.8 \text{ rad/s})(5 \text{ s}) + \frac{1}{2}(1.8 \text{ rad/s}^2)(5 \text{ s})^2 \\ \theta_5 &= 18.5 \text{ rad} \\ \theta_5 &= 18.5 \text{ rad}\end{aligned}$$

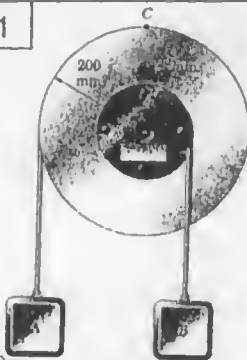
BLOCK A:

$$\begin{aligned}v_A &= r\omega_5 = (0.2 \text{ m})(8.2 \text{ rad/s}) = 1.64 \text{ m/s}; \quad v_A = 1.640 \text{ m/s} \\ s_A &= r\theta_5 = (0.2 \text{ m})(18.5 \text{ rad}) = 3.70 \text{ m}; \quad s_A = 3.70 \text{ m}\end{aligned}$$

BLOCK B:

$$\begin{aligned}v_B &= r\omega_5 = (0.09 \text{ m})(8.2 \text{ rad/s}) = 0.738 \text{ m/s}; \quad v_B = 0.738 \text{ m/s} \\ s_B &= r\theta_5 = (0.09 \text{ m})(18.5 \text{ rad}) = 1.665 \text{ m}; \quad s_B = 1.665 \text{ m}\end{aligned}$$

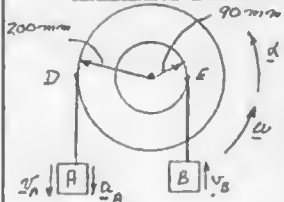
15.31



GIVEN: BLOCK A,  
 $v_0 = 120 \text{ mm/s}$ ,  
 $a_A = 75 \text{ mm/s}^2$

FIND:  
 (a) REVOLUTIONS OF  
 PULLEY IN 6 s.  
 (b) WHEN  $t = 6\text{ s}$  THE  
 VELOCITY AND  
 POSITION OF BLOCK B.  
 (c)  $\alpha_C$  WHEN  $t = 0$

MOTION OF PULLEY



UNIF. ACCEL. MOTION

$$\begin{aligned}v_0 &= r\omega_0, \quad 120 \text{ mm/s} = (200 \text{ mm})\omega_0 \\ \omega_0 &= 0.6 \text{ rad/s} \\ a_A &= r\alpha, \quad 75 \text{ mm/s}^2 = (200 \text{ mm})\alpha \\ \alpha &= 0.375 \text{ rad/s}^2 \\ \text{(a) PULLEY: WHEN } t &= 6\text{ s} \\ \text{(a) } \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ \theta &= 0 + (0.6 \text{ rad/s})(6 \text{ s}) + \frac{1}{2}(0.375 \text{ rad/s}^2)(6 \text{ s})^2 \\ \theta &= 10.35 \text{ rad} \\ \theta &= 10.35 \text{ rad} \left( \frac{\text{rev}}{2\pi \text{ rad}} \right); \quad \theta = 1.647 \text{ rev}\end{aligned}$$

$$\omega_6 = \omega_0 + \alpha t = 0.6 \text{ rad/s} + (0.375 \text{ rad/s}^2)(6 \text{ s}); \quad \omega_6 = 2.85 \text{ rad/s}$$

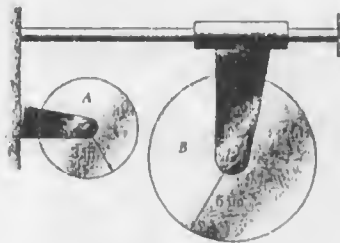
BLOCK B: WHEN  $t = 6\text{ s}$ 

$$\begin{aligned}v_B &= r\omega_6 = (90 \text{ mm})(2.85 \text{ rad/s}) \\ s_B &= r\theta = (90 \text{ mm})(10.35 \text{ rad})\end{aligned}$$

POINT C, WHEN  $t = 0$ 

$$\begin{aligned}a_C &= r\alpha = (200 \text{ mm})(0.375 \text{ rad/s}^2) = 75 \text{ mm/s}^2 \\ a_C &= r\omega^2 = (200 \text{ mm})(0.6 \text{ rad/s})^2 = 72 \text{ mm/s}^2\end{aligned}$$

15.32



GIVEN: WHEN  $t = 0$ ,  
 $(\omega_A)_0 = 450 \text{ rpm}$ ,  
 $(\omega_B)_0 = 0$   
 AFTER SLIPPAGE,  
 WHEN  $t = 6\text{ s}$   
 $\omega_A = 140 \text{ rpm}$

FIND: DURING SLIPPAGE  
 $\alpha_A$  AND  $\alpha_B$

$$\begin{aligned}\text{DISK A: } (\omega_A)_0 &= 450 \text{ rpm} = 47.124 \text{ rad/s} \\ \text{WHEN } t &= 6\text{ s: } \omega_A = 140 \text{ rpm} = 14.661 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}\omega_A &= (\omega_A)_0 + \alpha_A t \\ 14.661 \text{ rad/s} &= 47.124 \text{ rad/s} + \alpha_A(6 \text{ s}) \\ \alpha_A &= -5.41 \text{ rad/s}^2 \\ \alpha_A &= -5.41 \text{ rad/s}^2\end{aligned}$$

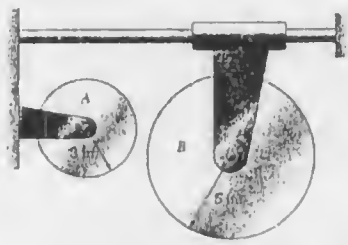
DISK B:  $\omega_B = 0$ WHEN  $t = 6\text{ s}$  (END OF SLIPPAGE)

$$\begin{aligned}r_A \omega_A &= r_B \omega_B; \quad (3 \text{ in.})(14.661 \text{ rad/s}) = (5 \text{ in.})(\omega_B) \\ \omega_B &= 8.796 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}\omega_B &= (\omega_B)_0 + \alpha_B t \\ 8.796 \text{ rad/s} &= 0 + \alpha_B(6 \text{ s}) \\ \alpha_B &= 1.466 \text{ rad/s}^2\end{aligned}$$

$$\alpha_B = 1.466 \text{ rad/s}^2$$

15.33



GIVEN:

DISK A:  $(\omega_A)_0 = 500 \text{ rpm}$   
 WILL COAST TO REST IN 60 s

DISK B:  $(\omega_B)_0 = 0$   
 $\alpha_B = 2.5 \text{ rad/s}^2$

FIND:

(a) WHEN DISKS CAN BE  
 BROUGHT TOGETHER  
 WITH NO SLIPPAGE  
 (b) FINAL  $\omega_A$  AND  $\omega_B$ .

$$\text{DISK A: } (\omega_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

DISK A WILL COAST TO REST IN 60 s

$$\begin{aligned}\omega_A &= (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 \text{ rad/s} + \alpha_A(60 \text{ s}) \\ \alpha_A &= -0.87266 \text{ rad/s}^2\end{aligned}$$

AT TIME  $t$ :

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad \omega_A = 52.36 - 0.87266 t \quad (1)$$

$$\text{DISK B: } \alpha_B = 2.5 \text{ rad/s}^2 \quad (\omega_B)_0 = 0$$

$$\text{AT TIME } t: \quad \omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5 t \quad (2)$$

$$\begin{aligned}\text{(a) BRING DISKS TOGETHER WHEN: } r_A \omega_A &= r_B \omega_B \\ (3 \text{ in.})(52.36 - 0.87266 t) &= (5 \text{ in.})(2.5 t) \\ 157.08 - 2.618 t &= 12.5 t \\ 157.08 &= 15.118 t \\ t &= 10.39 \text{ s}\end{aligned}$$

(b) WHEN CONTACT IS MADE ( $t = 10.39\text{ s}$ )

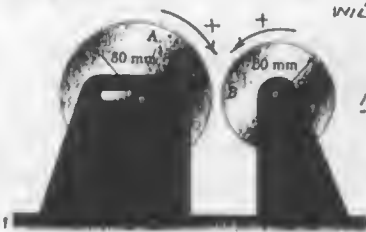
$$\begin{aligned}\text{EQ (1): } \omega_A &= 52.36 - 0.87266(10.39) \\ \omega_A &= 43.79 \text{ rad/s} \\ \omega_A &= 413 \text{ rpm}\end{aligned}$$

$$\text{EQ (2): } \omega_B = 2.5(10.39)$$

$$\omega_B = 25.975 \text{ rad/s} \quad \omega_B = 242 \text{ rpm}$$



15.34



GIVEN: DISK A:  $(\omega_A)_0 = 500 \text{ rpm}$   
WILL COAST TO REST IN 60S.

DISK B:  $(\omega_B)_0 = 0$   
 $\alpha_B = 2.5 \text{ rad/s}^2$

FIND:

- (a) WHEN DISKS CAN BE BROUGHT TOGETHER WITH NO SLIPAGE  
(b) FINAL  $\omega_A$  AND  $\omega_B$

DISK A:  $(\omega_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s}$

DISK A WILL COAST TO REST IN 60S

$\omega_A = (\omega_A)_0 + \alpha_A t$ ;  $0 = 52.36 + \alpha_A (60 \text{ s})$

$\alpha_A = -0.87266 \text{ rad/s}^2$

AT TIME  $t$ :

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad \omega_A = 52.36 - 0.87266 t \quad (1)$$

DISK B:  $\alpha_B = 2.5 \text{ rad/s}^2$   $(\omega_B)_0 = 0$

$$\text{AT TIME } t: \omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5 t \quad (2)$$

- (a) BRING DISKS TOGETHER WHEN:  $r_A \omega_A = r_B \omega_B$   
 $(60 \text{ mm})(52.36 - 0.87266 t) = (60 \text{ mm})(2.5 t)$   
 $4188.8 - 69.83 t = 150 t$   
 $4188.8 = 219.83 t$   
 $t = 19.056 \text{ s} \quad t = 19.06 \text{ s}$

(b) CONTACT IS MADE:

$$\text{EQ. (1): } \omega_A = 52.36 - 0.87266 (19.056)$$

$$\omega_A = 35.73 \text{ rad/s}$$

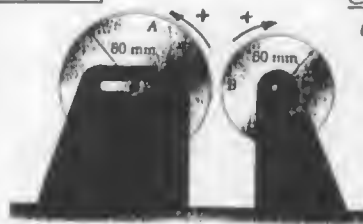
$$\omega_A = 341 \text{ rpm}$$

$$\text{EQ. (2): } \omega_B = 2.5 (19.056)$$

$$\omega_B = 47.64 \text{ rad/s}$$

$$\omega_B = 455 \text{ rpm}$$

15.35



GIVEN:

$(\omega_A)_0 = (\omega_B)_0 = 240 \text{ rpm}$

DISKS ARE BROUGHT TOGETHER, AFTER 8S OF SLIPAGE.

$\omega_A = 60 \text{ rpm}$

FIND: (a)  $\alpha_A$  AND  $\alpha_B$ .

(b) TIME AT WHICH  $\omega_B = 0$ .

(a) DISK A:  $(\omega_A)_0 = 240 \text{ rpm} = 25.133 \text{ rad/s}$

WHEN  $t = 8 \text{ s}$ ,  $\omega_A = 60 \text{ rpm} = 6.283 \text{ rad/s}$

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 6.283 \text{ rad/s} = 25.133 \text{ rad/s} + \alpha_A (8 \text{ s})$$

$$\alpha_A = -2.356 \text{ rad/s}^2$$

$$\alpha_A = 2.36 \text{ rad/s}^2$$

DISK B:  $(\omega_B)_0 = 240 \text{ rpm} = 25.123 \text{ rad/s}$

WHEN  $t = 8 \text{ s}$ : (SLIPAGE STOP)

$$r_A \omega_A = r_B \omega_B$$

$$(60 \text{ mm})(6.283 \text{ rad/s}) = (60 \text{ mm}) \omega_B$$

$$\omega_B = 6.283 \text{ rad/s}$$

$$\omega_B = 6.28 \text{ rad/s}$$

FOR  $t = 8 \text{ s}$ :  $\omega_B = (\omega_B)_0 + \alpha_B t$

$$6.283 \text{ rad/s} = 25.133 \text{ rad/s} + \alpha_B (8 \text{ s})$$

$$\alpha_B = -4.188 \text{ rad/s}^2$$

$$\alpha_B = 4.19 \text{ rad/s}^2$$

(b) TIME WHEN  $\omega_B = 0$

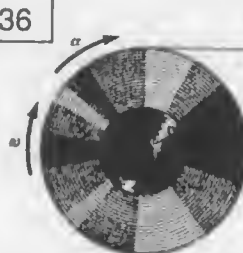
$$\text{FOR } t: \omega_B = (\omega_B)_0 + \alpha_B t$$

$$0 = 25.133 \text{ rad/s} + (-4.188 \text{ rad/s}^2) t$$

$$t = 6.00 \text{ s}$$

$$t = 6.00 \text{ s}$$

15.36

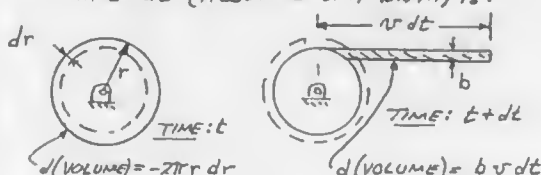


GIVEN: PAPER MOVES AT CONSTANT SPEED  $v$ .  
DERIVE AN EXPRESSION FOR  $\alpha$  OF ROLL.

ANG. VELOCITY IS  $\omega = v/r$ . SINCE  $v$  IS CONSTANT,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt} \left( \frac{v}{r} \right) = v \frac{d}{dt} \left( \frac{1}{r} \right) = - \frac{v}{r^2} \frac{dr}{dt} \quad (1)$$

WE NOTE THAT THE VOLUME OF PAPER UNROLLED IN TIME  $dt$  (ASSUMING UNIT WIDTH) IS:



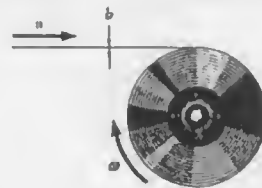
$$\text{THUS: } -2\pi r dr = b v dt$$

$$\frac{dr}{dt} = - \frac{b v}{2\pi r} \quad (2)$$

SUBSTITUTE FOR  $dr/dt$  FROM (2) INTO (1),

$$\alpha = - \frac{v}{r^2} \left( - \frac{b v}{2\pi r} \right) \quad \alpha = \frac{b v^2}{2\pi r^3}$$

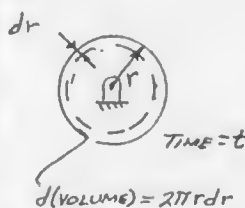
15.37



GIVEN:  $\omega = \omega_0$   
 $\alpha = 0$   
FIND: ACCEL.  $a$  OF TAPE

$$v = r \omega_0$$

$$\frac{v}{r} = \omega_0 \quad (1)$$



$$d(\text{VOLUME}) = b v dt$$

$$\text{THUS: } 2\pi r dr = b v dt$$

$$\frac{dr}{dt} = \frac{b v}{2\pi r} \quad (2)$$

$$v = r \omega_0; \quad a = \frac{dv}{dt} = \omega_0 \frac{dr}{dt}$$

SUBSTITUTE  $\frac{dr}{dt}$  FROM (2):

$$a = \omega_0 \frac{b v}{2\pi r} = \frac{\omega_0 b (v)}{2\pi (r)}$$

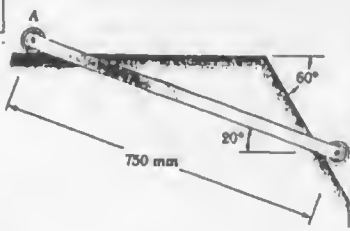
SUBSTITUTE  $v/r$  FROM (1):

$$a = \frac{\omega_0 b}{2\pi} \omega_0 \quad a = \frac{b \omega_0^2}{2\pi}$$

NOTE:  $a$  IS INDEPENDENT OF THE RADIUS  $r$ .

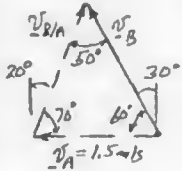
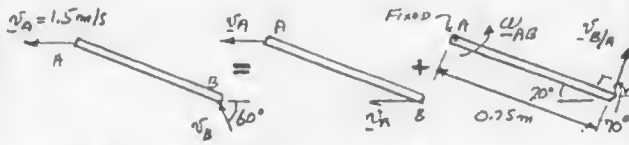


15.38



GIVEN:  $\underline{v}_A = 1.5 \text{ m/s} \leftarrow$

FIND: (a)  $\underline{\omega}_{AB}$   
(b)  $\underline{v}_B$



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

$$[\underline{v}_B \nearrow 60^\circ] = [1.5 \text{ m/s} \leftarrow] + [\underline{v}_{B/A} \nearrow 70^\circ]$$

LAW OF SINES

$$\frac{v_B}{\sin 70^\circ} = \frac{v_{B/A}}{\sin 60^\circ} = \frac{1.5 \text{ m/s}}{\sin 50^\circ}$$

$$\underline{v}_B = 1.840 \text{ m/s} \nearrow 60^\circ$$

$$\underline{v}_{B/A} = 1.696 \text{ m/s} \nearrow 70^\circ$$

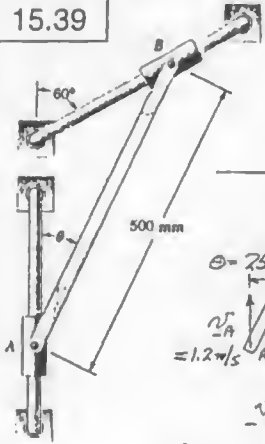
$$v_{B/A} = (AB) \omega_{AB}$$

$$1.696 \text{ m/s} = (0.75 \text{ m}) \omega_{AB}$$

$$\omega_{AB} = 2.261 \text{ rad/s}$$

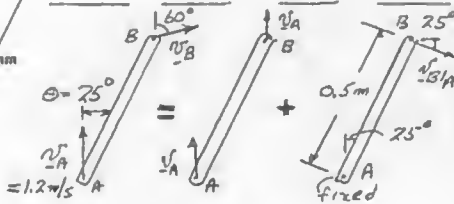
$$\underline{\omega}_{AB} = 2.26 \text{ rad/s}$$

15.39



GIVEN:  $\underline{v}_A = 1.2 \text{ m/s} \uparrow$   
 $\Theta = 25^\circ$

FIND: (a)  $\underline{\omega}_{AB}$   
(b)  $\underline{v}_B$



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

$$[\underline{v}_B \nearrow 30^\circ] = [1.2 \text{ m/s} \uparrow] + [\underline{v}_{B/A} \nearrow 25^\circ]$$

LAW OF SINES

$$\frac{v_B}{\sin 65^\circ} = \frac{v_{B/A}}{\sin 60^\circ} = \frac{1.2 \text{ m/s}}{\sin 55^\circ}$$

$$\underline{v}_B = 1.328 \text{ m/s} \nearrow 30^\circ$$

$$\underline{v}_{B/A} = 1.269 \text{ m/s} \nearrow 65^\circ$$

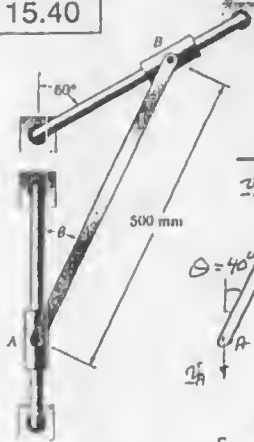
$$v_{B/A} = (AB) \omega_{AB}$$

$$1.269 \text{ m/s} = (0.5 \text{ m}) \omega_{AB}$$

$$\omega_{AB} = 2.538 \text{ rad/s}$$

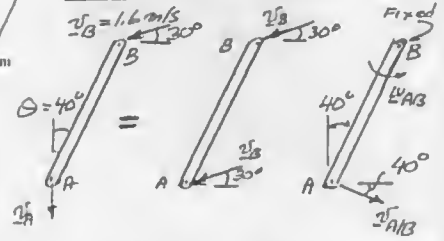
$$\underline{\omega}_{AB} = 2.54 \text{ rad/s}$$

15.40



GIVEN:  $\underline{v}_B = 1.6 \text{ m/s} \leftarrow$   
 $\Theta = 40^\circ$

FIND: (a)  $\underline{\omega}_{AB}$   
(b)  $\underline{v}_A$



$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

$$[\underline{v}_A \uparrow] = [1.6 \text{ m/s} \leftarrow 30^\circ] + [\underline{v}_{A/B} \nearrow 40^\circ]$$

LAW OF SINES

$$\frac{v_A}{\sin 70^\circ} = \frac{v_{A/B}}{\sin 60^\circ} = \frac{1.6 \text{ m/s}}{\sin 50^\circ}$$

$$\underline{v}_A = 1.963 \text{ m/s} \uparrow$$

$$\underline{v}_{A/B} = 1.809 \text{ m/s} \nearrow 40^\circ$$

$$v_{A/B} = (AB) \omega_{AB}$$

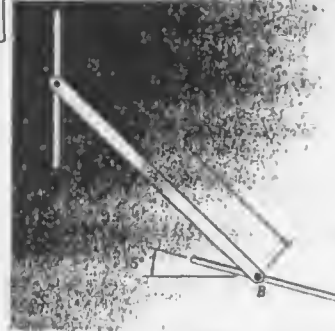
$$1.809 \text{ m/s} = (0.5 \text{ m}) \omega_{AB}$$

$$\omega_{AB} = 3.618 \text{ rad/s}$$

$$AB = 0.5 \text{ m}$$

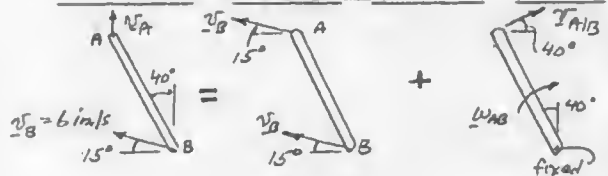
$$\underline{\omega}_{AB} = 3.62 \text{ rad/s}$$

15.41



GIVEN:  $\underline{v}_B = 6 \text{ in/s} \leftarrow$   
 $\Theta = 40^\circ$

FIND: (a)  $\underline{\omega}_{AB}$   
(b)  $\underline{v}_A$



$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

$$[\underline{v}_A \uparrow] = [6 \text{ in/s} \leftarrow 15^\circ] + [\underline{v}_{A/B} \nearrow 40^\circ]$$

LAW OF SINES

$$\frac{v_A}{\sin 55^\circ} = \frac{v_{A/B}}{\sin 75^\circ} = \frac{6 \text{ in/s}}{\sin 50^\circ}$$

$$\underline{v}_A = 6.42 \text{ in/s} \uparrow$$

$$\underline{v}_{A/B} = 7.566 \text{ in/s} \nearrow 40^\circ$$

$$v_{A/B} = (AB) \omega_{AB}$$

$$7.566 \text{ in/s} = (20 \text{ in}) \omega_{AB}$$

$$\omega_{AB} = 0.3783 \text{ rad/s}$$

$$AB = 20 \text{ in}$$

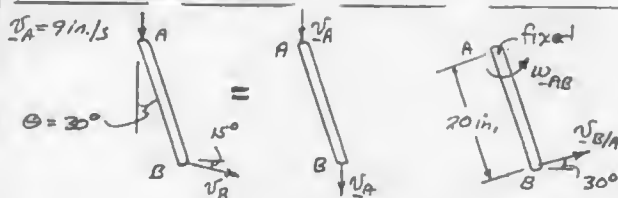
$$\underline{\omega}_{AB} = 0.378 \text{ rad/s}$$

15.42



GIVEN:  
 $v_A = 9 \text{ in./s}$   
 $\theta = 30^\circ$

FIND:  
 (a)  $\omega_{AB}$   
 (b)  $v_B$



$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$[v_B \angle 15^\circ] = [9 \text{ in./s} \angle 15^\circ] + [v_{B/A} \angle 30^\circ]$$

LAW OF SINES

$$\frac{v_B}{\sin 60^\circ} = \frac{v_{B/A}}{\sin 75^\circ} = \frac{9 \text{ in./s}}{\sin 45^\circ}$$

$$v_B = 11.02 \text{ in./s} \angle 15^\circ$$

$$v_{B/A} = 12.294 \text{ in./s} \angle 30^\circ$$

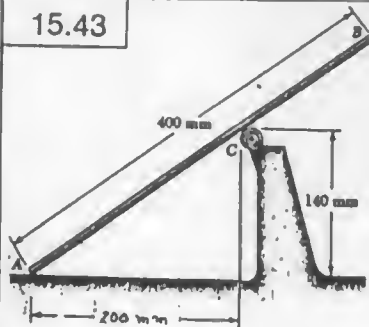
$$v_{B/A} = (AB) \omega_{AB}$$

$$12.294 \text{ in./s} = (20 \text{ in.}) \omega_{AB}$$

$$\omega_{AB} = 0.6147 \text{ rad/s}$$

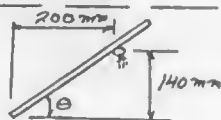
$$\omega_{AB} = 0.615 \text{ rad/s}$$

15.43

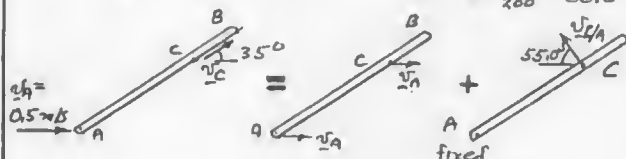


GIVEN:  
 $v_A = 0.5 \text{ m/s}$

FIND:  
 (a)  $\omega_{AB}$   
 (b)  $v_B$



$$\theta = \tan^{-1} \frac{140}{200} = 35.0^\circ$$



RIGHT TRIANGLE

$$v_{B/A} = (0.5 \text{ m/s}) \sin 35.0^\circ$$

$$= 0.2868 \text{ m/s}$$

$$AC^2 = (0.140)^2 + (0.200)^2; AC = 0.2441 \text{ m}$$

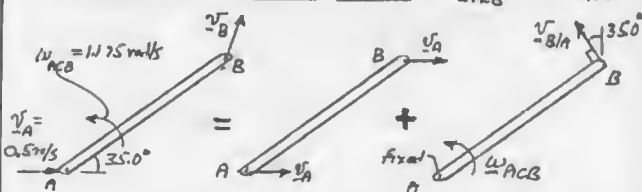
$$v_{B/A} = (AC) \omega_{ACB}$$

$$0.2868 \text{ m/s} = (0.2441 \text{ m}) \omega_{ACB}$$

$$\omega_{ACB} = 1.1747 \text{ rad/s} \quad \omega_{ACB} = 1.175 \text{ rad/s}$$

(CONTINUED)

15.43 CONTINUED

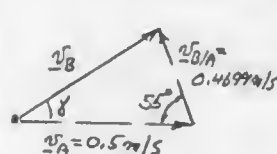
ROD ACB:  $\omega_{ACB} = 1.175 \text{ rad/s}$ 

$$AB = 0.4 \text{ m} \quad v_{B/A} = (AB) \omega_{ACB} = (0.4 \text{ m})(1.175 \text{ rad/s})$$

$$v_{B/A} = 0.4699 \text{ m/s} \angle 55^\circ$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$= [0.5 \text{ m/s} \angle 15^\circ] + [0.4699 \text{ m/s} \angle 55^\circ]$$



LAW OF COSINES

$$v_B^2 = v_A^2 + v_{B/A}^2 - 2v_A v_{B/A} \cos 55^\circ$$

$$= (0.5)^2 + (0.4699)^2 - 2(0.5)(0.4699) \cos 55^\circ$$

$$v_B = 0.449 \text{ m/s}$$

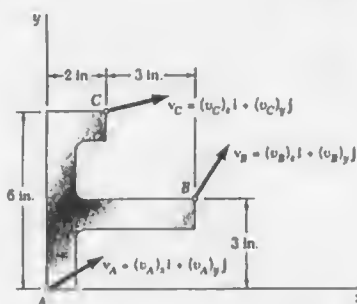
$$\text{LAW OF SINES} \quad \frac{0.4699 \text{ m/s}}{\sin \delta} = \frac{0.449 \text{ m/s}}{\sin 55^\circ}; \delta = 59.1^\circ$$

$$\vec{v}_B = 0.449 \text{ m/s} \angle 59.1^\circ$$

15.44

GIVEN:  $(v_A)_x = 4 \text{ in./s}$   
 $(v_A)_y = -3 \text{ in./s}$   
 $(v_C)_x = 16 \text{ in./s}$

FIND:  
 (a)  $\omega$  of plate  
 (b)  $\vec{v}_A$



$$\vec{v}_A = (4 \text{ in./s}) \hat{i} + (-3 \text{ in./s}) \hat{j} \quad \vec{v}_C = (16 \text{ in./s}) \hat{i} + (v_C)_y \hat{j}$$

$$\omega = \omega \hat{k}; \quad \vec{r}_{C/A} = (2 \text{ in.}) \hat{i} + (6 \text{ in.}) \hat{j}; \quad \vec{r}_{B/A} = (5 \text{ in.}) \hat{i} + (3 \text{ in.}) \hat{j}$$

$$\vec{v}_C = \vec{v}_A + \vec{v}_{C/A} = \vec{v}_A + \omega \times \vec{r}_{C/A} = \vec{v}_A + \omega \hat{k} \times (2 \hat{i} + 6 \hat{j})$$

$$16 \hat{i} + (v_C)_y \hat{j} = 4 \hat{i} + (-3 \text{ in./s}) \hat{j} + 2 \omega \hat{j} - 6 \omega \hat{i}$$

$$\text{COEFFICIENTS OF } \hat{i}: 16 = 4 - 6\omega; \quad \omega = -2$$

$$\omega = -(2 \text{ rad/s}) \hat{k}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \omega \times \vec{r}_{B/A} = \vec{v}_A + \omega \hat{k} \times (5 \hat{i} + 3 \hat{j})$$

$$(\vec{v}_B)_x \hat{i} - 3 \hat{j} = 4 \hat{i} + (-3 \text{ in./s}) \hat{j} + 5 \omega \hat{j} - 3 \omega \hat{i}$$

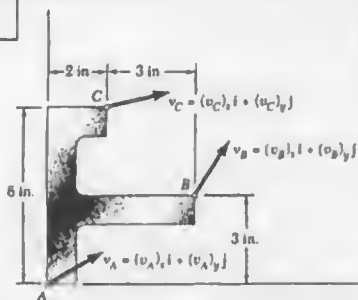
$$\text{COEFFICIENTS OF } \hat{j}: -3 = (-3 \text{ in./s}) + 5\omega$$

$$-3 = (-3 \text{ in./s}) + 5(-2); \quad (v_A)_y = 7 \text{ in./s}$$

$$\vec{v}_A = (v_A)_x \hat{i} + (v_A)_y \hat{j}$$

$$\vec{v}_A = (4 \text{ in./s}) \hat{i} + (7 \text{ in./s}) \hat{j}$$

15.45



GIVEN:

$$\begin{aligned} (v_A)_x &= 4 \text{ in./s} \\ (v_B)_y &= -3 \text{ in./s} \\ (v_C)_x &= 16 \text{ in./s} \end{aligned}$$

FIND:

LOCUS OF  
POINTS OF  
PLATE WITH  
 $v = 8 \text{ in./s}$

FROM THE ANSWER OF PROB. 15.44, WE HAVE

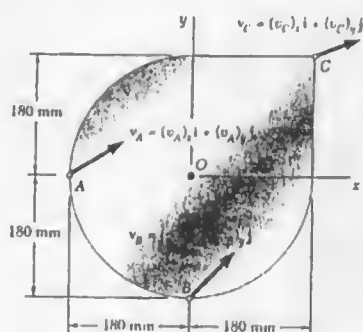
$$\begin{aligned} \omega &= -(2 \text{ rad/s}) \mathbf{k} & \mathbf{v}_A &= (4 \text{ in./s}) \mathbf{i} + (7 \text{ in./s}) \mathbf{j} \\ \text{LET } P &= x \mathbf{i} + y \mathbf{j} \text{ BE AN ARBITRARY POINT} \\ \text{THUS: } \mathbf{r}_{P/A} &= x \mathbf{i} + y \mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_P &= \mathbf{v}_A + \mathbf{v}_{PA} = \mathbf{v}_A + \omega \times \mathbf{r}_{P/A} = 4 \mathbf{i} + 7 \mathbf{j} + (-2 \mathbf{k}) \times (x \mathbf{i} + y \mathbf{j}) \\ \mathbf{v}_A &= (4 + 2y) \mathbf{i} + (7 - 2x) \mathbf{j} \\ (v_A)_x &= 4 + 2y & (v_A)_y &= 7 - 2x \\ v^2 &= (v_A)_x^2 + (v_A)_y^2 & \text{WE SEEK } v_A &= 8 \text{ in./s} \\ 8^2 &= (4 + 2y)^2 + (7 - 2x)^2 \end{aligned}$$

$$\text{SIMPLIFY: } (x - 3.5)^2 + (y + 2)^2 = 4^2$$

NOTE: LOCUS IS A CIRCLE OF RADIUS 4 in. WITH  
CENTER AT  $x = 3.5 \text{ in.}, y = -2 \text{ in.}$

15.46



GIVEN:

$$\begin{aligned} (v_A)_x &= 120 \text{ mm/s} \\ (v_B)_y &= 300 \text{ mm/s} \\ (v_C)_y &= -60 \text{ mm/s} \end{aligned}$$

FIND:

$$\begin{aligned} (a) \omega \\ (b) \mathbf{v}_A \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{C/B} &= (180 \text{ mm}) \mathbf{i} + (360 \text{ mm}) \mathbf{j} \\ \omega &= \omega \mathbf{k} \end{aligned}$$

$$\mathbf{v}_B = (v_B)_x \mathbf{i} + (300 \text{ mm/s}) \mathbf{j}; \quad \mathbf{v}_C = (v_C)_x \mathbf{i} - (60 \text{ mm/s}) \mathbf{j}$$

(a)

$$\begin{aligned} \mathbf{v}_C &= \mathbf{v}_B + \mathbf{v}_{CB} \\ (v_C)_x \mathbf{i} - (60 \text{ mm/s}) \mathbf{j} &= (v_B)_x \mathbf{i} + (300 \text{ mm/s}) \mathbf{j} + \omega \times \mathbf{r}_{C/B} \\ (v_C)_x \mathbf{i} - 60 \mathbf{j} &= (v_B)_x \mathbf{i} + 300 \mathbf{j} + \omega \mathbf{k} \times (180 \mathbf{i} + 360 \mathbf{j}) \\ (v_C)_x \mathbf{i} - 60 \mathbf{j} &= (v_B)_x \mathbf{i} + 300 \mathbf{j} + 180 \omega \mathbf{j} - 360 \omega \mathbf{i} \end{aligned}$$

COEFFICIENTS OF  $\mathbf{j}$ :  $-60 = 300 + 180 \omega$ 

$$\omega = -2 \text{ rad/s}$$

$$\omega = 2 \text{ rad/s}$$

(b) VELOCITY OF A:

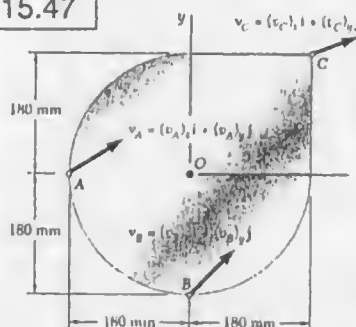
$$\mathbf{r}_{A/B} = -(180 \text{ mm}) \mathbf{i} + (180 \text{ mm}) \mathbf{j}$$

$$\begin{aligned} \mathbf{v}_A &= \mathbf{v}_B + \mathbf{v}_{AB} = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B} \\ 120 \mathbf{i} + (v_A)_y \mathbf{j} &= (v_B)_x \mathbf{i} + 300 \mathbf{j} + (-2 \mathbf{k}) \times (-180 \mathbf{i} + 180 \mathbf{j}) \\ 120 \mathbf{i} + (v_A)_y \mathbf{j} &= (v_B)_x \mathbf{i} + 300 \mathbf{j} + 360 \mathbf{j} + 360 \mathbf{i} \end{aligned}$$

COEFFICIENTS OF  $\mathbf{j}$ :  $(v_A)_y = 300 + 360 = 660 \text{ mm/s}$ 

$$\mathbf{v}_A = (120 \text{ mm/s}) \mathbf{i} + (660 \text{ mm/s}) \mathbf{j}$$

15.47



GIVEN:

$$\begin{aligned} (v_A)_x &= 120 \text{ mm/s} \\ (v_B)_y &= 300 \text{ mm/s} \\ (v_C)_y &= -60 \text{ mm/s} \end{aligned}$$

FIND:

(a)  $\mathbf{v}_B$   
(b) POINT OF  
ZERO VELOCITY

$$\mathbf{r}_{B/A} = (180 \text{ mm/s}) \mathbf{i} - (180 \text{ mm/s}) \mathbf{j}$$

FROM THE ANSWER OF PROB. 15.46, WE HAVE

$$\omega = -(2 \text{ rad/s}) \mathbf{k}; \quad \mathbf{v}_A = (120 \text{ mm/s}) \mathbf{i} + (660 \text{ mm/s}) \mathbf{j}$$

(a) VELOCITY OF B:

$$\begin{aligned} \mathbf{v}_B &= \mathbf{v}_A + \mathbf{v}_{BA} = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} \\ &= 120 \mathbf{i} + 660 \mathbf{j} - 2 \mathbf{k} \times (180 \mathbf{i} - 180 \mathbf{j}) \\ &= 120 \mathbf{i} + 660 \mathbf{j} - 360 \mathbf{j} - 360 \mathbf{i} \\ \mathbf{v}_B &= -(240 \text{ mm/s}) \mathbf{i} + (300 \text{ mm/s}) \mathbf{j} \end{aligned}$$

(b) POINT WITH  $v = 0$ :LET  $P = x \mathbf{i} + y \mathbf{j}$  BE AN ARBITRARY POINT

$$\text{THUS: } \mathbf{r}_{P/A} = (180 + x) \mathbf{i} + y \mathbf{j}$$

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{PA} = \mathbf{v}_A + \omega \times \mathbf{r}_{P/A}$$

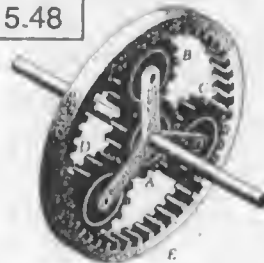
$$\mathbf{v}_P = 120 \mathbf{i} + 660 \mathbf{j} + (-2 \mathbf{k}) \times [(180 + x) \mathbf{i} + y \mathbf{j}]$$

$$\mathbf{v}_P = 120 \mathbf{i} + 660 \mathbf{j} - (360 + 2x) \mathbf{j} + 2y \mathbf{i}$$

$$\mathbf{v}_P = (120 + 2y) \mathbf{i} + (300 - 2x) \mathbf{j}$$

FOR  $\mathbf{v}_P = 0$   $120 + 2y = 0$  and  $300 - 2x = 0$ 
 $\mathbf{v} = 0$  AT:  $y = -60 \text{ mm}, x = 150 \text{ mm}$ 

15.48



GIVEN

$$r_A = r_B = r_C = 3 \text{ in.}; r_D = 9 \text{ in.}$$

$$\omega_A = 150 \text{ rpm}$$

$$\omega_E = 120 \text{ rpm}$$

FIND: (a)  $\omega_B$ (b)  $\omega_S = \omega_{\text{SPIDER}}$ 

GEAR A:

$$\begin{aligned} \mathbf{v}_D &= 3 \omega_A \\ \omega_A &= 150 \text{ rpm} \end{aligned}$$

OUTER GEAR E

$$\begin{aligned} \mathbf{v}_E &= 9 \omega_E \\ \omega_E &= 120 \text{ rpm} \end{aligned}$$

PLANETARY GEAR

$$\begin{aligned} \mathbf{v}_E &= 9 \omega_E \\ \mathbf{v}_D &= 3 \omega_D \\ \mathbf{v}_B &= 3 \omega_B \end{aligned}$$

$$\mathbf{v}_E = \mathbf{v}_D + 6 \omega_B$$

$$9 \omega_E = 3 \omega_D + 6 \omega_B$$

$$9(120 \text{ rpm}) = 3(150 \text{ rpm}) + 6 \omega_B$$

$$\omega_B = 105 \text{ rpm}$$

$$\omega_B = 105 \text{ rpm}$$

SPIDER:

$$\mathbf{v}_B = 6 \omega_S$$

$$\mathbf{v}_E = 6 \omega_S$$

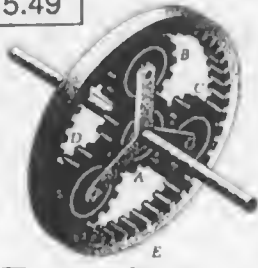
$$3(\omega_A + \omega_B) = 6 \omega_S$$

$$\omega_S = \frac{1}{2}(\omega_A + \omega_B) = \frac{1}{2}(150 \text{ rpm} + 105 \text{ rpm})$$

$$\omega_S = 127.5 \text{ rpm}$$

$$\omega_S = 127.5 \text{ rpm}$$

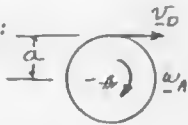
15.49



GIVEN:  $r_A = a$   
 $r_B = r_C = r_D = b$   
 $r_E = a + 2b$   
 $\omega_A = \omega_B$ ,  $\omega_E = 0$   
 FIND:  
 (a) RATIO  $b/a$  FOR WHICH:  
 $\omega_S = \omega_{SPIDER} = \frac{1}{5} \omega_A$   
 (b)  $\omega_S$

PLANETARY GEAR:

GEAR A:

GEAR A:  $v_D = a\omega_A \rightarrow$ 

PLANETARY GEAR:

 $v_D = v_E + v_{D/E}$ 

$$a\omega_A = 0 + (2b)\omega_B: \omega_B = \frac{a}{2b}\omega_A \quad (1)$$

$$v_B = v_E + v_{B/E} = 0 + b\omega_B = b\left(\frac{a}{2b}\omega_A\right)$$

$$v_B = \frac{1}{2}a\omega_A \rightarrow$$

SPIDER:  $v_B = (a+b)\omega_S$ 

$$\frac{1}{2}a\omega_A = (a+b)\omega_S \quad \omega_S = \frac{a}{2(a+b)}\omega_A \quad (2)$$

SUBSTITUTE DATA:  $\omega_S = \frac{1}{5}\omega_A$  INTO (2)

$$\frac{1}{5}\omega_A = \frac{a}{2(a+b)}\omega_A \quad 2a + 2b = 5a \quad \frac{b}{a} = 1.5$$

$$EQ(1): \omega_B = \frac{1}{2}\left(\frac{a}{b}\right)\omega_A = \frac{1}{2}\left(\frac{1}{1.5}\right)\omega_A \quad \omega_B = \frac{1}{3}\omega_A$$

15.50



GIVEN:  
 $\omega_A = 120 \text{ rpm}$   
 $\omega_{AB} = 90 \text{ rpm}$   
 FIND:  $\omega_B$

ARM AB:

$$AB = (60 + 90) \text{ mm} = 150 \text{ mm}$$

$$v_B = (AB)\omega_{AB} = 150\omega_{AB}$$

GEAR A:

$$r_A = 60 \text{ mm} \quad v_D = r_A\omega_A = 60\omega_A$$

GEAR B:

$$r_B = 90 \text{ mm} \quad v_B = r_B\omega_B = 90\omega_B$$

$$\rightarrow v_B = v_D + v_{B/D}; 150\omega_{AB} = 60\omega_A + 90\omega_B \quad (1)$$

(CONTINUED)

15.50 CONTINUED

$$EQ(1): \rightarrow 150\omega_{AB} = 60\omega_A + 90\omega_B$$

$$DATA: \omega_A = 120 \text{ rpm} \quad \omega_{AB} = 90 \text{ rpm}$$

$$150(90 \text{ rpm}) = 60(120 \text{ rpm}) + 90\omega_B$$

$$\omega_B = +70 \text{ rpm}$$

$$\omega_B = 70 \text{ rpm}$$

15.51



GIVEN:

$$\omega_{AB} = 42 \text{ rpm}$$

FIND:

(a)  $\omega_A$  FOR WHICH  $\omega_B$  IS 20 rpm(b)  $\omega_A$  FOR WHICH  $\omega_B = 0$  (CURVILINEAR TRANSLATION)

SEE FIRST PART OF SOLUTION OF PROB 15.50 FOR DERIVATION OF

$$\rightarrow 150\omega_{AB} = 60\omega_A + 90\omega_B \quad (1)$$

$$(a) \text{ For } \omega_B = 20 \text{ rpm}, \quad \omega_B = -20 \text{ rpm}$$

$$EQ(1): \rightarrow 150(42 \text{ rpm}) = 60\omega_A + 90(-20 \text{ rpm})$$

$$\omega_A = +135 \text{ rpm}$$

$$\omega_A = 135 \text{ rpm}$$

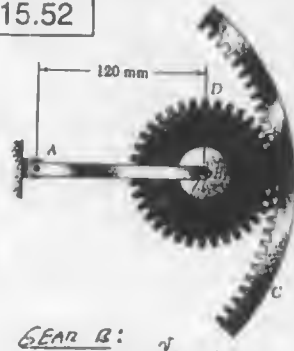
$$(b) \text{ For } \omega_B = 0:$$

$$EQ(1): \rightarrow 150(42 \text{ rpm}) = 60\omega_A + 0$$

$$\omega_A = +105 \text{ rpm}$$

$$\omega_A = 105 \text{ rpm}$$

15.52

GIVEN:  $\omega_{AB} = 20 \text{ rad/s}$ 

FIND:

(a)  $\omega_B$ (b)  $v_D$ 

$$\text{ARM AB: } \omega_{AB} = 20 \text{ rad/s} \quad v_B = (120 \text{ mm})(20 \text{ rad/s}) = 2.4 \text{ m/s} \uparrow$$

GEAR B:

$$v_B = 2.4 \text{ m/s} \quad v_D = 0 \quad v_E = 0 \quad v_{D/E} = 0$$

$$(a) \text{ BE} = 0.05 \text{ m}: \quad v_B = v_E + v_{B/E} = 0 + (BE)\omega_B$$

$$2.4 \text{ m/s} \uparrow = 0 + (0.05 \text{ m})\omega_B \uparrow$$

$$\omega_B = 48 \text{ rad/s}$$

$$\omega_B = 48 \text{ rad/s}$$

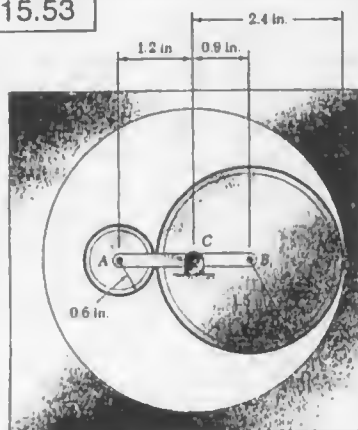
$$(b) \text{ DE} = (0.05\sqrt{2}): \quad v_D = v_E + v_{D/E} = 0 + (DE)\omega_B$$

$$v_D = 0 + (0.05\sqrt{2})(48)$$

$$v_D = 2.39 \text{ m/s}$$

$$v_D = 2.39 \text{ m/s} \angle 45^\circ$$

15.53

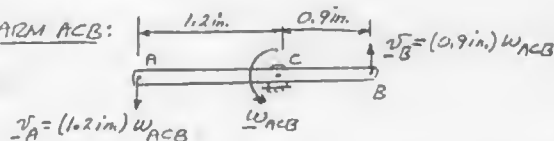


GIVEN:  
 $\omega_{ACB} = 40 \text{ rad/s}$

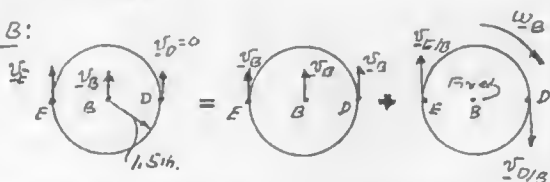
FIND:

- (a)  $\omega_A$   
 (b)  $\omega_B$

ARM ACB:



DISK B:



DISK ROLLS ON D:

$$v_D = v_B + v_{D/B} = v_B + (BD)\omega_B$$

$$0 = (0.9 \text{ in.})\omega_{ACB} - (1.5 \text{ in.})\omega_B$$

$$\omega_B = 0.6 \omega_{ACB} = 0.6(40 \text{ rad/s})$$

$$\omega_B = 24 \text{ rad/s}$$

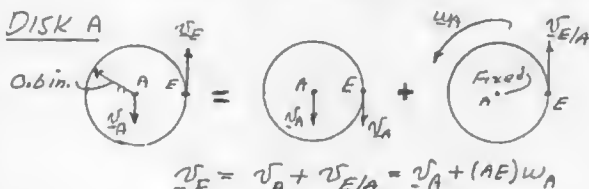
POINT OF CONTACT E OF THE DISKS:

$$v_E = v_B + v_{E/B} = v_B + (EB)\omega_B$$

$$v_E = (0.9 \text{ in.})\omega_{ACB} + (1.5 \text{ in.})(0.6\omega_{ACB})$$

$$v_E = (0.9 \text{ in.} + 0.9 \text{ in.})\omega_{ACB} = (1.8 \text{ in.})\omega_{ACB}$$

DISK A



$$v_E = v_A + v_{E/A} = v_A + (AE)\omega_A$$

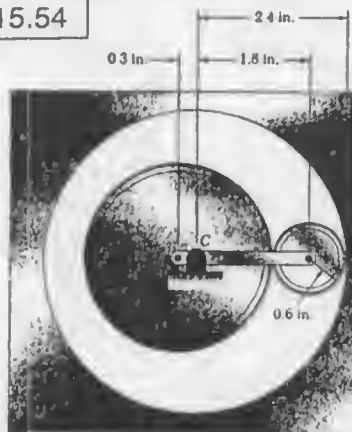
$$(1.8 \text{ in.})\omega_{ACB} = -(1.2 \text{ in.})\omega_{ACB} + (0.6 \text{ in.})\omega_A$$

$$\omega_A = \frac{1.8 + 1.2}{0.6} \omega_{ACB} = 5 \omega_{ACB}$$

$$\omega_A = 5(40 \text{ rad/s}) = 200 \text{ rad/s}$$

$$\omega_A = 200 \text{ rad/s}$$

15.54

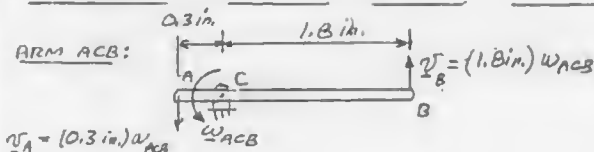


GIVEN:  
 $\omega_{ACB} = 40 \text{ rad/s}$

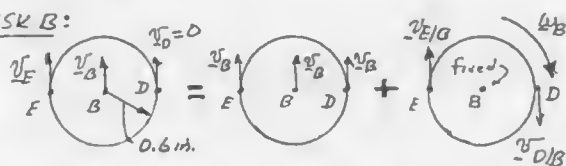
FIND:

- (a)  $\omega_A$   
 (b)  $\omega_B$

ARM ACB:



DISK B:



DISK ROLLS ON D:

$$v_D = v_B + v_{D/B} = v_B + (BD)\omega_B$$

$$0 = (1.8 \text{ in.})\omega_{ACB} - (0.6 \text{ in.})\omega_B$$

$$\omega_B = 3 \omega_{ACB} = 3(40 \text{ rad/s})$$

$$\omega_B = 120 \text{ rad/s}$$

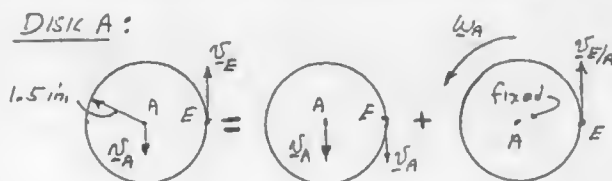
POINT OF CONTACT E OF THE DISKS:

$$v_E = v_B + v_{E/B} = v_B + (EB)\omega_B$$

$$v_E = (1.8 \text{ in.})\omega_{ACB} + (0.6 \text{ in.})(3\omega_{ACB})$$

$$v_E = (1.8 \text{ in.} + 1.8 \text{ in.})\omega_{ACB} = (3.6 \text{ in.})\omega_{ACB}$$

DISK A:



$$v_E = v_A + v_{E/A} = v_A + (AE)\omega_A$$

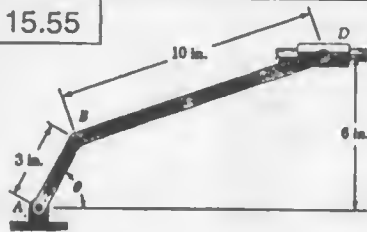
$$(3.6 \text{ in.})\omega_{ACB} = -(0.3 \text{ in.})\omega_{ACB} + (1.5 \text{ in.})\omega_A$$

$$\omega_A = \frac{3.6 + 0.3}{1.5} \omega_{ACB} = 2.6 \omega_{ACB}$$

$$\omega_A = 2.6(40 \text{ rad/s}) = 104 \text{ rad/s}$$

$$\omega_A = 104 \text{ rad/s}$$

15.55

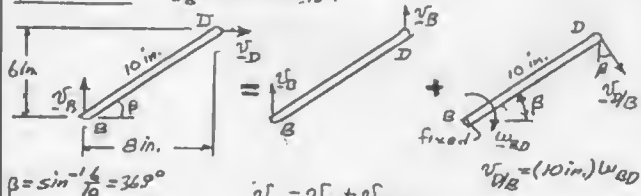


GIVEN:  
 $\omega_{AB} = 160 \text{ rpm}$   
 FIND:  $\omega_{BD}$  AND  $v_D$  WHEN  
 (a)  $\theta = 0$   
 (b)  $\theta = 90^\circ$

CRANK AB:

$$\omega_{AB} = 160 \text{ rpm} = 16.755 \text{ rad/s}$$

(a)  $\theta = 0$ :  $v_B = 50.27 \text{ in./s} \uparrow$



$$\beta = \sin^{-1} \frac{6}{10} = 36.9^\circ$$

$$v_D = v_B + v_{D/B}$$

$$[v_D \rightarrow] = [v_B \uparrow] + [10\omega_{BD}]$$

$$v_D = v_B \tan \beta = (50.27 \text{ in./s}) \tan 36.9^\circ$$

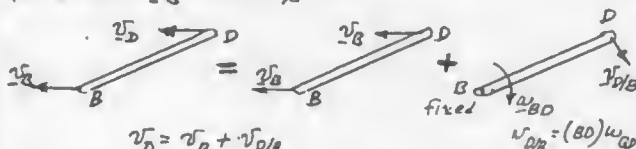
$$v_D = 37.7 \text{ m/s} \quad v_D = 377 \text{ in./s} \rightarrow$$

$$v_B = v_{D/B} \cos \beta$$

$$50.27 \text{ in./s} = (10\omega_{BD}) \cos 36.9^\circ$$

$$\omega_{BD} = 6.28 \text{ rad/s} = 60 \text{ rpm} \quad \omega_{BD} = 60 \text{ rpm} \rightarrow$$

(b)  $\theta = 90^\circ$ :  $v_B = 50.27 \text{ in./s} \rightarrow$



$$v_D = v_B + v_{D/B}$$

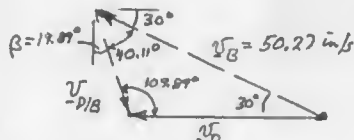
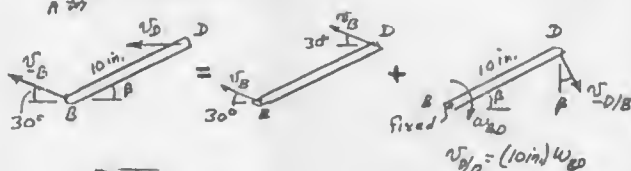
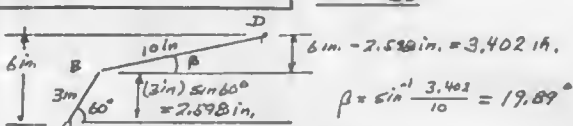
$$[v_D \rightarrow] = [v_B \rightarrow] + [(10)\omega_{BD} \downarrow]$$

$$\uparrow \text{ yields } (10)\omega_{BD} = 0 \quad \omega_{BD} = 0$$

$$\pm v_D = v_B + 0 = 50.27 \text{ in./s} \quad v_D = 50.3 \text{ in./s} \rightarrow$$

15.56 CONTINUED

ROD BD:



LAW OF SINES

$$\frac{v_D}{\sin 40.11^\circ} = \frac{v_{D/B}}{\sin 30^\circ} = \frac{50.27 \text{ in./s}}{\sin 109.89^\circ}$$

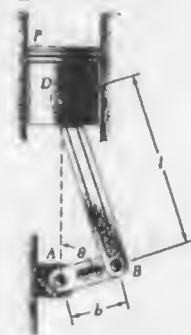
$$v_D = 34.44 \text{ in./s}$$

$$v_D = 34.4 \text{ in./s} \leftarrow$$

$$v_{D/B} = (10 \text{ in.}) \omega_{BD} = 26.73 \text{ in./s}$$

$$\omega_{BD} = 2.67 \text{ rad/s} \rightarrow$$

15.57



GIVEN:

$$l = 160 \text{ mm}$$

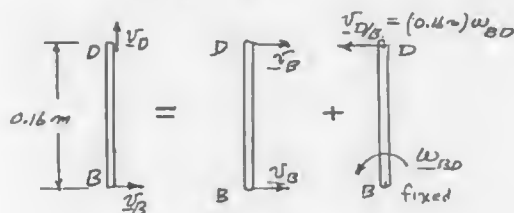
$$b = 60 \text{ mm}$$

$$\omega_{AB} = 1000 \text{ rpm} \rightarrow$$

FIND:  $v_P$  AND  $\omega_{BD}$   
 WHEN (a)  $\theta = 0$   
 (b)  $\theta = 90^\circ$

CRANK AB:  $\omega_{AB} = 1000 \text{ rpm} = 104.72 \text{ rad/s} \rightarrow$

(a)  $\theta = 0$ :  $v_B = (0.06 \text{ m})(104.72 \text{ rad/s})$   
 $v_B = 6.283 \text{ m/s} \rightarrow$



$$v_D = v_B + v_{D/B}$$

$$v_D \uparrow = [6.283 \text{ m/s} \rightarrow] + [v_{D/B} \leftarrow]$$

$$\uparrow v_D = 0; v_P = v_D; v_P = 0$$

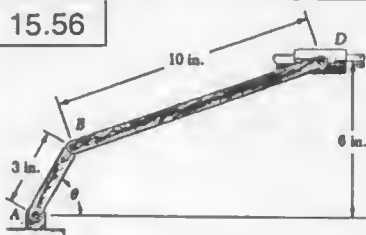
$$\pm 0 = 6.283 \text{ m/s} - v_{D/B}$$

$$v_{D/B} = 6.283 \text{ m/s} \leftarrow$$

$$(0.16 \text{ m}) \omega_{BD} = 6.283 \text{ m/s}; \omega_{BD} = 39.3 \text{ rad/s} \rightarrow$$

(CONTINUED)

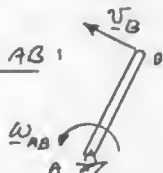
15.56



GIVEN:  
 $\omega_{AB} = 160 \text{ rpm}$   
 $\theta = 60^\circ$

FIND:  
 $\omega_{BD}$  AND  $v_D$

CRANK AB:



$$\omega_{AB} = 160 \text{ rpm} = 16.755 \text{ rad/s}$$

$$AB = 3 \text{ in.}$$

$$v_B = (AB) \omega_{AB}$$

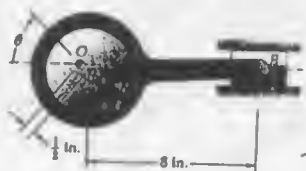
$$= (3 \text{ in.})(16.755 \text{ rad/s})$$

$$v_B = 50.27 \text{ in./s} \angle 30^\circ$$

(CONTINUED)

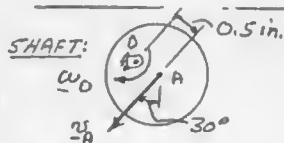


15.62



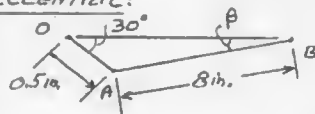
GIVEN:  $\theta = 30^\circ$   
 $\omega_D = 900 \text{ rpm}$   
 $OA = 0.5 \text{ in.}$

FIND:  $\underline{v}_B$



$\omega_D = 900 \text{ rpm} = 94.248 \text{ rad/s}$   
 $\underline{v}_A = (0.5 \text{ in.})(94.248 \text{ rad/s})$   
 $\underline{v}_A = 47.124 \text{ in/s}$   $\nearrow 60^\circ$

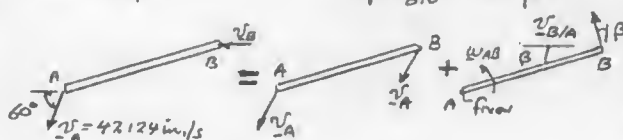
ECCENTRIC:



LAW OF SINES

$$\frac{\sin \beta}{0.5 \text{ in.}} = \frac{\sin 30^\circ}{8 \text{ in.}}$$

$$\sin \beta = \frac{0.5}{8.0} \sin 30^\circ; \beta = 1.79^\circ$$



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

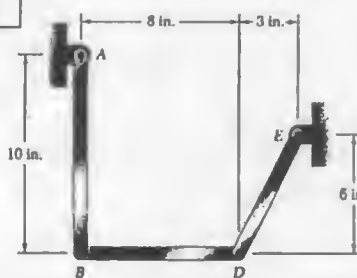
LAW OF SINES

$$\frac{\underline{v}_B}{\sin(30^\circ + \beta)} = \frac{\underline{v}_A}{\sin(90^\circ - \beta)}$$

$$\underline{v}_B = \frac{\sin(30^\circ + 1.79^\circ)}{\sin(90^\circ - 1.79^\circ)} (47.124 \text{ in/s})$$

$$\underline{v}_B = 24.8 \text{ in/s} \leftarrow$$

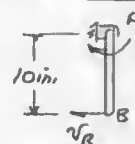
15.63



GIVEN:  
 $\omega_{AB} = 4 \text{ rad/s}$

FIND:  
 $\omega_{BD}$   
 $\omega_{DE}$

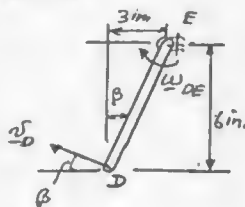
BAR AB:



$$\underline{v}_B = (10 \text{ in.})(4 \text{ rad/s})$$

$$\underline{v}_B = 40 \text{ in/s} \leftarrow$$

BAR DE:



$$\tan \beta = \frac{3 \text{ in.}}{6 \text{ in.}}; \beta = 26.57^\circ$$

$$DE = \frac{6 \text{ in.}}{\cos \beta} = 6.708 \text{ in.}$$

$$\underline{v}_D = (DE) \omega_{DE}$$

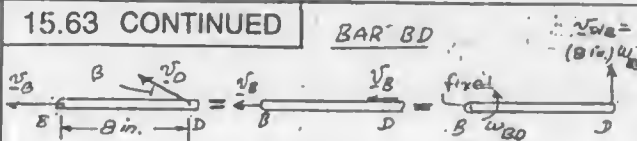
$$= (6.708 \text{ in.}) \omega_{DE}$$

$$\underline{v}_D = 6.708 \omega_{DE} \nearrow 26.57^\circ$$

(CONTINUED)

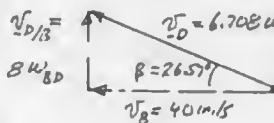
15.63 CONTINUED

BAR BD



$$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$$

$$[\underline{v}_D \nearrow \beta] = [\underline{v}_B \leftarrow] + [\underline{v}_{D/B} \uparrow]$$



$$\underline{v}_{D/B} = \underline{v}_B \tan \beta$$

$$8 \omega_{BD} = 40 \tan 26.57^\circ$$

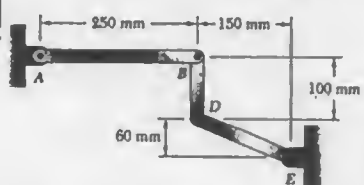
$$\omega_{BD} = 2.5 \text{ rad/s} \leftarrow$$

$$\underline{v}_B = \underline{v}_D \cos \beta$$

$$40 = 6.708 \omega_{DE} \cos 26.57^\circ$$

$$\omega_{DE} = 6.67 \text{ rad/s} \leftarrow$$

15.64



GIVEN:  
 $\omega_{AB} = 4 \text{ rad/s}$

FIND:  
 $\omega_{BD}$   
 $\omega_{DE}$

BAR AB:

$$\underline{v}_B = (0.25 \text{ m})(4 \text{ rad/s}) = 1 \text{ m/s}$$

$$\omega_{AB} = 4 \text{ rad/s}$$

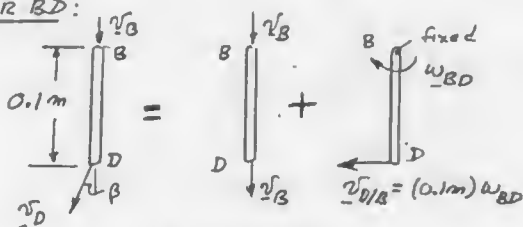
BAR DE:

$$\beta = \tan^{-1} \frac{0.06 \text{ m}}{0.15 \text{ m}} = 21.8^\circ$$

$$DE = \frac{0.15 \text{ m}}{\cos \beta} = 0.1616 \text{ m}$$

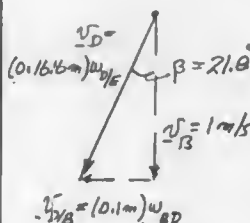
$$\underline{v}_D = (0.1616 \text{ m}) \omega_{DE}$$

BAR BD:



$$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$$

$$[\underline{v}_D \nearrow \beta] = [\underline{v}_B \uparrow] + [\underline{v}_{D/B} \leftarrow]$$



$$\underline{v}_{D/B} = \underline{v}_B \tan \beta$$

$$(0.1 \text{ m}) \omega_{BD} = (1 \text{ m/s}) \tan 21.8^\circ$$

$$\omega_{BD} = 4 \text{ rad/s} \leftarrow$$

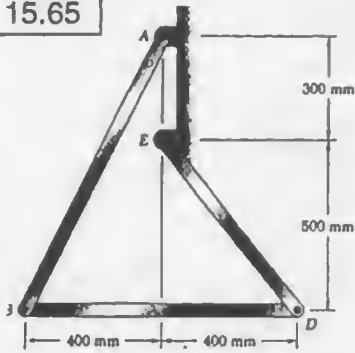
$$\underline{v}_D = \underline{v}_B / \cos \beta$$

$$(0.1616 \text{ m}) \omega_{DE} = (1 \text{ m/s}) / \cos 21.8^\circ$$

$$\omega_{DE} = 6.67 \text{ rad/s} \leftarrow$$



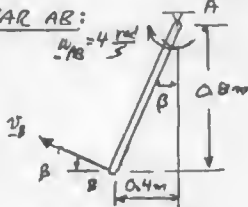
15.65



GIVEN:  
 $\omega_{AB} = 4 \text{ rad/s} \curvearrowright$

FIND:  
 $\omega_{BD}$   
 $\omega_{DE}$

BAR AB:



$$\beta = \tan^{-1} \frac{0.4}{0.8} = 26.56^\circ$$

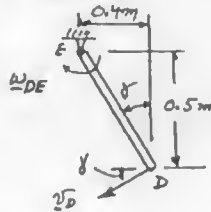
$$AB = \frac{0.8}{\cos \beta} = 0.8944 \text{ m}$$

$$v_B = (AB) \omega_{AB}$$

$$= (0.8944 \text{ m})(4 \text{ rad/s})$$

$$v_B = 3.578 \text{ m/s} \curvearrowright 26.56^\circ$$

BAR DE:



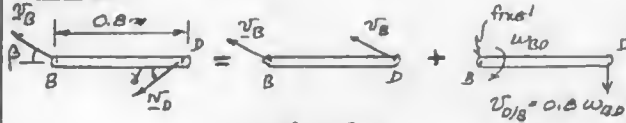
$$\gamma = \tan^{-1} \frac{0.4}{0.5} = 38.66^\circ$$

$$DE = \frac{0.5}{\cos \gamma} = 0.6403 \text{ m}$$

$$v_D = (DE) \omega_{DE}$$

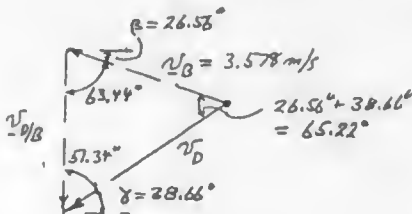
$$v_D = (0.6403 \text{ m}) \omega_{DE} \curvearrowright 38.66^\circ$$

BAR BD:



$$v_D = v_B + v_{D/B}$$

$$[v_D \curvearrowright \gamma] = [v_B \curvearrowright \beta] + [v_{D/B} \downarrow]$$



LAW OF SINES

$$\frac{v_D}{\sin 63.44^\circ} = \frac{v_{D/B}}{\sin 65.22^\circ} = \frac{3.578 \text{ m/s}}{\sin 51.34^\circ}$$

$$v_D = 4.099 \text{ m/s}$$

$$(0.6403 \text{ m}) \omega_{DE} = 4.099 \text{ m/s}$$

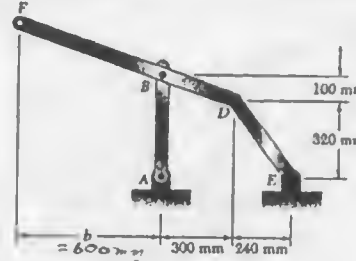
$$\omega_{DE} = 6.4 \text{ rad/s} \curvearrowright$$

$$v_{D/B} = 4.160 \text{ m/s}$$

$$(0.8 \text{ m}) \omega_{BD} = 4.16 \text{ m/s}$$

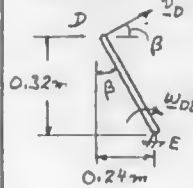
$$\omega_{BD} = 5.2 \text{ rad/s} \curvearrowright$$

15.66



GIVEN:  
 $\omega_{DE} = 15 \text{ rad/s} \curvearrowright$   
 $b = 600 \text{ mm}$   
 FIND:  
 (a)  $\omega_{FBD}$   
 (b)  $v_F$

BAR DE:



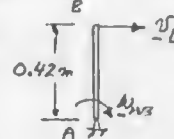
$$\beta = \tan^{-1} \frac{0.24 \text{ m}}{0.32 \text{ m}} = 36.87^\circ$$

$$DE = \frac{0.32 \text{ m}}{\cos \beta} = 0.4 \text{ m}$$

$$v_D = (DE) \omega_{DE} = (0.4 \text{ m})(15 \text{ rad/s})$$

$$v_D = 6 \text{ m/s} \curvearrowright 36.87^\circ$$

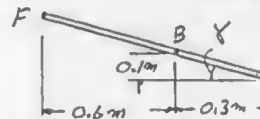
BAR AB:



$$v_B \rightarrow$$

BAR FBD:

GEOMETRY

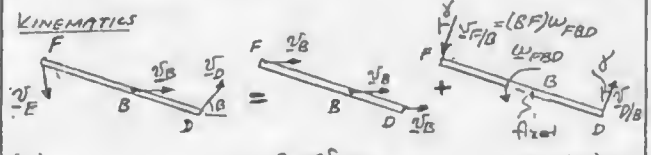


$$\gamma = \tan^{-1} \frac{0.1}{0.3} = 18.43^\circ$$

$$BD = \frac{0.3}{\cos \gamma} = 0.316 \text{ m}$$

$$BF = 2(BD) = 0.632 \text{ m}$$

KINEMATICS



(a)

$$v_D = v_B + v_{D/B}$$

$$[v_D \curvearrowright \gamma] = [v_B \rightarrow] + [v_{D/B} \curvearrowright \delta]$$

$$v_{D/B} = (BD) \omega_{FBD}$$

$$= (0.316 \text{ m}) \omega_{FBD}$$

$$90^\circ - \beta - \gamma = 34.7^\circ$$

$$v_D = 6 \text{ m/s}$$

$$\beta = 36.87^\circ$$

$$\gamma = 18.43^\circ$$

$$108.43^\circ$$

LAW OF SINES

$$\frac{v_B}{\sin 34.7^\circ} = \frac{v_{D/B}}{\sin 36.87^\circ} = \frac{6 \text{ m/s}}{\sin 108.43^\circ}$$

$$v_B = 3.6 \text{ m/s}$$

$$v_{D/B} = (0.316 \text{ m}) \omega_{FBD} = 3.795 \text{ m/s}$$

$$\omega_{FBD} = 12 \text{ rad/s} \curvearrowright$$

(b)

$$v_F = v_B + v_{F/B}$$

$$v_F = [v_B \rightarrow] + [v_{F/B} \curvearrowright \gamma]$$

$$v_{F/B} = (BF) \omega_{FBD}$$

$$= (0.632 \text{ m})(12 \text{ rad/s})$$

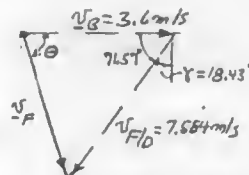
$$v_{F/B} = 7.584 \text{ m/s}$$

LAW OF COSINES

$$v_F^2 = (3.6)^2 + (7.584)^2$$

$$- 2(3.6)(7.584) \cos 71.57^\circ$$

$$v_F = 7.295 \text{ m/s}$$

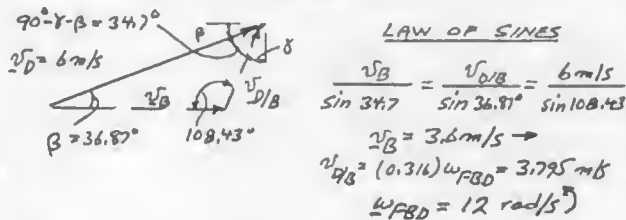
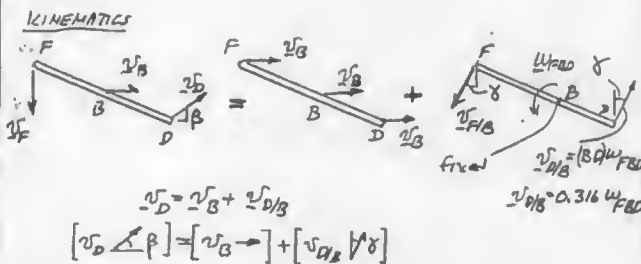
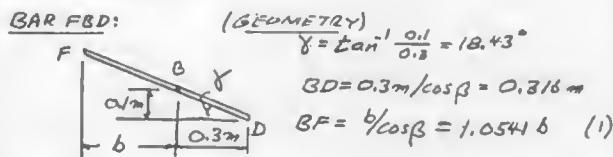
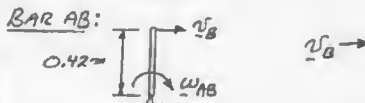
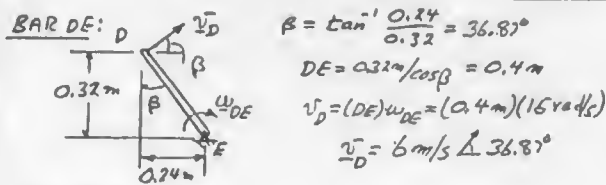
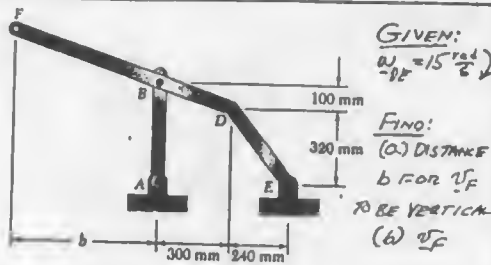


LAW OF SINES

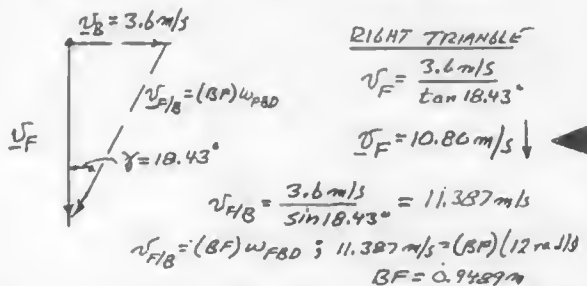
$$\frac{v_{F/B}}{\sin \theta} = \frac{v_F}{\sin 71.57^\circ} ; \frac{7.584 \text{ m/s}}{\sin \theta} = \frac{7.295 \text{ m/s}}{\sin 71.57^\circ} ; \theta = 80.5^\circ$$

$$v_F = 7.30 \text{ m/s} \curvearrowright 80.5^\circ$$

15.67

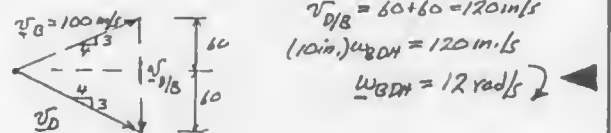
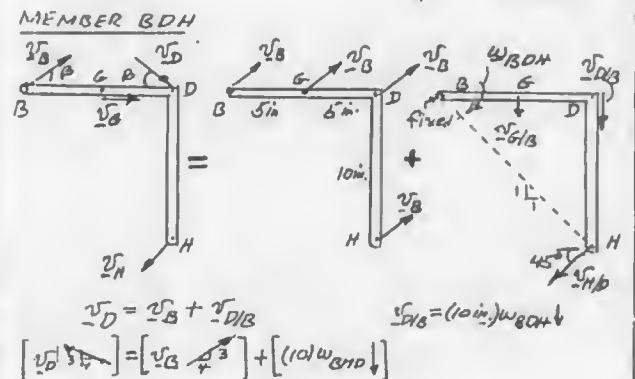
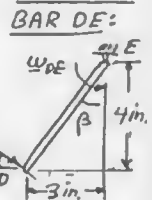
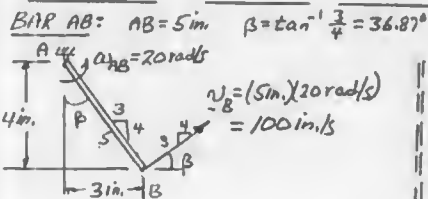
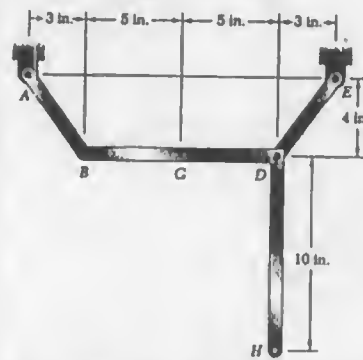


(a)  $\vec{v}_F = \vec{v}_B + \vec{v}_{F/B}$   
 $[\vec{v}_F \downarrow] = [\vec{v}_B \rightarrow] + [\vec{v}_{F/B} \angle \gamma]$   
 $[\vec{v}_F \downarrow] = [3.6 \text{ m/s} \rightarrow] + [(BF) \omega_{FBD} \angle 18.43^\circ]$



EQ.(1):  $BF = 1.0541 b$ ;  $0.9489 = 1.0541 b$ ;  $b = 0.900 \text{ m}$

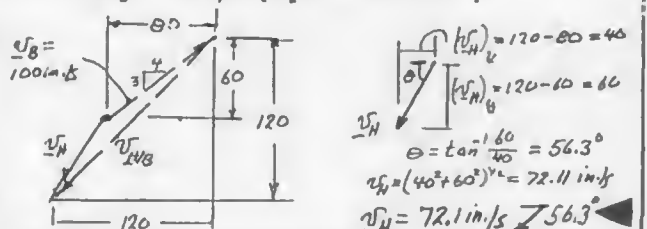
15.68 and 15.69

**PROBLEM 15.68**

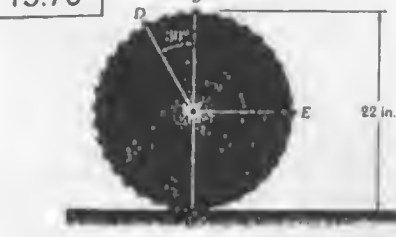
$\vec{v}_G = \vec{v}_B + \vec{v}_{G/B}$   
 $\vec{v}_G = [\vec{v}_B \angle \gamma] + [\vec{v}_{G/B} \downarrow]$   
 $\vec{v}_G = (100 \text{ in/s}) \frac{4}{5} + 0$   $\vec{v}_G = 80 \text{ in/s} \rightarrow$

**PROBLEM 15.69**

$\vec{v}_H = \vec{v}_B + \vec{v}_{H/B}$   
 $\vec{v}_H = [100 \text{ in/s} \angle \gamma] + [120 \sqrt{2} \text{ in/s} \angle \gamma]$



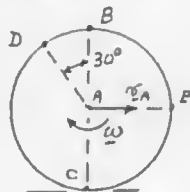
15.70



GIVEN:

$$v_A = 48 \text{ m/s} \rightarrow$$

FIND VELOCITIES  
OF POINTS  
B, C, D, AND E



$$\omega = \frac{v_A}{r} = \frac{48 \text{ m/s}}{11 \text{ in}} = 11 \text{ rad/s} \rightarrow$$

$$\omega = \frac{v_A}{r} = \frac{48 \text{ m/s}}{11 \text{ in}} = 11 \text{ rad/s} \rightarrow$$

$$r = 11 \text{ in}$$

$$v_B = v_A + v_{B/A} = [48 \text{ m/s} \rightarrow] + [r(\omega) \rightarrow] = 142.8 \text{ m/s} \rightarrow$$

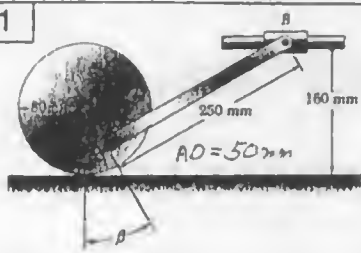
$$v_C = v_A + v_{C/A} = [48 \text{ m/s} \rightarrow] + [r(\omega) \leftarrow] = 0$$

$$v_D = v_A + v_{D/A} = [48 \text{ m/s} \rightarrow] + [r(\omega) \uparrow 30^\circ] = 136.0 \text{ m/s} \uparrow 15^\circ$$

$$v_E = v_A + v_{E/A} = [48 \text{ m/s} \rightarrow] + [r(\omega) \downarrow] = 70.4 \text{ m/s} \downarrow 45^\circ$$

$$v_E = v_A + v_{E/A} = [48 \text{ m/s} \rightarrow] + [r(\omega) \downarrow] = 70.4 \text{ m/s} \downarrow 45^\circ$$

15.71



GIVEN:

$$v_D = 900 \text{ mm/s} \leftarrow$$

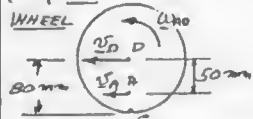
FIND:

$$v_B \text{ AND } \omega_{AB}$$

WHEN

$$(a) \beta = 0^\circ$$

$$(b) \beta = 90^\circ$$

(a)  $\beta = 0^\circ$ :

$$v_C = 0, \omega_{AD} = \frac{v_D}{r} = \frac{900 \text{ mm/s}}{80 \text{ mm}} = 11.25 \text{ rad/s}$$

$$v_A = v_D + v_{A/D} = [900 \text{ mm/s} \leftarrow] + [(80 \text{ mm})(11.25 \text{ rad/s}) \rightarrow]$$

$$v_A = 900 - 562.5 = 337.5 \text{ mm/s} \leftarrow$$

$$v_A = 900 - 562.5 = 337.5 \text{ mm/s} \leftarrow$$

$$\text{ROD AB: } v_B = v_A + v_{B/A}$$

$$[v_B \leftarrow] = [337.5 \text{ mm/s} \leftarrow] + [(AB)\omega_{AB} \uparrow]$$

$$+\uparrow 0 = (AB)\omega_{AB} \cos \theta \quad \omega_{AB} = 0$$

$$+\uparrow v_B = 337.5 \text{ mm/s} \leftarrow$$

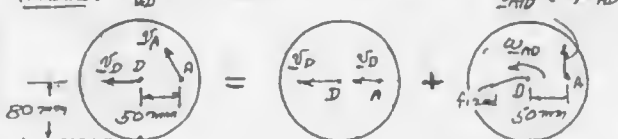
$$v_B = 337.5 \text{ mm/s} \leftarrow$$

(CONTINUED)

15.71 CONTINUED

(b)  $\beta = 90^\circ$ 

$$\text{WHEEL: } \omega_{AD} = 11.25 \text{ rad/s}$$



$$v_A = v_D + v_{A/D} = [900 \text{ mm/s} \leftarrow] + [(80 \text{ mm})(11.25 \text{ rad/s}) \uparrow]$$

$$= [900 \text{ mm/s} \leftarrow] + [562.5 \text{ mm/s} \uparrow]$$

$$v_A = 1061 \text{ mm/s} \uparrow 32.0^\circ$$

ROD AB GEOMETRY



$$\phi = \sin^{-1} \frac{80 \text{ mm}}{250 \text{ mm}}$$

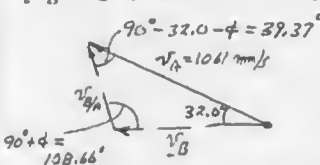
$$\phi = 18.66^\circ$$

KINEMATICS



$$v_B = v_A + v_{B/A}$$

$$[v_B \leftarrow] = [1061 \text{ mm/s} \uparrow 32.0^\circ] + [250 \omega_{AB} \uparrow \phi]$$



LAW OF SINES

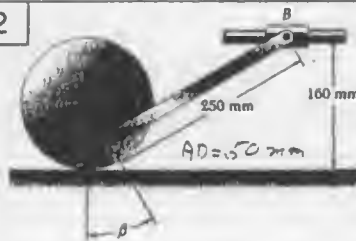
$$\frac{v_B}{\sin 39.37^\circ} = \frac{v_A}{\sin 32.0^\circ} = \frac{1061 \text{ mm/s}}{\sin 108.66^\circ}$$

$$v_B = 710 \text{ mm/s} \leftarrow$$

$$v_{B/A} = (250 \text{ mm}) \omega_{AB} = 593.4 \text{ mm/s}$$

$$\omega_{AB} = 2.37 \text{ rad/s} \rightarrow$$

15.72



GIVEN:

$$v_D = 900 \text{ mm/s} \leftarrow$$

FIND:

$$v_B \text{ AND } \omega_{AB}$$

WHEN

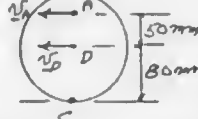
$$(a) \beta = 180^\circ$$

$$(b) \beta = 270^\circ$$

(a)  $\beta = 180^\circ$ 

$$v_C = 0, \omega_{AD} = \frac{v_D}{r} = \frac{900 \text{ mm/s}}{80 \text{ mm}} = 11.25 \text{ rad/s}$$

WHEEL



$$v_A = v_D + v_{A/D}$$

$$v_A = [900 \text{ mm/s} \leftarrow] + [(80 \text{ mm})(11.25 \text{ rad/s}) \rightarrow]$$

$$v_A = 900 + 562.5 = 1462.5 \text{ mm/s} \leftarrow$$

$$\text{ROD AB: } v_B = v_A + v_{B/A}$$

$$[v_B \leftarrow] = [1462.5 \text{ mm/s} \leftarrow] + [(AB)\omega_{AB} \uparrow]$$

$$+\uparrow 0 = (AB)\omega_{AB} \cos \theta \quad \omega_{AB} = 0$$

$$+\uparrow v_B = 1462.5 \text{ mm/s} \leftarrow$$

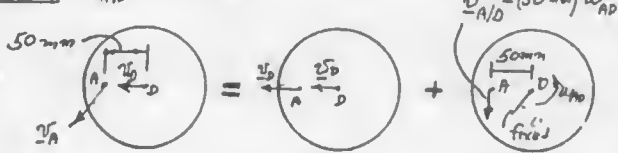
$$v_B = 1462.5 \text{ mm/s} \leftarrow$$

(CONTINUED)

# 15.72 CONTINUED

$$(b) \beta = 270^\circ$$

$$\text{WHEEL: } \omega_{AD} = 11.25 \text{ rad/s}$$



$$\begin{aligned} \underline{v}_A &= \underline{v}_D + \underline{v}_{A/D} = [900 \text{ mm/s} \leftarrow] + [(50 \text{ mm})(11.25 \text{ rad/s}) \downarrow] \\ &= [900 \text{ mm/s} \leftarrow] + [562.5 \text{ mm/s} \downarrow] \end{aligned}$$

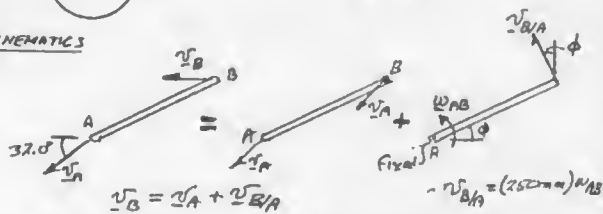
$$\underline{v}_D = 1061 \text{ mm/s} \nearrow 32.0^\circ$$

ROD AB: GEOMETRY

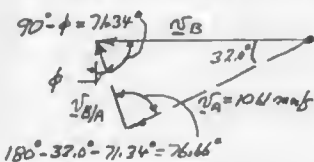


$$\begin{aligned} \phi &= \sin^{-1} \frac{80 \text{ mm}}{250 \text{ mm}} \\ \phi &= 18.66^\circ \end{aligned}$$

KINEMATICS



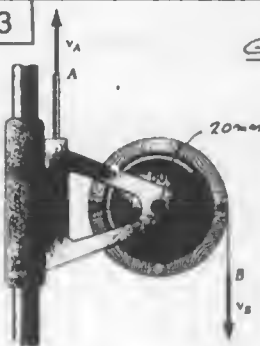
$$[\underline{v}_B \leftarrow] = [1061 \text{ mm/s} \nearrow 32.0^\circ] + [250 \omega_{AB} \uparrow \phi]$$



LAW OF SINES

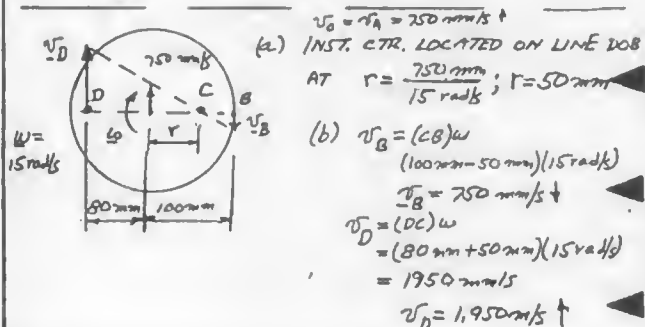
$$\begin{aligned} \frac{v_B}{\sin 76.34^\circ} &= \frac{v_{B/A}}{\sin 32.0^\circ} = \frac{1061 \text{ mm/s}}{\sin 71.34^\circ} \\ v_B &= 1090 \text{ mm/s} \\ v_{B/A} &= 1090 \text{ mm/s} \\ v_{B/A} &= (250 \text{ mm}) \omega_{AB} = 593.4 \text{ mm/s} \\ \omega_{AB} &= 2.37 \text{ rad/s} \end{aligned}$$

# 15.73



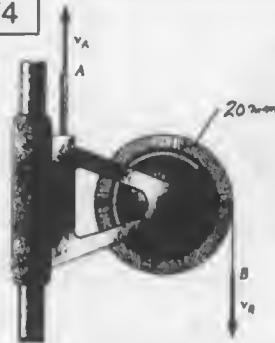
$$\begin{aligned} \text{GIVEN: } \underline{v}_A &= 750 \text{ mm/s} \uparrow \\ \omega &= 15 \text{ rad/s} \end{aligned}$$

FIND:  
(a) INST. CTR. OF ROTATION  
(b)  $\underline{v}_B$  AND  $\underline{v}_D$



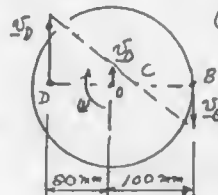
$$\begin{aligned} \underline{v}_D &= \underline{v}_A = 750 \text{ mm/s} \uparrow \\ \text{(a) INST. CTR. LOCATED ON LINE DOB} \\ \text{AT } r &= \frac{750 \text{ mm}}{15 \text{ rad/s}}; r = 50 \text{ mm} \\ \text{(b) } \underline{v}_B &= (CB) \omega \\ &= (100 \text{ mm} - 50 \text{ mm})(15 \text{ rad/s}) \\ &= 750 \text{ mm/s} \rightarrow \\ \underline{v}_D &= (DC) \omega \\ &= (80 \text{ mm} + 50 \text{ mm})(15 \text{ rad/s}) \\ &= 1950 \text{ mm/s} \uparrow \end{aligned}$$

# 15.74



$$\begin{aligned} \text{GIVEN: } \underline{v}_A &= 100 \text{ mm/s} \uparrow \\ \underline{v}_B &= 300 \text{ mm/s} \downarrow \end{aligned}$$

FIND:  
(a) INST. CTR. OF ROTATION  
(b)  $\underline{v}_D$



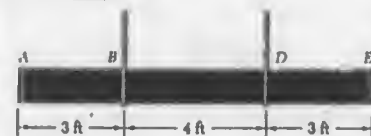
$\underline{v}_D = \underline{v}_A = 100 \text{ mm/s} \uparrow$   
(a) SINCE  $\underline{v}_D$  AND  $\underline{v}_B$  ARE PARALLEL, INST. CTR. C IS LOCATED AT INTERSECTION OF BC AND LINE JOINING END POINTS OF  $\underline{v}_D$  AND  $\underline{v}_B$   
SIMILAR TRIANGLES  
 $\frac{OC}{\underline{v}_D} = \frac{BC}{\underline{v}_B} = \frac{OC+BC}{\underline{v}_D + \underline{v}_B}$

$$\begin{aligned} OC &= \frac{\underline{v}_D}{\underline{v}_D + \underline{v}_B} (OC + BC) \\ OC &= \frac{100 \text{ mm/s}}{(100 + 300) \text{ mm/s}} (100 \text{ mm}) = 25 \text{ mm} \end{aligned}$$

$$\text{(b) } \frac{\underline{v}_D}{(DO) + (OC)} = \frac{\underline{v}_B}{(OC)}; \frac{\underline{v}_D}{(50 + 25) \text{ mm}} = \frac{100 \text{ mm/s}}{25 \text{ mm}}$$

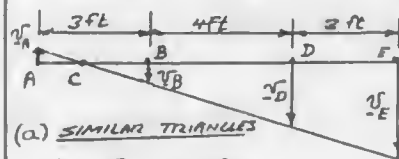
$$\underline{v}_D = 420 \text{ mm/s} \uparrow$$

# 15.75



$$\begin{aligned} \text{GIVEN: } \underline{v}_D &= 24 \text{ in/s} \downarrow \\ \underline{v}_E &= 36 \text{ in/s} \downarrow \end{aligned}$$

FIND:  
(a) INST. CTR. OF ROTATION  
(b)  $\underline{v}_A$



(a) SINCE  $\underline{v}_D$  AND  $\underline{v}_E$  ARE PARALLEL, THE INST. CTR. C IS LOCATED AT INTERSECTION OF AE AND LINE JOINING END POINTS OF  $\underline{v}_D$  AND  $\underline{v}_E$

$$\frac{\underline{v}_E}{CE} = \frac{\underline{v}_D}{CD} = \frac{\underline{v}_E - \underline{v}_D}{CE - CD}$$

$$\text{BUT: } CE - CD = 3 \text{ ft}$$

$$\frac{\underline{v}_D}{CD} = \frac{\underline{v}_E - \underline{v}_D}{CE - CD}; \frac{24 \text{ in/s}}{CD} = \frac{(36 - 24) \text{ in/s}}{3 \text{ ft}}; CD = 6 \text{ ft}$$

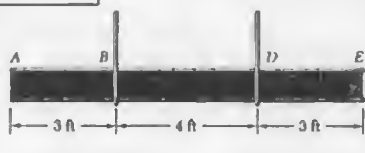
$$AC = AD - CD = 7 \text{ ft} - 6 \text{ ft} = 1 \text{ ft}$$

$$\text{INST. CTR. IS 1 FT TO RIGHT OF A}$$

$$\text{(b) } \frac{\underline{v}_A}{AC} = \frac{\underline{v}_D}{CD}$$

$$\frac{\underline{v}_A}{1 \text{ ft}} = \frac{24 \text{ in/s}}{6 \text{ ft}}; \underline{v}_A = 4 \text{ in/s} \uparrow$$

15.76



GIVEN:

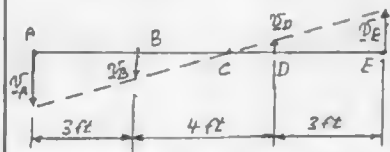
$$v_A = 13 \text{ in./s} \downarrow$$

$$v_E = 7 \text{ in./s} \uparrow$$

FIND:

(a) INST. CTR. OF ROTATION

$$(b) v_D$$



(a) SIMILAR TRIANGLES

$$\frac{AC}{v_A} = \frac{CE}{v_E} = \frac{AC+CE}{v_A+v_E}$$

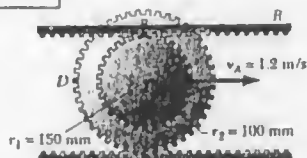
$$AC = \frac{v_A}{v_A+v_E} (AC+CE) = \frac{13 \text{ in./s}}{(13+7) \text{ in./s}} (10 \text{ ft}) = 6.5 \text{ ft}$$

$$CD = AD - AC = 7 \text{ ft} - 6.5 \text{ ft} = 0.5 \text{ ft}$$

INST. CTR. IS 0.5 FT TO LEFT OF D

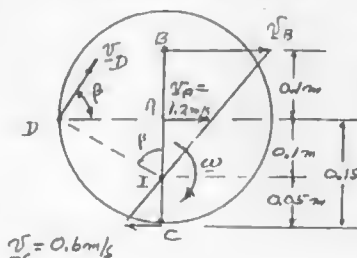
$$(b) \frac{v_D}{CD} = \frac{v_A}{AC}; \frac{v_D}{0.5 \text{ ft}} = \frac{13 \text{ in./s}}{6.5 \text{ ft}}; v_D = 1 \text{ in./s} \uparrow$$

15.77



GIVEN:

$$v_A = 1.2 \text{ m/s} \rightarrow$$

VELOCITY OF LOWER RACK IS  $v_C = 0.6 \text{ m/s} \leftarrow$ FIND: (a)  $\omega$ (b)  $v_R$  AND  $v_D$ 

SINCE  $v_A$  AND  $v_C$  ARE PARALLEL THE INST. CTR. OF ROTATION IS AT THE INTERSECTION OF  $BC$  AND THE LINE JOINING THE END POINTS OF  $v_A$  AND  $v_C$

(a) ANGULAR VELOCITY  $v_A = (AI)\omega$ 

$$1.2 \text{ m/s} = (0.15 \text{ m}) \omega$$

$$\omega = 12 \text{ rad/s} \curvearrowright$$

(b) UPPER RACK

$$v_R = v_B = (BI)\omega$$

$$v_R = (0.2 \text{ m})(12 \text{ rad/s})$$

$$v_R = 2.4 \text{ m/s} \rightarrow$$

VELOCITY OF POINT D:  $\beta = \tan^{-1} \frac{0.15 \text{ m}}{0.1 \text{ m}} = 56.3^\circ$ 

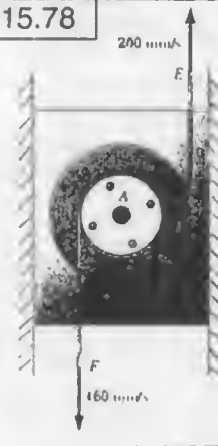
$$DI = \frac{DA}{\cos \beta} = \frac{0.15 \text{ m}}{\cos 56.3^\circ} = 0.1803 \text{ m}$$

$$v_D = (DI)\omega$$

$$v_D = (0.1803 \text{ m})(12 \text{ rad/s})$$

$$v_D = 2.16 \text{ m/s} \curvearrowright 56.3^\circ$$

15.78



GIVEN: INNER RADIUS = 30 mm

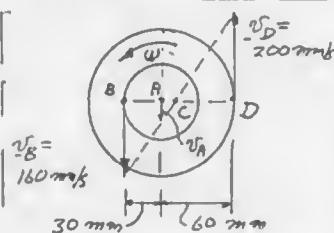
OUTER RADIUS = 60 mm

FIND: (a) INST. CTR. OF ROTATION

$$(b) v_{\text{BLOCK}} = v_A$$

(c) LENGTH OF CORD

WRAPPED OR UNWRAPPED PER SECOND ON EACH PULLEY.

(a) SINCE  $v_B$  AND  $v_D$  ARE PARALLEL, INST. CTR. IS LOCATED AT THE INTERSECTION OF  $BD$  AND LINE JOINING END POINTS OF  $v_B$  AND  $v_D$ 

$$\frac{BC}{160} = \frac{CD}{200} = \frac{BC+CD}{160+200}; \text{ BUT } BC+CD = 90 \text{ mm}$$

$$\frac{BC}{160} = \frac{90 \text{ mm}}{360}; BC = 40 \text{ mm}; AC = BC - AB = 40 \text{ mm} - 30 \text{ mm} = 10 \text{ mm}$$

INST. CTR. C IS 10 mm TO RIGHT OF A

$$(b) v_{\text{BLOCK}} = v_A; \omega = \frac{v_B}{BC} = \frac{160 \text{ mm/s}}{40 \text{ mm}}; \omega = 4 \text{ rad/s} \curvearrowright$$

(c) OUTER PULLEY:

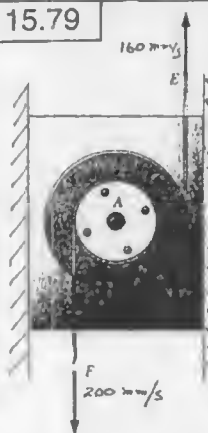
SINCE  $v_D \uparrow$  AND  $v_A \downarrow$ , CORD IS UNWRAPPED AT RATE  $(v_A + v_D)/s$ 

$$v_A + v_D = 40 + 200 = 240 \text{ mm/s}; 240 \text{ mm, UNWRAPPED/s}$$

INNER PULLEY:  $v_B \downarrow > v_A \downarrow$ , CORD IS UNWRAPPED AT RATE  $(v_B - v_A)/s$ 

$$v_B - v_A = 160 - 40 = 120 \text{ mm/s}; 120 \text{ mm, UNWRAPPED/s}$$

15.79



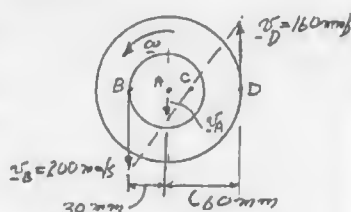
GIVEN: INNER PULLEY = 30 mm

OUTER PULLEY = 60 mm

FIND: (a) INST. CTR. OF ROTATION

$$(b) v_{\text{BLOCK}} = v_A$$

(c) LENGTH OF CORD WRAPPED OR UNWRAPPED PER SECOND ON EACH PULLEY

(a) SINCE  $v_B$  AND  $v_D$  ARE PARALLEL, INST. CTR. IS LOCATED AT THE INTERSECTION OF  $BD$  AND LINE JOINING END POINTS OF  $v_B$  AND  $v_D$ 

$$\frac{BC}{200} = \frac{CD}{160} = \frac{BC+CD}{200+160}; \text{ BUT } BC+CD = 90 \text{ mm}$$

$$\frac{BC}{200} = \frac{90 \text{ mm}}{360}; BC = 50 \text{ mm}; AC = BC - AB = 50 \text{ mm} - 30 \text{ mm} = 20 \text{ mm}$$

INST. CTR. C IS 20 mm TO RIGHT OF A

$$(b) v_{\text{BLOCK}} = v_A; \omega = v_B/BC = (200 \text{ mm/s})/50 \text{ mm}; \omega = 4 \text{ rad/s} \curvearrowright$$

$$v_A = (AC)\omega = (20 \text{ mm})(4 \text{ rad/s}) = 80 \text{ mm/s}; v_{\text{BLOCK}} = 80 \text{ mm/s} \uparrow$$

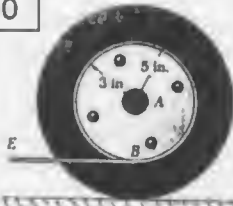
(c) OUTER PULLEY:  $v_D \uparrow$  AND  $v_A \downarrow$ , CORD IS UNWRAPPED AT  $(v_D + v_A)/s$ 

$$v_D + v_A = 160 + 80 = 240 \text{ mm/s}; 240 \text{ mm, UNWRAPPED/s}$$

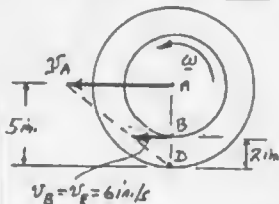
INNER PULLEY:  $v_B \downarrow > v_A \downarrow$ , CORD IS UNWRAPPED AT  $(v_B - v_A)/s$ 

$$v_B - v_A = 200 - 80 = 120 \text{ mm/s}; 120 \text{ mm, UNWRAPPED/s}$$

15.80

GIVEN:  $\underline{v_E} = 6 \text{ in/s} \leftarrow$ 

FIND: (a)  $\omega$   
 (b)  $\underline{v_A}$   
 (c) CORD UNWOUND OR  
 UNWOUND PER SECOND



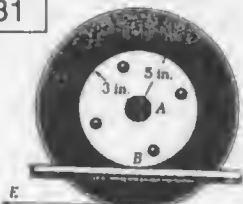
SINCE DRUM ROLLS WITHOUT SLIDING,  
 INST. CTR. OF ROTATION IS AT D.

(a)  $\underline{v_E} = (BC)\omega$   
 $6 \text{ in/s} = (2 \text{ in})\omega$ ;  $\omega = 3 \text{ rad/s}$

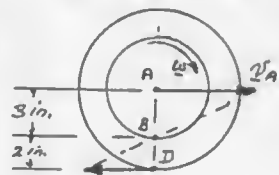
(b)  $\underline{v_A} = (AC)\omega$   
 $\underline{v_A} = (5 \text{ in})(3 \text{ rad/s})$ ;  $\underline{v_A} = 15 \text{ in/s}$

(c) SINCE  $\underline{v_A} > \underline{v_E}$ , DRUM GAINS  
 ON CORD AND CORD IS WOUND ON DRUM  
 AT RATE  $(\underline{v_A} - \underline{v_E}) = (15 \text{ in/s}) - (6 \text{ in/s}) = 9 \text{ in/s}$   
 CORD WOUND PER SECOND = 9 in.

15.81

GIVEN:  $\underline{v_F} = 6 \text{ in/s} \rightarrow$ 

FIND: (a)  $\omega$   
 (b)  $\underline{v_A}$   
 (c) CORD UNWOUND OR  
 UNWOUND PER SECOND



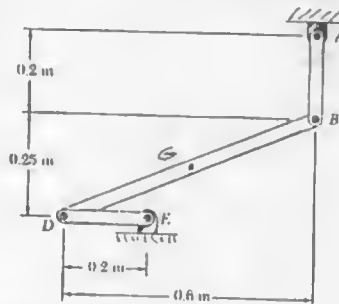
SINCE DRUM ROLLS WITHOUT SLIDING,  
 INST. CTR. OF ROTATION IS AT B.

(a)  $\underline{v_D} = (BD)\omega$   
 $6 \text{ in/s} = (2 \text{ in})\omega$ ;  $\omega = 3 \text{ rad/s}$

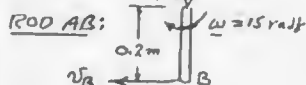
(b)  $\underline{v_A} = (AB)\omega$   
 $\underline{v_A} = (3 \text{ in})(3 \text{ rad/s})$   
 $\underline{v_A} = 9 \text{ in/s} \rightarrow$

(c) SINCE  $\underline{v_F} \rightarrow$  AND  $\underline{v_A} \rightarrow$ , CORD MOVES  
 TO LEFT AND DRUM MOVES TO THE RIGHT, CORD IS  
 UNWOUND FROM DRUM AT RATE  $(\underline{v_A} + \underline{v_F}) = (9 + 6) = 15 \text{ in/s}$   
 CORD UNWOUND PER SECOND = 15 in.

15.82

GIVEN:  
 $\underline{\omega_{AB}} = 15 \text{ rad/s}$ 

FIND:  
 (a)  $\omega_{BD}$   
 (b)  $\underline{v_G}$  WHERE  
 G IS MIDPOINT  
 OF BD



$\underline{v_B} = (AB)\omega = (0.2 \text{ m})(15 \text{ rad/s})$   
 $\underline{v_B} = 3 \text{ m/s} \leftarrow$

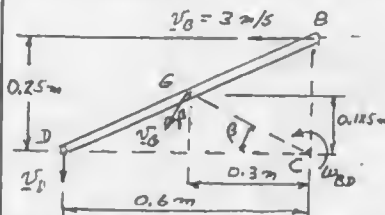
ROD DE:



(CONTINUED)

15.82 CONTINUED

ROD BD:



DRAW LINES  $\perp$  TO  
 $\underline{v_B}$  AND  $\underline{v_D}$  TO  
 LOCATE INST. CTR. C  
 (a)

$\underline{v_B} = (BC)\omega_{BD}$

$3 \text{ m/s} = (0.25 \text{ m})\omega_{BD}$

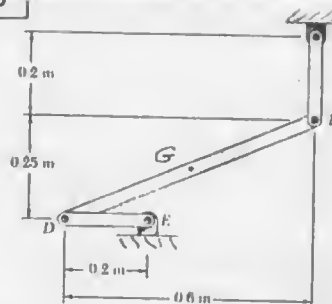
$\omega_{BD} = 12 \text{ rad/s}$

(b)  $\beta = \tan^{-1} \frac{0.125 \text{ m}}{0.3 \text{ m}} = 22.6^\circ$ ;  $CG = \frac{0.3 \text{ m}}{\cos \beta} = 0.325 \text{ m}$

$\underline{v_G} = (CG)\omega_{BD} = (0.325 \text{ m})(12 \text{ rad/s}) = 3.90 \text{ m/s}$

$\underline{v_G} = 3.90 \text{ m/s} \angle 22.6^\circ$ ;  $\underline{v_G} = 3.90 \text{ m/s} \angle 67.4^\circ$

15.83



GIVEN:

$\underline{v_D} = 2.4 \text{ m/s} \leftarrow$

FIND:

(a)  $\omega_{AB}$

(b)  $\underline{v_G}$  WHERE  
 G IS MIDPOINT  
 OF BD

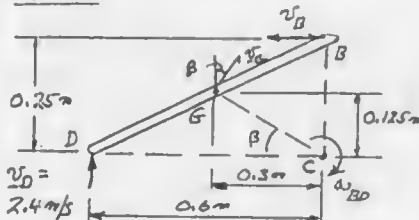
ROD AB:



$\underline{v_B} = (AB)\omega_{AB}$

$\underline{v_B} = (0.2 \text{ m})\omega_{AB} \rightarrow$  (1)

ROD BD:



DRAW LINES  $\perp$  TO  $\underline{v_B}$  AND  $\underline{v_D}$  TO  
 LOCATE INST. CTR. OF ROTATION C.

(a)  $\underline{v_D} = (CD)\omega_{BD}$

$2.4 \text{ m/s} = (0.6 \text{ m})\omega_{BD}$

$\omega_{BD} = 4 \text{ rad/s}$

$\underline{v_B} = (BC)\omega_{BD} = (0.25 \text{ m})(4 \text{ rad/s})$

$\underline{v_B} = 1 \text{ m/s} \rightarrow$

Eq (1):  $\underline{v_B} = (0.2 \text{ m})\omega_{AB}$

$1 \text{ m/s} = (0.2 \text{ m})\omega_{AB}$

$\omega_{AB} = 5 \text{ rad/s}$

(b)  $\beta = \tan^{-1} \frac{0.125 \text{ m}}{0.3 \text{ m}} = 22.6^\circ$

$CG = \frac{0.3 \text{ m}}{\cos \beta} = 0.325 \text{ m}$

$\underline{v_G} = (CG)\omega_{BD} = (0.325 \text{ m})(4 \text{ rad/s}) = 1.300 \text{ m/s}$

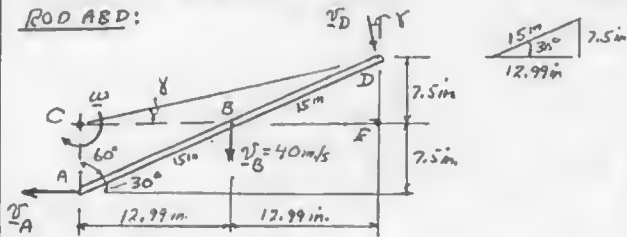
$\underline{v_G} = 1.300 \text{ m/s} \angle 22.6^\circ$   $\underline{v_G} = 1.300 \text{ m/s} \angle 67.4^\circ$

15.84



GIVEN:  
 $AB = BD = 15 \text{ in.}$   
 $\vec{v}_B = 40 \text{ in./s} \uparrow$   
 $\beta = 60^\circ$   
FIND: (a)  $\omega$   
 (b)  $\vec{v}_D$

ROD ABD:



WE LOCATE INST. CTR. C BY DRAWING LINES  $\perp \vec{v}_A$  AND  $\vec{v}_D$   
 (a) ANGULAR VELOCITY

$$\vec{v}_B = (BC)\omega$$

$$40 \text{ in./s} = (12.99 \text{ in.})\omega$$

$$\omega = 3.079 \text{ rad/s}$$

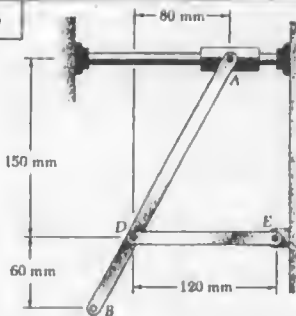
(b) VELOCITY OF D:

IN  $\triangle CDE$ :  $\gamma = \tan^{-1} \frac{7.5}{12.99} = 16.1^\circ$ ;  $CD = \frac{25.98}{\cos \gamma} = 27.04 \text{ in.}$

$$\vec{v}_D = (CD)\omega = (27.04 \text{ in.})(3.079 \text{ rad/s}) = 83.3 \text{ in./s}$$

$$\vec{v}_D = 83.3 \text{ in./s} \searrow 16.1^\circ; \vec{v}_D = 83.3 \text{ in./s} \searrow 73.9^\circ$$

15.85



GIVEN:  
 $\vec{v}_A = 900 \text{ mm/s} \leftarrow$   
FIND: (a)  $\omega_{ABD}$   
 (b)  $\vec{v}_B$

ROD DE:

ROD ABD: LOCATE INST. CTR. C  
 AT INTERSECTION OF LINES  
 DRAWN  $\perp \vec{v}_A$  AND  $\vec{v}_D$

(a) ANGULAR VELOCITY  $\omega_{ABD}$ 

$$\vec{v}_A = (CA)\omega_{ABD}$$

$$900 \text{ mm/s} = (150 \text{ mm})\omega_{ABD}$$

$$\omega_{ABD} = 6 \text{ rad/s} \uparrow$$

(b) VELOCITY OF D

IN  $\triangle BCF$ :  $\beta = \tan^{-1} \frac{32+80}{60} = 61.82^\circ$

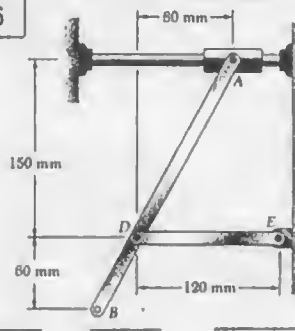
$$BC = \frac{32+80}{\sin \beta} = 127.06 \text{ mm}$$

$$\vec{v}_B = (BC)(\omega_{ABD})$$

$$= (127.06 \text{ mm})(6 \text{ rad/s}) = 762.4 \text{ mm/s}$$

$$\vec{v}_D = 762 \text{ mm/s} \searrow 61.8^\circ$$

15.86

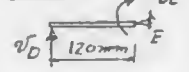


GIVEN:  
 $\omega_{DE} = 2.4 \text{ rad/s} \uparrow$

FIND:

(a)  $\vec{v}_A$   
 (b)  $\vec{v}_B$

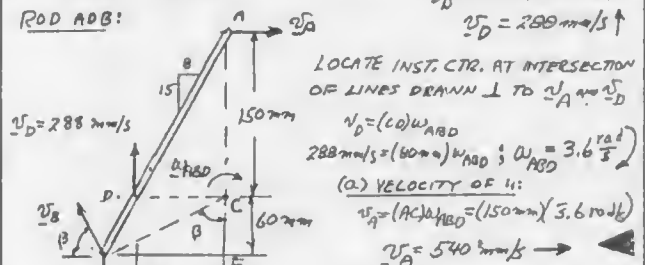
ROD DE:



$$\vec{v}_D = (120 \text{ mm})(2.4 \text{ rad/s})$$

$$\vec{v}_D = 288 \text{ mm/s} \uparrow$$

ROD ADB:



LOCATE INST. CTR. AT INTERSECTION  
 OF LINES DRAWN  $\perp \vec{v}_A$  AND  $\vec{v}_D$

$$\vec{v}_D = (CA)\omega_{ADB}$$

$$288 \text{ mm/s} = (150 \text{ mm})\omega_{ADB}; \omega_{ADB} = 3.6 \text{ rad/s} \uparrow$$

(a) VELOCITY OF A:

$$\vec{v}_A = (AC)\omega_{ADB} = (150 \text{ mm})(3.6 \text{ rad/s})$$

$$\vec{v}_A = 540 \text{ mm/s} \rightarrow$$

(b) VELOCITY OF B:

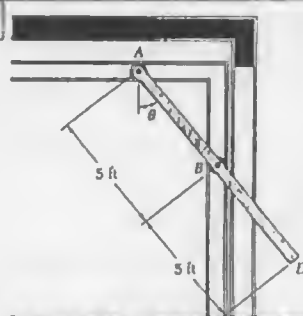
IN  $\triangle BCF$ :  $\beta = \tan^{-1} \frac{32+80}{60} = 61.82^\circ$

$$BC = \frac{32+80}{\sin \beta} = 127.06 \text{ mm}$$

$$\vec{v}_B = (BC)\omega_{ADB} = (127.06 \text{ mm})(3.6 \text{ rad/s}) = 457 \text{ mm/s}$$

$$\vec{v}_B = 457 \text{ mm/s} \searrow 61.8^\circ$$

15.87



GIVEN:  
 $\omega = 40^\circ$   
 $\vec{v}_B = 1.5 \text{ ft/s} \uparrow$

FIND:

(a)  $\omega$   
 (b)  $\vec{v}_D$

LOCATE INST. CTR. AT  
 INTERSECTION OF LINES  
 DRAWN  $\perp \vec{v}_A$  AND  $\vec{v}_B$

(a) ANGULAR VELOCITY

$$\vec{v}_B = (BC)\omega$$

$$1.5 \text{ ft/s} = (3.214 \text{ ft})\omega$$

$$\omega = 0.4667 \text{ rad/s}$$

(b) VELOCITY OF D:

IN  $\triangle CDE$ :  $\beta = \tan^{-1} \frac{6.427}{3.83} = 59.2^\circ$

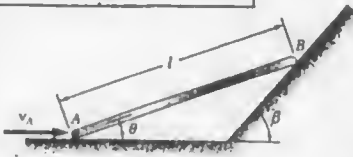
$$CD = \frac{6.427}{\sin \beta} = 7.482 \text{ ft}$$

$$\vec{v}_D = (CD)\omega = (7.482 \text{ ft})(0.4667 \text{ rad/s}) = 3.49 \text{ ft/s}$$

$$\vec{v}_D = 3.49 \text{ ft/s} \searrow 59.2^\circ$$



# 15.88 and 15.89



**PROBLEM 15.88**  
DERIVE AN EXPRESSION  
FOR (a)  $\omega$ , (b)  $v_B$ .

**PROBLEM 15.89**  
GIVEN:  $\theta = 20^\circ$ ,  
 $\beta = 50^\circ$ ,  $l = 0.6\text{ m}$ ,  $v_A = 3\text{ m/s}$   
FIND: (a)  $\omega$ , (b)  $v_B$

**PROBLEM 15.88**

LOCATE INST. CTR. AT  
INTERSECTION OF LINES  
DRAWN  $\perp$  TO  $v_A$  AND  $v_B$

**LAW OF SINES**

$$\frac{AC}{\sin(90^\circ - (\beta - \theta))} = \frac{BC}{\sin(90^\circ - \theta)} = \frac{l}{\sin \beta}$$

$$\frac{AC}{\cos(\beta - \theta)} = \frac{BC}{\cos \theta} = \frac{l}{\sin \beta}$$

$$AC = l \frac{\cos(\beta - \theta)}{\sin \beta}$$

$$BC = l \frac{\cos \theta}{\sin \beta}$$

(a) **ANGULAR VELOCITY:**  $v_A = (AC)\omega = l \frac{\cos(\beta - \theta)}{\sin \beta} \omega$

$$\omega = \frac{v_A}{l} \cdot \frac{\sin \beta}{\cos(\beta - \theta)}$$

(b) **VELOCITY OF B:**

$$v_B = (BC)\omega = l \frac{\cos \theta}{\sin \beta} \cdot \left[ \frac{v_A}{l} \cdot \frac{\sin \beta}{\cos(\beta - \theta)} \right]$$

$$v_B = v_A \frac{\cos \theta}{\cos(\beta - \theta)}$$

**PROBLEM 15.89: DATA**,  $\theta = 20^\circ$ ,  $\beta = 50^\circ$ ,  $l = 0.6\text{ m}$ ,  $v_A = 3\text{ m/s}$

(a)  $\omega = \frac{v_A}{l} \frac{\sin \beta}{\cos(\beta - \theta)} = \frac{3\text{ m/s}}{0.6\text{ m}} \frac{\sin 50^\circ}{\cos(50^\circ - 20^\circ)}$

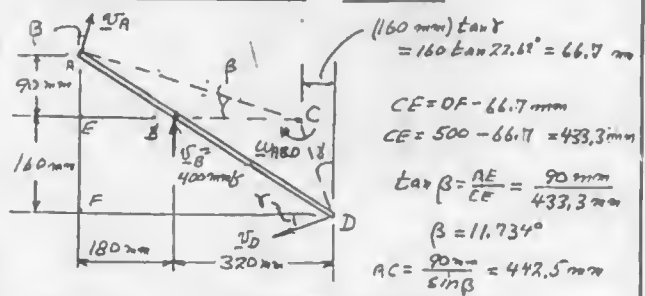
$$\omega = 4.423\text{ rad/s} \quad \omega = 4.42\text{ rad/s}$$

(b)  $v_B = v_A \frac{\cos \theta}{\cos(\beta - \theta)} = (3\text{ m/s}) \frac{\cos 20^\circ}{\cos(50^\circ - 20^\circ)}$

$$v_B = 3.2557\text{ m/s} \quad v_B = 3.26\text{ m/s} \angle 50^\circ$$

# 15.90 CONTINUED

**ARM ABD:**



INST. CTR. C IS LOCATED AT INTERSECTION  
OF LINES DRAWN  $\perp$  TO  $v_B$  AND  $v_D$ .

(a) **ANGULAR VELOCITY:**

$$v_B = (BC)\omega_{ABD}; \quad 400\text{ m/s} = [320\text{ mm} - 66.7\text{ mm}] \omega_{ABD}$$

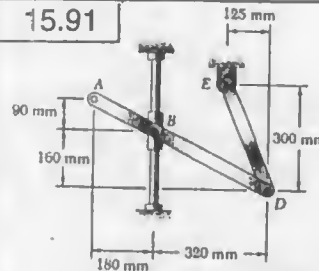
$$\omega_{ABD} = 1.579\text{ rad/s}$$

(b) **VELOCITY OF A:**

$$v_A = (AC)\omega_{ABD} = (442.5\text{ mm}) (1.579\text{ rad/s}) = 699\text{ mm/s}$$

$$v_A = 699\text{ mm/s} \angle 78.3^\circ$$

# 15.91

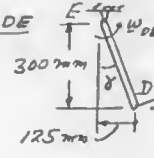


**GIVEN:**  
 $\omega_{DE} = 1.2\text{ rad/s}$

**FIND:**

(a)  $\omega_{ABD}$   
(b)  $v_A$

**CRANK DE**



$$\gamma = \tan^{-1} \frac{125}{300} = 22.62^\circ$$

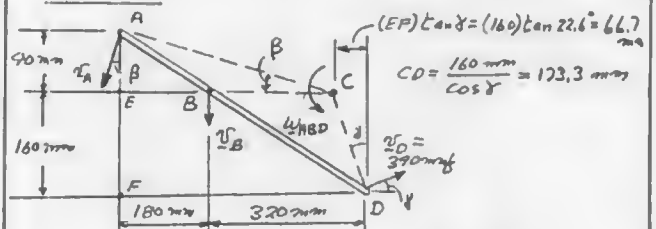
$$DE = \frac{300}{\cos \gamma} = 325\text{ mm}$$

$$v_D = (DE)\omega_{DE}$$

$$v_D = (325\text{ mm}) (1.2\text{ rad/s})$$

$$v_D = 390\text{ mm/s} \angle 22.6^\circ$$

**ARM ABD:**



INST. CTR. C IS LOCATED AT INTERSECTION  
OF LINES DRAWN  $\perp$  TO  $v_B$  AND  $v_D$

(a) **ANGULAR VELOCITY:**

$$v_D = (CD)\omega_{ABD}; \quad 390\text{ mm/s} = (173.3\text{ mm}) \omega_{ABD}$$

$$\omega_{ABD} = 2.25\text{ rad/s}; \quad \omega_{ABD} = 2.25\text{ rad/s}$$

(b) **VELOCITY OF A:**

$$CE = DF - 66.7\text{ mm} = 500\text{ mm} - 66.7\text{ mm} = 433.3\text{ mm}$$

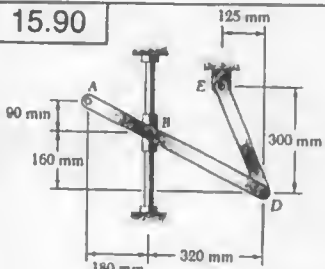
$$\beta = \tan^{-1} \frac{AE}{CE} = \tan^{-1} \frac{90\text{ mm}}{433.3\text{ mm}} = 11.734^\circ$$

$$AC = (CE)/\cos \beta = (433.3\text{ mm})/\cos 11.734^\circ = 442.5\text{ mm}$$

$$v_A = (AC)\omega_{ABD} = (442.5\text{ mm}) (2.25\text{ rad/s}) = 996\text{ mm/s}$$

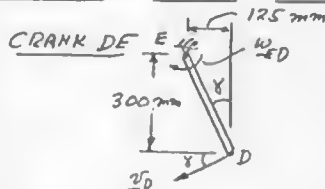
$$v_A = 996\text{ mm/s} \angle 78.3^\circ$$

# 15.90



**GIVEN:**  
 $v_B = 400\text{ mm/s} \uparrow$

**FIND:**  
(a)  $\omega_{ABD}$   
(b)  $v_A$



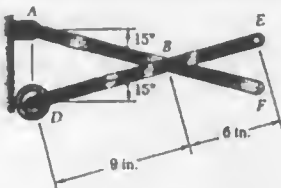
$$\gamma = \tan^{-1} \frac{125}{300} = 22.62^\circ$$

$$v_D = v_B \angle 22.62^\circ$$

(CONTINUED)



15.92

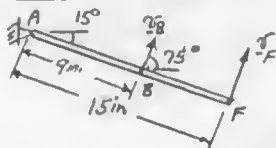


GIVEN:

$$\underline{v}_D = 10 \text{ in./s} \uparrow$$

FIND: (a)  $\underline{v}_E$   
(b)  $\underline{v}_F$ 

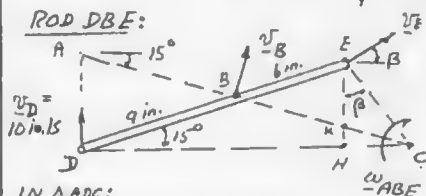
ROD ABF:



$$\underline{v}_B = \underline{v}_A \angle 75^\circ$$

$$\underline{v}_F = \frac{15}{9} \underline{v}_B \quad (1)$$

ROD DBE:

INST. CTR. C IS  
LOCATED AT INTER-  
SECTION OF LINES  
DRAWN  $\perp$   
TO  $\underline{v}_B$  AND  $\underline{v}_D$ IN  $\triangle ADC$ :

$$AD = 2(9 \text{ in.}) \sin 15^\circ = 4.6587 \text{ in.}$$

$$CD = AD / \tan 15^\circ = 17.387 \text{ in.}$$

IN  $\triangle DEH$ :

$$EH = (DE) \sin 15^\circ = (15 \text{ in.}) \sin 15^\circ = 3.8823 \text{ in.}$$

$$DH = (DE) \cos 15^\circ = (15 \text{ in.}) \cos 15^\circ = 14.489 \text{ in.}$$

IN  $\triangle CEH$ :

$$HC = CD - DH = 17.387 \text{ in.} - 14.489 \text{ in.} = 2.898 \text{ in.}$$

$$\beta = \tan^{-1} \frac{HC}{EH} = \tan^{-1} \frac{2.898 \text{ in.}}{3.8823 \text{ in.}} = 36.74^\circ$$

$$EC = (EH) / \cos \beta = (3.8823 \text{ in.}) / \cos \beta = 4.844 \text{ in.}$$

$$(\text{CHECK}) CK = (HC) / \sin 15^\circ = 3.000 \text{ in.} \quad \text{OK.}$$

$$BC = AB = 9 \text{ in.}$$

ANGULAR VELOCITY

$$\underline{v}_D = (CD) \omega_{ABE}$$

$$10 \text{ in./s} = (17.387 \text{ in.}) \omega_{ABE} \quad ; \quad \omega_{ABE} = 0.5751 \text{ rad/s}$$

(a) VELOCITY OF E:

$$\underline{v}_E = (EC) \omega_{ABE} = (4.844 \text{ in.}) (0.5751 \text{ rad/s}) = 2.79 \text{ in./s}$$

$$\underline{v}_E = 2.79 \text{ in./s} \angle 36.7^\circ$$

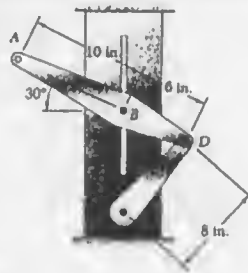
(b) VELOCITY OF F:

$$\underline{v}_B = (BC) \omega_{ABE} = (9 \text{ in.}) (0.5751 \text{ rad/s}) = 5.176 \text{ in./s}$$

$$\text{Eq. (1): } \underline{v}_F = \frac{15}{9} \underline{v}_B = \frac{15}{9} (5.176 \text{ in./s}) = 8.63 \text{ in./s}$$

$$\underline{v}_F = 8.63 \text{ in./s} \angle 75^\circ$$

15.93



GIVEN:

$$\underline{\omega}_{DE} = 3 \text{ rad/s} \curvearrowright$$

FIND:

(a)  $\underline{\omega}_{ABD}$   
(b)  $\underline{v}_A$ 

GEOMETRY

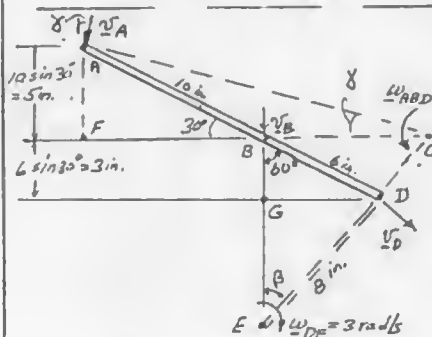
IN  $\triangle BDP$ 

$$\frac{\sin \beta}{6 \text{ in.}} = \frac{\sin 60^\circ}{8 \text{ in.}}$$

$$\sin \beta = \frac{3}{4} \sin 60^\circ$$

$$\sin \beta = 0.6495$$

$$\beta = 40.505^\circ$$



$$E \text{ of } \underline{\omega}_{DE} = 3 \text{ rad/s}$$

$$\text{IN } \triangle CDE: \quad EG = (CE) \cos \beta = (8 \text{ in.}) \cos \beta = 6.083 \text{ in.}$$

$$\text{IN } \triangle BCE: \quad BC = (BE) \tan \beta = [8.6 + EG] \tan \beta$$

$$= (3 \text{ in.} + 6.083 \text{ in.}) \tan \beta = 7.759 \text{ in.}$$

$$EC = (BE) / \cos \beta = (3 \text{ in.} + 6.083 \text{ in.}) / \cos \beta = 11.946 \text{ in.}$$

$$FB = (AB) \cos 30^\circ = (10 \text{ in.}) \cos 30^\circ = 8.660 \text{ in.}$$

$$FC = FB + BC = 8.660 \text{ in.} + 7.759 \text{ in.} = 16.419 \text{ in.}$$

$$\text{IN } \triangle AFC: \quad \gamma = \tan^{-1} \frac{AF}{FC} = \tan^{-1} \frac{\sin}{16.419 \text{ in.}} = 16.937^\circ$$

$$AC = \frac{FC}{\cos \gamma} = \frac{16.419 \text{ in.}}{\cos \gamma} = 17.163 \text{ in.}$$

$$\text{ARM DE: } \underline{v}_D = (DE) \omega_{DE} = (8 \text{ in.}) (3 \text{ rad/s})$$

$$\underline{v}_D = 24 \text{ in./s} \angle \beta$$

MEMBER ABD: THE INST. CTR. C IS LOCATED  
AT INTERSECTION OF LINES DRAWN  $\perp$  TO  $\underline{v}_B$  AND  $\underline{v}_D$ .

$$CD = EC - ED = 11.946 \text{ in.} - 8 \text{ in.} = 3.946 \text{ in.}$$

(a) ANGULAR VELOCITY  $\omega_{ABD}$ :

$$\underline{v}_D = (CD) \omega_{ABD}$$

$$24 \text{ in./s} = (3.946 \text{ in.}) \omega_{ABD}$$

$$\omega_{ABD} = 6.082 \text{ rad/s}$$

$$\underline{\omega}_{ABD} = 6.08 \text{ rad/s} \curvearrowright$$

(b) VELOCITY OF A:

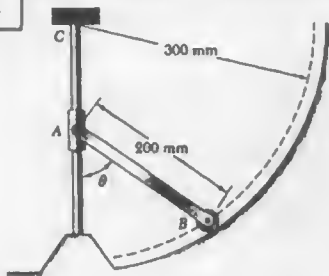
$$\underline{v}_A = (AC) \omega_{ABD} = (17.163 \text{ in.}) (6.082 \text{ rad/s})$$

$$\underline{v}_A = 104.4 \text{ in./s}$$

$$\underline{v}_A = 104.4 \text{ in./s} \angle \gamma = 104.4 \text{ in./s} \angle 16.9^\circ$$

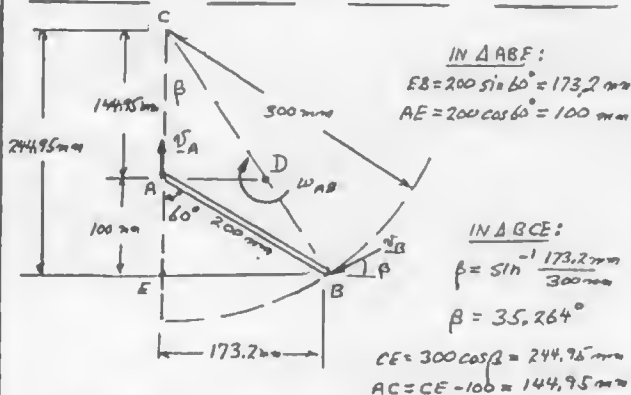
$$\underline{v}_A = 104.4 \text{ in./s} \angle 73.1^\circ$$

15.94



GIVEN:  
 $\theta = 60^\circ$   
 $v_A = 250 \text{ mm/s}$

Find:  
 (a)  $\omega_{AB}$   
 (b)  $v_B$



SIMILAR TRIANGLES:  $\triangle CAD$  AND  $\triangle CEB$

$$\frac{CD}{CB} = \frac{AD}{EB} = \frac{CA}{CE}$$

$$\frac{CD}{300 \text{ mm}} = \frac{AD}{173.2 \text{ mm}} = \frac{144.95 \text{ mm}}{244.95 \text{ mm}}$$

$$CD = 177.53 \text{ mm} \quad AD = 102.49 \text{ mm}$$

$$BD = CB - CD = 300 \text{ mm} - 177.53 \text{ mm} = 122.47 \text{ mm}$$

THE INST. CTR. IS LOCATED AT POINT D WHICH IS THE POINT OF INTERSECTION OF LINES DRAWN  $\perp$  TO  $v_A$  AND  $v_B$

(a) ANGULAR VELOCITY  $\omega_{AB}$ :

$$v_A = (AD) \omega_{AB}$$

$$250 \text{ mm/s} = (102.49 \text{ mm}) \omega_{AB}$$

$$\omega_{AB} = 2.439 \text{ rad/s} \quad \omega_{AB} = 2.44 \text{ rad/s} \quad \leftarrow$$

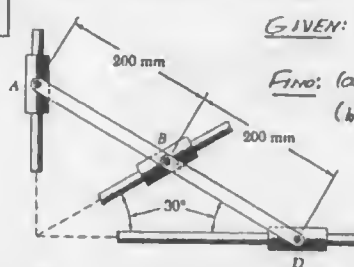
(b) VELOCITY OF B:

$$v_B = (BD) \omega_{AB} = (122.47 \text{ mm}) (2.439 \text{ rad/s})$$

$$v_B = 298.7 \text{ mm/s}$$

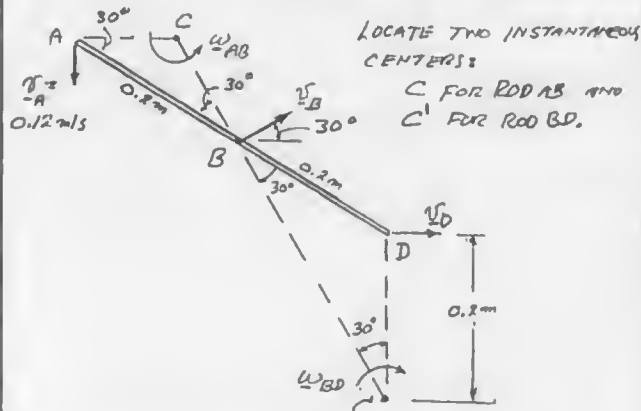
$$v_B = 299 \text{ mm/s} \angle 35.3^\circ \quad \leftarrow$$

15.95



GIVEN:  $v_A = 120 \text{ mm/s} \downarrow$

Find: (a)  $\omega_{AB}$ ,  $\omega_{BD}$   
 (b)  $v_D$



ISOSCELES  $\triangle ACB$ :

$$AC = BC = (0.1 \text{ m}) / \cos 30^\circ = 0.11547 \text{ m}$$

ISOSCELES  $\triangle BC'D$

$$BD = DC' = 0.2 \text{ m}$$

$$BC' = 2(0.2 \text{ m}) \cos 30^\circ = 0.3464 \text{ m}$$

(a) ROD AB:

$$v_A = (AC) \omega_{AB}$$

$$0.120 \text{ m/s} = (0.11547 \text{ m}) \omega_{AB}$$

$$\omega_{AB} = 1.0392 \text{ rad/s}$$

$$\omega_{AB} = 1.039 \text{ rad/s} \quad \leftarrow$$

$$v_B = (BC) \omega_{AB}$$

$$\text{SINCE } AC = BC, \quad v_B = v_A$$

$$v_B = 0.12 \text{ m/s} \quad \leftarrow$$

ROD BD:

$$v_B = (BC') \omega_{BD}$$

$$0.12 \text{ m/s} = (0.3464 \text{ m}) \omega_{BD}$$

$$\omega_{BD} = 0.3464 \text{ rad/s}$$

$$\omega_{BD} = 0.346 \text{ rad/s} \quad \leftarrow$$

(b) VELOCITY OF D:

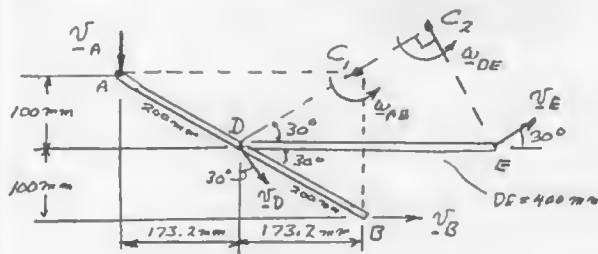
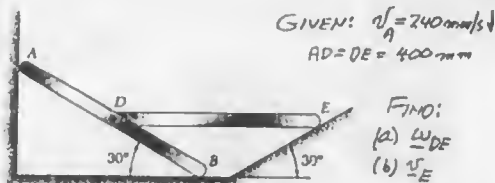
$$v_D = (DC') \omega_{BD}$$

$$= (0.2 \text{ m}) (0.3464 \text{ rad/s})$$

$$v_D = 0.06928 \text{ m/s}$$

$$v_D = 69.3 \text{ mm/s} \rightarrow \quad \leftarrow$$

15.96



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

- $C_1$ : FOR ROD AB, DRAW LINES  $\perp$  TO  $v_A$  &  $v_B$   
 $C_2$ : FOR ROD DE, DRAW LINES  $\perp$  TO  $v_D$  &  $v_E$

GEOMETRY:  $AC_1 = (400 \text{ mm}) \cos 30^\circ = 346.4 \text{ mm}$   
 $BC_1 = (400 \text{ mm}) \sin 30^\circ = 200 \text{ mm}$   
 $DC_2 = AD = 200 \text{ mm}$   
 $EC_2 = (DE) \cos 30^\circ = (400 \text{ mm}) \cos 30^\circ = 346.4 \text{ mm}$   
 $EC_2 = (DE) \sin 30^\circ = (400 \text{ mm}) \sin 30^\circ = 200 \text{ mm}$

ROD AB:

$$v_A = (AC_1) \omega_{AB}; 240 \text{ mm/s} = (346.4 \text{ mm}) \omega_{AB}$$

$$\omega_{AB} = 0.69284 \text{ rad/s}$$

$$v_D = (DC_1) \omega_{AB} = (200 \text{ mm}) (0.69284 \text{ rad/s})$$

$$v_D = 138.57 \text{ mm/s} \nearrow 30^\circ$$

ROD DE:

$$v_D = (DC_2) \omega_{DE}$$

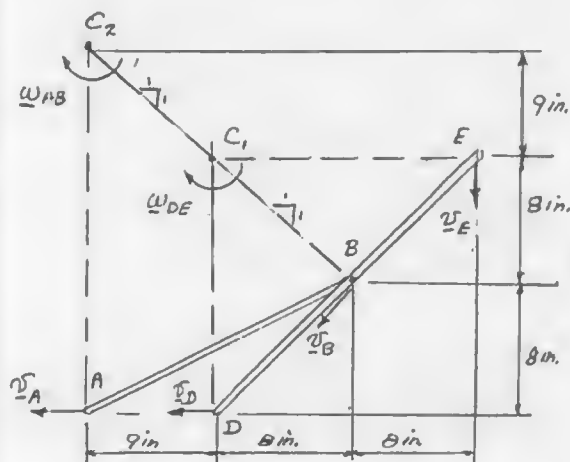
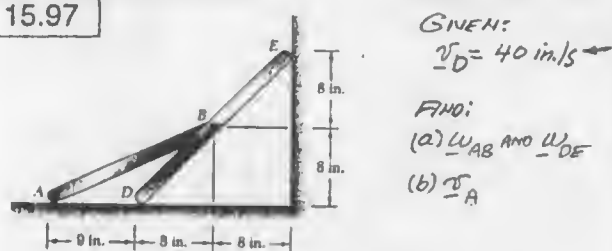
$$138.57 \text{ mm/s} = (346.4 \text{ mm}) \omega_{DE}$$

(a)  $\omega_{DE} = 0.400 \text{ rad/s}$   $\omega_{DE} = 0.4 \text{ rad/s}$

(b)  $v_E = (EC_2) \omega_{DE} = (200 \text{ mm}) (0.400 \text{ rad/s})$

$$v_E = 80 \text{ mm/s} \quad v_E = 80 \text{ mm/s} \nearrow 30^\circ$$

15.97



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

- $C_1$ : FOR ROD DE, DRAW LINES  $\perp$  TO  $v_D$  AND  $v_E$   
 $C_2$ : FOR ROD AB, DRAW LINES  $\perp$  TO  $v_A$  AND  $v_B$

GEOMETRY:  $BC_1 = (8 \text{ in.}) \sqrt{2} = 8\sqrt{2} \text{ in.}$   
 $DC_1 = 16 \text{ in.}$

$$BC_2 = (9 \text{ in.} + 8 \text{ in.}) \sqrt{2} = 17\sqrt{2} \text{ in.}$$

$$AC_2 = 25 \text{ in.}$$

(a) ROD DE:  $v_D = (DC_1) \omega_{DE}$   
 $40 \text{ in/s} = (16 \text{ in.}) \omega_{DE}$   
 $\omega_{DE} = 2.5 \text{ rad/s}$

$$\omega_{DE} = 2.5 \text{ rad/s}$$

$$v_B = (BC_2) \omega_{DE}$$

$$= (17\sqrt{2} \text{ in.}) (2.5 \text{ rad/s})$$

$$v_B = 20\sqrt{2} \text{ in/s} \nearrow 45^\circ$$

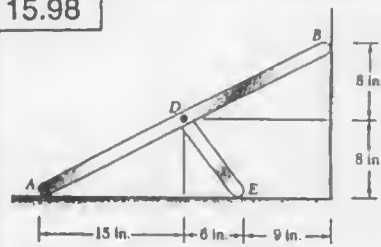
ROD AB:  $v_B = (BC_2) \omega_{AB}$   
 $20\sqrt{2} \text{ in/s} = (17\sqrt{2} \text{ in.}) \omega_{AB}$   
 $\omega_{AB} = \frac{20}{17} \text{ rad/s} = 1.1765 \text{ rad/s}$

$$\omega_{AB} = 1.176 \text{ rad/s}$$

(b)  $v_A = (AC_2) \omega_{AB}$   
 $= (25 \text{ in.}) (1.1765 \text{ rad/s})$

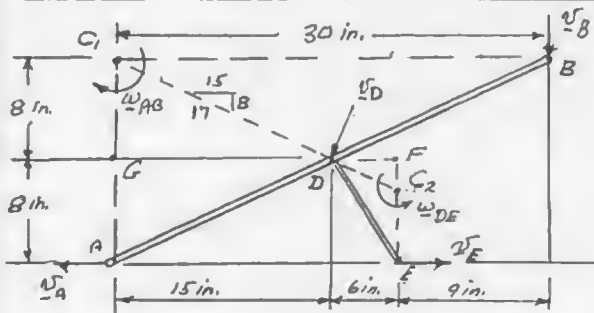
$$v_A = 29.41 \text{ in/s} \quad v_A = 29.4 \text{ in/s}$$

15.98



GIVEN:  
 $v_B = 60 \text{ in./s}$

FIND:  
 (a)  $\omega_{AB}$  and  $\omega_{DE}$   
 (b)  $v_E$



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

$C_1$ : FOR ROD AB DRAW LINES  $\perp$  TO  $v_A$  AND  $v_B$   
 $C_2$ : FOR ROD DE DRAW LINES  $\perp$  TO  $v_D$  AND  $v_E$

GEOMETRY:

$$OC_1 = (8^2 + 15^2)^{1/2} = 17 \text{ in.}$$

SINCE  $\triangle C_1DG$  AND  $\triangle DFC_2$  ARE SIMILAR,

$$\frac{C_2F}{8 \text{ in.}} = \frac{C_2D}{17 \text{ in.}} = \frac{6 \text{ in.}}{15 \text{ in.}}$$

$$C_2F = 3.2 \text{ in.} \quad C_2D = 6.8 \text{ in.}$$

$$EC_2 = 8 \text{ in.} - C_2F = 8 - 3.2 = 4.8 \text{ in.}$$

(a) ROD AB:  $v_B = (BC_1)\omega_{AB}$   
 $60 \text{ in./s} = (30 \text{ in.})\omega_{AB}$   
 $\omega_{AB} = 2 \text{ rad/s} \quad \omega_{AB} = 2 \text{ rad/s}$

$$v_D = (DC_1)\omega_{AB}$$

$$v_D = (17 \text{ in.})(2 \text{ rad/s}) = 34 \text{ in./s}$$

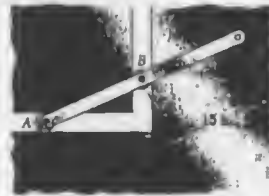
ROD DE:  $v_D = (DC_2)\omega_{DE}$   
 $34 \text{ in./s} = (6.8 \text{ in.})\omega_{DE}$

$$\omega_{DE} = 5 \text{ rad/s} \quad \omega_{DE} = 5 \text{ rad/s}$$

(b)  $v_E = (EC_2)\omega_{DE}$   
 $v_E = (4.8 \text{ in.})(5 \text{ rad/s})$   
 $v_E = 24 \text{ in./s}$

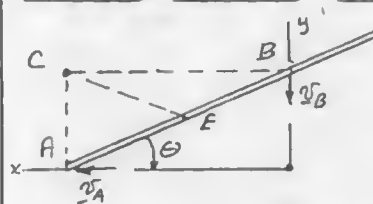
$$v_E = 24 \text{ in./s} \rightarrow$$

15.99



GIVEN:  
 $AB = BD = 15 \text{ in.}$

DESCRIBE THE SPACE CENTROID AND BODY CENTROID OF ROD ABD.



LET:  $AB = l = 15 \text{ in.}$

SPACE CENTROID: COORDINATES OF INST. CTR.

$$x = l \cos \theta \quad y = l \sin \theta$$

$$x^2 + y^2 = l^2 (\cos^2 \theta + \sin^2 \theta)$$

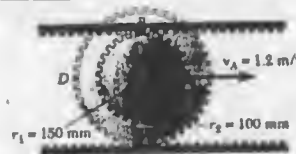
$$x^2 + y^2 = l^2$$

SPACE CENTROID IS A QUARTER CIRCLE OF  $l = 15 \text{ in.}$  RADIUS CENTERED AT INTERSECTION OF TRACES IN WHICH WHEELS A AND B MOVE

BODY CENTROID: DRAW LINE CB WHICH CONNECTS INST. CTR. C AND POINT E LOCATED MIDWAY BETWEEN A AND B.

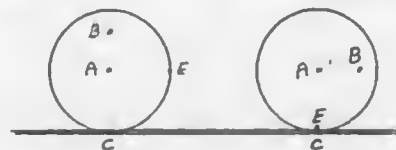
SINCE  $CE = AE = \frac{1}{2}l = 7.5 \text{ in.}$ , WE NOTE THAT BODY CENTROID IS A SEMI CIRCLE OF 7.5-in. RADIUS CENTERED AT E.

15.100



GIVEN: GEAR ROLLS ON STATIONARY LOWER RACK.

DESCRIBE THE SPACE CENTROID AND BODY CENTROID OF THE GEAR.



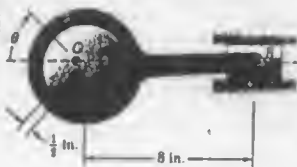
SINCE GEAR ROLLS ON LOWER RACK, THE INST. CTR. IS ALWAYS AT POINT OF CONTACT BETWEEN GEAR AND LOWER RACK.

SPACE CENTROID: LOWER RACK

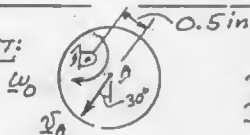
BODY CENTROID: CIRCUMFERENCE OF GEAR

15.101

(Prob. 15.62)

GIVEN:  $\theta = 30^\circ$  $\omega_0 = 900 \text{ rpm}$   
 $OA = 0.5 \text{ in.}$ FIND:  $\underline{v}_B$ 

SHAFT:

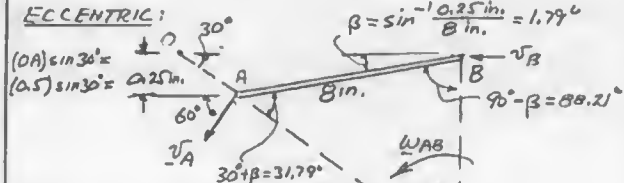


$$\omega_0 = 900 \text{ rpm} = 94.248 \text{ rad/s}$$

$$\underline{v}_A = (0.5 \text{ in.})(94.248 \text{ rad/s})$$

$$\underline{v}_A = 47.124 \text{ in./s} \angle 60^\circ$$

ECCENTRIC:

LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN  $\perp$  TO  $\underline{v}_A$  AND  $\underline{v}_B$ .

LAW OF SINES

$$\frac{AC}{\sin 88.21^\circ} = \frac{BC}{\sin 31.79^\circ} = \frac{8 \text{ in.}}{\sin 60^\circ}; \quad AC = 9.233 \text{ in.}$$

$$BC = 4.866 \text{ in.}$$

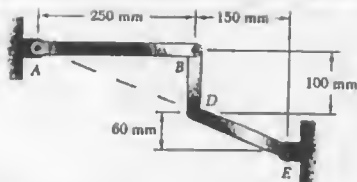
$$\underline{v}_A = (AC)\omega_{AB} \quad \omega_{AB} = \underline{v}_A / (AC)$$

$$\underline{v}_B = (BC)\omega_{AB} = (BC) \underline{v}_A / (AC) = \underline{v}_A \frac{BC}{AC}$$

$$\underline{v}_B = (47.124 \text{ in./s}) \frac{4.866 \text{ in.}}{9.233 \text{ in.}}; \quad \underline{v}_B = 24.8 \text{ in./s}$$

15.102

(Prob. 15.64)



GIVEN:

 $\underline{\omega}_{AB} = 4 \text{ rad/s}$ 

FIND:

 $\underline{\omega}_{BD}$  AND  $\underline{\omega}_{DE}$ BAR AB:  $\omega_{AB} = 4 \text{ rad/s}$ 

$$\underline{v}_B = (AB)\omega_{AB} = (0.25 \text{ m})(4 \text{ rad/s}) = 1 \text{ m/s}$$

BAR DE:

$$\beta = \tan^{-1} \frac{0.06 \text{ m}}{0.15 \text{ m}} = 21.8^\circ$$

$$DE = \frac{0.15 \text{ m}}{\cos \beta}$$

$$\underline{v}_D = (DE)\omega_{DE} = \frac{0.15 \text{ m}}{\cos \beta} \omega_{DE} \quad (1)$$

BAR BD:

LOCATE INST. CTR. C AT INTERSECTION OF LINES DRAWN  $\perp$  TO  $\underline{v}_B$  &  $\underline{v}_D$ .

$$BC = \frac{0.1 \text{ m}}{\tan \beta} = \frac{0.1 \text{ m}}{\tan 21.8^\circ}$$

$$BC = 0.25 \text{ m}$$

$$DC = \frac{0.25 \text{ m}}{\cos \beta}$$

$$\underline{v}_B = (BC)\omega_{BD}$$

$$1 \text{ m/s} = (0.25 \text{ m})\omega_{BD}$$

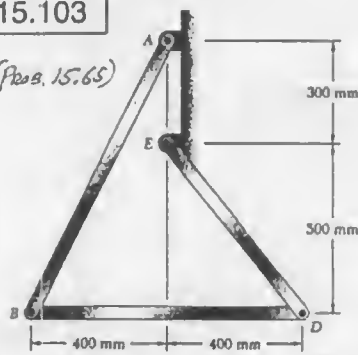
$$\omega_{BD} = \frac{0.25 \text{ m}}{\cos \beta} (4 \text{ rad/s})$$

$$\underline{v}_D = (DC)\omega_{BD} = \frac{0.25 \text{ m}}{\cos \beta} (4 \text{ rad/s})$$

$$\underline{v}_D = \frac{0.15 \text{ m}}{\cos \beta} \omega_{DE}; \quad \frac{1 \text{ m/s}}{\cos \beta} = \frac{0.15 \text{ m}}{\cos \beta} \omega_{DE}; \quad \omega_{DE} = 6.67 \text{ rad/s}$$

15.103

(Prob. 15.65)



GIVEN:

 $\omega_{AB} = 4 \text{ rad/s}$ 

FIND:

 $\underline{\omega}_{BD}$  $\underline{\omega}_{DE}$ 

BAR AB:

$$\beta = \tan^{-1} \frac{0.4 \text{ m}}{0.8 \text{ m}} = 26.56^\circ$$

$$AB = \frac{0.8 \text{ m}}{\cos \beta} = 0.8944 \text{ m}$$

$$\underline{v}_B = (AB)\omega_{AB} = (0.8944 \text{ m})(4 \text{ rad/s})$$

$$\underline{v}_B = 3.578 \text{ m/s} \angle 26.56^\circ$$

BAR DE:

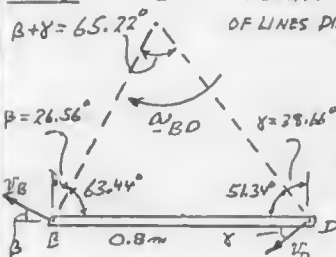
$$\gamma = \tan^{-1} \frac{0.4 \text{ m}}{0.5 \text{ m}} = 38.6^\circ$$

$$DE = \frac{0.5 \text{ m}}{\cos \gamma} = 0.6403 \text{ m}$$

$$\underline{v}_D = (DE)\omega_{DE}$$

$$\underline{v}_D = (0.6403 \text{ m})\omega_{DE} \angle 38.6^\circ$$

BAR BD:

LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN  $\perp$  TO  $\underline{v}_B$  AND  $\underline{v}_D$ 

$$\beta + \gamma = 65.22^\circ$$

$$\beta = 26.56^\circ$$

$$\gamma = 38.6^\circ$$

$$\underline{v}_B = 3.578 \text{ m/s} \angle 26.56^\circ$$

$$\underline{v}_D = 4.098 \text{ m/s} \angle 38.6^\circ$$

LAW OF SINES

$$\frac{BC}{\sin 51.34^\circ} = \frac{CD}{\sin 63.44^\circ} = \frac{0.8 \text{ m}}{\sin 65.22^\circ}$$

$$BC = 0.688 \text{ m}$$

$$CD = 0.788 \text{ m}$$

$$\underline{v}_B = (BC)\omega_{BD}$$

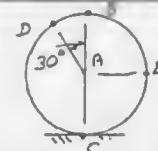
$$3.578 \text{ m/s} = (0.688 \text{ m})\omega_{BD}; \quad \omega_{BD} = 5.2 \text{ rad/s}$$

$$\underline{v}_D = (CD)\omega_{BD} = (0.788 \text{ m})(5.2 \text{ rad/s}) = 4.098 \text{ m/s}$$

$$\text{EQ (1)} \quad 4.098 \text{ m/s} = (0.6403 \text{ m})\omega_{DE}; \quad \omega_{DE} = 6.4 \text{ rad/s}$$

15.104

(Prob. 15.70)

GIVEN:  $\omega_A = 48 \text{ rad/s}$  $r = 11 \text{ in.}$ FIND:  $\underline{v}_B, \underline{v}_C, \underline{v}_D$  AND  $\underline{v}_E$  $\underline{v}_A = 48 \text{ rad/s} = 70.4 \text{ ft/s}$ 

FOR ROLLING INST. CTR. AT C

$$\omega = \frac{\underline{v}_A}{r} = \frac{70.4 \text{ ft/s}}{11 \text{ in.}}$$

$$\underline{v}_B = (BC)\omega = (2r) \frac{70.4}{r}$$

$$\underline{v}_B = 140.8 \text{ ft/s}$$

$$\underline{v}_C = 0$$

$$CD = 2r \cos 15^\circ$$

$$\underline{v}_D = (CD)\omega = (2r \cos 15^\circ) \frac{70.4}{r}$$

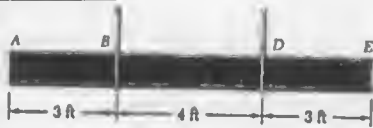
$$\underline{v}_D = 136.0 \text{ ft/s} \angle 15^\circ$$

$$CE = \sqrt{2}r$$

$$\underline{v}_E = (CE)\omega = (\sqrt{2}r) \frac{70.4}{r}$$

$$\underline{v}_E = 99.6 \text{ ft/s} \angle 45^\circ$$

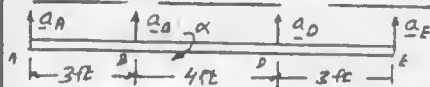
15.105



GIVEN:

$$a_B = 5 \text{ ft/s}^2 \uparrow$$

$$a_D = 3 \text{ ft/s}^2 \uparrow$$

FIND: (a)  $\alpha$   
(b)  $a_A$  and  $a_E$ 

$$(a) \quad a_D = a_B + a_{D/B} = a_B \uparrow + (BD)\alpha \downarrow$$

$$3 \text{ ft/s}^2 \uparrow = 5 \text{ ft/s}^2 \uparrow + (4 \text{ ft})\alpha \downarrow$$

$$2 \text{ ft/s}^2 \downarrow = (4 \text{ ft})\alpha \downarrow \quad \alpha = 0.5 \text{ rad/s}^2 \downarrow$$

$$(b) \quad a_A = a_B + a_{A/B} = a_B + (AB)\alpha$$

$$a_A = 5 \text{ ft/s}^2 \uparrow + (3 \text{ ft})(0.5 \text{ rad/s}^2) \uparrow$$

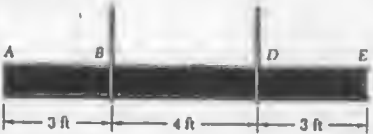
$$a_A = 6.5 \text{ ft/s}^2 \uparrow \quad a_B = 6.5 \text{ ft/s}^2 \uparrow$$

$$a_E = a_D + a_{E/D} = a_D + (DE)\alpha$$

$$a_E = 3 \text{ ft/s}^2 \uparrow + (3 \text{ ft})(0.5 \text{ rad/s}^2) \downarrow$$

$$a_E = 1.5 \text{ ft/s}^2 \uparrow \quad a_E = 1.5 \text{ ft/s}^2 \uparrow$$

15.106

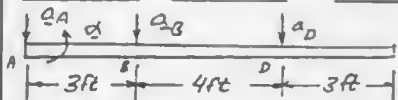


GIVEN:

$$a_A = 4 \text{ ft/s}^2 \downarrow$$

$$\alpha = 1.2 \text{ rad/s}^2 \downarrow$$

FIND:

(a)  $a_B$ (b)  $a_D$ 

$$(a) \quad a_B = a_A + a_{B/A} = a_A \downarrow + (AB)\alpha \uparrow$$

$$a_B = 4 \text{ ft/s}^2 \downarrow + (3 \text{ ft})(1.2 \text{ rad/s}^2) \uparrow$$

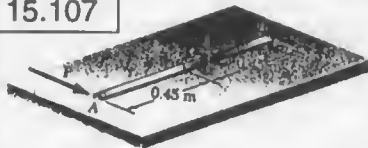
$$a_B = 4 \text{ ft/s}^2 \downarrow + 3.6 \text{ ft/s}^2 \uparrow \quad a_B = 0.4 \text{ ft/s}^2 \downarrow$$

$$a_D = a_A + a_{D/A} = a_A \downarrow + (AD)\alpha \uparrow$$

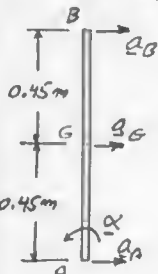
$$a_D = 4 \text{ ft/s}^2 \downarrow + (7 \text{ ft})(1.2 \text{ rad/s}^2) \uparrow$$

$$a_D = 4 \text{ ft/s}^2 \downarrow + 8.4 \text{ ft/s}^2 \uparrow \quad a_D = 4.4 \text{ ft/s}^2 \uparrow$$

15.107

GIVEN:  $a_A = 3.6 \text{ m/s}^2 \rightarrow$ 

$$\alpha = 6 \text{ rad/s}^2 \downarrow$$

FIND: (a)  $a_G$ (b)  $a_B$ 

$$a_G = a_A + a_{G/A} = a_A \rightarrow + (AG)\alpha \leftarrow$$

$$a_G = 3.6 \text{ m/s}^2 \rightarrow + (0.45 \text{ m})(6 \text{ rad/s}^2) \leftarrow$$

$$a_G = 3.6 \text{ m/s}^2 \rightarrow + 2.7 \text{ m/s}^2 \leftarrow$$

$$a_G = 0.9 \text{ m/s}^2 \rightarrow$$

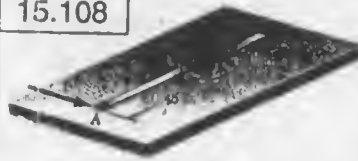
$$a_B = a_A + a_{B/A} = a_A \rightarrow + (AB)\alpha \leftarrow$$

$$a_B = 3.6 \text{ m/s}^2 \rightarrow + (0.9 \text{ m})(6 \text{ rad/s}^2) \leftarrow$$

$$a_B = 3.6 \text{ m/s}^2 \rightarrow + 5.4 \text{ m/s}^2 \leftarrow$$

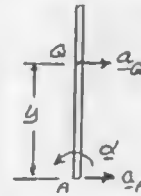
$$a_B = 1.8 \text{ m/s}^2 \leftarrow$$

15.108

GIVEN:  $a_A = 3.6 \text{ m/s}^2 \rightarrow$ 

$$\alpha = 6 \text{ rad/s}^2 \downarrow$$

FIND: POINT OF ROD FOR

(a)  $a = 0$ (b)  $a = 2.4 \text{ m/s}^2 \rightarrow$ (a) FOR  $a_G = 0$ 

$$a_G = a_A + a_{G/A} = a_A \rightarrow + (AG)\alpha \leftarrow$$

$$0 = 3.6 \text{ m/s}^2 \rightarrow + (y)(6 \text{ rad/s}^2) \leftarrow$$

$$y = \frac{3.6 \text{ m/s}^2}{6 \text{ rad/s}^2} = 0.6 \text{ m}$$

$$a = 0 \text{ AT } 0.6 \text{ m FROM A}$$

(b) FOR  $a_G = 2.4 \text{ m/s}^2 \rightarrow$ 

$$a_G = a_A + a_{G/A} = a_A \rightarrow + (AG)\alpha \leftarrow$$

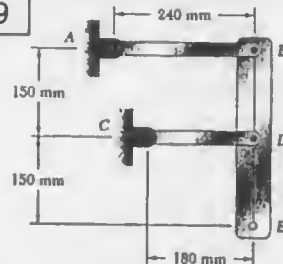
$$2.4 \text{ m/s}^2 \rightarrow = 3.6 \text{ m/s}^2 \rightarrow + (y)(6 \text{ rad/s}^2) \leftarrow$$

$$1.2 \text{ m/s}^2 \leftarrow = (y)(6 \text{ rad/s}^2) \leftarrow$$

$$y = 0.2 \text{ m}$$

$$a = 2.4 \text{ m/s}^2 \rightarrow \text{ AT } 0.2 \text{ m FROM A}$$

15.109



GIVEN:

$$\omega_{AB} = 3 \text{ rad/s} \downarrow$$

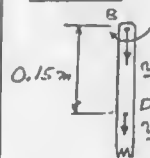
$$\alpha_{AB} = 0$$

FIND:

(a)  $a_D$ (b)  $a_E$ 

VELOCITY

$$v_B = (AB)\omega_{AB} = (0.24 \text{ m})(3 \text{ rad/s}) = 0.72 \text{ m/s} \downarrow$$



$$v_D = v_B + v_{D/B} = v_B \downarrow + (BD)\omega_{BD} \leftarrow$$

$$v_D \downarrow = 0.72 \text{ m/s} \downarrow + (0.15 \text{ m})\omega_{BD} \leftarrow$$

$$\omega_{BD} = 0$$

$$v_D = 0.72 \text{ m/s} \downarrow$$

$$v_D = (CD)\omega_{CD} \quad 0.72 \text{ m/s} \downarrow = (0.18 \text{ m})\omega_{CD} \downarrow$$

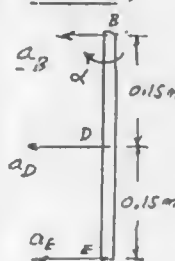
$$\omega_{CD} = 4 \text{ rad/s} \downarrow$$

ACCELERATIONS

$$\text{ROD AB: } a_B = (AB)\alpha_{AB} = (0.24 \text{ m})(3 \text{ rad/s})^2 = 2.16 \text{ m/s}^2 \leftarrow$$

$$\text{ROD CD: } a_D = (CD)\alpha_{CD} = (0.18 \text{ m})(4 \text{ rad/s}^2) = 2.88 \text{ m/s}^2 \leftarrow$$

ROD BDE:



$$a_D = a_B + a_{D/B} = a_B \leftarrow + (BD)\alpha \leftarrow$$

$$2.88 \text{ m/s}^2 \leftarrow = 2.16 \text{ m/s}^2 \leftarrow + (0.15 \text{ m})\alpha \leftarrow$$

$$0.72 \text{ m/s}^2 \leftarrow = (0.15 \text{ m})\alpha \leftarrow$$

$$\alpha = 4.8 \text{ rad/s}^2 \downarrow$$

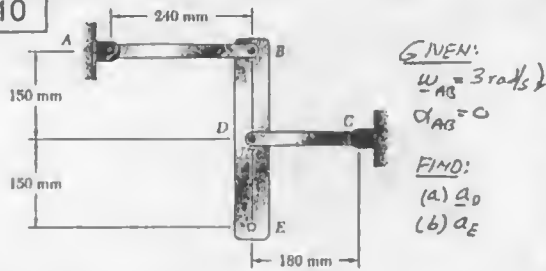
$$a_E = a_D + a_{E/D} = a_D \leftarrow + (DE)\alpha \leftarrow$$

$$a_E = 2.88 \text{ m/s}^2 \leftarrow + (0.15 \text{ m})(4.8 \text{ rad/s}^2) \leftarrow$$

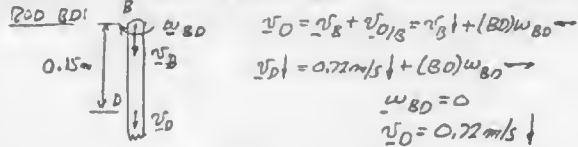
$$a_E = 2.88 \text{ m/s}^2 \leftarrow + 0.72 \text{ m/s}^2 \leftarrow$$

$$a_E = 3.6 \text{ m/s}^2 \leftarrow$$

15.110



VELOCITY: ROD AB  $\underline{v}_B = (AB)\omega_{AB} = (0.24\text{m})(3\text{rad/s}) = 0.72\text{m/s} \uparrow$



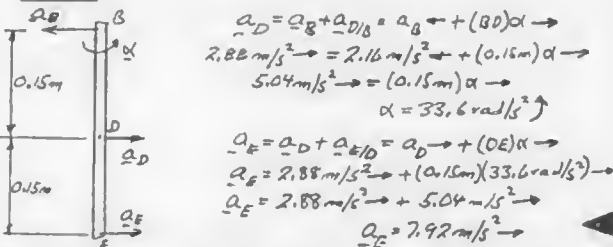
ROD DC:  $\underline{v}_D = (DC)\omega_{DC}$ ;  $0.72\text{m/s} \uparrow = (0.18\text{m})\omega_{DC}$   
 $\omega_{DC} = 4\text{rad/s} \uparrow$

ACCELERATION:

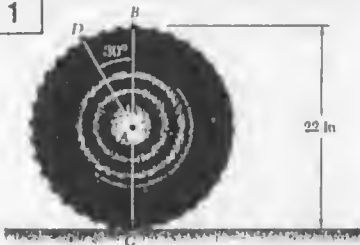
ROD AB:  $\underline{a}_B = (AB)\omega_{AB}^2 = (0.24\text{m})(3\text{rad/s})^2 = 2.16\text{m/s}^2 \leftarrow$

ROD DC:  $\underline{a}_D = (DC)\omega_{DC}^2 = (0.18\text{m})(4\text{rad/s})^2 = 2.88\text{m/s}^2 \leftarrow$

ROD BD:



15.111



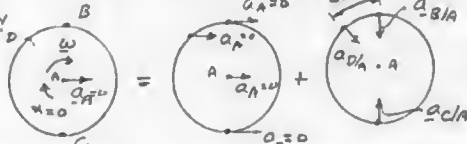
$$\underline{v}_A = 48 \frac{\text{mi}}{\text{h}} \cdot \frac{1}{3600 \text{ s}} \cdot \frac{5280 \text{ ft}}{\text{mi}} = 70.4 \text{ ft/s} \rightarrow$$

ROLLING WITH NO SLIDING, INST. CENTER IS AT C.

$$\therefore \underline{v}_A = (AC)\omega; 70.4 \text{ ft/s} = \left(\frac{1}{2} \text{ ft}\right)\omega$$

$$\omega = 76.8 \text{ rad/s} \uparrow$$

ACCELERATION



PLANE MOTION = TRANS. WITH A + ROTATION ABOUT A

$$\underline{a}_{B/A} = \underline{a}_{C/A} = \underline{a}_{D/A} = r\omega^2 = \left(\frac{1}{12} \text{ ft}\right)(76.8 \text{ rad/s})^2 = 5407 \text{ ft/s}^2$$

$$(a) \underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = 0 + 5407 \text{ ft/s}^2 \downarrow \quad \underline{a}_B = 5410 \text{ ft/s}^2 \downarrow$$

$$(b) \underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = 0 + 5407 \text{ ft/s}^2 \uparrow \quad \underline{a}_C = 5410 \text{ ft/s}^2 \uparrow$$

$$(c) \underline{a}_D = \underline{a}_A + \underline{a}_{D/A} = 0 + 5407 \text{ ft/s}^2 \uparrow \quad \underline{a}_D = 5410 \text{ ft/s}^2 \uparrow 30^\circ$$

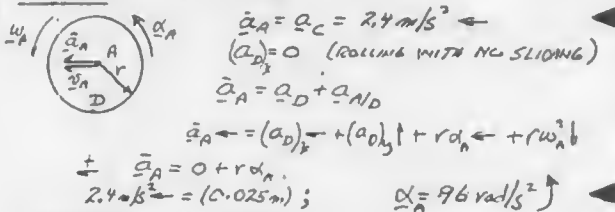
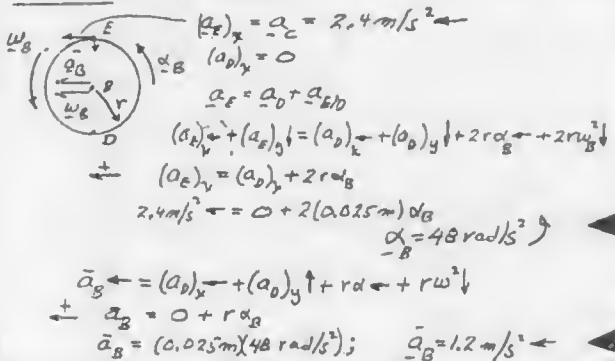
$$\underline{a}_D = 5410 \text{ ft/s}^2 \uparrow 60^\circ$$

15.112

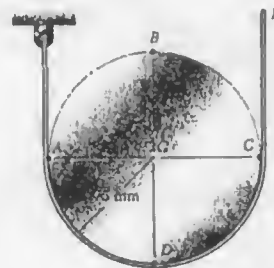
CASTER AND CYLINDER EACH OF 50-mm DIAM.



ROLLING OCCURS AT ALL SURFACES OF CONTACT  
 INST. CENTERS AT POINTS OF CONTACT WITH FLOOR

CASTER:  $r = 0.025 \text{ m}$ CYLINDER:  $r = 0.025 \text{ m}$ 

15.113 and 15.114



GIVEN:

$$\underline{v}_E = 300 \text{ mm/s} \uparrow$$

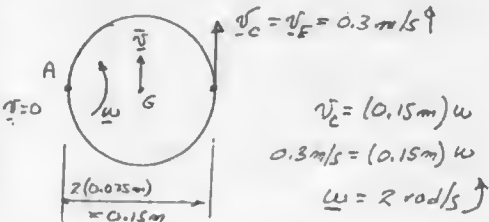
$$\underline{a}_E = 480 \text{ mm/s}^2 \uparrow$$

FIND:

$$(a) \underline{a}_A$$

$$(b) \underline{a}_B$$

VELOCITY:  $\underline{v}_A = 0$ , THUS INST. CENTER IS AT A.



$$\underline{v}_C = (0.15 \text{ m})\omega$$

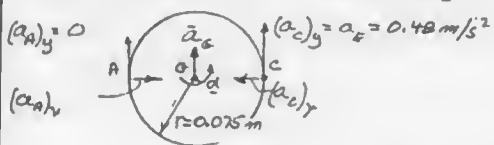
$$0.3 \text{ m/s} = (0.15 \text{ m})\omega$$

$$\omega = 2 \text{ rad/s} \uparrow$$

(CONTINUED)

## 15.113 and 15.114 CONTINUED

$\omega = 2 \text{ rad/s} \uparrow$

ANGULAR ACCELERATION AND  $\bar{a}_G$   $(a_A)_y = 0$ ;

$\bar{a}_C = \bar{a}_A + \bar{a}_{C/A}$

$(a_C)_x + (a_C)_y \uparrow = (a_A)_x + (a_C)_y \uparrow + 2r\alpha \uparrow$

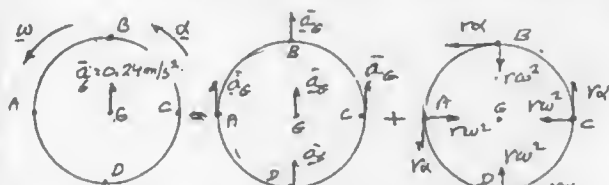
$0 = 0.48 \text{ m/s}^2 + 2(0.075 \text{ m})\alpha$

$\alpha = 3.2 \text{ rad/s}^2 \uparrow$

$\bar{a}_G = \bar{a}_A + \bar{a}_{G/A}$

$\bar{a}_G \uparrow = (a_A)_x + (0.075 \text{ m})\alpha$

$\bar{a}_G = (0.075 \text{ m})(3.2 \text{ rad/s}^2) = 0.24 \text{ m/s}^2 \uparrow$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$r\alpha = (0.075 \text{ m})(3.2 \text{ rad/s}^2) = 0.24 \text{ m/s}^2$

$r\omega^2 = (0.075 \text{ m})(2 \text{ rad/s})^2 = 0.3 \text{ m/s}^2$

FOR EACH POINT:

$\bar{a} = \bar{a}_G + r\alpha + r\omega^2$

POINT A:  $\bar{a}_A = 0.24 \text{ m/s}^2 \uparrow + 0.24 \text{ m/s}^2 \uparrow + 0.3 \text{ m/s}^2 \rightarrow$

$\bar{a}_A = 0.3 \text{ m/s}^2 \rightarrow$

POINT B:  $\bar{a}_B = 0.24 \text{ m/s}^2 \uparrow + 0.24 \text{ m/s}^2 \rightarrow + 0.3 \text{ m/s}^2 \downarrow$

$\bar{a}_B = 0.06 \text{ m/s}^2 \downarrow + 0.24 \text{ m/s}^2 \rightarrow$

$$\begin{array}{c} 0.24 \text{ m/s}^2 \\ \nearrow \\ \bar{a}_B \\ \nwarrow \\ 0.06 \text{ m/s}^2 \end{array} \quad \bar{a}_B = 0.247 \text{ m/s}^2 \nearrow 14.0^\circ$$

POINT C:  $\bar{a}_C = 0.24 \text{ m/s}^2 \uparrow + 0.24 \text{ m/s}^2 \uparrow + 0.3 \text{ m/s}^2 \leftarrow$

$\bar{a}_C = 0.48 \text{ m/s}^2 \uparrow + 0.3 \text{ m/s}^2 \leftarrow$

$$\begin{array}{c} \bar{a}_C \\ \nearrow \\ 0.48 \text{ m/s}^2 \\ \nwarrow \\ 0.3 \text{ m/s}^2 \end{array} \quad \bar{a}_C = 0.586 \text{ m/s}^2 \nearrow 58.0^\circ$$

POINT D:  $\bar{a}_D = 0.24 \text{ m/s}^2 \uparrow + 0.24 \text{ m/s}^2 \rightarrow + 0.3 \text{ m/s}^2 \uparrow$

$\bar{a}_D = 0.54 \text{ m/s}^2 \uparrow + 0.24 \text{ m/s}^2 \rightarrow$

$$\begin{array}{c} 0.54 \text{ m/s}^2 \\ \nearrow \\ \bar{a}_D \\ \nwarrow \\ 0.24 \text{ m/s}^2 \end{array} \quad \bar{a}_D = 0.591 \text{ m/s}^2 \nearrow 66.0^\circ$$

PROBLEM 15.113

$\bar{a}_A = 300 \text{ mm/s}^2 \rightarrow$

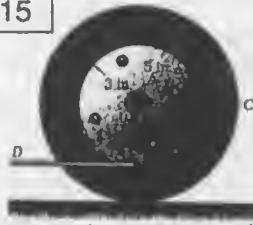
$\bar{a}_B = 247 \text{ mm/s}^2 \nearrow 14.0^\circ$

PROBLEM 15.114

$\bar{a}_C = 566 \text{ mm/s}^2 \nearrow 58.0^\circ$

$\bar{a}_D = 591 \text{ mm/s}^2 \nearrow 66.0^\circ$

## 15.115



GIVEN:

$\bar{v}_D = 8 \text{ in./s} \leftarrow$

$\bar{a}_D = 30 \text{ in./s}^2 \leftarrow$

FIND:

$\bar{a}_A, \bar{a}_B, \text{ AND } \bar{a}_C$

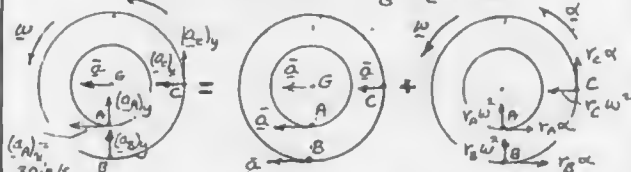
VELOCITY: INST. CENTER AT B.

$\bar{v}_D = (2 \text{ in.})\omega ; 8 \text{ in./s} = (2 \text{ in.})\omega$

$\omega = 4 \text{ rad/s} \uparrow$

ACCELERATION: FOR NO SLIDING:  $(a_B)_x = 0$ 

$r_A = 3 \text{ in.}, r_B = r_C = 5 \text{ in.}$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$$

$$\bar{a}_A = 30 \text{ in./s}^2 \leftarrow + (2 \text{ in.})\alpha ; \quad \alpha = 15 \text{ rad/s}^2 \uparrow$$

$\bar{a}_G = \bar{a}_B + \bar{a}_{G/B}$

$\bar{a}_G = 0 + (5 \text{ in.})(15 \text{ rad/s}^2) ; \quad \bar{a}_G = 75 \text{ in./s}^2 \leftarrow$

FOR EACH POINT:

$\bar{a} = \bar{a}_G + r\alpha + r\omega^2$

POINT A:  $\bar{a}_A = 75 \text{ in./s}^2 \leftarrow + (3 \text{ in.})(15 \text{ rad/s}^2) \rightarrow + (3 \text{ in.})(4 \text{ rad/s})^2 \uparrow$

$= 75 \text{ in./s}^2 \leftarrow + 45 \text{ in./s}^2 \rightarrow + 48 \text{ in./s}^2 \uparrow$

$\bar{a}_A = 30 \text{ in./s}^2 \leftarrow + 48 \text{ in./s}^2 \uparrow$

$$\begin{array}{c} \bar{a}_A \\ \nearrow \\ 48 \text{ in./s}^2 \\ \nwarrow \\ 30 \text{ in./s}^2 \end{array} \quad \bar{a}_A = 56.6 \text{ in./s}^2 \nearrow 58.0^\circ$$

POINT B:  $\bar{a}_B = 75 \text{ in./s}^2 \leftarrow + (5 \text{ in.})(15 \text{ rad/s}^2) \rightarrow + (5 \text{ in.})(4 \text{ rad/s})^2 \uparrow$

$= 75 \text{ in./s}^2 \leftarrow + 75 \text{ in./s}^2 \rightarrow + 80 \text{ in./s}^2 \uparrow$

$\bar{a}_B = 80 \text{ in./s}^2 \uparrow$

$\bar{a}_B = 80 \text{ in./s}^2 \uparrow$

POINT C:

$\bar{a}_C = 75 \text{ in./s}^2 \leftarrow + (5 \text{ in.})(15 \text{ rad/s}^2) \uparrow + (5 \text{ in.})(4 \text{ rad/s})^2 \leftarrow$

$= 75 \text{ in./s}^2 \leftarrow + 75 \text{ in./s}^2 \uparrow + 80 \text{ in./s}^2 \leftarrow$

$\bar{a}_C = 155 \text{ in./s}^2 \leftarrow + 75 \text{ in./s}^2 \uparrow$

$$\begin{array}{c} \bar{a}_C \\ \nearrow \\ 75 \text{ in./s}^2 \\ \nwarrow \\ 155 \text{ in./s}^2 \end{array} \quad \bar{a}_C = 172.2 \text{ in./s}^2 \nearrow 25.8^\circ$$



15.116



GIVEN:

$$v_D = 8 \text{ in./s} \leftarrow$$

$$a_D = 30 \text{ in./s}^2 \leftarrow$$

FIND:

$$a_A, a_B \text{ AND } a_C$$

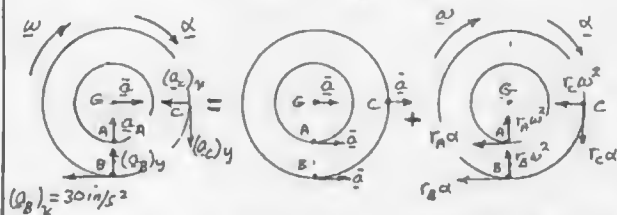
VELOCITY: INST. CENTER AT A

$$v_D = (2 \text{ in.})\omega; 8 \text{ in./s} = (2 \text{ in.})\omega$$

$$\omega = 4 \text{ rad/s} \curvearrowright$$

ACCELERATION: FOR NO SLIDING:  $(a_A)_x = 0$ 

$$r_A = 3 \text{ in.} \quad r_B = r_C = 5 \text{ in.}$$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$+ \quad a_B = a_G + a_{B/A}$$

$$+ \quad 30 \text{ in./s}^2 \leftarrow = 0 + (2 \text{ in.})\alpha; \quad \alpha = 15 \text{ rad/s}^2 \curvearrowright$$

$$a_G = a_A + a_{G/A}$$

$$+ \quad a_G = 0 + (3 \text{ in.})(15 \text{ rad/s}^2); \quad a_G = 45 \text{ in./s}^2 \rightarrow$$

FOR EACH POINT

$$a = a_G + r\alpha + r\omega^2$$

$$\text{POINT A: } a_A = 45 \text{ in./s}^2 \rightarrow + (3 \text{ in.})(15 \text{ rad/s}^2) \leftarrow + (3 \text{ in.})(4 \text{ rad/s})^2 \uparrow$$

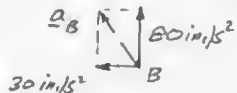
$$= 45 \text{ in./s}^2 \rightarrow + 45 \text{ in./s}^2 \leftarrow + 36 \text{ in./s}^2 \uparrow$$

$$a_A = 48 \text{ in./s}^2 \uparrow$$

$$\text{POINT B: } a_B = 45 \text{ in./s}^2 \rightarrow + (5 \text{ in.})(15 \text{ rad/s}^2) \leftarrow + (5 \text{ in.})(4 \text{ rad/s})^2 \uparrow$$

$$= 45 \text{ in./s}^2 \rightarrow + 75 \text{ in./s}^2 \leftarrow + 80 \text{ in./s}^2 \uparrow$$

$$a_B = 30 \text{ in./s}^2 \leftarrow + 80 \text{ in./s}^2 \uparrow$$

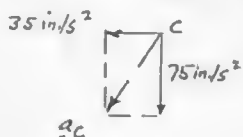


$$a_B = 85.4 \text{ in./s}^2 \nearrow 69.4^\circ$$

$$\text{POINT C: } a_C = 45 \text{ in./s}^2 \rightarrow + (5 \text{ in.})(15 \text{ rad/s}^2) \downarrow + (5 \text{ in.})(4 \text{ rad/s})^2 \leftarrow$$

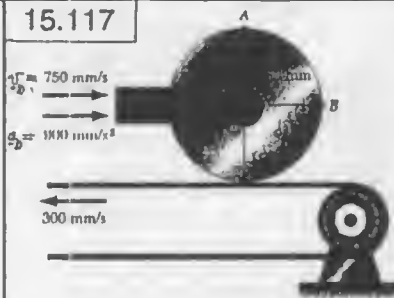
$$= 45 \text{ in./s}^2 \rightarrow + 75 \text{ in./s}^2 \downarrow + 80 \text{ in./s}^2 \leftarrow$$

$$= 35 \text{ in./s}^2 \leftarrow + 75 \text{ in./s}^2 \downarrow$$



$$a_C = 82.8 \text{ in./s}^2 \searrow 65.0^\circ$$

15.117



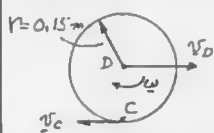
GIVEN:

$$v_{\text{BELT}} = 300 \text{ mm/s} \leftarrow$$

$$a_{\text{BELT}} = 0$$

FIND:

$$a_A, a_B, \text{ AND } a_C$$

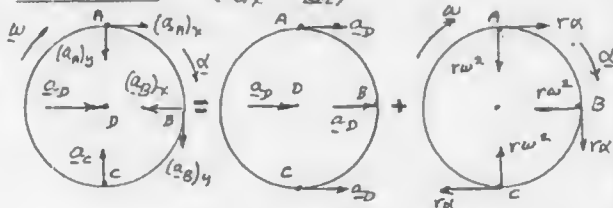
VELOCITY:  $v_C = v_{\text{BELT}} = 0.3 \text{ m/s} \leftarrow; \quad v_D = 0.75 \text{ m/s} \rightarrow$ 

$$v_D = v_C + v_{D/C} = v_C + r\omega$$

$$0.75 \text{ m/s} \rightarrow = 0.3 \text{ m/s} \leftarrow + r\omega \rightarrow$$

$$1.05 \text{ m/s} \rightarrow = (0.15 \text{ m})\omega$$

$$\omega = 7 \text{ rad/s} \curvearrowright$$

ACCELERATION:  $(a_C)_x = a_{\text{BELT}} = 0$ 

PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

$$a_D = a_C + a_{D/C}$$

$$= a_C + r\alpha$$

$$0.9 \text{ m/s}^2 \rightarrow = a_C \uparrow + (0.15 \text{ m})\alpha \rightarrow$$

$$\alpha = 6 \text{ rad/s}^2 \curvearrowright$$

$$r\alpha = (0.15 \text{ m})(6 \text{ rad/s}^2) = 0.9 \text{ m/s}^2$$

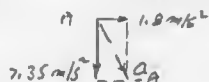
$$r\omega^2 = (0.15 \text{ m})(7 \text{ rad/s})^2 = 7.35 \text{ m/s}^2$$

FOR EACH POINT

$$a = a_D + r\alpha + r\omega^2$$

$$\text{POINT A: } a_A = 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \rightarrow + 7.35 \text{ m/s}^2 \downarrow$$

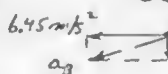
$$a_A = 1.8 \text{ m/s}^2 \rightarrow + 7.35 \text{ m/s}^2 \downarrow$$



$$a_A = 7.57 \text{ m/s}^2 \searrow 76.2^\circ$$

$$\text{POINT B: } a_B = 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow + 7.35 \text{ m/s}^2 \leftarrow$$

$$a_B = 6.45 \text{ m/s}^2 \leftarrow + 0.9 \text{ m/s}^2 \downarrow$$

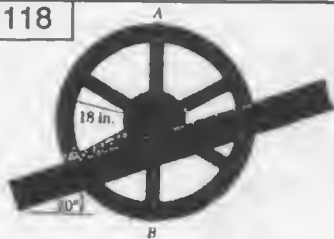


$$a_B = 6.51 \text{ m/s}^2 \searrow 7.9^\circ$$

$$\text{POINT C: } a_C = 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \rightarrow + 7.35 \text{ m/s}^2 \uparrow$$

$$a_C = 7.35 \text{ m/s}^2 \uparrow$$

15.118



GIVEN:

SHAFT:  $r = 1.5 \text{ in.}$  $\vec{v} = 1.2 \text{ in./s}$  $\vec{a} = 0.5 \text{ in./s}^2$ FIND: (a)  $\underline{a}_A$   
(b)  $\underline{a}_B$ 

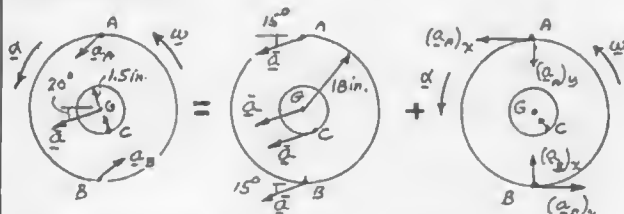
VELOCITY: SHAFT,

ROLLING, NO SLIDING

INST. CENTER AT C

 $\vec{v} = r\omega$  $1.2 \text{ in./s} = (1.5 \text{ in.})\omega$  $\omega = 0.8 \text{ rad/s}$ 

ACCELERATION:



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$\begin{aligned} \underline{a}_G &= \underline{a}_C + \underline{a}_{G/C} \\ \underline{a}_G &= 0 + r\alpha \\ 0.5 \text{ in./s}^2 &= 0 + (1.5 \text{ in.})\alpha; \quad \alpha = \frac{1}{3} \text{ rad/s}^2 \end{aligned}$$

FOR EACH POINT  $r_A = r_B = 1.5 \text{ in.}$  $\underline{a} = \underline{\bar{a}} + r\alpha + r\omega^2$ 

(a) POINT A:

$$\begin{aligned} \underline{a}_A &= (0.5 \text{ in./s}^2) \angle 20^\circ + (1.5 \text{ in.}) \left( \frac{1}{3} \text{ rad/s}^2 \right) \leftarrow + (1.5 \text{ in.}) (0.8 \text{ rad/s})^2 \downarrow \\ &= 0.470 \text{ in./s}^2 \leftarrow + 0.171 \text{ in./s}^2 \downarrow + 6 \text{ in./s}^2 \downarrow + 11.52 \text{ in./s}^2 \downarrow \end{aligned}$$

$$\underline{a}_A = 6.47 \text{ in./s}^2 \leftarrow + 11.69 \text{ in./s}^2 \downarrow$$

$$\underline{a}_A = 13.36 \text{ in./s}^2 \angle 61.0^\circ$$

(b) POINT B:

$$\begin{aligned} \underline{a}_B &= (0.5 \text{ in./s}^2) \angle 20^\circ + (1.5 \text{ in.}) \left( \frac{1}{3} \text{ rad/s}^2 \right) \leftarrow + (1.5 \text{ in.}) (0.8 \text{ rad/s})^2 \uparrow \\ &= 0.470 \text{ in./s}^2 \leftarrow + 0.171 \text{ in./s}^2 \downarrow + 6 \text{ in./s}^2 \uparrow + 11.52 \text{ in./s}^2 \uparrow \end{aligned}$$

$$\underline{a}_B = 5.53 \text{ in./s}^2 \leftarrow + 11.35 \text{ in./s}^2 \uparrow$$

$$\underline{a}_B = 12.62 \text{ in./s}^2 \angle 64.0^\circ$$

15.119



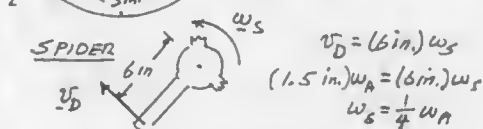
GIVEN:

 $r_A = r_B = r_C = 3 \text{ in.}; r_E = 9 \text{ in.}$  $\omega_A = 150 \text{ rpm}$ ,  $\alpha_A = 0$  $\omega_E = 0$ 

FIND: MAGNITUDE OF ACCELERATION OF TOOTH OF GEAR D IN CONTACT WITH (a) GEAR A, (b) GEAR E.

VELOCITY: T = TOOTH OF GEAR D IN CONTACT WITH GEAR A

GEARS

 $\vec{v}_T = r\omega_A = (3 \text{ in.})\omega_A$ SINCE  $\vec{v}_E = 0$ , E IS INST. CENTER OF GEAR D $\vec{v}_T = 2r\omega_D$  $(3 \text{ in.})\omega_A = 2(3 \text{ in.})\omega_D$  $\omega_D = \frac{1}{2}\omega_A$  $\vec{v}_D = r\omega_D = (3 \text{ in.}) \frac{1}{2}\omega_A = (1.5 \text{ in.})\omega_A$ 

SPIDER

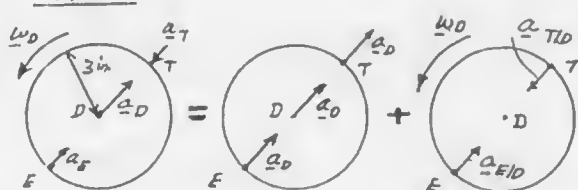
 $\vec{v}_D$  $\omega_A = 150 \text{ rpm} = 15.708 \text{ rad/s}$  $\omega_D = \frac{1}{2}\omega_A = 7.854 \text{ rad/s}$  $\omega_s = \frac{1}{4}\omega_A = 3.927 \text{ rad/s}$ 

ACCELERATION

SPIDER:

 $\omega_s = 3.927 \text{ rad/s}$  $\underline{a}_D = (AD)\omega_s^2 = (6 \text{ in.})(3.927 \text{ rad/s})^2$  $\underline{a}_D = 92.53 \text{ in./s}^2$ 

GEAR D:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

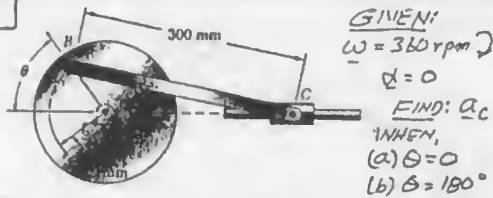
(a) TOOTH T IN CONTACT WITH GEAR A

 $\underline{a}_T = \underline{a}_D + \underline{a}_{T/D} = \underline{a}_D + (DT)\omega_D^2$  $= 92.53 \text{ in./s}^2 + (3 \text{ in.})(7.854 \text{ rad/s})^2$  $= 92.53 \text{ in./s}^2 + 185.06 \text{ in./s}^2$  $\underline{a}_T = 92.53 \text{ in./s}^2$  $\underline{a}_T = 92.5 \text{ in./s}^2$ 

(b) TOOTH E IN CONTACT WITH GEAR E

 $\underline{a}_E = \underline{a}_D + \underline{a}_{E/D} = \underline{a}_D + (ED)\omega_D^2$  $= 92.53 \text{ in./s}^2 + (3 \text{ in.})(7.854 \text{ rad/s})^2$  $= 92.53 \text{ in./s}^2 + 185.06 \text{ in./s}^2$  $\underline{a}_E = 277.6 \text{ in./s}^2$  $\underline{a}_E = 278 \text{ in./s}^2$

15.120



**DISK:**  $\omega = 360 \text{ rpm} = 37.7 \text{ rad/s}$

$$v_B = (AB)\omega = (0.075 \text{ m})(37.7 \text{ rad/s}) \quad v_B = 2.8275 \text{ m/s}$$

$$a_B = (AB)\omega^2 = (0.075 \text{ m})(37.7 \text{ rad/s})^2 \quad a_B = 106.59 \text{ m/s}^2$$

(a)  $\theta = 0$ :

$$v_C = (BC)\omega_{BC}$$

$$2.8275 \text{ m/s} = (0.3 \text{ m})\omega_{BC}$$

$$\omega_{BC} = 9.425 \text{ rad/s}$$

PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = \underline{a}_B + (BC)\omega_{BC}^2$$

$$= 106.59 \text{ m/s}^2 + (0.3 \text{ m})(9.425 \text{ rad/s})^2$$

$$= 106.59 \text{ m/s}^2 + 26.65 \text{ m/s}^2$$

$$\underline{a}_C = 79.94 \text{ m/s}^2 \quad \underline{a}_C = 79.9 \text{ m/s}^2$$

(b)  $\theta = 180^\circ$ :

$$v_C = (BC)\omega_{BC}$$

$$2.8275 \text{ m/s} = (0.3 \text{ m})\omega_{BC}$$

$$\omega_{BC} = 9.425 \text{ rad/s}$$

PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

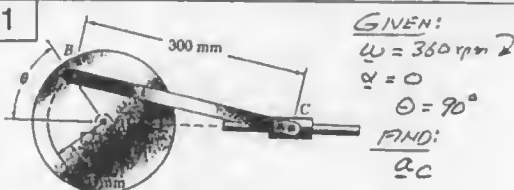
$$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = \underline{a}_B + (BC)\omega_{BC}^2$$

$$= 106.59 \text{ m/s}^2 + (0.3 \text{ m})(9.425 \text{ rad/s})^2$$

$$= 106.59 \text{ m/s}^2 + 26.65 \text{ m/s}^2$$

$$\underline{a}_C = 133.24 \text{ m/s}^2 \quad \underline{a}_C = 133.2 \text{ m/s}^2$$

15.121



**DISK:**  $\omega = 360 \text{ rpm} = 37.7 \text{ rad/s}$

$$\underline{a}_B = (AB)\omega^2 = (0.075 \text{ m})(37.7 \text{ rad/s})^2 = 106.59 \text{ m/s}^2$$

**ROD:**  $\beta = 14.477^\circ$  INST. CENTER AT O  
 $\therefore \omega_{BC} = 0$

$$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B}$$

$$[\underline{a}_C] = [\underline{a}_B] + [\underline{a}_{C/B}]$$

$$[\underline{a}_C] = [106.59 \text{ m/s}^2] + [a_{C/B} \angle 14.477^\circ]$$

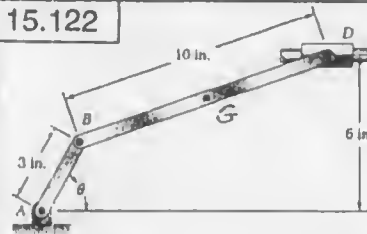
VECTOR DIAGRAM

$$a_C = (106.59 \text{ m/s}^2) \tan 14.477^\circ$$

$$\underline{a}_C = 27.52 \text{ m/s}^2$$

$$\underline{a}_C = 27.5 \text{ m/s}^2$$

15.122



VELOCITY:

$$v_D = (AB)\omega_{AB}$$

$$= (3 \text{ in.})(16 \text{ rad/s})$$

$$v_D = 48 \text{ in./s}$$

$$v_D = (BC)\omega_{BD}$$

$$48 \text{ in./s} = (8 \text{ in.})\omega_{BD}$$

$$\omega_{BD} = 6 \text{ rad/s}$$

ACCELERATION

ROD AB:  $\underline{a}_B = (AB)\omega_{AB}^2 = (3 \text{ in.})(16 \text{ rad/s})^2 = 768 \text{ in./s}^2$

PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

(a)  $\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (a_{D/B})_t + (a_{D/B})_n$

$$\underline{a}_D = \underline{a}_B + (BD)\alpha + (BD)\omega^2$$

$$\underline{a}_D = 768 \text{ in./s}^2 + (10 \text{ in.})\alpha + (10 \text{ in.})(6 \text{ rad/s})^2$$

$$\underline{a}_D = 768 \text{ in./s}^2 + (10 \text{ in.})\alpha + 360 \text{ in./s}^2$$

VECTOR DIAGRAM

$$(10 \text{ in.})\alpha = 216 \text{ in./s}^2 \quad \alpha = 27 \text{ rad/s}^2$$

$$\underline{a}_D = 768 + 288 + \frac{3}{5}(10 \text{ in.})\alpha$$

$$= 768 + 288 + \frac{3}{5}(10)(27) = 768 + 288 + 162 = 1218 \text{ in./s}^2$$

$$\underline{a}_D = 1218 \text{ in./s}^2$$

$$(10 \text{ in.})\alpha = 216 \text{ in./s}^2 \quad \alpha = 27 \text{ rad/s}^2$$

$$\underline{a}_D = 768 + 288 + \frac{3}{5}(10 \text{ in.})\alpha$$

$$= 768 + 288 + \frac{3}{5}(10)(27) = 768 + 288 + 162 = 1218 \text{ in./s}^2$$

$$\underline{a}_D = 1218 \text{ in./s}^2$$

(b)  $\underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + (a_{G/B})_t + (a_{G/B})_n$

$$\underline{a}_G = \underline{a}_B + (BG)\alpha + (BG)\omega^2$$

$$\underline{a}_G = 768 \text{ in./s}^2 + (5 \text{ in.})(27 \text{ rad/s})^2 + (5 \text{ in.})(6 \text{ rad/s})^2$$

$$\underline{a}_G = 768 \text{ in./s}^2 + 135 \text{ in./s}^2 + 180 \text{ in./s}^2$$

+ COMPONENTS:  $(a_G)_y = 135(\frac{4}{5}) - 180(\frac{3}{5})$

$$= 108 \text{ in./s}^2 - 108 \text{ in./s}^2$$

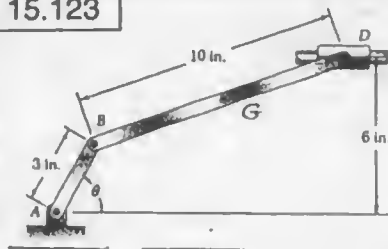
$$(a_G)_y = 0$$

+ COMPONENTS:  $(a_G)_x = 768 + 135(\frac{3}{5}) + 180(\frac{4}{5})$

$$= 768 + 81 + 144 = 993 \text{ in./s}^2$$

$$\underline{a}_G = 993 \text{ in./s}^2$$

15.123

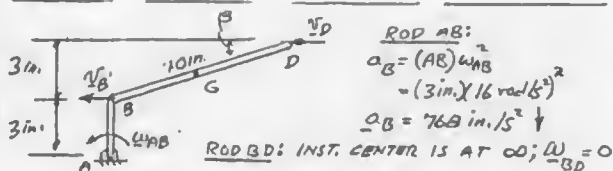


GIVEN:

$$\theta = 90^\circ$$

$$\omega_{AB} = 16 \text{ rad/s}$$

$$\alpha_{AB} = 0$$

FIND: (a)  $a_D$ (b)  $a_G$ 

ROD AB:

$$a_B = (AB)\omega_{AB}^2$$

$$= (3 \text{ in.} \times 16 \text{ rad/s})^2$$

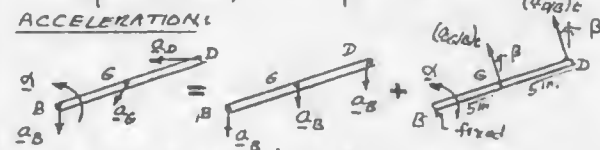
$$a_B = 768 \text{ in./s}^2 \downarrow$$

$$a_D = 0$$

ROD BD: INST. CENTER IS AT  $\infty$ ;  $\omega_{BD} = 0$ 

$$\sin \beta = (3 \text{ in.}) / (10 \text{ in.}) = 0.3; \beta = 17.46^\circ$$

ACCELERATION:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

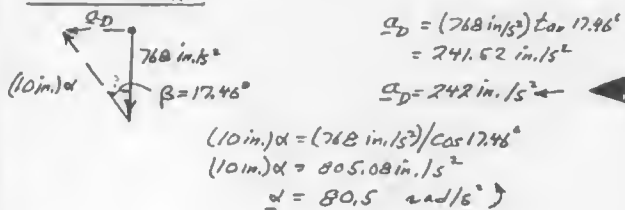
$$(a) \quad a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$= a_B + (BD)\alpha \uparrow \beta + (BD)\omega_{BD}^2 \rightarrow \beta$$

$$= 768 \text{ in./s}^2 \downarrow + (10 \text{ in.})\alpha \uparrow \beta + (10 \text{ in.})(0)^2 \rightarrow \beta$$

$$a_D \rightarrow = 768 \text{ in./s}^2 \downarrow + (10 \text{ in.})\alpha \uparrow \beta$$

VECTOR DIAGRAM:



$$a_D = (768 \text{ in./s}^2) \tan 17.46^\circ$$

$$= 241.52 \text{ in./s}^2$$

$$a_D = 242 \text{ in./s}^2 \leftarrow$$

$$(10 \text{ in.})\alpha = (768 \text{ in./s}^2) / \cos 17.46^\circ$$

$$(10 \text{ in.})\alpha = 805.08 \text{ in./s}^2$$

$$\alpha = 80.5 \text{ rad/s}^2$$

$$(b) \quad a_G = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$$

$$= a_B + (BG)\alpha \uparrow \beta + (BG)\omega_{BD}^2 \rightarrow \beta$$

$$= 768 \text{ in./s}^2 \downarrow + (5 \text{ in.})(80.5 \text{ rad/s}^2) \uparrow \beta + (5 \text{ in.})(0)^2 \rightarrow \beta$$

$$a_G = 768 \text{ in./s}^2 \downarrow + 402.5 \text{ in./s}^2 \uparrow 17.46^\circ$$

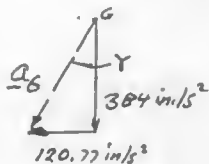
$$\uparrow \text{ COMPONENTS: } (a_G)_x = (402.5 \text{ in./s}^2) \sin 17.46^\circ$$

$$(a_G)_x = 120.77 \text{ in./s}^2$$

$$\downarrow \text{ COMPONENTS: } (a_G)_y = 768 \text{ in./s}^2 - (402.5 \text{ in./s}^2) \cos 17.46^\circ$$

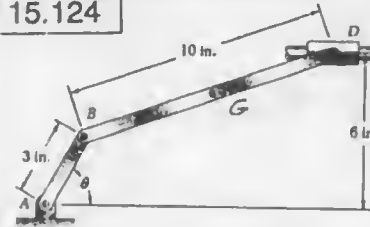
$$= 768 \text{ in./s}^2 - 384 \text{ in./s}^2$$

$$(a_G)_y = 384 \text{ in./s}^2 \downarrow$$



$$a_G = 403 \text{ in./s}^2 \uparrow 72.5^\circ$$

15.124

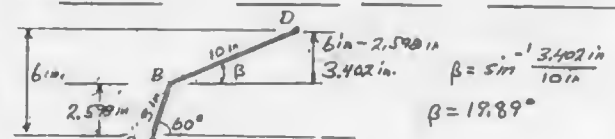


GIVEN:

$$\theta = 60^\circ$$

$$\omega_{AB} = 16 \text{ rad/s}$$

$$\alpha_{AB} = 0$$

FIND:  $a_D$ 

$$\text{VELOCITY: } v_B = (AB)\omega_{AB} = (3 \text{ in.} \times 16 \text{ rad/s}) = 48 \text{ in./s} \angle 30^\circ$$

ROD BD:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$$v_D = v_B + v_{D/B} = v_B + (BD)\omega_{BD} \uparrow \beta$$

$$v_D \rightarrow = 48 \text{ in./s} \angle 30^\circ + (10 \text{ in.})\omega_{BD} \uparrow 19.89^\circ$$

$$\uparrow \text{ COMPONENTS: } (48 \text{ in./s}) \sin 30^\circ - (10 \text{ in.})\omega_{BD} \cos 19.89^\circ$$

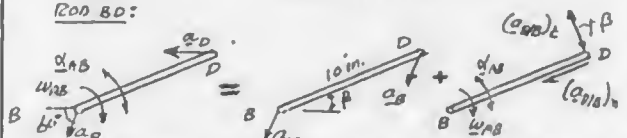
$$\omega_{BD} = \frac{(48 \text{ in./s}) \sin 30^\circ}{(10 \text{ in.}) \cos 19.89^\circ} = 2.552 \text{ rad/s}$$

ACCELERATION:

$$\text{ROD AB: } a_B = (AB)\omega_{AB}^2 \angle 60^\circ = (3 \text{ in.} \times 16 \text{ rad/s})^2 \angle 60^\circ$$

$$a_B = 768 \text{ in./s}^2 \angle 60^\circ$$

ROD BD:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

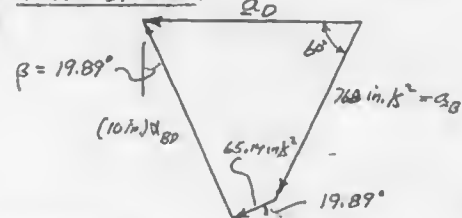
$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$a_D \rightarrow = a_B \angle 60^\circ + (BD)\alpha_{BD} \uparrow \beta + (BD)\omega_{BD}^2 \angle \beta$$

$$= 768 \text{ in./s}^2 \angle 60^\circ + (10 \text{ in.})\alpha_{BD} \uparrow \beta + (10 \text{ in.})(2.552 \text{ rad/s})^2 \angle \beta$$

$$a_D \rightarrow = 768 \text{ in./s}^2 \angle 60^\circ + (10 \text{ in.})\alpha_{BD} \uparrow 19.89^\circ + 65.14 \text{ in./s}^2 \angle 19.89^\circ$$

VECTOR DIAGRAM



y COMPONENTS

$$\uparrow 768 \sin 60^\circ + 65.14 \sin 19.89^\circ - 10 \alpha_{BD} \cos 19.89^\circ = 0$$

$$\alpha_{BD} = 73.09 \text{ rad/s}^2$$

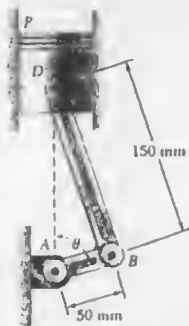
x COMPONENTS:

$$\rightarrow a_D = 768 \cos 60^\circ + 65.14 \cos 19.89^\circ + (10)(73.09) \sin 19.89^\circ$$

$$a_D = 693.9 \text{ in./s}^2$$

$$a_D = 694 \text{ in./s}^2 \leftarrow$$

15.125



GIVEN:

$$\theta = 60^\circ$$

$$\omega_{AB} = 900 \text{ rpm} \uparrow$$

$$\alpha_{AB} = 0$$

FIND:

$$a_P$$

VELOCITY

$$\beta = \sin^{-1} \frac{43.30 \text{ mm}}{150 \text{ mm}} = 16.78^\circ$$

$$\omega_{AB} = 900 \text{ rpm} = 94.248 \text{ rad/s} \uparrow$$

$$AB = 50 \text{ mm}$$

$$(AB) \cos 60^\circ = (150 \text{ mm}) \cos 60^\circ = 43.30 \text{ mm}$$

$$\text{ROD AB: } v_B = (AB) \omega_{AB} = (0.05 \text{ m})(94.248 \text{ rad/s}) = 4.712 \text{ m/s} \angle 60^\circ$$

ROD BD:

$$v_D = v_B + v_{D/B}$$

$$v_D = 4.712 \angle 60^\circ + (0.15 \text{ m}) \omega_{BD}$$

$$v_D = 4.712 \angle 60^\circ + 0.15 \omega_{BD} \angle 16.78^\circ$$

$$v_D = 4.712 \text{ m/s}$$

VECTOR DIAGRAM

x COMPONENTS:

$$+ \uparrow (4.712 \text{ m/s}) \cos 60^\circ - (0.15 \text{ m}) \omega_{BD} \cos 16.78^\circ = 0$$

$$\omega_{BD} = 16.41 \text{ rad/s} \uparrow$$

ACCELERATION:

$$\text{ROD AB: } a_B = (AB) \omega_{AB}^2 = (0.05 \text{ m})(94.248 \text{ rad/s})^2 = 444.1 \text{ m/s}^2 \angle 30^\circ$$

ROD BD:

$$a_D = a_B + a_{D/B}$$

$$a_D = 444.1 \angle 30^\circ + (0.15 \text{ m}) \alpha_{BD}$$

$$a_D = 444.1 \angle 30^\circ + (0.15 \text{ m}) \alpha_{BD} \angle 16.78^\circ$$

$$a_D = 444.1 \text{ m/s}^2$$

x COMPONENTS:

$$+ \uparrow : 444.1 \cos 30^\circ - (0.15 \text{ m}) \alpha_{BD} \cos 16.78^\circ - 40.4 \sin 16.78^\circ = 0$$

$$\alpha_{BD} = 2597 \text{ rad/s}^2 \uparrow$$

y COMPONENTS:

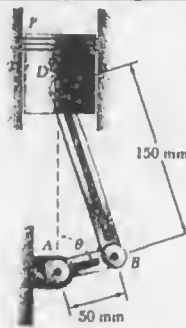
$$+ \uparrow a_D = 444.1 \sin 30^\circ - (0.15)(2597) \sin 16.78^\circ + 40.4 \cos 16.78^\circ$$

$$a_D = 148.3 \text{ m/s}^2$$

$$a_P = a_D$$

$$a_P = 148.3 \text{ m/s}^2 \downarrow$$

15.126



GIVEN:

$$\theta = 120^\circ$$

$$\omega_{AB} = 900 \text{ rpm} \uparrow$$

$$\alpha_{AB} = 0$$

FIND:

$$a_P$$

VELOCITY

$$\beta = \sin^{-1} \frac{43.30 \text{ mm}}{150 \text{ mm}} = 16.78^\circ$$

$$\omega_{AB} = 900 \text{ rpm} = 94.248 \text{ rad/s} \uparrow$$

$$AB = 50 \text{ mm}$$

$$(AB) \cos 120^\circ = (150 \text{ mm}) \cos 120^\circ = -43.30 \text{ mm}$$

$$\text{ROD AB: } v_B = (AB) \omega_{AB} = (0.05 \text{ m})(94.248 \text{ rad/s}) = 4.712 \text{ m/s} \angle 60^\circ$$

ROD BD:

$$v_D = v_B + v_{D/B}$$

$$v_D = 4.712 \angle 60^\circ + (0.15 \text{ m}) \omega_{BD}$$

$$v_D = 4.712 \angle 60^\circ + 0.15 \omega_{BD} \angle 16.78^\circ$$

$$v_D = 4.712 \text{ m/s}$$

VECTOR DIAGRAM

x COMPONENTS:

$$+ \uparrow -(4.712 \text{ m/s}) \cos 60^\circ + (0.15 \text{ m}) \omega_{BD} \cos 16.78^\circ = 0$$

$$\omega_{BD} = 16.41 \text{ rad/s} \uparrow$$

ACCELERATION:

$$\text{ROD AB: } a_B = (AB) \omega_{AB}^2 = (0.05 \text{ m})(94.248 \text{ rad/s})^2 = 444.1 \text{ m/s}^2 \angle 30^\circ$$

ROD BD:

$$a_D = a_B + a_{D/B}$$

$$a_D = 444.1 \angle 30^\circ + (0.15 \text{ m}) \alpha_{BD}$$

$$a_D = 444.1 \angle 30^\circ + (0.15 \text{ m}) \alpha_{BD} \angle 16.78^\circ$$

$$a_D = 444.1 \text{ m/s}^2$$

x COMPONENTS:

$$+ \uparrow : 444.1 \cos 30^\circ - (0.15 \text{ m}) \alpha_{BD} \cos 16.78^\circ - 40.4 \sin 16.78^\circ = 0$$

$$\alpha_{BD} = 2597 \text{ rad/s}^2 \uparrow$$

y COMPONENTS:

$$+ \uparrow a_D = 444.1 \sin 30^\circ - (0.15)(2597) \sin 16.78^\circ - 40.4 \cos 16.78^\circ$$

$$a_D = 296 \text{ m/s}^2$$

$$a_P = a_D$$

$$a_P = 296 \text{ m/s}^2 \uparrow$$

# 15.127 and 15.128

GIVEN:  $\alpha_{AB} = 0$

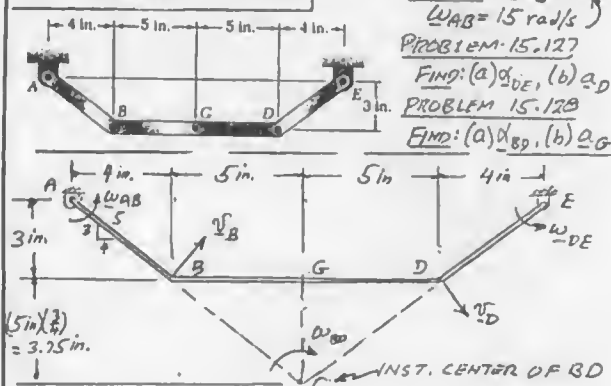
$\omega_{AB} = 15 \text{ rad/s}$

PROBLEM 15.127

FIND: (a)  $\alpha_{DE}$ , (b)  $a_D$

PROBLEM 15.128

FIND: (a)  $\alpha_{BD}$ , (b)  $a_G$



$$BC = (3.75^2 + 5^2)^{1/2} = 6.25 \text{ in.}$$

$$v_B = (AB)\omega_{AB} = (5 \text{ in.})(15 \text{ rad/s})$$

$$\omega_{BD} = \frac{v_B}{BC} = \frac{5 \text{ in.}}{6.25 \text{ in.}} \omega_{AB} = \frac{5 \text{ in.}}{6.25 \text{ in.}} (15 \text{ rad/s})$$

$$\omega_{BD} = 12 \text{ rad/s}$$

$$v_D = v_B, \text{ thus } \omega_{DE} = \omega_{AB}; \quad \omega_{DE} = 15 \text{ rad/s}$$

ACCELERATION: ROD AB:  $\alpha_{AB} = 0$

$$a_B = (AB)\alpha_{AB} = (5 \text{ in.})(0) = 0$$

ROD DE:

$$a_D = (DE)\alpha_{DE} = (5 \text{ in.})(15 \text{ rad/s}^2) = 1125 \text{ in./s}^2$$

$$a_D = (a_D)_t + (a_D)_n$$

$$= (5 \text{ in.})\alpha_{DE} + (5 \text{ in.})(15 \text{ rad/s})^2$$

$$a_D = (5 \text{ in.})\alpha_{DE} + 1125 \text{ in./s}^2$$

ROD BD:  $\omega_{BD} = 12 \text{ rad/s}$ ,  $\alpha_{BD} = ?$

$$a_B = a_G = a_D$$

$$a_B = a_G = a_D$$

$$a_B = a_G = a_D$$

PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$a_D = a_B + (10 \text{ in.})\alpha_{BD} + (10 \text{ in.})(12 \text{ rad/s})^2$$

$$(5 \text{ in.})\alpha_{DE} + 1125 \text{ in./s}^2 = 1125 \text{ in./s}^2 + (10 \text{ in.})\alpha_{BD} + 1440 \text{ in./s}^2$$

$$(1)$$

X COMPONENTS:

$$(5 \text{ in.})\alpha_{DE} \left(\frac{3}{5}\right) + (1125 \text{ in./s}^2) \left(\frac{4}{5}\right) = (1125 \text{ in./s}^2) \left(\frac{4}{5}\right) + 1440 \text{ in./s}^2$$

$$3\alpha_{DE} + 900 = -900 - 1440$$

$$\alpha_{DE} = -1080 \text{ rad/s}^2$$

WE ASSUMED  $\alpha_{DE}$  MINUS SIGN SHOWS  $\alpha_{DE}$  IS

$$\alpha_{DE} = 1080 \text{ rad/s}^2$$

Y COMPONENTS (USING  $\alpha_{DE} = -1080 \text{ rad/s}^2$ )

$$(5 \text{ in.})(-1080 \text{ rad/s}^2) \left(\frac{4}{5}\right) - (1125 \text{ in./s}^2) \left(\frac{3}{5}\right) = -(1125 \text{ in./s}^2) \left(\frac{3}{5}\right) - (10 \text{ in.})\alpha_{BD}$$

$$-4320 \text{ in./s}^2 - 675 \text{ in./s}^2 = -675 \text{ in./s}^2 - (10 \text{ in.})\alpha_{BD}$$

$$\alpha_{BD} = 432 \text{ rad/s}^2$$

WE ASSUMED  $\alpha_{BD}$  MINUS SIGN SHOWS  $\alpha_{BD}$  IS

$$\alpha_{BD} = 432 \text{ rad/s}^2$$

# 15.127 and 15.128 CONTINUED

ACCELERATION OF E: WE KNOW  $\alpha_{DE} = 1080 \text{ rad/s}^2$  AND

$$\omega_{DE} = 15 \text{ rad/s}$$

$$a_D = (a_D)_t + (a_D)_n$$

$$= (DE)\alpha_{DE} + (DE)\omega_{DE}^2$$

$$a_D = (5 \text{ in.})(1080 \text{ rad/s}^2) + (5 \text{ in.})(15 \text{ rad/s})^2$$

$$a_D = 5400 \text{ in./s}^2 + 1125 \text{ in./s}^2$$

$$(a_D)_x = 5400 \left(\frac{3}{5}\right) - 1125 \left(\frac{4}{5}\right) = +2340 \text{ in./s}^2$$

$$(a_D)_y = 5400 \left(\frac{4}{5}\right) + 1125 \left(\frac{3}{5}\right) = +4995 \text{ in./s}^2$$

$$a_D = 5570 \text{ in./s}^2 \angle 64.9^\circ$$

$$(a_D)_x = 2340$$

ACCELERATION OF G: WE AGAIN USE THE FREE-BODY EQUATION.

$$a_G = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$$

$$a_G = a_B + (BG)\alpha_{BD} + (BG)\omega_{BD}^2$$

RECALL WE FOUND  $\alpha_{BD} = +432 \text{ rad/s}^2$  AND USE THIS VALUE HERE TOGETHER WITH  $\omega_{BD} = 12 \text{ rad/s}$  AND

$$a_B = 1125 \text{ in./s}^2$$

$$a_G = 1125 \text{ in./s}^2 + (5 \text{ in.})(432 \text{ rad/s}^2) + (5 \text{ in.})(12 \text{ rad/s})^2$$

$$= [900 \text{ in./s}^2 + 675 \text{ in./s}^2] + 720 \text{ in./s}^2$$

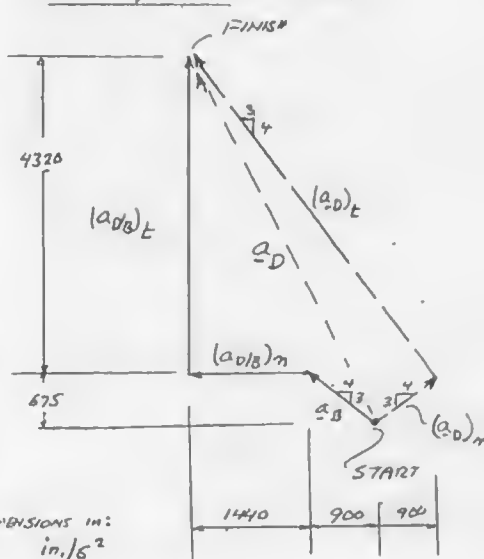
$$a_G = 2835 \text{ in./s}^2$$

$$a_G = 2835 \text{ in./s}^2$$

$$a_G = 3270 \text{ in./s}^2 \angle 60.3^\circ$$

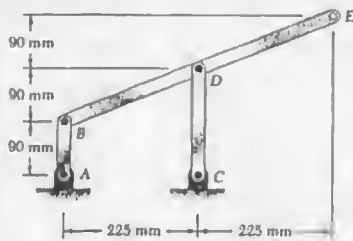
$$1620 \text{ in./s}^2$$

VECTOR DIAGRAM OF  $a_G$



DIMENSIONS in:  
in./s<sup>2</sup>

# 15.129 and 15.130

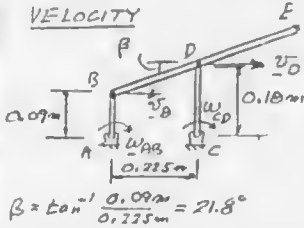


GIVEN:  
 $\omega_{AB} = 6 \text{ rad/s} \downarrow$   
 $\alpha_{AB} = 0$

PROBLEM 15.129  
 FIND:  $\alpha_D$

PROBLEM 15.130  
 FIND: (a)  $\alpha_{BDE}$   
 (b)  $\alpha_E$

VELOCITY



INST. CENTER OF

BDE AT O.  $\omega_{BDE} = 0$

$v_B = v_D$

$(AB)\omega_{AB} = (CD)\omega_{CD}$

$\omega_{CD} = \frac{(AB)}{(CD)} \omega_{AB} = \frac{0.09 \text{ m}}{0.18 \text{ m}} \omega_{AB}$

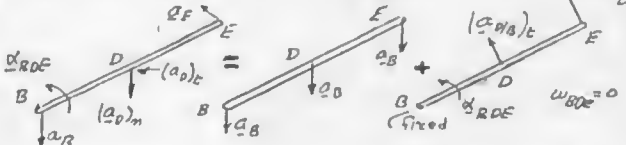
$\omega_{CD} = \frac{1}{2} \omega_{AB} = 3 \text{ rad/s} \downarrow$

ACCELERATION  $\alpha_{AB} = 0$

ROD AB:  $a_B = (AB)\omega_{AB}^2 \downarrow = (0.09 \text{ m})(6 \text{ rad/s})^2 = 3.24 \text{ m/s}^2 \downarrow$

ROD CD:  $a_D = (CD)\omega_{CD}^2 \downarrow = (0.18 \text{ m})(3 \text{ rad/s})^2 = 1.62 \text{ m/s}^2 \downarrow$   
 $a_D = (0.18 \text{ m})\alpha_{CD} + 1.62 \text{ m/s}^2 \downarrow$

ROD BDE:  $a_{BDE} = 0$ ;  $BD = \frac{0.225 \text{ m}}{\cos 21.8^\circ} = 0.2423 \text{ m}$



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$a_D = a_B + a_{D/B} = a_B + (BD)\alpha_{BDE}$

$(0.18 \text{ m})\alpha_{CD} + 1.62 \text{ m/s}^2 \downarrow = 3.24 \text{ m/s}^2 \downarrow + (BD)\alpha_{BDE}$

$(0.18 \text{ m})\alpha_{CD} + 1.62 \text{ m/s}^2 \downarrow = 3.24 \text{ m/s}^2 \downarrow + (0.2423 \text{ m})\alpha_{BDE}$

VECTOR DIAGRAM

$(a_{D/B})_t = 0.2423 \alpha_{BDE} = \frac{1.62 \text{ m/s}^2}{\cos \beta}$

$\alpha_{BDE} = 7.2 \text{ rad/s}^2 \downarrow$

$(a_{D/B})_n = (1.62 \text{ m/s}^2) \tan \beta = 0.648 \text{ m/s}^2 \downarrow$

$a_D = 0.648 \text{ m/s}^2 \downarrow + 1.62 \text{ m/s}^2 \downarrow$

$a_D = 1.745 \text{ m/s}^2 \swarrow 66.8^\circ$

$BE = 2 \frac{0.225}{\cos \beta} = 0.4847 \text{ m}$

$a_E = a_B + a_{E/B} = a_B + (BE)\alpha_{BDE}$

$= a_B + (BE)\alpha_{BDE}$

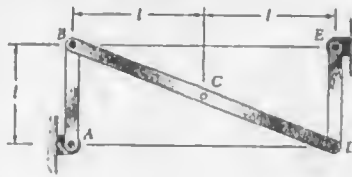
$= 3.2 \text{ m/s}^2 \downarrow + (0.4847 \text{ m})(7.2 \text{ rad/s}^2) \swarrow 21.8^\circ$

$= 3.2 \text{ m/s}^2 \downarrow + 3.4896 \text{ m/s}^2 \swarrow 21.8^\circ$

$a_E = 3.2 \text{ m/s}^2 \downarrow + 4.296 \text{ m/s}^2 \swarrow + 3.2 \text{ m/s}^2 \uparrow$

$a_E = 1.296 \text{ m/s}^2 \swarrow$

# 15.131 and 15.132



GIVEN:  
 $\omega_{AB} = \omega_0$ ,  $\alpha_{AB} = 0$

PROBLEM 15.131

FIND: (a)  $\alpha_{DE}$

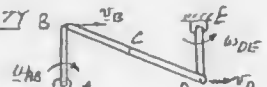
(b)  $a_D$

PROBLEM 15.132

FIND:  $a_C$  KNOWING

$\omega_0 = 8 \text{ rad/s}$  AND  $l = 0.3 \text{ m}$

VELOCITY



INST. CENTER OF

BD IS AT O

THUS,  $\omega_{BD} = 0$

$v_D = v_B$ ;  $(AB)\omega_{AB} = (DE)\omega_{DE}$ ;  $\omega_{DE} = \omega_{AB} = \omega_0$

ACCELERATION  $\alpha_{AB} = 0$

ROD AB:  $a_B = (AB)\omega_{AB}^2 \downarrow = l\omega_0^2 \downarrow$

ARM DE:

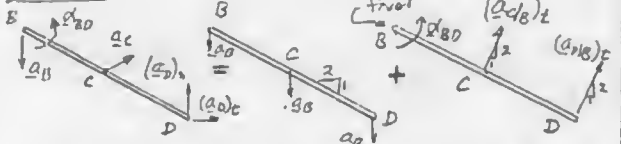
$a_D = (a_D)_t + (a_D)_n$

$(a_D)_t = l\alpha_{DE}$

$(a_D)_n = l\omega_{DE}^2$

$a_D = l\alpha_{DE} + l\omega_0^2$

ROD BCD:



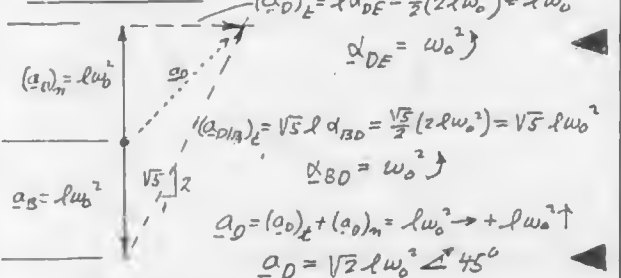
PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$a_D = a_B + a_{D/B} = a_B + (BD)\alpha_{BDE}$

$l\alpha_{DE} + l\omega_0^2 \downarrow = l\omega_0^2 \downarrow + (BD)\alpha_{BDE}$

$l\alpha_{DE} + l\omega_0^2 \downarrow + l\omega_0^2 \downarrow + \sqrt{5}l\alpha_{BD} \swarrow$

VECTOR DIAGRAM



$a_C = a_B + a_{C/B}$

$= a_B + (BC)\alpha_{BDE}$

$= l\omega_0^2 \downarrow + (\sqrt{5}l\frac{1}{2})\omega_0^2 \swarrow$

$a_C = l\omega_0^2 \downarrow + \frac{1}{2}l\omega_0^2 \swarrow + l\omega_0^2 \uparrow$

$a_C = \frac{1}{2}l\omega_0^2 \uparrow$

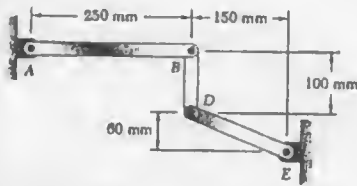
PROBLEM 15.132:

For  $\omega_0 = 8 \text{ rad/s}$  and  $l = 0.3 \text{ m}$ , WE HAVE

$a_C = \frac{1}{2}l\omega_0^2 = (\frac{0.3 \text{ m}}{2})(8 \text{ rad/s})^2 = 9.6 \text{ m/s}^2$

$a_C = 9.6 \text{ m/s}^2 \rightarrow$

15.133



GIVEN:

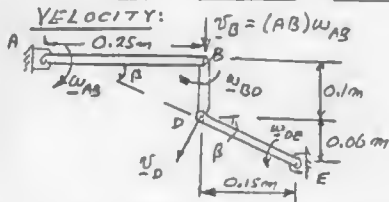
$$\omega_{AB} = 4 \text{ rad/s} \quad \downarrow$$

$$\alpha_{AB} = 0$$

FIND:

$$(a) \alpha_{BD}$$

$$(b) \alpha_{DE}$$



INST. CENTER OF BD IS AT A, THUS

$$\omega_{BD} = \omega_{AB}$$

$$\omega_{BD} = 4 \text{ rad/s}$$

$$AD = (0.25 \text{ m}) / \cos \beta \quad DE = (0.15 \text{ m}) / \cos \beta$$

$$v_D = (AD) \omega_{AB} = \frac{(AD)}{(DE)} \omega_{AB} = \frac{(0.25 / \cos \beta)}{(0.15 / \cos \beta)} (4 \text{ rad/s}) = 6.667 \text{ m/s}$$

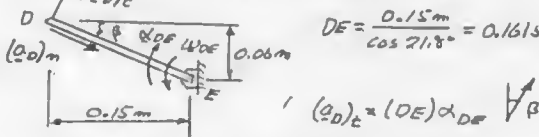
ACCELERATION:

$$\text{BAR AB: } \alpha_{AB} = (AB) \omega_{AB}^2 = (0.25 \text{ m}) (4 \text{ rad/s})^2 = 4 \text{ m/s}^2 \quad \leftarrow$$

BAR DE:

$$\beta = \tan^{-1} \frac{0.06}{0.15} = 21.8^\circ$$

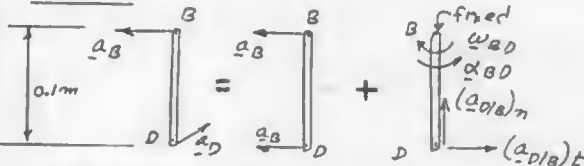
$$DE = \frac{0.15 \text{ m}}{\cos 21.8^\circ} = 0.16155 \text{ m}$$



$$(a_D)_n = (DE) \omega_{DE}^2 = (0.16155 \text{ m}) (6.667 \text{ rad/s})^2$$

$$(a_D)_n = 7.1801 \text{ m/s}^2 \quad \nwarrow \beta$$

BAR BD:



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$(a_D)_t \uparrow \beta + (a_D)_n \nwarrow \beta = a_B \leftarrow + (BD) \alpha_{BD} \rightarrow + (BD) \omega_{BD}^2 \uparrow$$

$$(0.16155 \text{ m}) \alpha_{DE} \uparrow \beta + 7.1803 \text{ m/s}^2 \nwarrow \beta = 4 \text{ m/s}^2 \leftarrow + (0.1 \text{ m}) \alpha_{BD} \rightarrow + (0.1 \text{ m}) (4 \text{ rad/s})^2 \uparrow$$

y COMPONENTS:  $\beta = 21.8^\circ$ 

$$+ \uparrow (0.16155 \text{ m}) \alpha_{DE} \cos \beta - (7.1801 \text{ m/s}^2) \sin \beta = 1.6 \text{ m/s}^2$$

$$0.15 \alpha_{DE} - 2.6665 = 1.6$$

$$\alpha_{DE} = 28.445 \text{ rad/s}^2$$

$$\alpha_{DE} = 28.4 \text{ rad/s}^2 \quad \nwarrow$$

x COMPONENTS:

$$+ \rightarrow (0.16155 \text{ m}) (28.445 \text{ rad/s}^2) \sin \beta + (7.1801 \text{ m/s}^2) \cos \beta$$

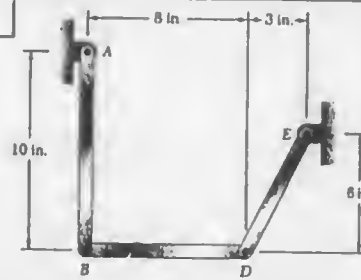
$$= -4 \text{ m/s}^2 + (0.1 \text{ m}) \alpha_{BD}$$

$$1.7066 + 6.6667 = -4 + 0.1 \alpha_{BD}$$

$$\alpha_{BD} = 123.73 \text{ rad/s}^2$$

$$\alpha_{BD} = 123.7 \text{ rad/s}^2 \quad \nwarrow$$

15.134



GIVEN:

$$\omega_{AB} = 4 \text{ rad/s} \quad \downarrow$$

$$\alpha_{AB} = 0$$

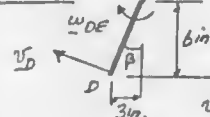
FIND:

$$(a) \alpha_{BD}$$

$$(b) \alpha_{DE}$$

$$\text{VELOCITY: BAR AB: } v_B = (10 \text{ in.}) (4 \text{ rad/s}) = 40 \text{ in./s} \quad \leftarrow$$

BAR DE:

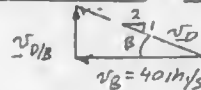


$$\beta = \tan^{-1} \frac{3}{6} = 26.56^\circ$$

$$DE = \frac{6}{\cos \beta} = 6.708 \text{ in.}$$

$$v_D = (DE) \omega_{DE} = (6.708 \text{ in.}) \omega_{DE} \quad v_{D/B} = (8 \text{ in.}) \omega_{BD}$$

$$v_D = v_B + v_{D/B} \quad \text{PLANE MOTION = TRANS WITH B + ROTATION ABOUT B}$$



$$v_{D/B} = \frac{1}{2} v_B; (8 \text{ in.}) \omega_{BD} = \frac{1}{2} (40 \text{ in./s}); \omega_{BD} = 2.5 \text{ rad/s} \quad \uparrow$$

$$v_D = \frac{40 \text{ in./s}}{\cos \beta}; (6.708 \text{ in.}) \omega_{DE} = \frac{40 \text{ in./s}}{\cos \beta}; \omega_{DE} = 6.667 \text{ rad/s} \quad \nwarrow$$

$$\text{ACCELERATIONS BAR AB: } a_B = (AB) \omega_{AB}^2 = (10 \text{ in.}) (4 \text{ rad/s})^2 = 160 \text{ in./s}^2 \quad \leftarrow$$

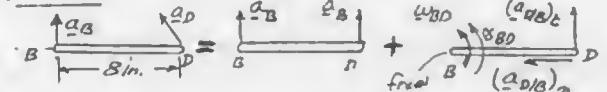
BAR DE:

$$(a_D)_t = (DE) \alpha_{DE} = (6.708 \text{ in.}) \alpha_{DE} \quad \nwarrow \beta$$

$$(a_D)_n = (DE) \omega_{DE}^2 = (6.708 \text{ in.}) (6.667 \text{ rad/s})^2$$

$$(a_D)_n = 298.1 \text{ in./s}^2 \quad \nwarrow \beta$$

BAR BD:



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$(a_D)_t \nwarrow \beta + (a_D)_n \nwarrow \beta = a_B \leftarrow + (8 \text{ in.}) \alpha_{BD} \uparrow + (8 \text{ in.}) (2.5 \text{ rad/s})^2 \nwarrow$$

$$(6.708 \text{ in.}) \alpha_{DE} \nwarrow \beta + 298.1 \text{ in./s}^2 \nwarrow \beta = 160 \text{ in./s}^2 \leftarrow + (8 \text{ in.}) \alpha_{BD} \uparrow + 50 \text{ in./s}^2 \nwarrow$$

x COMPONENTS:  $\beta = 26.56^\circ$ 

$$+ \rightarrow (6.708 \text{ in.}) \alpha_{DE} \cos \beta - (298.1 \text{ in./s}^2) \sin \beta = 160 \text{ in./s}^2 + (8 \text{ in.}) \alpha_{BD}$$

$$6.000 \alpha_{DE} - 133.31 = -50$$

$$\alpha_{DE} = 30.55 \text{ rad/s}^2$$

$$\alpha_{DE} = 30.6 \text{ rad/s}^2 \quad \nwarrow$$

y COMPONENTS:

$$+ \uparrow (6.708 \text{ in.}) \alpha_{DE} \sin \beta + (298.1 \text{ in./s}^2) \cos \beta = 160 \text{ in./s}^2 + (8 \text{ in.}) \alpha_{BD}$$

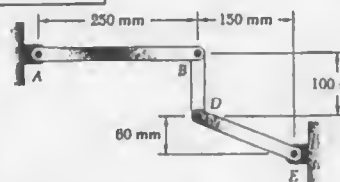
$$(6.708) (30.55) \sin \beta + 298.1 \cos \beta = 160 + 8 \alpha_{BD}$$

$$\alpha_{BD} = 24.8 \text{ rad/s}^2$$

$$\alpha_{BD} = 24.8 \text{ rad/s}^2 \quad \nwarrow$$

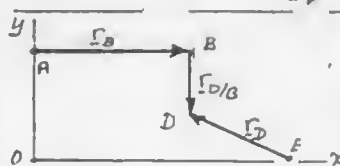


15.135

GIVEN:  $\omega_{AB} = 4 \text{ rad/s}$  $\alpha_{AB} = 0$ 

FIND:

(a)  $\alpha_{BD}$ , (b)  $\alpha_{DE}$   
 [USE FULL VECTOR  
 APPROACH AS DONE  
 IN SAMPLE PROBS 15.8]



$$\begin{aligned} \mathbf{r}_B &= (0.25\text{ m})\hat{i} \\ \mathbf{r}_D &= (-0.15\text{ m})\hat{i} + (0.06\text{ m})\hat{j} \\ \mathbf{r}_{D/B} &= (-0.1)\hat{j} \end{aligned}$$

$$\omega_{AB} = -(4 \text{ rad/s})\hat{k} \quad \omega_{BD} = \omega_{BD}\hat{k} \quad \omega_{DE} = \omega_{DE}\hat{k}$$

VELOCITY:

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$\omega_{DE}\hat{k} \times \mathbf{r}_D = \omega_{AB}\hat{k} \times \mathbf{r}_B + \omega_{BD}\hat{k} \times \mathbf{r}_{D/B}$$

$$\omega_{DE}\hat{k} \times (-0.15\hat{i} + 0.06\hat{j}) = -4\hat{k} \times 0.25\hat{i} + \omega_{BD}\hat{k} \times (-0.1\hat{j})$$

$$-0.15\omega_{DE}\hat{j} - 0.06\omega_{DE}\hat{i} = -\hat{j} + 0.1\omega_{BD}\hat{i}$$

COEFFICIENTS OF  $\hat{j}$ :  $-0.15\omega_{DE} = -1$   $\omega_{DE} = 6.667 \text{ rad/s}$

COEFFICIENTS OF  $\hat{i}$ :  $-0.06\omega_{DE} = 0.1\omega_{BD}$   
 $-0.06(6.667) = 0.1\omega_{BD}$   $\omega_{BD} = 4 \text{ rad/s}$

ACCELERATION:

$$\alpha_{AB} = 0 \quad \alpha_{BD} = \alpha_{BD}\hat{k} \quad \alpha_{DE} = \alpha_{DE}\hat{k}$$

$$\alpha_D = \alpha_B + \alpha_{D/B} \quad (1)$$

$$\alpha_D = \alpha_{DE}\hat{k} \times \mathbf{r}_D - \omega_{DE}^2 \mathbf{r}_D$$

$$= \alpha_{DE}\hat{k} \times (-0.15\hat{i} + 0.06\hat{j}) - (6.667)^2(-0.15\hat{i} + 0.06\hat{j})$$

$$\alpha_D = -0.15\alpha_{DE}\hat{j} - 0.06\alpha_{DE}\hat{i} + 6.667\hat{i} - 2.667\hat{j}$$

$$\alpha_B = \alpha_{AB}\hat{k} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$= 0 - (4)^2(0.25\hat{i})$$

$$\alpha_B = -4\hat{i}$$

$$\alpha_{D/B} = \alpha_{BD}\hat{k} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$= \alpha_{BD}\hat{k} \times (-0.1\hat{j}) - (4)^2(-0.1\hat{j})$$

$$\alpha_{D/B} = -0.1\alpha_{BD}\hat{i} + 1.6\hat{j}$$

SUBSTITUTE FOR  $\alpha_D, \alpha_B, \alpha_{D/B}$  IN EQ. (1),

$$\alpha_D = \alpha_B + \alpha_{D/B}$$

$$-0.15\alpha_{DE}\hat{j} - 0.06\alpha_{DE}\hat{i} + 6.667\hat{i} - 2.667\hat{j} = -4\hat{i} - 0.1\alpha_{BD}\hat{i} + 1.6\hat{j}$$

COEFFICIENTS OF  $\hat{j}$ :

$$-0.15\alpha_{DE} - 2.667 = 1.6$$

$$\alpha_{DE} = -28.44 \text{ rad/s}^2$$

$$\alpha_{DE} = 28.4 \text{ rad/s}^2$$

COEFFICIENTS OF  $\hat{i}$ :

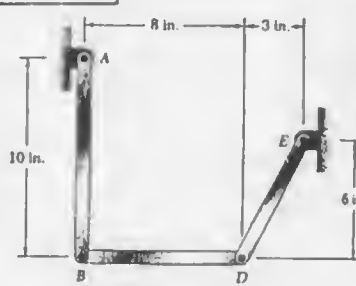
$$-0.06\alpha_{DE} + 6.667 = -4 - 0.1\alpha_{BD}$$

$$-(0.06)(-28.44) + 6.667 = -4 - 0.1\alpha_{BD}$$

$$\alpha_{BD} = 127.73 \text{ rad/s}^2$$

$$\alpha_{BD} = 127.7 \text{ rad/s}^2$$

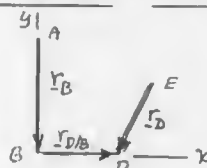
15.136

GIVEN:  $\omega_{AB} = 4 \text{ rad/s}$  $\alpha_{AB} = 0$ 

FIND:

(a)  $\alpha_{BD}$ , (b)  $\alpha_{DE}$

[USE FULL VECTOR  
 APPROACH AS DONE  
 IN SAMPLE PROBS 15.8]



$$\mathbf{r}_B = -(10 \text{ in.})\hat{j}$$

$$\mathbf{r}_D = -(3 \text{ in.})\hat{i} - (6 \text{ in.})\hat{j}$$

$$\mathbf{r}_{D/B} = (8 \text{ in.})\hat{i}$$

$$\omega_{AB} = -(4 \text{ rad/s})\hat{k} \quad \omega_{BD} = \omega_{BD}\hat{k} \quad \omega_{DE} = \omega_{DE}\hat{k}$$

VELOCITY:  $\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$ 

$$\omega_{DE}\hat{k} \times \mathbf{r}_D = \omega_{AB}\hat{k} \times \mathbf{r}_B + \omega_{BD}\hat{k} \times \mathbf{r}_{D/B}$$

$$\omega_{DE}\hat{k} \times (-3\hat{i} - 6\hat{j}) = -4\hat{k} \times (-10\hat{j}) + \omega_{BD}\hat{k} \times 8\hat{i}$$

$$-3\omega_{DE}\hat{j} + 6\omega_{DE}\hat{i} = -40\hat{i} + 8\omega_{BD}\hat{j}$$

COEFFICIENTS OF  $\hat{i}$ :  $6\omega_{DE} = -40$   $\omega_{DE} = -6.667 \text{ rad/s}$

COEFFICIENTS OF  $\hat{j}$ :  $-3\omega_{DE} = 8\omega_{BD}$

$$-3(-6.667) = 8\omega_{BD} \quad \omega_{BD} = 2.5 \text{ rad/s}$$

ACCELERATION:

$$\alpha_{AB} = 0 \quad \alpha_{BD} = \alpha_{BD}\hat{k} \quad \alpha_{DE} = \alpha_{DE}\hat{k}$$

$$\alpha_D = \alpha_B + \alpha_{D/B} \quad (1)$$

$$\alpha_D = \alpha_{DE}\hat{k} \times \mathbf{r}_D - \omega_{DE}^2 \mathbf{r}_D$$

$$= \alpha_{DE}\hat{k} \times (-3\hat{i} - 6\hat{j}) - (-6.667)^2(-3\hat{i} - 6\hat{j})$$

$$= -3\alpha_{DE}\hat{j} + 6\alpha_{DE}\hat{i} + 133.3\hat{i} + 266.7\hat{j}$$

$$\alpha_B = \alpha_{AB}\hat{k} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$= 0 - (4)^2(-10\hat{j})$$

$$\alpha_B = 160\hat{j}$$

$$\alpha_{D/B} = \alpha_{BD}\hat{k} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$= \alpha_{BD}\hat{k} \times 8\hat{i} - (2.5)^2(8\hat{i})$$

$$\alpha_{D/B} = 8\alpha_{BD}\hat{j} - 50\hat{i}$$

EQ. (1):  $\alpha_D = \alpha_B + \alpha_{D/B}$

$$-3\alpha_{DE}\hat{j} + 6\alpha_{DE}\hat{i} + 133.3\hat{i} + 266.7\hat{j} = 160\hat{j} + 8\alpha_{BD}\hat{j} - 50\hat{i}$$

COEFFICIENTS OF  $\hat{i}$ :

$$+6\alpha_{DE} + 133.3 = -50$$

$$\alpha_{DE} = -30.55 \text{ rad/s}^2$$

$$\alpha_{DE} = 30.6 \text{ rad/s}^2$$

COEFFICIENTS OF  $\hat{j}$ :

$$-3\alpha_{DE} + 266.7 = 160 + 8\alpha_{BD}$$

$$-3(-30.55) + 266.7 = 160 + 8\alpha_{BD}$$

$$\alpha_{BD} = 24.8 \text{ rad/s}^2$$

$$\alpha_{BD} = 24.8 \text{ rad/s}^2$$

15.137



INSTANTANEOUS CENTER OF ROTATION AT C

(a) SHOW THAT

$$\underline{r}_C = \underline{r}_A + \frac{\underline{\omega} \times \underline{r}_A}{\omega^2}$$

(b) SHOW THAT  $\underline{a}_C = 0$ , IF, AND ONLY IF,

$$\underline{a}_A = \frac{\alpha}{\omega} \underline{r}_A + \underline{\omega} \times \underline{v}_A$$

$$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C} = \underline{v}_C + \underline{\omega} \times (\underline{r}_{A/C})$$

$$\underline{v}_A = \underline{v}_C + \underline{\omega} \times (\underline{r}_A - \underline{r}_C)$$

BUT  $\underline{v}_C = 0$ :  $\underline{v}_A = \underline{\omega} \times (\underline{r}_A - \underline{r}_C)$

CROSS MULTIPLY EACH MEMBER BY  $\underline{\omega}$

$$\underline{\omega} \times \underline{v}_A = \underline{\omega} \times [\underline{\omega} \times (\underline{r}_A - \underline{r}_C)]$$

SINCE  $\underline{\omega} \perp$  TO PLANE CONTAINING  $(\underline{r}_A - \underline{r}_C)$ , CROSS MULTIPLYING TWICE BY  $\underline{\omega}$  IS EQUIVALENT TO MULTIPLYING  $(\underline{r}_A - \underline{r}_C)$  BY  $\omega^2$  AND ROTATING IT THROUGH  $180^\circ$ . THUS,

$$\underline{\omega} \times \underline{v}_A = -\omega^2 (\underline{r}_A - \underline{r}_C)$$

SOLVING FOR  $\underline{r}_C$ :  $\underline{r}_C = \underline{r}_A + \frac{\underline{\omega} \times \underline{v}_A}{\omega^2}$  (Q.E.D.)

(b) SINCE WE WANT  $\underline{a}_C = 0$ , WE SHALL WRITE

$$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = 0 \quad (1)$$

USING EQ. 15.11, PAGE 891

$$\underline{a}_{C/A} = \underline{\alpha} \times \underline{r}_{C/A} - \omega^2 \underline{r}_{C/A} \quad (2)$$

FROM PART a:  $\underline{r}_{C/A} = \underline{r}_C - \underline{r}_A = \frac{\underline{\omega} \times \underline{v}_A}{\omega^2}$

EQ(2):  $\underline{a}_{C/A} = \underline{\alpha} \times \frac{\underline{\omega} \times \underline{v}_A}{\omega^2} - (\underline{\omega} \times \underline{v}_A)$

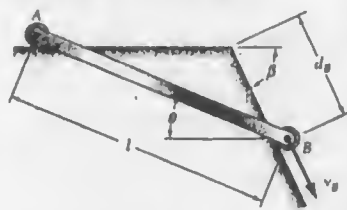
BUT  $\underline{\alpha} = \underline{\alpha} \underline{\hat{r}}$  AND  $\underline{\omega} = \omega \underline{\hat{r}}$ , AND SINCE  $\underline{\hat{r}} \perp \underline{v}_A$

$$\begin{aligned} \underline{a}_{C/A} &= \frac{\alpha}{\omega} \left[ \underline{\hat{r}} \times (\underline{\hat{r}} \times \underline{v}_A) \right] - \underline{\omega} \times \underline{v}_A \\ &= -\frac{\alpha}{\omega} \underline{v}_A - \underline{\omega} \times \underline{v}_A \end{aligned}$$

SUBSTITUTING INTO (1) AND SOLVING FOR  $\underline{a}_A$ , WE HAVE FOR  $\underline{a}_C = 0$

$$\underline{a}_A = \frac{\alpha}{\omega} \underline{v}_A + \underline{\omega} \times \underline{v}_A \quad (Q.E.D.)$$

\*15.138 and 15.139



PROBLEM 15.138

EXPRESS  $\omega$  OF ROD IN TERMS OF  $\underline{v}_B$ ,  $\theta$ ,  $l$ , AND  $\beta$

PROBLEM 15.139

GIVEN:  $\underline{a}_B = 0$ ,

EXPRESS  $\alpha$  OF ROD IN TERMS OF  $\underline{v}_B$ ,  $\theta$ ,  $l$ , AND  $\beta$

PROBLEM 15.138



LAW OF SINES

$$\frac{d_B}{\sin \theta} = \frac{l}{\sin \beta}$$

$$d_B = \frac{l}{\sin \beta} \sin \theta$$

$$\underline{v}_B = \frac{d}{dt}(d_B) = \frac{l}{\sin \beta} \cos \theta \frac{d\theta}{dt} = \frac{l}{\sin \beta} \cos \theta \omega$$

$$\omega = \frac{\underline{v}_B \sin \beta}{l \cos \theta}$$

PROBLEM 15.139

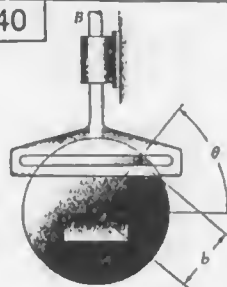
NOTE THAT  $\underline{a}_B = \frac{d\underline{v}_B}{dt} = 0$ .

$$\underline{a} = \frac{d\omega}{dt} = \frac{\underline{v}_B \sin \beta}{l} \cdot \frac{\sin \theta}{\cos^2 \theta} \cdot \frac{d\theta}{dt}$$

$$\alpha = \frac{\underline{v}_B \sin \beta \sin \theta}{l \cos^2 \theta} \cdot \frac{\underline{v}_B \sin \beta}{l \cos \theta}$$

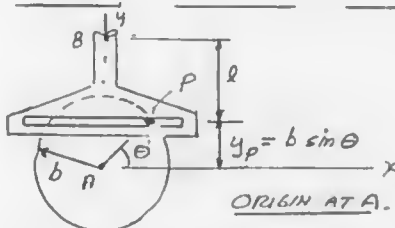
$$\alpha = \left[ \frac{\underline{v}_B \sin \beta}{l} \right]^2 \frac{\sin \theta}{\cos^3 \theta}$$

\*15.140



GIVEN: FOR DISK,  $\underline{\omega}$  AND  $\underline{\alpha}$  ARE

DERIVE EXPRESSIONS FOR  $\underline{v}_B$  AND  $\underline{a}_B$



ORIGIN AT A.

$$y_B = l + y_P = l + b \sin \theta$$

$$\underline{v}_B = \dot{y}_B = b \cos \theta \dot{\theta} = b \cos \theta \omega$$

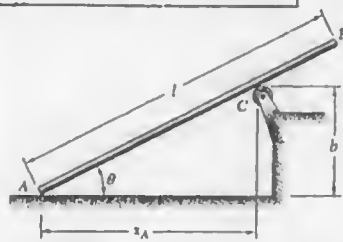
$$\underline{v}_B = b \omega \cos \theta$$

$$\underline{a}_B = \ddot{y}_B = \frac{d}{dt} \underline{v}_B = \frac{d}{dt} (b \cos \theta \dot{\theta})$$

$$\underline{a}_B = -b \sin \theta \dot{\theta}^2 + b \cos \theta \ddot{\theta}$$

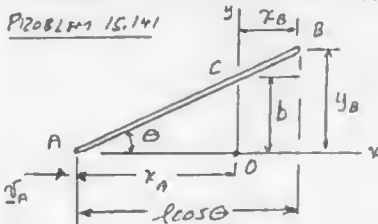
$$\underline{a}_B = b \alpha \cos \theta - b \omega^2 \sin \theta$$

\* 15.141 and 15.142



GIVEN:  $a_A = 0$   
 $\dot{x}_A = \dot{x}_B \rightarrow$   
 DERIVE EXPRESSIONS,  
 PROBLEM 15.141  
 $\alpha$  AND  $\omega$   
 PROBLEM 15.142  
 $(\dot{v}_B)_x$  AND  $(\dot{v}_B)_y$

PROBLEM 15.141



$$x_A = \frac{b}{\tan \theta}$$

$$\dot{x}_A = -\dot{v}_A$$

$$x_A = \frac{b}{\tan \theta} = b \frac{\cos \theta}{\sin \theta} \quad \dot{x}_A = -\dot{v}_A$$

$$\dot{x}_A = -\dot{v}_A = b \frac{-\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta} \dot{\theta} - \frac{b}{\sin^2 \theta} \omega$$

$$\dot{v}_A = \frac{b}{\sin^2 \theta} \omega \quad \omega = \frac{\dot{v}_A}{b} \sin^2 \theta$$

$$\alpha = \dot{\omega} = \frac{\dot{v}_A}{b} 2 \sin \theta \cos \theta \dot{\theta} + \frac{\sin^2 \theta}{b} \ddot{v}_A$$

BUT  $\dot{v}_A = a_A = 0$ ;  $\alpha = \frac{\dot{v}_A}{b} 2 \sin \theta \cos \theta \left[ \frac{\dot{v}_A}{b} \sin^2 \theta \right]$

$$\alpha = 2 \left( \frac{\dot{v}_A}{b} \right)^2 \sin^2 \theta \cos \theta$$

PROBLEM 15.142

$$x_B = l \cos \theta - x_A$$

$$\dot{x}_B = -l \sin \theta \dot{\theta} - \dot{x}_A$$

$$(\dot{v}_B)_x = -l \sin \theta \omega + \frac{b}{\sin^2 \theta} \omega$$

$$(\dot{v}_B)_x = \left( -l \sin \theta + \frac{b}{\sin^2 \theta} \right) \left( \frac{\dot{v}_A}{b} \sin^2 \theta \right)$$

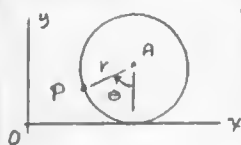
$$(\dot{v}_B)_x = \dot{v}_A \left( 1 - \frac{l}{b} \sin^2 \theta \right)$$

$$y_B = l \sin \theta; \quad \dot{y}_B = l \cos \theta \dot{\theta}$$

$$\dot{y}_B = (\dot{v}_B)_y = l \cos \theta \left( \frac{\dot{v}_A}{b} \sin^2 \theta \right)$$

$$(\dot{v}_B)_y = \dot{v}_A \frac{l}{b} \cos \theta \sin^2 \theta$$

\* 15.143



GIVEN:  $\dot{v}_A = v \rightarrow$ ,  $a_A = 0$   
 AT  $t=0$ , P IS ON GROUND AT O.  
 FIND:  $\dot{x}_P$  AND  $\dot{y}_P$  AT ANY TIME t

$$x_P = x_A - r \sin \theta = r \dot{\theta} - r \sin \theta$$

$$y_P = y_A - r \cos \theta = r - r \cos \theta$$

$$\dot{x}_P = r(\dot{\theta} + \cos \theta \dot{\theta}) = r \dot{\theta}(1 + \cos \theta)$$

$$\dot{y}_P = r(\sin \theta \dot{\theta}) = r \dot{\theta} \sin \theta$$

ROLLING MOTION:  $\theta = \frac{v}{r} t$ ;  $\dot{\theta} = \frac{v}{r}$ ;  $r \dot{\theta} = v$

$$\dot{x}_P = \dot{x}_P$$

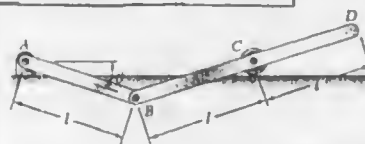
$$v_x = v(1 + \cos \frac{v}{r} t)$$

$$\dot{y}_P = \dot{y}_P$$

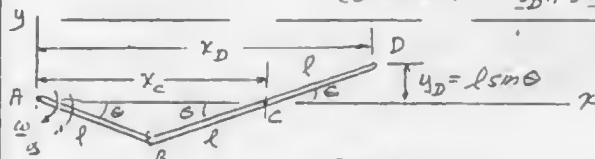
$$v_y = v \sin \frac{v}{r} t$$

\* 15.144 and 15.145

GIVEN:  $\omega_{AB} = \omega$   
 $\alpha_{AB} = \alpha$



DERIVE EXPRESSIONS,  
 PROBLEM 15.144  
 $\dot{v}_A$  AND  $a_C$   
 PROBLEM 15.145  
 COMPONENTS OF  $\dot{v}_D$  AND  $a_D$



PROBLEM 15.144

$$x_C = l \cos \theta$$

$$\dot{x}_C = -l \sin \theta \dot{\theta}$$

$$\dot{v}_C = -2l \omega \sin \theta$$

$$\ddot{x}_C = -2l \sin \theta \ddot{\theta} - 2l \cos \theta \dot{\theta}^2$$

$$a_C = -2l \alpha \sin \theta - 2l \omega^2 \cos \theta$$

PROBLEM 15.145

$$x_D = 3l \cos \theta$$

$$\dot{x}_D = -3l \sin \theta \dot{\theta}$$

$$(\dot{v}_D)_x = -3l \omega \sin \theta$$

$$\ddot{x}_D = -3l \sin \theta \ddot{\theta} - 3l \cos \theta \dot{\theta}^2$$

$$(a_D)_x = -3l \alpha \sin \theta - 3l \omega^2 \cos \theta$$

$$y_D = l \sin \theta$$

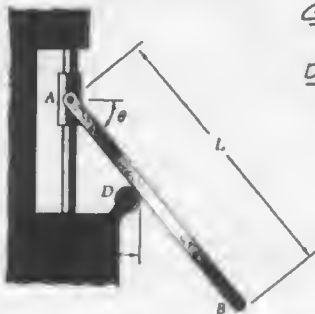
$$\dot{y}_D = l \cos \theta \dot{\theta}$$

$$(\dot{v}_D)_y = l \omega \cos \theta$$

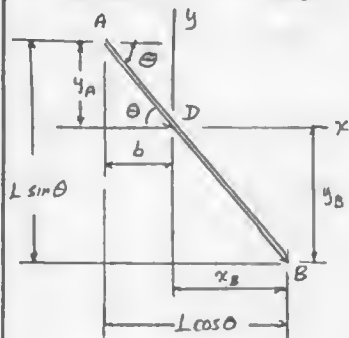
$$\ddot{y}_D = l \cos \theta \ddot{\theta} - l \sin \theta \dot{\theta}^2$$

$$(a_D)_y = l \alpha \cos \theta - l \omega^2 \sin \theta$$

• 15.146 and 15.147



GIVEN:  $v_A = v_A \uparrow$   
 $a_A = 0$   
 DERIVE EXPRESSIONS,  
 PROBLEM 15.146  
 (a)  $\omega_{AB}$   
 (b) COMPONENTS OF  $v_B$   
 PROBLEM 15.147  
 $\alpha_{AB}$



POSITIVE  $\theta$  IS  $\curvearrowright$

PROBLEM 15.146  $y_A = b \tan \theta$   
 $v_A' = \dot{y}_A = b \frac{1}{\cos^2 \theta} \dot{\theta} = \frac{b\omega}{\cos^2 \theta}$   
 $\omega = \frac{v_A}{b} \cos^2 \theta$

$x_B = L \cos \theta - b$   
 $\dot{x}_B = -L \sin \theta \dot{\theta} = -L\omega \sin \theta$   
 $= -L \left( \frac{v_A}{b} \cos^2 \theta \right) \sin \theta$   
 $\rightarrow (v_B)_x = \dot{x}_B = -v_A \frac{L}{b} \sin \theta \cos^2 \theta$

$y_B = L \sin \theta - y_A = L \sin \theta - b \tan \theta$   
 $\dot{y}_B = L \cos \theta \dot{\theta} - b \frac{1}{\cos^2 \theta} \dot{\theta}$   
 $= \left( L \cos \theta - \frac{b}{\cos^2 \theta} \right) \left( \frac{v_A}{b} \cos^2 \theta \right)$

$\uparrow (v_B)_y = \dot{y}_B = v_A \left( \frac{L}{b} \cos^3 \theta - 1 \right)$

PROBLEM 15.147

RECALL THAT  $a_A = \dot{v}_A = 0$  AND

$\omega = \dot{\theta} = \frac{v_A}{b} \cos^2 \theta$

$\alpha = \dot{\omega} = \frac{v_A}{b} (-2 \cos \theta \sin \theta) \dot{\theta}$

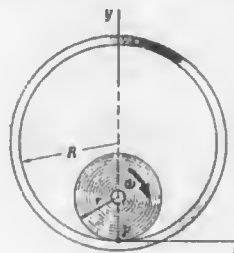
$\alpha = -2 \frac{v_A}{b} \cos \theta \sin \theta \left( \frac{v_A}{b} \cos^2 \theta \right)$

$\alpha = -2 \left( \frac{v_A}{b} \right)^2 \sin \theta \cos^3 \theta$

NOTE: SINCE POSITIVE  $\theta$  IS  $\curvearrowright$ , THE DIRECTION OF  $\alpha$  IS  $\curvearrowleft$ .

$\alpha = 2 \left( \frac{v_A}{b} \right)^2 \sin \theta \cos^3 \theta$

• 15.148 and 15.149



GIVEN: POSITION SHOWN IS WHEN  $t = 0$

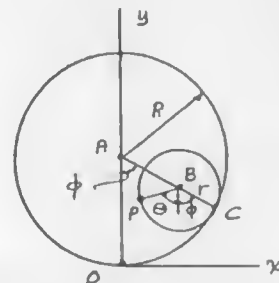
$\omega = \text{CONSTANT} (\alpha = 0)$

PROBLEM 15.148

DERIVE EXPRESSIONS FOR  $(v_P)_x$  AND  $(v_P)_y$

PROBLEM 15.149

WHEN  $r = R/2$  SHOW THAT PATH OF P IS y AXIS AND DERIVE EXPRESSIONS FOR  $v_P$  AND  $a_P$



$\phi = \angle OAB$

$\theta = \text{ANGLE BP FORMS WITH THE VERTICAL}$

$\theta = \omega t; \dot{\theta} = \omega \quad (1)$

$v_B = (AB)\dot{\phi}$

$v_B = (R-r)\dot{\phi}$

SINCE C IS INSTANTANEOUS CENTER,  $v_B = r\omega$   
 EQUATING THE TWO EXPRESSIONS OBTAINED FOR  $v_B$

$(R-r)\dot{\phi} = r\omega \quad \dot{\phi} = \frac{r}{R-r} \omega \quad (2)$

$x_P = (R-r) \sin \phi - r \sin \theta$   
 $y_P = R - (R-r) \cos \phi - r \cos \theta$

DIFFERENTIATING AND USING (1) AND (2):

$\dot{x}_P = (R-r) \cos \phi \dot{\phi} - r \cos \theta \dot{\theta}$

$\dot{y}_P = (R-r) \sin \phi \dot{\phi} + r \sin \theta \dot{\theta}$

$\dot{x}_P = (R-r) \cos \phi \left( \frac{r}{R-r} \right) \omega - r \cos \theta \omega$

$\dot{y}_P = (R-r) \sin \phi \left( \frac{r}{R-r} \right) \omega + r \sin \theta \omega$

$\dot{x}_P = r\omega (\cos \phi - \cos \theta)$

$\dot{y}_P = r\omega (\sin \phi + \sin \theta)$

$(v_P)_x = \dot{x}_P = r\omega \left[ \cos \frac{r\omega t}{R-r} - \cos \omega t \right]$

$(v_P)_y = \dot{y}_P = r\omega \left[ \sin \frac{r\omega t}{R-r} + \sin \omega t \right]$

PROBLEM 15.149 FOR  $r = R/2$

$\dot{x}_P = r\omega (\cos \omega t - \cos \omega t) = 0$

THUS P MOVES ALONG THE y AXIS

$v = \dot{y}_P = r\omega (\sin \omega t + \sin \omega t)$

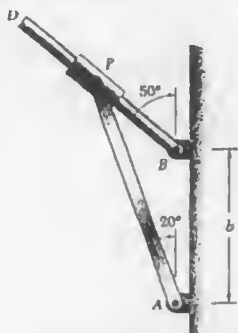
$v = 2r\omega \sin \omega t$

$v = (R\omega \sin \omega t) \hat{j}$

$a = \frac{dv}{dt} = 2r\omega (\omega \cos \omega t) \quad [\text{RECALL } \omega = \text{CONSTANT}]$

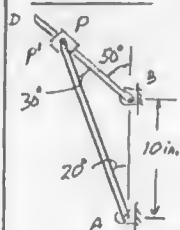
$a = (R\omega^2 \cos \omega t) \hat{j}$

15.150



GIVEN:  
 $b = 10 \text{ in.}$   
 $\omega_B = 5 \text{ rad/s}$

FIND:  
 (a)  $\omega_{AP}$   
 (b)  $\dot{\theta}_{P/B}$



GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{10 \text{ in.}}{\sin 30^\circ}$$

$$AP = 15.32 \text{ in.}$$

$$BP = 6.84 \text{ in.}$$

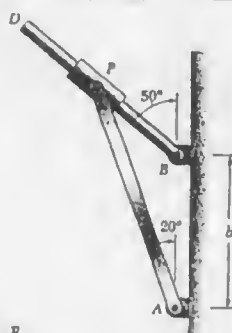
NOTE: POINT P' OF BP COINCIDES WITH P

VELOCITIES:  $\dot{\theta}_P = (BP)\omega_B = (6.84 \text{ in.})(5 \text{ rad/s}) = 34.2 \text{ in/s} \angle 40^\circ$

$$\begin{aligned} \dot{\theta}_P &= \dot{\theta}_P + \dot{\theta}_{P/B} \\ [34.2 \angle 20^\circ] &= [34.2 \angle 40^\circ] + [\dot{\theta}_{P/B} \angle 40^\circ] \\ \text{(a) } \dot{\theta}_P &= \frac{\dot{\theta}_P}{\cos 30^\circ} = \frac{34.2}{\cos 30^\circ} = 39.49 \text{ in/s} \\ \omega_{AP} &= \frac{\dot{\theta}_P}{AP} = \frac{39.49 \text{ in/s}}{15.32 \text{ in.}} = 2.58 \text{ rad/s} \end{aligned}$$

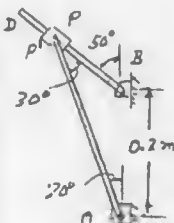
$$\begin{aligned} \text{(b) } \dot{\theta}_{P/B} &= \dot{\theta}_P \tan 30^\circ = (39.49 \text{ in/s}) \tan 30^\circ \\ \dot{\theta}_{P/B} &= 19.75 \text{ in/s} \angle 40^\circ \end{aligned}$$

15.152



GIVEN:  
 $b = 200 \text{ mm}$   
 $\dot{\theta}_B = 300 \text{ rad/s} \angle 50^\circ$

FIND:  
 $\omega_{AP}$   
 $\omega_{BP}$



GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 30^\circ} = \frac{BP}{\sin 20^\circ} = \frac{0.2 \text{ m}}{\sin 20^\circ}$$

$$AP = 0.3064 \text{ m} \quad BP = 0.1368 \text{ m}$$

NOTE: POINT P' OF BP COINCIDES WITH P

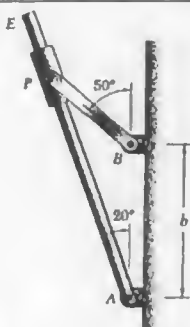
VELOCITIES:  $\dot{\theta}_P = (BP)\omega_B = (0.1368 \text{ m})\omega_B \angle 40^\circ$

$$\begin{aligned} \dot{\theta}_P &= \dot{\theta}_P + \dot{\theta}_{P/B} \\ [\dot{\theta}_P \angle 20^\circ] &= [\dot{\theta}_P \angle 40^\circ] + [\dot{\theta}_{P/B} \angle 40^\circ] \\ \dot{\theta}_P &= \frac{\dot{\theta}_{P/B}}{\tan 30^\circ} = \frac{0.3 \text{ m/s}}{\tan 30^\circ} = 0.5196 \text{ m/s} \\ \omega_{BP} &= \frac{\dot{\theta}_P}{BP} = \frac{0.5196 \text{ m/s}}{0.1368 \text{ m}} = 3.80 \text{ rad/s} \\ \omega_{BP} &= 3.80 \text{ rad/s} \end{aligned}$$

$$\dot{\theta}_P = \frac{\dot{\theta}_{P/B}}{\sin 30^\circ} = \frac{0.3 \text{ m/s}}{\sin 30^\circ} = 0.6 \text{ m/s}$$

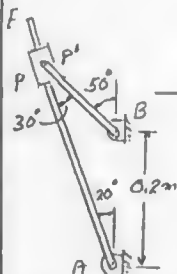
$$\omega_{AP} = \frac{\dot{\theta}_P}{AP} = \frac{0.6 \text{ m/s}}{0.3064 \text{ m}} = 1.958 \text{ rad/s}$$

15.151



GIVEN:  
 $b = 200 \text{ mm}$   
 $\omega_B = 9 \text{ rad/s}$

FIND:  
 (a)  $\omega_{AP}$   
 (b)  $\dot{\theta}_{P/AE}$



GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{0.2 \text{ m}}{\sin 30^\circ}$$

$$AP = 0.3064 \text{ m} \quad BP = 0.1368 \text{ m}$$

NOTE: POINT P' OF AE COINCIDES WITH P

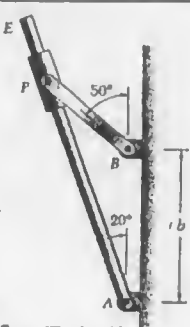
VELOCITIES:  $\dot{\theta}_P = (BP)\omega_B = (0.1368 \text{ m})(9 \text{ rad/s}) = 1.231 \text{ m/s} \angle 40^\circ$

$$\begin{aligned} \dot{\theta}_P &= \dot{\theta}_P + \dot{\theta}_{P/AE} \\ [1.231 \angle 40^\circ] &= [\dot{\theta}_P \angle 20^\circ] + [\dot{\theta}_{P/AE} \angle 20^\circ] \\ \text{(a) } \dot{\theta}_P &= \dot{\theta}_P \cos 30^\circ = (1.231 \text{ m/s}) \cos 30^\circ = 1.066 \text{ m/s} \\ \omega_{AE} &= \frac{\dot{\theta}_P}{AP} = \frac{1.066 \text{ m/s}}{0.3064 \text{ m}} = 3.48 \text{ rad/s} \end{aligned}$$

$$\text{(b) } \dot{\theta}_{P/AE} = \dot{\theta}_P \sin 30^\circ = (1.231 \text{ m/s}) \sin 30^\circ$$

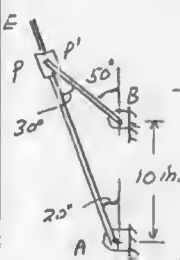
$$\dot{\theta}_{P/AE} = 0.616 \text{ m/s} \angle 70^\circ$$

15.153



GIVEN:  
 $b = 10 \text{ in.}$   
 $\dot{\theta}_B = 15 \text{ in/s} \angle 20^\circ$

FIND:  
 $\omega_{AP}$   
 $\omega_{BP}$



GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{10 \text{ in.}}{\sin 30^\circ}$$

$$AP = 15.32 \text{ in.} \quad BP = 6.84 \text{ in.}$$

NOTE: POINT P' OF AE COINCIDES WITH P

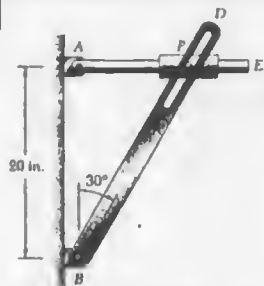
VELOCITIES:

$$\begin{aligned} \dot{\theta}_P &= \dot{\theta}_P + \dot{\theta}_{P/AE} \\ [\dot{\theta}_P \angle 40^\circ] &= [\dot{\theta}_P \angle 20^\circ] + [\dot{\theta}_{P/AE} \angle 20^\circ] \\ \dot{\theta}_P &= \frac{\dot{\theta}_{P/AE}}{\tan 30^\circ} = \frac{15 \text{ in/s}}{\tan 30^\circ} = 25.98 \text{ in/s} \\ \omega_{AE} &= \frac{\dot{\theta}_P}{AP} = \frac{25.98 \text{ in/s}}{15.32 \text{ in.}} = 1.696 \text{ rad/s} \\ \omega_{AE} &= 1.696 \text{ rad/s} \end{aligned}$$

$$\dot{\theta}_P = \frac{\dot{\theta}_{P/AE}}{\sin 30^\circ} = \frac{15 \text{ in/s}}{\sin 30^\circ} = 30 \text{ in/s}$$

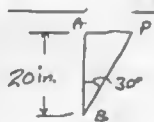
$$\omega_{BP} = \frac{\dot{\theta}_P}{BP} = \frac{30 \text{ in/s}}{6.84 \text{ in.}} = 4.39 \text{ rad/s} \quad \omega_{BP} = 4.39 \text{ rad/s}$$

15.154



GIVEN:  
 $\omega_{AE} = 4 \text{ rad/s}$   
 $\omega_{BD} = 1.5 \text{ rad/s}$

FIND:  
 $\underline{v}_P$



GEOMETRY:  
 $AP = (20 \text{ in.}) \tan 30^\circ = 11.547 \text{ in.}$   
 $BP = (20 \text{ in.}) / \cos 30^\circ = 23.094 \text{ in.}$

ROD AE AND COLLAR:

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/AE}$$

$$\underline{v}_P = [(AP)\omega_{AE} \downarrow] + [\underline{v}_{P/AE} \rightarrow]$$

$$\underline{v}_P = [(11.547 \text{ in.})(4 \text{ rad/s}) \downarrow] + [\underline{v}_{P/AE} \rightarrow]$$

$$\underline{v}_P = [46.188 \text{ in./s} \downarrow] + [\underline{v}_{P/AE} \rightarrow] \quad (1)$$

ROD BD AND COLLAR:

$$\underline{v}_P = \underline{v}_B + \underline{v}_{P/BD} \quad (2)$$

$$\underline{v}_P = [(BP)\omega_{BD} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

$$\underline{v}_P = [(23.094 \text{ in.})(1.5 \text{ rad/s}) \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

$$\underline{v}_P = [34.64 \text{ in./s} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ] \quad (3)$$

$$\underline{v}_{P/B} = (BP)\omega_{BD} = (23.094 \text{ in.})(1.5 \text{ rad/s}); \quad \underline{v}_{P/B} = 34.64 \text{ in./s} \searrow 30^\circ$$

EQUATE EQS. (1) AND (3):

$$[46.188 \text{ in./s} \downarrow] + [\underline{v}_{P/AE} \rightarrow] = [34.64 \text{ in./s} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

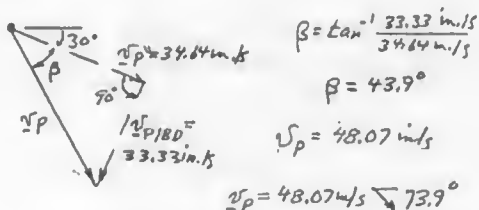
$$+\downarrow y \text{ COMPONENTS: } 46.188 = 34.64 \sin 30^\circ + \underline{v}_{P/BD} \cos 30^\circ$$

$$\underline{v}_{P/BD} = 33.33 \text{ in./s} \quad \underline{v}_{P/BD} = 33.33 \text{ in./s} \nearrow 30^\circ$$

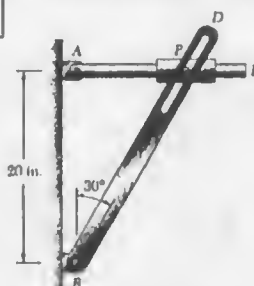
VECTOR DIAGRAM FOR EQ. (2):

$$\underline{v}_P = \underline{v}_B + \underline{v}_{P/BD}$$

$$\underline{v}_P = [34.64 \text{ in./s} \searrow 30^\circ] + [33.33 \text{ in./s} \nearrow 30^\circ]$$

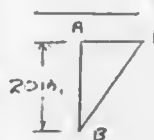


15.155



GIVEN:  
 $\omega_{AE} = 3.5 \text{ rad/s}$   
 $\omega_{BD} = 2.4 \text{ rad/s}$

FIND:  
 $\underline{v}_P$



GEOMETRY:  
 $AP = (20 \text{ in.}) \tan 30^\circ = 11.547 \text{ in.}$   
 $BP = (20 \text{ in.}) / \cos 30^\circ = 23.094 \text{ in.}$

ROD AE AND COLLAR:

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/AE}$$

$$\underline{v}_P = [(AP)\omega_{AE} \downarrow] + [\underline{v}_{P/AE} \rightarrow]$$

$$\underline{v}_P = [(11.547 \text{ in.})(3.5 \text{ rad/s}) \downarrow] + [\underline{v}_{P/AE} \rightarrow]$$

$$\underline{v}_P = [40.415 \text{ in./s} \downarrow] + [\underline{v}_{P/AE} \rightarrow] \quad (1)$$

ROD BD AND COLLAR:

$$\underline{v}_P = \underline{v}_B + \underline{v}_{P/BD} \quad (2)$$

$$\underline{v}_P = [(BP)\omega_{BD} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

$$\underline{v}_P = [(23.094 \text{ in.})(2.4 \text{ rad/s}) \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

$$\underline{v}_P = [55.426 \text{ in./s} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ] \quad (3)$$

$$\underline{v}_{P/B} = (BP)\omega_{BD} = (23.094 \text{ in.})(2.4 \text{ rad/s}); \quad \underline{v}_{P/B} = 55.426 \text{ in./s} \searrow 30^\circ$$

EQUATE EQS. (1) AND (3):

$$[40.415 \text{ in./s} \downarrow] + [\underline{v}_{P/AE} \rightarrow] = [55.426 \text{ in./s} \searrow 30^\circ] + [\underline{v}_{P/BD} \nearrow 30^\circ]$$

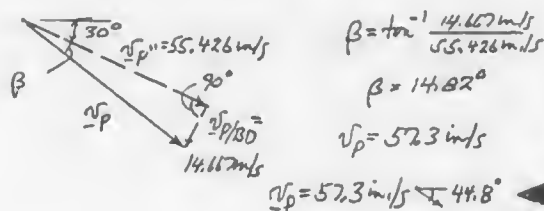
$$+\downarrow y \text{ COMPONENTS: } 40.415 = 55.426 \sin 30^\circ + \underline{v}_{P/BD} \cos 30^\circ$$

$$\underline{v}_{P/BD} = 14.667 \text{ in./s} \quad \underline{v}_{P/BD} = 14.667 \text{ in./s} \nearrow 30^\circ$$

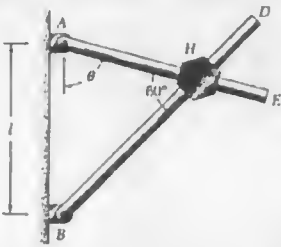
VECTOR DIAGRAM FOR EQ. (2):

$$\underline{v}_P = \underline{v}_B + \underline{v}_{P/BD}$$

$$\underline{v}_P = [55.426 \text{ in./s} \searrow 30^\circ] + [14.667 \text{ in./s} \nearrow 30^\circ]$$



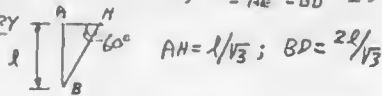
15.156



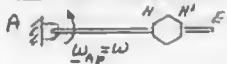
GIVEN:  
 $W_{AE} = W$

FIND:  
 $V_{H/AE}$  AND  $V_{H/BD}$   
WHEN  
(a)  $\theta = 90^\circ$   
(b)  $\theta = 60^\circ$

ANGLE BETWEEN RODS IS CONSTANT,  $\therefore W_{AE} = W_{BD} = W$   
(a)  $\theta = 90^\circ$ : GEOMETRY



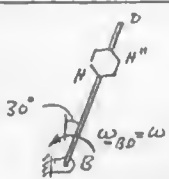
ROD AE AND BLOCK H



$$V_H = V_{H1} + V_{H/AE} = (AH)W \uparrow + V_{H/AE} \rightarrow$$

$$V_H = \frac{l}{\sqrt{3}} W \uparrow + V_{H/AE} \rightarrow \quad (1)$$

ROD BD AND BLOCK H



$$V_H = V_{H1} + V_{H/BD}$$

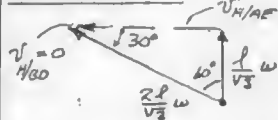
$$V_H = (BD)W \Delta 30^\circ + V_{H/BD} \Delta 30^\circ$$

$$V_H = \frac{2l}{\sqrt{3}} W \Delta 30^\circ + V_{H/BD} \Delta 30^\circ \quad (2)$$

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) + (2)

$$\frac{l}{\sqrt{3}} W \uparrow + V_{H/AE} \rightarrow = \frac{2l}{\sqrt{3}} W \Delta 30^\circ + V_{H/BD} \Delta 30^\circ$$

VECTOR DIAGRAM:



$$V_{H/AE} = \frac{l}{\sqrt{3}} W \tan 60^\circ = \frac{l}{\sqrt{3}} W (\sqrt{3})$$

$$V_{H/AE} = lW \leftarrow$$

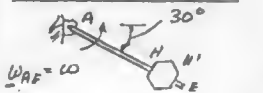
$$V_{H/BD} = 0$$

(b)  $\theta = 60^\circ$ : GEOMETRY:



EQUILATERAL TRIANGLE  
 $AH = BH = l$

ROD AE AND BLOCK H:

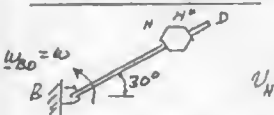


$$V_H = V_{H1} + V_{H/AE}$$

$$= (AH)W \Delta 60^\circ + [V_{H/AE} \Delta 30^\circ]$$

$$V_H = [lW \Delta 60^\circ] + [V_{H/AE} \Delta 30^\circ] \quad (1)$$

ROD BD AND BLOCK H:



$$V_H = V_{H1} + V_{H/BD}$$

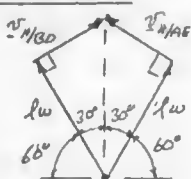
$$= [(BH)W \Delta 60^\circ] + [V_{H/BD} \Delta 30^\circ]$$

$$V_H = [lW \Delta 60^\circ] + [V_{H/BD} \Delta 30^\circ] \quad (2)$$

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2):

$$[lW \Delta 60^\circ] + [V_{H/AE} \Delta 30^\circ] = [lW \Delta 60^\circ] + [V_{H/BD} \Delta 30^\circ]$$

VECTOR DIAGRAM

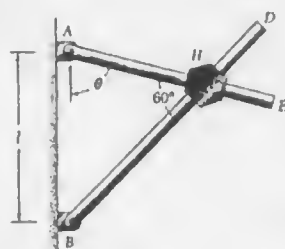


$$V_{H/BD} = V_{H/AE} = lW \tan 30^\circ = \frac{lW}{\sqrt{3}}$$

$$V_{H/AE} = \frac{lW}{\sqrt{3}} \Delta 30^\circ$$

$$V_{H/BD} = \frac{lW}{\sqrt{3}} \Delta 30^\circ$$

15.157



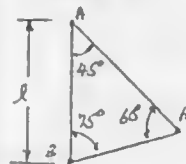
GIVEN:  
 $W_{AE} = W$   
 $\theta = 45^\circ$

FIND:  
 $V_{H/AE}$   
 $V_{H/BD}$

ANGLE BETWEEN RODS IS CONSTANT,  $\therefore W_{AE} = W_{BD} = W$

GEOMETRY:

LAW OF SINES

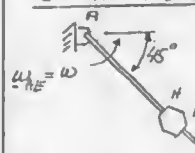


$$\frac{AH}{\sin 75^\circ} = \frac{BH}{\sin 45^\circ} = \frac{l}{\sin 60^\circ}$$

$$AH = 1.115 l$$

$$BH = 0.8165 l$$

ROD AE AND BLOCK H:

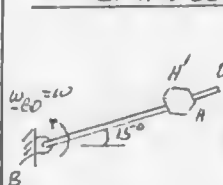


$$V_H = V_{H1} + V_{H/AE}$$

$$= [(AH)W \Delta 45^\circ] + [V_{H/AE} \Delta 45^\circ]$$

$$V_H = [1.115 lW \Delta 45^\circ] + [V_{H/AE} \Delta 45^\circ] \quad (1)$$

ROD BD AND BLOCK H:



$$V_H = V_{H1} + V_{H/BD}$$

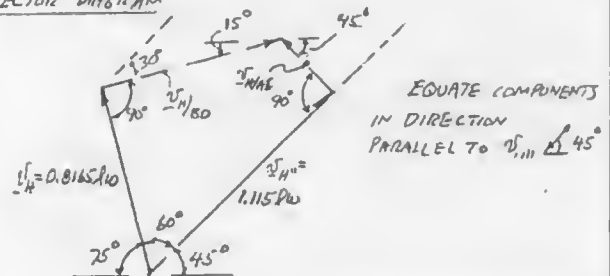
$$= [(BH)W \Delta 75^\circ] + [V_{H/BD} \Delta 15^\circ]$$

$$V_H = [0.8165 lW \Delta 75^\circ] + [V_{H/BD} \Delta 15^\circ] \quad (2)$$

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2):

$$[1.115 lW \Delta 45^\circ] + [V_{H/AE} \Delta 45^\circ] = [0.8165 lW \Delta 75^\circ] + [V_{H/BD} \Delta 15^\circ]$$

VECTOR DIAGRAM



EQUATE COMPONENTS IN DIRECTION PARALLEL TO  $V_{H1} \Delta 45^\circ$   
 $(0.8165 lW) \cos 60^\circ + V_{H/BD} \cos 30^\circ = 1.115 lW$

$$V_{H/BD} = +0.816 lW$$

$$V_{H/BD} = 0.816 lW \Delta 15^\circ$$

EQUATE COMPONENTS IN DIRECTION PERPENDICULAR TO  $V_{H1} \Delta 45^\circ$

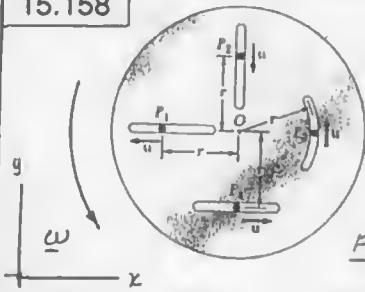
$$(0.8165 lW) \sin 60^\circ - V_{H/BD} \sin 30^\circ = V_{H/AE}$$

$$(0.8165 lW) \sin 60^\circ - (0.816 lW) \sin 30^\circ = V_{H/AE}$$

$$V_{H/AE} = +0.299 lW$$

$$V_{H/AE} = 0.299 lW \Delta 45^\circ$$

15.158



GIVEN:

CONSTANT ANGULAR  
VELOCITY =  $\omega$ CONSTANT SPEED OF  
PINS RELATIVE TO  
PLATE =  $u$ FIND: ACCELERATION  
OF EACH PIN.FOR EACH PIN:  $\underline{a}_p = \underline{a}_{p1} + \underline{a}_{p/g} + \underline{a}_c$ ACCELERATION OF COINCIDING POINT  $P'$ :FOR EACH PIN:  $\underline{a}_{p1} = r\omega^2$  TOWARD CENTER  $O$ 

ACCELERATION OF PIN WITH RESPECT TO PLATE:

FOR  $P_1, P_2$ , AND  $P_4$ :  $\underline{a}_{p/g} = 0$ FOR  $P_3$ :  $\underline{a}_{p/g} = u^2/r$  TOWARD CENTER  $O$ CORIOLIS ACCELERATION FOR EACH PIN  $\underline{a}_c = 2\omega \underline{u}$ ,  
WITH  $\underline{a}_c$  IN A DIRECTION OBTAINED BY ROTATING  $\underline{u}$   
THROUGH  $90^\circ$  IN THE SENSE OF  $\omega$ .

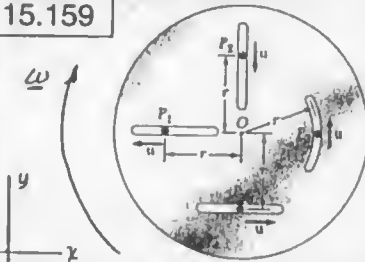
$$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \uparrow]; \quad \underline{a}_1 = r\omega^2 \underline{i} - 2u\omega \underline{j}$$

$$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \rightarrow]; \quad \underline{a}_2 = 2u\omega \underline{i} - r\omega^2 \underline{j}$$

$$\underline{a}_3 = [r\omega^2 \rightarrow] + [\frac{u^2}{r} \rightarrow] + [2u\omega \rightarrow]; \quad \underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} + 2u\omega) \underline{i}$$

$$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \uparrow]; \quad \underline{a}_4 = (r\omega^2 + 2u\omega) \underline{j}$$

15.159



GIVEN:

CONSTANT ANGULAR  
VELOCITY =  $\omega$ CONSTANT SPEED OF  
PINS RELATIVE TO  
PLATE =  $u$ FOR EACH PIN:  $\underline{a}_p = \underline{a}_{p1} + \underline{a}_{p/g} + \underline{a}_c$ ACCELERATION OF COINCIDING POINT  $P'$ :FOR EACH PIN:  $\underline{a}_{p1} = r\omega^2$  TOWARD CENTER  $O$ 

ACCELERATION OF PIN WITH RESPECT TO PLATE:

FOR  $P_1, P_2$ , AND  $P_4$ :  $\underline{a}_{p/g} = 0$ FOR  $P_3$ :  $\underline{a}_{p/g} = u^2/r$  TOWARD CENTER  $O$ .CORIOLIS ACCELERATION: FOR EACH PIN,  $\underline{a}_c = 2\omega \underline{u}$ ,  
WITH  $\underline{a}_c$  IN A DIRECTION OBTAINED BY ROTATING  $\underline{u}$   
THROUGH  $90^\circ$  IN THE SENSE OF  $\omega$ .

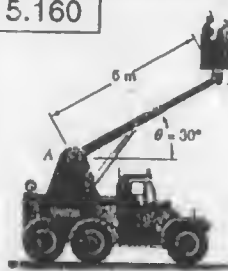
$$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \uparrow]; \quad \underline{a}_1 = r\omega^2 \underline{i} + 2u\omega \underline{j}$$

$$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \rightarrow]; \quad \underline{a}_2 = -2u\omega \underline{i} - r\omega^2 \underline{j}$$

$$\underline{a}_3 = [r\omega^2 \rightarrow] + [\frac{u^2}{r} \rightarrow] + [2u\omega \rightarrow]; \quad \underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} + 2u\omega) \underline{i}$$

$$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \uparrow]; \quad \underline{a}_4 = (r\omega^2 + 2u\omega) \underline{j}$$

15.160



GIVEN:

$$\omega_{AB} = 0.08 \text{ rad/s} \downarrow$$

$$\underline{a}_{AB} = 0$$

$$\underline{v}_{B/A} = 0.2 \text{ m/s} \nearrow 30^\circ$$

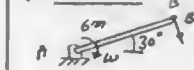
$$\underline{a}_{B/A} = 0$$

FIND:

$$(a) \underline{v}_B$$

$$(b) \underline{a}_B$$

(a) VELOCITY:



$$\underline{v}_{B/g} = \underline{v}_{B/A} = 0.2 \text{ m/s} \nearrow 30^\circ$$

$$\underline{v}_B = \underline{v}_{B1} + \underline{v}_{B/g}$$

$$\underline{v}_B = [(AB)\omega \nearrow 60^\circ] + [0.2 \text{ m/s} \nearrow 30^\circ]$$

$$= [(6 \text{ m})(0.08 \text{ rad/s}) \nearrow 60^\circ] + [0.2 \text{ m/s} \nearrow 30^\circ]$$

$$\underline{v}_B = 0.48 \text{ m/s} \nearrow 60^\circ + 0.2 \text{ m/s} \nearrow 30^\circ$$

$$\beta = 22.6^\circ \quad \underline{v}_B = 0.52 \text{ m/s} \nearrow 82.6^\circ$$

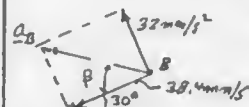
(b) ACCELERATION:

$$\underline{a}_B = \underline{a}_{B1} + \underline{a}_{B/g} + \underline{a}_c; \quad \underline{a}_{B/g} = \underline{a}_{B/A} = 0$$

$$\underline{a}_{B1} = (AB)\omega^2 = (6 \text{ m})(0.08 \text{ rad/s})^2 = 0.0384 \text{ m/s}^2 = 38.4 \text{ mm/s}^2 \nearrow 30^\circ$$

$$\underline{a}_c = 2\omega \underline{u} = 2(0.2 \text{ m/s})(0.08 \text{ rad/s}) = 0.032 \text{ m/s}^2 = 32 \text{ mm/s}^2 \nearrow 60^\circ$$

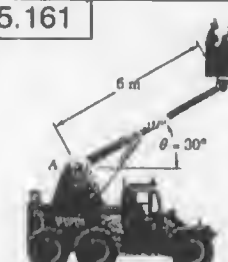
$$\underline{a}_B = [38.4 \text{ mm/s}^2 \nearrow 30^\circ] + 0 + [32 \text{ mm/s}^2 \nearrow 60^\circ]$$



$$\beta = 39.8^\circ \quad \underline{a}_B = 50.0 \text{ mm/s}^2$$

$$\underline{a}_B = 50.0 \text{ mm/s}^2 \nearrow 9.8^\circ$$

15.161



GIVEN:

$$\omega_{AB} = 0.08 \text{ rad/s} \downarrow$$

$$\underline{a}_{AB} = 0$$

$$\underline{v}_{B/A} = 0.2 \text{ m/s} \nearrow 30^\circ$$

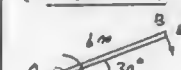
$$\underline{a}_{B/A} = 0$$

FIND:

$$(a) \underline{v}_B$$

$$(b) \underline{a}_B$$

(a) VELOCITY



$$\underline{v}_{B/g} = \underline{v}_{B/A} = 0.2 \text{ m/s} \nearrow 30^\circ$$

$$\underline{v}_B = \underline{v}_{B1} + \underline{v}_{B/g}$$

$$\underline{v}_B = [(AB)\omega \nearrow 60^\circ] + [\underline{v}_{B/g} \nearrow 30^\circ]$$

$$= [(6 \text{ m})(0.08 \text{ rad/s}) \nearrow 60^\circ] + [0.2 \text{ m/s} \nearrow 30^\circ]$$

$$\underline{v}_B = 0.48 \text{ m/s} \nearrow 60^\circ + 0.2 \text{ m/s} \nearrow 30^\circ$$

$$\gamma = 90^\circ - 30^\circ - 22.6^\circ = 37.4^\circ; \quad \underline{v}_B = 0.52 \text{ m/s} \nearrow 37.4^\circ$$

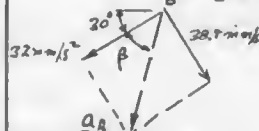
(b) ACCELERATION

$$\underline{a}_B = \underline{a}_{B1} + \underline{a}_{B/g} + \underline{a}_c; \quad \underline{a}_{B/g} = 0$$

$$\underline{a}_{B1} = (AB)\omega^2 = (6 \text{ m})(0.08 \text{ rad/s})^2 = 0.0384 \text{ m/s}^2 = 38.4 \text{ mm/s}^2 \nearrow 30^\circ$$

$$\underline{a}_c = 2\omega \underline{u} = 2(0.2 \text{ m/s})(0.08 \text{ rad/s}) = 0.032 \text{ m/s}^2 = 32 \text{ mm/s}^2 \nearrow 60^\circ$$

$$\underline{a}_B = [38.4 \text{ mm/s}^2 \nearrow 30^\circ] + 0 + [32 \text{ mm/s}^2 \nearrow 60^\circ]$$

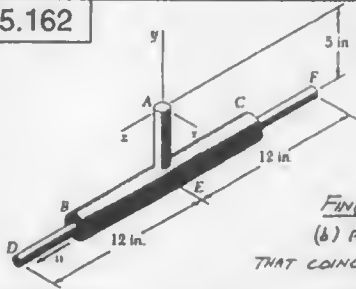


$$\beta = 39.8^\circ \quad \underline{a}_B = 50.0 \text{ mm/s}^2$$

$$\underline{a}_B = 50.0 \text{ mm/s}^2 \nearrow 69.8^\circ$$



15.162



GIVEN:  
 $\omega_{ABC} = \omega = (3 \text{ rad/s}) \hat{i}$   
 $\dot{\omega}_{ABC} = 0$   
 $\vec{r}_{D/C} = (16 \text{ in./s}) \hat{j}$   
 $\dot{\omega}_{D/C} = 0$

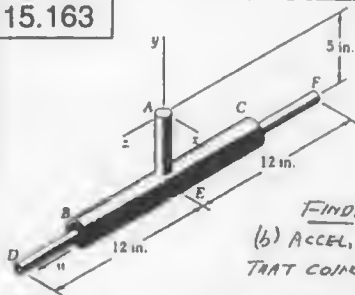
FIND: (a)  $\underline{a}_D$   
 (b) ACCEL. OF POINT P OF DF THAT COINCIDES WITH E.

(a) POINT D:  $\vec{r}_{D/A} = \vec{r}_{D/C} = (16 \text{ in./s}) \hat{j}$ ;  $\dot{\omega}_{D/A} = 0$   
 $\vec{AD} = -(5 \text{ in.}) \hat{j} + (12 \text{ in.}) \hat{i}$   
 $\underline{a}_D = \omega \times \omega \times \vec{AD} = -\omega^2 (\vec{AD}) = -(3 \text{ rad/s})^2 \vec{AD} = -(45 \text{ in./s}^2) \hat{j} - (108 \text{ in./s}^2) \hat{i}$   
 $\underline{a}_C = 2\omega \times \vec{r}_{D/C} = 2(3 \text{ rad/s}) \hat{i} \times (16 \text{ in./s}) \hat{j} = -(96 \text{ in./s}^2) \hat{j}$   
 $\underline{a}_D = \underline{a}_D + \underline{a}_{D/C} + \underline{a}_C$   
 $= [(45 \text{ in./s}^2) \hat{j} + (108 \text{ in./s}^2) \hat{i}] + 0 + [-(96 \text{ in./s}^2) \hat{j}]$   
 $\underline{a}_D = -(51 \text{ in./s}^2) \hat{j} + (108 \text{ in./s}^2) \hat{i}$

(b) POINT P OF DF THAT COINCIDES WITH E

$\vec{r}_{P/A} = \vec{r}_{P/C} = (16 \text{ in./s}) \hat{j}$ ;  $\dot{\omega}_{P/A} = 0$   
 $\vec{AE} = -(5 \text{ in.}) \hat{j}$   
 $\underline{a}_P = \omega \times \omega \times \vec{AE} = -\omega^2 \vec{AE} = -(3 \text{ rad/s})^2 \vec{AE} = (45 \text{ in./s}^2) \hat{j}$   
 $\underline{a}_C = 2\omega \times \vec{r}_{D/C} = 2(3 \text{ rad/s}) \hat{i} \times (16 \text{ in./s}) \hat{j} = -(96 \text{ in./s}^2) \hat{j}$   
 $\underline{a}_P = \underline{a}_P + \underline{a}_{P/C} + \underline{a}_C$   
 $= [(45 \text{ in./s}^2) \hat{j}] + 0 + [-(96 \text{ in./s}^2) \hat{j}]$   
 $\underline{a}_P = -(51 \text{ in./s}^2) \hat{j}$

15.163



GIVEN:  
 $\omega_{ABC} = \omega = (3 \text{ rad/s}) \hat{j}$   
 $\dot{\omega}_{ABC} = 0$   
 $\vec{r}_{D/C} = (16 \text{ in./s}) \hat{i}$   
 $\dot{\omega}_{D/C} = 0$

FIND: (a)  $\underline{a}_D$   
 (b) ACCEL. OF POINT P OF DF THAT COINCIDES WITH E

(a) POINT D:  $\vec{r}_{D/A} = \vec{r}_{D/C} = (16 \text{ in./s}) \hat{i}$ ;  $\dot{\omega}_{D/A} = 0$   
 $\vec{AD} = -(5 \text{ in.}) \hat{j} + (12 \text{ in.}) \hat{i}$   
 $\underline{a}_D = \omega \times \omega \times \vec{AD} = (3 \text{ rad/s})^2 \hat{j} \times [-(5 \text{ in.}) \hat{j} + (12 \text{ in.}) \hat{i}] = (36 \text{ in./s}^2) \hat{i}$   
 $\underline{a}_C = 2\omega \times \vec{r}_{D/C} = 2(3 \text{ rad/s}) \hat{j} \times (16 \text{ in./s}) \hat{i} = -(108 \text{ in./s}^2) \hat{j}$   
 $\underline{a}_D = \underline{a}_D + \underline{a}_{D/C} + \underline{a}_C$   
 $= (36 \text{ in./s}^2) \hat{i} + 0 + [-(108 \text{ in./s}^2) \hat{j}]$   
 $\underline{a}_D = (36 \text{ in./s}^2) \hat{i} - (108 \text{ in./s}^2) \hat{j}$

(b) POINT P OF DF THAT COINCIDES WITH E

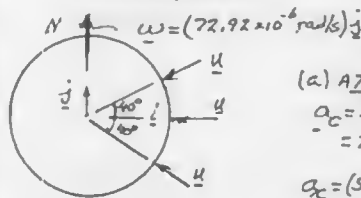
$\vec{r}_{P/A} = \vec{r}_{P/C} = (16 \text{ in./s}) \hat{i}$ ;  $\dot{\omega}_{P/A} = 0$   
 $\vec{AE} = -(5 \text{ in.}) \hat{j}$ ;  $\underline{a}_P = \omega \times \omega \times \vec{AE} = (3 \text{ rad/s})^2 \hat{j} \times [-(5 \text{ in.}) \hat{j}] = 0$   
 $\underline{a}_C = 2\omega \times \vec{r}_{D/C} = 2(3 \text{ rad/s}) \hat{j} \times (16 \text{ in./s}) \hat{i} = -(108 \text{ in./s}^2) \hat{j}$   
 $\underline{a}_P = \underline{a}_P + \underline{a}_{P/C} + \underline{a}_C$   
 $\underline{a}_P = 0 + 0 + [-(108 \text{ in./s}^2) \hat{j}]$   
 $\underline{a}_P = -(108 \text{ in./s}^2) \hat{j}$

15.164

GIVEN: ELEVATOR MOVES DOWNWARD AT 40 ft/s  
FIND: CORIOLIS ACCELERATION OF ELEVATOR IF IT IS LOCATED AT: (a) EQUATOR, (b) 40° NORTH, (c) 40° SOUTH.

EARTH MAKES ONE REVOLUTION IN 23 h 56 m = 23,933 h

$$\omega = \frac{2\pi \text{ rad}}{(23,933 \text{ h})(3600 \text{ s/h})} = 72.92 \times 10^{-6} \text{ rad/s}$$



(a) AT EQUATOR:

$\underline{a}_C = 2\omega \times \underline{u}$   
 $= 2(72.92 \times 10^{-6} \text{ rad/s}) \hat{j} \times (-40 \text{ ft/s}) \hat{i}$   
 $\underline{a}_C = (5.83 \times 10^{-3} \text{ ft/s}^2) \hat{j}$   
 $\underline{a}_C = 5.83 \times 10^{-3} \text{ ft/s}^2 \text{ WEST}$

(b) AT 40° NORTH:

$\underline{u} = 40 \text{ ft/s} (-\cos 40^\circ \hat{i} - \sin 40^\circ \hat{j})$   
 $\underline{a}_C = 2\omega \times \underline{u} = 2(72.92 \times 10^{-6} \text{ rad/s}) \hat{j} \times (40 \text{ ft/s}) (-\cos 40^\circ \hat{i} - \sin 40^\circ \hat{j})$   
 $\underline{a}_C = (4.47 \text{ ft/s}^2) \hat{j}$   
 $\underline{a}_C = 4.47 \text{ ft/s}^2 \text{ WEST}$

(c) AT 40° SOUTH:

$\underline{u} = 40 \text{ ft/s} (-\cos 40^\circ \hat{i} + \sin 40^\circ \hat{j})$   
 $\underline{a}_C = 2\omega \times \underline{u} = 2(72.92 \times 10^{-6} \text{ rad/s}) \hat{j} \times (40 \text{ ft/s}) (-\cos 40^\circ \hat{i} + \sin 40^\circ \hat{j})$   
 $\underline{a}_C = (4.47 \times 10^{-3}) \text{ ft/s}^2$   
 $\underline{a}_C = (4.47 \times 10^{-3}) \text{ ft/s}^2 \text{ WEST}$

\* NOTE: EARTH ROTATES COUNTERCLOCKWISE WHEN OBSERVED FROM ABOVE THE NORTH POLE.

15.165

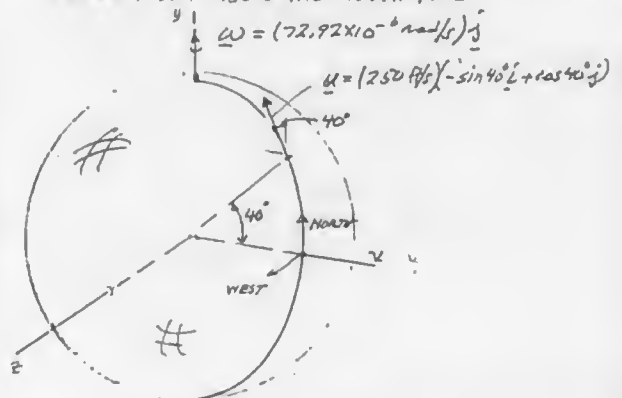
GIVEN: TEST SLED MOVING DUE NORTH AT 900 ft/m/h. AT 40° NORTH LATITUDE.  
FIND: CORIOLIS ACCELERATION OF SLED

EARTH MAKES ONE REVOLUTION IN 23 h 56 m OR 23,933 h.

$$\omega = \frac{(2\pi \text{ rad}) \hat{j}}{(23,933 \text{ h})(3600 \text{ s/h})} = (72.92 \times 10^{-6} \text{ rad/s}) \hat{j}$$

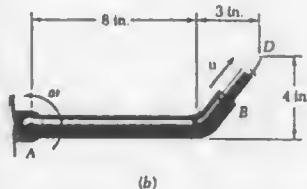
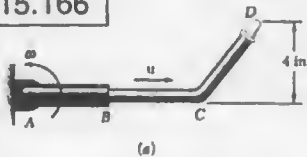
$$u = 900 \text{ ft/m/h} = 250 \text{ m/s}$$

NOTE: EARTH ROTATES COUNTERCLOCKWISE WHEN VIEWED FROM ABOVE THE NORTH POLE.



$\underline{a}_C = 2\omega \times \underline{u} = 2(72.92 \times 10^{-6} \text{ rad/s}) \hat{j} \times (250 \text{ m/s}) (-\sin 40^\circ \hat{i} + \cos 40^\circ \hat{j})$   
 $\underline{a}_C = (23.4 \times 10^{-3}) \text{ m/s}^2 \hat{j} = (23.4 \times 10^{-3}) \text{ m/s}^2 \text{ WEST}$   
 $\underline{a}_C = (23.4 \times 10^{-3}) \text{ ft/s}^2 \text{ TO LEFT OF SLED}$

15.166



GIVEN:

$$\begin{aligned}\omega_{AB} &= \omega = 2.4 \text{ rad/s} \\ a_{AB} &= 0 \\ u &= 10 \text{ in/s} \\ \dot{\theta} &= 0\end{aligned}$$

FIND:  $a_D$  FOR EACH ARRANGEMENTFOR EACH ARRANGEMENT:  $\omega = (2.4 \text{ rad/s}) \hat{k}$ 

$$\begin{aligned}\vec{AD} &= (11 \text{ in.})\hat{i} + (4 \text{ in.})\hat{j} \\ a_D &= \omega \times \omega \times \vec{AD} = -\omega^2 \vec{AD} \\ &= -(2.4 \text{ rad/s})^2 [(11 \text{ in.})\hat{i} + (4 \text{ in.})\hat{j}] \\ a_{D1} &= -(63.36 \text{ in/s}^2)\hat{i} - (23.04 \text{ in/s}^2)\hat{j}\end{aligned}$$

$$\begin{aligned}\text{(a)} \quad u &= v_{D/x} = (10 \text{ in/s})\hat{i} \quad a_{D/x} = 0 \\ a_c &= 2\omega \times v_{D/y} = 2(2.4 \text{ rad/s}) \hat{k} \times (10 \text{ in/s})\hat{j} = (48 \text{ in/s}^2)\hat{i} \\ a_D &= a_{D1} + a_{D/y} + a_c \\ &= [-(63.36 \text{ in/s}^2)\hat{i} - (23.04 \text{ in/s}^2)\hat{j}] + 0 + (48 \text{ in/s}^2)\hat{i} \\ a_D &= -(63.36 \text{ in/s}^2)\hat{i} + (24.96 \text{ in/s}^2)\hat{j}\end{aligned}$$

$$a_D = \sqrt{63.36^2 + 24.96^2} = 68.1 \text{ in/s}^2 \quad \phi = 21.5^\circ \quad a_D = 68.1 \text{ in/s}^2 \angle 21.5^\circ$$

$$\text{(b)} \quad u = 10 \text{ in/s}^2 \quad u = \frac{x}{r}(\omega)\hat{i} + \frac{y}{r}(\omega)\hat{j}$$

$$u = v_{D/y} = (6 \text{ in/s})\hat{i} + (8 \text{ in/s})\hat{j} \quad a_{D/y} = 0$$

$$a_c = 2\omega \times v_{D/y} = 2(2.4 \text{ rad/s}) \hat{k} \times [(6 \text{ in/s})\hat{i} + (8 \text{ in/s})\hat{j}]$$

$$a_c = -(38.4 \text{ in/s}^2)\hat{i} + (28.8 \text{ in/s}^2)\hat{j}$$

$$\begin{aligned}a_D &= a_{D1} + a_{D/y} + a_c \\ &= -(63.36 \text{ in/s}^2)\hat{i} - (23.04 \text{ in/s}^2)\hat{j} + 0 - (38.4 \text{ in/s}^2)\hat{i} \\ &\quad + (28.8 \text{ in/s}^2)\hat{j}\end{aligned}$$

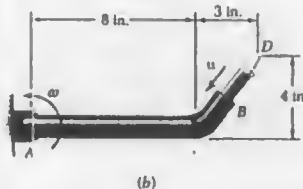
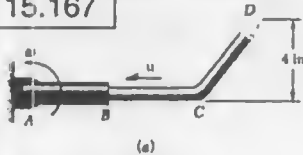
$$a_D = -(101.76 \text{ in/s}^2)\hat{i} + (5.76 \text{ in/s}^2)\hat{j}$$

$$a_D = \sqrt{101.76^2 + 5.76^2} = 101.9 \text{ in/s}^2$$

$$\beta = 3.2^\circ \quad a_D = 101.9 \text{ in/s}^2$$

$$a_D = 101.9 \text{ in/s}^2 \angle 3.2^\circ$$

15.167



GIVEN:

$$\begin{aligned}\omega_{AB} &= \omega = 2.4 \text{ rad/s} \\ a_{AB} &= 0 \\ u &= 10 \text{ in/s} \\ \dot{\theta} &= 0\end{aligned}$$

FIND:  $a_D$  FOR EACH ARRANGEMENT

FOR EACH ARRANGEMENT:

$$\begin{aligned}AD &= (4^2 + 11^2)^{1/2} = 11.705 \text{ in} \\ \gamma &= 19.98^\circ \\ a_{D1} &= (AD)\omega^2 \angle 19.98^\circ \\ &= (11.705 \text{ in})(2.4 \text{ rad/s})^2 \angle 19.98^\circ \\ a_{D1} &= 67.42 \text{ in/s}^2 \angle 19.98^\circ\end{aligned}$$

$$\begin{aligned}\text{(a)} \quad u &= v_{D/y} = 10 \text{ in/s} \rightarrow ; \quad a_{D/y} = 0 \\ a_c &= 2\omega \times v_{D/y} = 2(2.4 \text{ rad/s})(10 \text{ in/s}) = 48 \text{ in/s}^2 \\ \text{IN DIRECTION } 90^\circ \text{ FROM } v_{D/y}: \quad a_c &= 48 \text{ in/s}^2 \downarrow\end{aligned}$$

$$\begin{aligned}a_D &= a_{D1} + a_{D/y} + a_c \\ a_D &= 67.42 \text{ in/s}^2 \angle 19.98^\circ + 0 + 48 \text{ in/s}^2 \downarrow\end{aligned}$$

$$\begin{aligned}\text{VECTOR DIAGRAM} \\ a_D &= \sqrt{67.42^2 + 48^2} = 82.01 \text{ in/s}^2 \\ a_D &= 82.01 \text{ in/s}^2 \angle 48.3^\circ\end{aligned}$$

$$\text{(b)} \quad u = 10 \text{ in/s} \quad u = v_{D/y} = 10 \text{ in/s} \quad a_{D/y} = 0$$

$$\begin{aligned}a_c &= 2\omega \times v_{D/y} = 2(2.4 \text{ rad/s})(10 \text{ in/s}) = 48 \text{ in/s}^2 \\ \text{IN DIRECTION } 90^\circ \text{ FROM } v_{D/y}: \quad a_c &= 48 \text{ in/s}^2 \angle 53.13^\circ = 48 \text{ in/s}^2 \angle 36.87^\circ\end{aligned}$$

$$\begin{aligned}a_D &= a_{D1} + a_{D/y} + a_c \\ a_D &= 67.42 \text{ in/s}^2 \angle 19.98^\circ + 0 + 48 \text{ in/s}^2 \angle 36.87^\circ\end{aligned}$$

VECTOR DIAGRAM

$$\begin{aligned}a_D &= \sqrt{67.42^2 + 48^2} = 82.01 \text{ in/s}^2 \\ a_D &= 82.01 \text{ in/s}^2 \angle 48.3^\circ\end{aligned}$$

LAW OF COSINES

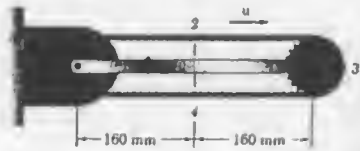
$$a_D^2 = (67.42)^2 + (48)^2 - 2(67.42)(48)\cos 56.85^\circ; \quad a_D = 57.53 \text{ in/s}^2$$

LAW OF SINES

$$\frac{a_D}{\sin 56.85^\circ} = \frac{48}{\sin \beta}; \quad \sin \beta = \frac{48}{57.53} \sin 56.85^\circ; \quad \beta = 44.3^\circ$$

$$a_D = 57.53 \angle (19.98^\circ + 44.3^\circ); \quad a_D = 57.5 \text{ in/s}^2 \angle 64.3^\circ$$

# 15.168 and 15.169



GIVEN:  
 $\omega_{AB} = \omega = 0.75 \text{ rad/s}$   
 $\alpha_{AB} = 0$   
 $u = 80 \text{ mm/s}$   
 $a = 0$

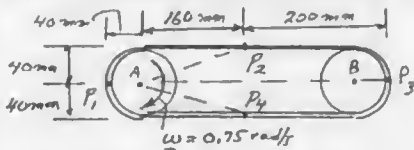
FIND: ACCEL. OF LINKS

PROB. 15.168: LINKS 1 + 2  
 PROB. 15.169: LINKS 3 + 4

$$a_p = a_{p1} + a_{p/g} + a_c \quad (1)$$

EACH TERM IS COMPUTED SEPARATELY FOR EACH LINK

$a_{p1}$  ACCELERATION OF COINCIDING POINT P'



$$a_{p1} = (AP_1) \omega^2 = 40 \times 0.75^2 = 22.5 \text{ mm/s}^2 \rightarrow$$

$$a_{p2} = (AP_2) \omega^2 = 160 \times 0.75^2 + 40 \times 0.75^2$$

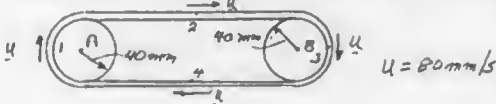
$$a_{p2} = 90 \text{ mm/s}^2 \leftarrow + 22.5 \text{ mm/s}^2 \downarrow$$

$$a_{p3} = (AP_3) \omega^2 = (160 + 200) 0.75^2 = 202.5 \text{ mm/s}^2 \leftarrow$$

$$a_{p4} = (AP_4) \omega^2 = 160 \times 0.75^2 + 40 \times 0.75^2$$

$$a_{p4} = 90 \text{ mm/s}^2 \leftarrow + 22.5 \text{ mm/s}^2 \uparrow$$

$a_{p/g}$  ACCELERATION OF P RELATIVE TO ROTATING FRAME



$$a_{p/g} = u^2/r = (80)^2/40 = 160 \text{ mm/s}^2 \rightarrow$$

$$a_{p/g} = a_{p/g} = 0$$

$$a_{p/g} = u^2/r = (80)^2/40 = 160 \text{ mm/s}^2 \leftarrow$$

$a_c$  CORIOLIS ACCELERATION

MAGNITUDE FOR ALL LINKS

$$a_c = 2\omega u = 2(0.75 \text{ rad/s})(80 \text{ mm/s}) = 120 \text{ mm/s}^2$$

DIRECTION: ROTATE  $u$  THROUGH  $90^\circ$

$$\text{LINK 1: } a_c = 120 \text{ mm/s}^2 \rightarrow$$

$$\text{LINK 2: } a_c = 120 \text{ mm/s}^2 \downarrow$$

$$\text{LINK 3: } a_c = 120 \text{ mm/s}^2 \leftarrow$$

$$\text{LINK 4: } a_c = 120 \text{ mm/s}^2 \uparrow$$

$$a_p = a_{p1} + a_{p/g} + a_c$$

PROBLEM 15.168:

$$\text{LINK 1: } a_{p1} = (22.5 \text{ mm/s}^2 \rightarrow) + (160 \text{ mm/s}^2 \rightarrow) + (120 \text{ mm/s}^2 \rightarrow)$$

$$a_{p1} = 302.5 \text{ mm/s}^2 \rightarrow$$

$$\text{LINK 2: } a_{p2} = (90 \text{ mm/s}^2 \leftarrow + 22.5 \text{ mm/s}^2 \downarrow) + 0 + (120 \text{ mm/s}^2 \downarrow)$$

$$a_{p2} = 90 \text{ mm/s}^2 \leftarrow + 142.5 \text{ mm/s}^2 \downarrow = 168.5 \text{ mm/s}^2 \angle 57.7^\circ$$

PROBLEM 15.169:

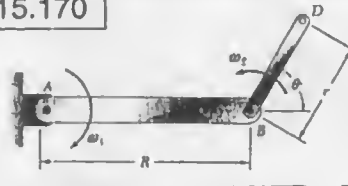
$$\text{LINK 3: } a_{p3} = (202.5 \text{ mm/s}^2 \leftarrow) + (160 \text{ mm/s}^2 \leftarrow) + (120 \text{ mm/s}^2 \leftarrow)$$

$$a_{p3} = 482.5 \text{ mm/s}^2 \leftarrow$$

$$\text{LINK 4: } a_{p4} = (90 \text{ mm/s}^2 \leftarrow + 22.5 \text{ mm/s}^2 \uparrow) + 0 + (120 \text{ mm/s}^2 \uparrow)$$

$$a_{p4} = 90 \text{ mm/s}^2 \leftarrow + 142.5 \text{ mm/s}^2 \uparrow = 168.5 \text{ mm/s}^2 \angle 57.7^\circ$$

# 15.170



GIVEN:  
 $\omega_2 = 2\omega_1$   
 SHOW THAT  $a_D$   
 PASSES THROUGH A  
 AND THAT THE RESULT  
 IS INDEPENDENT OF  $R, r, \theta$

$$a_D = a_{D1} + a_{D/g} + a_c$$

$$\vec{BD} = r \cos \theta \vec{i} + r \sin \theta \vec{j}$$

$$\vec{AD} = (R + r \cos \theta) \vec{i} + r \sin \theta \vec{j}$$

$$a_{D1} = -\omega_1^2 (\vec{AD}) = -(R + r \cos \theta) \omega_1^2 \vec{i} - r \omega_1^2 \sin \theta \vec{j}$$

$$a_{D/g} = \omega_2 \times (\vec{BD}) = 2\omega_1 \times (r \cos \theta \vec{i} + r \sin \theta \vec{j})$$

$$a_{D/g} = -2\omega_1 r \sin \theta \vec{i} + 2\omega_1 r \cos \theta \vec{j}$$

$$a_{D/g} = -\omega_2^2 (\vec{BD}) = -(2\omega_1)^2 (r \cos \theta \vec{i} + r \sin \theta \vec{j})$$

$$a_{D/g} = -4\omega_1^2 r \cos \theta \vec{i} - 4\omega_1^2 r \sin \theta \vec{j}$$

$$a_c = 2\omega_1 \times a_{D/g} = 2(-\omega_1 \vec{i}) \times (-2\omega_1 r \sin \theta \vec{i} + 2\omega_1 r \cos \theta \vec{j})$$

$$a_c = +4\omega_1^2 r \cos \theta \vec{i} + 4\omega_1^2 r \sin \theta \vec{j}$$

$$a_D = a_{D1} + a_{D/g} + a_c$$

$$= -(R + r \cos \theta) \omega_1^2 \vec{i} - r \omega_1^2 \sin \theta \vec{j} - 4\omega_1^2 r \cos \theta \vec{i} - 4\omega_1^2 r \sin \theta \vec{j} + 4\omega_1^2 r \cos \theta \vec{i} + 4\omega_1^2 r \sin \theta \vec{j}$$

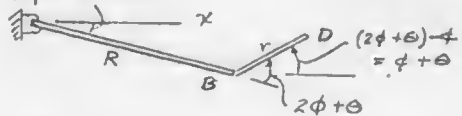
$$a_D = -\omega_1^2 [(R + r \cos \theta) \vec{i} - r \sin \theta \vec{j}]$$

$$a_D = -\omega_1^2 (\vec{AD}) \quad QED$$

ALTERNATIVE SOLUTION

AT ANY TIME  $t$ :

$$A \rightarrow \phi = \omega_1 t$$



FOR POINT D:

$$x = R \cos \phi + r \cos(\phi + \theta) = R \cos \omega_1 t + r \cos(\omega_1 t + \theta)$$

$$y = -R \sin \phi + r \sin(\phi + \theta) = -R \sin \omega_1 t + r \sin(\omega_1 t + \theta)$$

$$\dot{x} = -R \omega_1 \sin \omega_1 t - r \omega_1 \sin(\omega_1 t + \theta)$$

$$\dot{y} = -R \omega_1 \cos \omega_1 t + r \omega_1 \cos(\omega_1 t + \theta)$$

$$\ddot{x} = -R \omega_1^2 \cos \omega_1 t - r \omega_1^2 \cos(\omega_1 t + \theta)$$

$$\ddot{y} = +R \omega_1^2 \sin \omega_1 t - r \omega_1^2 \sin(\omega_1 t + \theta)$$

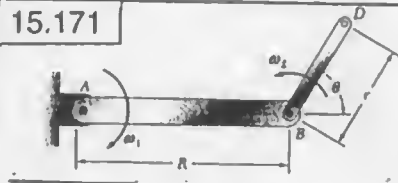
$$\ddot{x} = -\omega_1^2 (R \cos \omega_1 t + r \cos(\omega_1 t + \theta)) = -\omega_1^2 x$$

$$\ddot{y} = -\omega_1^2 (-R \sin \omega_1 t + r \sin(\omega_1 t + \theta)) = -\omega_1^2 y$$

$$\therefore a_D = -\omega_1^2 (\vec{AD})$$

WHEN  $\omega_2 = 2\omega_1$ ,  $a_D$  PASSES THROUGH POINT A DURING ENTIRE MOTION

15.171

GIVEN:  $R = 15 \text{ in.}$  $r = 8 \text{ in.}, \theta = 60^\circ$  $\omega_1 = 5 \text{ rad/s}$  $\omega_2 = 3 \text{ rad/s}$ FIND:  $a_D$ 

$$\vec{r}_D = \vec{AD} = (15 + 8 \cos 60^\circ) \hat{i} + 8 \sin 60^\circ \hat{j} = (19 \text{ in.}) \hat{i} + (6.928 \text{ in.}) \hat{j}$$

$$\vec{a}_D = -\omega_1^2 \vec{r}_D = -5^2 (19 \hat{i} + 6.928 \hat{j}) = (-475 \text{ in./s}^2) \hat{i} - (173.2 \text{ in./s}^2) \hat{j}$$

A.D. ACCELERATION OF COINCIDING POINT D

$$\vec{a}_D = -\omega_1^2 \vec{r}_D = -5^2 (19 \hat{i} + 6.928 \hat{j}) = (-475 \text{ in./s}^2) \hat{i} - (173.2 \text{ in./s}^2) \hat{j}$$

MOTION OF D RELATIVE TO FRAME

$$\vec{v}_{D/F} = \omega_2 \times \vec{r}_{D/B} = (3 \hat{k}) \times (4 \hat{i} + 6.928 \hat{j}) = (-20.78 \text{ in./s}) \hat{i} + (12 \text{ in./s}) \hat{j}$$

$$\vec{a}_{D/F} = -\omega_2^2 \vec{r}_{D/B} = -3^2 (4 \hat{i} + 6.928 \hat{j}) = (-36 \text{ in./s}^2) \hat{i} - (62.35 \text{ in./s}^2) \hat{j}$$

C.C. CORIOLIS ACCELERATION

$$\vec{a}_C = 2 \omega_1 \times \vec{v}_{D/F} = 2(-5 \hat{k}) \times (-20.78 \hat{i} + 12 \hat{j})$$

$$\vec{a}_C = (120 \text{ in./s}^2) \hat{i} + (207.8 \text{ in./s}^2) \hat{j}$$

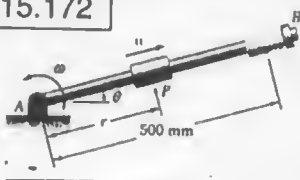
$$\vec{a}_D = \vec{a}_D + \vec{a}_{D/F} + \vec{a}_C$$

$$= (-475 \text{ in./s}^2) \hat{i} - (173.2 \text{ in./s}^2) \hat{j} - (36 \text{ in./s}^2) \hat{i} - (62.35 \text{ in./s}^2) \hat{j} + (120 \text{ in./s}^2) \hat{i} + (207.8 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_D = (-391 \text{ in./s}^2) \hat{i} - (27.75 \text{ in./s}^2) \hat{j}$$

$$291 \text{ in./s}^2 \quad 27.75 \text{ in./s}^2 \quad a_D = 392 \text{ in./s}^2 \angle 4.05^\circ$$

15.172

GIVEN:  $\omega = 20 \text{ rpm}$  $r = 250 \text{ mm}$  WHEN  $\theta = 0$ 

AND COLLAR REACHES B

WHEN  $\theta = 90^\circ$ FIND:  $a_D$  JUST AS COLLAR REACHES B.

$$\omega = 20 \text{ rpm} = 2.094 \text{ rad/s}$$

ROD ROTATES  $90^\circ = \pi/2 \text{ radians}$ 

$$\omega t = \frac{\pi}{2}; t = \frac{\pi}{2\omega} = \frac{\pi}{2(2.094 \text{ rad/s})}$$

$$t = 0.75 \text{ s}$$

COLLAR MOVES 0.25 m IN  $t = 0.75 \text{ s}$ 

$$u t = 0.25 \text{ m}; u = \frac{0.25 \text{ m}}{0.75 \text{ s}} = 0.333 \text{ m/s}$$

A.B. ACCEL. OF COINCIDING POINT B'

$$\vec{a}_{B'} = L \omega^2 \hat{i} = 0.5 \text{ m} (2.094 \text{ rad/s})^2 = 2.19 \text{ m/s}^2$$

A.B./G = 0, SINCE  $u = \text{CONSTANT}$ 

CORIOLIS ACCELERATION

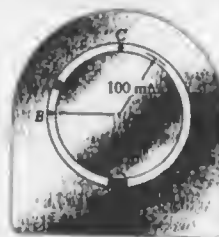
$$\vec{a}_C = 2 u \omega \hat{i} = 2(0.333 \text{ m/s})(2.094 \text{ rad/s})$$

$$\vec{a}_C = 1.395 \text{ m/s}^2$$

$$\vec{a}_B^2 = (2.19 \text{ m/s}^2)^2 + (1.395 \text{ m/s}^2)^2$$

$$a_B = 2.60 \text{ m/s}^2$$

15.173 and 15.174

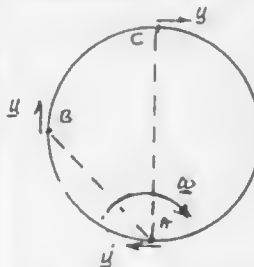
GIVEN:  $\omega = 3 \text{ rad/s}$  $u = 90 \text{ mm/s}, \dot{u} = 0$ PROBLEM 15.173: FOR  $\alpha = 0$ FIND:  $a_P$  WHEN PIN IS AT

(a) POINT A, (b) POINT B, (c) POINT C.

PROBLEM 15.174: SOLVE

SAME PROBLEM IF  $\alpha = 5 \text{ rad/s}^2$ 

AS PIN IS AT POINTS A, B, + C.

PROBLEM 15.173:  $\vec{AB} = 0.1 \text{ m} \hat{i} + 0.1 \text{ m} \hat{j}; \vec{AC} = 0.2 \text{ m} \hat{i}$ 

ACCELERATIONS OF COINCIDING POINTS

$$\vec{a}_A = 0$$

$$\vec{a}_B = -\omega^2 (\vec{AB}) = -(3^2) (\vec{AB}) = 0.9 \text{ m/s}^2 \rightarrow +0.9 \text{ m/s}^2 \downarrow$$

$$\vec{a}_C = -\omega^2 (\vec{AC}) = -(3^2) (\vec{AC}) = 1.8 \text{ m/s}^2 \downarrow$$

ACCELERATIONS OF PIN RELATIVE TO THE

ROTATING FRAME  $= u^2/r = (0.09 \text{ m/s})^2 / (0.1 \text{ m}) = 0.081 \text{ m/s}^2$ 

WE HAVE:

$$\vec{a}_{A/G} = 0.081 \text{ m/s}^2 \uparrow$$

$$\vec{a}_{B/G} = 0.081 \text{ m/s}^2 \rightarrow$$

$$\vec{a}_{C/G} = 0.081 \text{ m/s}^2 \downarrow$$

CORIOLIS ACCELERATIONS

$$\text{POINT A: } \vec{a}_C = 2 u \omega = 2(0.09 \text{ m/s})(3 \text{ rad/s}) = 0.54 \text{ m/s}^2 \uparrow$$

$$\text{POINT B: SAME MAGNITUDE } \vec{a}_C = 0.54 \text{ m/s}^2 \rightarrow$$

$$\text{POINT C: } \vec{a}_C = 0.54 \text{ m/s}^2 \downarrow$$

$$\vec{a}_P = \vec{a}_P + \vec{a}_{P/G} + \vec{a}_C$$

$$\text{POINT A: } \vec{a}_A = 0 + 0.081 \text{ m/s}^2 \uparrow + 0.54 \text{ m/s}^2 \uparrow = 0.621 \text{ m/s}^2 \uparrow$$

$$\text{POINT B: } \vec{a}_B = 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow + 0.081 \text{ m/s}^2 \rightarrow + 0.54 \text{ m/s}^2 \rightarrow = 1.521 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow = 1.767 \text{ m/s}^2 \angle 30.6^\circ$$

$$\text{POINT C: } \vec{a}_C = 1.8 \text{ m/s}^2 \downarrow + 0.081 \text{ m/s}^2 \downarrow + 0.54 \text{ m/s}^2 \downarrow = 2.421 \text{ m/s}^2 \downarrow$$

PROBLEM 15.174 WE NOW ALSO HAVE  $\alpha = 5 \text{ rad/s}^2$ 

THIS ADDITION CHANGES ONLY THE

ACCELERATIONS OF THE COINCIDING POINT BY ADDING THE TERM  $\alpha \times \vec{r}$ AT POINT A:  $r = 0$  and  $\alpha \times \vec{r} = 0$ AT POINT B:  $\alpha \times \vec{r} = \alpha (\vec{AB}) = (5 \text{ rad/s}^2) \vec{AB}$ 

$$= 0.5 \text{ m/s}^2 \rightarrow + 0.5 \text{ m/s}^2 \downarrow$$

AT POINT C:  $\alpha \times \vec{r} = \alpha (\vec{AC}) = (5 \text{ rad/s}^2) (0.2 \text{ m}) = 1 \text{ m/s}^2 \rightarrow$ WE NOW ADD  $\alpha \times \vec{r}$  TO RESULTS OF PROB. 15.173

$$\text{POINT A: } \vec{a}_A = 0 + 0.621 \text{ m/s}^2 \uparrow \quad \vec{a}_A = 0.621 \text{ m/s}^2 \uparrow$$

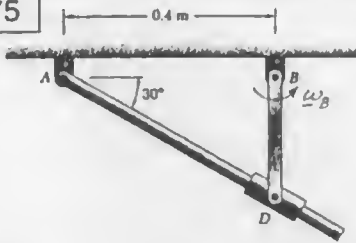
$$\text{POINT B: } \vec{a}_B = 0.5 \text{ m/s}^2 \rightarrow + 0.5 \text{ m/s}^2 \downarrow + 1.521 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow = 1.021 \text{ m/s}^2 \rightarrow + 1.4 \text{ m/s}^2 \downarrow$$

$$\vec{a}_B = 1.733 \text{ m/s}^2 \angle 53.9^\circ$$

$$\text{POINT C: } \vec{a}_C = 1 \text{ m/s}^2 \rightarrow + 2.421 \text{ m/s}^2 \downarrow$$

$$\vec{a}_C = 2.62 \text{ m/s}^2 \angle 67.6^\circ$$

15.175



GIVEN:

$$\omega_B = 6 \text{ rad/s}$$

$$\alpha_B = 0$$

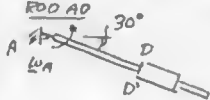
FIND:

$$\omega_A$$

$$\alpha_A$$

GEOMETRY:  $BD = (0.4 \text{ m}) \tan 30^\circ = 0.23094 \text{ m}$   
 $AD = (0.4 \text{ m}) / \cos 30^\circ = 0.4618 \text{ m}$

VELOCITY: ROD BD:  $v_D = (BD)\omega_B = (0.2309 \text{ m})(6 \text{ rad/s}) = 1.3856 \text{ m/s} \rightarrow$



$$v_{D1} = (AD)\omega_A = (0.4618 \text{ m})\omega_A \angle 60^\circ \quad (1)$$

$$v_{D1} = v_D \angle 30^\circ$$

VECTOR DIAGRAM:

$$v_D = 1.3856 \text{ m/s}$$

$$v_{D1} = 1.3856 \sin 30^\circ = 0.6928 \text{ m/s}$$

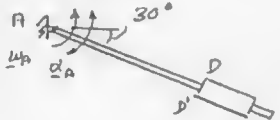
$$E.G.: 0.6928 \text{ m/s} = (0.4618 \text{ m})\omega_A$$

$$\omega_A = 1.5 \text{ rad/s}$$

$$v_{D1} = (1.3856 \text{ m/s}) \cos 30^\circ = 1.2 \text{ m/s} \angle 30^\circ$$

ACCELERATION: ROD BD  $a_D = (BD)\omega_B^2 = (0.23094 \text{ m})(6 \text{ rad/s})^2$   
 $a_D = 8.314 \text{ m/s}^2 \uparrow$

ROD AD:



$$(\omega_A = 1.5 \text{ rad/s})$$

$$v_{D1} = 1.2 \text{ m/s} \angle 30^\circ$$

$$a_{D1} = (AD)\omega_A^2 \angle 30^\circ + (AD)\alpha_A \angle 60^\circ$$

$$= (0.4618 \text{ m})(1.5 \text{ rad/s})^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ$$

$$a_{D1} = 1.0391 \text{ m/s}^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ$$

$$a_{D1} = 30^\circ$$

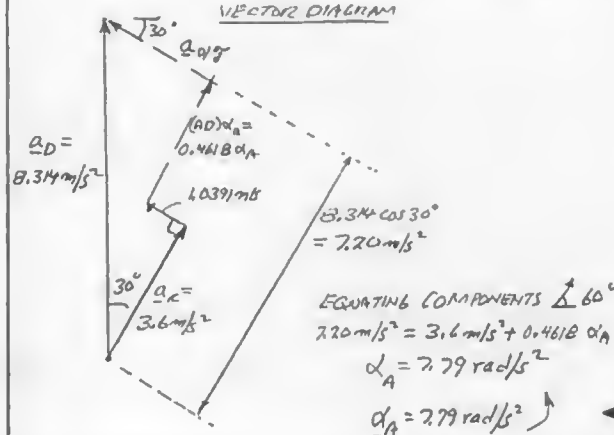
$$a_C = 2\omega_A v_{D1} = 2(1.5 \text{ rad/s})(1.2 \text{ m/s}) = 3.6 \text{ m/s}^2 \angle 60^\circ$$

$$a_D = a_{D1} + a_{D1} + a_C$$

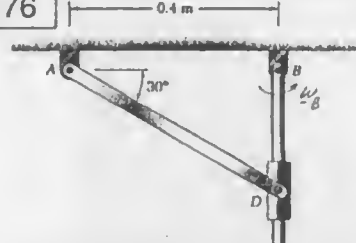
$$[8.314 \text{ m/s}^2 \uparrow] = [1.0391 \text{ m/s}^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ]$$

$$+ [a_{D1} \angle 30^\circ] + [3.6 \text{ m/s}^2 \angle 60^\circ]$$

VECTOR DIAGRAM



15.176



GIVEN:

$$\omega_B = 6 \text{ rad/s}$$

$$\alpha_B = 0$$

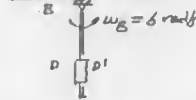
FIND:

$$\omega_A$$

$$\alpha_A$$

GEOMETRY:  $BD = (0.4 \text{ m}) \tan 30^\circ = 0.23094 \text{ m}$   
 $AD = (0.4 \text{ m}) / \cos 30^\circ = 0.4618 \text{ m}$

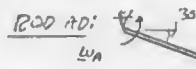
VELOCITY ROD BD:



$$v_{D1} = (BD)\omega_B = (0.2309 \text{ m})(6 \text{ rad/s})$$

$$v_{D1} = 1.3856 \text{ m/s} \rightarrow$$

$$v_{D1} = \downarrow$$



$$v_D = (AD)\omega_A = (0.4618 \text{ m})\omega_A \angle 60^\circ \quad (1)$$

VECTOR DIAGRAM:

$$v_D = v_{D1} + v_{D1}$$

$$[v_D \angle 60^\circ] = [1.3856 \text{ m/s} \rightarrow] + [v_{D1} \downarrow]$$

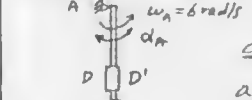
$$v_D = (1.3856) / \cos 60^\circ = 2.7712 \text{ m/s}$$

$$E.G.: 2.7712 \text{ m/s} = (0.4618 \text{ m})\omega_A$$

$$\omega_A = 6 \text{ rad/s}$$

$$v_{D1} = (1.3856 \text{ m/s}) \tan 60^\circ = 2.4 \text{ m/s} \uparrow$$

ACCELERATION: ROD BD

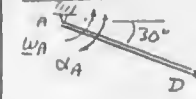


$$a_{D1} = (BD)\omega_B^2 \uparrow$$

$$a_{D1} = (0.23094 \text{ m})(6 \text{ rad/s})^2 = 8.314 \text{ m/s}^2 \uparrow$$

$$a_{D1} = \downarrow$$

ROD AD:



$$a_D = (AD)\omega_A^2 \angle 30^\circ + (AD)\alpha_A \angle 60^\circ$$

$$= (0.4618 \text{ m})(6 \text{ rad/s})^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ$$

$$a_D = 16.625 \text{ m/s}^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ$$

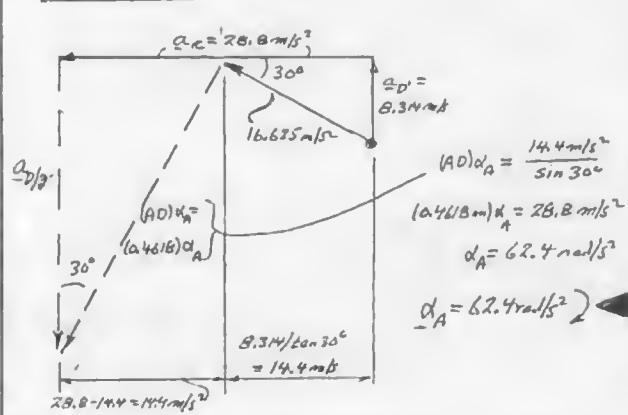
$$a_C = 2\omega_A v_{D1} = 2(6 \text{ rad/s})(2.4 \text{ m/s}) = 28.8 \text{ m/s}^2 \leftarrow$$

$$a_D = a_{D1} + a_{D1} + a_C$$

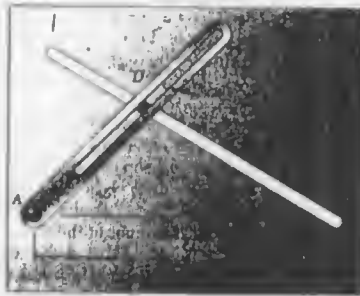
$$[16.625 \text{ m/s}^2 \angle 30^\circ + (0.4618 \text{ m})\alpha_A \angle 60^\circ] = [8.314 \text{ m/s}^2 \uparrow]$$

$$+ [a_{D1} \downarrow] + [28.8 \text{ m/s}^2 \leftarrow]$$

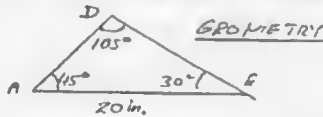
VECTOR DIAGRAM



15.177



GIVEN:  
 $\omega_A = 3 \text{ rad/s}$   
 $\alpha_A = 5 \text{ rad/s}^2$   
FIND:  
 $\underline{a}_D$

GEOMETRYLAW OF SINES

$$\frac{AD}{\sin 30^\circ} = \frac{20 \text{ in}}{\sin 105^\circ}$$

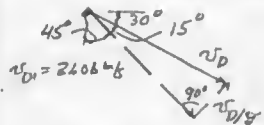
$$AD = 10.353 \text{ in.}$$

VELOCITY:

$$\underline{v}_D = (AD)\omega_A = (10.353 \text{ in.})(3 \text{ rad/s}) = 31.06 \text{ in/s} \angle 45^\circ$$

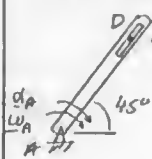
$$\underline{v}_D = \underline{v}_{D1} + \underline{v}_{D2}$$

$$[v_D \angle 30^\circ] = [31.06 \text{ in/s} \angle 45^\circ] + [v_{D2} \angle 45^\circ]$$

VECTOR DIAGRAM

$$v_{D2} = (31.06 \text{ in/s}) \tan 15^\circ$$

$$v_{D2} = 8.322 \text{ in/s} \angle 45^\circ$$

ACCELERATION

$$\underline{a}_D = (AD)\omega_A^2 \angle 45^\circ + (AD)\alpha_A \angle 45^\circ$$

$$= (10.353 \text{ in.})(3 \text{ rad/s})^2 \angle 45^\circ$$

$$+ (10.353 \text{ in.})(5 \text{ rad/s}^2) \angle 45^\circ$$

$$\underline{a}_D = 93.177 \text{ in/s}^2 \angle 45^\circ + 51.765 \text{ in/s}^2 \angle 45^\circ$$

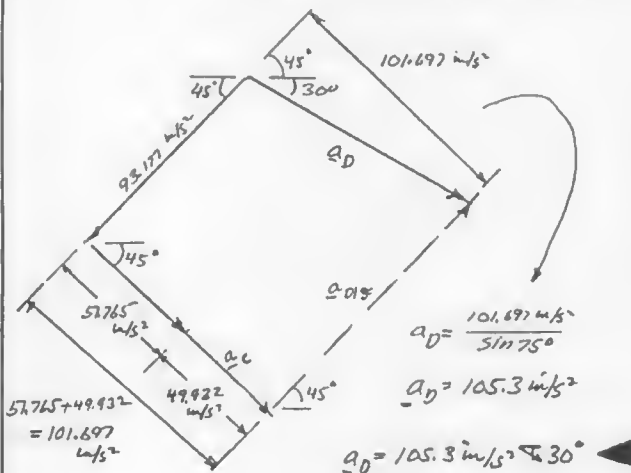
$$\underline{a}_c = 2\omega_A v_{D2} = 2(3 \text{ rad/s})(8.322 \text{ in/s})$$

$$\underline{a}_c = 49.932 \text{ in/s}^2 \angle 45^\circ$$

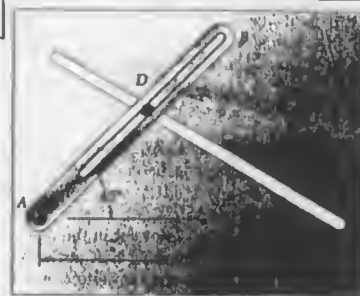
$$\underline{a}_D = \underline{a}_{D1} + \underline{a}_{D2} + \underline{a}_c$$

$$[a_D \angle 30^\circ] = [93.177 \text{ in/s}^2 \angle 45^\circ + 51.765 \text{ in/s}^2 \angle 45^\circ]$$

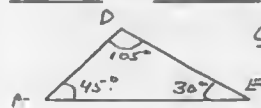
$$+ [49.932 \text{ in/s}^2 \angle 45^\circ]$$



15.178



GIVEN:  
 $\omega_A = 3 \text{ rad/s}$   
 $\alpha_A = 5 \text{ rad/s}^2$   
FIND:  
 $\underline{a}_D$

GEOMETRYLAW OF SINES

$$\frac{AD}{\sin 30^\circ} = \frac{20 \text{ in}}{\sin 105^\circ}$$

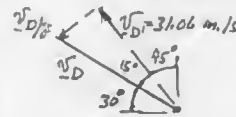
$$AD = 10.353 \text{ in}$$

VELOCITY:

$$\underline{v}_D = (AD)\omega_A = (10.353 \text{ in.})(3 \text{ rad/s}) = 31.06 \text{ in/s} \angle 45^\circ$$

$$\underline{v}_D = \underline{v}_{D1} + \underline{v}_{D2}$$

$$[v_D \angle 30^\circ] = [31.06 \text{ in/s} \angle 45^\circ] + [v_{D2} \angle 45^\circ]$$

VECTOR DIAGRAM

$$v_D = (31.06 \text{ in/s}) \tan 15^\circ$$

$$v_{D2} = 8.322 \text{ in/s} \angle 45^\circ$$

ACCELERATION:

$$\underline{a}_D = (AD)\omega_A^2 \angle 45^\circ + (AD)\alpha_A \angle 45^\circ$$

$$= (10.353 \text{ in.})(3 \text{ rad/s})^2 \angle 45^\circ$$

$$+ (10.353 \text{ in.})(5 \text{ rad/s}^2) \angle 45^\circ$$

$$\underline{a}_D = 93.177 \text{ in/s}^2 \angle 45^\circ + 51.765 \text{ in/s}^2 \angle 45^\circ$$

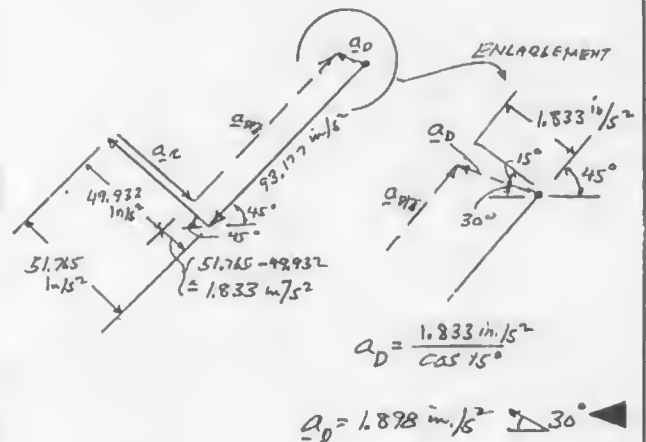
$$\underline{a}_c = 2\omega_A v_{D2} = 2(3 \text{ rad/s})(8.322 \text{ in/s})$$

$$\underline{a}_c = 49.932 \text{ in/s}^2 \angle 45^\circ$$

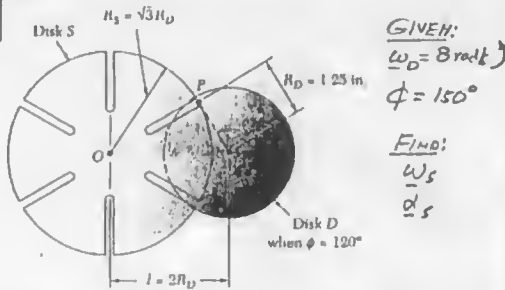
$$\underline{a}_D = \underline{a}_{D1} + \underline{a}_{D2} + \underline{a}_c$$

$$[a_D \angle 30^\circ] = [93.177 \text{ in/s}^2 \angle 45^\circ + 51.765 \text{ in/s}^2 \angle 45^\circ]$$

$$+ [49.932 \text{ in/s}^2 \angle 45^\circ]$$

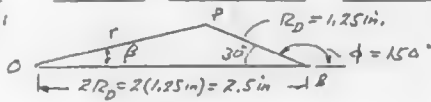


15.179



GIVEN:  
 $\omega_D = 8 \text{ rad/s}$   
 $\phi = 150^\circ$   
 FIND:  
 $\omega_S$   
 $\alpha_S$

GEOMETRY:



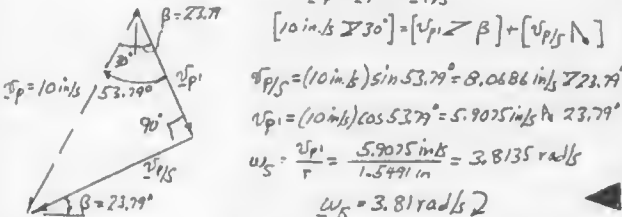
$$r^2 = (1.25)^2 + (2.5)^2 - 2(1.25)(2.5)\cos 30^\circ$$

$$r = 1.5491 \text{ in}$$

$$\frac{\sin \beta}{1.25 \text{ in}} = \frac{\sin 30^\circ}{1.5491 \text{ in}}$$

$$\beta = 23.79^\circ$$

VELOCITY:  $v_P = (8 \text{ rad/s}) R_D \angle 30^\circ = (1.25 \text{ in})(8 \text{ rad/s}) = 10 \text{ in/s} \angle 30^\circ$   
 $v_{P/S} = v_{P/S} \angle \beta$   
 $v_P = v_{P/S} \angle \beta$ , WHERE P IS COINCIDING POINT ON S  
 $v_P = v_{P/S} + v_{P/S}$

ACCELERATION:  $a_P = (8 \text{ rad/s}) R_D \angle 30^\circ = (1.25 \text{ in})(8 \text{ rad/s}) = 80 \text{ in/s}^2 \angle 30^\circ$ 

$$a_P = r \omega_S \angle \beta + r \alpha_S \angle \beta$$

$$= (1.5491 \text{ in})(3.8135 \text{ rad/s}) \angle \beta + (1.5491 \text{ in}) \alpha_S \angle \beta$$

$$a_P = 22.528 \text{ in/s}^2 \angle 23.79^\circ + (1.5491 \text{ in}) \alpha_S \angle 23.79^\circ$$

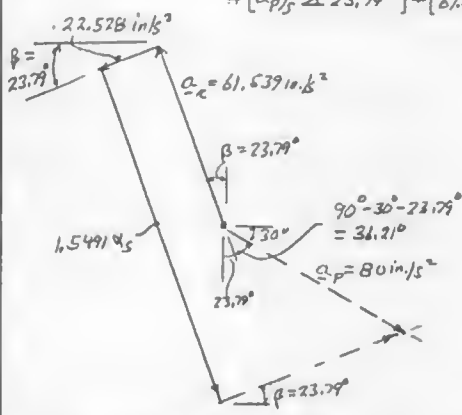
$$a_{P/S} = a_{P/S} \angle 23.79^\circ$$

$$a_C = 2 \omega_S v_{P/S} = 2(3.8135 \text{ rad/s})(8.086 \text{ in/s})$$

$$= 61.539 \text{ in/s}^2 \angle 23.79^\circ$$

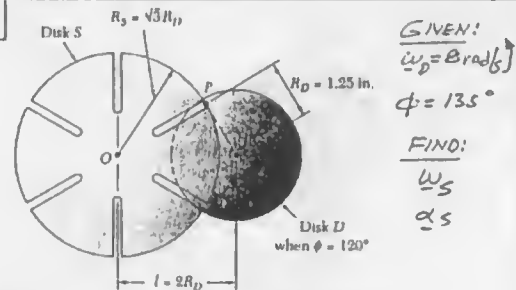
$$a_P = a_{P/S} + a_{P/S} + a_C$$

$$[80 \text{ in/s}^2 \angle 30^\circ] = [22.528 \text{ in/s}^2 \angle 23.79^\circ] + [(1.5491 \text{ in}) \alpha_S \angle 23.79^\circ] + [61.539 \text{ in/s}^2 \angle 23.79^\circ]$$



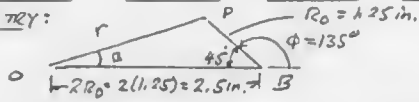
SUM COMPONENTS  $A_\beta$  (THAT IS, SUM  $\perp$  TO SLOT)  
 $+A_\beta: (80 \text{ in/s}^2) \cos 36.21^\circ = (1.5491 \text{ in}) \alpha_S - (61.539 \text{ in/s}^2)$   
 $\alpha_S = 81.39 \text{ rad/s}^2$   $\alpha_S = 81.4 \text{ rad/s}^2$

15.180



GIVEN:  
 $\omega_D = 8 \text{ rad/s}$   
 $\phi = 135^\circ$   
 FIND:  
 $\omega_S$   
 $\alpha_S$

GEOMETRY:



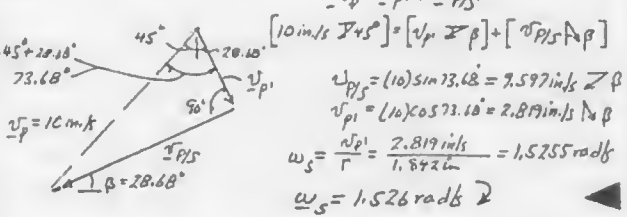
$$r^2 = (1.25)^2 + (2.5)^2 - 2(1.25)(2.5)\cos 45^\circ$$

$$r = 1.842 \text{ in}$$

$$\frac{\sin \beta}{1.25 \text{ in}} = \frac{\sin 45^\circ}{1.842 \text{ in}}$$

$$\beta = 28.68^\circ$$

VELOCITY:  $v_P = (8 \text{ rad/s}) R_D \angle 45^\circ = (1.25 \text{ in})(8 \text{ rad/s}) = 10 \text{ in/s} \angle 45^\circ$   
 $v_{P/S} = v_{P/S} \angle \beta$   
 $v_P = v_{P/S} \angle \beta$ , WHERE P IS COINCIDING POINT ON S  
 $v_P = v_{P/S} + v_{P/S}$

ACCELERATION:  $a_P = (8 \text{ rad/s}) R_D \angle 45^\circ = (1.25 \text{ in})(8 \text{ rad/s}) = 80 \text{ in/s}^2 \angle 45^\circ$ 

$$a_P = r \omega_S \angle \beta + r \alpha_S \angle \beta$$

$$= (1.842 \text{ in})(1.526 \text{ rad/s}) \angle \beta + (1.842 \text{ in}) \alpha_S \angle \beta$$

$$a_P = 4.289 \text{ in/s}^2 \angle 28.68^\circ + (1.842 \text{ in}) \alpha_S \angle \beta$$

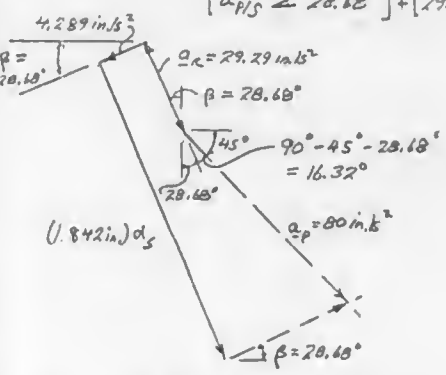
$$a_{P/S} = a_{P/S} \angle 28.68^\circ$$

$$a_C = 2 \omega_S v_{P/S} = 2(1.526 \text{ rad/s})(7.597 \text{ in/s})$$

$$= 29.29 \text{ in/s}^2 \angle 28.68^\circ$$

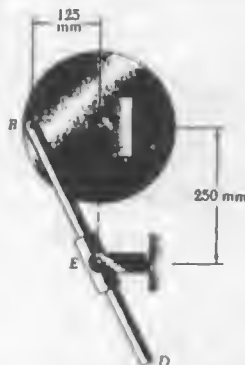
$$a_P = a_{P/S} + a_{P/S} + a_C$$

$$[80 \text{ in/s}^2 \angle 45^\circ] = [4.289 \text{ in/s}^2 \angle 28.68^\circ] + [(1.842 \text{ in}) \alpha_S \angle 28.68^\circ] + [29.29 \text{ in/s}^2 \angle 28.68^\circ]$$



SUM COMPONENTS  $A_\beta$  (THAT IS, SUM  $\perp$  TO SLOT)  
 $+A_\beta: (80 \text{ in/s}^2) \cos 16.32^\circ = (1.842 \text{ in}) \alpha_S - (29.29 \text{ in/s}^2)$   
 $\alpha_S = 57.58 \text{ rad/s}^2$   $\alpha_S = 57.6 \text{ rad/s}^2$

15.181



GIVEN:

$$\omega_A = 12 \text{ rad/s}$$

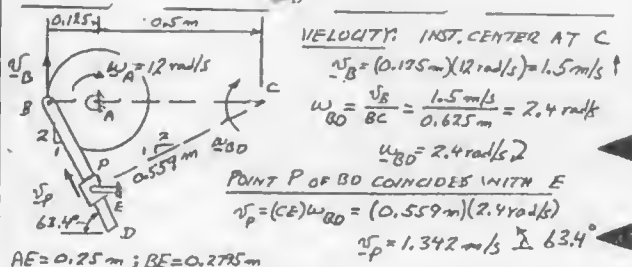
$$\alpha_A = 0$$

FIND:

(a)  $\omega_{BD}$  AND  $\alpha_{BD}$ 

(b) VELOCITY AND

ACCELERATION OF POINT OF BD THAT COINCIDES WITH E



$$AE = 0.25 \text{ m}; BE = 0.275 \text{ m}$$

$$\text{ACCELERATION: } a_B = (R)\omega_A^2 = (0.125 \text{ m})(12 \text{ rad/s})^2 = 18 \text{ m/s}^2 \rightarrow$$

$$a_P = a_B + a_{P/B}$$

$$= [18 \text{ m/s}^2] + [(0.275 \text{ m})\alpha_{BD} \angle 63.4^\circ + (0.275 \text{ m})\omega_{BD}^2 \angle 26.6^\circ]$$

$$= [18 \text{ m/s}^2] + [(0.275)(\alpha_{BD}) \angle 63.4^\circ + (0.275)(\omega_{BD}^2) \angle 26.6^\circ]$$

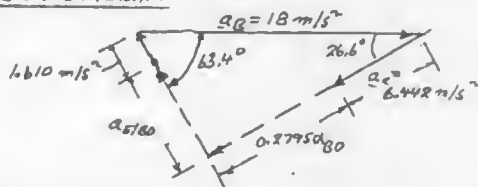
$$= [18 \text{ m/s}^2] + [1.610 \text{ m/s}^2 \angle 63.4^\circ + 0.275 \omega_{BD}^2 \angle 26.6^\circ]$$

$$a_{E/BD} = a_{E/BD} \angle 63.4^\circ = a_{E/BD} \angle 26.6^\circ$$

$$a_C = 2a_{E/BD} \angle 63.4^\circ \quad \text{NOTE: } v_{E/BD} = -v_P = 1.342 \text{ m/s} \angle 63.4^\circ$$

$$a_C = 2(2.4 \text{ rad/s})(1.342 \text{ m/s}) = 6.442 \text{ m/s}^2 \angle 63.4^\circ$$

VECTOR DIAGRAM

SUM COMPONENTS PARALLEL TO  $a_C$ 

$$+ \angle 26.6^\circ: (18 \text{ m/s}^2) \cos 26.6^\circ - 0.275 \alpha_{BD} - 6.442 \text{ m/s}^2 = 0$$

$$\alpha_{BD} = 34.54 \text{ rad/s}^2 \quad \alpha_{BD} = 34.5 \text{ rad/s}^2$$

SUM COMPONENTS  $\perp$  TO  $a_C$ 

$$+ \angle 26.6^\circ: a_{E/BD} + 1.610 \text{ m/s}^2 - (18 \text{ m/s}^2) \sin 26.6^\circ = 0$$

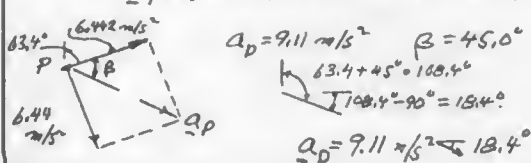
$$a_{E/BD} = 6.44 \text{ m/s}^2 \angle 63.4^\circ$$

SINCE POINT E IS FIXED, WE NOTE THAT

$$a_P = a_{P/E} = -a_{E/P}$$

$$= -[a_{E/BD} \angle 63.4^\circ + a_C \angle 63.4^\circ]$$

$$a_P = [6.45 \text{ m/s}^2 \angle 63.4^\circ + 6.442 \text{ m/s}^2 \angle 63.4^\circ]$$



$$a_P = 9.11 \text{ m/s}^2 \quad \beta = 45.0^\circ$$

$$63.4^\circ + 45^\circ = 108.4^\circ$$

$$108.4^\circ - 90^\circ = 18.4^\circ$$

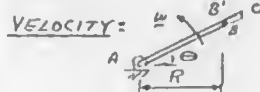
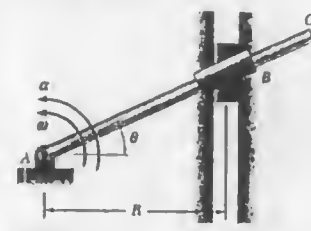
$$a_P = 9.11 \text{ m/s}^2 \angle 18.4^\circ$$

15.182 and 15.183

PROBLEM 15.182

DERIVE EXPRESSIONS FOR  $v_B$  AND  $a_B$ 

PROBLEM 15.183

GIVEN:  $R = 15 \text{ in.}$ ,  $\theta = 25^\circ$ ,  $\omega = 3 \text{ rad/s}$ ,  $\alpha = 8 \text{ rad/s}^2$ FIND:  $v_B$  AND  $a_B$ 

$$AB = R / \cos \theta$$

$$v_B = (AB)\omega = R\omega / \cos \theta \quad \angle \theta$$

$$v_B = v_B + v_{B/P}$$

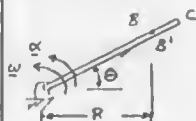
$$[v_B \angle \theta] = [R\omega / \cos \theta \angle \theta] + [v_{B/P} \angle \theta]$$

VECTOR DIAGRAM:

$$v_B = \frac{v_{B'}}{\cos \theta} = \frac{R\omega}{\cos^2 \theta} \quad \uparrow$$

$$v_{B/P} = v_B \tan \theta = \frac{R\omega}{\cos \theta} \tan \theta \quad \angle \theta$$

ACCELERATION:



$$a_B = (AB)\omega^2 \angle \theta + (AB)\alpha \angle \theta$$

$$= \frac{R\omega^2}{\cos \theta} \angle \theta + \frac{R\alpha}{\cos \theta} \angle \theta$$

$$a_{B/P} = a_B \tan \theta \quad \angle \theta$$

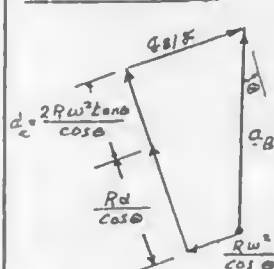
$$a_C = 2\omega v_{B/P} = 2\omega \frac{R\omega}{\cos \theta} \tan \theta$$

$$a_C = \frac{2R\omega^2}{\cos \theta} \tan \theta \quad \angle \theta$$

$$a_B = a_{B'} + a_{B/P} + a_C$$

$$[a_B \angle \theta] = [\frac{R\omega^2}{\cos \theta} \angle \theta + \frac{R\alpha}{\cos \theta} \angle \theta] + [a_{B/P} \angle \theta] + [\frac{2R\omega^2 \tan \theta}{\cos \theta} \angle \theta]$$

VECTOR DIAGRAM:

SUM COMPONENTS PARALLEL TO  $a_C$  $\angle \theta$ :

$$a_B \cos \theta - \frac{2R\omega^2 \tan \theta}{\cos \theta} - \frac{R\alpha}{\cos \theta} = 0$$

$$a_B = \frac{2R\omega^2 \tan \theta}{\cos^2 \theta} + \frac{R\alpha}{\cos^2 \theta}$$

$$a_B = \frac{R}{\cos^2 \theta} (\alpha + 2\omega^2 \tan \theta) \quad \uparrow$$

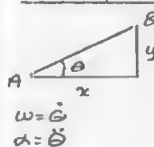
PROBLEM 15.183  $R = 15 \text{ in.}$ ,  $\theta = 25^\circ$ ,  $\omega = 3 \text{ rad/s}$ ,  $\alpha = 8 \text{ rad/s}^2$ 

$$v_B = \frac{R\omega}{\cos^2 \theta} = \frac{(15 \text{ in.})(3 \text{ rad/s})}{\cos^2 25^\circ} = 54.78 \text{ in/s}; \quad v_B = 54.8 \text{ in/s} \quad \uparrow$$

$$a_B = \frac{R}{\cos^2 \theta} (\alpha + 2\omega^2 \tan \theta) = \frac{15 \text{ in.}}{\cos^2 25^\circ} (8 \text{ rad/s}^2 + 2(3 \text{ rad/s})^2 \tan 25^\circ)$$

$$a_B = 18.262(8 + 6.333) = 299 \text{ in/s}^2; \quad a_B = 299 \text{ in/s}^2 \quad \uparrow$$

ALTERNATIVE DERIVATION USING A PARAMETER, SOL 15.9



$$y = R \tan \theta$$

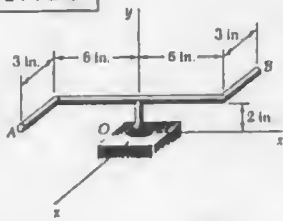
$$v_B = \dot{y} = (R / \cos^2 \theta) \dot{\theta} = \frac{R\omega}{\cos^2 \theta}$$

$$a_B = \ddot{y} = \frac{R}{\cos^2 \theta} \ddot{\theta} + (2R \sin \theta / \cos^3 \theta) \dot{\theta}^2$$

$$a_B = \frac{R}{\cos^2 \theta} (\alpha + 2\omega^2 \tan \theta)$$



15.184



GIVEN:  $\omega_y = 30 \text{ rad/s}$   
 $\underline{v}_A = (100 \text{ in./s})\underline{i} + (6 \text{ in./s})\underline{j}$   
 $+ (1 \text{ in./s})\underline{k}$

FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{v}_B$

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \quad \underline{r}_A = -(6 \text{ in.})\underline{i} + (3 \text{ in.})\underline{j} + (2 \text{ in.})\underline{k}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 30 & \omega_z \\ -6 & 2 & 3 \end{vmatrix}$$

$$\underline{v}_A = (90 - 2\omega_z)\underline{i} + (-3\omega_x - 6\omega_z)\underline{j} + (2\omega_x + 180)\underline{k}$$

$$(v_A)_x = 90 - 2\omega_z = 100 \quad \omega_z = -5 \text{ rad/s}$$

$$(v_A)_y = -3\omega_x - 6(-5) = 6$$

$$-3\omega_x + 30 = 6 \quad \omega_x = 8 \text{ rad/s}$$

(a)  $\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$

$$\underline{\omega} = (8 \text{ rad/s})\underline{i} + (30 \text{ rad/s})\underline{j} - (5 \text{ rad/s})\underline{k}$$

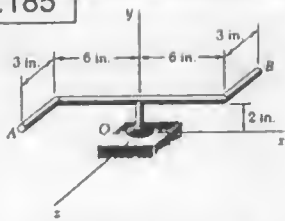
(b)  $\underline{r}_B = (6 \text{ in.})\underline{i} + (2 \text{ in.})\underline{j} - (3 \text{ in.})\underline{k}$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 8 & 30 & -5 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= (-90 + 10)\underline{i} + (-30 + 24)\underline{j} + (16 - 180)\underline{k}$$

$$\underline{v}_B = (-80 \text{ in./s})\underline{i} - (6 \text{ in./s})\underline{j} - (164 \text{ in./s})\underline{k}$$

15.185



GIVEN:  $\omega_y = 40 \text{ rad/s}$   
 $\underline{v}_A = (100 \text{ in./s})\underline{i}$   
 $+ (6 \text{ in./s})\underline{j} + (1 \text{ in./s})\underline{k}$

FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{v}_B$

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \quad \underline{r}_A = -(6 \text{ in.})\underline{i} + (2 \text{ in.})\underline{j} + (3 \text{ in.})\underline{k}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 40 & \omega_z \\ -6 & 2 & 3 \end{vmatrix}$$

$$\underline{v}_A = (120 - 2\omega_z)\underline{i} + (-3\omega_x - 6\omega_z)\underline{j} + (2\omega_x + 240)\underline{k}$$

$$(v_A)_x = 120 - 2\omega_z = 100 \quad \omega_z = 10$$

$$(v_A)_y = -3\omega_x - 6(10) = 6$$

$$-3\omega_x - 60 = 6 \quad \omega_x = -22$$

(a)  $\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$

$$\underline{\omega} = (-22 \text{ rad/s})\underline{i} + (40 \text{ rad/s})\underline{j} + (10 \text{ rad/s})\underline{k}$$

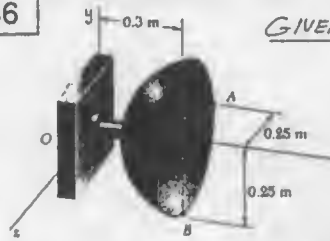
(b)  $\underline{r}_B = (6 \text{ in.})\underline{i} + (2 \text{ in.})\underline{j} - (3 \text{ in.})\underline{k}$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -22 & 40 & 10 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= (-120 - 20)\underline{i} + (60 - 66)\underline{j} + (-44 - 240)\underline{k}$$

$$\underline{v}_B = (-140 \text{ in./s})\underline{i} - (6 \text{ in./s})\underline{j} - (284 \text{ in./s})\underline{k}$$

15.186



GIVEN:  $(v_A)_y = 300 \text{ mm/s}$   
 $(v_B)_y = 180 \text{ mm/s}$   
 $(v_B)_z = 360 \text{ mm/s}$

FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{v}_A$

$$\underline{r}_A = (0.3 \text{ m})\underline{i} - (0.25 \text{ m})\underline{k}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$(v_A)_x = -0.25\omega_y \quad (1)$$

$$(v_A)_y = 0.3\omega_z + 0.25\omega_x \quad (2)$$

$$(v_A)_z = -0.3\omega_y \quad (3)$$

$$\underline{r}_B = (0.3 \text{ m})\underline{i} - (0.25 \text{ m})\underline{j}$$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x = 0.25\omega_z \quad (4)$$

$$(v_B)_y = 0.3\omega_z \quad (5)$$

$$(v_B)_z = -0.25\omega_x - 0.3\omega_y \quad (6)$$

Eq 5:  $(v_B)_y = 0.18 \text{ m/s} = (0.3 \text{ m})\omega_z$  ;  $\omega_z = 0.6 \text{ rad/s}$

Eq 2:  $(v_A)_y = 0.3 \text{ m/s} = (0.3)(0.6) + 0.25\omega_x$  ;  $\omega_x = 0.48 \text{ rad/s}$

Eq 6:  $(v_B)_z = 0.36 \text{ m/s} = (-0.25)(0.48) - 0.3\omega_y$  ;  $\omega_y = -1.6 \text{ rad/s}$

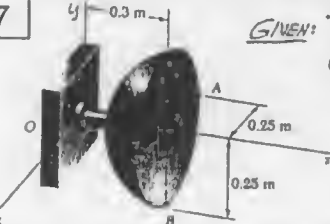
(a)  $\underline{\omega} = (0.48 \text{ rad/s})\underline{i} - (1.6 \text{ rad/s})\underline{j} + (0.6 \text{ rad/s})\underline{k}$

(b)  $\underline{v}_A = (v_A)_x \underline{i} + (v_A)_y \underline{j} + (v_A)_z \underline{k} = -0.25(1.6)\underline{j} + 0.3(-1.6)\underline{k}$

$$\underline{v}_A = (0.4 \text{ m/s})\underline{i} + (0.3 \text{ m/s})\underline{j} + (0.48 \text{ m/s})\underline{k}$$

$$\underline{v}_A = (400 \text{ mm/s})\underline{i} + (300 \text{ mm/s})\underline{j} + (480 \text{ mm/s})\underline{k}$$

15.187



GIVEN:  $(v_A)_x = 100 \text{ mm/s}$   
 $(v_A)_y = -90 \text{ mm/s}$   
 $(v_B)_z = 120 \text{ mm/s}$

FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{v}_A$

$$\underline{r}_A = (0.3 \text{ m})\underline{i} - (0.25 \text{ m})\underline{k}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$(v_A)_x = -0.25\omega_y \quad (1)$$

$$(v_A)_y = 0.3\omega_z + 0.25\omega_x \quad (2)$$

$$(v_A)_z = -0.3\omega_y \quad (3)$$

$$\underline{r}_B = (0.3 \text{ m})\underline{i} - (0.25 \text{ m})\underline{j}$$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x = 0.25\omega_z \quad (4)$$

$$(v_B)_y = 0.3\omega_z \quad (5)$$

$$(v_B)_z = -0.25\omega_x - 0.3\omega_y \quad (6)$$

Eq (1):  $(v_A)_x = 0.1 \text{ m/s} = -0.25\omega_y$  ;  $\omega_y = -0.4 \text{ rad/s}$

Eq (6):  $(v_B)_z = 0.12 \text{ m/s} = -0.25(\omega_x) - 0.3(-0.4)$  ;  $\omega_x = 0$

Eq (2):  $(v_A)_y = -0.09 \text{ m/s} = 0.3\omega_z + 0.25(0)$  ;  $\omega_z = -0.3 \text{ rad/s}$

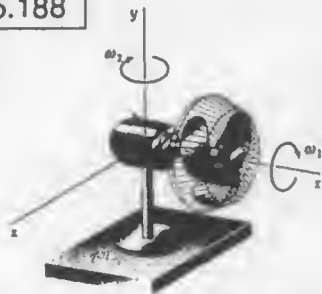
(a)  $\underline{\omega} = (-0.4 \text{ rad/s})\underline{j} - (0.3 \text{ rad/s})\underline{k}$

(b)  $\underline{v}_A = (v_A)_x \underline{i} + (v_A)_y \underline{j} + (v_A)_z \underline{k} = 0.1 \underline{i} - 0.09 \underline{j} + 0.3(-0.3) \underline{k}$

$$\underline{v}_A = (0.1 \text{ m/s})\underline{i} - (0.09 \text{ m/s})\underline{j} + (0.09 \text{ m/s})\underline{k}$$

$$\underline{v}_A = (100 \text{ mm/s})\underline{i} - (90 \text{ mm/s})\underline{j} + (120 \text{ mm/s})\underline{k}$$

15.188



GIVEN:

$$\omega_1 = (360 \text{ rpm}) \hat{i}$$

$$\alpha_1 = 0$$

$$\omega_2 = -(2.5 \text{ rpm}) \hat{j}$$

$$\alpha_2 = 0$$

FIND: FOR HOUSING  
OF MOTOR

(a)  $\omega_H$

(b)  $\alpha_H$

$$\omega_1 = -(360 \text{ rpm}) \hat{i} = -(12\pi \text{ rad/s}) \hat{i}$$

$$\omega_2 = -(2.5 \text{ rpm}) \hat{j} = -(\pi/12 \text{ rad/s}) \hat{j}$$

 $\omega_2$  = ROTATION OF FRAME  $Oxyz$ 

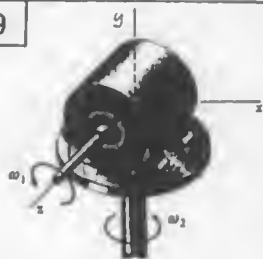
$$\alpha = (\dot{\omega}_1 + \dot{\omega}_2) = (\dot{\omega}_1 + \dot{\omega}_2)_{Oxyz} + \omega_2 \times (\omega_1 + \omega_2)$$

$$\alpha = \omega_2 \times \omega_1 = (-\pi/12 \text{ rad/s}) \hat{j} \times (-12\pi \text{ rad/s}) \hat{i}$$

$$\alpha = -(9.8696 \text{ rad/s}^2) \hat{k}$$

$$\alpha = -(9.87 \text{ rad/s}^2) \hat{k}$$

15.189

GIVEN:  $\omega_1 = 1800 \text{ rpm}$ 

$$\alpha_1 = 0$$

$$\omega_2 = 6 \text{ rpm}$$

$$\alpha_2 = 0$$

FIND: FOR ROTOR  
OF MOTOR,  $\alpha$ 

$$\omega_1 = (1800 \text{ rpm}) \hat{k} = (60\pi \text{ rad/s}) \hat{k}$$

$$\omega_2 = (6 \text{ rpm}) \hat{j} = (\pi/5 \text{ rad/s}) \hat{j}$$

 $\omega_2$  = ROTATION OF FRAME  $Oxyz$ 

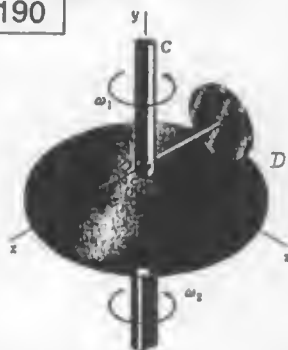
$$\alpha = (\dot{\omega}_1 + \dot{\omega}_2) = (\dot{\omega}_1 + \dot{\omega}_2)_{Oxyz} + \omega_2 \times (\omega_1 + \omega_2)$$

$$\alpha = \omega_2 \times \omega_1 = (\pi/5 \text{ rad/s}) \hat{j} \times (60\pi \text{ rad/s}) \hat{k}$$

$$\alpha = (118.4 \text{ rad/s}^2) \hat{i}$$

$$\alpha = (118.4 \text{ rad/s}^2) \hat{i}$$

15.190

GIVEN:  $\omega_1 = \omega_1 \hat{k}$ 

$$\alpha_1 = 0$$

$$\omega_2 = \alpha_2 = 0$$

FIND: FOR DISK

(a)  $\vec{v}_A$

(b)  $\alpha_A$

DISK A: (IN ROTATION ABOUT O)

SINCE  $\omega_y = \omega_1$   $\omega_A = \omega_x \hat{i} + \omega_1 \hat{j} + \omega_z \hat{k}$

POINT D IS POINT OF CONTACT OF WHEEL &amp; DISK

$$\vec{r}_{DB} = -r \hat{j} - R \hat{k}$$

$$\vec{v}_D = \omega_A \times \vec{r}_{DB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$$\vec{v}_D = (-R\omega_1 + r\omega_z) \hat{i} + R\omega_1 \hat{j} - r\omega_x \hat{k}$$

(CONTINUED)

15.190 CONTINUED

SINCE  $\omega_z = 0$ ,  $\vec{v}_D = 0$ EACH COMPONENT OF  $\vec{v}_D$  IS ZERO

$$(\vec{v}_D)_x = r\omega_z = 0$$

$$\omega_x = 0$$

$$(\vec{v}_D)_y = -R\omega_1 + r\omega_z = 0$$

$$\omega_z = (R/r)\omega_1$$

(a)

$$\omega_A = \omega_1 \hat{j} + (R/r)\omega_1 \hat{k}$$

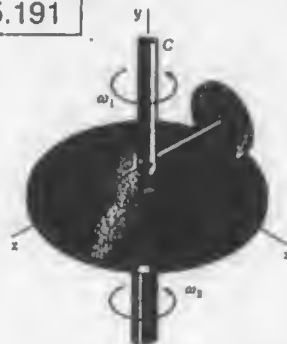
(b)

DISK A: ROTATES ABOUT y AXIS AT RATE  $\omega_1$ 

$$\alpha_A = \frac{d\omega_A}{dt} = \omega_y \times \omega_A = \omega_1 \hat{j} \times (\omega_1 \hat{j} + \frac{R}{r}\omega_1 \hat{k})$$

$$\alpha_A = \frac{R}{r}\omega_1^2 \hat{i}$$

15.191

GIVEN:  $\omega_1 = \omega_1 \hat{k}$ 

$$\alpha_1 = 0$$

$$\omega_2 = \omega_2 \hat{k}$$

$$\alpha_2 = 0$$

FIND: FOR DISK

(a)  $\vec{v}_A$

(b)  $\alpha_A$

DISK A: (IN ROTATION ABOUT O)

SINCE  $\omega_y = \omega_1$

$$\omega_A = \omega_x \hat{i} + \omega_1 \hat{j} + \omega_z \hat{k}$$

POINT D IS POINT OF CONTACT OF WHEEL AND DISK

$$\vec{r}_{DB} = -r \hat{j} - R \hat{k}$$

$$\vec{v}_D = \omega_A \times \vec{r}_{DB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$$\vec{v}_D = (-R\omega_1 + r\omega_z) \hat{i} + R\omega_1 \hat{j} - r\omega_x \hat{k} \quad (1)$$

DISK B:  $\omega_B = \omega_2 \hat{k}$ 

$$\vec{v}_D = \omega_B \times \vec{r}_{DB} = \omega_2 \hat{k} \times (-r \hat{j} - R \hat{k}) = -R\omega_2 \hat{i} \quad (2)$$

FROM EGS 1 AND 2:

$$\vec{v}_D = \vec{v}_D: (-R\omega_1 + r\omega_z) \hat{i} + R\omega_1 \hat{j} - r\omega_x \hat{k} = -R\omega_2 \hat{i}$$

COMP. OF  $\hat{i}$ :  $-R\omega_x = 0$ 

$$\omega_x = 0$$

COMP. OF  $\hat{j}$ :  $(-R\omega_1 + r\omega_z) = -R\omega_2$ 

$$\omega_z = \frac{R}{r}(\omega_1 - \omega_2)$$

(a)

$$\omega_A = \omega_1 \hat{j} + \frac{R}{r}(\omega_1 - \omega_2) \hat{k}$$

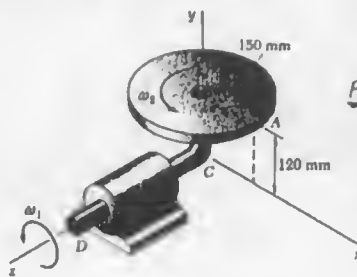
(b)

DISK A ROTATES ABOUT y AXIS AT RATE  $\omega_1$ 

$$\alpha_A = \frac{d\omega_A}{dt} = \omega_y \times \omega_A = \omega_1 \hat{j} \times \left[ \omega_1 \hat{j} + \frac{R}{r}(\omega_1 - \omega_2) \hat{k} \right]$$

$$\alpha_A = \frac{R}{r}\omega_1(\omega_1 - \omega_2) \hat{i}$$

# 15.192 and 15.193



GIVEN:

$$\omega_1 = 5 \text{ rad/s}, \alpha_1 = 0$$

$$\omega_2 = 4 \text{ rad/s}, \alpha_2 = 0$$

PROBLEM 15.192

FIND:  $\alpha_{\text{DISK}} = \alpha$

PROBLEM 15.193

FIND: (a)  $\underline{v}_A$

(b)  $\underline{a}_A$

DISK:  $\underline{\omega} = \omega_2 \underline{j} + \omega_1 \underline{k} = (4 \text{ rad/s}) \underline{j} + (5 \text{ rad/s}) \underline{k}$

PROBLEM 15.192:

DISK ROTATES ABOUT Z AXIS AT RATE  $\omega_1 = \omega_1 \underline{k}$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (5 \text{ rad/s}) \underline{k} \times [(4 \text{ rad/s}) \underline{j} + (5 \text{ rad/s}) \underline{k}]$$

$$\underline{\alpha} = -(20 \text{ rad/s}^2) \underline{i}$$

PROBLEM 15.193:

$$\underline{r}_A = (0.15 \text{ m}) \underline{i} + (0.12 \text{ m}) \underline{j}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 5 \\ 0.15 & 0.12 & 0 \end{vmatrix} = -0.6 \underline{i} + 0.75 \underline{j} - 0.6 \underline{k}$$

$$\underline{v}_A = -(0.6 \text{ m/s}) \underline{i} + (0.75 \text{ m/s}) \underline{j} - (0.6 \text{ m/s}) \underline{k}$$

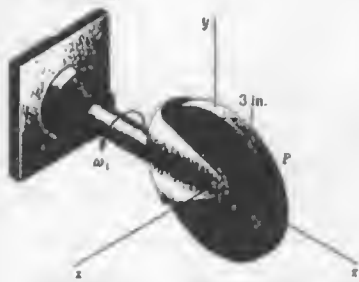
$$\underline{a}_A = \underline{\alpha} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A$$

$$= -20 \underline{i} \times (0.15 \underline{i} + 0.12 \underline{j}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 5 \\ -0.6 & 0.75 & -0.6 \end{vmatrix}$$

$$\underline{a}_A = -2.4 \underline{k} + (-2.4 \underline{i} - 3.75 \underline{j} - 3 \underline{j} + 2.4 \underline{k})$$

$$\underline{a}_A = -(6.15 \text{ m/s}^2) \underline{i} - (3 \text{ m/s}^2) \underline{j}$$

# 15.194



GIVEN:

$$\omega_1 = 5 \text{ rad/s}, \alpha_1 = 0$$

$$\omega_2 = 4 \text{ rad/s}, \alpha_2 = 0$$

FIND:

(a)  $\alpha_{\text{DISK}} = \alpha$

(b)  $\underline{a}_P$  WHEN  $\theta = 0$

(c)  $\underline{a}_P$  WHEN  $\theta = 90^\circ$

$$\underline{\omega} = \omega_1 \underline{i} + \omega_2 \underline{k} = (5 \text{ rad/s}) \underline{i} + (4 \text{ rad/s}) \underline{k}$$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (5 \text{ rad/s}) \underline{i} \times [(5 \text{ rad/s}) \underline{i} + (4 \text{ rad/s}) \underline{k}]$$

$$(a) \quad \underline{\alpha} = -(20 \text{ rad/s}^2) \underline{j}$$

$$(b) \quad \theta = 0: \underline{r}_P = (3 \text{ in.}) \underline{i}$$

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (5 \underline{i} + 4 \underline{k}) \times 3 \underline{i}; \quad \underline{v}_P = (12 \text{ in./s}) \underline{j}$$

$$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$$

$$= -20 \underline{j} \times 3 \underline{i} + (5 \underline{i} + 4 \underline{k}) \times 12 \underline{j}$$

$$= 60 \underline{k} + 60 \underline{k} - 48 \underline{i} = -48 \underline{i} + 120 \underline{k}$$

$$\underline{a}_P = -(48 \text{ in./s}^2) \underline{i} + (120 \text{ in./s}^2) \underline{k}$$

(CONTINUED)

# 15.194 CONTINUED

$$(c) \quad \theta = 90^\circ: \underline{r}_P = (3 \text{ in.}) \underline{j}$$

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (5 \underline{i} + 4 \underline{k}) \times 3 \underline{j}; \quad \underline{v}_P = -(12 \text{ in./s}) \underline{i} + (15 \text{ in./s}) \underline{k}$$

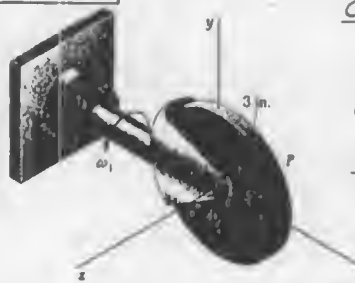
$$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$$

$$= -20 \underline{j} \times 3 \underline{j} + (5 \underline{i} + 4 \underline{k}) \times (-12 \underline{i} + 15 \underline{k})$$

$$= 0 - 75 \underline{j} - 48 \underline{j} = -123 \underline{j}$$

$$\underline{a}_P = -(123 \text{ in./s}^2) \underline{j}$$

# 15.195



GIVEN:

$$\omega_1 = 5 \text{ rad/s}, \alpha_1 = 0$$

$$\omega_2 = 4 \text{ rad/s}, \alpha_2 = 0$$

$$\theta = 30^\circ$$

FIND:  $\underline{a}_P$

$$\underline{\omega} = \omega_1 \underline{i} + \omega_2 \underline{k} = (5 \text{ rad/s}) \underline{i} + (4 \text{ rad/s}) \underline{k}$$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (5 \text{ rad/s}) \underline{i} \times [(5 \text{ rad/s}) \underline{i} + (4 \text{ rad/s}) \underline{k}]$$

$$\underline{\alpha} = -(20 \text{ rad/s}^2) \underline{j}$$

FOR  $\theta = 30^\circ$

$$r = 3 \text{ in.}$$

$$(\underline{r}_P)_x = r \cos 30^\circ = (3 \text{ in.}) \cos 30^\circ = 2.598 \text{ in.}$$

$$(\underline{r}_P)_y = r \sin 30^\circ = (3 \text{ in.}) \sin 30^\circ = 1.5 \text{ in.}$$

$$\underline{r}_P = (2.598 \text{ in.}) \underline{i} + (1.5 \text{ in.}) \underline{j}$$

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 0 & 4 \\ 2.598 & 1.5 & 0 \end{vmatrix}$$

$$= -6 \underline{i} + 10.392 \underline{j} + 7.5 \underline{k}$$

$$\underline{v}_P = -(6 \text{ in./s}) \underline{i} + (10.392 \text{ in./s}) \underline{j} + (7.5 \text{ in./s}) \underline{k}$$

$$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$$

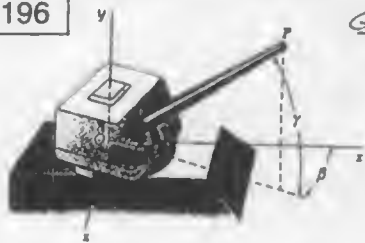
$$\underline{a}_P = -20 \underline{j} \times (2.598 \underline{i} + 1.5 \underline{j}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 0 & 4 \\ -6 & 10.392 & 7.5 \end{vmatrix}$$

$$= 51.96 \underline{k} - 41.57 \underline{i} + (-24 - 37.5) \underline{j} + 51.96 \underline{k}$$

$$\underline{a}_P = -41.57 \underline{i} - 61.5 \underline{j} + 103.92 \underline{k}$$

$$\underline{a}_P = -(41.6 \text{ in./s}^2) \underline{i} - (61.5 \text{ in./s}^2) \underline{j} + (103.9 \text{ in./s}^2) \underline{k}$$

15.196



GIVEN:  $OP = 4\text{ m}$   
 $\frac{d\theta}{dt} = 30^\circ/\text{s}$   
 $\frac{d^2\theta}{dt^2} = 10^\circ/\text{s}^2$   
 $\beta = 90^\circ$   
 $\gamma = 30^\circ$   
 FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{\alpha}$   
 (c)  $\underline{v}_P$  AND  $\underline{a}_P$

$$\underline{\omega}_1 = -\frac{d\beta}{dt} \underline{j} = -(30^\circ/\text{s}) \underline{j} = -\left(\frac{\pi}{6} \text{ rad/s}\right) \underline{j}$$

$$\underline{\omega}_2 = -\frac{d\gamma}{dt} \underline{i} = -(10^\circ/\text{s}) \underline{i} = -\left(\frac{\pi}{18} \text{ rad/s}\right) \underline{i}$$

$$\underline{r}_P = (4\text{ m}) \sin 30^\circ \underline{j} + (4\text{ m}) \cos 30^\circ \underline{k}$$

$$\underline{r}_P = (2\text{ m}) \underline{j} + (3.464\text{ m}) \underline{k}$$

(a)  $\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = -\frac{\pi}{18} \underline{i} - \frac{\pi}{6} \underline{j} = -0.17453 \underline{i} - 0.5236 \underline{j}$   
 $\underline{\omega} = -(0.1745 \text{ rad/s}) \underline{i} - (0.524 \text{ rad/s}) \underline{j}$

(b) NOTE TURRET ROTATES ABOUT  $y$  AXIS AT RATE  $\underline{\omega}$ ,  
 $\underline{\alpha} = \underline{\omega} \times \underline{\omega} = -\frac{\pi}{6} \underline{j} \times \left(-\frac{\pi}{18} \underline{i} - \frac{\pi}{6} \underline{j}\right) = -\frac{\pi^2}{(6)(18)} \underline{k}$   
 $\underline{\alpha} = -(0.0914 \text{ rad/s}^2) \underline{k}$

(c)  $\underline{v}_P = \underline{\omega} \times \underline{r}_P = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.17453 & -0.5236 & 0 \\ 0 & 2 & 3.464 \end{vmatrix}$

$$\underline{v}_P = -1.8138 \underline{i} + 0.6046 \underline{j} - 0.34906 \underline{k}$$

$$\underline{v}_P = -(1.814 \text{ m/s}) \underline{i} + (0.605 \text{ m/s}) \underline{j} - (0.349 \text{ m/s}) \underline{k}$$

$$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times (\underline{\omega} \times \underline{r}_P) = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$$

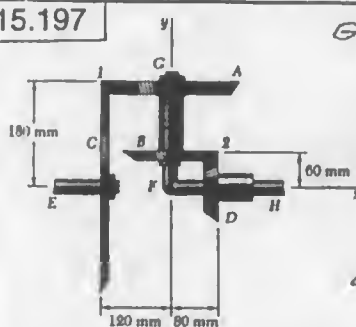
$$\underline{a}_P = -0.0914 \underline{k} \times (2 \underline{j} + 3.464 \underline{k}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.17453 & -0.5236 & 0 \\ -1.814 & 0.605 & -0.349 \end{vmatrix}$$

$$\underline{a}_P = 0.1828 \underline{i} + 0.1828 \underline{i} - 0.0609 \underline{j} + (-0.1056 - 0.9498) \underline{k}$$

$$= 0.3656 \underline{i} - 0.0609 \underline{j} - 1.0554 \underline{k}$$

$$\underline{a}_P = (0.366 \text{ m/s}^2) \underline{i} - (0.0609 \text{ m/s}^2) \underline{j} - (1.055 \text{ m/s}^2) \underline{k}$$

15.197



GIVEN:  $\omega_A = (15 \text{ rad/s}) \underline{i}$   
 $\omega_D = (30 \text{ rad/s}) \underline{i}$

FIND: (a)  $\underline{\omega}_{AB}$   
 (b)  $\underline{\alpha}_{AB}$   
 (c) ACCELERATION OF TOOTH OF GEAR B IN CONTACT WITH GEAR D

$$\underline{r}_1 = -(0.12 \text{ m}) \underline{i} + (0.18 \text{ m}) \underline{j}; \quad \underline{r}_2 = (0.08 \text{ m}) \underline{i} + (0.06 \text{ m}) \underline{j}$$

$$\underline{v}_1 = \underline{\omega}_A \times \underline{r}_1 = 15 \underline{i} \times (-0.12 \underline{i} + 0.18 \underline{j}) = (2.7 \text{ m/s}) \underline{k} \quad (1)$$

$$\underline{v}_2 = \underline{\omega}_D \times \underline{r}_2 = 30 \underline{i} \times (0.08 \underline{i} + 0.06 \underline{j}) = (1.8 \text{ m/s}) \underline{k} \quad (2)$$

MOTION OF GEARS A & B  $\underline{\omega}_{AB} = \underline{\omega}_x \underline{i} + \underline{\omega}_y \underline{j} + \underline{\omega}_z \underline{k}$   
 VELOCITY:

$$\underline{v}_1 = \underline{\omega}_{AB} \times \underline{r}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -0.12 & 0.18 & 0 \end{vmatrix}$$

$$\underline{v}_1 = -0.18 \omega_z \underline{i} - 0.12 \omega_z \underline{j} + (0.18 \omega_x + 0.12 \omega_y) \underline{k}$$

FROM EQ 1:  $\underline{v}_1 = (2.7 \text{ m/s}) \underline{k}$

$$2.7 \underline{k} = -0.18 \omega_z \underline{i} - 0.12 \omega_z \underline{j} + (0.18 \omega_x + 0.12 \omega_y) \underline{k}$$

COEFFICIENTS OF  $\underline{i}$ :  $\omega_z = 0$

COEFFICIENTS OF  $\underline{k}$ :  $2.7 = 0.18 \omega_x + 0.12 \omega_y$  (3)

( $\omega_z = 0$ )

$$\underline{v}_2 = \underline{\omega}_{AB} \times \underline{r}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & 0 \\ 0.08 & 0.06 & 0 \end{vmatrix} = (0.06 \omega_x - 0.08 \omega_y) \underline{k}$$

FROM EQ 2:  $\underline{v}_2 = (1.8 \text{ m/s}) \underline{k} = (0.06 \omega_x - 0.08 \omega_y) \underline{k}$

COEFFICIENTS OF  $\underline{k}$ :  $1.8 = 0.06 \omega_x - 0.08 \omega_y$  (4)

SOLVING SIMULTANEOUSLY EQS 1 AND 2, WE FIND

$$\omega_x = 20 \quad \omega_y = -7.5$$

(a)  $\underline{\omega}_{AB} = (20 \text{ rad/s}) \underline{i} - (7.5 \text{ rad/s}) \underline{j}$

ACCELERATION GEARS A & B ROTATE ABOUT  $x$  AXIS AT RATE  $\underline{\omega}_{FH} = (\omega_{AB})_x = (20 \text{ rad/s}) \underline{i}$

$$\underline{\alpha}_{AB} = \underline{\omega}_{FH} \times \underline{\omega}_{AB} = (20 \underline{i}) \times (20 \underline{i} - 7.5 \underline{j}) = -150 \underline{k}$$

(b)  $\underline{\alpha}_{AB} = -(150 \text{ rad/s}^2) \underline{k}$

(c) FOR TOOTH OF GEAR B IN CONTACT WITH GEAR D

$$\underline{r}_2 = (0.08 \text{ m}) \underline{i} + (0.06 \text{ m}) \underline{j}; \quad \underline{v}_2 = (1.8 \text{ m/s}) \underline{k}$$

$$\underline{a}_2 = \underline{\alpha}_{AB} \times \underline{r}_2 + \underline{\omega}_{AB} \times \underline{v}_2$$

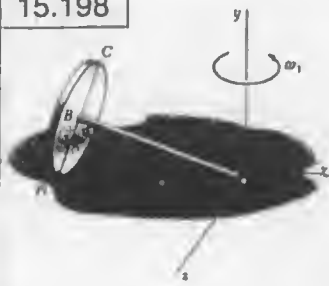
$$= (-150 \underline{k}) \times (0.08 \underline{i} + 0.06 \underline{j}) + (20 \underline{i} - 7.5 \underline{j}) \times (1.8 \underline{k})$$

$$= -12 \underline{j} + 9 \underline{i} - 36 \underline{j} - 13.5 \underline{i}$$

$$\underline{a}_2 = -4.5 \underline{i} - 48 \underline{j}$$

$$\underline{a}_2 = -(4.5 \text{ m/s}^2) \underline{i} - (48 \text{ m/s}^2) \underline{j}$$

15.198



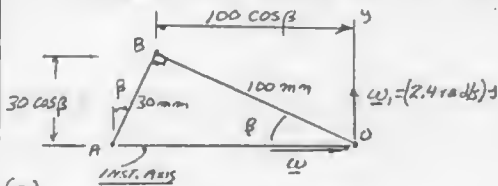
GIVEN:

$$\begin{aligned} r &= AB = BC = 30 \text{ mm} \\ l &= OB = 100 \text{ mm} \\ \omega_1 &= (2.4 \text{ rad/s}) \hat{j} \\ \alpha_1 &= 0 \end{aligned}$$

FIND: FOR WHEEL,

- (a)  $\omega$   
(b)  $\alpha$   
(c)  $\underline{a}_C$

FOR WHEEL-ROD UNIT: ANGULAR VELOCITY =  $\omega$   
INSTANTANEOUS AXIS OF ROTATION IS THE X AXIS



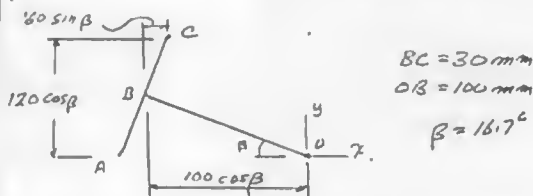
(a)

$$\begin{aligned} \tan \beta &= \frac{30}{100} ; \beta = 16.7^\circ \\ \text{CONSIDER MOTION ABOUT Y AXIS: } \underline{v}_B &= (100 \cos \beta)(2.4) \\ \text{CONSIDER MOTION ABOUT INST. AXIS: } \underline{v}_B &= (30 \cos \beta) \omega \\ \underline{v}_B &= \underline{v}_B: (100 \cos \beta)(2.4) = (30 \cos \beta) \omega \\ \omega &= \frac{100}{30}(2.4) \quad \omega = (8 \text{ rad/s}) \hat{j} \end{aligned}$$

(b)  $\alpha = \underline{\omega} \times \underline{\omega} = (2.4 \text{ rad/s}) \hat{j} \times (8 \text{ rad/s}) \hat{j}$   
 $\alpha = -(19.2 \text{ rad/s}^2) \hat{k}$

(c) POINT C:  $CA = 2r = 0.06 \text{ m}$   
 $\underline{r}_{CA} = (0.06 \text{ m})(-\sin \beta \hat{i} + \cos \beta \hat{j})$

$$\begin{aligned} \underline{v}_C &= \underline{\omega} \times \underline{r}_{CA} = (8 \text{ rad/s}) \hat{j} \times (0.06 \text{ m})(-\sin \beta \hat{i} + \cos \beta \hat{j}) \\ &= 0.48 \cos \beta \hat{k} = 0.48 \cos 16.7^\circ \hat{k} \\ \underline{v}_C &= (0.4598 \text{ m/s}) \hat{k} \end{aligned}$$

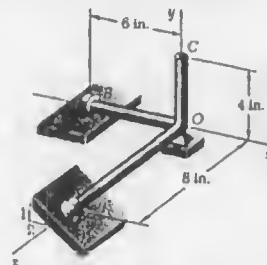


$$\begin{aligned} \underline{r}_C &= -(100 \cos \beta - 30 \sin \beta) \hat{i} + (100 \sin \beta) \hat{j} \\ &= -(95.782 - 0.621) \hat{i} + 57.47 \hat{j} \\ \underline{r}_C &= -(87.16 \text{ mm}) \hat{i} + (57.47 \text{ mm}) \hat{j} \\ \underline{r}_C &= (0.08716 \text{ m}) \hat{i} + (0.05747 \text{ m}) \hat{j} \end{aligned}$$

$$\begin{aligned} \underline{a}_C &= \alpha \times \underline{r}_C + \underline{\omega} \times \underline{v}_C \\ &= -(19.2) \hat{k} \times (0.08716 \hat{i} + 0.05747 \hat{j}) + 8 \hat{j} \times 0.4598 \hat{k} \\ &= +1.673 \hat{i} + 1.103 \hat{i} - 3.678 \hat{j} \\ &= 1.103 \hat{i} - 2.005 \hat{j} \end{aligned}$$

$$\underline{a}_C = (1.103 \text{ m/s}^2) \hat{i} - (2.005 \text{ m/s}^2) \hat{j}$$

15.199 and 15.200



GIVEN:  $\underline{v}_B = -(15 \text{ m/s}) \hat{k}$   
 $(\underline{a}_B)_z = 0$

PROBLEM 15.199

FIND: (a)  $\omega$   
(b)  $\underline{v}_C$

PROBLEM 15.200

FIND: (a)  $\alpha$   
(b)  $\underline{a}_C$

PROBLEM 15.199

$$\begin{aligned} \underline{r}_B &= -(6 \text{ in.}) \hat{i} \\ \underline{r}_A &= (8 \text{ in.}) \hat{k} \\ \underline{\omega} &= \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} \\ \underline{v}_B &= \underline{\omega} \times \underline{r}_B = (\omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}) \times (-6 \text{ in.}) \hat{i} \\ &= -(15 \text{ in/s}) \hat{k} = (6 \text{ in.}) \omega_y \hat{k} - (6 \text{ in.}) \omega_z \hat{j} \end{aligned}$$

COEF. OF  $\hat{k}$ :  $-15 \text{ in/s} = (6 \text{ in.}) \omega_y$   $\omega_y = -2.5 \text{ rad/s}$   
COEF. OF  $\hat{j}$ :  $0 = (6 \text{ in.}) \omega_z$   $\omega_z = 0$

$$\begin{aligned} \underline{v}_A &= \underline{\omega} \times \underline{r}_A = (\omega_x \hat{i} - 2.5 \hat{j}) \times 8 \hat{k} \\ \underline{v}_A &= -(20 \text{ in/s}) \hat{i} - (8 \text{ in.}) \omega_x \hat{j} \end{aligned} \quad (1)$$

BUT  $(\underline{v}_A)_x = -2(\underline{v}_A)_y$ :  $-20 \text{ in/s} = -2(-8 \text{ in.}) \omega_x$   
 $\omega_x = -1.25 \text{ rad/s}$

$$\underline{\omega} = -(1.25 \text{ rad/s}) \hat{i} - (2.5 \text{ rad/s}) \hat{j}$$

$$\begin{aligned} \underline{r}_C &= (4 \text{ in.}) \hat{j} \\ \underline{v}_C &= \underline{\omega} \times \underline{r}_C = (-1.25 \hat{i} - 2.5 \hat{j}) \times 4 \hat{j} = -5 \hat{k} \\ \underline{v}_C &= -(5 \text{ in/s}) \hat{k} \end{aligned}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = (-1.25 \hat{i} - 2.5 \hat{j}) \times 8 \hat{k} = -(20 \text{ in/s}) \hat{i} + (10 \text{ in.}) \hat{j}$$

PROBLEM 15.200 SINCE  $(\underline{a}_B)_z = 0$ ,  $\underline{a}_B = (\underline{a}_B)_x \hat{i}$ 

$$\begin{aligned} \underline{a}_B &= \alpha \times \underline{r}_B + \underline{\omega} \times \underline{v}_B \\ &= (\alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}) \times 6 \hat{i} + (-1.25 \hat{i} - 2.5 \hat{j}) \times (-15 \hat{k}) \\ &= +6 \alpha_y \hat{k} - 6 \alpha_z \hat{j} - 18.75 \hat{j} + 37.5 \hat{i} \end{aligned}$$

COEF. OF  $\hat{k}$ :  $0 = \alpha_y$   $\alpha_y = 0$   
COEF. OF  $\hat{j}$ :  $0 = -6 \alpha_z - 18.75$   $\alpha_z = -3.125 \text{ rad/s}^2$

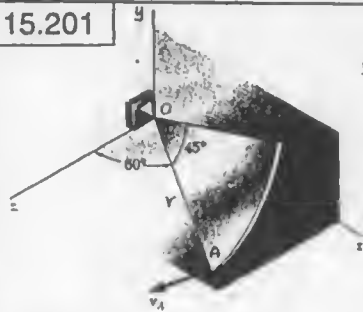
$$\begin{aligned} \underline{a}_A &= \alpha \times \underline{r}_A + \underline{\omega} \times \underline{v}_A \\ &= (\alpha_x \hat{i} - 3.125 \hat{k}) \times 8 \hat{k} + (-1.25 \hat{i} - 2.5 \hat{j}) \times (-20 \hat{i} + 10 \hat{j}) \\ \underline{a}_A &= -8 \alpha_x \hat{j} - 12.5 \hat{k} - 50 \hat{k} \end{aligned}$$

$$\begin{aligned} (\underline{a}_A)_x &= 0 \quad (\underline{a}_A)_y = -8 \alpha_x \quad (\underline{a}_A)_z = -62.5 \\ \text{SINCE } (\underline{v}_A)_x &= -2(\underline{v}_A)_y, \quad (\underline{a}_A)_x = -2(\underline{a}_A)_y \\ 0 &= -8 \alpha_x \quad \alpha_x = 0 \end{aligned}$$

$$\begin{aligned} \underline{r}_C &= (4 \text{ in.}) \hat{j} \\ \underline{a}_C &= \alpha \times \underline{r}_C + \underline{\omega} \times \underline{v}_C \\ &= (-3.125 \hat{k}) \times 4 \hat{j} + (-1.25 \hat{i} - 2.5 \hat{j}) \times (-5 \hat{k}) \\ &= 12.5 \hat{i} - 6.25 \hat{j} + 12.5 \hat{i} \end{aligned}$$

$$\underline{a}_C = (25 \text{ in./s}^2) \hat{i} - (6.25 \text{ in./s}^2) \hat{j}$$

15.201



GIVEN:  $r = 10 \text{ in.}$   
 $\dot{\theta} = 60 \text{ in./s}$

FIND: (a)  $\alpha$   
 (b)  $\dot{\theta}_B$

FIND OB:  $\vec{OA} = (10 \text{ in.}) \sin 60^\circ \hat{i} + (10 \text{ in.}) \cos 60^\circ \hat{j}$   
 $\vec{r}_A = \vec{OA} = (8.6603 \text{ in.}) \hat{i} + (5 \text{ in.}) \hat{j}$   
 $\vec{OB} = (r_B)_x \hat{i} + (r_B)_y \hat{j}$

SCALAR PRODUCT:

$\vec{OA} \cdot \vec{OB} = (OA)(OB) \sin 45^\circ$   
 $(8.6603 \hat{i} + 5 \hat{j}) \cdot (r_B)_x \hat{i} + (r_B)_y \hat{j} = (10)(10) \sin 45^\circ$   
 $8.6603(r_B)_x = 70.711 \quad (r_B)_x = 8.165 \text{ in.}$   
 $(r_B)_y = \sqrt{OB^2 - (r_B)_x^2} = \sqrt{10^2 - 8.165^2} = 5.773 \text{ in.}$   
 $\vec{r}_B = (8.165 \text{ in.}) \hat{i} + (5.773 \text{ in.}) \hat{j}$

SINCE  $\vec{v}_A \perp \vec{OA}$ ,  $\vec{v}_A$  FORMS  $30^\circ$  ANGLE WITH  $x$  AXIS

$\vec{v}_A = (60 \text{ in./s})(-\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})$   
 $\vec{v}_A = (-30 \text{ in./s}) \hat{i} + (51.96 \text{ in./s}) \hat{j}$

PLATE CIR:  $\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

$\vec{v}_A = \vec{\omega} \times \vec{r}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 8.6603 & 0 & 5 \end{vmatrix}$

$-30 \hat{i} + 51.96 \hat{j} = 5\omega_y \hat{i} + (8.6603\omega_z - 5\omega_x) \hat{j} - 8.6603\omega_y \hat{k}$

COEF  $\hat{i}$ :  $-30 = 5\omega_y \rightarrow \omega_y = -6 \text{ rad/s}$

COEF  $\hat{j}$ :  $0 = 8.6603\omega_z - 5\omega_x \rightarrow \omega_z = 0.57735\omega_x$  (1)

COEF  $\hat{k}$ :  $51.96 = -8.6603\omega_y \rightarrow \omega_y = -6 \text{ rad/s}$

$\vec{v}_B = \vec{\omega} \times \vec{r}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 8.165 & 5.773 & 0 \end{vmatrix}$

$\vec{v}_B = -5.773\omega_x \hat{i} + 8.165\omega_z \hat{j} + (5.773\omega_x - 8.165\omega_y) \hat{k}$

SINCE POINT B MOVES IN  $xy$  PLANE

$(v_B)_z = 0 = 5.773\omega_x - 8.165\omega_y$   
 $0 = 5.773\omega_x - 8.165(-6)$

$\omega_x = -8.486 \text{ rad/s}$

EQ(1):  $\omega_z = 0.57735(-8.486) = -4.899 \text{ rad/s}$

$\vec{\omega} = (-8.49 \text{ rad/s}) \hat{i} - (6 \text{ rad/s}) \hat{j} - (4.90 \text{ rad/s}) \hat{k}$

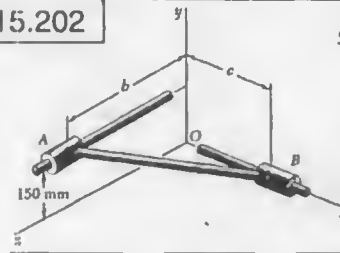
$(v_B)_x = -5.773(-4.899) = 28.3 \text{ in./s}$

$(v_B)_y = 8.165(-4.899) = -40.0 \text{ in./s}$

$(v_B)_z = 0$

$\vec{v}_B = (28.3 \text{ in./s}) \hat{i} - (40.0 \text{ in./s}) \hat{j}$

15.202



GIVEN:  $r = 175 \text{ mm}$   
 $\dot{\theta}_B = -(180 \text{ mm/s}) \hat{i}$   
 $AB = 275 \text{ mm}$   
 FIND:  $\dot{\theta}_A$

$C = 175 \text{ mm}$ ;  $275^2 = 150^2 + 175^2 + b^2$ ;  $b = 150 \text{ mm}$   
 $\vec{r}_B = -(180 \text{ mm/s}) \hat{i}$ ;  $\vec{v}_A = \vec{v}_B \hat{k}$ ;  $\vec{r}_{AB} = -175 \hat{i} + 150 \hat{j} + 150 \hat{k}$   
 $\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{AB}$   
 $\vec{v}_A \hat{k} = -180 \hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -175 & 150 & 150 \end{vmatrix}$

$\vec{v}_A \hat{k} = -180 \hat{i} + (150\omega_y - 150\omega_z) \hat{i} + (-175\omega_z - 150\omega_x) \hat{j} + (150\omega_x + 175\omega_y) \hat{k}$

COEF OF  $\hat{i}$ :  $+180 = 150\omega_y - 150\omega_z$  (1)

COEF OF  $\hat{j}$ :  $0 = -175\omega_z - 150\omega_x$  (2)

COEF OF  $\hat{k}$ :  $\vec{v}_A = 150\omega_x + 175\omega_y$  (3)

(EQ 2 + EQ 3)  $\frac{6}{5} \vec{v}_A = 0 + 150\omega_z - 150\omega_z$  (4)

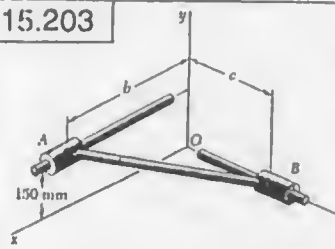
EQ 1 - EQ 4  $180 - 6\vec{v}_A = 0$ ;  $\vec{v}_A = (30 \text{ mm/s}) \hat{k}$

FOR USE IN PROB. 15.214: WE CALCULATE A POSSIBLE  $\vec{\omega}$ .

WE SHALL ASSUME  $\omega_x = 0$ . FROM EQ(2), WE HAVE  $\omega_z = 0$ .

EQ(1):  $180 = 0 + 150\omega_y$   $\omega_y = (1.2 \text{ rad/s}) \hat{j}$

15.203



GIVEN:  $r = 50 \text{ mm}$   
 $\dot{\theta}_B = -(180 \text{ mm/s}) \hat{i}$   
 $AB = 275 \text{ mm}$   
 FIND:  $\dot{\theta}_A$

$C = 50 \text{ mm}$ ;  $275^2 = 150^2 + 50^2 + b^2$ ;  $b = 225 \text{ mm}$   
 $\vec{r}_B = -(180 \text{ mm/s}) \hat{i}$ ;  $\vec{v}_A = \vec{v}_B \hat{k}$ ;  $\vec{r}_{AB} = -50 \hat{i} + 150 \hat{j} + 225 \hat{k}$

$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{AB}$

$\vec{v}_A \hat{k} = -180 \hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -50 & 150 & 225 \end{vmatrix}$

$\vec{v}_A \hat{k} = -180 \hat{i} + (225\omega_y - 150\omega_z) \hat{i} + (-50\omega_z - 225\omega_x) \hat{j} + (150\omega_x + 50\omega_y) \hat{k}$

COEF OF  $\hat{i}$ :  $+180 = 225\omega_y - 150\omega_z$  (1)

COEF OF  $\hat{j}$ :  $0 = -225\omega_x - 50\omega_z$  (2)

COEF OF  $\hat{k}$ :  $\vec{v}_A = 150\omega_x + 50\omega_y$  (3)

[EQ. 1 + 3]  $180 - 45\vec{v}_A = 0$

$\vec{v}_A = (40 \text{ mm/s}) \hat{k}$

FOR USE IN PROB. 15.215:

WE SHALL CALCULATE A POSSIBLE  $\vec{\omega}$ . SINCE  $\vec{\omega}$  IS

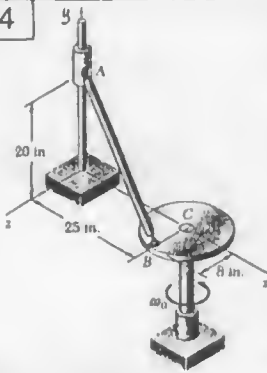
INDETERMINATE, WE CAN ASSUME A VALUE FOR

ANY COMPONENT OF  $\vec{\omega}$ . WE ASSUME  $\omega_x = 0$ ,

FROM EQ(2), WE FIND  $\omega_z = 0$

EQ(1):  $+180 = 225\omega_y$   $\omega_y = (0.8 \text{ rad/s}) \hat{j}$

15.204



GIVEN:  
 $\omega_0 = (3 \text{ rad/s}) \hat{j}$

FIND:  $\vec{v}_A$

$$\vec{r}_{A/B} = (-25 \text{ in}) \hat{i} + (20 \text{ in}) \hat{j} - (8 \text{ in}) \hat{k}$$

DISK:  $\vec{v}_B = \omega_0 \times \vec{r}_{B/C} = (3 \text{ rad/s}) \hat{j} \times (8 \text{ in}) \hat{k} = (24 \text{ in/s}) \hat{i}$

ROD AB:  $\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$

$$\vec{v}_A \hat{j} = (24 \text{ in/s}) \hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -25 & 20 & -8 \end{vmatrix}$$

$$\vec{v}_A \hat{j} = 24 \hat{i} + (-8\omega_y - 20\omega_z) \hat{i} + (-25\omega_z + 8\omega_x) \hat{j} + (20\omega_x + 25\omega_y) \hat{k}$$

COEF OF  $\hat{i}$ :  $-24 = -8\omega_y - 20\omega_z$  (1)

COEF OF  $\hat{j}$ :  $\vec{v}_A = 8\omega_x - 25\omega_z$  (2)

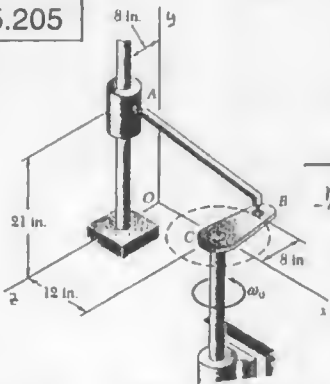
COEF OF  $\hat{k}$ :  $0 = 20\omega_x + 25\omega_y$  (3)

SINCE DETERMINANT OF  $\omega_x, \omega_y, \omega_z$  IS ZERO,  $\omega$  IS INDETERMINATE. WE CAN ASSUME ANY ONE COMPONENT ASSUME  $\omega_x = 0$ , EQ. 3 YIELDS  $\omega_y = 0$ .

EQ. 1:  $-24 = 0 - 20\omega_z$ ;  $\omega_z = 1.2 \text{ rad/s}$

EQ. 2:  $\vec{v}_A = 0 - 25(1.2) = -30$ ;  $\vec{v}_A = -(30 \text{ in/s}) \hat{j}$

15.205



GIVEN:  
 $\omega_0 = (10 \text{ rad/s}) \hat{j}$

FIND:  $\vec{v}_A$

$$\vec{r}_{A/B} = (-12 \text{ in}) \hat{i} + (21 \text{ in}) \hat{j} + (8 \text{ in}) \hat{k}$$

CRANK BC:  
 $\vec{r}_{B/C} = (8 \text{ in}) \hat{k}$

$$\vec{v}_B = \omega_0 \times \vec{r}_{B/C} = (10 \text{ rad/s}) \hat{j} \times (8 \text{ in}) \hat{k} = (80 \text{ in/s}) \hat{i}$$

$$\vec{v}_B = (80 \text{ in/s}) \hat{i}$$

ROD AB:  $\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$

$$\vec{v}_A \hat{j} = -80 \hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -12 & 21 & 8 \end{vmatrix}$$

COEF OF  $\hat{i}$ :  $-80 = 16\omega_y - 21\omega_z$  (1)

COEF OF  $\hat{j}$ :  $\vec{v}_A = -16\omega_x - 12\omega_z$  (2)

COEF OF  $\hat{k}$ :  $0 = 21\omega_x + 12\omega_y$  (3)

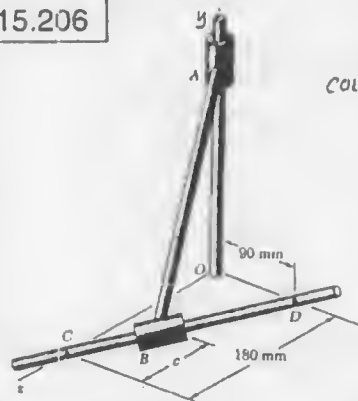
SINCE DETERMINANT OF  $\omega_x, \omega_y, \omega_z$  IS ZERO, THE ANGULAR VELOCITY IS INDETERMINATE, WE CAN ASSUME VALUE OF ANY ONE COMPONENT.

ASSUME  $\omega_x = 0$ , EQ. 2 YIELDS  $\omega_z = 0$

EQ. (1):  $-80 = 0 - 21\omega_z$ ;  $\omega_z = -\frac{80}{21} \text{ rad/s}$

EQ. (2):  $\vec{v}_A = 0 - 12(-\frac{80}{21})$ ;  $\vec{v}_A = (45.7 \text{ in/s}) \hat{j}$

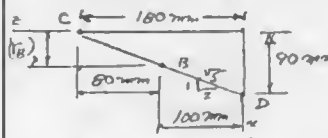
15.206



GIVEN:  $AB = 300 \text{ mm}$   
 COLLAR B MOVES TOWARD  
 POINT D AT  $50 \text{ mm/s}$   
 $r = 80 \text{ mm}$

FIND:  $\vec{v}_A$

LOCATION OF B IN XY PLANE



$$(r_B)_x = \frac{1}{2} r = \frac{80 \text{ mm}}{2}$$

$$(r_B)_x = 40 \text{ mm}$$

ROD AB:  $AB = 300 \text{ mm}$

$$(300 \text{ mm})^2 = (100 \text{ mm})^2 + (40 \text{ mm})^2 + (r_B)_y^2$$

$$\vec{r}_A = (280 \text{ mm}) \hat{j}$$

VELOCITY OF B:

$$\vec{v}_B = 50 \text{ mm/s}$$

$$\vec{r}_{CD} = \frac{1}{\sqrt{5}} (-2 \hat{i} + \hat{j})$$

$$\vec{v}_B = \vec{v}_B = \frac{50}{\sqrt{5}} (-2 \hat{i} + \hat{j}) = -(44.72 \text{ mm/s}) \hat{i} + (22.36 \text{ mm/s}) \hat{j}$$

$$\vec{r}_{A/B} = -(40 \text{ mm}) \hat{i} + (280 \text{ mm}) \hat{j} - (100 \text{ mm}) \hat{k}$$

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$$

$$\vec{v}_A \hat{j} = -44.72 \hat{i} + 22.36 \hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -40 & 280 & -100 \end{vmatrix}$$

$$\vec{v}_A \hat{j} = -44.72 \hat{i} + 22.36 \hat{j} + (-100\omega_y - 280\omega_z) \hat{i} + (-40\omega_z + 100\omega_x) \hat{j} + (280\omega_x + 40\omega_y) \hat{k}$$

COEF OF  $\hat{i}$ :  $-44.72 = -100\omega_y - 280\omega_z$  (1)

COEF OF  $\hat{j}$ :  $\vec{v}_A = 100\omega_x - 40\omega_z$  (2)

COEF OF  $\hat{k}$ :  $0 = 280\omega_x + 40\omega_y$  (3)

SINCE DETERMINANT OF  $\omega_x, \omega_y, \omega_z$  IS ZERO, THE ANGULAR VELOCITY IS INDETERMINATE, WE CAN ASSUME VALUE OF ANY COMPONENT.

ASSUME  $\omega_x = 0$

EQ. 3:  $44.72 = 0 + 40\omega_y$ ;  $\omega_y = 1.118 \text{ rad/s}$

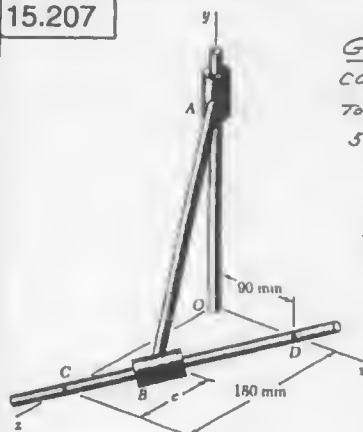
EQ. 1:  $-44.72 = -100(1.118) - 280\omega_z$   
 $\omega_z = -0.3194 \text{ rad/s}$

$$\omega = (1.118 \text{ rad/s}) \hat{j} - (0.3194 \text{ rad/s}) \hat{k}$$

EQ. 2:  $\vec{v}_A = 0 - 40(-0.3194) = 12.777 \text{ mm/s}$

$$\vec{v}_A = (12.78 \text{ mm/s}) \hat{j}$$

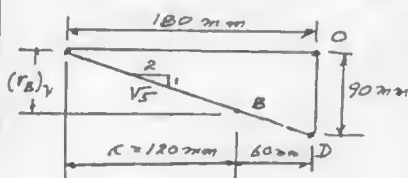
15.207



GIVEN:  $AB = 300 \text{ mm}$   
 COLLAR B MOVES  
 TOWARD POINT D AT  
 $50 \text{ mm/s}$ .  
 $r = 120 \text{ mm}$

FIND:  $\vec{v}_A$

LOCATION OF B IN THE PLANE



$$(r_B)_x = \frac{r}{2} = \frac{120 \text{ mm}}{2}$$

$$(r_B)_y = 60 \text{ mm}$$

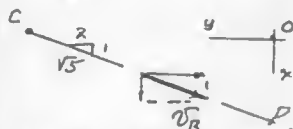
$$(r_B)_z = 60 \text{ mm}$$

ROD  $AB = 300 \text{ mm}$

$$(300 \text{ mm})^2 = (60 \text{ mm})^2 + (60 \text{ mm})^2 + (r_A)^2$$

$$r_A = 287.75 \text{ mm}$$

$$\vec{r}_{A/B} = -(60 \text{ mm})\hat{i} + (287.75 \text{ mm})\hat{j} - (60 \text{ mm})\hat{k}$$



VELOCITY OF B:

$$v_B = 50 \text{ mm/s}$$

$$\vec{v}_{C/D} = \frac{1}{\sqrt{5}}(-2\hat{i} + \hat{j})$$

$$\vec{v}_B = v_B \vec{v}_{C/D} = \frac{50}{\sqrt{5}}(\hat{i} - 2\hat{j}) = +22.36 \text{ mm/s}\hat{i} - 44.72 \text{ mm/s}\hat{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \omega \times \vec{r}_{A/B}$$

$$\vec{v}_A \hat{j} = 22.36\hat{i} - 44.72\hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -60 & 287.75 & -60 \end{vmatrix}$$

$$\vec{v}_A \hat{j} = 22.36\hat{i} - 44.72\hat{j} + (-60\omega_y - 287.75\omega_z)\hat{i} + (-60\omega_z + 60\omega_x)\hat{j} + (287.75\omega_x + 60\omega_y)\hat{k}$$

$$\text{COEF. OF } \hat{i}: -22.36 = -60\omega_y - 287.75\omega_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: v_A = 60\omega_x - 60\omega_z \quad (2)$$

$$\text{COEF. OF } \hat{k}: 44.72 = 287.75\omega_x + 60\omega_y \quad (3)$$

SINCE DETERMINANT OF  $\omega_x, \omega_y, \omega_z$  IS ZERO, THE ANGULAR VELOCITY IS INDETERMINANT. WE CAN THUS ASSUME THE VALUE OF ANY COMPONENT.

ASSUME  $\omega_x = 0$ :

$$\text{EQ. 3: } 44.72 = 0 + 60\omega_y; \quad \omega_y = 0.7453 \text{ rad/s}$$

$$\text{EQ. 1: } -22.36 = -60(0.7453) - 287.75\omega_z$$

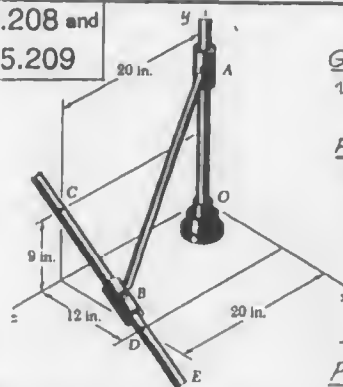
$$\omega_z = -0.0777 \text{ rad/s}$$

$$\omega = (0.7453 \text{ rad/s})\hat{j} - (0.0777 \text{ rad/s})\hat{k}$$

$$\text{EQ. 2: } v_A = 0 - 60(-0.0777) = 4.66$$

$$\vec{v}_A = (4.66 \text{ mm/s})\hat{j}$$

15.208 and 15.209



GIVEN:  $AB = 25 \text{ in}$ .  
 $v_B = 20 \text{ in/s}$  TOWARD E

FIND:  $\vec{v}_A$  AS COLLAR B  
 PASSES POINT D

PROBLEM 15.208

COLLAR AT D:  $AB^2 = 25^2 = 12^2 + 20^2 + r_A^2$ ;  $r_A = 9 \text{ in}$ .

$$\vec{r}_{A/B} = -(12 \text{ in})\hat{i} + (9 \text{ in})\hat{j} - (20 \text{ in})\hat{k}$$

$$\vec{v}_B = 20 \text{ in/s} \quad \vec{v}_B = v_B \vec{v}_{C/D} = v_B(0.8\hat{i} - 0.6\hat{j})$$

$$\vec{v}_B = (16 \text{ in/s})\hat{i} - (12 \text{ in/s})\hat{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \omega \times \vec{r}_{A/B}$$

$$\vec{v}_A \hat{j} = 16\hat{i} - 12\hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -12 & 9 & -20 \end{vmatrix}$$

$$\text{COEF. OF } \hat{i}: -16 = -20\omega_y - 9\omega_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: v_A + 12 = 20\omega_x - 12\omega_z \quad (2)$$

$$\text{COEF. OF } \hat{k}: 0 = 9\omega_x + 12\omega_y \quad (3)$$

SINCE DETERMINANT OF  $\omega_x, \omega_y, \omega_z$  IS ZERO, THE ANGULAR VELOCITY IS INDETERMINANT. WE CAN THUS ASSUME THE VALUE OF ANY COMPONENT

ASSUME  $\omega_x = 0$ , EQ. 3, YIELDS  $\omega_y = 0$

$$\text{EQ. 1: } -16 = 0 - 9\omega_z \quad \omega_z = \frac{16}{9} \text{ rad/s}$$

$$\text{EQ. 2: } v_A + 12 = 0 - 12\left(\frac{16}{9}\right); \quad v_A + 12 = -21.33$$

$$v_A = -(33.33 \text{ in/s})\hat{j}$$

PROBLEM 15.209

COLLAR AT C:  $AB^2 = 25^2 = 20^2 + r_A^2$ ;  $r_A = 15 \text{ in}$ .

$$\vec{r}_{A/B} = (15 \text{ in})\hat{j} - (20 \text{ in})\hat{k}$$

$$\vec{v}_B = (16 \text{ in/s})\hat{i} - (12 \text{ in/s})\hat{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \omega \times \vec{r}_{A/B}$$

$$\vec{v}_A \hat{j} = 16\hat{i} - 12\hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 15 & -20 \end{vmatrix}$$

$$\text{COEF. OF } \hat{i}: 16 = -20\omega_y - 15\omega_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: v_A + 12 = 20\omega_x \quad (2)$$

$$\text{COEF. OF } \hat{k}: 0 = 15\omega_x \quad (3)$$

$$\text{EQ. 3: } \omega_x = 0$$

$$\text{EQ. 2: } v_A + 12 = 0 \quad v_A = -12 \text{ in/s}$$

$$\vec{v}_A = -(12 \text{ in/s})\hat{j}$$

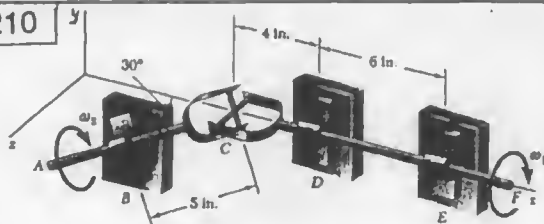
NOTE:  $\omega$  IS INDETERMINATE. ANY VALUE CAN BE CHOSEN FOR EITHER  $\omega_y$  OR  $\omega_z$

FOR EXAMPLE, IF  $\omega_x = 0$ , THEN

$$\text{EQ. 1: } 16 = -20\omega_y \quad \omega_y = -0.8 \text{ rad/s}$$

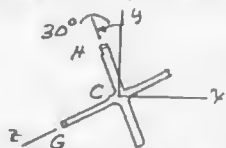


15.210



WHEN ARM OF CROSSPIECE ATTACHED TO SHAFT CF IS HORIZONTAL, FIND  $\omega_2$  OF SHAFT AC.

PLACE ORIGIN AT CENTER OF CROSSPIECE AND DENOTE BY  $l$  THE LENGTH OF EACH ARM.



$$\begin{aligned} \underline{r}_G &= l \underline{i} \\ \underline{r}_H &= -l \sin 30^\circ \underline{i} + l \cos 30^\circ \underline{j} \\ \underline{\omega}_1 &= -\omega_1 \underline{i} \\ \underline{\omega}_2 &= -\omega_2 \cos 30^\circ \underline{i} - \omega_2 \sin 30^\circ \underline{j} \end{aligned}$$

SHAFT CF:  $\underline{v}_G = \underline{\omega}_1 \times \underline{r}_G = -\omega_1 l \underline{i} \times l \underline{i} = \underline{0}$  (1)

SHAFT AC:  $\underline{v}_H = \underline{\omega}_2 \times \underline{r}_H$   
 $= (-\omega_2 \cos 30^\circ \underline{i} - \omega_2 \sin 30^\circ \underline{j}) \times (-l \sin 30^\circ \underline{i} + l \cos 30^\circ \underline{j})$   
 $\underline{v}_H = -\omega_2 \cos^2 30^\circ l \underline{j} - \omega_2 \sin^2 30^\circ l \underline{i}$   
 $\underline{v}_H = -\omega_2 l (\cos^2 30^\circ + \sin^2 30^\circ) \underline{i} = -\omega_2 l \underline{i}$  (2)

CROSSPIECE:  $\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$   
 $\underline{v}_G = \underline{\omega} \times \underline{r}_G = (\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \times l \underline{i}$   
 $\underline{v}_G = -l \omega_y \underline{j} + l \omega_z \underline{k}$  (3)

EQ. 1 = EQ. 3:  $\underline{v}_G = \underline{v}_G$   
 $l \omega_x \underline{j} = -l \omega_y \underline{j} + l \omega_z \underline{k}$   
 COEF. OF  $\underline{j}$ :  $l \omega_x = -l \omega_y$   $\omega_x = -\omega_y$  (4)  
 COEF. OF  $\underline{k}$ :  $0 = l \omega_z$   $\omega_z = 0$  (5)

$$\underline{v}_H = \underline{\omega} \times \underline{r}_H = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -l \sin 30^\circ & l \cos 30^\circ & 0 \end{vmatrix}$$

$$= -l \omega_z \cos 30^\circ \underline{i} - l \omega_z \sin 30^\circ \underline{j} + (l \omega_x \cos 30^\circ + l \omega_y \sin 30^\circ) \underline{k}$$

SUBSTITUTE FROM EGS. 4 AND 5:  $\omega_x = -\omega_y$  AND  $\omega_z = 0$   
 $\underline{v}_H = -l \omega_z \cos 30^\circ \underline{i} - l \omega_z \sin 30^\circ \underline{j} - l \omega_y \cos 30^\circ \underline{k}$

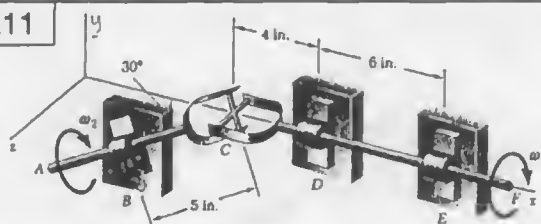
SUBSTITUTE FOR  $\underline{v}_H$  FROM EQ. 2.

$$-l \omega_y \underline{k} = -l \omega_z \cos 30^\circ \underline{i} - l \omega_z \sin 30^\circ \underline{j} - l \omega_y \cos 30^\circ \underline{k}$$

COEF. OF  $\underline{j}$ :  $0 = -l \omega_z \sin 30^\circ$   $\omega_z = 0$   
 COEF. OF  $\underline{k}$ :  $-l \omega_y = -l \omega_y \cos 30^\circ$

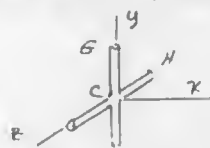
$$\omega_y = \omega_1 \cos 30^\circ$$

15.211



WHEN ARM OF CROSSPIECE ATTACHED TO SHAFT CF IS VERTICAL, FIND  $\omega_2$  OF SHAFT AC.

PLACE ORIGIN AT CENTER OF CROSSPIECE AND DENOTE BY  $l$  THE LENGTH OF EACH ARM.



$$\begin{aligned} \underline{r}_G &= l \underline{j} \\ \underline{r}_H &= -l \underline{i} \\ \underline{\omega}_1 &= -\omega_1 \underline{i} \\ \underline{\omega}_2 &= -\omega_2 \cos 30^\circ \underline{i} - \omega_2 \sin 30^\circ \underline{j} \end{aligned}$$

SHAFT CF:  $\underline{v}_G = \underline{\omega}_1 \times \underline{r}_G = -\omega_1 \underline{i} \times l \underline{j} = -l \omega_1 \underline{k}$  (1)

SHAFT AC:  $\underline{v}_H = \underline{\omega}_2 \times \underline{r}_H$   
 $= (-\omega_2 \cos 30^\circ \underline{i} - \omega_2 \sin 30^\circ \underline{j}) \times (-l \underline{i})$   
 $\underline{v}_H = -l \omega_2 \cos 30^\circ \underline{j} + l \omega_2 \sin 30^\circ \underline{k}$  (2)

CROSSPIECE:  $\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$   
 $\underline{v}_G = \underline{\omega} \times \underline{r}_G = (\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \times l \underline{j}$   
 $\underline{v}_G = l \omega_x \underline{k} - l \omega_z \underline{i}$  (3)

$\underline{v}_H = \underline{\omega} \times \underline{r}_H = (\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \times (-l \underline{i})$   
 $\underline{v}_H = -l \omega_y \underline{j} - l \omega_z \underline{k}$  (4)

EQ. 1 = EQ. 3:  $\underline{v}_G = \underline{v}_G$   
 $-l \omega_1 \underline{k} = l \omega_x \underline{k} - l \omega_z \underline{i}$   
 COEF. OF  $\underline{k}$ :  $-l \omega_1 = l \omega_x$   $\omega_x = -\omega_1$  (4)  
 COEF. OF  $\underline{i}$ :  $0 = -l \omega_z$   $\omega_z = 0$  (5)

EQ. 2 = EQ. 4:  $\underline{v}_H = \underline{v}_H$   
 $-l \omega_2 \cos 30^\circ \underline{j} + l \omega_2 \sin 30^\circ \underline{k} = -l \omega_y \underline{j} - l \omega_z \underline{k}$

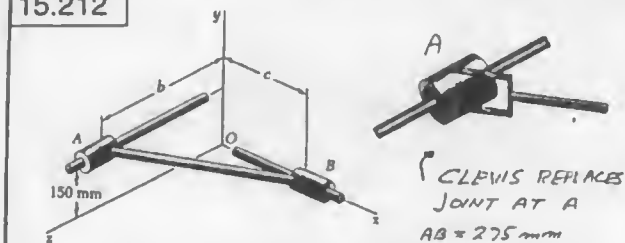
COEF. OF  $\underline{j}$ :  $-l \omega_2 \cos 30^\circ = -l \omega_y$   $\omega_y = \omega_2 \cos 30^\circ$   
 COEF. OF  $\underline{k}$ :  $l \omega_2 \sin 30^\circ = -l \omega_z$   $\omega_z = -\omega_2 \sin 30^\circ$

FROM EQ. 5:  $\omega_x = -\omega_1$

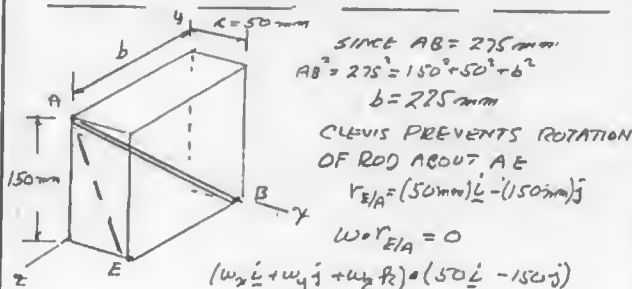
THUS,  $\omega_2 = -\frac{(-\omega_1)}{\cos 30^\circ}$

$$\omega_2 = \frac{\omega_1}{\cos 30^\circ}$$

15.212



GIVEN:  $c = 60 \text{ mm}$ ,  $\underline{v}_B = -(180 \text{ mm/s})\underline{i}$ .  
 FIND: (a)  $\underline{\omega}$ , (b)  $\underline{v}_A$



SINCE  $AB = 275 \text{ mm}$   
 $AB^2 = 150^2 + 60^2 + b^2$   
 $b = 225 \text{ mm}$

CLEVIS PREVENTS ROTATION OF ROD ABOUT AE

$$\underline{r}_{E/A} = (50 \text{ mm})\underline{i} - (150 \text{ mm})\underline{j}$$

$$\underline{\omega} \cdot \underline{r}_{E/A} = 0$$

$$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot (50 \underline{i} - 150 \underline{j}) = 0$$

$$50\omega_x - 150\omega_y = 0$$

$$\omega_y = \frac{1}{3}\omega_x \quad (1)$$

$$\underline{r}_{A/B} = -(60 \text{ mm})\underline{i} + (150 \text{ mm})\underline{j} + (225 \text{ mm})\underline{k}$$

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$$

$$\underline{v}_A \underline{k} = -180 \underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -60 & 150 & 225 \end{vmatrix}$$

$$\underline{v}_A \underline{k} = -180 \underline{i} + (225\omega_y - 150\omega_z)\underline{i} + (-50\omega_z - 225\omega_x)\underline{j} + (150\omega_x + 50\omega_z)\underline{k}$$

COEF. OF  $\underline{i}$ :  $+180 = 225\omega_y - 150\omega_z \quad (2)$

COEF. OF  $\underline{j}$ :  $0 = -225\omega_x - 50\omega_z \quad (3)$

COEF. OF  $\underline{k}$ :  $0 = 150\omega_x + 50\omega_z \quad (4)$

3 (EQ 3):  $0 = -675\omega_x - 150\omega_z \quad (5)$

EQ 1 - EQ 5:  $180 = 675\omega_x + 225\omega_y$

SUBSTITUTE  $\omega_y = \frac{1}{3}\omega_x$  FROM EQ 2 INTO EQ 4:

$$180 = 675\omega_x + 225(\frac{1}{3}\omega_x)$$

$$180 = 750\omega_x \quad \omega_x = 0.24 \text{ rad/s}$$

$$\omega_y = \frac{1}{3}\omega_x = \frac{1}{3}(0.24) \quad \omega_y = 0.08 \text{ rad/s}$$

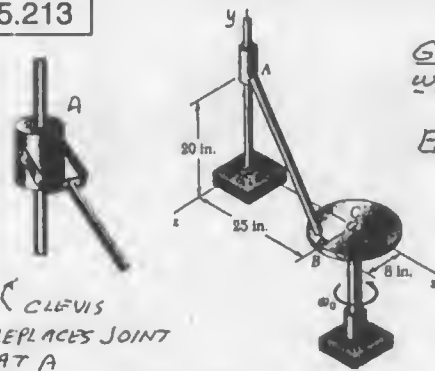
EQ 3:  $0 = -225(0.24) - 50\omega_z \quad \omega_z = -1.08 \text{ rad/s}$

(a)  $\underline{\omega} = (0.24 \text{ rad/s})\underline{i} + (0.08 \text{ rad/s})\underline{j} - (1.08 \text{ rad/s})\underline{k}$

(b) EQ 4:  $\underline{v}_A = 150(0.24) + 50(0.08)$   
 $= 36 + 4$

$$\underline{v}_A = (40 \text{ mm/s})\underline{k}$$

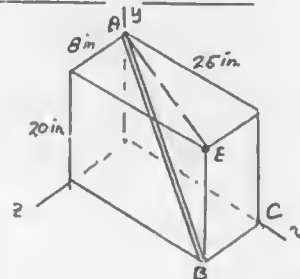
15.213



GIVEN:  $\underline{\omega}_0 = (3 \text{ rad/s})\underline{j}$

FIND: (a)  $\underline{\omega}$   
 (b)  $\underline{v}_A$

CLEVIS REPLACES JOINT AT A



CLEVIS PREVENTS ROTATION OF AB ABOUT AE.

$$\underline{r}_{E/A} = (25 \text{ in})\underline{i} + (8 \text{ in})\underline{k}$$

$$\underline{\omega} \cdot \underline{r}_{E/A} = 0$$

$$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot (25 \underline{i} + 8 \underline{k}) = 0$$

$$25\omega_x + 8\omega_z = 0$$

$$\omega_z = -\frac{25}{8}\omega_x \quad (1)$$

$$\underline{r}_{A/B} = -(25 \text{ in})\underline{i} + (20 \text{ in})\underline{j} - (8 \text{ in})\underline{k}$$

DISK:  $\underline{v}_B = \underline{\omega}_0 \times \underline{r}_{B/C} = (3 \text{ rad/s})\underline{j} \times (8 \text{ in})\underline{k} = (24 \text{ in/s})\underline{i}$

ROD:  $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$\underline{v}_A \underline{j} = 24 \underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -25 & 20 & -8 \end{vmatrix}$$

COEF. OF  $\underline{i}$ :  $-24 = -8\omega_y - 20\omega_z \quad (2)$

COEF. OF  $\underline{j}$ :  $\underline{v}_A = 8\omega_x - 25\omega_z \quad (3)$

COEF. OF  $\underline{k}$ :  $0 = 20\omega_x + 25\omega_z \quad (4)$

SUBSTITUTE  $\omega_z = -\frac{25}{8}\omega_x$  FROM EQ. 1 INTO EQ. 2

$$-24 = -8\omega_y - 20(-\frac{25}{8}\omega_x)$$

$$-24 = -8\omega_y + 62.5\omega_x \quad (5)$$

FROM EQ. 4:  $\omega_y = -\frac{20}{8}\omega_x = -2.5\omega_x \quad (6)$

SUBSTITUTE INTO EQ. 5:

$$-24 = -8(-2.5\omega_x) + 62.5\omega_x$$

$$\omega_x = -0.3483 \text{ rad/s}$$

EQ. 6:  $\omega_y = -2.5(-0.3483) \quad \omega_y = 0.8707 \text{ rad/s}$

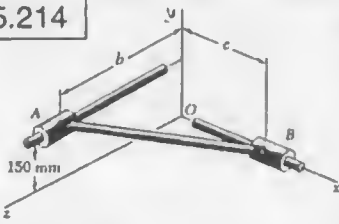
EQ. 1:  $\omega_z = -\frac{25}{8}(-0.3483) \quad \omega_z = 1.0885 \text{ rad/s}$

(a)  $\underline{\omega} = (-0.348 \text{ rad/s})\underline{i} + (0.871 \text{ rad/s})\underline{j} + (1.089 \text{ rad/s})\underline{k}$

(b) EQ. 3:  $\underline{v}_A = 8(-0.3483) - 25(1.0885)$   
 $= -2.79 - 27.21 = -30$

$$\underline{v}_A = -(30 \text{ in./s})\underline{j}$$

15.214

GIVEN:  $c = 175 \text{ mm}$ 

$$\vec{v}_B = -(180 \text{ mm/s}) \hat{i}$$

$$\vec{\omega}_B = 0$$

$$AB = 275 \text{ mm}$$

FIND:  $\vec{\omega}_A$ 

FROM SOLUTION OF PROB. 15.202 WE RECALL

$$b = 150 \text{ mm}; \quad \vec{r}_{A/B} = -(175 \text{ mm}) \hat{i} + (150 \text{ mm}) \hat{j} + (150 \text{ mm}) \hat{k}$$

$$\vec{\omega} = + (1.2 \text{ rad/s}) \hat{j}$$

WE NOW CALCULATE:

$$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B} = -1.2 \hat{j} \times (-175 \hat{i} + 150 \hat{j} + 150 \hat{k})$$

$$\vec{v}_{A/B} = + (210 \text{ mm/s}) \hat{k} + (180 \text{ mm/s}) \hat{i}$$

$$\vec{\omega}_A = \vec{\omega}_B + \vec{\omega}_{A/B} = \vec{\omega}_B + \alpha \times \vec{r}_{A/B} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\alpha_A \hat{k} = 0 + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -175 & 150 & 150 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.2 & 0 \\ 180 & 0 & 210 \end{vmatrix}$$

$$\alpha_A \hat{k} = (150 \alpha_y - 150 \alpha_z) \hat{i} + (-175 \alpha_z - 150 \alpha_x) \hat{j} + (150 \alpha_x + 175 \alpha_y) \hat{k}$$

$$+ 252 \hat{i} - 216 \hat{k}$$

$$\text{COEF. OF } \hat{i}: -252 = 150 \alpha_y - 150 \alpha_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: 0 = -150 \alpha_z - 175 \alpha_x \quad (2)$$

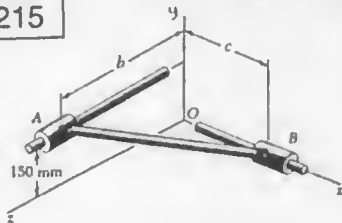
$$\text{COEF. OF } \hat{k}: \alpha_A + 216 = 150 \alpha_x + 175 \alpha_y \quad (3)$$

$\alpha$  IS INDETERMINATE: ASSUME  $\alpha_z = 0$ , FROM EQ. 2,  $\alpha_x = 0$   
 THEN EQ. 1, YIELDS:  $-252 = 150 \alpha_y$ ;  $\alpha_y = -1.68$

$$\text{EQ. 3: } \alpha_A + 216 = 0 + 175(-1.68) = -294$$

$$\alpha_A = -216 - 294 = -510 \quad \alpha_A = -(510 \text{ mm/s}^2) \hat{k}$$

15.215

GIVEN:  $c = 50 \text{ mm}$ 

$$\vec{v}_B = -(180 \text{ mm/s}) \hat{i}$$

$$\vec{\omega}_B = 0$$

$$AB = 275 \text{ mm}$$

FIND:  $\vec{\omega}_A$ 

FROM SOLUTION OF PROB. 15.203, WE RECALL:

$$b = 225 \text{ mm}; \quad \vec{r}_{A/B} = -(50 \text{ mm}) \hat{i} + (150 \text{ mm}) \hat{j} + (225 \text{ mm}) \hat{k}$$

$$\vec{\omega} = (0.8 \text{ rad/s}) \hat{j}$$

$$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B} = 0.8 \hat{j} \times (-50 \hat{i} + 150 \hat{j} + 225 \hat{k})$$

$$\vec{v}_{A/B} = (40 \text{ mm/s}) \hat{k} + (180 \text{ mm/s}) \hat{i}$$

$$\vec{\omega}_A = \vec{\omega}_B + \vec{\omega}_{A/B} = \vec{\omega}_B + \alpha \times \vec{r}_{A/B} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\alpha_A \hat{k} = 0 + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -50 & 150 & 225 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.8 & 0 \\ 180 & 0 & 40 \end{vmatrix}$$

$$\alpha_A \hat{k} = (225 \alpha_y - 150 \alpha_z) \hat{i} + (-50 \alpha_z - 225 \alpha_x) \hat{j} + (150 \alpha_x + 50 \alpha_y) \hat{k}$$

$$+ 32 \hat{i} - 144 \hat{k}$$

(CONTINUED)

15.215 CONTINUED

$$\text{COEF. OF } \hat{i}: -32 = 225 \alpha_y - 150 \alpha_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: 0 = -225 \alpha_x - 50 \alpha_z \quad (2)$$

$$\text{COEF. OF } \hat{k}: \alpha_A + 144 = 150 \alpha_x + 50 \alpha_y \quad (3)$$

$\alpha$  IS INDETERMINATE: ASSUME  $\alpha_x = 0$ , FROM EQ. 2,  $\alpha_z = 0$

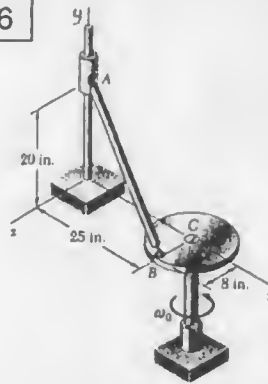
$$\text{THEN EQ. 1, YIELDS: } -32 = 225 \alpha_y \quad \alpha_y = -22/225$$

$$\text{EQ. 3: } \alpha_A + 144 = 0 + 50(-22/225)$$

$$\alpha_A = -144 - 7.111$$

$$\alpha_A = -(151.1 \text{ mm/s}^2) \hat{k}$$

15.216



GIVEN:

$$\vec{\omega}_B = (3 \text{ rad/s}) \hat{j}$$

$$\vec{\omega}_C = 0$$

FIND:  $\vec{\omega}_A$ FROM PROB. 15.204, WE RECALL:  $\vec{v}_B = (24 \text{ in./s}) \hat{i}$ 

$$\vec{r}_{A/B} = -(25 \text{ in.}) \hat{i} + (20 \text{ in.}) \hat{j} - (8 \text{ in.}) \hat{k}; \quad \vec{\omega} = (3 \text{ rad/s}) \hat{j}$$

WE NOW CALCULATE:  $\vec{\omega}_B = \vec{\omega}_C + \vec{\omega}_{B/C} = 3 \hat{j} \times 24 \hat{i} = -(72 \text{ in./s}) \hat{k}$ 

$$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B} = 3 \hat{j} \times (-25 \hat{i} + 20 \hat{j} - 8 \hat{k})$$

$$\vec{v}_{A/B} = -(30 \text{ in./s}) \hat{j} - (24 \text{ in./s}) \hat{i}$$

$$\vec{\omega}_A = \vec{\omega}_B + \alpha \times \vec{r}_{A/B} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\alpha_A \hat{k} = -72 \hat{k} - \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -25 & 20 & -8 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1.2 \\ -24 & -30 & 0 \end{vmatrix}$$

$$\alpha_A \hat{k} = -72 \hat{k} + (-8 \alpha_y - 20 \alpha_z) \hat{i} + (-25 \alpha_z + 8 \alpha_x) \hat{j} + (20 \alpha_x + 25 \alpha_y) \hat{k}$$

$$+ 36 \hat{i} - 28.8 \hat{j}$$

$$\text{COEF. OF } \hat{i}: -36 = -8 \alpha_y - 20 \alpha_z \quad (1)$$

$$\text{COEF. OF } \hat{j}: \alpha_A + 28.8 = 8 \alpha_x - 25 \alpha_z \quad (2)$$

$$\text{COEF. OF } \hat{k}: 72 = 20 \alpha_x + 25 \alpha_y \quad (3)$$

$\alpha$  IS INDETERMINATE: ASSUME  $\alpha_y = 0$ :

$$\text{EQ. 3: } 72 = 20 \alpha_x + 0 \quad \alpha_x = 3.6$$

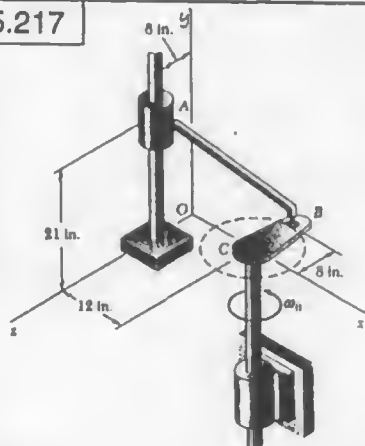
$$\text{EQ. 1: } -36 = 0 - 20 \alpha_z \quad \alpha_z = 1.8$$

$$\text{EQ. 2: } \alpha_A + 28.8 = 8(3.6) - 25(1.8)$$

$$\alpha_A = -28.8 + 28.8 - 45$$

$$\alpha_A = -(45 \text{ in./s}^2) \hat{k}$$

15.217



GIVEN:

$$\omega_B = (10 \text{ rad/s})\underline{j}$$

$$\alpha_B = 0$$

FIND:  $\alpha_A$ FROM SOLUTION OF PROB. 15.205:  $\omega = -(\frac{80}{21} \text{ rad/s})\underline{k}$ 

$$\underline{r}_{A/B} = -(12 \text{ in.})\underline{i} + (21 \text{ in.})\underline{j} + (16 \text{ in.})\underline{k}$$

WE NOW CALCULATE:  $\underline{v}_B = \omega \times \underline{r}_{B/O} = 10\underline{j} \times 8\underline{k} = -(80 \text{ in./s})\underline{i}$ 

$$\alpha_B = \omega \times \underline{v}_B = 10\underline{j} \times -80\underline{i} = (800 \text{ in./s}^2)\underline{k}$$

$$\underline{v}_{A/B} = \omega \times \underline{r}_{A/B} = -\frac{80}{21}\underline{k} \times (-(12)\underline{i} + (21)\underline{j} + (16)\underline{k})$$

$$\underline{v}_{A/B} = (45.714 \text{ in./s})\underline{j} - (80 \text{ in./s})\underline{i}$$

$$\alpha_A = \alpha_B + \underline{\omega} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$\alpha_A \underline{j} = 800\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -12 & 21 & 16 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -80/21 \\ -80 & 45.714 & 0 \end{vmatrix}$$

$$\alpha_A \underline{j} = 800\underline{k} + (16\alpha_y - 21\alpha_z)\underline{i} + (-12\alpha_z - 16\alpha_x)\underline{j} + (21\alpha_x + 12\alpha_y)\underline{k} + 174.15\underline{i} + 304.76\underline{j}$$

$$\text{COEF. OF } \underline{i}: -174.15 = 16\alpha_y - 21\alpha_z \quad (1)$$

$$\text{COEF. OF } \underline{j}: \alpha_A - 304.76 = -12\alpha_z - 16\alpha_x \quad (2)$$

$$\text{COEF. OF } \underline{k}: -800 = 21\alpha_x + 12\alpha_y \quad (3)$$

 $\alpha$  IS INDETERMINANT:ASSUME  $\alpha_y = 0$ 

$$\text{EQ. 1: } -174.15 = 0 - 21\alpha_z \quad \alpha_z = 8.293$$

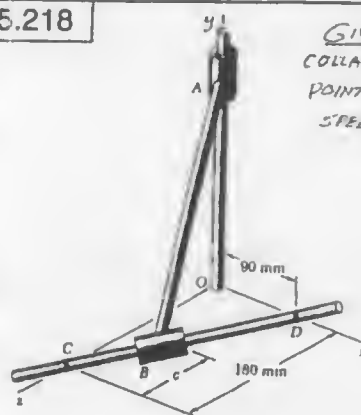
$$\text{EQ. 3: } -800 = 21\alpha_x + 0 \quad \alpha_x = -38.095$$

$$\text{EQ. 2: } \alpha_A - 304.76 = -12(-38.095) - 12(8.293)$$

$$\alpha_A - 304.76 = 609.53 - 99.50$$

$$\alpha_A = (815 \text{ in./s}^2)\underline{j}$$

15.218



GIVEN:  $AB = 300 \text{ mm}$   
COLLAR B MOVES TOWARD  
POINT D AT CONSTANT  
SPEED OF  $50 \text{ mm/s}$   
 $\kappa = 80 \text{ mm}$

FIND:  $\alpha_A$ 

FROM PROB. 15.206 WE RECALL:

$$\underline{v}_B = (22.36 \text{ mm/s})\underline{i} - (44.72 \text{ mm/s})\underline{k}$$

$$\underline{\omega} = (1.118 \text{ rad/s})\underline{j} - (0.3194 \text{ rad/s})\underline{k}$$

$$\underline{r}_{A/B} = -(40 \text{ mm})\underline{i} + (280 \text{ mm})\underline{j} - (100 \text{ mm})\underline{k}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1.118 & -0.3194 \\ -40 & 280 & -100 \end{vmatrix} = (-111.8 + 87.432)\underline{i} + 12.77\underline{j} + 44.72\underline{k}$$

$$\underline{v}_{A/B} = (-22.37 \text{ mm/s})\underline{i} + (12.77 \text{ mm/s})\underline{j} + (44.72 \text{ mm/s})\underline{k}$$

$$\alpha_A = \alpha_B + \underline{\omega} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$\alpha_A \underline{j} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -40 & 280 & -100 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1.118 & -0.3194 \\ -22.37 & 12.77 & 44.72 \end{vmatrix}$$

$$\alpha_A \underline{j} = (-100\alpha_y - 280\alpha_z)\underline{i} + (-40\alpha_z + 100\alpha_x)\underline{j} + (280\alpha_x + 40\alpha_y)\underline{k} + (50.0 + 4.08)\underline{i} + 7.15\underline{j} + 25.5\underline{k}$$

$$\text{COEF. OF } \underline{i}: -54.08 = -100\alpha_y - 280\alpha_z \quad (1)$$

$$\text{COEF. OF } \underline{j}: \alpha_A - 7.15 = 100\alpha_x - 40\alpha_z \quad (2)$$

$$\text{COEF. OF } \underline{k}: -25.5 = 280\alpha_x + 40\alpha_y \quad (3)$$

 $\alpha$  IS INDETERMINANT: ASSUME  $\alpha_y = 0$ 

$$\text{EQ. 1: } -54.08 = 0 - 280\alpha_z \quad \alpha_z = 0.19314$$

$$\text{EQ. 3: } -25.5 = 280\alpha_x + 0 \quad \alpha_x = -0.0893$$

$$\underline{\alpha} = -(0.0893 \text{ rad/s}^2)\underline{i} + (0.19314 \text{ rad/s}^2)\underline{k}$$

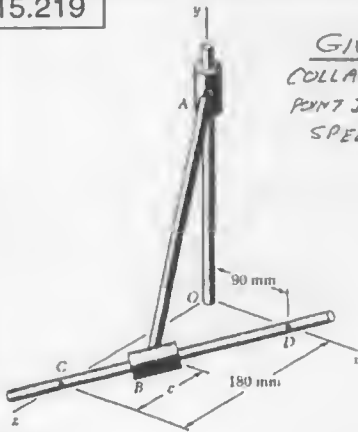
$$\text{EQ. 2: } \alpha_A - 7.15 = 100(-0.0893) - 40(0.19314)$$

$$\alpha_A - 7.15 = -8.93 - 7.73$$

$$\alpha_A = -9.51$$

$$\underline{\alpha}_A = -(9.51 \text{ mm/s}^2)\underline{j}$$

15.219



GIVEN:  $AB = 300 \text{ mm}$   
COLLAR B MOVES TOWARD  
POINT D AT CONSTANT  
SPEED OF  $50 \text{ mm/s}$   
 $\alpha = 120^\circ$

FIND:  $a_D$

$\alpha = 120^\circ$ : FROM PROB. 15.207 WE RECALL:

$$\underline{v}_B = (+22.36 \text{ mm/s})\underline{i} - (44.72 \text{ mm/s})\underline{j}$$

$$\underline{\omega} = (0.7453 \text{ rad/s})\underline{j} - (0.0777 \text{ rad/s})\underline{k}$$

$$\underline{r}_{A/B} = -(60 \text{ mm})\underline{i} + (287.75 \text{ mm})\underline{j} - (60 \text{ mm})\underline{k}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.7453 & -0.0777 \\ -60 & 287.75 & -60 \end{vmatrix}$$

$$= (-44.718 + 22.358)\underline{i} + 4.662\underline{j} + 44.718\underline{k}$$

$$\underline{v}_{A/B} = -(22.36 \text{ mm/s})\underline{i} + (4.662 \text{ mm/s})\underline{j} + (44.718 \text{ mm/s})\underline{k}$$

$$\underline{a}_B = 0$$

$$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$\underline{a}_A \underline{j} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -60 & 287.75 & -60 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.7453 & -0.0777 \\ -22.36 & 4.662 & 44.718 \end{vmatrix}$$

$$\underline{a}_A \underline{j} = (-60\alpha_y - 287.75\alpha_z)\underline{i} + (-60\alpha_z + 60\alpha_x)\underline{j} + (287.75\alpha_x + 60\alpha_y)\underline{k}$$

$$+ (33.33 + 0.363)\underline{i} + 1.737\underline{j} + 16.66\underline{k}$$

$$\text{COEF. OF } \underline{i}: -33.69 = -60\alpha_y - 287.75\alpha_z \quad (1)$$

$$\text{COEF. OF } \underline{j}: \alpha_y - 1.737 = 60\alpha_x - 60\alpha_z \quad (2)$$

$$\text{COEF. OF } \underline{k}: -16.66 = 287.75\alpha_x + 60\alpha_y \quad (3)$$

$\alpha$  IS INDETERMINANT! ASSUME  $\alpha_y = 0$

$$\text{EQ. 1: } -33.69 = 0 - 287.75\alpha_z; \quad \alpha_z = 0.1171$$

$$\text{EQ. 3: } -16.66 = 287.75\alpha_x + 0; \quad \alpha_x = -0.0579$$

$$\underline{\alpha} = -(0.0579 \text{ rad/s})\underline{i} + (0.1171 \text{ rad/s})\underline{k}$$

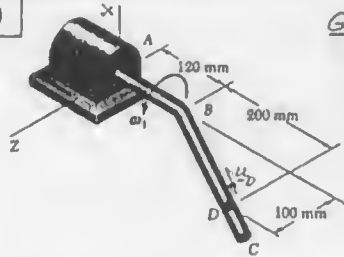
$$\text{EQ. 2: } \alpha_y - 1.737 = 60(-0.0579) - 60(0.1171)$$

$$\alpha_y - 1.737 = -3.474 - 7.026$$

$$\alpha_y = -8.76$$

$$\underline{a}_y = -(8.76 \text{ mm/s}^2)\underline{j}$$

15.220



GIVEN:

$$\omega_1 = 5 \text{ rad/s}$$

$$\alpha_1 = 0$$

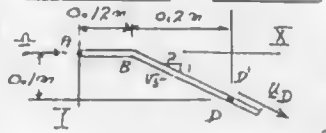
$$u_D = 1.5 \text{ m/s}$$

$$\alpha_D = 0$$

FIND:

$$(a) \underline{v}_D$$

$$(b) \underline{a}_D$$



$$\underline{v}_{D/B} = \underline{v}_D - \underline{v}_B$$

$$\underline{v}_{D/B} = 1.5 \frac{2\underline{i} + \underline{k}}{\sqrt{5}}$$

$$\underline{v}_{D/B} = (1.3416 \text{ m/s})\underline{i} + (0.6708 \text{ m/s})\underline{k}$$

$$\underline{v}_{D/A} = (0.320 \text{ m})\underline{i} + (0.1 \text{ m})\underline{k}$$

$$\underline{\omega} = \omega_1 = (5 \text{ rad/s})\underline{k}$$

$$(a) \text{ VELOCITY OF D: } \underline{v}_D = \underline{\omega} \times \underline{r}_{D/A} = 5\underline{k} \times (0.32\underline{i} + 0.1\underline{k})$$

$$\underline{v}_D = -(0.5 \text{ m/s})\underline{j}$$

$$\underline{v}_D = \underline{v}_{D/B} + \underline{v}_B = -0.5\underline{j} + 1.3416\underline{i} + 0.6708\underline{k}$$

$$\underline{v}_D = (1.342 \text{ m/s})\underline{i} - (0.5 \text{ m/s})\underline{j} + (0.671 \text{ m/s})\underline{k}$$

$$(b) \text{ ACCELERATION OF D: } \underline{a}_D = 0; \quad \underline{\alpha} = 0$$

$$\underline{a}_D = \underline{\alpha} \times \underline{r}_{D/A} + \underline{\omega} \times \underline{v}_{D/A}$$

$$= 0 + \underline{\omega} \times \underline{v}_D = 5\underline{k} \times (-0.5\underline{j}) = -(2.5 \text{ m/s}^2)\underline{i}$$

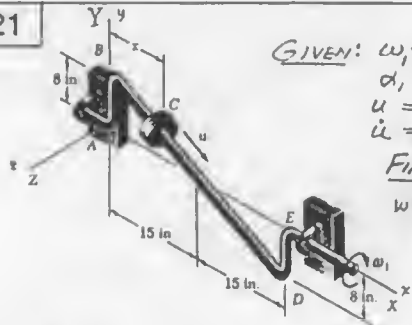
$$\underline{a}_D = 2\underline{\omega} \times \underline{v}_{D/B} = 2(5\underline{k}) \times (1.3416\underline{i} + 0.6708\underline{k}) = -(6.71 \text{ m/s}^2)\underline{j}$$

$$\underline{a}_D = \underline{a}_{D/B} + \underline{a}_B$$

$$= -2.5\underline{i} + 0 - 6.71\underline{j}$$

$$\underline{a}_D = -(6.71 \text{ m/s}^2)\underline{j} - (2.5 \text{ m/s}^2)\underline{i}$$

15.221



GIVEN:  $\omega_1 = 3 \text{ rad/s}$

$$\alpha_1 = 0$$

$$u_D = 34 \text{ in/s}$$

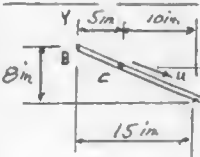
$$\alpha_D = 0$$

FIND:  $\underline{v}_C$  AND  $\underline{a}_C$

WHEN:

$$(a) \theta = 5^\circ$$

$$(b) \theta = 15^\circ$$



$$(a) \theta = 5^\circ$$

$$\frac{y_C}{8 \text{ in.}} = \frac{10 \text{ in.}}{15 \text{ in.}}; \quad y_C = \frac{16}{3} \text{ in.}$$

$$\underline{v}_{C/B} = u_D \underline{\lambda}_{BC} = 34 \frac{15\underline{i} - 8\underline{j}}{17}$$

$$\underline{v}_{C/B} = (30 \text{ in/s})\underline{i} - (16 \text{ in/s})\underline{j}$$

$$\underline{r}_{C/A} = (5 \text{ in.})\underline{i} + \left(\frac{16}{3} \text{ in.}\right)\underline{j}$$

$$\underline{\omega} = \omega_1 \underline{k} = -(3 \text{ rad/s})\underline{k}$$

(CONTINUED)

## 15.221 continued

 $x = 5 \text{ in.}$  VELOCITY:

$$\vec{v}_C = \vec{\omega} \times \vec{r}_{C/A} = (-3 \text{ rad/s}) \hat{k} \times \left[ (5 \text{ in.}) \hat{i} + \left( \frac{16}{3} \text{ in.} \right) \hat{j} \right] = -(16 \text{ in./s}) \hat{k}$$

$$\vec{v}_C = \vec{v}_C + \vec{v}_{C/\mathcal{F}} = (-16 \text{ in./s}) \hat{k} + (30 \text{ in./s}) \hat{j} = -(16 \text{ in./s}) \hat{j}$$

$$\vec{v}_C = (30 \text{ in./s}) \hat{i} - (16 \text{ in./s}) \hat{j} - (16 \text{ in./s}) \hat{k}$$

ACCELERATION:  $\vec{a}_{C/\mathcal{F}} = 0$ ;  $\dot{\vec{\omega}} = 0$ 

$$\vec{a}_C = \dot{\vec{\omega}} \times \vec{r}_{C/A} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{C/A} = \vec{\omega} \times \vec{\omega} \times \vec{r}_{C/A} + \vec{\omega} \times \vec{v}_C$$

$$\vec{a}_C = 0 + (-3 \text{ rad/s}) \hat{k} \times (16 \text{ in./s}) \hat{k} = -(48 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{C/\mathcal{F}} = 2(-3 \text{ rad/s}) \hat{k} \times [(30 \text{ in./s}) \hat{i} - (16 \text{ in./s}) \hat{j}]$$

$$\vec{a}_C = (96 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_C = \vec{a}_C + \vec{a}_{C/\mathcal{F}} + \vec{a}_C$$

$$= -(48 \text{ in./s}^2) \hat{j} + 0 + (96 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_C = -(48 \text{ in./s}^2) \hat{j} + (96 \text{ in./s}^2) \hat{k}$$

(b) FOR  $x = 15 \text{ in.}$  (COLLAR C IS IN  $xz$  PLANE)VELOCITY: FROM PART a:  $\vec{v}_{C/\mathcal{F}} = (30 \text{ in./s}) \hat{i} - (16 \text{ in./s}) \hat{j}$ 

$$\vec{r}_{C/A} = (15 \text{ in.}) \hat{i}; \quad \vec{v}_C = \vec{\omega} \times \vec{r}_{C/A} = -3 \hat{k} \times 15 \hat{i} = 0$$

$$\vec{v}_C = \vec{v}_C + \vec{v}_{C/\mathcal{F}} = 0 + \vec{v}_{C/\mathcal{F}}; \quad \vec{v}_C = (30 \text{ in./s}) \hat{i} - (16 \text{ in./s}) \hat{j}$$

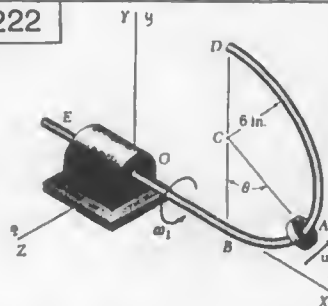
ACCELERATION:  $\vec{a}_{C/\mathcal{F}} = 0$ ;  $\dot{\vec{\omega}} = 0$ 

$$\vec{a}_C = \dot{\vec{\omega}} \times \vec{r}_{C/A} + \vec{\omega} \times \vec{v}_C = 0 + 0; \quad \vec{a}_C = 0$$

$$\vec{a}_C \text{ IS SAME AS IN PART a: } \vec{a}_C = (96 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_C = \vec{a}_C + \vec{a}_{C/\mathcal{F}} + \vec{a}_C = 0 + 0 + \vec{a}_C; \quad \vec{a}_C = (96 \text{ in./s}^2) \hat{k}$$

## 15.222



GIVEN:

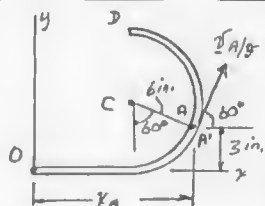
$$\omega_1 = 8 \text{ rad/s}$$

$$\alpha_1 = 0$$

$$u = 30 \text{ in./s}$$

$$\dot{u} = 0$$

$$\theta = 60^\circ$$

FIND: (a)  $\vec{v}_A$ (b)  $\vec{a}_A$ 

$$\vec{r}_{A/O} = x_A \hat{i} + (3 \text{ in.}) \hat{j}$$

$$\vec{v}_{A/\mathcal{F}} = u \angle 60^\circ = (30 \text{ in./s}) \angle 60^\circ$$

$$\vec{v}_{A/\mathcal{F}} = (15 \text{ in./s}) \hat{i} + (25.98 \text{ in./s}) \hat{j}$$

$$\vec{\omega} = \omega_1 \hat{k} = (8 \text{ rad/s}) \hat{k}$$

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/O} = 8 \hat{k} \times (x_A \hat{i} + 3 \hat{j}) = (24 \text{ in./s}) \hat{k}$$

(a) VELOCITY:  $\vec{v}_A = \vec{v}_A + \vec{v}_{A/\mathcal{F}} = 24 \hat{k} + 15 \hat{i} + 25.98 \hat{j}$ 

$$\vec{v}_A = (15 \text{ in./s}) \hat{i} + (26.0 \text{ in./s}) \hat{j} + (24 \text{ in./s}) \hat{k}$$

(CONTINUED)

## 15.222 continued

(b) ACCELERATION:  $\dot{\vec{\omega}} = 0$ 

$$\vec{a}_{A/\mathcal{F}} = \frac{u^2}{r} \angle 30^\circ = \frac{(30 \text{ in./s})^2}{6 \text{ in.}} \angle 30^\circ = 150 \text{ in./s}^2 \angle 30^\circ$$

$$\vec{a}_{A/\mathcal{F}} = -(129.9 \text{ in./s}^2) \hat{i} + (75 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_A = \dot{\vec{\omega}} \times \vec{r}_{A/O} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{A/O} = \vec{\omega} \times \vec{\omega} \times \vec{r}_{A/O} + \vec{\omega} \times \vec{v}_A$$

$$\vec{a}_A = 0 + (8 \text{ rad/s}) \hat{k} \times (24 \text{ in./s}) \hat{k} = -(192 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{A/\mathcal{F}} = 2(8 \text{ rad/s}) \hat{k} \times [(15 \text{ in./s}) \hat{i} + (25.98 \text{ in./s}) \hat{j}]$$

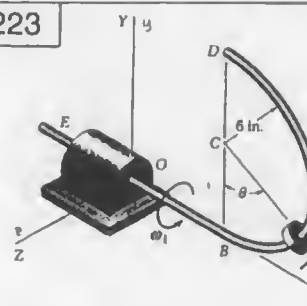
$$\vec{a}_C = (415.7 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_A = \vec{a}_A + \vec{a}_{A/\mathcal{F}} + \vec{a}_C$$

$$= -(192 \text{ in./s}^2) \hat{j} - (129.9 \text{ in./s}^2) \hat{i} + (75 \text{ in./s}^2) \hat{j} + (415.7 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_A = -(129.9 \text{ in./s}^2) \hat{i} - (17 \text{ in./s}^2) \hat{j} + (416 \text{ in./s}^2) \hat{k}$$

## 15.223



GIVEN:

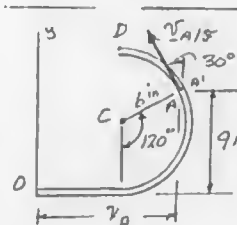
$$\omega_1 = 8 \text{ rad/s}$$

$$\alpha_1 = 0$$

$$u = 30 \text{ in./s}$$

$$\dot{u} = 0$$

$$\theta = 120^\circ$$

FIND: (a)  $\vec{v}_A$ (b)  $\vec{a}_A$ 

$$\vec{r}_{A/O} = x_A \hat{i} + (9 \text{ in.}) \hat{j}$$

$$\vec{v}_{A/\mathcal{F}} = u \angle 30^\circ = (30 \text{ in./s}) \angle 30^\circ$$

$$\vec{v}_{A/\mathcal{F}} = -(15 \text{ in./s}) \hat{i} + (25.98 \text{ in./s}) \hat{j}$$

$$\vec{\omega} = \omega_1 \hat{k} = (8 \text{ rad/s}) \hat{k}$$

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/O} = 8 \hat{k} \times (x_A \hat{i} + 9 \hat{j}) = (72 \text{ in./s}) \hat{k}$$

(a) VELOCITY:  $\vec{v}_A = \vec{v}_A + \vec{v}_{A/\mathcal{F}}$ 

$$= 72 \hat{k} - 15 \hat{i} + 25.98 \hat{j}$$

$$\vec{v}_A = -(15 \text{ in./s}) \hat{i} + (26.0 \text{ in./s}) \hat{j} + (72 \text{ in./s}) \hat{k}$$

(b) ACCELERATION:  $\dot{\vec{\omega}} = 0$ 

$$\vec{a}_{A/\mathcal{F}} = \frac{u^2}{r} \angle 30^\circ = \frac{(30 \text{ in./s})^2}{6 \text{ in.}} \angle 30^\circ = 150 \text{ in./s}^2 \angle 30^\circ$$

$$\vec{a}_{A/\mathcal{F}} = -(129.9 \text{ in./s}^2) \hat{i} - (75 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_A = \dot{\vec{\omega}} \times \vec{r}_{A/O} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{A/O} = \vec{\omega} \times \vec{\omega} \times \vec{r}_{A/O} + \vec{\omega} \times \vec{v}_A$$

$$\vec{a}_A = 0 + (8 \text{ rad/s}) \hat{k} \times (72 \text{ in./s}) \hat{k} = -(576 \text{ in./s}^2) \hat{j}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{A/\mathcal{F}} = 2(8 \text{ rad/s}) \hat{k} \times [-(15 \text{ in./s}) \hat{i} + (25.98 \text{ in./s}) \hat{j}]$$

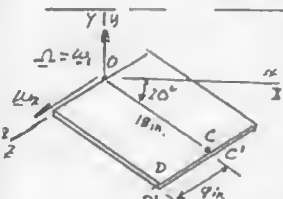
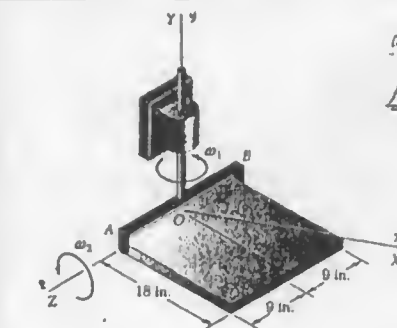
$$\vec{a}_C = (415.7 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_A = \vec{a}_A + \vec{a}_{A/\mathcal{F}} + \vec{a}_C$$

$$= -(576 \text{ in./s}^2) \hat{j} - (129.9 \text{ in./s}^2) \hat{i} - (75 \text{ in./s}^2) \hat{j} + (415.7 \text{ in./s}^2) \hat{k}$$

$$\vec{a}_A = -(129.9 \text{ in./s}^2) \hat{i} - (651 \text{ in./s}^2) \hat{j} + (416 \text{ in./s}^2) \hat{k}$$

# 15.224 and 15.225



**PROBLEM 15.224**

**GIVEN:**  $\omega_1 = 3 \text{ rad/s}$   
 $\omega_2 = 4 \text{ rad/s}$   
 $\alpha_1 = \alpha_2 = 0$

**FIND:** (a)  $\vec{v}_C$   
 (b)  $\vec{a}_C$

**PROBLEM 15.225**

**FIND:** (a)  $\vec{v}_D$   
 (b)  $\vec{a}_D$

$\vec{r}_{C/O} = (18 \text{ in.}) \cos 20^\circ \hat{i} - (9 \text{ in.}) \sin 20^\circ \hat{j}$   
 $\vec{r}_{C/O} = (16.914 \text{ in.}) \hat{i} - (6.158 \text{ in.}) \hat{j}$   
 $\vec{r}_{D/O} = \vec{r}_{C/O} + (9 \text{ in.}) \hat{i}$

**PROBLEM 15.224**

$\vec{\omega}_1 = \omega_1 \hat{k} = (3 \text{ rad/s}) \hat{k}$   
 $\vec{v}_{C/\mathcal{F}} = \vec{\omega}_1 \times \vec{r}_{C/O} = (3 \text{ rad/s}) \hat{k} \times [(16.914 \text{ in.}) \hat{i} - (6.158 \text{ in.}) \hat{j}]$   
 $\vec{v}_{C/\mathcal{F}} = (67.258 \text{ in./s}) \hat{j} + (24.625 \text{ in./s}) \hat{i}$   
 $\vec{v}_C = \vec{\omega}_1 \times \vec{r}_{C/O} = (3 \text{ rad/s}) \hat{k} \times [(16.914 \text{ in.}) \hat{i} - (6.158 \text{ in.}) \hat{j}]$   
 $\vec{v}_C = (50.74 \text{ in./s}) \hat{i} - (147.75 \text{ in./s}) \hat{j}$   
 $\vec{a}_C = \vec{a}_{C/\mathcal{F}} + \vec{a}_{C/\mathcal{F}} = -50.74 \hat{i} + 67.258 \hat{j} + 24.625 \hat{i}$   
 $\vec{a}_C = (24.6 \text{ in./s}^2) \hat{i} + (67.2 \text{ in./s}^2) \hat{j} - (50.7 \text{ in./s}^2) \hat{i}$   
 $\vec{a}_C = (24.6 \text{ in./s}^2) \hat{i} + (67.2 \text{ in./s}^2) \hat{j} - (50.7 \text{ in./s}^2) \hat{i}$   
 $\vec{a}_C = (24.6 \text{ in./s}^2) \hat{i} + (67.2 \text{ in./s}^2) \hat{j} - (50.7 \text{ in./s}^2) \hat{i}$

**PROBLEM 15.224**

**VELOCITY:**

$\vec{v}_{D/\mathcal{F}} = \vec{\omega}_2 \times \vec{r}_{D/O} = \omega_2 \hat{k} \times (\vec{r}_{C/O} + 9 \hat{i}) = \omega_2 \times \vec{r}_{C/O}$   
 THUS  $\vec{v}_{D/\mathcal{F}} = \vec{v}_{C/\mathcal{F}} = (67.258 \text{ in./s}) \hat{j} + (24.625 \text{ in./s}) \hat{i}$   
 $\vec{v}_D = \vec{\omega}_1 \times \vec{r}_{D/O} = \omega_1 \hat{k} \times (\vec{r}_{C/O} + 9 \hat{i}) = \vec{v}_{C/\mathcal{F}} + (27 \text{ in./s}) \hat{i}$   
 $\vec{v}_D = (27 \text{ in./s}) \hat{i} - (50.74 \text{ in./s}) \hat{j} + 67.258 \hat{j} + 24.625 \hat{i}$   
 $\vec{v}_D = (51.6 \text{ in./s}) \hat{i} + (67.7 \text{ in./s}) \hat{j} - (50.7 \text{ in./s}) \hat{j}$

(CONTINUED)

# 15.225 continued

ACCELERATION  $\dot{\omega} = 0$

$\vec{a}_{D/\mathcal{F}} = \vec{\omega}_2 \times \vec{v}_{D/\mathcal{F}} = (4 \text{ rad/s}) \hat{k} \times [(67.258 \text{ in./s}) \hat{j} + (24.625 \text{ in./s}) \hat{i}]$   
 $\vec{a}_{D/\mathcal{F}} = -(270.63 \text{ in./s}^2) \hat{i} + (98.5 \text{ in./s}^2) \hat{j}$   
 NOTE:  $\vec{a}_{D/\mathcal{F}} = \vec{a}_{C/\mathcal{F}}$  SINCE  $\vec{v}_{D/\mathcal{F}} = \vec{v}_{C/\mathcal{F}}$

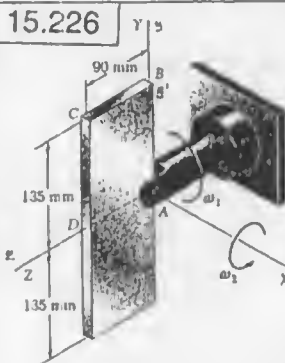
$\vec{a}_{D/\mathcal{F}} = \vec{\omega}_1 \times \vec{r}_{D/O} + \vec{\omega}_2 \times \vec{v}_{D/\mathcal{F}}$   
 $= 0 + (3 \text{ rad/s}) \hat{k} \times [(27 \text{ in./s}) \hat{i} - (50.74 \text{ in./s}) \hat{j}]$   
 $\vec{a}_{D/\mathcal{F}} = -(81 \text{ in./s}^2) \hat{j} - (152.2 \text{ in./s}^2) \hat{i}$

$\vec{a}_C = 2 \vec{\omega}_1 \times \vec{v}_{D/\mathcal{F}}$  BUT WE KNOW THAT  $\vec{v}_{D/\mathcal{F}} = \vec{v}_C/8$   
 THUS CORIOLIS ACCELERATION  $\vec{a}_C$  FOR POINT D IS SAME AS  $\vec{a}_C$  FOR POINT C.  
 $\vec{a}_C = -(147.75 \text{ in./s}^2) \hat{j}$

$\vec{a}_D = \vec{a}_{D/\mathcal{F}} + \vec{a}_{D/\mathcal{F}} + \vec{a}_C$   
 $= -81 \hat{j} - 152.2 \hat{i} - 270.63 \hat{i} + 98.5 \hat{j} - 147.75 \hat{j}$

$\vec{a}_D = -(423 \text{ in./s}^2) \hat{i} + (98.5 \text{ in./s}^2) \hat{j} - (227 \text{ in./s}^2) \hat{j}$

# 15.226



**GIVEN:**  
 $\omega_1 = 9 \text{ rad/s}$   
 $\omega_2 = 12 \text{ rad/s}$   
 $\alpha_1 = \alpha_2 = 0$

**FIND:**  $\vec{v}_B$   
 $\vec{a}_B$

$\vec{r}_{B/A} = 0.135 \text{ m} \hat{j}$   
 $\vec{\omega}_1 = \omega_1 \hat{k} = (9 \text{ rad/s}) \hat{k}$   
 $\vec{v}_{B/\mathcal{F}} = \vec{\omega}_1 \times \vec{r}_{B/A} = (9 \text{ rad/s}) \hat{k} \times (0.135 \text{ m}) \hat{j} = -(1.215 \text{ m/s}) \hat{i}$   
 $\vec{v}_{B/\mathcal{F}} = \vec{\omega}_2 \times \vec{r}_{B/A} = (12 \text{ rad/s}) \hat{k} \times (0.135 \text{ m}) \hat{j} = -(1.62 \text{ m/s}) \hat{i}$   
 $\vec{v}_B = \vec{v}_{B/\mathcal{F}} + \vec{v}_{B/\mathcal{F}}$   
 $\vec{v}_B = -(1.215 \text{ m/s}) \hat{i} + (1.62 \text{ m/s}) \hat{j}$

**ACCELERATION:**

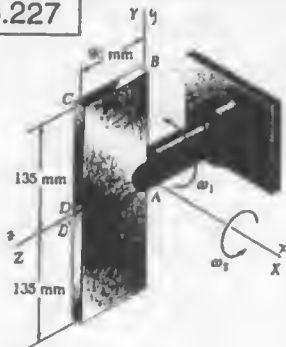
$\vec{a}_{B/\mathcal{F}} = \vec{\omega}_1 \times \vec{v}_{B/\mathcal{F}} = \omega_1 \hat{k} \times \vec{v}_{B/\mathcal{F}}$   
 $= (9 \text{ rad/s}) \hat{k} \times (-1.215 \text{ m/s}) \hat{i} = -(10.935 \text{ m/s}^2) \hat{j}$

$\vec{a}_{B/\mathcal{F}} = \vec{\omega}_2 \times \vec{v}_{B/\mathcal{F}} = \omega_2 \hat{k} \times \vec{v}_{B/\mathcal{F}}$   
 $= (12 \text{ rad/s}) \hat{k} \times (-1.62 \text{ m/s}) \hat{i} = -(19.44 \text{ m/s}^2) \hat{j}$

$\vec{a}_C = 2 \vec{\omega}_1 \times \vec{v}_{B/\mathcal{F}} = 2(9 \text{ rad/s}) \hat{k} \times (-1.62 \text{ m/s}) \hat{i} = 0$

$\vec{a}_B = \vec{a}_{B/\mathcal{F}} + \vec{a}_{B/\mathcal{F}} + \vec{a}_C$   
 $= -(10.935 \text{ m/s}^2) \hat{j} - (19.44 \text{ m/s}^2) \hat{j}$   
 $\vec{a}_B = -(30.4 \text{ m/s}^2) \hat{j}$

15.227



GIVEN:  $\omega_1 = 9 \text{ rad/s}$   
 $\omega_2 = 12 \text{ rad/s}$   
 $d_1 = d_2 = 0$

FIND:  $\vec{v}_D$   
 $\vec{a}_D$

$$\vec{r}_{D/A} = (0.09 \text{ m}) \hat{j}$$

$$\vec{\omega} = \omega_1 \hat{k} = (9 \text{ rad/s}) \hat{k}$$

VELOCITY:

$$\vec{v}_D = \vec{\omega} \times \vec{r}_{D/A} = (9 \text{ rad/s}) \hat{k} \times (0.09 \text{ m}) \hat{j} = 0$$

$$\vec{v}_{D/2} = \omega_2 \times \vec{r}_{D/A} = (12 \text{ rad/s}) \hat{k} \times (0.09 \text{ m}) \hat{j} = -(1.08 \text{ m/s}) \hat{j}$$

$$\vec{v}_D = \vec{v}_D + \vec{v}_{D/2}$$

$$\vec{v}_D = -(1.08 \text{ m/s}) \hat{j}$$

ACCELERATION

$$\vec{a}_D = \vec{\omega} \times \vec{\omega} \times \vec{r}_{D/A} = \vec{\omega} \times \vec{v}_D = \vec{\omega} \times 0 = 0$$

$$\vec{a}_{D/2} = \omega_2 \times \omega_2 \times \vec{r}_{D/A} = \omega_2 \times \vec{v}_{D/2}$$

$$\vec{a}_{D/2} = (12 \text{ rad/s}) \hat{k} \times (-1.08 \text{ m/s}) \hat{j} = -(12.96 \text{ m/s}^2) \hat{j}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{D/2} = 2(9 \text{ rad/s}) \hat{k} \times (-1.08 \text{ m/s}) \hat{j}$$

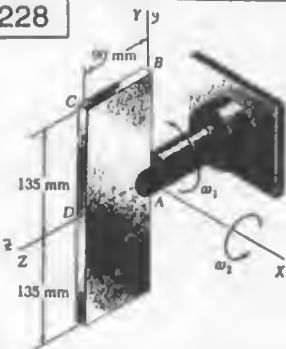
$$\vec{a}_C = (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_D = \vec{a}_D + \vec{a}_{D/2} + \vec{a}_C$$

$$= 0 - (12.96 \text{ m/s}^2) \hat{j} + (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_D = (19.44 \text{ m/s}^2) \hat{i} - (12.96 \text{ m/s}^2) \hat{j}$$

15.228



GIVEN:  $\omega_1 = 9 \text{ rad/s}$   
 $\omega_2 = 12 \text{ rad/s}$   
 $d_1 = d_2 = 0$

FIND:  $\vec{v}_C$   
 $\vec{a}_C$

$$\vec{r}_{C/A} = (0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}$$

$$\vec{\omega} = \omega_1 \hat{k} = (9 \text{ rad/s}) \hat{k}$$

VELOCITY:  $\vec{v}_C = \vec{\omega} \times \vec{r}_{C/A} = (9 \text{ rad/s}) \hat{k} \times [(0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}]$

$$\vec{v}_C = -(1.215 \text{ m/s}) \hat{i}$$

$$\vec{v}_{C/2} = \omega_2 \times \vec{r}_{C/A} = (12 \text{ rad/s}) \hat{k} \times [(0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}]$$

$$\vec{v}_{C/2} = (1.62 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j}$$

$$\vec{v}_C = \vec{v}_C + \vec{v}_{C/2} = \vec{v}_C = -(1.215 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j} + (1.62 \text{ m/s}) \hat{i}$$

ACCELERATION:

$$\vec{a}_C = \vec{\omega} \times \vec{\omega} \times \vec{r}_{C/A} = \vec{\omega} \times \vec{v}_C = (9 \text{ rad/s}) \hat{k} \times [-(1.215 \text{ m/s}) \hat{i}] = -(10.95 \text{ m/s}^2) \hat{j}$$

$$\vec{a}_{C/2} = \omega_2 \times \omega_2 \times \vec{r}_{C/A} = \omega_2 \times \vec{v}_{C/2}$$

$$\vec{a}_{C/2} = (12 \text{ rad/s}) \hat{k} \times [1.62 \hat{i} - 1.08 \hat{j}] = -(19.44 \text{ m/s}^2) \hat{j} - (12.96 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{C/2} = 2(9 \text{ rad/s}) \hat{k} \times [(1.62 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j}]$$

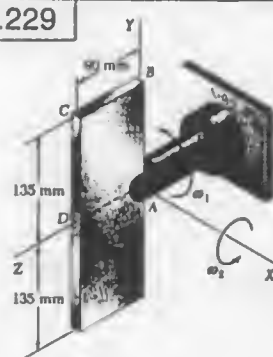
$$\vec{a}_C = (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = \vec{a}_C + \vec{a}_{C/2} + \vec{a}_C$$

$$= -(10.95 \text{ m/s}^2) \hat{j} - (19.44 \text{ m/s}^2) \hat{j} - (12.96 \text{ m/s}^2) \hat{i} + (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = (19.44 \text{ m/s}^2) \hat{i} - (30.4 \text{ m/s}^2) \hat{j} - (12.96 \text{ m/s}^2) \hat{i}$$

15.229



GIVEN:  $\omega_1 = 9 \text{ rad/s}$   
 $d_1 = -45 \text{ rad/s}^2$   
 $\omega_2 = 12 \text{ rad/s}$   
 $d_2 = -60 \text{ rad/s}^2$

FIND:  $\vec{v}_C$   
 $\vec{a}_C$

$$\vec{\omega} = \omega_1 \hat{k} = (9 \text{ rad/s}) \hat{k}$$

$$\vec{\omega} = \omega_1 \hat{k} = (-45 \text{ rad/s}^2) \hat{k}$$

$$\omega_2 \hat{k} = (12 \text{ rad/s}) \hat{k}$$

$$\omega_2 \hat{k} = (-60 \text{ rad/s}^2) \hat{k}$$

$$\vec{r}_{C/A} = (0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}$$

VELOCITY:  $\vec{v}_C = \vec{\omega} \times \vec{r}_{C/A} = (9 \text{ rad/s}) \hat{k} \times [(0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}]$

$$\vec{v}_C = -(1.215 \text{ m/s}) \hat{i}$$

$$\vec{v}_{C/2} = \omega_2 \times \vec{r}_{C/A} = (12 \text{ rad/s}) \hat{k} \times [(0.135 \text{ m}) \hat{j} + (0.09 \text{ m}) \hat{k}]$$

$$\vec{v}_{C/2} = (1.62 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j}$$

$$\vec{v}_C = \vec{v}_C + \vec{v}_{C/2}$$

$$\vec{v}_C = -(1.215 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j} + (1.62 \text{ m/s}) \hat{i}$$

ACCELERATION

$$\vec{a}_C = \vec{\omega} \times \vec{v}_C + \vec{\omega} \times \vec{\omega} \times \vec{r}_{C/A} = \vec{\omega} \times \vec{v}_C + \vec{\omega} \times \vec{v}_C$$

$$= (-45 \text{ rad/s}^2) \hat{k} \times [-(1.215 \text{ m/s}) \hat{i}] + (9 \text{ rad/s}) \hat{k} \times [-(1.215 \text{ m/s}) \hat{i}]$$

$$+ (9 \text{ rad/s}) \hat{k} \times [-(1.215 \text{ m/s}) \hat{i}]$$

$$\vec{a}_C = (6.075 \text{ m/s}^2) \hat{j} - (10.94 \text{ m/s}^2) \hat{j}$$

$$\vec{a}_{C/2} = \omega_2 \times \vec{v}_{C/2} + \omega_2 \times \omega_2 \times \vec{r}_{C/A} = \omega_2 \times \vec{v}_{C/2} + \omega_2 \times \vec{v}_{C/2}$$

$$= (-60 \text{ rad/s}^2) \hat{k} \times [(1.62 \text{ m/s}) \hat{i} + (0.09 \text{ m}) \hat{k}] + (12 \text{ rad/s}) \hat{k} \times [(1.62 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j}]$$

$$= -(9.10 \text{ m/s}^2) \hat{j} + (5.4 \text{ m/s}^2) \hat{j} - (19.44 \text{ m/s}^2) \hat{j} - (12.96 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_{C/2} = -(14.04 \text{ m/s}^2) \hat{j} - (21.06 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = 2 \vec{\omega} \times \vec{v}_{C/2} = 2(9 \text{ rad/s}) \hat{k} \times [(1.62 \text{ m/s}) \hat{i} - (1.08 \text{ m/s}) \hat{j}]$$

$$\vec{a}_C = (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = \vec{a}_C + \vec{a}_{C/2} + \vec{a}_C$$

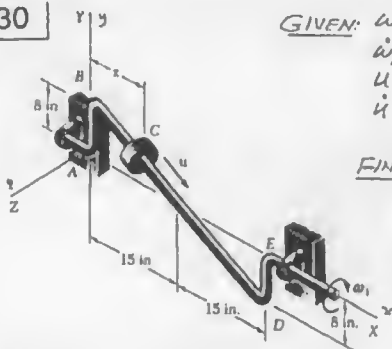
$$= (6.075 \text{ m/s}^2) \hat{j} - (10.94 \text{ m/s}^2) \hat{j}$$

$$- (14.04 \text{ m/s}^2) \hat{j} - (21.06 \text{ m/s}^2) \hat{i} + (19.44 \text{ m/s}^2) \hat{i}$$

$$\vec{a}_C = (25.5 \text{ m/s}^2) \hat{i} - (25.0 \text{ m/s}^2) \hat{j} - (21.1 \text{ m/s}^2) \hat{i}$$

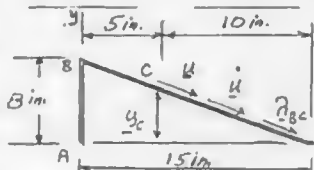


15.230



GIVEN:  $\omega_1 = 3 \text{ rad/s}$   
 $\dot{\omega}_1 = 12 \text{ rad/s}^2$   
 $u = 34 \text{ in/s}$   
 $\dot{u} = -85 \text{ in/s}^2$

FIND:  $\dot{v}_C$  AND  $a_C$   
 WHEN,  
 (a)  $\gamma = 5 \text{ in.}$   
 (b)  $\gamma = 15 \text{ in.}$



(a)  $\gamma = 5 \text{ in.}$

$$\frac{y_C}{8 \text{ in.}} = \frac{10 \text{ in.}}{15 \text{ in.}}; y_C = \frac{16}{3} \text{ in.}$$

$$\frac{17}{15} \quad \frac{15i - 8j}{17}$$

$$\dot{v}_{C/\mathcal{F}} = u \dot{r}_{BC} = (34 \text{ in/s}) \frac{15i - 8j}{17} = (30 \text{ in/s})i - (16 \text{ in/s})j$$

$$a_{C/\mathcal{F}} = \dot{u} \dot{r}_{BC} = (-85 \text{ in/s}^2) \frac{15i - 8j}{17} = -(75 \text{ in/s}^2)i + (40 \text{ in/s}^2)j$$

$$\underline{\Omega} = \omega_1 \dot{r}_{BC} = (3 \text{ rad/s})i; \quad \dot{\underline{\Omega}} = \dot{\omega}_1 \dot{r}_{BC} = (12 \text{ rad/s}^2)i$$

$$r_{C/A} = (5 \text{ in.})i + (16/3 \text{ in.})j$$

VELOCITY:

$$\dot{v}_C = \underline{\Omega} \times r_{C/A} = (3 \text{ rad/s})i \times [(5 \text{ in.})i + (16/3 \text{ in.})j] = -(16 \text{ in/s})j$$

$$\dot{v}_C = \dot{v}_{C/\mathcal{F}} + \dot{v}_{C/\mathcal{F}} = -(16 \text{ in/s})j + (30 \text{ in/s})i - (16 \text{ in/s})j$$

$$\dot{v}_C = (30 \text{ in/s})i - (16 \text{ in/s})j - (16 \text{ in/s})j$$

ACCELERATION:  $a_{C/\mathcal{F}}$ , SEE ABOVE

$$\underline{a}_C = -\underline{\Omega} \times r_{C/A} + \underline{\Omega} \times \underline{r}_{C/A} = -\underline{\Omega} \times r_{C/A} + \underline{\Omega} \times \dot{r}_{C/A}$$

$$= -(12 \text{ rad/s}^2)i \times [(5 \text{ in.})i + (16/3 \text{ in.})j] + (3 \text{ rad/s})i \times (16 \text{ in/s})j$$

$$\underline{a}_C = -(64 \text{ in/s}^2)j - (48 \text{ in/s}^2)j$$

$$\underline{a}_C = 2 \underline{\Omega} \times \dot{r}_{C/\mathcal{F}} = 2(3 \text{ rad/s})i \times [(30 \text{ in/s})i - (16 \text{ in/s})j] = (96 \text{ in/s}^2)j$$

$$\underline{a}_C = \underline{a}_{C/\mathcal{F}} + \underline{a}_{C/\mathcal{F}} + \underline{a}_C$$

$$= -(64 \text{ in/s}^2)j - (48 \text{ in/s}^2)j - (75 \text{ in/s}^2)i + (40 \text{ in/s}^2)j + (96 \text{ in/s}^2)j$$

$$\underline{a}_C = -(75 \text{ in/s}^2)i - (8 \text{ in/s}^2)j + (32 \text{ in/s}^2)j$$

(b)  $\gamma = 15 \text{ in.}$  (COLLAR C IS IN XZ PLANE):  $\dot{v}_C = 0$

VELOCITY:  $\dot{v}_{C/\mathcal{F}} = \text{SAME AS IN PART A ABOVE}$

$$\dot{v}_C = \dot{v}_{C/\mathcal{F}} + \dot{v}_{C/\mathcal{F}} = 0 + \dot{v}_{C/\mathcal{F}}; \quad \dot{v}_C = (30 \text{ in/s})i - (16 \text{ in/s})j$$

ACCELERATION:

$$\underline{a}_{C/\mathcal{F}} = \text{SAME AS IN PART A ABOVE}$$

$$\underline{a}_C = 0; \text{ SINCE COLLAR LIES ON AXIS OF ROTATION}$$

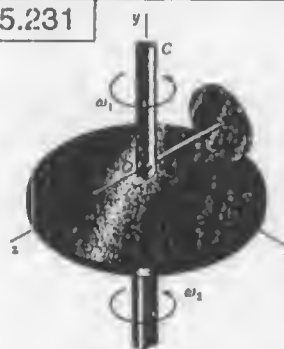
$$\underline{a}_C = 2 \underline{\Omega} \times \dot{r}_{C/\mathcal{F}} = \text{SAME AS IN PART A ABOVE}$$

$$\underline{a}_C = \underline{a}_{C/\mathcal{F}} + \underline{a}_{C/\mathcal{F}} + \underline{a}_C$$

$$= 0 - (75 \text{ in/s}^2)i + (40 \text{ in/s}^2)j + (96 \text{ in/s}^2)j$$

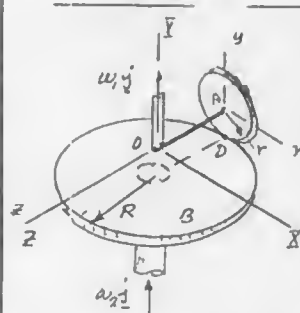
$$\underline{a}_C = -(75 \text{ in/s}^2)i + (40 \text{ in/s}^2)j + (96 \text{ in/s}^2)j$$

15.231



GIVEN:  $\omega_1 = \omega_1 \hat{k}$   
 $\dot{\omega}_1 = 0$   
 $\omega_2 = \omega_2 \hat{k}$   
 $\dot{\omega}_2 = 0$

FIND: FOR DISK,  
 (a)  $\omega_A$   
 (b)  $a_A$



MOVING FRAME  $\mathcal{F}$  AT  $\mathcal{F}$   
 ROTATES WITH  
 ANGULAR VELOCITY  $\underline{\Omega} = \omega_1 \hat{j}$   
 $\underline{\omega}_{\text{Disk}/\mathcal{F}} = \omega_1 \hat{i} + \omega_2 \hat{k}$   
 $\underline{r}_{D/A} = -r \hat{j} - R \hat{k}$

(a) TOTAL ANGULAR VELOCITY OF DISK A:

$$\underline{\omega} = \omega_1 \hat{j} + \omega_{\text{Disk}/\mathcal{F}} = \omega_1 \hat{j} + \omega_1 \hat{i} + \omega_2 \hat{k} \quad (1)$$

Denote by D POINT OF CONTACT OF DISKS

CONSIDER DISK B:

$$\underline{v}_D = \omega_2 \hat{j} \times (-R \hat{k}) = -R \omega_2 \hat{i} \quad (2)$$

CONSIDER SYSTEM OC, OA, AND DISK A.

$$\underline{v}_D = \underline{\Omega} \times r_{D/A} = \omega_1 \hat{j} \times (-r \hat{j} - R \hat{k}) = -R \omega_1 \hat{i}$$

$$\underline{v}_D/\mathcal{F} = \underline{\omega}_{\text{Disk}/\mathcal{F}} \times \underline{r}_{D/A} = (\omega_1 \hat{i} + \omega_2 \hat{k}) \times (-r \hat{j} - R \hat{k})$$

$$= -r \omega_1 \hat{k} + R \omega_2 \hat{j} + r \omega_2 \hat{i}$$

$$\underline{v}_D = \underline{v}_D + \underline{v}_D/\mathcal{F} = -R \omega_1 \hat{i} - r \omega_1 \hat{k} + R \omega_2 \hat{j} + r \omega_2 \hat{i} \quad (3)$$

EQUATE  $\underline{v}_D = \underline{v}_D$  FROM EQ. 2 AND EQ. 3

$$-R \omega_2 \hat{i} = -R \omega_1 \hat{i} + r \omega_2 \hat{j} + R \omega_2 \hat{j} - r \omega_2 \hat{k}$$

COEF. OF  $\hat{j}$ :  $0 = R \omega_2 \rightarrow \omega_2 = 0$

COEF. OF  $\hat{k}$ :  $-R \omega_2 = -R \omega_1 + r \omega_2; \quad \omega_2 = \frac{R}{r}(\omega_1 - \omega_2)$

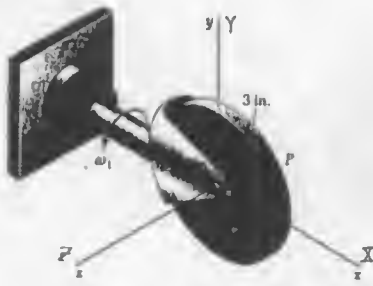
$$\underline{\omega} = \omega_1 \hat{j} + \frac{R}{r}(\omega_1 - \omega_2) \hat{k}$$

(b) DISK A ROTATES ABOUT y AXIS AT RATE  $\omega_1$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1 \hat{j} \times \left[ \omega_1 \hat{j} + \frac{R}{r}(\omega_1 - \omega_2) \hat{k} \right]$$

$$\underline{\alpha} = \omega_1 (\omega_1 - \omega_2) \frac{R}{r} \hat{i}$$

15.232



GIVEN:  
 $\omega_1 = 5 \text{ rad/s}$ ,  $\alpha_1 = 0$   
 $\omega_2 = 4 \text{ rad/s}$ ,  $\alpha_2 = 0$   
 $\theta = 30^\circ$

FIND:  $\underline{a}_P$

FRAME OXYZ IS FIXED. MOVING FRAME Oxyz ROTATES WITH ANGULAR VELOCITY  $\underline{\Omega} = \omega_1 \underline{i} = (5 \text{ rad/s}) \underline{i}$

$$\underline{r}_{P/O} = (3 \text{ in.}) \cos 30^\circ \underline{i} + (3 \text{ in.}) \sin 30^\circ \underline{j}$$

$$= (2.598 \text{ in.}) \underline{i} + (1.5 \text{ in.}) \underline{j}$$

$$\underline{\omega}_{\text{Disk}} = \omega_2 \underline{k} = (4 \text{ rad/s}) \underline{k}$$

$$\underline{v}_{P/\mathcal{F}} = \underline{\omega}_{\text{Disk}} \times \underline{r}_{P/O} = (4 \text{ rad/s}) \underline{k} \times (2.598 \underline{i} + 1.5 \underline{j})$$

$$\underline{v}_{P/\mathcal{F}} = (10.392 \text{ in./s}) \underline{j} - (6 \text{ in./s}) \underline{i}$$

$$\underline{v}_P = \underline{\Omega} \times \underline{r}_{P/O} = (5 \text{ rad/s}) \underline{i} \times (2.598 \underline{i} + 1.5 \underline{j})$$

$$\underline{v}_P = (7.5 \text{ in./s}) \underline{j}$$

ACCELERATION:  $\underline{a}_P = \underline{\Omega} \times \underline{v}_P = (5 \text{ rad/s}) \underline{i} \times (7.5 \text{ in./s}) \underline{j} = (37.5 \text{ in./s}^2) \underline{j}$

$$\underline{a}_{P/\mathcal{F}} = \underline{\omega}_{\text{Disk}} \times \underline{v}_{P/\mathcal{F}} = (4 \text{ rad/s}) \underline{k} \times (10.392 \underline{j} - 6 \underline{i})$$

$$\underline{a}_{P/\mathcal{F}} = -(41.569 \text{ in./s}^2) \underline{i} - (24 \text{ in./s}^2) \underline{j}$$

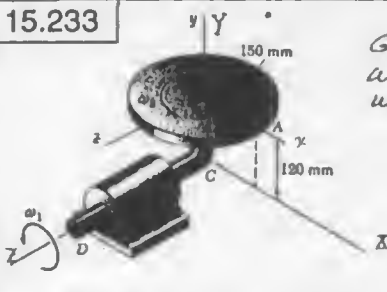
$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{P/\mathcal{F}} = 2(5 \text{ rad/s}) \underline{i} \times (10.392 \underline{j} - 6 \underline{i})$$

$$\underline{a}_C = (103.92 \text{ in./s}^2) \underline{j}$$

$$\underline{a}_P = \underline{a}_C + \underline{a}_{P/\mathcal{F}} = 37.5 \underline{j} - 41.569 \underline{i} - 24 \underline{j} + 103.92 \underline{j}$$

$$\underline{a}_P = -(41.6 \text{ in./s}^2) \underline{i} - (61.5 \text{ in./s}^2) \underline{j} + (103.92 \text{ in./s}^2) \underline{j}$$

15.233



GIVEN:  
 $\omega_1 = 5 \text{ rad/s}$ ,  $\alpha_1 = 0$   
 $\omega_2 = 4 \text{ rad/s}$ ,  $\alpha_2 = 0$

FIND:  $\underline{a}_{\text{Disk}}$

FRAME OXYZ IS FIXED

MOVING FRAME Bxyz ROTATES WITH ANGULAR VELOCITY  $\underline{\Omega} = \omega_1 \underline{k} = (5 \text{ rad/s}) \underline{k}$  ABOUT Z AXIS

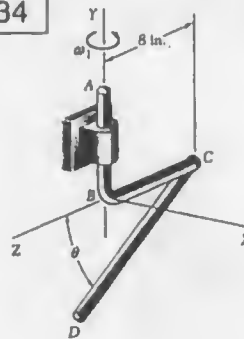
$$\underline{\omega}_{\text{Disk}} = \omega_1 \underline{k} + \omega_2 \underline{j}$$

$$\underline{a}_{\text{Disk}} = \underline{\Omega} \times \underline{\omega}_{\text{Disk}} = \omega_1 \underline{k} \times (\omega_1 \underline{k} + \omega_2 \underline{j}) = -\omega_1 \omega_2 \underline{i}$$

$$\underline{a}_{\text{Disk}} = -(5 \text{ rad/s})(4 \text{ rad/s}) \underline{i}$$

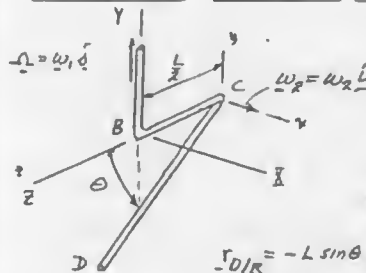
$$\underline{a}_{\text{Disk}} = -(20 \text{ rad/s}^2) \underline{i}$$

15.234



GIVEN:  $CD = L = 16 \text{ in.}$   
 $\theta = 30^\circ$   
 $\omega_1 = 4 \text{ rad/s}$   
 $\omega_2 = \frac{d\theta}{dt} = 3 \text{ rad/s}$

FIND:  $\underline{v}_D$  AND  $\underline{a}_D$



FRAME BXYZ IS FIXED.  
 MOVING FRAME Cxyz  
 ROTATES ABOUT Y AXIS  
 WITH  $\underline{\Omega} = \omega_2 \underline{j}$

$$\underline{r}_{D/B} = -L \sin \theta \underline{j} + (L \cos \theta - \frac{1}{2}) \underline{k}$$

$$\underline{r}_{D/C} = -L \sin \theta \underline{j} + L \cos \theta \underline{k}$$

VELOCITY:  $\underline{v}_D = \underline{\Omega} \times \underline{r}_{D/B} = \omega_2 \underline{j} \times [-L \sin \theta \underline{j} + (L \cos \theta - \frac{1}{2}) \underline{k}]$

$$\underline{v}_D = L \omega_2 (\cos \theta - \frac{1}{2}) \underline{i}$$

$$\underline{v}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{D/C} = \omega_2 \underline{j} \times (-L \sin \theta \underline{j} + L \cos \theta \underline{k})$$

$$\underline{v}_{D/\mathcal{F}} = -L \omega_2 \sin \theta \underline{k} - L \omega_2 \cos \theta \underline{i}$$

$$\underline{v}_D = \underline{v}_D + \underline{v}_{D/\mathcal{F}}$$

$$\underline{v}_D = L \omega_1 (\cos \theta - \frac{1}{2}) \underline{i} - L \omega_2 \cos \theta \underline{j} - L \omega_2 \sin \theta \underline{k}$$

ACCELERATION:  $\underline{a}_D = \underline{\Omega} \times \underline{v}_D = \omega_2 \underline{j} \times L \omega_1 (\cos \theta - \frac{1}{2}) \underline{i}$

$$\underline{a}_D = -L \omega_1^2 (\cos \theta - \frac{1}{2}) \underline{k}$$

$$\underline{a}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{D/\mathcal{F}} = \omega_2 \underline{j} \times [-L \omega_2 \sin \theta \underline{k} - L \omega_2 \cos \theta \underline{i}]$$

$$\underline{a}_{D/\mathcal{F}} = +L \omega_2^2 \sin \theta \underline{j} - L \omega_2^2 \cos \theta \underline{k}$$

$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2 \omega_2 \underline{j} \times (-L \omega_2 \sin \theta \underline{k} - L \omega_2 \cos \theta \underline{i})$$

$$\underline{a}_C = -2L \omega_2 \omega_1 \sin \theta \underline{i}$$

$$\underline{a}_D = \underline{a}_D + \underline{a}_{D/\mathcal{F}} + \underline{a}_C$$

$$= -L \omega_1^2 (\cos \theta - \frac{1}{2}) \underline{k} + L \omega_2^2 \sin \theta \underline{j} - L \omega_2^2 \cos \theta \underline{k} - 2L \omega_1 \omega_2 \sin \theta \underline{i}$$

$$\underline{a}_D = -2L \omega_1 \omega_2 \sin \theta \underline{i} + L \omega_2^2 \sin \theta \underline{j} + (-L \omega_1^2 (\cos \theta - \frac{1}{2}) - L \omega_2^2 \cos \theta) \underline{k}$$

DATA:  $\theta = 30^\circ$ ,  $L = 16 \text{ in.}$ ,  $\omega_1 = 4 \text{ rad/s}$ ,  $\omega_2 = 3 \text{ rad/s}$

$$\underline{v}_D = 16(4)(\cos 30^\circ - \frac{1}{2}) \underline{i} - 16(3) \cos 30^\circ \underline{j} - 16(3) \sin 30^\circ \underline{k}$$

$$\underline{v}_D = (23.4 \text{ in./s}) \underline{i} - (41.6 \text{ in./s}) \underline{j} - (24 \text{ in./s}) \underline{k}$$

$$\underline{a}_D = -2(16)(3)(4) \sin 30^\circ \underline{i} + 16(3)^2 \sin 30^\circ \underline{j}$$

$$+ (-16(4)^2 (\cos 30^\circ - \frac{1}{2}) - 16(3)^2 \cos 30^\circ) \underline{k}$$

$$\underline{a}_D = -(192 \text{ in./s}^2) \underline{i} + (72 \text{ in./s}^2) \underline{j} - (218 \text{ in./s}^2) \underline{k}$$

# 15.235 and 15.236

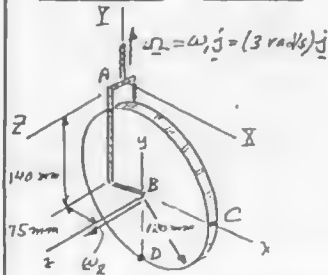
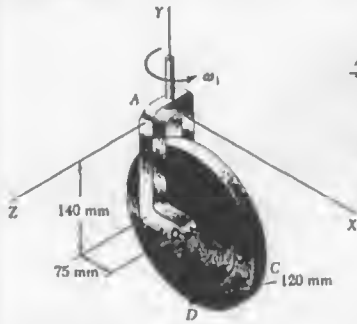
GIVEN:  $\omega_1 = 3 \text{ rad/s}$   
 $\omega_2 = 5 \text{ rad/s}$

PROBLEM 15.235:

FIND:  $\vec{v}_C$  AND  $\vec{a}_C$

PROBLEM 15.236:

FIND:  $\vec{v}_D$  AND  $\vec{a}_D$



FRAME  $AXYZ$  IS FIXED  
 MOVING FRAME  $Bxyz$   
 ROTATES ABOUT  $Y$  AXIS  
 WITH  $\Omega = (3 \text{ rad/s})\hat{j}$   
 $\omega_2 = (5 \text{ rad/s})\hat{e}$

PROBLEM 15.235: FOR POINT C

$$\vec{r}_{C/A} = (195 \text{ mm})\hat{i} - (140 \text{ mm})\hat{j}; \quad \vec{r}_{C/B} = (120 \text{ mm})\hat{i}$$

VELOCITY:

$$\vec{v}_C = \Omega \times \vec{r}_{C/A} = (3 \text{ rad/s})\hat{j} \times (195\hat{i} - 140\hat{j}) = -(585 \text{ mm/s})\hat{e}$$

$$\vec{v}_C = \omega_2 \times \vec{r}_{C/B} = (5 \text{ rad/s})\hat{e} \times (120\hat{i}) = +(600 \text{ mm/s})\hat{j}$$

$$\vec{v}_C = \vec{v}_C + \vec{v}_{C/B} \quad \vec{v}_C = (600 \text{ mm/s})\hat{j} - (585 \text{ mm/s})\hat{e}$$

ACCELERATION

$$\vec{a}_C = \Omega \times \vec{v}_C = (3 \text{ rad/s})\hat{j} \times (600\hat{j} - 585\hat{e}) = -(1.755 \text{ m/s}^2)\hat{i}$$

$$\vec{a}_C = \omega_2 \times \vec{v}_C = (5 \text{ rad/s})\hat{e} \times (600\hat{j}) = -(3.00 \text{ m/s}^2)\hat{i}$$

$$\vec{a}_C = 2\Omega \times \vec{r}_{C/B} = 2(3 \text{ rad/s})\hat{j} \times (600\hat{j}) = 0$$

$$\vec{a}_C = \vec{a}_C + \vec{a}_{C/B} + \vec{a}_C = -(1.755 \text{ m/s}^2)\hat{i} - (3.00 \text{ m/s}^2)\hat{i}$$

$$\vec{a}_C = -(4.76 \text{ m/s}^2)\hat{i}$$

PROBLEM 15.236: FOR POINT D

$$\vec{r}_{D/A} = (75 \text{ mm})\hat{i} - (260 \text{ mm})\hat{j}; \quad \vec{r}_{D/B} = -(120 \text{ mm})\hat{j}$$

VELOCITY

$$\vec{v}_D = \Omega \times \vec{r}_{D/A} = (3 \text{ rad/s})\hat{j} \times (75\hat{i} - 260\hat{j}) = -(225 \text{ mm/s})\hat{e}$$

$$\vec{v}_D = \omega_2 \times \vec{r}_{D/B} = (5 \text{ rad/s})\hat{e} \times (-120\hat{j}) = (600 \text{ mm/s})\hat{i}$$

$$\vec{v}_D = \vec{v}_D + \vec{v}_{D/B}$$

$$\vec{v}_D = (600 \text{ mm/s})\hat{i} - (225 \text{ mm/s})\hat{e}$$

ACCELERATION

$$\vec{a}_D = \Omega \times \vec{v}_D = (3 \text{ rad/s})\hat{j} \times (600\hat{i} - 225\hat{e}) = -(0.675 \text{ m/s}^2)\hat{i}$$

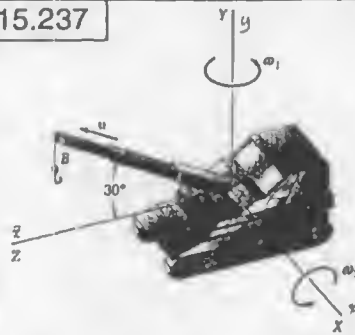
$$\vec{a}_D = \omega_2 \times \vec{v}_D = (5 \text{ rad/s})\hat{e} \times (600\hat{j}) = +(3.00 \text{ m/s}^2)\hat{i}$$

$$\vec{a}_D = 2\Omega \times \vec{r}_{D/B} = 2(3 \text{ rad/s})\hat{j} \times (600\hat{j}) = -(3.60 \text{ m/s}^2)\hat{e}$$

$$\vec{a}_D = \vec{a}_D + \vec{a}_{D/B} + \vec{a}_D$$

$$\vec{a}_D = -(0.675 \text{ m/s}^2)\hat{i} + (3.00 \text{ m/s}^2)\hat{i} - (3.60 \text{ m/s}^2)\hat{e}$$

# 15.237



GIVEN:

$$\omega_1 = 0.25 \text{ rad/s}, \quad \alpha_1 = 0$$

$$\omega_2 = 0.40 \text{ rad/s}, \quad \alpha_2 = 0$$

$$AB = 20 \text{ ft}$$

$$L = 1.5 \text{ ft/s}$$

$$\dot{\theta} = 0$$

FIND:  $\vec{v}_B$  AND  $\vec{a}_B$

FRAME  $AXYZ$  IS FIXED. MOVING FRAME  $Oxyz$  ROTATES ABOUT  $Y$  AXIS WITH  $\Omega = \omega_1\hat{j} = (0.25 \text{ rad/s})\hat{j}$

$$\omega_2 = (0.40 \text{ rad/s})\hat{e}; \quad \hat{e}_{AB} = \sin 30^\circ\hat{j} + \cos 30^\circ\hat{e}$$

$$\vec{r}_{B/A} = (AB)\hat{e}_{AB} = (10 \text{ ft})\hat{j} + (17.32 \text{ ft})\hat{e}$$

$$\vec{v}_B = \Omega \times \vec{r}_{B/A} = (1.5 \text{ ft/s})\hat{j} \times (10\hat{j} + 17.32\hat{e}) = (2.599 \text{ ft/s})\hat{e}$$

VELOCITY:

$$\vec{v}_B = \Omega \times \vec{r}_{B/A} = (0.25 \text{ rad/s})\hat{j} \times (10\hat{j} + 17.32\hat{e}) = (4.33 \text{ ft/s})\hat{e}$$

$$\vec{v}_B = \vec{v}_B + \vec{v}_{B/A} = (4.33 \text{ ft/s})\hat{e} + (2.599 \text{ ft/s})\hat{e} + (0.40 \text{ ft/s}) \times (10\hat{j} + 17.32\hat{e})$$

$$= 0.25\hat{j} + 2.599\hat{e} + 4\hat{j} - 6.928\hat{j}$$

$$\vec{v}_B = (-6.178 \text{ ft/s})\hat{j} + (5.299 \text{ ft/s})\hat{e}$$

$$\vec{v}_B = \vec{v}_B + \vec{v}_{B/A} = (4.33 \text{ ft/s})\hat{e} - (6.178 \text{ ft/s})\hat{j} + (5.299 \text{ ft/s})\hat{e}$$

$$\vec{v}_B = (4.33 \text{ ft/s})\hat{e} - (6.18 \text{ ft/s})\hat{j} + (5.30 \text{ ft/s})\hat{e}$$

ACCELERATION:

$$\vec{a}_B = \Omega \times \vec{v}_B = (0.25 \text{ rad/s})\hat{j} \times (4.33\hat{e}) = -(1.083 \text{ ft/s}^2)\hat{e}$$

$$\vec{a}_B = \omega_2 \times \vec{v}_B + 2\Omega \times \vec{r}_{B/A}$$

ACCELERATION OF

COINCIDING POINT AS AB

ROTATES WITH  $\omega_2$  ABOUT

$Y$  AXIS

CORIOLIS ACCELERATION

OF B MOVING AT  $L$  AS

AB ROTATES ABOUT  $X$  AXIS

$$\vec{a}_B = (0.4\hat{e}) \times (0.4\hat{e} \times (10\hat{j} + 17.32\hat{e})) + 2(0.4\hat{e}) \times (1.5\hat{j} + 2.599\hat{e})$$

$$= (0.4\hat{e}) \times (4\hat{j} - 6.928\hat{j}) + 0.6\hat{e} - 1.0392\hat{j}$$

$$= -1.6\hat{j} - 2.771\hat{e} + 0.6\hat{e} - 1.0392\hat{j}$$

$$\vec{a}_B = (-2.639 \text{ ft/s}^2)\hat{j} - (2.171 \text{ ft/s}^2)\hat{e}$$

$\vec{a}_C$  = CORIOLIS ACCELERATION OF B MOVING WITH

FRAME  $AXYZ$  IN ROTATION ABOUT  $Y$  WITH

ANGULAR VELOCITY  $\Omega$

$$\vec{a}_C = 2\Omega \times \vec{v}_{B/A}$$

$$= 2(0.25 \text{ rad/s})\hat{j} \times (6.178 \text{ ft/s})\hat{j} + (5.299 \text{ ft/s})\hat{e}$$

$$= (2.650 \text{ ft/s}^2)\hat{i}$$

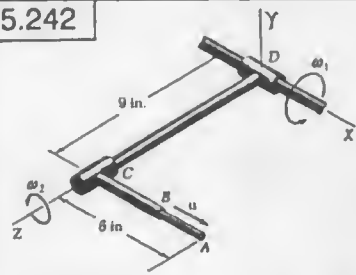
$$\vec{a}_B = \vec{a}_B + \vec{a}_{B/A} + \vec{a}_C$$

$$= -(1.083 \text{ ft/s}^2)\hat{e} - (2.639 \text{ ft/s}^2)\hat{j} - (2.171 \text{ ft/s}^2)\hat{e} + (2.650 \text{ ft/s}^2)\hat{i}$$

$$\vec{a}_B = (2.65 \text{ ft/s}^2)\hat{i} - (2.64 \text{ ft/s}^2)\hat{j} - (3.25 \text{ ft/s}^2)\hat{e}$$

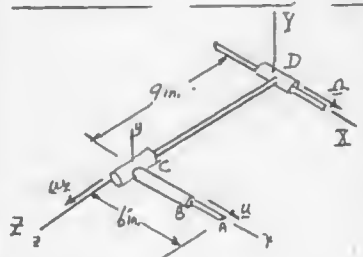


15.242



GIVEN:

$$\begin{aligned}\omega_1 &= 1.2 \text{ rad/s}, \quad \alpha_1 = 0 \\ \omega_2 &= 1.5 \text{ rad/s}, \quad \alpha_2 = 0 \\ u &= 3 \text{ in./s}, \quad \dot{u} = 0\end{aligned}$$

FIND:  $\vec{v}_A$  AND  $\vec{a}_A$ 

FRAME DXYZ IS FIXED.  
MOVING FRAME Cxyz  
ROTATES ABOUT THE  
Y AXIS WITH  
 $\underline{\Omega} = \omega_1 \hat{i} = (1.2 \text{ rad/s}) \hat{i}$ .

$$\begin{aligned}\underline{\omega}_2 &= \omega_2 \hat{k} = (1.5 \text{ rad/s}) \hat{k} \\ u &= u \hat{i} = (3 \text{ in./s}) \hat{i}\end{aligned}$$

$$\underline{r}_{AD} = (6 \text{ in.}) \hat{j} + (9 \text{ in.}) \hat{k}$$

$$\underline{r}_{AC} = (6 \text{ in.}) \hat{i}$$

VELOCITY:

$$\underline{v}_{A1} = \underline{\Omega} \times \underline{r}_{AD} = (1.2 \text{ rad/s}) \hat{i} \times [(6 \text{ in.}) \hat{j} + (9 \text{ in.}) \hat{k}] = -(10.8 \text{ in./s}) \hat{j}$$

$$\underline{v}_{A2} = \underline{\omega}_2 \times \underline{r}_{AC} + u = (1.5 \text{ rad/s}) \hat{k} \times (6 \text{ in.}) \hat{i} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_{A2} = (9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_A = \underline{v}_{A1} + \underline{v}_{A2} = -(10.8 \text{ in./s}) \hat{j} + (9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_A = (3 \text{ in./s}) \hat{i} - (1.8 \text{ in./s}) \hat{j}$$

ACCELERATION

$$\underline{a}_{A1} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{AD} = \underline{\Omega} \times \underline{v}_{A1} = (1.2 \text{ rad/s}) \hat{i} \times (-10.8 \text{ in./s}) \hat{j}$$

$$\underline{a}_{A1} = -(12.96 \text{ in./s}^2) \hat{k}$$

$\underline{a}_{A2}$ : NOTE, SINCE POINT A MOVES IN THE  
ROTATING FRAME Cxyz THERE IS A  
CORIOLIS ACCELERATION.

$$\begin{aligned}\underline{a}_{A2} &= \underline{\omega}_2 \times \underline{\omega}_2 \times \underline{r}_{AC} + 2 \underline{\omega}_2 \times u \\ &= \omega_2 \times (1.5 \text{ rad/s}) \hat{k} \times (6 \text{ in.}) \hat{i} + 2(1.5 \text{ rad/s}) \hat{k} \times (3 \text{ in./s}) \hat{i} \\ &= (1.5 \text{ rad/s}) \hat{k} \times (9 \text{ in./s}) \hat{j} + (9 \text{ in./s}^2) \hat{j}\end{aligned}$$

$$\underline{a}_{A2} = -(13.5 \text{ in./s}^2) \hat{i} + (9 \text{ in./s}^2) \hat{j}$$

$\underline{a}_C$ : CORIOLIS ACCELERATION DUE TO A MOVING  
WITH VELOCITY  $\underline{v}_{A2}$

$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{A2} = 2(1.2 \text{ rad/s}) \hat{i} \times [(9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}]$$

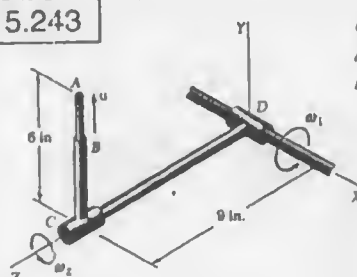
$$\underline{a}_C = (21.6 \text{ in./s}^2) \hat{k}$$

$$\underline{a}_A = \underline{a}_{A1} + \underline{a}_{A2} + \underline{a}_C$$

$$= -(12.96 \text{ in./s}^2) \hat{k} - (13.5 \text{ in./s}^2) \hat{i} + (9 \text{ in./s}^2) \hat{j} + (21.6 \text{ in./s}^2) \hat{k}$$

$$\underline{a}_A = -(13.5 \text{ in./s}^2) \hat{i} + (9 \text{ in./s}^2) \hat{j} + (8.64 \text{ in./s}^2) \hat{k}$$

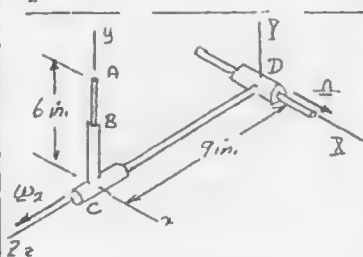
15.243



GIVEN:

$$\begin{aligned}\omega_1 &= 1.2 \text{ rad/s}, \quad \alpha_1 = 0 \\ \omega_2 &= 1.5 \text{ rad/s}, \quad \alpha_2 = 0 \\ u &= 3 \text{ in./s}, \quad \dot{u} = 0\end{aligned}$$

FIND:

 $\vec{v}_A$  AND  $\vec{a}_A$ 

FRAME DXYZ IS FIXED.  
MOVING FRAME Cxyz  
ROTATES ABOUT THE  
Y AXIS WITH  
 $\underline{\Omega} = \omega_1 \hat{i} = (1.2 \text{ rad/s}) \hat{i}$ .

$$\begin{aligned}\underline{\omega}_2 &= \omega_2 \hat{k} = (1.5 \text{ rad/s}) \hat{k} \\ u &= u \hat{i} = (3 \text{ in./s}) \hat{i}\end{aligned}$$

$$\underline{r}_{AD} = (6 \text{ in.}) \hat{j} + (9 \text{ in.}) \hat{k}$$

$$\underline{r}_{AC} = (6 \text{ in.}) \hat{i}$$

VELOCITY:

$$\underline{v}_{A1} = \underline{\Omega} \times \underline{r}_{AD} = (1.2 \text{ rad/s}) \hat{i} \times [(6 \text{ in.}) \hat{j} + (9 \text{ in.}) \hat{k}]$$

$$\underline{v}_{A1} = (7.2 \text{ in./s}) \hat{k} - (10.8 \text{ in./s}) \hat{j}$$

$$\underline{v}_{A2} = \underline{\omega}_2 \times \underline{r}_{AC} = (1.5 \text{ rad/s}) \hat{k} \times (6 \text{ in.}) \hat{i} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_{A2} = (9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_A = \underline{v}_{A1} + \underline{v}_{A2} = (7.2 \text{ in./s}) \hat{k} - (10.8 \text{ in./s}) \hat{j} - (9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}$$

$$\underline{v}_A = (3 \text{ in./s}) \hat{i} - (19.8 \text{ in./s}) \hat{j} + (7.2 \text{ in./s}) \hat{k}$$

ACCELERATION:

$$\underline{a}_{A1} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{AD} = \underline{\Omega} \times \underline{v}_{A1} = (1.2 \text{ rad/s}) \hat{i} \times [(7.2 \text{ in./s}) \hat{k} - (10.8 \text{ in./s}) \hat{j}]$$

$$\underline{a}_{A1} = -(8.64 \text{ in./s}^2) \hat{j} - (12.96 \text{ in./s}^2) \hat{k}$$

$\underline{a}_{A2}$ : NOTE, SINCE POINT A MOVES IN THE  
ROTATING FRAME Cxyz THERE IS A  
CORIOLIS ACCELERATION

$$\begin{aligned}\underline{a}_{A2} &= \underline{\omega}_2 \times \underline{\omega}_2 \times \underline{r}_{AC} + 2 \underline{\omega}_2 \times u \\ &= \omega_2 \times (1.5 \text{ rad/s}) \hat{k} \times (6 \text{ in.}) \hat{i} + 2(1.5 \text{ rad/s}) \hat{k} \times (3 \text{ in./s}) \hat{i} \\ &= (1.5 \text{ rad/s}) \hat{k} \times (9 \text{ in./s}) \hat{j} - (9 \text{ in./s}^2) \hat{j}\end{aligned}$$

$$\underline{a}_{A2} = -(13.5 \text{ in./s}^2) \hat{i} - (9 \text{ in./s}^2) \hat{j}$$

$\underline{a}_C$ : CORIOLIS ACCELERATION DUE TO A MOVING  
WITH VELOCITY  $\underline{v}_{A2}$

$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{A2} = 2(1.2 \text{ rad/s}) \hat{i} \times [(9 \text{ in./s}) \hat{j} + (3 \text{ in./s}) \hat{i}]$$

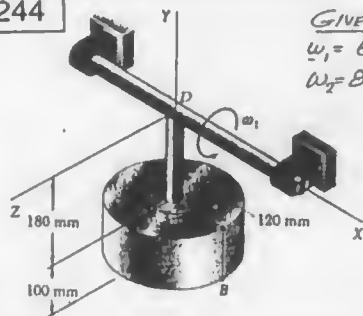
$$\underline{a}_C = (21.6 \text{ in./s}^2) \hat{k}$$

$$\underline{a}_A = \underline{a}_{A1} + \underline{a}_{A2} + \underline{a}_C$$

$$= -(8.64 \text{ in./s}^2) \hat{j} - (12.96 \text{ in./s}^2) \hat{k} - (13.5 \text{ in./s}^2) \hat{i} + (21.6 \text{ in./s}^2) \hat{k}$$

$$\underline{a}_A = -(13.5 \text{ in./s}^2) \hat{i} - (22.1 \text{ in./s}^2) \hat{j} - (5.76 \text{ in./s}^2) \hat{k}$$

15.244



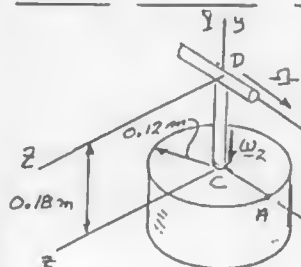
GIVEN:

$$\omega_1 = 6 \text{ rad/s}, \alpha_1 = 0$$

$$\omega_2 = 8 \text{ rad/s}, \alpha_2 = 0$$

FIND:

$$\underline{v}_A \text{ AND } \underline{a}_A$$



FRAME DXYZ IS FIXED.

MOVING FRAME, Cxyz,

ROTATES ABOUT THE

Y AXIS WITH

$$\underline{\Omega} = \omega_1 \underline{\hat{i}} = (6 \text{ rad/s}) \underline{\hat{i}}$$

$$\underline{\omega}_2 = \omega_2 \underline{\hat{j}} = (8 \text{ rad/s}) \underline{\hat{j}}$$

$$\underline{r}_{A/D} = +(0.12 \text{ m}) \underline{\hat{i}} - (0.18 \text{ m}) \underline{\hat{j}}$$

$$\underline{r}_{A/C} = (0.12 \text{ m}) \underline{\hat{i}}$$

VELOCITY:

$$\underline{v}_{A/D} = \underline{\Omega} \times \underline{r}_{A/D} = (6 \text{ rad/s}) \underline{\hat{i}} \times [(0.12 \text{ m}) \underline{\hat{i}} - (0.18 \text{ m}) \underline{\hat{j}}]$$

$$\underline{v}_A = -(1.08 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_{A/C} = \underline{\omega}_2 \times \underline{r}_{A/C} = (8 \text{ rad/s}) \underline{\hat{j}} \times (0.12 \text{ m}) \underline{\hat{i}} = (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_A = \underline{v}_{A/D} + \underline{v}_{A/C} = -(1.08 \text{ m/s}) \underline{\hat{k}} + (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_A = -(0.12 \text{ m/s}) \underline{\hat{k}}$$

ACCELERATION:

$$\underline{a}_A = \underline{\Omega} \times \underline{v}_A = (6 \text{ rad/s}) \underline{\hat{i}} \times (-(0.12 \text{ m/s}) \underline{\hat{k}})$$

$$\underline{a}_A = (0.72 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_{A/C} = \underline{\omega}_2 \times \underline{v}_{A/C} = (8 \text{ rad/s}) \underline{\hat{j}} \times (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{a}_{A/C} = -(7.68 \text{ m/s}^2) \underline{\hat{i}}$$

$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{A/C} = 2(6 \text{ rad/s}) \underline{\hat{i}} \times (0.96 \text{ m/s}) \underline{\hat{k}}$$

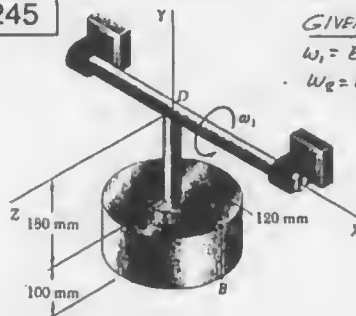
$$\underline{a}_C = -(11.52 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_A = \underline{a}_A + \underline{a}_{A/C} + \underline{a}_C$$

$$= (0.72 \text{ m/s}^2) \underline{\hat{j}} - (7.68 \text{ m/s}^2) \underline{\hat{i}} - (11.52 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_A = -(7.68 \text{ m/s}^2) \underline{\hat{i}} - (10.80 \text{ m/s}^2) \underline{\hat{j}}$$

15.245



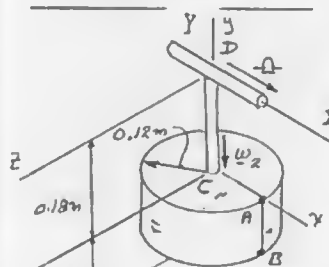
GIVEN:

$$\omega_1 = 6 \text{ rad/s}, \alpha_1 = 0$$

$$\omega_2 = 8 \text{ rad/s}, \alpha_2 = 0$$

FIND:

$$\underline{v}_B \text{ AND } \underline{a}_B$$



FRAME DXYZ IS FIXED.

MOVING FRAME, Cxyz,

ROTATES ABOUT THE

Y AXIS WITH

$$\underline{\Omega} = \omega_1 \underline{\hat{i}} = (6 \text{ rad/s}) \underline{\hat{i}}$$

$$\underline{\omega}_2 = \omega_2 \underline{\hat{j}} = (8 \text{ rad/s}) \underline{\hat{j}}$$

$$\underline{r}_{B/D} = +(0.12 \text{ m}) \underline{\hat{i}} - (0.20 \text{ m}) \underline{\hat{j}}$$

$$\underline{r}_{B/C} = (0.12 \text{ m}) \underline{\hat{i}} - (0.1 \text{ m}) \underline{\hat{j}}$$

VELOCITY:

$$\underline{v}_{B/D} = \underline{\Omega} \times \underline{r}_{B/D} = (6 \text{ rad/s}) \underline{\hat{i}} \times [(0.12 \text{ m}) \underline{\hat{i}} - (0.20 \text{ m}) \underline{\hat{j}}]$$

$$\underline{v}_B = -(1.68 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_{B/C} = \underline{\omega}_2 \times \underline{r}_{B/C} = (8 \text{ rad/s}) \underline{\hat{j}} \times [(0.12 \text{ m}) \underline{\hat{i}} - (0.1 \text{ m}) \underline{\hat{j}}]$$

$$\underline{v}_{B/C} = (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_B = \underline{v}_B + \underline{v}_{B/C} = -(1.68 \text{ m/s}) \underline{\hat{k}} + (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{v}_B = -(0.72 \text{ m/s}) \underline{\hat{k}}$$

ACCELERATION:

$$\underline{a}_B = \underline{\Omega} \times \underline{v}_B = (6 \text{ rad/s}) \underline{\hat{i}} \times (-(0.72 \text{ m/s}) \underline{\hat{k}})$$

$$\underline{a}_B = (0.432 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_{B/C} = \underline{\omega}_2 \times \underline{v}_{B/C} = (8 \text{ rad/s}) \underline{\hat{j}} \times (0.96 \text{ m/s}) \underline{\hat{k}}$$

$$\underline{a}_{B/C} = -(7.68 \text{ m/s}^2) \underline{\hat{i}}$$

$$\underline{a}_C = 2 \underline{\Omega} \times \underline{v}_{B/C} = 2(6 \text{ rad/s}) \underline{\hat{i}} \times (0.96 \text{ m/s}) \underline{\hat{k}}$$

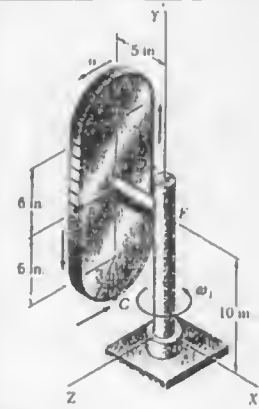
$$\underline{a}_C = -(11.52 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_B = \underline{a}_B + \underline{a}_{B/C} + \underline{a}_C$$

$$= (0.432 \text{ m/s}^2) \underline{\hat{j}} - (7.68 \text{ m/s}^2) \underline{\hat{i}} - (11.52 \text{ m/s}^2) \underline{\hat{j}}$$

$$\underline{a}_B = -(7.68 \text{ m/s}^2) \underline{\hat{i}} - (11.088 \text{ m/s}^2) \underline{\hat{j}}$$

## 15.246 and 15.247



**GIVEN:**  
 $\omega_1 = 1.6 \text{ rad/s}$ ,  $\alpha_1 = 0$   
 LINK BELT MOVES AROUND  
 PERIMETER AT CONSTANT  
 SPEED  $u = 4.5 \text{ in./s}$ .

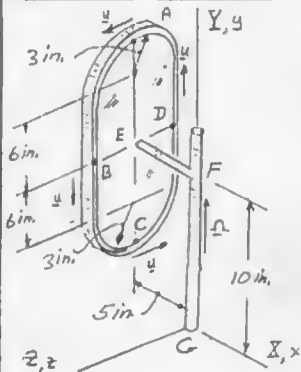
**PROBLEM 15.246**

**FIND:** (a)  $\underline{a}_A$   
 (b)  $\underline{a}_B$

**PROBLEM 15.247**

**FIND:** (a)  $\underline{a}_C$   
 (b)  $\underline{a}_D$

FRAME GXYZ IS FIXED.  
 MOVING FRAME, Gxyz,  
 ROTATES ABOUT THE  
 Y AXIS WITH  
 $\underline{\omega} = \omega_1 \underline{j} = (1.6 \text{ rad/s}) \underline{j}$

**PROBLEM 15.246: (a) POINT A:  $\underline{u} = (4.5 \text{ in./s}) \underline{i}$** 

$$\underline{r}_{A/G} = (-5 \text{ in.}) \underline{i} + (19 \text{ in.}) \underline{j}$$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_{A/G} = (1.6 \text{ rad/s}) \underline{j} \times [(-5 \text{ in.}) \underline{i} + (19 \text{ in.}) \underline{j}] = (8 \text{ in./s}) \underline{i}$$

$$\underline{v}_{A/F} = \underline{u} = (4.5 \text{ in./s}) \underline{i}$$

$$\underline{v}_A = \underline{v}_A + \underline{v}_{A/F} = (8 \text{ in./s}) \underline{i} + (4.5 \text{ in./s}) \underline{i}$$

$$\underline{v}_A = (12.5 \text{ in./s}) \underline{i}$$

$$\underline{a}_A = \underline{\omega} \times \underline{v}_{A/F} = (1.6 \text{ rad/s}) \underline{j} \times (4.5 \text{ in./s}) \underline{i} = (7.2 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_{A/F} = -\frac{u^2}{r} \underline{j} = -\frac{(4.5 \text{ in./s})^2}{(3 \text{ in.})} \underline{j} = -(6.75 \text{ in./s}^2) \underline{j}$$

$$\underline{a}_A = 2 \underline{\omega} \times \underline{u} = 2(1.6 \text{ rad/s}) \underline{j} \times (4.5 \text{ in./s}) \underline{i} = (14.4 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_A = \underline{a}_A + \underline{a}_{A/F} + \underline{a}_A$$

$$\underline{a}_A = (12.80 \text{ in./s}^2) \underline{i} - (6.75 \text{ in./s}^2) \underline{j} + (14.4 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_A = (27.2 \text{ in./s}^2) \underline{i} - (6.75 \text{ in./s}^2) \underline{j}$$

(CONTINUED)

## 15.246 and 15.247 continued

**PROBLEM 15.246: (b) POINT B:  $\underline{u} = -(4.5 \text{ in./s}) \underline{j}$** 

$$\underline{r}_{B/G} = (-5 \text{ in.}) \underline{i} + (10 \text{ in.}) \underline{j} + (3 \text{ in.}) \underline{k}$$

$$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/G} = (1.6 \text{ rad/s}) \underline{j} \times [(-5 \text{ in.}) \underline{i} + (10 \text{ in.}) \underline{j} + (3 \text{ in.}) \underline{k}]$$

$$\underline{v}_B = (8 \text{ in./s}) \underline{i} + (4.8 \text{ in./s}) \underline{k}$$

$$\underline{v}_{B/F} = \underline{u} = -(4.5 \text{ in./s}) \underline{j}$$

$$\underline{v}_B = \underline{v}_B + \underline{v}_{B/F} = (8 \text{ in./s}) \underline{i} + (4.8 \text{ in./s}) \underline{k} - (4.5 \text{ in./s}) \underline{j}$$

$$\underline{v}_B = (4.8 \text{ in./s}) \underline{i} - (4.5 \text{ in./s}) \underline{j} + (8 \text{ in./s}) \underline{k}$$

$$\underline{a}_B = \underline{\omega} \times \underline{v}_B = (1.6 \text{ rad/s}) \underline{j} \times [(4.8 \text{ in./s}) \underline{i} + (8 \text{ in./s}) \underline{k}]$$

$$\underline{a}_B = (12.8 \text{ in./s}^2) \underline{i} - (7.68 \text{ in./s}^2) \underline{k}$$

$$\underline{a}_{B/F} = 0$$

$$\underline{a}_B = 2 \underline{\omega} \times \underline{v}_{B/F} = 2(1.6 \text{ rad/s}) \underline{j} \times (-4.5 \text{ in./s}) \underline{j} = 0$$

$$\underline{a}_B = \underline{a}_B + \underline{a}_{B/F} + \underline{a}_B = (12.8 \text{ in./s}^2) \underline{i} - (7.68 \text{ in./s}^2) \underline{k} + 0 + 0$$

$$\underline{a}_B = (12.8 \text{ in./s}^2) \underline{i} - (7.68 \text{ in./s}^2) \underline{k}$$

**PROBLEM 15.247 (a) POINT C:  $\underline{u} = -(4.5 \text{ in./s}) \underline{i}$** 

$$\underline{r}_{C/G} = (-5 \text{ in.}) \underline{i} + (1 \text{ in.}) \underline{j}$$

$$\underline{v}_C = \underline{\omega} \times \underline{r}_{C/G} = (1.6 \text{ rad/s}) \underline{j} \times [(-5 \text{ in.}) \underline{i} + (1 \text{ in.}) \underline{j}] = (8 \text{ in./s}) \underline{i}$$

$$\underline{v}_{C/F} = \underline{u} = -(4.5 \text{ in./s}) \underline{i}$$

$$\underline{v}_C = \underline{v}_C + \underline{v}_{C/F} = (8 \text{ in./s}) \underline{i} - (4.5 \text{ in./s}) \underline{i}$$

$$\underline{v}_C = (3.5 \text{ in./s}) \underline{i}$$

$$\underline{a}_C = \underline{\omega} \times \underline{v}_C = (1.6 \text{ rad/s}) \underline{j} \times (3.5 \text{ in./s}) \underline{i} = (5.6 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_{C/F} = \frac{u^2}{r} \underline{j} = \frac{(4.5 \text{ in./s})^2}{(3 \text{ in.})} \underline{j} = (6.75 \text{ in./s}^2) \underline{j}$$

$$\underline{a}_C = 2 \underline{\omega} \times \underline{v}_{C/F} = 2(1.6 \text{ rad/s}) \underline{j} \times (-4.5 \text{ in./s}) \underline{i} = (14.4 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_C = \underline{a}_C + \underline{a}_{C/F} + \underline{a}_C$$

$$\underline{a}_C = (5.6 \text{ in./s}^2) \underline{i} + (6.75 \text{ in./s}^2) \underline{j} + (14.4 \text{ in./s}^2) \underline{i}$$

$$\underline{a}_C = (20.75 \text{ in./s}^2) \underline{i} + (6.75 \text{ in./s}^2) \underline{j}$$

**(b) POINT D:  $\underline{u} = (4.5 \text{ in./s}) \underline{j}$** 

$$\underline{r}_{D/G} = (-5 \text{ in.}) \underline{i} + (10 \text{ in.}) \underline{j} - (3 \text{ in.}) \underline{k}$$

$$\underline{v}_D = \underline{\omega} \times \underline{r}_{D/G} = (1.6 \text{ rad/s}) \underline{j} \times [(-5 \text{ in.}) \underline{i} + (10 \text{ in.}) \underline{j} - (3 \text{ in.}) \underline{k}]$$

$$\underline{v}_D = (8 \text{ in./s}) \underline{i} - (4.8 \text{ in./s}) \underline{k}$$

$$\underline{v}_{D/F} = \underline{u} = (4.5 \text{ in./s}) \underline{j}$$

$$\underline{v}_D = \underline{v}_D + \underline{v}_{D/F} = (8 \text{ in./s}) \underline{i} - (4.8 \text{ in./s}) \underline{k} + (4.5 \text{ in./s}) \underline{j}$$

$$\underline{v}_D = (8 \text{ in./s}) \underline{i} + (4.5 \text{ in./s}) \underline{j} - (4.8 \text{ in./s}) \underline{k}$$

$$\underline{a}_D = \underline{\omega} \times \underline{v}_D = (1.6 \text{ rad/s}) \underline{j} \times [(8 \text{ in./s}) \underline{i} + (4.5 \text{ in./s}) \underline{j}]$$

$$\underline{a}_D = (12.8 \text{ in./s}^2) \underline{i} + (7.68 \text{ in./s}^2) \underline{k}$$

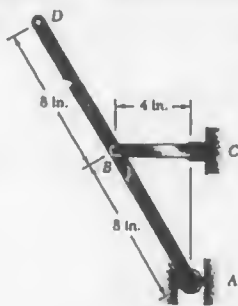
$$\underline{a}_{D/F} = 0$$

$$\underline{a}_D = 2 \underline{\omega} \times \underline{v}_{D/F} = 2(1.6 \text{ rad/s}) \underline{j} \times (4.5 \text{ in./s}) \underline{j} = 0$$

$$\underline{a}_D = \underline{a}_D + \underline{a}_{D/F} + \underline{a}_D = (12.8 \text{ in./s}^2) \underline{i} + (7.68 \text{ in./s}^2) \underline{k} + 0 + 0$$

$$\underline{a}_D = (12.8 \text{ in./s}^2) \underline{i} + (7.68 \text{ in./s}^2) \underline{k}$$

15.248



GIVEN:  
 $\omega_{BC} = 45 \text{ rpm}$ ,  $\alpha_{AB}$

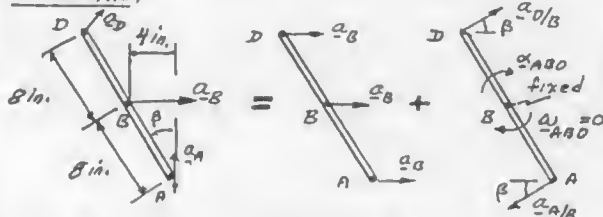
FIND:  
 (a)  $a_A$   
 (b)  $a_D$

CRANK BC:  $\omega_{BC} = (45 \text{ rpm}) \frac{2\pi}{60} = 4.7124 \text{ rad/s}$

$a_B = (BC)\omega_{BC}^2 = (4 \text{ in.})(4.7124 \text{ rad/s})^2$   
 $a_B = 88.83 \text{ in./s}^2$

BAR ABD:  $\vec{v}_B = (BC)\omega_{BC}$   
 VELOCITY: INST. CENTER AT  $\infty$   
 THUS,  $\omega_{ABD} = 0$

ACCELERATION



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$\beta = \sin^{-1} \frac{4 \text{ in.}}{8 \text{ in.}} = 30^\circ$

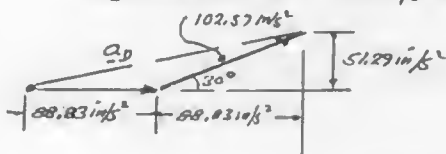
$a_A = a_B + a_{A/B}$   
 $a_A \downarrow = [88.83 \text{ in./s}^2 \rightarrow] + a_{A/B} \uparrow \beta$

$a_B = 88.83 \text{ in./s}^2$   
 $a_{A/B} = (88.83 \text{ in./s}^2) / \cos 30^\circ$   
 $a_{A/B} = 102.57 \text{ in./s}^2 \uparrow 30^\circ$   
 $a_A = (102.57) \sin 30^\circ = 51.3 \text{ in./s}^2 \downarrow$

$a_{AB} = (AB)\alpha_{ABD} = (8 \text{ in.})\alpha_{ABD}$   
 $a_{DB} = (BD)\alpha_{ABD} = (8 \text{ in.})\alpha_{ABD}$   
 NOTE  $a_{AD} = a_{AB}$

POINT D:  $a_D = a_B + a_{D/B} = a_B + a_{D/B} \angle 30^\circ$

$a_D = 88.83 \text{ in./s}^2 \rightarrow + 102.57 \text{ in./s}^2 \angle 30^\circ$



$a_D = 184.9 \text{ in./s}^2$   
 $\gamma = 16.1^\circ$

$a_D = 184.9 \text{ in./s}^2 \angle 16.1^\circ$

15.249

GIVEN: ROTOR IN UNIFORMLY  
 ACCELERATED MOTION

$t = 0$ ,  $\omega_0 = 1800 \text{ rpm}$ ,  $\theta = 0$

$\omega = 0$ ,  $\theta = 1550 \text{ rev}$  velocity

FIND: (a)  $\alpha$ , (b)  $t$  REQUIRED TO COME TO REST.

$\omega_0 = 1800 \text{ rpm} \left( \frac{2\pi}{60} \right) = 188.50 \text{ rad/s}$

$\theta = 1550 \text{ rev} (2\pi) = 9739 \text{ rad}$

(a) ANGULAR ACCELERATION: (USE LAST OF EQS. 15.16)

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$0 = (188.50 \text{ rad/s})^2 + 2\alpha(9739 \text{ rad} - 0)$

$\alpha = -1.824 \text{ rad/s}^2$

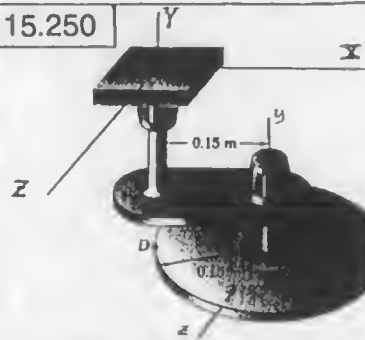
(b) TIME REQUIRED TO STOP: (USE FIRST OF EQS. 15.16)

$\omega = \omega_0 + \alpha t$

$0 = 188.50 \text{ rad/s} - (1.824 \text{ rad/s}^2)t$

$t = 103.3 \text{ s}$

15.250



GIVEN:  
 $\omega_1 = \omega_2 = 3 \text{ rad/s}$   
 $\alpha_1 = \alpha_2 = 0$

FIND:

(a)  $\vec{v}_D$  AND  $\vec{a}_D$   
 (b)  $\vec{v}_F$  AND  $\vec{a}_F$

FRAME AXYZ IS FIXED.  
 MOVING FRAME Exy

ROTATES ABOUT Y AXIS AT  $\omega_2 = \omega_2 \hat{j} = (3 \text{ rad/s}) \hat{j}$

(a) POINT D:  $\omega_2 = \omega_2 \hat{j} = (3 \text{ rad/s}) \hat{j}$

$\vec{r}_{D/A} = 0$ ;  $\vec{r}_{D/E} = -(0.15 \text{ m}) \hat{i}$

$\vec{v}_D = \omega_2 \times \vec{r}_{D/A} = 0$

$\vec{v}_{D/E} = \omega_2 \times \vec{r}_{D/E} = (3 \text{ rad/s}) \hat{j} \times (-0.15 \text{ m}) \hat{i} = (0.45 \text{ m/s}) \hat{k}$

$\vec{v}_D = \vec{v}_{D/E} + \vec{v}_E$

$\vec{v}_D = (0.45 \text{ m/s}) \hat{k}$

$a_D = \omega_2 \times \vec{v}_D = 0$

$a_{D/E} = \omega_2 \times \vec{v}_{D/E} = (3 \text{ rad/s}) \hat{j} \times (0.45 \text{ m/s}) \hat{k} = (1.35 \text{ m/s}^2) \hat{i}$

$a_E = 2\omega_2 \times \vec{v}_{D/E} = 2(3 \text{ rad/s}) \hat{j} \times (0.45 \text{ m/s}) \hat{k} = (2.70 \text{ m/s}^2) \hat{i}$

$a_D = a_{D/E} + a_E = 0 + (1.35 \text{ m/s}^2) \hat{i} + (2.70 \text{ m/s}^2) \hat{i}$

$a_D = (4.05 \text{ m/s}^2) \hat{i}$

(b) POINT F:  $\omega_2 = \omega_2 \hat{j} = (3 \text{ rad/s}) \hat{j}$

$\vec{r}_{F/A} = (0.3 \text{ m}) \hat{i}$ ;  $\vec{r}_{F/E} = (0.15 \text{ m}) \hat{i}$

$\vec{v}_F = \omega_2 \times \vec{r}_{F/A} = (3 \text{ rad/s}) \hat{j} \times (0.3 \text{ m}) \hat{i} = -(0.9 \text{ m/s}) \hat{k}$

$\vec{v}_{F/E} = \omega_2 \times \vec{r}_{F/E} = (3 \text{ rad/s}) \hat{j} \times (0.15 \text{ m}) \hat{i} = -(0.45 \text{ m/s}) \hat{k}$

$\vec{v}_F = \vec{v}_{F/E} + \vec{v}_E$

$\vec{v}_F = -(0.9 \text{ m/s}) \hat{k} - (0.45 \text{ m/s}) \hat{k}$

$\vec{v}_F = -(1.35 \text{ m/s}) \hat{k}$

$a_F = \omega_2 \times \vec{v}_F = (3 \text{ rad/s}) \hat{j} \times (-1.35 \text{ m/s}) \hat{k} = -(2.7 \text{ m/s}^2) \hat{i}$

$a_{F/E} = \omega_2 \times \vec{v}_{F/E} = (3 \text{ rad/s}) \hat{j} \times (-0.45 \text{ m/s}) \hat{k} = -(1.35 \text{ m/s}^2) \hat{i}$

$a_E = 2\omega_2 \times \vec{v}_{F/E} = 2(3 \text{ rad/s}) \hat{j} \times (-0.45 \text{ m/s}) \hat{k} = -(2.7 \text{ m/s}^2) \hat{i}$

$a_F = a_{F/E} + a_E + a_E$

$= -(2.7 \text{ m/s}^2) \hat{i} - (1.35 \text{ m/s}^2) \hat{i} - (2.7 \text{ m/s}^2) \hat{i}$

$a_F = -(6.75 \text{ m/s}^2) \hat{i}$



15.251

**GIVEN:**  
 AUTO:  $\vec{v} = (12 \text{ km/h}) \hat{i}$   
 FAN:  $\omega = (2500 \text{ rpm}) \hat{k}$   
 FIND:  $\alpha$  OF FAN

$\vec{v} = (12 \text{ km/h}) \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1}{3600 \text{ s}} \hat{i} = (3.333 \text{ m/s}) \hat{i}$   
 $\omega = (2500 \text{ rpm}) \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = (261.8 \text{ rad/s}) \hat{k}$   
 $\omega_2 = \text{ANGULAR VELOCITY OF AUTO}$   
 $\omega_2 = -\left(\frac{v}{r}\right) \hat{j} = -\left(\frac{3.333 \text{ m/s}}{12 \text{ m}}\right) \hat{j} = (-0.2778 \text{ rad/s}) \hat{j}$   
 $\alpha = \omega_2 \times \omega_1 = (-0.2778 \text{ rad/s}) \hat{j} \times (261.8 \text{ rad/s}) \hat{k}$   
 $\alpha = (72.7 \text{ rad/s}^2) \hat{i}$   
 $\alpha = 72.7 \text{ rad/s}^2 \text{ TOWARD THE EAST}$

15.252

**GIVEN:**  $v_F = 0$   
 $v_E = 15 \text{ in/s} \rightarrow, a_E = 0$   
 FIND: (a)  $v_A$   
 (b)  $a_D$

**(a)** INST. CENTER OF ROTATION IS AT POINT D.

$v_E = (BD)\omega; 15 \text{ in/s} = (3 \text{ in.})\omega; \omega = 5 \text{ rad/s}$   
 $v_A = (AD)\omega = (7.5 \text{ in.} \times 5 \text{ rad/s}) = 37.5 \text{ in/s} \rightarrow$

**(b)**  $\omega = 5 \text{ rad/s}$

PLANE MOTION = TRANS. WITH A + ROTATION ABOUT A

$a_D = a_A + a_{D/A}$   
 $(a_D)_x + (a_D)_y \hat{j} = a_A \rightarrow + (a_{D/A})_x \hat{i} + (a_{D/A})_y \hat{j}$   
 $0 + (a_D)_y \hat{j} = 0 + r\alpha \leftarrow + r\omega^2 \uparrow$

$\uparrow x \text{ COMPONENTS: } r\alpha = 0, \alpha = 0$

$\uparrow y \text{ COMPONENTS: } (a_D)_y = r\omega^2 = (7.5 \text{ in.} \times 5 \text{ rad/s})^2$   
 $(a_D)_y = 187.5 \text{ in/s}^2 \uparrow$

$a_D = (a_D)_y; a_D = 187.5 \text{ in/s}^2 \uparrow$

15.253

**GIVEN:**  $v_F = 9 \text{ in/s} \rightarrow$   
 $v_E = 15 \text{ in/s} \rightarrow$   
 $a_E = a_F = 0$   
 FIND: (a)  $v_D$   
 (b)  $a_D$

INST. CENTER OF ROTATION IS AT POINT C.

SIMILAR TRIANGLES

$\frac{y}{9 \text{ in/s}} = \frac{y+3 \text{ in.}}{15 \text{ in/s}}$   
 $15y = 9y + 27$   
 $y = 4.5 \text{ in.}$

$\frac{v_A}{AC} = \frac{v_F}{CD}; \frac{v_A}{(4.5+3+4.5) \text{ in.}} = \frac{9 \text{ in/s}}{4.5 \text{ in.}}$   
 $v_A = 24 \text{ in/s} \rightarrow$

$\omega = \frac{v_F}{y} = \frac{9 \text{ in/s}}{4.5 \text{ in.}} = 2 \text{ rad/s}$

**(b)**  $\omega = 2 \text{ rad/s}$

PLANE MOTION = TRANS WITH A + ROTATION ABOUT A

$a_D = a_A + a_{D/A}; (a_D)_x + (a_D)_y \hat{j} = a_A + (a_{D/A})_x \hat{i} + (a_{D/A})_y \hat{j}$   
 $0 + (a_D)_y \hat{j} = 0 + r\omega^2 \hat{i} + r\alpha \leftarrow$   
 $\uparrow x \text{ COMPONENTS: } (a_D)_y = r\omega^2 = (7.5 \text{ in.}) (2 \text{ rad/s})^2$   
 $a_D = (a_D)_y; a_D = 30 \text{ in/s}^2 \uparrow$

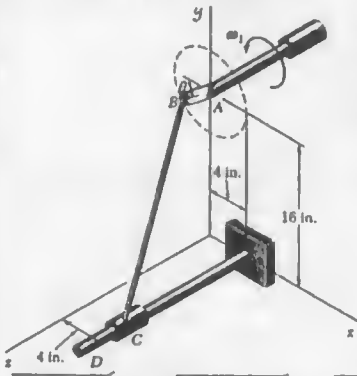
15.254

**GIVEN:**  
 $\omega = 90 \text{ rpm}, \alpha = 0$   
 VELOCITY OF WATER RELATIVE TO PIPE =  $8 \text{ m/s}$   
 FIND:  $a_P$  OF WATER AT P.

$AP = (0.5 \text{ m}) V = 0.707 \text{ m}$   
 $u = 8 \text{ m/s} \leftarrow$   
 $\omega = 90 \text{ rpm} \cdot \frac{2\pi}{60} = 9.425 \text{ rad/s}$   
 $a_P = a_P + a_{P/g} + a_C$

$a_{P/g} = (AP)\omega^2 \angle 45^\circ = (0.707 \text{ m}) (9.425 \text{ rad/s})^2 = 62.8 \text{ m/s}^2 \angle 45^\circ$   
 $a_{P/g} = u^2/r = (8 \text{ m/s})^2 / 0.5 \text{ m} \downarrow = 128 \text{ m/s}^2 \downarrow$   
 $a_C = 2\omega \times u = 2(9.425 \text{ rad/s}) (8 \text{ m/s}) = 150.8 \text{ m/s}^2 \uparrow$   
 $a_P = 62.8 \text{ m/s}^2 \angle 45^\circ + 128 \text{ m/s}^2 \downarrow + 150.8 \text{ m/s}^2 \uparrow$   
 $\uparrow x \text{ COMPONENTS: } (a_P)_x = -62.8 \sin 45^\circ - 128 + 150.8 = -21.61 \text{ m/s}^2$   
 $\uparrow y \text{ COMPONENTS: } (a_P)_y = +62.8 \cos 45^\circ = 44.41 \text{ m/s}^2$   
 $\beta = 26.0^\circ; a_P = 49.4 \text{ m/s}^2$   
 $a_P = 49.4 \text{ m/s}^2 \angle 26.0^\circ$

# 15.255 and 15.256



**GIVEN:**

$$\omega_1 = 10 \text{ rad/s}, \alpha_1 = 0$$

$$AB = 4 \text{ in.}$$

$$BC = 24 \text{ in.}$$

**FIND:**  $\vec{v}_C$

**PROBLEM 15.255**  
WHEN  $\theta = 0$

**PROBLEM 15.256**  
WHEN  $\theta = 90^\circ$

**PROBLEM 15.255**  $\theta = 0$ :  $\vec{r}_{B/A} = -(4 \text{ in.})\hat{i}$

$$\vec{v}_B = \omega_1 \times \vec{r}_{B/A} = (10 \text{ rad/s})\hat{k} \times (-4 \text{ in.})\hat{i} = -(40 \text{ in./s})\hat{j}$$

**PROBLEM 15.256**  $\theta = 90^\circ$ :  $\vec{r}_{B/A} = -(4 \text{ in.})\hat{j}$

$$\vec{v}_B = \omega_1 \times \vec{r}_{B/A} = (10 \text{ rad/s})\hat{k} \times (-4 \text{ in.})\hat{j} = (40 \text{ in./s})\hat{i}$$

**ROD BC:**

$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} = \vec{v}_B + \omega \times \vec{r}_{C/B}$$

$$\vec{v}_C = -(40 \text{ in./s})\hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 8 & -16 & 16 \end{vmatrix}$$

$$\vec{v}_C = -(40 \text{ in./s})\hat{j} + (16\omega_y + 16\omega_z)\hat{i} + (-16\omega_x - 8\omega_z)\hat{k}$$

**X COMPONENTS:**  $0 = 16\omega_y + 16\omega_z$  (1)

**Y COMPONENTS:**  $40 = -16\omega_x + 8\omega_z$  (2)

**Z COMPONENTS:**  $\vec{v}_C = -16\omega_x - 8\omega_z$  (3)

**LET**  $\omega_y = 0$ , EQ (1) YIELDS  $\omega_z = 0$ .

EQ (2):  $40 = -16\omega_x$ ;  $\omega_x = -2.5 \text{ rad/s}$

EQ (3):  $\vec{v}_C = -16(-2.5) = 40$

$\vec{v}_C = (40 \text{ in./s})\hat{i}$

**PROBLEM 15.256**  $\theta = 90^\circ$   $\vec{r}_{B/A} = -(4 \text{ in.})\hat{j}$

$\vec{v}_B = \omega \times \vec{r}_{B/A} = (10 \text{ rad/s})\hat{k} \times (-4 \text{ in.})\hat{j} = (40 \text{ in./s})\hat{i}$

$\vec{v}_B = (40 \text{ in./s})\hat{i}$

$BC = 24 \text{ in.}$

$(BC)_x = 4 \text{ in.}$

$(BC)_y = 12 \text{ in.}$

$BC^2 = (BC)_x^2 + (BC)_y^2 + (BC)_z^2$

$24^2 = 4^2 + 12^2 + (BC)_z^2$

$(BC)_z = 20.396 \text{ in.}$

$\vec{r}_{C/B} = (4 \text{ in.})\hat{i} - (12 \text{ in.})\hat{j} + (20.396 \text{ in.})\hat{k}$

(CONTINUED)

# 15.256 continued

$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$

**ROD BC:**  $\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} = \vec{v}_B + \omega \times \vec{r}_{C/B}$

$\vec{v}_C = (40 \text{ in./s})\hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 4 & -12 & 20.396 \end{vmatrix}$

$\vec{v}_C = 40\hat{i} + (20.396\omega_y + 12\omega_z)\hat{j} + (-12\omega_x - 4\omega_z)\hat{k}$

**X COMPONENTS:**  $-40 = 20.396\omega_y + 12\omega_z$  (1)

**Y COMPONENTS:**  $0 = -20.396\omega_x + 4\omega_z$  (2)

**Z COMPONENTS:**  $\vec{v}_C = -12\omega_x - 4\omega_z$  (3)

**LET:**  $\omega_z = 0$ , EQ (2) YIELDS  $\omega_x = 0$

EQ (1):  $-40 = 20.396\omega_y$   $\omega_y = -1.9612 \text{ rad/s}$

EQ (3):  $\vec{v}_C = -4(-1.9612) = 7.8447 \text{ in./s}$

$\vec{v}_C = (7.84 \text{ in./s})\hat{k}$

# 15.257 and 15.258

**GIVEN:**

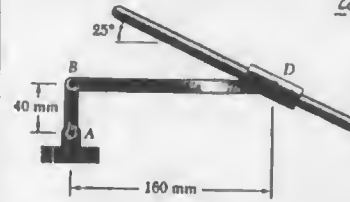
$\omega_{AB} = 1.5 \text{ rad/s}$ ,  $\alpha_{AB} = 0$

**PROBLEM 15.257**

**FIND:** (a)  $\omega_{BD}$ , (b)  $\vec{v}_D$

**PROBLEM 15.258**

**FIND:** (a)  $\omega_{BD}$ , (b)  $\vec{a}_D$



**CRANK AB:**  $\vec{v}_B = (AB)\omega_{AB} = (40 \text{ mm})(1.5 \text{ rad/s})$

$$\vec{v}_B = 60 \text{ mm/s}$$

$\vec{a}_B = (AB)\alpha_{AB} = (40 \text{ mm})(0 \text{ rad/s}^2)$

$$\vec{a}_B = 0 \text{ mm/s}^2$$

**PROBLEM 15.257:**

**ROD BD:**

$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B} = \vec{v}_B + (\omega_{BD})\vec{r}_{D/B}$

$\vec{v}_D \angle 25^\circ = 60 \text{ mm/s} + (160 \text{ mm})\omega_{BD} \hat{i}$

$\vec{v}_{D/B} = 60 \text{ mm/s} \angle 25^\circ = 27.978 \text{ mm/s}$

$\omega_{BD} = \frac{27.978 \text{ mm/s}}{160 \text{ mm}} = 0.17487 \text{ rad/s}$

$\omega_{BD} = 0.1749 \text{ rad/s}$

$\vec{v}_D = \frac{60 \text{ mm/s}}{\cos 25^\circ} = 66.202 \text{ mm/s}$   $\vec{v}_D = 66.2 \text{ mm/s} \angle 25^\circ$

**PROBLEM 15.258**

$(\vec{a}_{D/B})_t = (\omega_{BD})\vec{r}_{D/B}$

$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$

$$\vec{a}_D \angle 25^\circ = \vec{a}_B + (\omega_{BD})\vec{r}_{D/B} + (\omega_{BD})\vec{r}_{D/B}$$

**PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B**

$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$

$\vec{a}_D \angle 25^\circ = \vec{a}_B + (\omega_{BD})\vec{r}_{D/B} + (\omega_{BD})\vec{r}_{D/B}$

$\vec{a}_D \angle 25^\circ = 90 \text{ mm/s}^2 + (160 \text{ mm})\omega_{BD} \hat{i} + (160 \text{ mm})\omega_{BD} \hat{j}$

**± COMPONENTS:**  $+a_D \cos 25^\circ = (160 \text{ mm})\omega_{BD}$

$a_D \cos 25^\circ = 4.8925 \text{ mm/s}^2$

$a_D = 5.398 \text{ mm/s}^2$   $\vec{a}_D = 5.40 \text{ mm/s}^2 \angle 25^\circ$

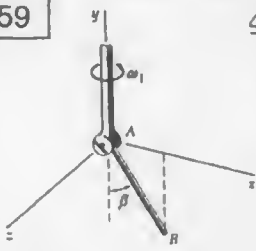
**± COMPONENTS:**  $+a_D \sin 25^\circ = 90 \text{ mm/s}^2 + (160 \text{ mm})\omega_{BD}$

$(5.398 \text{ mm/s}^2) \sin 25^\circ = 90 \text{ mm/s}^2 + (160 \text{ mm})\omega_{BD}$

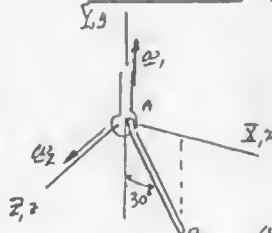
$\omega_{BD} = 0.5768 \text{ rad/s}^2$

$\vec{a}_{BD} = 0.577 \text{ rad/s}^2$

15.259



GIVEN:  $AB = 125 \text{ mm}$   
 $\omega_1 = 5 \text{ rad/s}$ ,  $\dot{\alpha}_1 = 0$   
 $d\beta/dt = 3 \text{ rad/s}$ ,  $d^2\beta/dt^2 = 0$   
 $\beta = 30^\circ$   
 FIND:  $\underline{v}_B$  AND  $\underline{a}_B$



FRAME  $AXYZ$  IS FIXED.  
 MOVING FRAME  $Ax'y'z'$   
 ROTATES ABOUT THE  
 $Y$  AXIS WITH  
 $\underline{\omega} = \omega_1 = (5 \text{ rad/s})\underline{j}$   
 $\underline{\omega}_2 = (d\beta/dt)\underline{k} = (3 \text{ rad/s})\underline{k}$

VELOCITY

$$\underline{r}_{B/A} = (0.125 \text{ m})[\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j}] = (0.0625 \text{ m})\underline{i} - (0.10826 \text{ m})\underline{j}$$

$$\underline{v}_{B/A} = \underline{\omega} \times \underline{r}_{B/A} = (5 \text{ rad/s})\underline{j} \times [(0.0625 \text{ m})\underline{i} - (0.10826 \text{ m})\underline{j}]$$

$$\underline{v}_{B/A} = -(0.3125 \text{ m/s})\underline{k}$$

$$\underline{v}_{B/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{B/A} = (3 \text{ rad/s})\underline{k} \times [(0.0625 \text{ m})\underline{i} - (0.10826 \text{ m})\underline{j}]$$

$$\underline{v}_{B/\mathcal{F}} = (0.1875 \text{ m/s})\underline{i} + (0.32476 \text{ m/s})\underline{j}$$

$$\underline{v}_B = \underline{v}_{B/A} + \underline{v}_{B/\mathcal{F}} = -0.3125 \underline{k} + 0.1875 \underline{i} + 0.32476 \underline{j}$$

$$\underline{v}_B = (0.325 \text{ m/s})\underline{i} + (0.1875 \text{ m/s})\underline{j} - (0.313 \text{ m/s})\underline{k}$$

ACCELERATION

$$\underline{a}_{B/A} = \underline{\omega} \times \underline{v}_{B/A} = (5 \text{ rad/s})\underline{j} \times (-0.3125 \text{ m/s})\underline{k}$$

$$\underline{a}_{B/A} = -(1.5625 \text{ m/s}^2)\underline{i}$$

$$\underline{a}_{B/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{B/\mathcal{F}} = (3 \text{ rad/s})\underline{k} \times [(0.1875 \text{ m/s})\underline{i} + (0.32476 \text{ m/s})\underline{j}]$$

$$\underline{a}_{B/\mathcal{F}} = -(0.5625 \text{ m/s}^2)\underline{i} + (0.9743 \text{ m/s}^2)\underline{j}$$

$$\underline{a}_B = \underline{a}_{B/A} + \underline{a}_{B/\mathcal{F}} = 2(5 \text{ rad/s})\underline{j} \times [(0.1875 \text{ m/s})\underline{i} + (0.32476 \text{ m/s})\underline{j}]$$

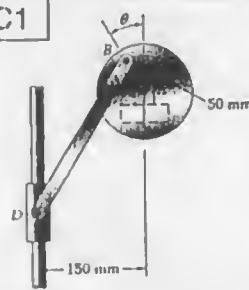
$$\underline{a}_B = -(3.248 \text{ m/s}^2)\underline{k}$$

$$\underline{a}_B = \underline{a}_{B/A} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$$

$$= -1.5625 \underline{i} - 0.5625 \underline{i} + 0.9743 \underline{j} - 3.248 \underline{k}$$

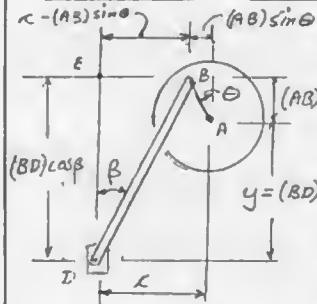
$$\underline{a}_B = -(2.13 \text{ m/s}^2)\underline{i} + (0.974 \text{ m/s}^2)\underline{j} - (3.25 \text{ m/s}^2)\underline{k}$$

15.C1



GIVEN:  
 $\omega_{\text{disk}} = \omega = 500 \text{ rpm}$   
 $\dot{\omega} = 0$   
 $BD = 250 \text{ mm}$

FIND: (a)  $\underline{v}_D$  AND  $\omega_{BD}$  FOR  
 $\theta = 0$  TO  $360^\circ$  USING  
 $30^\circ$  INCREMENTS.  
 (b) TWO VALUES OF  $\theta$   
 FOR WHICH  $\underline{v}_D = 0$



ANALYSIS

SIGNS:  $\downarrow$   
 $+ \omega_{BD}$ 

$$y = (BD) \cos \beta - (AB) \cos \theta$$

FROM  $\Delta BDE$ 

$$\beta = \sin^{-1} \left[ \frac{\kappa - (AB) \sin \theta}{BD} \right] \quad (2)$$

$$\text{FROM (2): } \sin \beta = \frac{\kappa - (AB) \sin \theta}{BD}$$

$$\frac{d}{d\theta} : \cos \beta \frac{d\beta}{d\theta} = -\frac{AB}{BD} \cos \theta \frac{d\theta}{d\theta}$$

$$\text{BUT: } \frac{dy}{d\theta} = \omega \quad \text{AND} \quad \frac{d\beta}{d\theta} = \omega_{BD}$$

$$\omega_{BD} = -\frac{AB}{BD} \frac{\cos \theta}{\cos \beta} \omega \quad (3)$$

$$\text{FROM (1): } \underline{v}_D = \frac{dy}{d\theta} = -(BD) \sin \beta \frac{d\beta}{d\theta} + (AB) \sin \theta \frac{d\theta}{d\theta}$$

$$\underline{v}_D = -(BD) \sin \beta \omega_{BD} + (AB) \sin \theta \omega \quad (4)$$

$$\text{DATA: } \frac{d\theta}{d\theta} = \omega = 500 \text{ rpm} = 500 \left( \frac{2\pi}{60} \right) \text{ rad/s}$$

$$AB = 50 \text{ mm}; \quad BD = 250 \text{ mm}; \quad \kappa = 160 \text{ mm}$$

OUTLINE OF PROGRAM: FOR EACH VALUE OF  $\theta$ 

1. DETERMINE  $\beta$  BY USING EQ 2, THEN
2. USE EQ (3) TO DETERMINE  $\omega_{BD}$ , FINALLY
3. DETERMINE  $\underline{v}_D$  (+) BY USING EQ (4)

theta deg	beta deg	yD mm	vD m/s	omega BD rad/s
0.000	36.870	150.000	1.963	13.090
30.000	30.000	173.205	2.618	10.472
60.000	25.264	201.087	2.885	5.790
90.000	23.578	229.129	2.618	0.000
120.000	25.264	251.087	1.649	-5.790
150.000	30.000	259.808	0.000	-10.472
180.000	36.870	250.000	-1.963	-13.090
210.000	44.427	221.837	-3.531	-12.699
240.000	50.643	183.539	-3.863	-8.257
270.000	53.130	150.000	-2.618	-0.000
300.000	50.643	133.539	-0.671	8.257
330.000	44.427	135.234	0.913	12.699
360.000	36.870	150.000	1.963	13.090

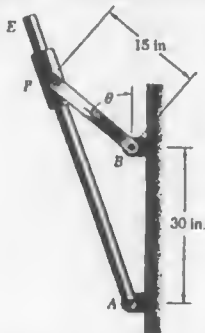
Theta [for  $v_D = 0$ ]

Theta deg	beta deg	yD mm	vD m/s
149.900	29.980	259.807	0.006
150.000	30.000	259.808	0.000
150.100	30.020	259.807	-0.006

Theta [for  $v_D = 0$ ]

Theta deg	beta deg	yD mm	vD m/s
311.400	48.592	132.288	-0.001
311.410	48.590	132.288	0.000
311.420	48.588	132.288	0.001

## 15.C2



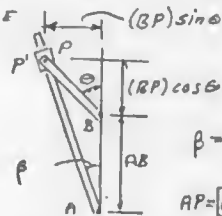
GIVEN:

$$\omega_{BP} = 6 \text{ rad/s}, \quad \alpha_{BP} = 0$$

FIND:

- (1)  $\omega_{AE}$  AND  $\alpha_{AE}$  FOR  $\theta = 0^\circ$  TO  $180^\circ$  AT  $15^\circ$  INCREMENTS,  
 (2)  $(\alpha_{AE})_{\text{minimum}}$  AND CORRESPONDING VALUE OF  $\theta$ .

GEOMETRY:



$$\beta = \tan^{-1} \frac{(BP) \sin \theta}{AB + (BP) \cos \theta} \quad (1)$$

$$AP = [(BP) \sin \theta] / \sin \beta \quad (2)$$

VELOCITY:

$$\text{ROD BP: } \vec{v}_P = (BP) \omega_{BP} \vec{\Delta} \theta \quad (3)$$

$$\text{ROD AE: } \vec{v}_{P1} = (AP) \omega_{AE} \vec{\Delta} \beta \quad (4)$$

$$\vec{v}_P = \vec{v}_{P1} + \vec{v}_{P/g}$$

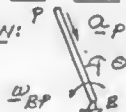
$$[\vec{v}_P \vec{\Delta} \theta] = [\vec{v}_{P1} \vec{\Delta} \beta] + [\vec{v}_{P/g} \Delta \beta]$$

$$\vec{v}_{P/g} = \vec{v}_P \sin(\theta - \beta) \vec{\Delta} \beta$$

$$\vec{v}_{P1} = \vec{v}_P \cos(\theta - \beta) \vec{\Delta} \beta$$

$$\omega_{AE} = \vec{v}_{P1} / (AP) \quad (5)$$

ACCELERATION:



$$\vec{a}_P = (BP) \omega_{BP}^2 \vec{\Delta} \theta \quad (6)$$

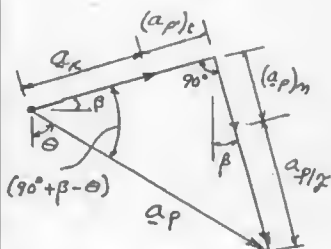
$$(\vec{a}_{P1})_t = (AP) \alpha_{AE} \vec{\Delta} \beta$$

$$(\vec{a}_{P1})_n = (AP) \omega_{AE}^2 \vec{\Delta} \beta$$

$$\vec{a}_{P/g} = \vec{a}_{P1/g} \vec{\Delta} \beta$$

$$\vec{a}_K = 2 \omega_{AE} \vec{v}_{P1/g} \vec{\Delta} \beta \quad (7)$$

$$\vec{a}_P = \vec{a}_{P1} + \vec{a}_{P/g} + \vec{a}_K$$



$$\text{RIGHT TRIANGLE: } a_K + (\vec{a}_P)_t = a_P \cos(90^\circ + \beta - \theta)$$

$$a_K + (AP) \alpha_{AE} = a_P \cos(90^\circ + \beta - \theta)$$

$$\alpha_{AE} = \frac{1}{AP} [a_P \cos(90^\circ + \beta - \theta) - a_K] \quad (8)$$

(CONTINUED)

## 15.C2 continued

DATA:  $\omega_{BP} = 6 \text{ rad/s}$   
 $BP = 15 \text{ in.}; AB = 30 \text{ in}$ 

OUTLINE OF PROGRAM:

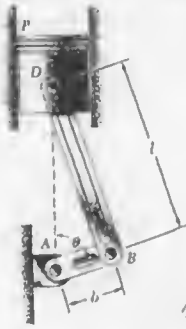
1. USE EQS. (1) AND (2) TO FIND  $\beta$  AND  $AP$ .
2. USE EQS. (3) AND (4) TO FIND  $\vec{v}_P$  AND  $\vec{v}_{P1}$ .
3. DETERMINE  $\omega_{AE}$  BY USING EQ. (5).
4. USE EQ. (6) TO FIND  $\vec{a}_P$ .
5. USE EQ. (7) TO FIND  $\vec{a}_K$ .
6. DETERMINE  $\alpha_{AE}$  BY USING EQ. (8).

theta deg.	beta deg.	omega AE rad/s	alpha rad/s^2
0	0.00	2.000	0.000
15	4.99	1.985	0.712
30	9.90	1.937	1.508
45	14.84	1.850	2.492
60	19.11	1.714	3.618
75	23.18	1.509	5.128
90	26.57	1.200	6.640
105	29.02	0.730	13.273
120	30.00	0.000	20.785
135	28.68	-1.144	32.388
150	23.79	-2.880	45.782
165	14.05	-4.920	43.288
180	0.00	-8.000	0.000

theta for maximum alpha

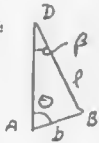
theta deg.	alpha rad/s^2
157.0800	48.58893
157.0900	48.58894
157.1000	48.58894
157.1100	48.58893

15.C3

GIVEN:  $\omega_{AB} = 1000 \text{ rpm}$  $\alpha_{AB} = 0$  $l = 160 \text{ mm}$  $b = 60 \text{ mm}$ FOR VALUES OF  $\theta$   
FROM 0 TO 180° AT  
10° INTERVALS:FIND: (a)  $\omega_{BD}$  AND  $\alpha_{BD}$ (b)  $v_D$  AND  $a_D$ 

NOTE: MOTION OF D + P ARE EQUAL

GEOMETRY:

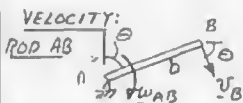


LAW OF SINES

$$\frac{\sin \beta}{b} = \frac{\sin \theta}{l}$$

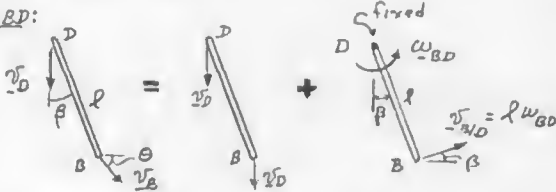
$$\sin \beta = \frac{b}{l} \sin \theta \quad (1)$$

VELOCITY:



$$v_B = b \omega_{AB} \quad (2)$$

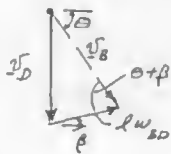
ROD BD:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

$$v_B = v_D + v_{B/D}$$

$$[v_B \angle \theta] = [v_D \downarrow] + [l \omega_{BD} \angle \beta]$$



LAW OF SINES

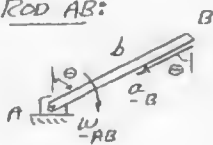
$$\frac{v_D}{\sin(\theta + \beta)} = \frac{v_B}{\sin(90^\circ - \beta)} = \frac{l \omega_{BD}}{\sin(90^\circ - \theta)}$$

$$v_D = v_B \frac{\sin(\theta + \beta)}{\sin(90^\circ - \beta)} = v_B \frac{\sin(\theta + \beta)}{\cos \beta} \quad (3)$$

$$\omega_{BD} = \frac{v_B}{l} \frac{\sin(90^\circ - \theta)}{\sin(90^\circ - \beta)} = \frac{v_B}{l} \cos \theta \quad (4)$$

ACCELERATION:

ROD AB:

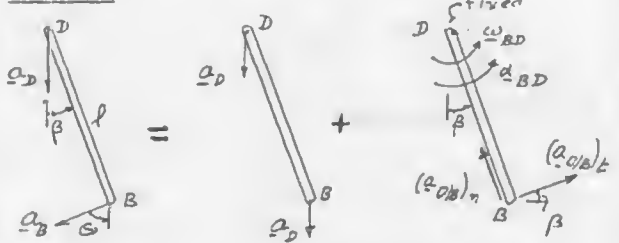


$$a_B = b \alpha_{AB} \quad (5)$$

(CONTINUED)

15.C3 continued

ROD BD:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

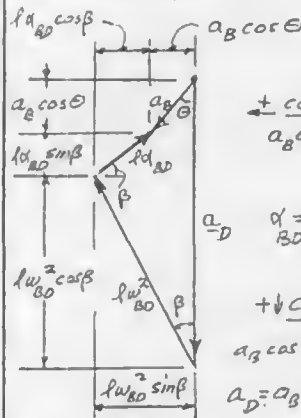
$$(a_{B/D})_t = l \alpha_{BD}$$

$$(a_{B/D})_n = l \omega_{BD}^2$$

$$a_B = a_D + a_{B/D}$$

$$a_B = a_D + (a_{B/D})_t + (a_{B/D})_n$$

$$[a_B \angle \beta] = [a_D \downarrow] + [l \alpha_{BD} \angle \beta] + [l \omega_{BD}^2 \angle \beta]$$



VECTOR DIAGRAM

COMPONENTS

$$a_B \cos \theta = l \omega_{BD}^2 \sin \beta - l \alpha_{BD} \cos \beta$$

$$a_D = \frac{l \omega_{BD}^2 \sin \beta - a_B \cos \theta}{l \cos \beta} \quad (6)$$

COMPONENTS

$$a_B \cos \theta = a_D - l \omega_{BD}^2 \cos \beta - l \alpha_{BD} \sin \beta$$

$$a_D = a_B \cos \theta + l \omega_{BD}^2 \cos \beta + l \alpha_{BD} \sin \beta \quad (7)$$

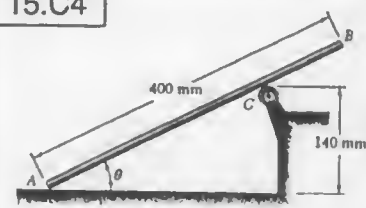
OUTLINE OF PROGRAM: FOR EACH VALUE OF  $\theta$ :

VELOCITY: USE EQS 1, 2, 3, AND 4, IN SEQUENCE

TO OBTAIN  $\beta$ ,  $v_B$ ,  $v_D$ , AND  $\omega_{BD}$ ACCELERATION: USE EQ(5) TO FIND  $a_B$ , RECALLVALUES OF  $\beta$  AND  $\omega_{BD}$ , AND FROM EQS (6) AND (7)FIND  $\alpha_{BD}$  AND  $a_D$ .DATA:  $\omega_{AB} = 1000 \text{ rpm} = 1000 \left( \frac{2\pi}{60} \right) \text{ rad/s}$  $l = 0.16 \text{ m}$   $b = 0.06 \text{ m}$ 

theta deg.	beta deg.	omega rad/s	alpha rad/s^2	vD m/s	aD m/s^2
0	0.00	39.27	0	0.000	905
10	3.73	38.78	-618	1.495	881
20	7.37	37.21	-1239	2.913	613
30	10.81	34.62	-1864	4.180	702
40	13.95	31.00	-2485	5.234	557
50	16.69	26.35	-3081	6.024	386
60	18.95	20.78	-3616	6.520	208
70	20.63	14.35	-4062	6.713	27
80	21.67	7.34	-4337	6.621	-134
90	22.02	0.00	-4436	6.283	-266
100	21.67	-7.34	-4337	5.754	-362
110	20.63	-14.35	-4062	5.095	-423
120	18.95	-20.78	-3616	4.363	-452
130	16.69	-26.35	-3081	3.602	-458
140	13.95	-31.00	-2485	2.643	-451
150	10.81	-34.62	-1864	2.103	-437
160	7.37	-37.21	-1239	1.365	-424
170	3.73	-38.78	-618	0.667	-416
180	0.00	-39.27	0	0.000	-411

## 15.C4

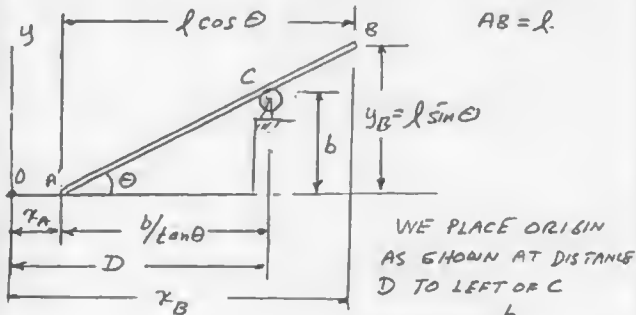


GIVEN:

$$v_A = 180 \text{ mm/s} \rightarrow$$

$$a_A = 0$$

FIND: (1)  $v_B$  AND  $a_B$   
FOR VALUES OF  $\theta$   
FROM  $20^\circ$  TO  $90^\circ$   
AT  $5^\circ$  INCREMENTS  
(2)  $\theta$  AND  $d$  FOR  $a_{\text{MAX}}$ .



WE PLACE ORIGIN  
AS SHOWN AT DISTANCE  
D TO LEFT OF C

$$x_A = D - \frac{b}{\tan \theta}$$

$$v_A = \dot{x}_A = + \frac{b}{\sin^2 \theta} \dot{\theta} \quad \omega = \dot{\theta} = \frac{v_A \sin^2 \theta}{b} \quad (1)$$

$$x_B = D - \frac{b}{\tan \theta} + l \cos \theta$$

$$(v_B)_x = \dot{x}_B = \frac{b}{\sin^2 \theta} \dot{\theta} - l \sin \theta \dot{\theta} = \left( \frac{b}{\sin^2 \theta} - l \sin \theta \right) \frac{v_A \sin^2 \theta}{b} \quad (2)$$

$$(v_B)_x = v_A \left( 1 - \frac{l}{b} \sin^3 \theta \right)$$

$$y_B = l \sin \theta; \quad (v_B)_y = \dot{y}_B = l \cos \theta \dot{\theta} = l \cos \theta \frac{v_A \sin^2 \theta}{b} \quad (3)$$

$$(v_B)_y = v_A \frac{l}{b} \cos \theta \sin \theta$$

ACCELERATION

$$a_A = \ddot{x}_A = 0; \quad a_A = \frac{b}{\sin^2 \theta} \ddot{\theta} - \frac{2 \cos \theta}{\sin^3 \theta} \dot{\theta}^2$$

$$a_A = 0 \Rightarrow \frac{b}{\sin^2 \theta} \left[ \alpha - \frac{2}{\tan \theta} \omega^2 \right]$$

$$\alpha = \frac{2}{\tan \theta} \left( \frac{v_A}{b} \sin^2 \theta \right) \quad \alpha = 2 \left( \frac{v_A}{b} \right)^2 \sin^2 \theta \cos \theta \quad (4)$$

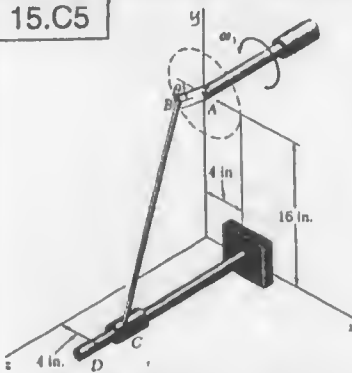
OUTLINE OF PROGRAM FOR EACH VALUE OF  $\theta$ :USE EQS (1) AND (4) TO CALCULATE  $\omega$  AND  $\alpha$ ,USE EQS (2) AND (3) TO CALCULATE  $(v_B)_x$  AND  $(v_B)_y$ 

$$v_B^2 = (v_B)_x^2 + (v_B)_y^2; \quad \gamma = \tan^{-1} \frac{(v_B)_y}{(v_B)_x}$$

theta deg.	omega rad/s	alpha rad/s <sup>2</sup>	vx mm/s	vy mm/s	vel mm/s	gamma deg.
20	0.150	0.124	169.42	56.53	180.15	19.52
25	0.230	0.225	141.18	83.25	163.90	30.53
30	0.321	0.358	115.71	111.35	150.59	43.90
35	0.423	0.511	82.95	138.60	151.52	59.10
40	0.531	0.673	43.41	162.78	158.47	75.07
45	0.643	0.827	-1.53	181.83	181.84	-89.42
50	0.754	0.956	-51.19	193.99	200.93	-75.22
55	0.863	1.042	-102.65	197.94	222.68	-82.58
60	0.954	1.074	-154.04	192.88	246.82	-51.39
65	1.056	1.040	-202.55	178.53	270.22	-41.36
70	1.135	0.938	-248.74	155.32	291.55	-32.19
75	1.200	0.771	-253.49	124.19	309.49	-23.56
80	1.247	0.545	-311.20	55.51	323.03	-15.55
85	1.275	0.285	-328.44	44.48	331.44	-7.71
90	1.285	0.000	-334.29	0.00	334.29	-0.00

theta [deg.]	maximum alpha [rad/s <sup>2</sup> ]
59.900	1.073682
60.000	1.073695
60.100	1.073682

## 15.C5

GIVEN:  $BC = 24 \text{ in.}$ 

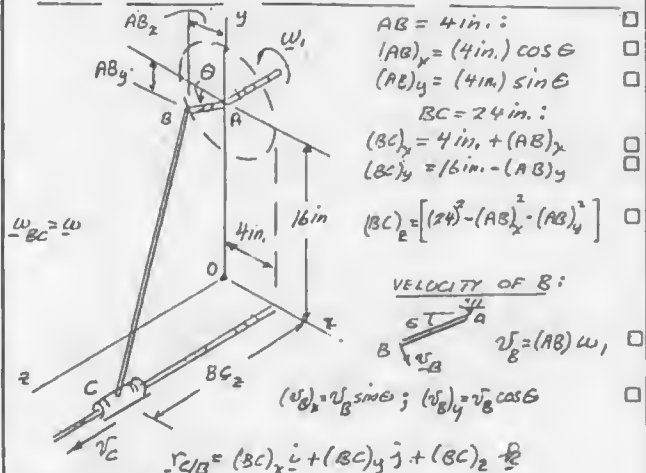
$$AB = 4 \text{ in.}$$

$$\omega_1 = 10 \text{ rad/s}$$

$$a_1 = 0$$

FIND: (1)  $v_C$  FOR  
VALUES OF  $\theta$  FROM  
0 TO  $360^\circ$  AT  
 $30^\circ$  INCREMENTS.

(2) TWO VALUES  
OF  $\theta$  FOR WHICH  
 $v_C = 0$



$$AB = 4 \text{ in.}$$

$$(AB)_x = (4 \text{ in.}) \cos \theta$$

$$(AB)_y = (4 \text{ in.}) \sin \theta$$

$$BC = 24 \text{ in.}$$

$$(BC)_x = 4 \text{ in.} + (AB)_x$$

$$(BC)_y = 16 \text{ in.} - (AB)_y$$

$$(BC)_z = \left[ (24)^2 - (AB)_x^2 - (AB)_y^2 \right]^{1/2}$$

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$$(BC)_z = \left[ (24)^2 - (AB)_x^2 - (AB)_y^2 \right]^{1/2}$$

VELOCITY OF C:

$$v_C = v_B + v_{C/B} = v_B + \omega_{BC} \times r_{C/B}$$

$$v_C = (v_B)_x i + (v_B)_y j + \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ (BC)_x & (BC)_y & (BC)_z \end{vmatrix}$$

$$x \text{ COMPONENTS: } -(v_B)_x = (BC)_z \omega_y - (BC)_y \omega_z \quad (a)$$

$$y \text{ COMPONENTS: } -(v_B)_y = -(BC)_z \omega_x + (BC)_x \omega_z \quad (b)$$

$$z \text{ COMPONENTS: } v_C = (BC)_y \omega_x - (BC)_x \omega_y \quad (c)$$

DETERMINANT  $(\omega_x, \omega_y, \omega_z)$  IS ZERO. CHOOSE  $\omega_z = 0$ 

$$\text{EQ (a) YIELDS: } \omega_y = -(v_B)_x / (BC)_z$$

$$\text{EQ (b) YIELDS: } \omega_x = (v_B)_y / (BC)_z$$

$$\text{THEN USE EQ (c): } v_C = (BC)_y \omega_x - (BC)_x \omega_y$$

OUTLINE OF PROGRAM:

FOR INITIAL VALUE  $\theta = 0$ , START

AT TOP OF SOLUTION AND IN SEQUENCE

SHOWN, PROGRAM EACH EQUATION

DESIGNATED BY A "D". EVALUATE AND STORE

LEFT HAND MEMBER OF EACH EQUATION.

PRINT VALUES OF  $\theta$ , COMPONENTS OF  $BC$ , AND  $v_C$ INCREASE VALUE OF  $\theta$  BY  $30^\circ$  ANDREPEAT EVALUATION UNTIL  $\theta = 360^\circ$ .

(CONTINUED)

### 15.C5 continued

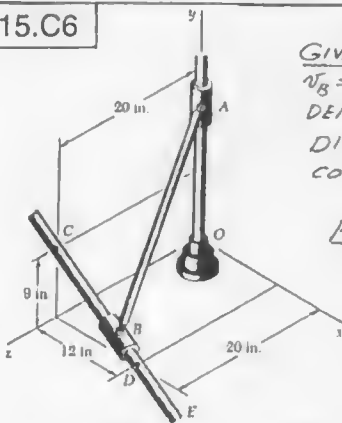
theta deg	Components of rod BC			Velocity of C in./s
	x in.	y in.	z in.	
0.000	8.000	-16.000	16.000	40.000
30.000	7.484	-14.000	16.008	35.221
60.000	6.000	-12.536	19.587	23.436
90.000	4.000	-12.000	20.396	7.845
120.000	2.000	-12.536	20.368	-8.908
150.000	0.536	-14.000	19.486	-24.338
180.000	0.000	-16.000	17.869	-35.777
210.000	0.536	-16.000	15.865	-39.977
240.000	2.000	-19.464	13.898	-32.995
270.000	4.000	-20.000	12.649	-12.649
300.000	6.000	-19.464	12.694	14.293
330.000	7.484	-16.000	14.010	33.851
360.000	8.000	-16.000	16.000	40.000

Determination of values of theta for  $v_C = 0$

theta	Components of rod BC			Velocity of C
	x	y	z	
104.034	3.030	-12.119	20.492	0.001
104.035	3.030	-12.119	20.492	0.001
104.036	3.030	-12.119	20.492	0.000
104.037	3.030	-12.119	20.492	-0.000

theta	Components of rod BC			Velocity of C
	x	y	z	
264.020	4.969	-19.881	12.492	-0.015
264.030	4.970	-19.881	12.492	-0.008
264.040	4.970	-19.881	12.492	0.003

### 15.C6



GIVEN:  $AB = 25 \text{ in.}$   
 $v_B = 20 \text{ in/s}$  TOWARD POINT E  
 DENOTE BY  $d$  THE  
 DISTANCE BC OF  
 COLLAR FROM POINT C

FIND:  $v_A$  FOR VALUES  
 OF  $d$  FROM  
 0 TO 15 in. AT  
 1-in. INCREMENTS

GEOMETRY:

$$CD = 15 \text{ in.}$$

$$B_x = \frac{4}{5}d = 0.8d$$

ROD AB:  $(AB)_x = -B_x = -0.8d$  (1)  
 $AB = 25 \text{ in.}$   $(AB)_z = -20 \text{ in.}$  (2)

$$AB^2 = (AB)_x^2 + (AB)_y^2 + (AB)_z^2$$

$$(AB)_y = [AB^2 - (AB)_x^2 - (AB)_z^2]^{1/2}$$

$$(AB)_y = [25^2 - (0.8d)^2 - 20^2]^{1/2} = [225 - 0.64d^2]^{1/2} \quad (3)$$

$$v_{A/B} = + (AB)_x \dot{i} + (AB)_y \dot{j} + (AB)_z \dot{k}$$

(CONTINUED)

### 15.C6 continued

VELOCITY OF B:

$$v_B = 20 \text{ in/s}$$

$$\hat{e}_{CD} = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

$$v_B = v_B \hat{e}_{CD}$$

$$v_B = (20 \text{ in/s}) \left( \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j} \right)$$

$$(v_B)_x = 16 \text{ in/s} \quad (4)$$

$$(v_B)_y = -12 \text{ in/s} \quad (5)$$

VELOCITY OF A:

$$v_A = v_B + v_{A/B} = v_B + \omega \times r_{AB}$$

$$v_A \hat{j} = (v_B)_x \hat{i} + (v_B)_y \hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ (AB)_x & (AB)_y & (AB)_z \end{vmatrix}$$

$x$  COMPONENTS:  $-(v_B)_x = + (AB)_z \omega_y + (AB)_y \omega_z$  (a)

$y$  COMPONENTS:  $v_A - (v_B)_y = - (AB)_z \omega_x + (AB)_x \omega_z$  (b)

$z$  COMPONENTS:  $0 = (AB)_y \omega_x - (AB)_x \omega_y$  (c)

DETERMINATE OF  $(\omega_x, \omega_y, \omega_z)$  IS ZERO.

CHOOSE  $\omega_x = 0$

EQ. (c) YIELDS:  $\omega_y = 0$

EQ. (a):  $-(v_B)_x = 0 + (AB)_y \omega_z$   
 $\omega_z = -(v_B)_x / (AB)_y$  (6)

EQ. (b):  $v_A - (v_B)_y = 0 + (AB)_x \omega_z$   
 $v_A = (v_B)_y + (AB)_x \omega_z$  (7)

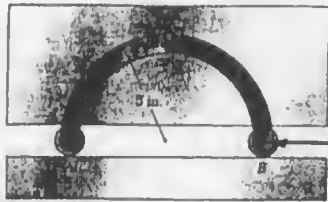
OUTLINE OF PROGRAM:

FOR INITIAL VALUE  $d = 0$ , PROGRAM, IN  
 SEQUENCES, EQUATIONS (1) THROUGH (7)  
 EVALUATE LEFT-HAND MEMBER OF EACH  
 EQUATION AND PRINT VALUES OF  
 $d$ , COMPONENTS OF  $v_{A/B}$  AND  $v_C$ .  
 INCREASE VALUE OF  $d$  BY 1 in. AND  
 REPEAT PROCESS UNTIL  $d = CD = 15 \text{ in.}$

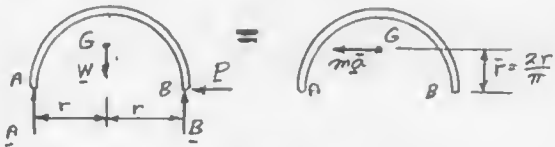
d in.	Components of AB			Velocity $v_A$ in/s
	x in.	y in.	z in.	
0.000	0.000	15.000	-20.00	-12.000
1.000	-0.800	14.979	-20.00	-12.855
2.000	-1.600	14.914	-20.00	-13.716
3.000	-2.400	14.807	-20.00	-14.593
4.000	-3.200	14.655	-20.00	-15.494
5.000	-4.000	14.457	-20.00	-16.427
6.000	-4.800	14.211	-20.00	-17.404
7.000	-5.600	13.915	-20.00	-18.439
8.000	-6.400	13.566	-20.00	-19.548
9.000	-7.200	13.159	-20.00	-20.754
10.000	-8.000	12.689	-20.00	-22.088
11.000	-8.800	12.147	-20.00	-23.591
12.000	-9.600	11.526	-20.00	-25.327
13.000	-10.400	10.809	-20.00	-27.394
14.000	-11.200	9.978	-20.00	-29.860
15.000	-12.000	9.000	-20.00	-33.353

# 16.1 and 16.2

GIVEN:  $W = 316$



PROBLEM 16.1:  
FOR  $P = 516$ ,  
FIND: (a)  $\alpha$ ,  
(b) REACTIONS.  
PROBLEM 16.2:  
FOR  $A = 0$ ,  
FIND: (a)  $P$ , (b)  $\alpha$ .



$$\begin{aligned} \uparrow \Sigma F_x = \Sigma (F_x)_{eff}: & P = m\bar{a} & (1) \\ \uparrow \Sigma M_A = \Sigma (M_A)_{eff}: & B(2r) - Wr = m\bar{a}\left(\frac{2r}{\pi}\right) & (2) \\ \uparrow \Sigma F_y = \Sigma (F_y)_{eff}: & A + B - W = 0 & (3) \end{aligned}$$

PROBLEM 16.1:  $P = 516$ ,  $W = 316$ ,  $m = W/g$

$$\begin{aligned} \text{EQ(1): } P &= (W/g)\bar{a} \\ \bar{a} &= (P/W)g = \frac{516}{316}(32.2 \text{ ft/s}^2) \\ \bar{a} &= 53.17 \text{ ft/s}^2; \quad \bar{\alpha} = 53.7 \text{ ft/s}^2 \end{aligned}$$

$$\begin{aligned} \text{EQ(2): } B(2r) - Wr &= \frac{W}{g}\left(\frac{P}{W}\right)\left(\frac{2r}{\pi}\right) \\ B &= \frac{1}{2}W + \frac{P}{\pi} = \frac{1}{2}(316) + \frac{516}{\pi} \\ B &= 3.09216 \quad \underline{B = 3.0916} \end{aligned}$$

$$\begin{aligned} \text{EQ(3): } A + 3.09216 - 316 &= 0 \\ A &= -0.09216 \quad \underline{A = 0.09216} \end{aligned}$$

PROBLEM 16.2:  $A = 0$ ,  $W = 316$ ,  $m = W/g$

$$\begin{aligned} \text{EQ(2): } 0 - Wr &= \frac{W}{g}\bar{\alpha}\left(\frac{2r}{\pi}\right) \\ \bar{\alpha} &= \frac{\pi}{2}g = \frac{\pi}{2}(32.2 \text{ ft/s}^2) \\ \bar{\alpha} &= 50.58 \text{ ft/s}^2; \quad \underline{\bar{\alpha} = 50.6 \text{ ft/s}^2} \end{aligned}$$

$$\begin{aligned} \text{EQ(1): } P &= \frac{W}{g}\bar{\alpha} \\ P &= \frac{W}{g}\left(\frac{\pi}{2}g\right) = \frac{\pi}{2}W = 4.71216 \\ \underline{P = 4.7116} \end{aligned}$$

# 16.3 and 16.4

GIVEN:

Rod:  $m = 2.5 \text{ kg}$ ,  $AB = 300 \text{ mm}$

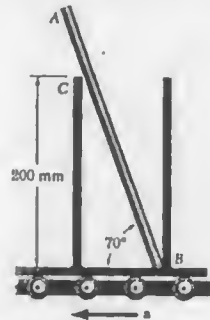
PROBLEM 16.3:

For  $\alpha = 1.5 \text{ m/s}^2$

FIND: (a)  $C$ , (b)  $B$

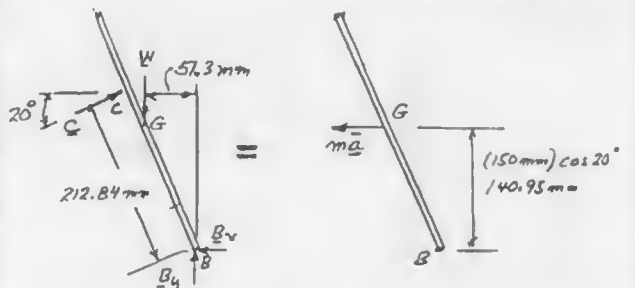
PROBLEM 16.4:

FIND:  $\alpha_{\max}$  FOR ROD TO REMAIN IN POSITION



GEOMETRY

$$\begin{aligned} \frac{200 \text{ mm}}{\cos 20^\circ} &= 212.84 \text{ mm} \\ (0.15 \text{ m}) \sin 20^\circ &= 51.3 \text{ mm} \\ W &= mg = (2.5 \text{ kg})(9.81 \text{ m/s}^2) = 24.525 \text{ N} \end{aligned}$$



$$\begin{aligned} \uparrow \Sigma M_B = \Sigma (M_B)_{eff}: & C(212.84 \text{ mm}) - W(51.3 \text{ mm}) = -m\bar{a}(140.95 \text{ mm}) \\ C &= 0.241W - 0.6621m\bar{a} \\ C &= 0.241(24.525 \text{ N}) - 0.6622(2.5 \text{ kg})(\alpha) \\ C &= 5.911 \text{ N} - 1.656\alpha & (1) \end{aligned}$$

PROBLEM 16.3:  $\alpha = 1.5 \text{ m/s}^2$

$$\begin{aligned} \text{EQ(1): } C &= 5.911 - 1.656(1.5); \quad \underline{C = 3.43 \text{ N}} \\ \uparrow \Sigma F_y = \Sigma (F_y)_{eff}: & B_y - W + C \sin 20^\circ = 0 \\ B_y &= 24.525 \text{ N} - (3.43 \text{ N}) \sin 20^\circ = 23.35 \text{ N} \uparrow \\ \uparrow \Sigma F_x = \Sigma (F_x)_{eff}: & B_x - C \cos 20^\circ = m\bar{a} \\ B_x - (3.43 \text{ N}) \cos 20^\circ &= (2.5 \text{ kg})(1.5 \text{ m/s}^2) \\ B_x &= 3.22 + 3.75 = 6.97 \text{ N} \leftarrow \\ \underline{B} &= 24.4 \text{ N} \searrow 73.4^\circ \end{aligned}$$

PROBLEM 16.4: For  $\alpha_{\max}$ ,  $C = 0$

$$\begin{aligned} \text{EQ(1): } C &= 5.911 \text{ N} - 1.656\alpha \\ 0 &= 5.911 \text{ N} - 1.656\alpha_{\max} \\ \alpha_{\max} &= 3.57 \text{ m/s}^2 \end{aligned}$$

$$\underline{\alpha_{\max} = 3.57 \text{ m/s}^2}$$

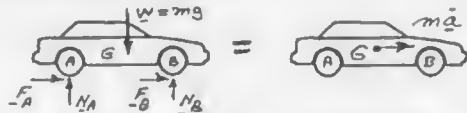


16.5



GIVEN:  $\mu_s = 0.80$   
 FIND:  $\alpha_{\max}$  ASSUMING  
 (a) FOUR-WHEEL DRIVE  
 (b) REAR-WHEEL DRIVE  
 (c) FRONT-WHEEL DRIVE

(a) FOUR-WHEEL DRIVE:



$$+\uparrow \Sigma F_y = 0: N_A + N_B - W = 0 \quad N_A + N_B = W = mg$$

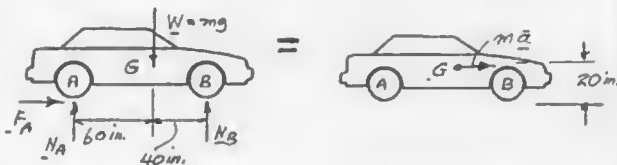
$$\text{THIS: } F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s (N_A + N_B) = \mu_s W = 0.80 mg$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_A + F_B = m\bar{a}$$

$$0.80 mg = m\bar{a}$$

$$\bar{a} = 0.80g = 0.80(32.2 \text{ ft/s}^2) \quad \bar{a} = 25.8 \text{ ft/s}^2 \rightarrow$$

(b) REAR-WHEEL DRIVE:



$$+\uparrow \Sigma M_B = \Sigma (M_B)_{\text{eff}}: (40 \text{ in.})W - (100 \text{ in.})N_A = -(20 \text{ in.})m\bar{a}$$

$$N_A = 0.4W + 0.2m\bar{a}$$

$$\text{THUS: } F_A = \mu_s N_B = 0.80(0.4W + 0.2m\bar{a}) = 0.32mg + 0.16m\bar{a}$$

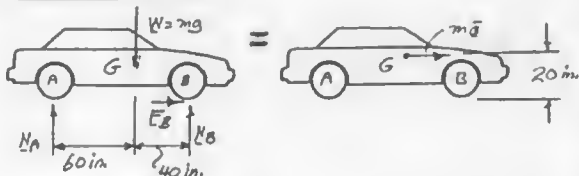
$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_A = m\bar{a}$$

$$0.32mg + 0.16m\bar{a} = m\bar{a}$$

$$0.32g = 0.84\bar{a}$$

$$\bar{a} = \frac{0.32}{0.84}(32.2 \text{ ft/s}^2) \quad \bar{a} = 12.27 \text{ ft/s}^2 \rightarrow$$

(c) FRONT-WHEEL DRIVE:



$$+\uparrow \Sigma M_A = \Sigma (M_A)_{\text{eff}}: (100 \text{ in.})N_B - (60 \text{ in.})W = -(20 \text{ in.})m\bar{a}$$

$$N_B = 0.6W - 0.2m\bar{a}$$

$$\text{THUS: } F_B = \mu_s N_B = 0.80(0.6W - 0.2m\bar{a}) = 0.48mg - 0.16m\bar{a}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_B = m\bar{a}$$

$$0.48mg - 0.16m\bar{a} = m\bar{a}$$

$$0.48g = 1.16\bar{a}$$

$$\bar{a} = \frac{0.48}{1.16}(32.2 \text{ ft/s}^2)$$

$$\bar{a} = 13.32 \text{ ft/s}^2 \rightarrow$$

16.6



GIVEN:  $v_0 = 30 \text{ ft/s} \rightarrow$   
 FROM SAMPLE PROB 16.1

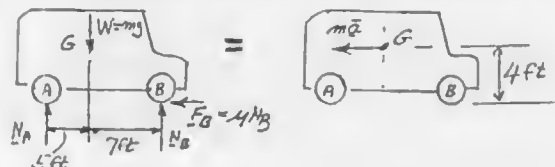
$$\mu_k = 0.699$$

FIND: DISTANCE REQUIRED  
 TO STOP IF

(a) REAR-WHEEL BRAKES  
 FAIL TO OPERATE

(b) FRONT-WHEEL BRAKES  
 FAIL TO OPERATE

(a) IF REAR-WHEEL BRAKES FAIL TO OPERATE



$$+\uparrow \Sigma M_A = \Sigma (M_A)_{\text{eff}}: N_B(12 \text{ ft}) - W(5 \text{ ft}) = m\bar{a}(4 \text{ ft})$$

$$N_B = \frac{5}{12}W + \frac{1}{3}W\bar{a}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_B = m\bar{a}$$

$$\mu_k N_B = \frac{W}{g}\bar{a}$$

$$0.699\left(\frac{5}{12}W + \frac{1}{3}W\bar{a}\right) = \frac{W}{g}\bar{a}$$

$$\bar{a} = \frac{0.699\left(\frac{5}{12}\right)(32.2 \text{ ft/s}^2)}{1 - 0.233}$$

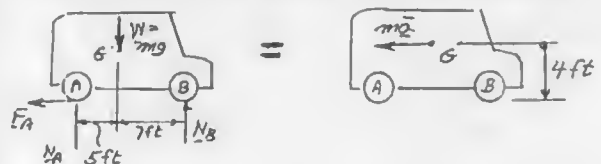
$$\bar{a} = 12.227 \text{ ft/s}^2 \leftarrow$$

UNIFORMLY ACCELERATED MOTION

$$v^2 = v_0^2 + 2ax \quad 0 = (30 \text{ ft/s})^2 - 2(12.227 \text{ ft/s}^2)x$$

$$x = 36.8 \text{ ft} \leftarrow$$

(b) IF FRONT-WHEEL BRAKES FAIL TO OPERATE



$$+\uparrow \Sigma M_B = \Sigma (M_B)_{\text{eff}}: W(7 \text{ ft}) - N_A(12 \text{ ft}) = m\bar{a}(4 \text{ ft})$$

$$N_A = \frac{7}{12}W - \frac{1}{3}W\bar{a}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_A = m\bar{a}$$

$$\mu_k N_A = \frac{W}{g}\bar{a}$$

$$0.699\left(\frac{7}{12}W - \frac{1}{3}W\bar{a}\right) = \frac{W}{g}\bar{a}$$

$$\bar{a} = \frac{0.699\left(\frac{7}{12}\right)(32.2 \text{ ft/s}^2)}{1 + 0.233}$$

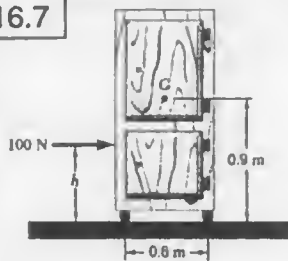
$$\bar{a} = 10.648 \text{ ft/s}^2 \leftarrow$$

UNIFORMLY ACCELERATED MOTION

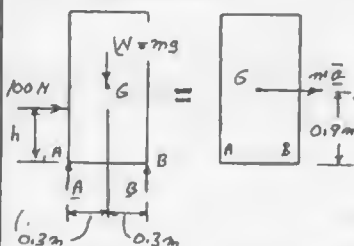
$$v^2 = v_0^2 + 2ax \quad 0 = (30 \text{ ft/s})^2 - 2(10.648 \text{ ft/s}^2)x$$

$$x = 42.3 \text{ ft} \leftarrow$$

16.7

GIVEN:  $\mu = 0$  $m = 20 \text{ kg}$ FIND: (a)  $\bar{a}$   
(b) RANGE OF VALUES OF  $h$  FOR NO TIPPING

(a) ACCELERATION



$$+\uparrow \Sigma F_x = \Sigma (F_x)_{eff}: 100 \text{ N} = m\bar{a}$$

$$100 \text{ N} = (20 \text{ kg})\bar{a}$$

$$\bar{a} = 5 \text{ m/s}^2 \rightarrow$$

(b) FOR TIPPING TO IMPEND:  $\bar{a} = 0$ 

$$+\uparrow \Sigma M_B = \Sigma (M_B)_{eff}: (100 \text{ N})h - mg(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h - (20 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = (100 \text{ N})(0.9 \text{ m})$$

$$h = 1.489 \text{ m}$$

FOR TIPPING TO IMPEND:  $\bar{a} = 0$ 

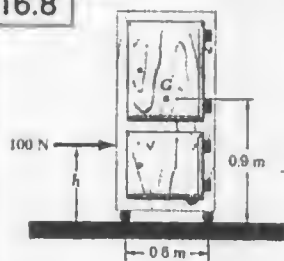
$$+\uparrow \Sigma M_A = \Sigma (M_A)_{eff}: (100 \text{ N})h + mg(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h + (20 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = (100 \text{ N})(0.9 \text{ m})$$

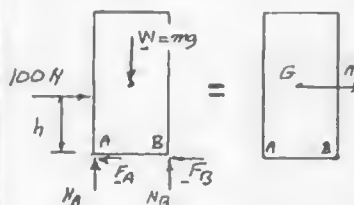
$$h = 0.311 \text{ m}$$

CABINET WILL NOT TIP:  $0.311 \text{ m} \leq h \leq 1.489 \text{ m}$ 

16.8

GIVEN:  $\mu = 0.25$  $m = 20 \text{ kg}$ FIND: (a)  $\bar{a}$   
(b) RANGE OF VALUES OF  $h$  FOR NO TIPPING

(a) ACCELERATION



$$+\uparrow \Sigma F_y = 0$$

$$N_A + N_B - W = 0$$

$$N_A + N_B = mg$$

BUT,  $F = \mu N$ , THUS

$$F_A + F_B = \mu (mg)$$

$$+\uparrow \Sigma F_x = \Sigma (F_x)_{eff}: 100 \text{ N} - (F_A + F_B) = m\bar{a}$$

$$100 \text{ N} - \mu mg = m\bar{a}$$

$$100 \text{ N} - 0.25(20 \text{ kg})(9.81 \text{ m/s}^2) = (20 \text{ kg})\bar{a}$$

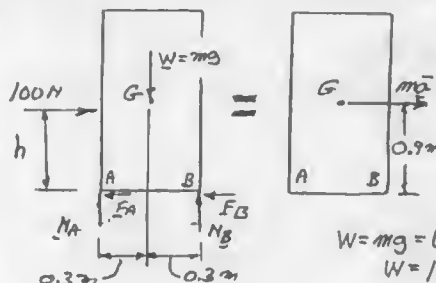
$$\bar{a} = 2.548 \text{ m/s}^2$$

$$\bar{a} = 2.55 \text{ m/s}^2 \rightarrow$$

(CONTINUED)

16.8 continued

(b) TIPPING OF CABINET



$$\bar{a} = 2.548 \text{ m/s}^2$$

$$W = mg = (20 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 196.2 \text{ N}$$

FOR TIPPING TO IMPEND:  $N_A = 0$ 

$$+\uparrow \Sigma M_B = \Sigma (M_B)_{eff}: (100 \text{ N})h - W(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h - (196.2 \text{ N})(0.3 \text{ m}) = (20 \text{ kg})(2.548 \text{ m/s}^2)(0.9 \text{ m})$$

$$h = 1.047 \text{ m}$$

FOR TIPPING TO IMPEND:  $N_B = 0$ 

$$+\uparrow \Sigma M_A = \Sigma (M_A)_{eff}: (100 \text{ N})h + W(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h + (196.2 \text{ N})(0.3 \text{ m}) = (20 \text{ kg})(9.81 \text{ m/s}^2)(0.9 \text{ m})$$

$$h = -0.130 \text{ m (IMPOSSIBLE)}$$

CABINET WILL NOT TIP:

$$h \leq 1.047 \text{ m}$$

16.9



GIVEN:

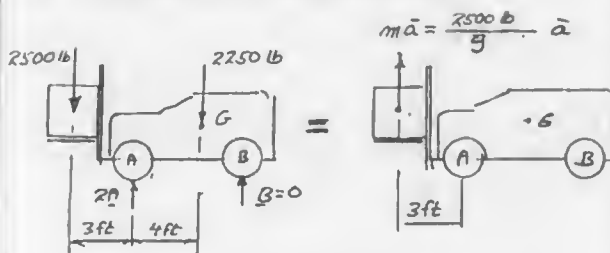
2250-lb TRUCK

2500-lb CRATE

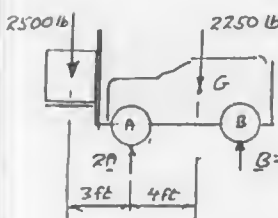
FIND:

(a)  $\bar{a}$  OFCRATE FOR  $B = 0$ (b)  $\bar{a}$  AT EACH

FRONT WHEEL

(a) ACCELERATION OF CRATE FOR  $B = 0$ 

$$m\bar{a} = \frac{2500 \text{ lb}}{g} \bar{a}$$



$$+\uparrow \Sigma M_A = \Sigma (M_A)_{eff}: (2500 \text{ lb})(3 \text{ ft}) - (2250 \text{ lb})(4 \text{ ft}) = -m\bar{a}(3 \text{ ft})$$

$$7500 - 9000 = \frac{2500}{g} \bar{a} (3)$$

$$\bar{a} = \frac{1}{5}g = \frac{1}{5}(32.2 \text{ ft/s}^2)$$

$$\bar{a} = 6.44 \text{ ft/s}^2 \uparrow$$

(b) REACTION AT A:

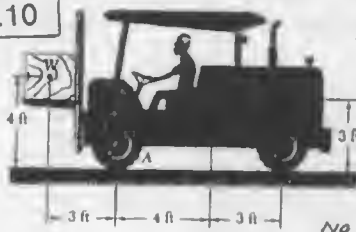
$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: R_A - 2500 \text{ lb} - 2250 \text{ lb} = m\bar{a}$$

$$R_A - 4750 \text{ lb} = \frac{2500 \text{ lb}}{g} \left(\frac{g}{5}\right)$$

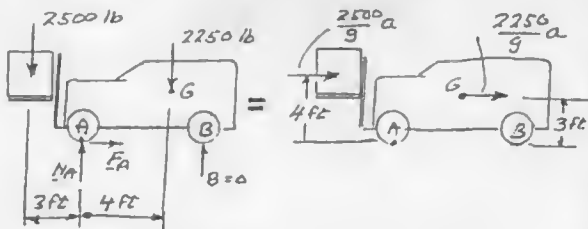
$$R_A = 5250 \text{ lb} \text{ FOR ONE WHEEL:}$$

$$A = 2625 \text{ lb} \uparrow$$

16.10



GIVEN:  $\mu = 0.30$   
 2250-lb TRUCK  
 2500-lb CRATE  
 $v_0 = 10 \text{ ft/s}$   
 FIND: SMALLEST  
 DISTANCE FOR  
 TRUCK TO STOP WITH  
 NO TIPPING OR SLIDING.



ASSUME CRATE DOES NOT SLIDE AND THAT  
 TIPPING IMPENDS ABOUT A. ( $B = 0$ )

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$(2500 \text{ lb})(3 \text{ ft}) - (2250 \text{ lb})(4 \text{ ft}) = -(2500 \frac{a}{g})(4 \text{ ft}) - (2250 \frac{a}{g})(3 \text{ ft})$$

$$7500 - 9000 = -(10000 + 6750) \frac{a}{g}$$

$$\frac{a}{g} = 0.09; a = 0.09(32.2 \text{ ft/s}^2); a = 2.884 \text{ ft/s}^2 \rightarrow$$

UNIFORMLY ACCELERATED MOTION

$$v^2 = v_0^2 + 2ax; 0 = (10 \text{ ft/s})^2 - 2(2.884 \text{ ft/s}^2)x$$

$$x = 17.34 \text{ ft}$$

CHECK WHETHER CRATE SLIDES

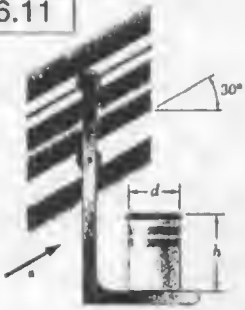
$$N = W$$

$$F = ma = \frac{W}{g} a$$

$$\mu_{\text{REQ}} = \frac{F}{N} = \frac{a}{g} = \frac{2.884 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}$$

$$\mu_{\text{REQ}} = 0.09 < 0.30. \text{ CRATE DOES NOT SLIDE}$$

16.11



(a) SLIDING IMPENDS

$$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{\text{eff}}; F = ma \cos 30^\circ$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}; N - mg = ma \sin 30^\circ$$

$$N = m(g + a \sin 30^\circ)$$

$$\mu_s = \frac{F}{N}; 0.25 = \frac{ma \cos 30^\circ}{m(g + a \sin 30^\circ)}; g + a \sin 30^\circ = 4a \cos 30^\circ$$

$$\frac{a}{g} = \frac{1}{4 \cos 30^\circ - \sin 30^\circ}; a = 0.337g \angle 30^\circ$$

(CONTINUED)

16.11 continued

(b) TIPPING IMPENDS

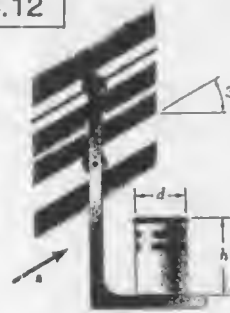
$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}};$$

$$F(\frac{h}{2}) - N(\frac{d}{2}) = 0$$

$$\frac{F}{N} = \frac{d}{h}$$

$$\mu = \frac{F}{N}; 0.25 = \frac{d}{h}; \frac{h}{d} = 4$$

16.12

GIVEN:  $\mu = 0.25$ 

FIND: (a)  $a$  FOR CAN  
 TO SLIDE  
 (b) SMALLEST  
 RATIO  $h/d$  FOR TIPPING  
 BEFORE CAN SLIDE.

(a) SLIDING IMPENDS:

$$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{\text{eff}}; F = ma \cos 30^\circ$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}; N - mg = -ma \sin 30^\circ$$

$$N = m(g - a \sin 30^\circ)$$

$$\mu_s = \frac{F}{N}; 0.25 = \frac{ma \cos 30^\circ}{m(g - a \sin 30^\circ)}$$

$$g - a \sin 30^\circ = 4a \cos 30^\circ$$

$$\frac{a}{g} = \frac{1}{4 \cos 30^\circ + \sin 30^\circ} = 0.252$$

$$a = 0.252g$$

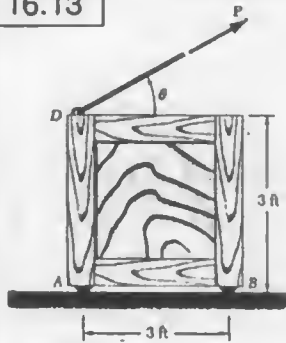
(b) TIPPING IMPENDS:

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}};$$

$$F(\frac{h}{2}) = W(\frac{d}{2}); \frac{F}{N} = \frac{d}{h}$$

$$\mu = \frac{F}{N}; 0.25 = \frac{d}{h}; \frac{h}{d} = 4$$

16.13



GIVEN: 100-lb CRATE

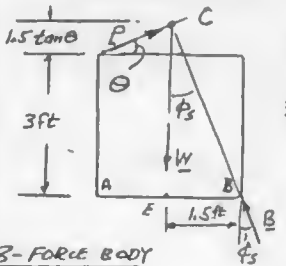
$$\mu_s = 0.40$$

$$\mu_k = 0.30$$

FIND:

- (a) VALUES OF  $\theta$  AND  $P$  FOR BOTH SLIDING AND TIPPING IMPENDING  
(b) ACCELERATION OF CRATE IF  $P$  IS SLIGHTLY INCREASED

(a) CRATE IS IN EQUILIBRIUM: FREE-BODY DIAGRAM

TIPPING IMPENDING:  $A = 0$ 

SLIDING IMPENDING:

$$\tan \phi_s = \mu_s = 0.40$$

IN  $\triangle BCE$ 

$$\tan \phi_s = \frac{1.5}{3 + 1.5 \tan \theta}$$

$$0.40 = \frac{1}{2 + \tan \theta}$$

$$\tan \theta = \frac{1}{2}; \theta = 26.57^\circ; \theta = 26.6^\circ$$

$$+\circlearrowleft \Sigma M_B = 0: (P \cos \theta)(3 \text{ ft}) + (P \sin \theta)(3 \text{ ft}) - W(1.5 \text{ ft}) = 0$$

$$P(\cos \theta + \sin \theta) = \frac{1}{2} W$$

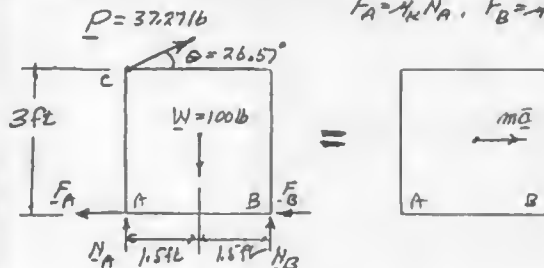
$$\theta = 26.6^\circ \quad P(\cos 26.57^\circ + \sin 26.57^\circ) = \frac{1}{2}(100 \text{ lb})$$

$$P = 37.27 \text{ lb}$$

$$P = 37.3 \text{ lb}$$

(b) FOR  $P$  SLIGHTLY  $> 37.26 \text{ lb}$ , CRATE MOVES,  $\mu = \mu_k$ 

$$F_A = \mu_k N_A, \quad F_B = \mu_k N_B$$



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N_A + N_B - 100 \text{ lb} - (37.27 \text{ lb}) \sin 26.57^\circ = 0$$

$$N_A + N_B = 100 - 16.67 = 83.33 \text{ lb}$$

$$F_A + F_B = \mu_k (N_A + N_B) = 0.30(83.33 \text{ lb}) = 25.0 \text{ lb}$$

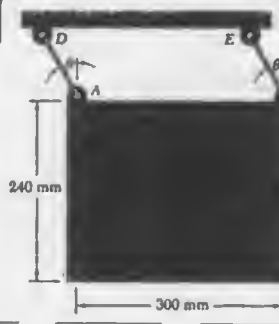
$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$(37.27 \text{ lb}) \cos 26.57^\circ - (F_A + F_B) = m \bar{a}$$

$$33.33 - 25.0 = \frac{100 \text{ lb}}{32.2 \text{ ft/s}^2} \bar{a}$$

$$\bar{a} = 2.68 \text{ ft/s}^2 \rightarrow$$

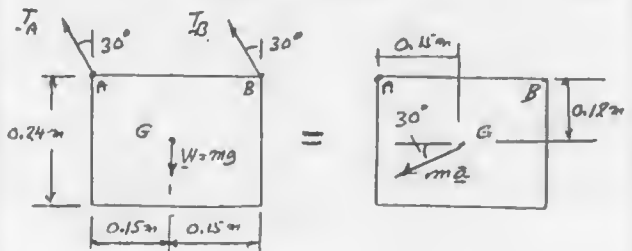
16.14

GIVEN:  $\theta = 30^\circ$ 

$$m = 5 \text{ kg}$$

CUT CF:

FIND:

(a)  $\bar{a}$ (b)  $T_{AD}$  AND  $T_{BE}$ (a) ACCELERATION  $+\circlearrowleft \Sigma \tau = \Sigma \tau_{\text{eff}}:$ 

$$mg \sin 30^\circ = m \bar{a}$$

$$\bar{a} = 0.5g = 4.905 \text{ m/s}^2$$

$$\bar{a} = 4.91 \text{ m/s}^2 \swarrow 30^\circ$$

(b) TENSION IN ROPES

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$(T_B \cos 30^\circ)(0.3 \text{ m}) - mg(0.15 \text{ m}) = -m \bar{a}(\cos 30^\circ)(0.12 \text{ m})$$

$$-m \bar{a}(\sin 30^\circ)(0.15 \text{ m})$$

$$0.2598 T_B - (5 \text{ kg})(9.81 \text{ m/s}^2)(0.15 \text{ m}) = -(5 \text{ kg})(4.905 \text{ m/s}^2)(0.1639 + 0.07)$$

$$0.2598 T_B - 7.3575 = 4.388$$

$$T_B = +11.43 \text{ N}$$

$$T_{BE} = 11.43 \text{ N}$$

$$+\circlearrowleft \Sigma \tau = \Sigma \tau_{\text{eff}}: T_A + 11.43 \text{ N} - mg \cos 30^\circ = 0$$

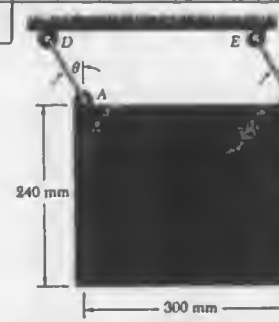
$$T_A + 11.43 \text{ N} - (5 \text{ kg})(9.81 \text{ m/s}^2) \cos 30^\circ = 0$$

$$T_A + 11.43 \text{ N} - 42.46 \text{ N} = 0$$

$$T_A = 31.04 \text{ N}$$

$$T_{AD} = 31.0 \text{ N}$$

16.15

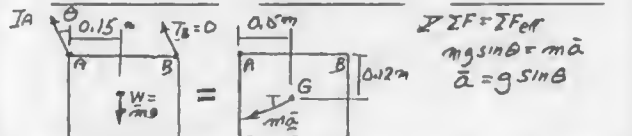
FIND: LARGEST  $\theta$ 

FOR WHICH

ROPE REMAIN

TAUT WHEN

CF IS CUT.



$$+\circlearrowleft \Sigma \tau = \Sigma \tau_{\text{eff}}:$$

$$mg \sin \theta = m \bar{a}$$

$$\bar{a} = g \sin \theta$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: mg(0.15 \text{ m}) = m \bar{a} \cos \theta (0.12 \text{ m}) + m \bar{a} \sin \theta (0.15 \text{ m})$$

$$mg(0.15) = mg \sin \theta (0.12 \cos \theta + 0.15 \sin \theta)$$

$$1 = 0.8 \sin \theta \cos \theta - \sin^2 \theta$$

$$1 - \sin^2 \theta = 0.8 \sin \theta \cos \theta; \cos^2 \theta = 0.8 \sin \theta \cos \theta$$

$$1 = 0.8 \sin \theta / \cos \theta; \tan \theta = 1.25; \theta = 51.3^\circ$$

16.16

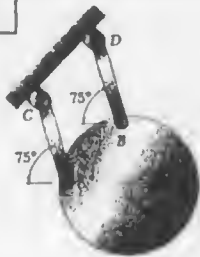
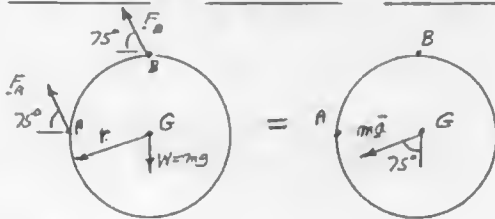
GIVEN:  $m = 3 \text{ kg}$ 

PLATE IS RELEASED FROM REST.

FIND: (a)  $\bar{a}$   
(b) TENSION IN EACH LINK

(a) ACCELERATION

$$\sum F = \Sigma F_{\text{eff}}: mg \cos 75^\circ = m\bar{a}$$

$$\bar{a} = g \cos 75^\circ \quad \bar{a} = 2.54 \text{ m/s}^2 \angle 15^\circ$$

(b) TENSION IN EACH LINK

$$\sum M_G = \Sigma (M_G)_{\text{eff}}: (F_A \cos 75^\circ)r + (F_A \sin 75^\circ)r = (m\bar{a} \sin 75^\circ)r$$

$$F_A (\cos 75^\circ + \sin 75^\circ) = (3 \text{ kg})(2.54 \text{ m/s}^2) \sin 75^\circ$$

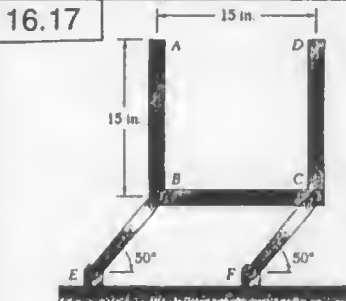
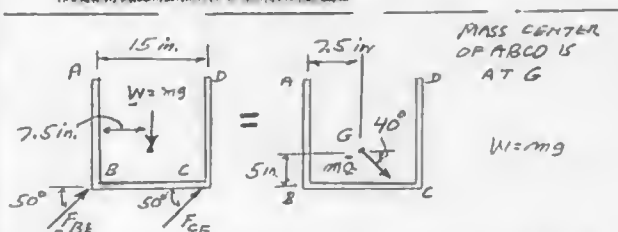
$$F_A = 6.009 \text{ N} \quad F_{AC} = 6.01 \text{ N} \cdot \text{T}$$

$$+\Delta 75^\circ \Sigma F = \Sigma F_{\text{eff}}: F_A + F_B - mg \sin 75^\circ = 0$$

$$6.009 \text{ lb} + F_B - (3 \text{ kg})(9.81 \text{ m/s}^2) \sin 75^\circ = 0$$

$$F_B = 22.42 \text{ N} \quad F_{BD} = 22.4 \text{ N} \cdot \text{T}$$

16.17

GIVEN: FOR EACH BAR  
 $W = 8 \text{ lb}$ NEGLECTING WEIGHT  
OF LINKS BE AND CF  
FIND: FORCE IN  
EACH LINK IMMEDIATELY  
AFTER RELEASE

$$+\Delta 40^\circ \Sigma F = \Sigma F_{\text{eff}}: mg \cos 50^\circ = m\bar{a}$$

$$(24 \text{ lb}) \cos 50^\circ = m\bar{a} \quad m\bar{a} = 15.427 \text{ lb}$$

$$+\Delta \Sigma M_G = \Sigma (M_G)_{\text{eff}}:$$

$$(F_{CF} \sin 50^\circ)(15 \text{ in.}) - (24 \text{ lb})(7.5 \text{ in.}) =$$

$$-m\bar{a} \sin 40^\circ (7.5 \text{ in.}) - m\bar{a} \cos 40^\circ (5 \text{ in.})$$

$$11.491 F_{CF} - 180 = -m\bar{a} (4.821 + 3.83)$$

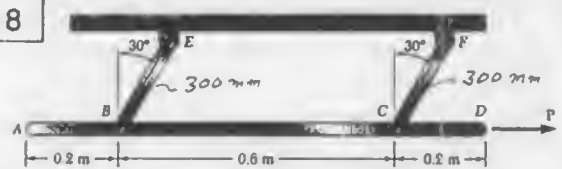
$$11.491 F_{CF} - 180 = -(15.427 \text{ lb})(8.651)$$

$$F_{CF} = +4.05 \text{ lb} \quad F_{CF} = 4.05 \text{ lb} \cdot \text{C}$$

$$+\Delta 50^\circ \Sigma F = \Sigma F_{\text{eff}}: F_{BE} + 4.05 \text{ lb} - (24 \text{ lb}) \sin 50^\circ = 0$$

$$F_{BE} = +14.33 \text{ lb} \quad F_{BE} = 14.33 \text{ lb} \cdot \text{C}$$

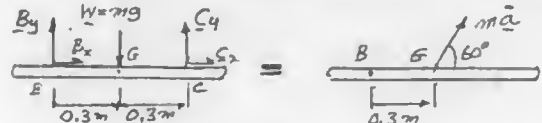
16.18

GIVEN:  $m_{AD} = 6 \text{ kg}$ ,  $P = 0$ ,  $\omega_{BE} = \omega_{CF} = 90 \text{ rpm}$ ,  $\alpha_{BE} = \alpha_{CF} = 0$   
FIND:  $B_y$  AND  $C_y$ 

$$\omega = 90 \text{ rpm} \left( \frac{2\pi}{60} \right) = 3\pi \text{ rad/s}$$

$$\text{BAR AD IS IN TRANSLATION}$$

$$\bar{a} = a_B = a_C = \dot{r}\omega = (0.3 \text{ m})(3\pi) = 2.646 \text{ m/s}^2 \angle 60^\circ$$



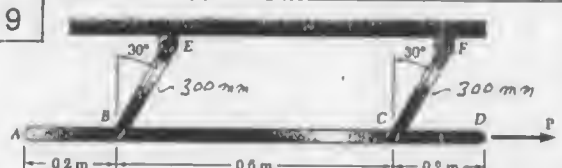
$$+\Delta \Sigma M_G = \Sigma (M_G)_{\text{eff}}: C_y(0.3 \text{ m}) - B_y(0.3 \text{ m}) = 0 \quad B_y = C_y$$

$$+\Delta \Sigma F_y = \Sigma (F_y)_{\text{eff}}: B_y + C_y - mg = m\bar{a} \sin 60^\circ$$

$$2B_y - (6 \text{ kg})(9.81 \text{ m/s}^2) = (6 \text{ kg})(2.646 \text{ m/s}^2) \sin 60^\circ$$

$$B_y = +98.66 \text{ N} \quad B_y = C_y = 98.7 \text{ N}$$

16.19

GIVEN:  $m_{AD} = 6 \text{ kg}$ ,  $\omega_{BE} = \omega_{CF} = 6 \text{ rad/s}$ ,  $\alpha_{BE} = \alpha_{CF} = 12 \text{ rad/s}^2$   
FIND: (a)  $P$ , (b)  $F_{BE}$  AND  $F_{CF}$ .

$$\text{LINKS:}$$

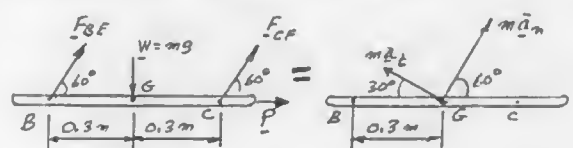
$$\omega = 6 \text{ rad/s} \quad \alpha = 12 \text{ rad/s}^2$$

$$r = 0.3 \text{ m} \quad a_n = r\omega^2 = (0.3 \text{ m})(6 \text{ rad/s})^2$$

$$a_n = 10.80 \text{ m/s}^2 \angle 60^\circ$$

$$a_t = r\alpha = (0.3 \text{ m})(12 \text{ rad/s}^2)$$

$$a_t = 3.6 \text{ m/s}^2 \angle 30^\circ$$

BAR AD IS IN TRANSLATION  $\bar{a} = a_B = a_C$ 

$$+\Delta \Sigma M_G = \Sigma (M_G)_{\text{eff}}: F_{CF} \cos 60^\circ (0.3 \text{ m}) - F_{BE} \cos 60^\circ (0.3 \text{ m}) = 0$$

$$F_{CF} = F_{BE}$$

$$+\Delta \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$F_{CF} \sin 60^\circ (0.6 \text{ m}) - mg(0.3 \text{ m}) = m a_{nB} \sin 30^\circ (0.3 \text{ m}) + m a_{tB} \sin 60^\circ (0.3 \text{ m})$$

$$0.5196 F_{CF} - (6 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = + (6 \text{ kg})(3.6 \text{ m/s}^2) \sin 30^\circ (0.3 \text{ m})$$

$$+ (6 \text{ kg})(10.80 \text{ m/s}^2) \sin 60^\circ (0.3 \text{ m})$$

$$0.5196 F_{CF} - 17.658 = +3.24 + 16.736$$

$$F_{CF} = +72.62 \text{ N} \quad F_{CF} = F_{BE} = 72.6 \text{ N} (\text{T})$$

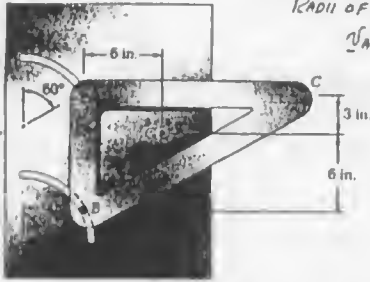
$$+\Delta \Sigma F_x = \Sigma (F_x)_{\text{eff}}: (F_{BE} + F_{CF}) \cos 60^\circ + P = -m a_{nB} \cos 30^\circ + m a_{tB} \cos 60^\circ$$

$$2(72.62 \text{ N}) \cos 60^\circ + P = -(6 \text{ kg})(3.6 \text{ m/s}^2) \cos 30^\circ + (6 \text{ kg})(10.80 \text{ m/s}^2) \cos 60^\circ$$

$$72.62 + P = -18.706 + 37.40$$

$$P = -58.9 \text{ N} \quad P = 58.9 \text{ N}$$

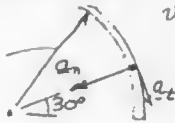
16.20



GIVEN:  $W = 16 \text{ lb}$   
 RADIUS OF SLOTS:  $r = 6 \text{ in.}$   
 $v_A = v_B = 30 \text{ in/s}$

AT INSTANT SHOWN:  
 FIND: (a)  $\bar{a}$   
 (b) REACTIONS  
 AT A AND B

SLOT:  
 $r = 6 \text{ in.}$



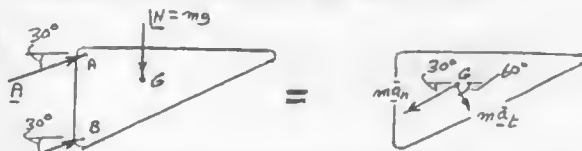
$$v = 30 \text{ in/s}$$

$$a_n = \frac{v^2}{r} = \frac{(30 \text{ in/s})^2}{6 \text{ in}} = 150 \text{ in/s}^2$$

$$a_n = 12.5 \text{ ft/s}^2 \nearrow 30^\circ$$

$$a_t = a_n \searrow 60^\circ$$

WELDMENT IS IN TRANSLATION  $\bar{a}_n = 12.5 \text{ ft/s}^2$



$$\sum 60^\circ \Sigma F = \Sigma F_{\text{eff}}: mg \cos 30^\circ = m a_t$$

$$\bar{a}_t = 27.886 \text{ ft/s}^2 \searrow 60^\circ$$

(a) ACCELERATION

$$\beta = \tan^{-1} \frac{\bar{a}_n}{\bar{a}_t} = \tan^{-1} \frac{12.5}{27.886} = 24.14^\circ$$

$$\bar{a}_n = 12.5 \text{ ft/s}^2$$

$$\bar{a}_t = 27.886 \text{ ft/s}^2$$

$$\bar{a}^2 = a_t^2 + a_n^2$$

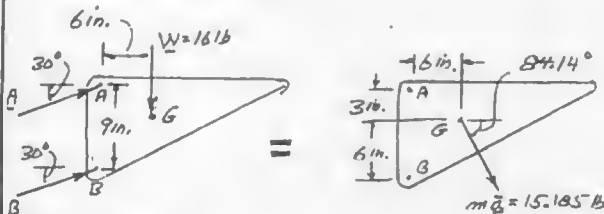
$$= (27.886)^2 + (12.5)^2$$

$$\bar{a} = 30.56 \text{ ft/s}^2 \searrow 84.1^\circ$$

$$\bar{a} = 30.6 \text{ ft/s}^2 \searrow 84.1^\circ$$

(b) REACTIONS

$$m \bar{a} = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} (30.56 \text{ ft/s}^2) = 15.185 \text{ lb}$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$B \cos 30^\circ (9 \text{ in.}) - (16 \text{ lb}) (6 \text{ in.}) = (15.185 \text{ lb}) (\cos 84.1^\circ) (3 \text{ in.})$$

$$- (15.185 \text{ lb}) (\sin 84.1^\circ) (6 \text{ in.})$$

$$7.794 B - 96 = +4.651 - 90.634$$

$$B = +1.285 \text{ lb}$$

$$B = 1.285 \text{ lb} \searrow 30^\circ$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$A \cos 30^\circ + B \cos 30^\circ = m \bar{a} \cos 84.14^\circ$$

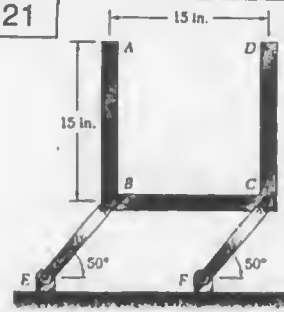
$$A \cos 30^\circ + (1.285 \text{ lb}) \cos 30^\circ = (15.185 \text{ lb}) \cos 84.14^\circ$$

$$A \cos 30^\circ + 1.113 \text{ lb} = 1.550 \text{ lb}$$

$$A = +0.505 \text{ lb}$$

$$A = 0.505 \text{ lb} \nearrow 30^\circ$$

\* 16.21



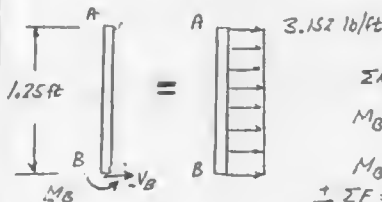
GIVEN:  
 $W_{AB} = 8 \text{ lb}$   
 FROM PROB. 16.17  
 $\bar{a} = 20.7 \text{ ft/s}^2 \searrow 40^\circ$

DRAW V AND M  
 DIAGRAMS FOR AB

DISTRIBUTED WEIGHT PER UNIT LENGTH:  $w = \frac{8 \text{ lb}}{(15/2 \text{ ft})} = 6.4 \text{ lb/ft}$

HORIZONTAL COMP. OF EFFECTIVE FORCES

$$\frac{w}{g} \bar{a}_x = \frac{6.4 \text{ lb/ft}}{32.2 \text{ ft/s}^2} (20.7 \text{ ft/s}^2) \cos 40^\circ = 3.1538 \text{ lb/ft}$$



$$\Sigma M_B = \Sigma (M_B)_{\text{eff}}$$

$$M_B = (3.1538 \text{ lb/ft}) \left( \frac{(1.25 \text{ ft})^2}{2} \right)$$

$$M_B = 2.46 \text{ lb-ft}$$

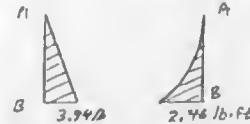
$$+\rightarrow \Sigma F = \Sigma (F)_{\text{eff}}$$

$$V_B = (3.1538 \text{ lb/ft}) (1.25 \text{ ft})$$

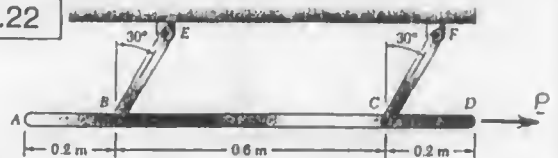
$$V_B = 3.94 \text{ lb}$$

WE NOTE THAT  $V_A = M_A = 0$  AND SKETCH THE

V AND M DIAGRAMS



\* 16.22

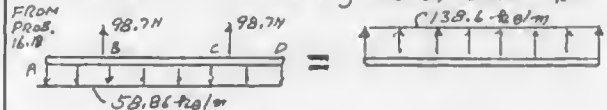


GIVEN:  $m_{AD} = 6 \text{ kg}$ , FROM PROB. 16.18  $\bar{a} = 26.648 \text{ m/s}^2 \searrow 60^\circ$   
 DRAW V AND M DIAGRAMS FOR BAR AD

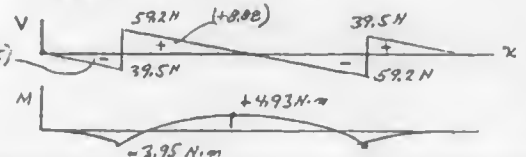
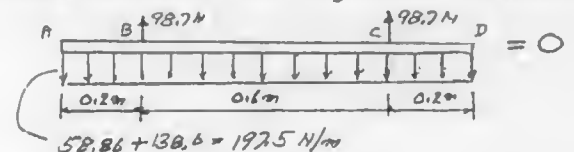
DISTRIBUTED MASS PER UNIT LENGTH:  $m' = \frac{6 \text{ kg}}{1 \text{ m}} = 6 \text{ kg/m}$

VERT. COMP. OF EFFECTIVE FORCES:  $m' a_y = 6 (26.648) \sin 60^\circ = 138.6 \text{ N/m}$

DISTRIBUTED WEIGHTS:  $w = m' g = 6 (9.81) = 58.86 \text{ N/m}$



DYNAMIC EQUILIBRIUM (ADD  $-m' a_y$  TO LEFT-HAND SIDE)

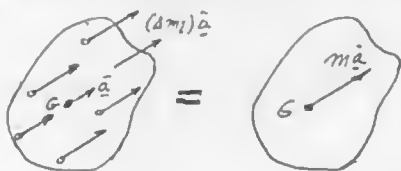


16.23



FOR TRANSLATION  
SHOW THAT EFFECTIVE  
FORCES ARE  $(\Delta m_i)\bar{a}$   
ATTACHED TO PARTICLES  
AND ARE REDUCED TO  
 $m\bar{a}$  ATTACHED AT G

SINCE SLAB IS IN TRANSLATION, EACH PARTICLE  
HAS SAME ACCELERATION AS G, NAMELY  $\bar{a}$ .  
THE EFFECTIVE FORCES CONSIST OF  $(\Delta m_i)\bar{a}$ .



THE SUM OF THESE VECTORS IS:  $\sum (\Delta m_i)\bar{a} = (\sum \Delta m_i)\bar{a}$   
OR SINCE  $\sum \Delta m_i = m$ ,

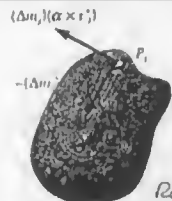
$$\sum (\Delta m_i)\bar{a} = m\bar{a}$$

THE SUM OF THE MOMENTS ABOUT G IS:

$$\sum \mathbf{r}_{iG} \times (\Delta m_i)\bar{a} = (\sum \Delta m_i \mathbf{r}_{iG}) \times \bar{a} \quad (1)$$

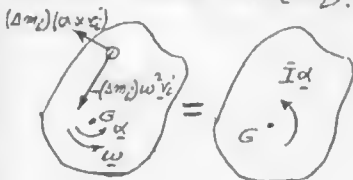
BUT,  $\sum \Delta m_i \mathbf{r}_{iG} = m\bar{\mathbf{r}} = 0$ , BECAUSE G IS THE MASS  
CENTER. IT FOLLOWS THAT THE RIGHT-HAND  
MEMBER OF EQ.(1) IS ZERO. THUS, THE MOMENT  
ABOUT G OF  $m\bar{a}$  MUST ALSO BE ZERO, WHICH MEANS  
THAT ITS LINE OF ACTION PASSES THROUGH G AND  
THAT IT MAY BE ATTACHED AT G.

16.24



FOR CENTROIDAL ROTATION,  
SHOW THAT EFFECTIVE  
FORCES CONSIST OF VECTORS  
 $(\Delta m_i)\omega^2 \mathbf{r}_{iG}'$  AND  $(\Delta m_i)(\mathbf{a} \times \mathbf{r}_{iG}')$   
ATTACHED TO PARTICLES AND  
REDUCE TO A COUPLE  $\bar{I}\alpha$ .

FOR CENTROIDAL ROTATION:  $\mathbf{a}_i = (\mathbf{a}_i)_t + (\mathbf{a}_i)_n = \mathbf{a} \times \mathbf{r}_{iG}' - \omega^2 \mathbf{r}_{iG}'$   
EFFECTIVE FORCES ARE:  $(\Delta m_i)\mathbf{a}_i = (\Delta m_i)(\mathbf{a} \times \mathbf{r}_{iG}') - (\Delta m_i)\omega^2 \mathbf{r}_{iG}'$



$$\sum (\Delta m_i)\mathbf{a}_i = \sum (\Delta m_i)(\mathbf{a} \times \mathbf{r}_{iG}') - \sum (\Delta m_i)\omega^2 \mathbf{r}_{iG}'$$

$$= \mathbf{a} \times \sum (\Delta m_i)\mathbf{r}_{iG}' - \omega^2 \sum (\Delta m_i)\mathbf{r}_{iG}'$$

SINCE G IS THE MASS CENTER,  $\sum (\Delta m_i)\mathbf{r}_{iG}' = 0$   
∴ EFFECTIVE FORCES REDUCE TO A COUPLE.

SUMMING MOMENTS ABOUT G

$$\sum (\mathbf{r}_{iG}' \times \Delta m_i \mathbf{a}_i) = \sum [\mathbf{r}_{iG}' \times (\Delta m_i)(\mathbf{a} \times \mathbf{r}_{iG}')] - \sum \mathbf{r}_{iG}' \times (\Delta m_i)\omega^2 \mathbf{r}_{iG}'$$

$$\text{BUT, } \sum \mathbf{r}_{iG}' \times (\Delta m_i)\omega^2 \mathbf{r}_{iG}' = \omega^2 (\Delta m_i) (\mathbf{r}_{iG}' \times \mathbf{r}_{iG}') = 0$$

$$\text{AND, } \mathbf{r}_{iG}' \times (\Delta m_i)(\mathbf{a} \times \mathbf{r}_{iG}') = (\Delta m_i) \mathbf{r}_{iG}'^2 \alpha$$

$$\text{THUS, } \sum (\mathbf{r}_{iG}' \times \Delta m_i \mathbf{a}_i) = \sum (\Delta m_i) \mathbf{r}_{iG}'^2 \alpha = [\sum (\Delta m_i) \mathbf{r}_{iG}'^2] \alpha$$

$$\text{SINCE } \sum (\Delta m_i) \mathbf{r}_{iG}'^2 = \bar{I},$$

THE MOMENT OF THE COUPLE IS  $\bar{I}\alpha$

16.25

FLYWHEEL:  $W = 6000 \text{ lb}$   $\bar{R} = 36 \text{ in.}$

AT  $t = 0$ ,  $\omega_0 = 300 \text{ rpm}$ , AT  $t = 10 \text{ min.}$ ,  $\omega = 0$

FIND COUPLE DUE TO KINETIC FRICTION, (UNIF. ACCEL. MOTION)

$$\bar{I} = m\bar{R}^2 = \left( \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (3 \text{ ft})^2 = 1677.0 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\omega_0 = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$$

$$\omega = \omega_0 + \alpha t; \quad 0 = 10\pi \text{ rad/s} + \alpha (600 \text{ s})$$

$$\alpha = -0.05236 \text{ rad/s}^2$$

$$M = \bar{I}\alpha = (1677 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (-0.05236 \text{ rad/s}^2) = -87.81 \text{ lb} \cdot \text{ft}$$

$$M = 87.8 \text{ lb} \cdot \text{ft}$$

16.26

ROTOR:  $m = 50 \text{ kg}$ ,  $\bar{R} = 180 \text{ mm}$

FRICTION COUPLE:  $M = 3.5 \text{ N} \cdot \text{m}$

$\Theta = 0$ ,  $\omega_0 = 3600 \text{ rpm}$  (UNIF. ACCEL. MOTION)

FIND: REVOLUTIONS AS ROTOR COASTS TO REST

$$\bar{I} = m\bar{R}^2 = (50 \text{ kg})(0.180 \text{ m})^2 = 1.620 \text{ kg} \cdot \text{m}^2$$

$$M = \bar{I}\alpha; \quad 3.5 \text{ N} \cdot \text{m} = (1.620 \text{ kg} \cdot \text{m}^2) \alpha$$

$$\alpha = 2.1605 \text{ rad/s}^2 \text{ (DECELERATION)}$$

$$\omega_0 = 3600 \text{ rpm} \left( \frac{2\pi}{60} \right) = 120\pi \text{ rad/s}$$

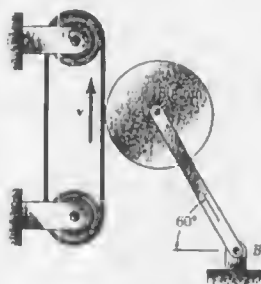
$$\omega^2 = \omega_0^2 + 2\alpha\Theta; \quad 0 = (120\pi \text{ rad/s})^2 + 2(-2.1605 \text{ rad/s}^2)\Theta$$

$$\Theta = 32.871 \times 10^3 \text{ rad} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right); \quad \Theta = 5234.8 \text{ rev}$$

$$\Theta = 5230 \text{ rev}$$

16.27

GIVEN:  $\mu_k = 0.40$



FIND:  $\alpha$  FOR DIRECTION  
OF MOTION OF BELT SHOWN

BELT:  $N \uparrow \quad F \leftarrow \quad N \quad F = \mu_k N$

DISK:  $W = mg$

$\sum F_x = \sum (F_x)_R: \quad N - F_{AB} \cos \Theta = 0$

$F_{AB} \cos \Theta = N \quad (1)$

$\sum F_y = \sum (F_y)_R: \quad \mu_k N + F_{AB} \sin \Theta - mg = 0$

$F_{AB} \sin \Theta = mg - \mu_k N \quad (2)$

EQ.(2):  $\tan \Theta = \frac{mg - \mu_k N}{N}$

$N \tan \Theta = mg - \mu_k N; \quad N = \frac{mg}{\tan \Theta + \mu_k}; \quad F = \mu_k N = \frac{mg \mu_k}{\tan \Theta + \mu_k}$

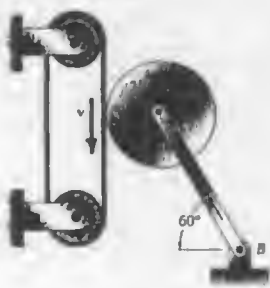
$\sum M_A = \sum (M_A)_R: \quad Fr = \bar{I}\alpha$

$\alpha = \frac{r}{\bar{I}} F = \frac{r}{\frac{1}{2} m r^2} \cdot \frac{mg \mu_k}{\tan \Theta + \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan \Theta + \mu_k}$

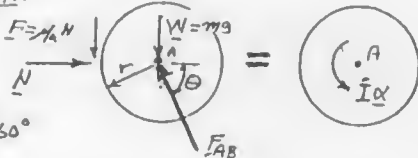
DATA:  $r = 0.18 \text{ m}$ ,  $\Theta = 60^\circ$ ,  $\mu_k = 0.40$

$\alpha = \frac{2(9.81 \text{ m/s}^2) \cdot 0.40}{0.18 \text{ m} \cdot \tan 60^\circ + 0.40} \quad \alpha = 20.4 \frac{\text{rad}}{\text{s}^2}$

16.28

GIVEN:  $\mu_k = 0.40$ FIND:  $\alpha$  FOR  
DIRECTION OF  
MOTION OF BELT  
SHOWINBELT:  $\downarrow F$   $\uparrow N$   $F = \mu_k N$ 

DISK:

 $\theta = 60^\circ$ 

$$\pm \Sigma F_x = \Sigma (F_x)_{eff}: N - F_{AB} \cos \theta; F_{AB} \cos \theta = N \quad (1)$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: F_{AB} \sin \theta - mg - \mu_k N = 0$$

$$F_{AB} \sin \theta = mg + \mu_k N \quad (2)$$

$$\text{EQ. (2)}: \tan \theta = \frac{mg + \mu_k N}{N}$$

EQ. (1)

$$N \tan \theta = mg + \mu_k N; N = \frac{mg}{\tan \theta - \mu_k}$$

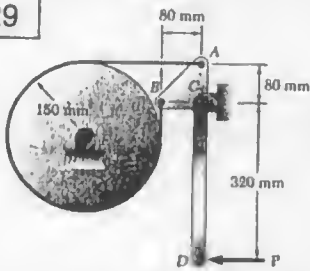
$$+\uparrow \Sigma M_A = \Sigma (M_A)_{eff}: Fr = I\alpha$$

$$\alpha = \frac{Fr}{I} = \frac{\mu_k N r}{\frac{1}{2} m r^2} = \frac{\mu_k}{\frac{1}{2} m r} \cdot \frac{mg}{\tan \theta - \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan \theta - \mu_k}$$

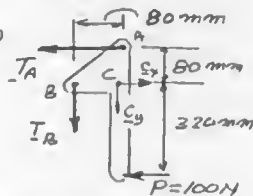
$$\text{DATA: } r = 0.18 \text{ m}, \theta = 60^\circ, \mu_k = 0.40$$

$$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60^\circ - 0.40}; \alpha = 32.7 \text{ rad/s}^2$$

16.29

GIVEN:  $I = 75 \text{ kg} \cdot \text{m}^2$   
 $P = 100 \text{ N}$   
 $\mu_k = 0.25$   
 $\omega_0 = 240 \text{ rpm}$ FIND: TIME REQUIRED  
FOR DISK TO  
COME TO REST

LEVER ABCD



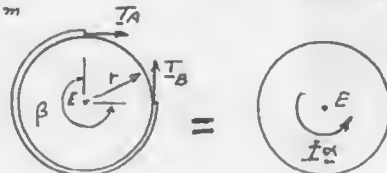
STATIC EQUILIBRIUM:

$$+\uparrow \Sigma M_C = 0: T_A(80 \text{ mm}) + T_B(80 \text{ mm}) - (100 \text{ N})(320 \text{ mm}) = 0$$

$$T_A + T_B = 400 \text{ N} \quad (1)$$

(CONTINUED)

16.29 continued

 $\omega_0 = 240 \text{ rpm} \left( \frac{2\pi}{60} \right) = 8\pi \text{ rad/s}$   
THUS  $\alpha$  WILL BE  $\downarrow$  $r = 0.15 \text{ m}$ 

$$+\uparrow \Sigma M_E = \Sigma (M_E)_{eff}: T_B r - T_A r = I\alpha$$

$$T_B - T_A = \frac{I}{r} \alpha \quad (2)$$

BELT FRICTION:

$$\beta = 270^\circ = \frac{3}{2}\pi \text{ rad} \quad \frac{T_B}{T_A} = e^{\mu_k \beta} = e^{(0.25) \frac{3}{2}\pi} = e^{1.178} = 3.248$$

$$T_B = 3.248 T_A \quad (3)$$

$$\text{EQ. (1)}: T_A + T_B = 400 \text{ N}; T_A + 3.248 T_A = 400 \text{ N}$$

$$T_A = 94.16 \text{ N} \quad T_B = 3.248(94.16 \text{ N}) = 305.9 \text{ N}$$

$$\text{EQ. (2)}: T_B - T_A = \frac{I}{r} \alpha; 305.9 \text{ N} - 94.16 \text{ N} = \frac{75 \text{ kg} \cdot \text{m}^2}{0.15 \text{ m}} \alpha$$

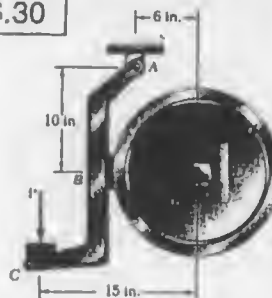
$$\alpha = 0.423 \text{ rad/s}^2$$

UNIF. ACCEL. MOTION

$$\omega_0 = \alpha t: 8\pi \text{ rad/s} = (0.423 \text{ rad/s}^2) t; t = 59.45$$

NOTE: IF  $\alpha$  IS REVERSED THEN  $T_A$  AND  $T_B$  ARE  
INTERCHANGED. THIS CAUSES NO CHANGE IN EQ. (1)  
AND EQ. (2). THUS FROM EQ. (3),  $\alpha$  IS NOT CHANGED.

16.30



GIVEN:

$$I = 14 \text{ lb} \cdot \text{ft}^2$$

$$\mu_k = 0.35$$

$$P = 75 \text{ lb}$$

$$\omega_0 = 360 \text{ rpm}$$

FIND: NUMBER OF  
REVOLUTIONS OF DRUM  
BEFORE IT COMES  
TO RESTLEVER ABC: STATIC EQUILIBRIUM (FRICTION FORCE  $\downarrow$ )

$$F = \mu_k N = 0.35 N$$

$$+\uparrow \Sigma M_A = 0:$$

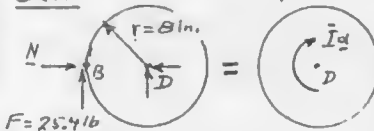
$$N(10 \text{ in}) - F(2 \text{ in}) - (75 \text{ lb})(9 \text{ in}) = 0$$

$$10N - 2(0.35N) - 675 = 0$$

$$N = 72.58 \text{ lb}$$

$$F = \mu_k N = 0.35(72.58 \text{ lb}) = 25.40 \text{ lb}$$

DRUM



$$F = 25.4 \text{ lb}$$

$$r = 6 \text{ in} = \frac{1}{2} \text{ ft}$$

$$\omega_0 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right)$$

$$\omega_0 = 12\pi \text{ rad/s}$$

$$+\uparrow \Sigma M_D = \Sigma (M_D)_{eff}: Fr = I\alpha$$

$$(25.4 \text{ lb}) \left( \frac{1}{2} \text{ ft} \right) = (14 \text{ lb} \cdot \text{ft}^2) \alpha$$

$$\alpha = 1.2097 \text{ rad/s}^2 \text{ (DECELERATION)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta; 0 = (12\pi \text{ rad/s})^2 + 2(-1.2097 \text{ rad/s}^2)\theta$$

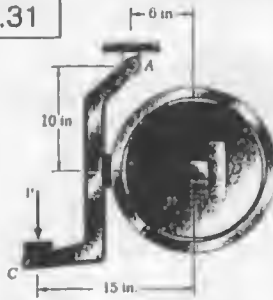
$$\theta = 587.4 \text{ rad}$$

$$\theta = 587.4 \text{ rad} \left( \frac{1}{2\pi} \right) = 93.49 \text{ rev}$$

$$\theta = 93.5 \text{ rev}$$



16.31

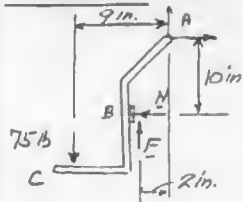


GIVEN:

$$\begin{aligned} \bar{I} &= 14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ \mu_k &= 0.35 \\ P &= 75 \text{ lb} \\ \omega_0 &= 360 \text{ rpm} \end{aligned}$$

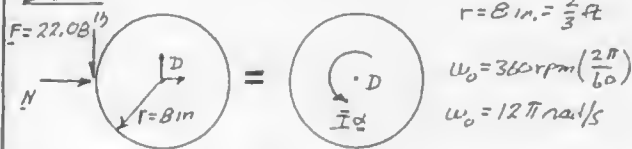
FIND: NUMBER OF REVOLUTIONS OF DRUM BEFORE IT COMES TO REST.

LEVER ABC: STATIC EQUILIBRIUM (FRICTION FORCE ↑)



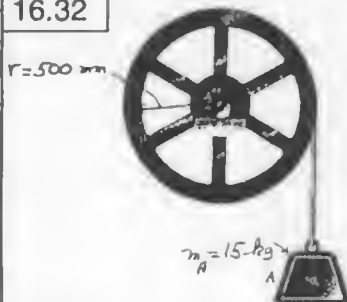
$$\begin{aligned} F &= \mu_k N = 0.35N \\ +\circlearrowleft \Sigma M_A &= 0 \\ N(10 \text{ in}) + F(2 \text{ in}) - (75 \text{ lb})(9 \text{ in}) &= 0 \\ 10N + 2(0.35N) - 675 &= 0 \\ N &= 63.08 \text{ lb} \\ F = \mu_k N &= 0.35(63.08 \text{ lb}) = 22.08 \text{ lb} \end{aligned}$$

DRUM:



$$\begin{aligned} +\circlearrowleft \Sigma M_D &= \Sigma (M_D)_{\text{eff}} = 0 \\ F r &= \bar{I} \alpha \\ (22.08 \text{ lb})(\frac{2}{3} \text{ ft}) &= (14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) \alpha \\ \alpha &= 1.5015 \text{ rad/s}^2 \text{ (DECELERATION)} \\ \omega^2 &= \omega_0^2 + 2\alpha \theta; \quad 0 = (12\pi \text{ rad/s})^2 + 2(-1.5015 \text{ rad/s}^2) \theta \\ \theta &= 675.8 \text{ rad} \\ \theta &= 675.8 \text{ rad} (\frac{1}{2\pi}) = 107.56 \text{ rev}; \quad \theta = 107.6 \text{ rev} \end{aligned}$$

16.32



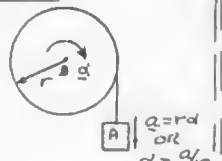
GIVEN:

$$\begin{aligned} \text{FLYWHEEL } m_F &= 120 \text{ kg} \\ r_F &= 375 \text{ mm} \\ \mu &= 0 \\ v_0 &= 0 \text{ AT } S = 0 \end{aligned}$$

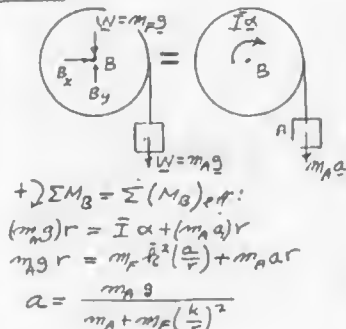
FIND:

- (a)  $a$  OF BLOCK.  
(b)  $v_A$  AFTER IT HAS MOVED 1.5 m.

KINEMATICS



KINETICS



$$a = \frac{(15 \text{ kg})(9.8 \text{ m/s}^2)}{15 \text{ kg} + (120 \text{ kg})(\frac{375 \text{ mm}}{500 \text{ mm}})^2} = 1.7836 \text{ m/s}^2 \quad (\text{CONTINUED})$$

16.32 continued

$$(a) \alpha = 1.7836 \text{ m/s}^2 \downarrow$$

$$\alpha = \frac{a}{r} = \frac{1.7836 \text{ m/s}^2}{0.5 \text{ m}} = 3.567 \text{ rad/s}^2$$

$$\alpha = 3.57 \text{ rad/s}^2 \downarrow$$

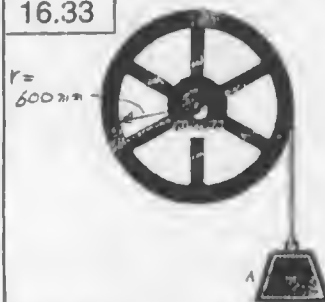
$$(b) v_A^2 = v_0^2 + 2a s$$

$$\text{FOR } s = 1.5 \text{ m: } v_A^2 = 0 + 2(1.7836 \text{ m/s}^2)(1.5 \text{ m})$$

$$v_A = 2.313 \text{ m/s}$$

$$v_A = 2.31 \text{ m/s} \downarrow$$

16.33



GIVEN: SYSTEM

RELEASED FROM REST:

1. IF  $m_A = 12 \text{ kg}$ , BLOCK

FALLS 3 m IN 4.6 s

2. IF  $m_A = 24 \text{ kg}$ , BLOCK

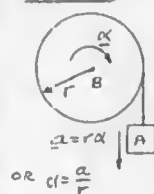
FALLS 3 m IN 3.1 s

ASSUME CONSTANT  $M_f$ 

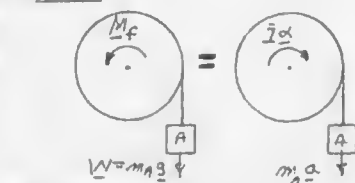
DUE TO AXLE FRICTION.

FIND:  $\bar{I}$ 

KINEMATICS



KINETICS



$$\begin{aligned} +\circlearrowleft \Sigma M_B &= \Sigma (M_B)_{\text{eff}} \\ (m_A g) r - M_f &= \bar{I} \alpha + (m_A a) r \\ m_A g r - M_f &= \bar{I} \frac{a}{r} + m_A a r \quad (1) \end{aligned}$$

CASE 1:  $y = 3 \text{ m}$ ,  $t = 4.6 \text{ s}$ 

$$y = \frac{1}{2} a t^2; \quad 3 \text{ m} = \frac{1}{2} a (4.6 \text{ s})^2; \quad a = 0.2836 \text{ m/s}^2$$

$$m_A = 12 \text{ kg}$$

SUBSTITUTE INTO EQ (1)

$$(12 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) - M_f = \bar{I} \left( \frac{0.2836 \text{ m/s}^2}{0.6 \text{ m}} \right) + (12 \text{ kg})(0.2836 \text{ m/s}^2)(0.6 \text{ m})$$

$$70.632 - M_f = \bar{I}(0.4727) + 2.0419 \quad (2)$$

CASE 2:  $y = 3 \text{ m}$ ,  $t = 3.1 \text{ s}$ 

$$y = \frac{1}{2} a t^2; \quad 3 \text{ m} = \frac{1}{2} a (3.1 \text{ s})^2; \quad a = 0.6243 \text{ m/s}^2$$

$$m_A = 24 \text{ kg}$$

SUBSTITUTE INTO EQ (1):

$$(24 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) - M_f = \bar{I} \left( \frac{0.6243 \text{ m/s}^2}{0.6 \text{ m}} \right) + (24 \text{ kg})(0.6243 \text{ m/s}^2)(0.6 \text{ m})$$

$$141.264 - M_f = \bar{I}(1.0406) + 8.9899 \quad (3)$$

SUBTRACT EQ (1) FROM EQ (2), TO ELIMINATE  $M_f$ 

$$70.632 = \bar{I}(1.0406 - 0.4727) + 6.948$$

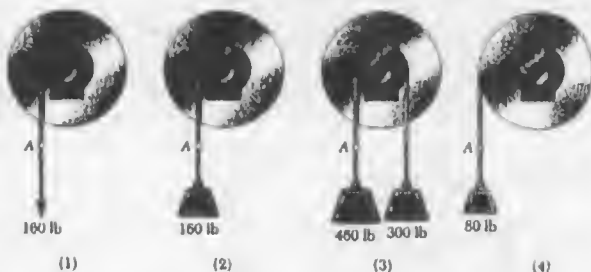
$$63.684 = \bar{I}(0.5679)$$

$$\bar{I} = 112.14 \text{ kg} \cdot \text{m}^2$$

$$\bar{I} = 112.1 \text{ kg} \cdot \text{m}^2$$

# 16.34

GIVEN: FOR EACH PULLEY,  $\bar{I} = 15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
INNER RADIUS = 9 in.; OUTER RADIUS = 18 in.



FIND: FOR EACH PULLEY: (a)  $\alpha$ , (b)  $\omega$  WHEN  $y_A = 10 \text{ ft}$

CASE 1:

$$\begin{aligned} (a) + \sum M_O &= \sum (M_O)_{\text{eff}} \\ (160 \text{ lb}) \left( \frac{9}{12} \text{ ft} \right) &= (15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) \alpha \\ \alpha &= 8 \text{ rad/s}^2 \\ (b) \theta &= \frac{10 \text{ ft}}{(9/12 \text{ ft})} = 13.333 \text{ rad} \\ \omega^2 &= 2\alpha\theta = 2(8 \text{ rad/s}^2)(13.333 \text{ rad}) \\ \omega &= 14.61 \text{ rad/s} \end{aligned}$$

CASE 2:

$$\begin{aligned} (a) + \sum M_O &= \sum (M_O)_{\text{eff}} \\ (160) \left( \frac{9}{12} \right) &= 15\alpha + m a \left( \frac{9}{12} \right) \\ 120 &= 15\alpha + \frac{160}{32.2} \left( \frac{9}{12} \alpha \right) \\ 120 &= (15 + 2.795) \alpha \\ \alpha &= 6.7435 \text{ rad/s}^2 \\ \omega &= 6.74 \text{ rad/s} \\ (b) \theta &= \frac{10 \text{ ft}}{(9/12 \text{ ft})} = 13.333 \text{ rad} \\ \omega^2 &= 2\alpha\theta = 2(6.7435 \text{ rad/s}^2)(13.333 \text{ rad}) \\ \omega &= 13.41 \text{ rad/s} \end{aligned}$$

CASE 3:

$$\begin{aligned} (a) + \sum M_O &= \sum (M_O)_{\text{eff}} \\ (460) \left( \frac{9}{12} \right) - (300) \left( \frac{9}{12} \right) &= 15\alpha + \frac{460}{32.2} a \left( \frac{9}{12} \right) + \frac{300}{32.2} a \left( \frac{9}{12} \right) \\ 120 &= 15\alpha + \frac{460}{32.2} \left( \frac{9}{12} \alpha \right) + \frac{300}{32.2} \left( \frac{9}{12} \alpha \right) \\ \alpha &= 4.2437 \text{ rad/s}^2 \\ \omega &= 4.24 \text{ rad/s} \\ (b) \theta &= \frac{10 \text{ ft}}{(9/12 \text{ ft})} = 13.333 \text{ rad} \\ \omega^2 &= 2\alpha\theta = 2(4.2437)(13.333) \\ \omega &= 10.64 \text{ rad/s} \end{aligned}$$

CASE 4:

$$\begin{aligned} (a) + \sum M_O &= \sum (M_O)_{\text{eff}} \\ (80) \left( \frac{18}{12} \right) &= 15\alpha + \frac{80}{32.2} a \left( \frac{18}{12} \right) \\ 120 &= 15\alpha + \frac{80}{32.2} \left( \frac{18}{12} \alpha \right) \\ 120 &= (15 + 5.570) \alpha \\ \alpha &= 5.828 \text{ rad/s}^2 \\ \omega &= 5.83 \text{ rad/s} \\ (b) \theta &= \frac{10 \text{ ft}}{(18/12 \text{ ft})} = 6.667 \text{ rad} \\ \omega^2 &= 2\alpha\theta = 2(5.828 \text{ rad/s}^2)(6.667 \text{ rad}) \\ \omega &= 8.82 \text{ rad/s} \end{aligned}$$

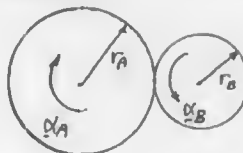
# 16.35



GIVEN:  
 $W_A = 10 \text{ lb}$ ,  $r_A = 4.5 \text{ in}$   
 $W_B = 4 \text{ lb}$ ,  $r_B = 3 \text{ in}$   
 $M = 5 \text{ lb} \cdot \text{in} \cdot \text{s}^2$   
FIND:  
(a)  $\alpha_A$  AND  $\alpha_B$

(b) FRICTION FORCE  
EXERTED ON B.

KINEMATICS:



SINCE THE TANGENTIAL  
ACCELERATIONS OF THE  
OUTSIDE OF THE DISKS  
ARE EQUAL,

$$r_A \alpha_A = r_B \alpha_B \quad (1)$$

KINETICS:

DISK A:

$$\begin{aligned} + \sum M_A &= \sum (M_A)_{\text{eff}} \\ M - F r_A &= \bar{I}_A \alpha_A \end{aligned} \quad (2)$$

DISK B:

$$\begin{aligned} + \sum M_B &= \sum (M_B)_{\text{eff}} \\ F r_B &= \bar{I}_B \alpha_B \\ F r_B &= \left( \frac{1}{2} m_B r_B^2 \right) \alpha_B \quad F = \frac{1}{2} m_B r_B \alpha_B \end{aligned} \quad (3)$$

SUBSTITUTE FOR F FROM EQ(3) INTO EQ(2):

$$M - \left( \frac{1}{2} m_B r_B \alpha_B \right) r_A = \bar{I}_A \alpha_A$$

SUBSTITUTE FOR F FROM EQ(3), AND FOR  $\alpha_B$  FROM EQ(1):

$$M - \frac{1}{2} m_B r_B r_A \left( \frac{r_A}{r_B} \alpha_A \right) = \bar{I}_A \alpha_A$$

$$M = \frac{1}{2} (m_A + m_B) r_A^2 \alpha_A$$

$$\alpha_A = \frac{2M}{(m_A + m_B) r_A^2} = \frac{2Mg}{(W_A + W_B) r_A^2}$$

DATA:  $W_A = 10 \text{ lb}$ ,  $W_B = 4 \text{ lb}$

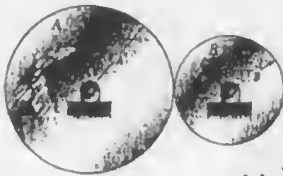
$$\begin{aligned} (a) \quad r_A &= 4.5 \text{ in} = 0.375 \text{ ft}; \quad M = 5 \text{ lb} \cdot \text{in} = \frac{5}{12} \text{ lb} \cdot \text{ft} \\ \alpha_A &= \frac{2 \left( \frac{5}{12} \text{ lb} \cdot \text{ft} \right) (32.2 \text{ ft/s}^2)}{(10 \text{ lb} + 4 \text{ lb}) (0.375 \text{ ft})^2} \quad \alpha_A = 13.63 \text{ rad/s}^2 \end{aligned}$$

$$\begin{aligned} \text{EQ(1): } \alpha_B &= \frac{r_A}{r_B} \alpha_A = \frac{4.5 \text{ in}}{3 \text{ in}} (13.63 \text{ rad/s}^2) \\ \alpha_B &= 20.44 \text{ rad/s}^2 \end{aligned}$$

$$(b) \text{ EQ(3): } F = \frac{1}{2} m_B r_B \alpha_B = \frac{1}{2} \frac{W_B}{g} r_B \alpha_B$$

$$F = \frac{1}{2} \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{3}{12} \text{ ft} \right) (20.44 \text{ rad/s}^2); \quad F = 0.317 \text{ lb}$$

16.36



GIVEN:

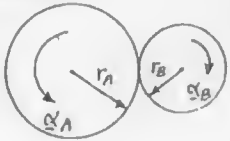
$$W_A = 10 \text{ lb}, r_A = 4.5 \text{ in.}$$

$$W_B = 4 \text{ lb}, r_B = 3 \text{ in.}$$

$$M = 5 \text{ lb} \cdot \text{in.}$$

FIND: (a)  $\alpha_A$  AND  $\alpha_B$   
(b) FRICTION FORCE  
EXERTED ON B.

KINEMATICS:



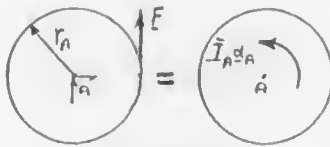
SINCE THE TANGENTIAL  
ACCELERATION OF THE  
OUTSIDE OF THE DISKS  
ARE EQUAL.

$$r_A \alpha_A = r_B \alpha_B$$

$$\alpha_A = \frac{r_B}{r_A} \alpha_B \quad (1)$$

KINETICS:

DISK A:



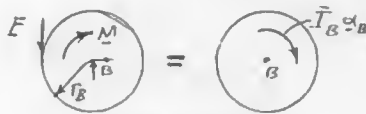
$$\bar{I}_A = \frac{1}{2} m_A r_A^2$$

$$+\sum M_A = \sum (M_A)_{eff}$$

$$F r_A = \bar{I}_A \alpha_A$$

$$F r_A = \frac{1}{2} m_A r_A^2 \alpha_A \quad F = \frac{1}{2} m_A r_A \alpha_A \quad (2)$$

DISK B:



$$+\sum M_B = \sum (M_B)_{eff}$$

$$M - F r_B = \bar{I}_B \alpha_B \quad (3)$$

SUBSTITUTE FOR F FROM EQ (2) INTO EQ (3)

$$M - \left(\frac{1}{2} m_A r_A \alpha_A\right) r_B = \bar{I}_B \alpha_B$$

SUBSTITUTE FOR  $\alpha_A$  FROM EQ (1), AND  $I_B = \frac{1}{2} m_B r_B^2$ 

$$M - \frac{1}{2} m_A r_A r_B \left(\frac{r_B}{r_A} \alpha_B\right) = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$M = \frac{1}{2} (m_A + m_B) r_B^2 \alpha_B$$

$$\alpha_B = \frac{2M}{(m_A + m_B) r_B^2} = \frac{2Mg}{(W_A + W_B) r_B^2}$$

DATA:  $W_A = 10 \text{ lb}$ ,  $W_B = 4 \text{ lb}$ ,  $r_A = 4.5 \text{ in.}$ 

$$r_B = 3 \text{ in.} = 0.25 \text{ ft}; M = 5 \text{ lb} \cdot \text{in.} = \frac{5}{12} \text{ lb} \cdot \text{ft}$$

$$(a) \alpha_B = \frac{2 \left( \frac{5}{12} \text{ lb} \cdot \text{ft} \right) \left( 32.2 \text{ ft/s}^2 \right)}{(10 \text{ lb} + 4 \text{ lb}) (0.25 \text{ ft})^2} = 30.667 \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_B = 30.7 \frac{\text{rad}}{\text{s}^2}$$

$$\text{EQ (1): } \alpha_A = \frac{r_B}{r_A} \alpha_B = \left( \frac{3 \text{ in.}}{4.5 \text{ in.}} \right) \left( 30.667 \frac{\text{rad}}{\text{s}^2} \right) = 20.44 \frac{\text{rad}}{\text{s}^2}$$

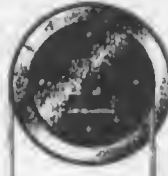
$$\alpha_A = 20.4 \frac{\text{rad}}{\text{s}^2}$$

$$(b) \text{EQ (2)} F = \frac{1}{2} m_A r_A \alpha_A = \frac{1}{2} \frac{W_A}{g} r_A \alpha_A$$

$$F = \frac{1}{2} \left( \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} \right) \left( \frac{4.5}{12} \text{ ft} \right) (20.44 \text{ rad/s}^2) = 1.190 \text{ lb}$$

FRICTION FORCE ON DISK B:  $F = 1.190 \text{ lb}$ 

16.37



GIVEN:

$$\text{DISK A: } W = 20 \text{ lb}, r_A = 8 \text{ in.}$$

$$\text{DISK B: } W = 12 \text{ lb}, r_B = 6 \text{ in.}$$

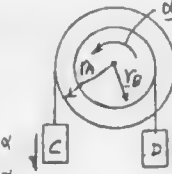
FIND: (a)  $a_C$   
(b)  $a_D$

$$\text{TOTAL } \bar{I}: \bar{I} = \frac{1}{2} m_A r_A^2 + \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2} \frac{20 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{8}{12} \text{ ft} \right)^2 + \frac{1}{2} \frac{12 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{6}{12} \text{ ft} \right)^2$$

$$= 0.13803 + 0.04658 = 0.18461 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

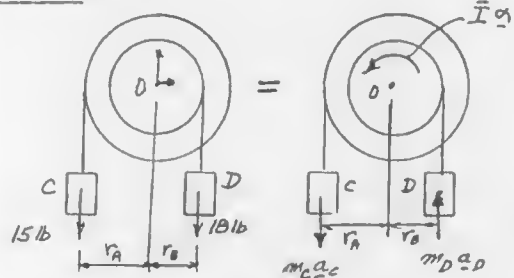
KINEMATICS:



$$a_C = r \alpha = \left( \frac{8}{12} \text{ ft} \right) \alpha$$

$$a_D = r \alpha = \left( \frac{6}{12} \text{ ft} \right) \alpha = \frac{1}{2} \alpha$$

KINETICS



$$+\sum M_O = \sum (M_O)_{eff}$$

$$(15 \text{ lb}) r_A - (18 \text{ lb}) r_B = \bar{I} \alpha + m_C a_C r_A + m_D a_D r_B$$

$$(15 \text{ lb}) \left( \frac{8}{12} \text{ ft} \right) - (18 \text{ lb}) \left( \frac{6}{12} \text{ ft} \right) = 0.18461 \alpha + \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{8}{12} \text{ ft} \right) \left( \frac{8}{12} \text{ ft} \right) \alpha$$

$$+ \frac{18 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{6}{12} \text{ ft} \right) \left( \frac{6}{12} \text{ ft} \right) \alpha$$

$$10 - 9 = (0.18461 + 0.20704 + 0.13975) \alpha$$

$$1 = 0.5314 \alpha$$

$$\alpha = 1.8818 \text{ rad/s}^2$$

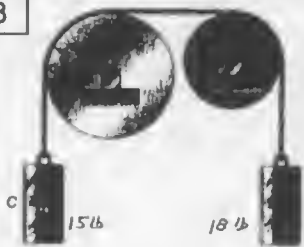
$$(a) a_C = \frac{8}{12} \alpha = \frac{8}{12} (1.8818 \text{ rad/s}^2)$$

$$a_C = 1.255 \text{ ft/s}^2 \downarrow$$

$$(b) a_D = \frac{1}{2} \alpha = \frac{1}{2} (1.8818 \text{ rad/s}^2)$$

$$a_D = 0.941 \text{ ft/s}^2 \downarrow$$

16.38



GIVEN:

DISK A:

$$W_A = 20 \text{ lb}, r_A = 8 \text{ in}$$

DISK B:

$$W_B = 18 \text{ lb}, r_B = 6 \text{ in}$$

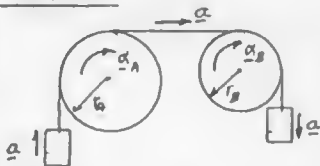
FIND: (a)  $\underline{a}_C$   
(b)  $\underline{a}_D$ 

MOMENTS OF INERTIA

$$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2} \frac{20 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{8}{12} \text{ ft} \right)^2 = 0.13803 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2} \frac{18 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{6}{12} \text{ ft} \right)^2 = 0.04658 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

KINEMATICS

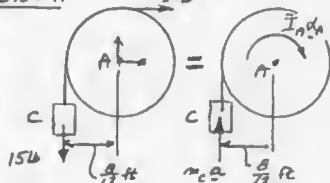


$$\alpha_A = \frac{a}{r_A} = \frac{a}{(8/12) \text{ ft}} = 1.5a$$

$$\alpha_B = \frac{a}{r_B} = \frac{a}{(6/12) \text{ ft}} = 2a$$

KINETICS

DISK A:

F<sub>AB</sub> = TENSION IN CORD  
BETWEEN DISKS

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: F_{AB} \left( \frac{8}{12} \text{ ft} \right) - 15 \text{ lb} \left( \frac{8}{12} \text{ ft} \right) = \bar{I}_A \alpha_A + m_A a \left( \frac{8}{12} \text{ ft} \right)$$

$$\frac{2}{3} F_{AB} - 10 = (0.13803)(1.5a) + \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2} a \left( \frac{2}{3} \right)$$

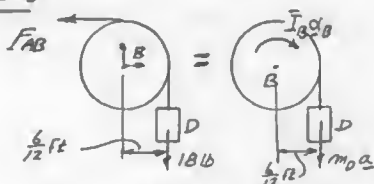
$$\frac{2}{3} F_{AB} - 10 = 0.20705a + 0.31058a$$

$$\frac{2}{3} F_{AB} - 10 = 0.51763a$$

$$F_{AB} = 15 + 0.77641a$$

(1)

DISK B:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$(18 \text{ lb}) \left( \frac{6}{12} \text{ ft} \right) - F_{AB} \left( \frac{6}{12} \text{ ft} \right) = \bar{I}_B \alpha_B + m_B a \left( \frac{6}{12} \text{ ft} \right)$$

$$9 - 0.5 F_{AB} = (0.04658)(2a) + \frac{18 \text{ lb}}{32.2 \text{ ft/s}^2} a \left( \frac{1}{2} \right)$$

$$9 - 0.5 F_{AB} = 0.09316a + 0.2795a$$

SUBSTITUTE FOR F<sub>AB</sub> FROM EQ (1)

$$9 - 0.5(15 + 0.77641a) = 0.37266a$$

$$9 - 7.5 - 0.3882a = 0.37266a$$

$$1.5 = 0.76086a$$

$$a = 1.971 \text{ ft/s}^2$$

BOTH  $\underline{a}_C$  AND  $\underline{a}_D$  HAVE THE SAME MAGNITUDE

$$\underline{a}_C = 1.971 \text{ ft/s}^2 \uparrow$$

$$\underline{a}_D = 1.971 \text{ ft/s}^2 \downarrow$$

16.39 and 16.40

GIVEN:  $m_A = 6 \text{ kg}$ ;  $m_B = 3 \text{ kg}$ 

$$N = 20 \text{ N}, \mu_k = 0.15$$

PROBLEM 16.39:

$$(\omega_A)_0 = 360 \text{ rpm}; (\omega_B)_0 = 0$$

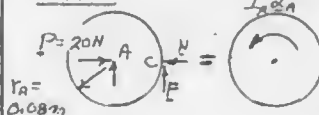
PROBLEM 16.40:

$$(\omega_A)_0 = 0; (\omega_B)_0 = 360 \text{ rpm}$$

FOR EACH PROBLEM:

FIND: (a)  $\underline{a}_A$  AND  $\underline{a}_B$ . (b) FINAL VELOCITIES  $\underline{v}_A$  AND  $\underline{v}_B$ WHILE SLIPPING OCCURS, A FRICTION FORCE  $F \uparrow$  IS APPLIED TO DISK A, AND  $F \downarrow$  TO DISK B.

DISK A:



$$\bar{I}_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2} (6 \text{ kg}) (0.08 \text{ m})^2$$

$$= 0.0192 \text{ kg} \cdot \text{m}^2$$

$$\Sigma F: N = P = 20 \text{ N}$$

$$F = \mu N = 0.15(20) = 3 \text{ N}$$

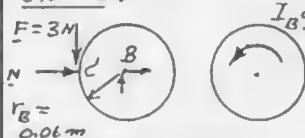
$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: F r_A = \bar{I}_A \alpha_A$$

$$(3 \text{ N})(0.08 \text{ m}) = (0.0192 \text{ kg} \cdot \text{m}^2) \alpha_A$$

$$\alpha_A = 12.5 \text{ rad/s}^2$$

$$\underline{a}_A = 12.5 \text{ rad/s}^2 \uparrow$$

DISK B:



$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2} (3 \text{ kg}) (0.06 \text{ m})^2$$

$$= 0.0054 \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: F r_B = \bar{I}_B \alpha_B$$

$$(3 \text{ N})(0.06 \text{ m}) = (0.0054 \text{ kg} \cdot \text{m}^2) \alpha_B$$

$$\alpha_B = 33.33 \text{ rad/s}^2$$

$$\underline{a}_B = 33.3 \text{ rad/s}^2 \uparrow$$

PROBLEM 16.39:  $(\omega_A)_0 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right) = 12\pi \text{ rad/s}$ ;  $(\omega_B)_0 = 0$ DISKS WILL STOP SLIDING, WHEN  $\underline{v}_C = \underline{v}_D$ , THAT IS WHEN

$$\omega_A r_A = \omega_B r_B$$

$$[(\omega_A)_0 - \alpha_A t] r_A = \alpha_B t r_B$$

$$(12\pi - 12.5 t)(0.08) = (33.33 t)(0.06)$$

$$3.0159 - t = 2t; \quad t = 1.0053 \text{ s}$$

$$+\circlearrowleft \omega_A = (\omega_A)_0 - \alpha_A t = 12\pi - 12.5(1.0053) = 25.132 \text{ rad/s}$$

$$\omega_A = 25.132 \text{ rad/s} \left( \frac{60}{2\pi} \right) = 240 \text{ rpm}$$

$$\underline{\omega}_A = 240 \text{ rpm} \uparrow$$

$$+\circlearrowleft \omega_B = \alpha_B t = (33.33)(1.0053) = 33.507 \text{ rad/s}$$

$$\omega_B = 33.507 \text{ rad/s} \left( \frac{60}{2\pi} \right) = 320 \text{ rpm}$$

$$\underline{\omega}_B = 320 \text{ rpm} \uparrow$$

PROBLEM 16.40:  $(\omega_A)_0 = 0$ ;  $(\omega_B)_0 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right) = 12\pi \text{ rad/s}$ SLIDING STOPS WHEN  $\underline{v}_C = \underline{v}_D$ , THAT IS WHEN

$$\omega_A r_A = \omega_B r_B$$

$$(\alpha_A t) r_A = [(\omega_B)_0 - \alpha_B t] r_B$$

$$(12.5 t)(0.08) = (12\pi - 33.33 t)(0.06)$$

$$t = 2.26195 - 2t; \quad t = 0.75398 \text{ s}$$

$$+\circlearrowleft \omega_A = \alpha_A t = (12.5)(0.75398) = 9.4248 \text{ rad/s}$$

$$\omega_A = 9.4248 \text{ rad/s} \left( \frac{60}{2\pi} \right) = 90 \text{ rpm}$$

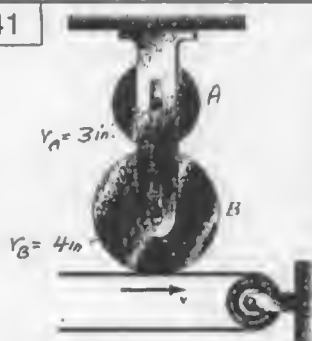
$$\underline{\omega}_A = 90 \text{ rpm} \uparrow$$

$$+\circlearrowleft \omega_B = (\omega_B)_0 - \alpha_B t = 12\pi - (33.33)(0.75398) = 12.589 \text{ rad/s}$$

$$\omega_B = 12.589 \text{ rad/s} \left( \frac{60}{2\pi} \right) = 120 \text{ rpm}$$

$$\underline{\omega}_B = 120 \text{ rpm} \uparrow$$

16.41



GIVEN:

$$\begin{aligned} W_A &= 4 \text{ lb} \\ W_B &= 9 \text{ lb} \\ \mu_k &= 0.20 \text{ AT} \\ &\text{ALL SURFACES} \end{aligned}$$

FIND: INITIAL  
ANGULAR ACCELERATION  
OF EACH DISK

ASSUME THAT SLIPPING OCCURS BETWEEN  
DISKS A AND B.

DISK A:

$$\begin{aligned} \bar{I}_A &= \frac{1}{2} m_A r_A^2 \\ &= \frac{1}{2} \frac{4 \text{ lb}}{32.2} \left( \frac{3}{12} \text{ ft} \right)^2 \\ \uparrow \Sigma F = \Sigma F_{\text{net}}: N_A &= 4 \text{ lb} \quad F_A = \mu_k N = 0.2(4 \text{ lb}) = 0.8 \text{ lb} \\ + \Sigma M_A = \Sigma (M_A)_{\text{net}}: F_A r_A &= \bar{I}_A \alpha_A \\ (0.8 \text{ lb}) \left( \frac{3}{12} \text{ ft} \right) &= \frac{1}{2} \frac{4 \text{ lb}}{32.2} \left( \frac{3}{12} \text{ ft} \right)^2 \alpha_A \\ \alpha_A &= 51.52 \text{ rad/s}^2 \end{aligned}$$

DISK B:

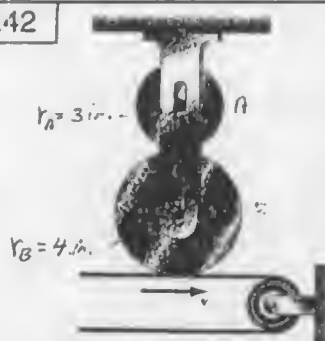
$$\begin{aligned} \bar{I}_B &= \frac{1}{2} m_B r_B^2 \\ &= \frac{1}{2} \frac{9 \text{ lb}}{32.2} \left( \frac{4}{12} \text{ ft} \right)^2 \\ \uparrow \Sigma F = \Sigma F_{\text{net}}: N_B &= 4 + 9 = 13 \text{ lb} \\ F_B &= \mu_k N_B = 0.20(13 \text{ lb}) = 2.6 \text{ lb} \\ + \Sigma M_B = \Sigma (M_B)_{\text{net}}: (F_B - F_A) r_B &= \bar{I}_B \alpha_B \\ (2.6 \text{ lb} - 0.8 \text{ lb}) \left( \frac{4}{12} \text{ ft} \right) &= \frac{1}{2} \frac{9 \text{ lb}}{32.2} \left( \frac{4}{12} \text{ ft} \right)^2 \alpha_B \\ \alpha_B &= 38.64 \text{ rad/s}^2 \end{aligned}$$

KINEMATICS:

$$\begin{aligned} (a_C)_t &= r_A \alpha_A = \left( \frac{3}{12} \text{ ft} \right) (51.52 \text{ rad/s}^2) \\ &= 12.88 \text{ ft/s}^2 \\ (a_C')_t &= r_B \alpha_B = \left( \frac{4}{12} \text{ ft} \right) (38.64 \text{ rad/s}^2) \\ &= 12.88 \text{ ft/s}^2 \end{aligned}$$

WE FIND THAT SLIPPING DOES NOT  
OCCUR BETWEEN DISKS, BUT SINCE  
 $(a_C)_t = (a_C')_t$  SLIPPING IMPENDS AND THAT  $F_A = \mu_k N_A = 0.8 \text{ lb}$   
AND ABOVE RESULTS ARE VALID.

16.42



GIVEN:

$$\begin{aligned} W_A &= 4 \text{ lb} \\ W_B &= 9 \text{ lb} \\ \mu_k &= 0.10 \text{ BETWEEN} \\ &\text{THE DISKS} \\ \mu_k &= 0.20 \text{ BETWEEN} \\ &\text{BELT AND DISK B} \end{aligned}$$

FIND: INITIAL  
ANGULAR ACCELERATION  
OF EACH DISK.

ASSUME THAT SLIPPING OCCURS BETWEEN  
DISKS A AND B.

DISK A:

$$\begin{aligned} \bar{I}_A &= \frac{1}{2} m_A r_A^2 \\ &= \frac{1}{2} \frac{4 \text{ lb}}{32.2} \left( \frac{3}{12} \text{ ft} \right)^2 \\ + \Sigma F = \Sigma (F_A)_{\text{net}}: N_A &= 4 \text{ lb} \quad F_A = \mu_k N_A = 0.1(4 \text{ lb}) = 0.4 \text{ lb} \\ + \Sigma M_A = \Sigma (M_A)_{\text{net}}: F_A r_A &= \bar{I}_A \alpha_A \\ (0.4 \text{ lb}) \left( \frac{3}{12} \text{ ft} \right) &= \frac{1}{2} \frac{4 \text{ lb}}{32.2} \left( \frac{3}{12} \text{ ft} \right)^2 \alpha_A \\ \alpha_A &= 25.76 \text{ rad/s}^2 \end{aligned}$$

DISK B:

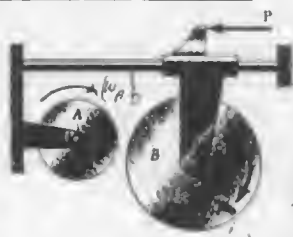
$$\begin{aligned} \bar{I}_B &= \frac{1}{2} m_B r_B^2 \\ &= \frac{1}{2} \frac{9 \text{ lb}}{32.2} \left( \frac{4}{12} \text{ ft} \right)^2 \\ \Sigma F = \Sigma F_{\text{net}}: N_B &= 4 + 9 = 13 \text{ lb} \\ F_B &= \mu_k N_B = 0.20(13 \text{ lb}) = 2.6 \text{ lb} \\ + \Sigma M_B = \Sigma (M_B)_{\text{net}}: (F_B - F_A) r_B &= \bar{I}_B \alpha_B \\ (2.6 \text{ lb} - 0.4 \text{ lb}) \left( \frac{4}{12} \text{ ft} \right) &= \frac{1}{2} \frac{9 \text{ lb}}{32.2} \left( \frac{4}{12} \text{ ft} \right)^2 \alpha_B \\ \alpha_B &= 47.23 \text{ rad/s}^2 \end{aligned}$$

KINEMATICS:

$$\begin{aligned} (a_C)_t &= r_A \alpha_A = \left( \frac{3}{12} \text{ ft} \right) (25.76 \text{ rad/s}^2) \\ &= 6.44 \text{ ft/s}^2 \\ (a_C')_t &= r_B \alpha_B = \left( \frac{4}{12} \text{ ft} \right) (47.23 \text{ rad/s}^2) \\ &= 15.74 \text{ ft/s}^2 \end{aligned}$$

SINCE  $(a_C')_t > (a_C)_t$ , WE  
CONFIRM THAT ASSUMPTION OF  
SLIPPING BETWEEN DISKS IS TRUE

# 16.43 and 16.44



GIVEN:

$$P = 2.5 \text{ lb}, \mu_k = 0.25$$

$$W_A = 6 \text{ lb}, r_A = 2 \text{ in.}$$

$$W_B = 15 \text{ lb}, r_B = 5 \text{ in.}$$

FIND:

- $\alpha_A$  AND  $\alpha_B$
- FINAL  $\omega_A$  AND  $\omega_B$

PROBLEM 16.43:

$$(\omega_A)_0 = 375 \text{ rpm}, (\omega_B)_0 = 0$$

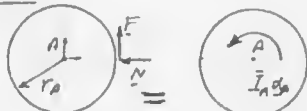
PROBLEM 16.44:

$$(\omega_A)_0 = 0, (\omega_B)_0 = 375 \text{ rpm}$$

WHILE SLIPPING OCCURS:

$$F = \mu_k N = \mu_k P = 0.25(2.5 \text{ lb}) = 0.625 \text{ lb}$$

DISK A:



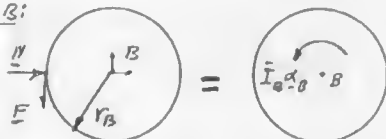
$$+\circlearrowleft \Sigma M_P = \Sigma (M_P)_{eff}; \quad F r_A = \bar{I}_A \alpha_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$F = \frac{1}{2} m_A r_A \alpha_A$$

$$0.625 \text{ lb} = \frac{1}{2} \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{3}{12} \text{ ft} \right) \alpha_A$$

$$\alpha_A = 26.833 \text{ rad/s}^2 \quad \alpha_A = 26.8 \text{ rad/s}^2 \quad \leftarrow$$

DISK B:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}; \quad F r_B = \bar{I}_B \alpha_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$F = \frac{1}{2} m_B r_B \alpha_B$$

$$0.625 \text{ lb} = \frac{1}{2} \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{5}{12} \text{ ft} \right) \alpha_B$$

$$\alpha_B = 6.44 \text{ rad/s}^2 \quad \alpha_B = 6.44 \text{ rad/s}^2 \quad \leftarrow$$

PROBLEM 16.43:

$$(\omega_A)_0 = 375 \text{ rpm} \left( \frac{2\pi}{60} \right) = 39.27 \text{ rad/s}; (\omega_B)_0 = 0$$



WHEN DISKS STOP SLIDING

$$v_P = v_P; \quad \omega_A r_A = \omega_B r_B \quad (1)$$

$$[(\omega_A)_0 - \alpha_A t] r_A = (\alpha_B t) r_B$$

$$(39.27 - 26.833 t)(3 \text{ in.}) = (6.44 t)(5 \text{ in.})$$

$$t = 1.0454 \text{ s}$$

$$\omega_A = (\omega_A)_0 - \alpha_A t = 39.27 - (26.833)(1.0454)$$

$$\omega_A = 11.22 \text{ rad/s} \left( \frac{50}{2\pi} \right) \quad \omega_A = 107.1 \text{ rpm} \quad \leftarrow$$

$$\text{EQ (1): } (107.2 \text{ rpm})(3 \text{ in.}) = \omega_B (5 \text{ in.}) \quad \omega_B = 64.3 \text{ rpm} \quad \leftarrow$$

PROBLEM 16.44:  $(\omega_A)_0 = 0; (\omega_B)_0 = 375 \text{ rpm} \left( \frac{2\pi}{60} \right) = 39.27 \text{ rad/s}$

$$\text{EQ (1): } \omega_A r_A = \omega_B r_B; \quad \alpha_A t r_A = [(\omega_B)_0 - \alpha_B t] r_B$$

$$(26.833 t)(3 \text{ in.}) = [39.27 - 6.44 t](5 \text{ in.})$$

$$t = 1.742 \text{ s}$$

$$\omega_A = \alpha_A t = (26.833)(1.742) = 46.74 \text{ rad/s} \left( \frac{60}{2\pi} \right)$$

$$\omega_A = 446 \text{ rpm} \quad \leftarrow$$

$$\text{EQ (1): } (446 \text{ rpm})(3 \text{ in.}) = \omega_B (5 \text{ in.}) \quad \omega_B = 268 \text{ rpm} \quad \leftarrow$$

# 16.45



GIVEN:

$$(\omega_B)_0 = \omega_0$$

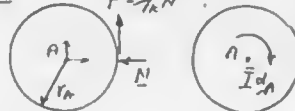
$$(\omega_A)_0 = 0$$

SHOW THAT:

- FINAL  $\omega_A$  AND  $\omega_B$  ARE INDEPENDENT OF  $\mu_k$
- FINAL  $\omega_B = f(\omega_0, m_A/m_B)$

WHILE SLIPPING OCCURS:  $F = \mu_k N = \mu_k P$

DISK A:



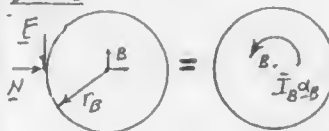
$$\Sigma M_A = \Sigma (M_A)_{eff};$$

$$F r_A = \bar{I}_A \alpha_A$$

$$\mu_k P r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\alpha_A = \frac{2 \mu_k P}{m_A r_A} \quad (1)$$

DISK B:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff};$$

$$F r_B = \bar{I}_B \alpha_B$$

$$\mu_k P r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\alpha_B = \frac{2 \mu_k P}{m_B r_B} \quad (2)$$

AT ANY TIME t

$$+\circlearrowleft \omega_A = 0 + \alpha_A t = \frac{2 \mu_k P}{m_A r_A} t \quad (3)$$

$$+\circlearrowleft \omega_B = \omega_0 - \alpha_B t = \omega_0 - \frac{2 \mu_k P}{m_B r_B} t \quad (4)$$

SLIPPING ENDS WHEN  $\omega_A r_A = \omega_B r_B$

$$\alpha_A t r_A = (\omega_0 - \alpha_B t) r_B$$

$$(\alpha_A r_A + \alpha_B r_B) t = \omega_0 r_B$$

SUBSTITUTE FROM EQS (1) + (2):  $\left[ \frac{2 \mu_k P}{m_A r_A} r_A + \frac{2 \mu_k P}{m_B r_B} r_B \right] t = \omega_0 r_B$

$$2 \mu_k P \left( \frac{1}{m_A} + \frac{1}{m_B} \right) t = \omega_0 r_B; \quad t = \frac{\omega_0 r_B}{2 \mu_k P} \cdot \frac{1}{\frac{1}{m_A} + \frac{1}{m_B}}$$

$$t = \frac{\omega_0 r_B}{2 \mu_k P} \cdot \frac{m_A m_B}{m_A + m_B}$$

$$\text{EQ (3): } \omega_A = \alpha_A t = \frac{2 \mu_k P}{m_A r_A} \left[ \frac{\omega_0 r_B}{2 \mu_k P} \cdot \frac{m_A m_B}{m_A + m_B} \right]$$

$$\omega_A = \frac{r_B}{r_A} \cdot \frac{m_B}{m_A + m_B} \omega_0$$

( $\omega_A$  IS INDEPENDENT OF  $\mu_k$ , Q.E.D.)

$$\text{EQ (4): } \omega_B = \omega_0 - \alpha_B t = \omega_0 - \frac{2 \mu_k P}{m_B r_B} \left[ \frac{\omega_0 r_B}{2 \mu_k P} \cdot \frac{m_A m_B}{m_A + m_B} \right]$$

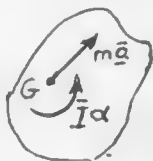
$$\omega_B = \omega_0 \left\{ 1 - \frac{m_A}{m_A + m_B} \right\}$$

$$\omega_B = \omega_0 \frac{m_A + m_B - m_A}{m_A + m_B} = \omega_0 \frac{m_B}{m_A + m_B}$$

$$\omega_B = \frac{\omega_0}{\frac{m_A}{m_B} + 1}$$

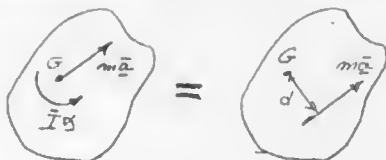
$\omega_B$  DEPENDS ONLY UPON  $\omega_0$  AND  $\frac{m_A}{m_B}$  (Q.E.D.)

16.46



SHOW THAT  
SYSTEM OF  
EFFECTIVE FORCES  
FOR A SLAB  
REDUCES TO  $m\vec{a}$   
AND EXPRESS DISTANCE  
FROM ITS LINE OF ACTION  
TO G IN TERMS OF  $\vec{r}_i, \vec{a}$ , AND  $\alpha$ .

WE KNOW THAT THE SYSTEM OF EFFECTIVE  
FORCES CAN BE REDUCED TO THE VECTOR  $m\vec{a}$   
AT G AND THE COUPLE  $\vec{I}\alpha$ . WE FURTHER KNOW  
FROM CHAPTER 3 OF STATICS THAT A  
FORCE-COUPLE SYSTEM IN A PLANE CAN BE  
FURTHER REDUCED TO A SINGLE FORCE.



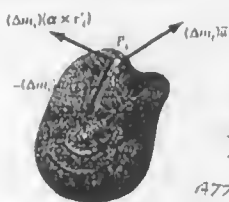
THE PERPENDICULAR DISTANCE  $d$  FROM G TO THE  
LINE OF ACTION OF THE SINGLE VECTOR  $m\vec{a}$  IS  
EXPRESSED BY WRITING

$$\downarrow \Sigma M_G = \Sigma (M_G)_{eff}: \quad \vec{I}\alpha = (m\vec{a})d$$

$$d = \frac{\vec{I}\alpha}{m\vec{a}} = \frac{m\vec{r}_i^2 \alpha}{m\vec{a}}$$

$$d = \frac{\vec{r}_i^2 \alpha}{\vec{a}}$$

16.47



SHOW THAT THE  
SYSTEM OF EFFECTIVE  
FORCES OF A RIGID  
SLAB CONSISTS OF  
THE VECTORS SHOWN  
ATTACHED TO THE PARTICLES  
 $P_i$  OF THE SLAB. FURTHER

SHOW THAT THE EFFECTIVE FORCES REDUCE TO  
 $m\vec{a}$  ATTACHED AT G AND A COUPLE  $\vec{I}\alpha$ .

KINEMATICS

THE ACCELERATION OF  $P_i$  IS

$$\vec{a}_i = \vec{a} + \alpha \vec{r}_i / \rho$$

$$\vec{a}_i = \vec{a} + \alpha \times \vec{r}_i + \omega \times (\omega \times \vec{r}_i)$$

$$= \vec{a} + \alpha \times \vec{r}_i - \omega^2 \vec{r}_i$$

NOTE THAT  $\alpha \times \vec{r}_i$  IS  $\perp$  TO  $\vec{r}_i$



THUS, THE EFFECTIVE FORCES ARE AS SHOWN  
IN FIG P16.47 (also shown above). WE WRITE

$$(\Delta m_i) \vec{a}_i = (\Delta m_i) \vec{a} + (\Delta m_i) (\alpha \times \vec{r}_i) - (\Delta m_i) \omega^2 \vec{r}_i$$

THE SUM OF THE EFFECTIVE FORCES IS

$$\Sigma (\Delta m_i) \vec{a}_i = \Sigma (\Delta m_i) \vec{a} + \Sigma (\Delta m_i) (\alpha \times \vec{r}_i) - \Sigma (\Delta m_i) \omega^2 \vec{r}_i$$

$$\Sigma (\Delta m_i) \vec{a}_i = \vec{a} \Sigma (\Delta m_i) + \alpha \times \Sigma (\Delta m_i) \vec{r}_i - \omega^2 \Sigma (\Delta m_i) \vec{r}_i$$

(CONTINUED)

16.47 continued

WE NOTE THAT

$$\Sigma (\Delta m_i) = m. \text{ AND SINCE G IS THE MASS CENTER}$$

$$\Sigma (\Delta m_i) \vec{r}_i = m \vec{r}_G = 0$$

$$\text{THUS, } \Sigma (\Delta m_i) \vec{a}_i = m \vec{a} \quad (1)$$

THE SUM OF THE MOMENTS ABOUT G OF THE  
EFFECTIVE FORCES IS:

$$\Sigma (\vec{r}_i \times \Delta m_i \vec{a}_i) = \Sigma \vec{r}_i \times \Delta m_i \vec{a} + \Sigma \vec{r}_i \times (\Delta m_i) (\alpha \times \vec{r}_i) - \Sigma \vec{r}_i \times (\Delta m_i) \omega^2 \vec{r}_i$$

$$\Sigma (\vec{r}_i \times \Delta m_i \vec{a}_i) = \Sigma \vec{r}_i \times \Delta m_i \vec{a} + \Sigma [\vec{r}_i \times (\alpha \times \vec{r}_i) \Delta m_i] - \omega^2 \Sigma (\vec{r}_i \times \vec{r}_i) \Delta m_i$$

SINCE G IS THE MASS CENTER,  $\Sigma \vec{r}_i \times \Delta m_i = 0$

ALSO, FOR EACH PARTICLE,  $\vec{r}_i \times \vec{r}_i = 0$

THUS

$$\Sigma (\vec{r}_i \times \Delta m_i \vec{a}_i) = \Sigma [\vec{r}_i \times (\alpha \times \vec{r}_i) \Delta m_i]$$

SINCE  $\alpha \perp \vec{r}_i$ , WE HAVE  $\vec{r}_i \times (\alpha \times \vec{r}_i) = \vec{r}_i^2 \alpha$  AND

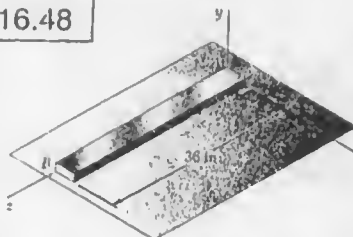
$$\Sigma (\vec{r}_i \times \Delta m_i \vec{a}_i) = \Sigma \vec{r}_i^2 (\Delta m_i) \alpha = (\Sigma \vec{r}_i^2 \Delta m_i) \alpha$$

SINCE  $\Sigma \vec{r}_i^2 \Delta m_i = \vec{I}$

$$\Sigma (\vec{r}_i \times \Delta m_i \vec{a}_i) = \vec{I} \alpha \quad (2)$$

FROM EQS. (1) AND (2) WE CONCLUDE THAT  
SYSTEM OF EFFECTIVE FORCES REDUCE TO  
 $m\vec{a}$  ATTACHED AT G AND A COUPLE  $\vec{I}\alpha$ .

16.48

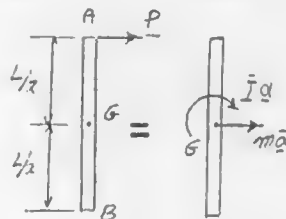


GIVEN: 1.75-16 ROD AB

$$P = 0.25 \text{ lb}$$

$$L = 36 \text{ in}$$

FIND: ACCELERATION  
(a) OF A.  
(b) OF B.



$$m = \frac{W}{g}$$

$$\vec{I} = \frac{1}{12} \frac{W}{g} L^2$$

$$\uparrow \Sigma F_x = \Sigma (F_x)_{eff}: \quad P = m\vec{a} = \frac{W}{g} \vec{a}$$

$$\vec{a} = \frac{P}{W} g = \frac{0.25 \text{ lb}}{1.75 \text{ lb}} g = \frac{1}{7} g \rightarrow$$

$$+ \Sigma M_G = \Sigma (M_G)_{eff}: \quad P \frac{L}{2} = \vec{I} \alpha = \frac{1}{12} \frac{W}{g} L^2 \alpha$$

$$\alpha = 6 \frac{P}{W} \frac{g}{L} = 6 \frac{0.25 \text{ lb}}{1.75 \text{ lb}} \frac{g}{L} = \frac{6}{7} \frac{g}{L}$$

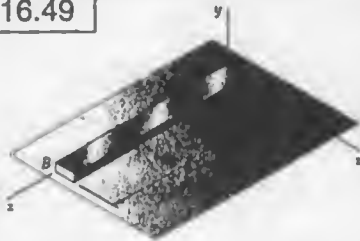
$$(a) \uparrow a_A = \vec{a} + \frac{L}{2} \alpha = \frac{1}{7} g + \frac{L}{2} \cdot \frac{6}{7} \frac{g}{L} = \frac{4}{7} g = \frac{4}{7} (32.2 \text{ ft/s}^2)$$

$$a_A = 18.40 \text{ ft/s}^2 \rightarrow$$

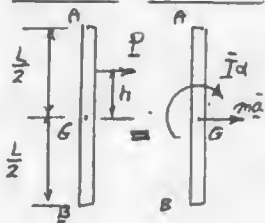
$$(b) \uparrow a_B = \vec{a} - \frac{L}{2} \alpha = \frac{1}{7} g - \frac{L}{2} \cdot \frac{6}{7} \frac{g}{L} = -\frac{2}{7} g = -\frac{2}{7} (32.2 \text{ ft/s}^2)$$

$$a_B = 9.2 \text{ ft/s}^2 \leftarrow$$

16.49



GIVEN: 1.75-lb ROD AB  
 $P = 0.25 \text{ lb}$   
 $L = 3 \text{ ft}$   
 FIND: (a) WHERE  $P$  SHOULD BE APPLIED FOR  $a_B = 0$ .  
 (b) CORRESPONDING ACCEL. OF POINT A.



$$\begin{aligned} \sum F_x &= \sum (F_x)_\text{eff} \\ P &= m\bar{a} = \frac{W}{g} \bar{a} \\ \bar{a} &= \frac{P}{W} g \rightarrow \\ +\sum M_G &= \sum (M_G)_\text{eff}: \\ Ph &= \bar{I} \alpha = \frac{1}{12} \frac{W}{g} L^2 \alpha \\ \alpha &= \frac{12Ph}{WL^2} g \end{aligned}$$

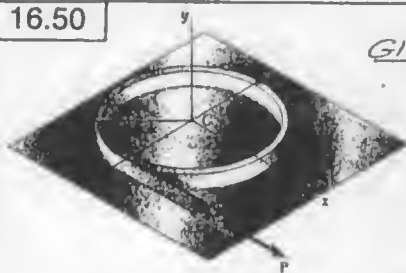
$$\begin{aligned} \text{(a)} \quad \pm a_B &= \bar{a} - \frac{L}{2} \alpha \\ 0 &= \frac{P}{W} g - \frac{L}{2} \cdot \frac{12Ph}{WL^2} g; \quad h = \\ h &= \frac{L}{6} = \frac{36 \text{ in.}}{6} = 6 \text{ in.} \end{aligned}$$

THUS,  $P$  IS LOCATED 12 in. FROM END A.

$$\text{FOR } h = \frac{L}{6}: \alpha = \frac{12P(L/6)}{WL^2} g = 2 \frac{P}{W} \cdot \frac{g}{L}$$

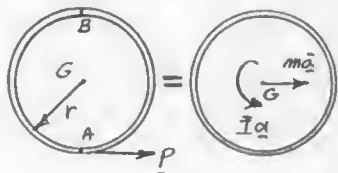
$$\begin{aligned} \text{(b)} \quad \pm a_A &= \bar{a} + \frac{L}{2} \alpha = \frac{P}{W} g + \frac{L}{2} \cdot 2 \frac{P}{W} \frac{g}{L} = 2 \frac{P}{W} g \\ a_A &= 2 \frac{0.25 \text{ lb}}{1.75 \text{ lb}} (32.2 \text{ ft/s}^2); \quad a_A = 9.2 \text{ ft/s}^2 \rightarrow \end{aligned}$$

16.50



GIVEN:  $P = 3 \text{ N}$   
 $m = 2.4 \text{ kg}$

FIND:  
 (a)  $a_A$   
 (b)  $a_B$



$$\begin{aligned} \text{HOOP: } \bar{I} &= mr^2 \\ \sum F_x &= \sum (F_x)_\text{eff}: \\ P &= m\bar{a} \\ \bar{a} &= \frac{P}{m} \rightarrow \end{aligned}$$

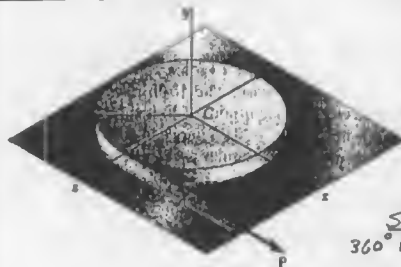
$$\begin{aligned} +\sum M_G &= \sum (M_G)_\text{eff}: \quad Pr = \bar{I} \alpha = mr^2 \alpha \\ \alpha &= \frac{P}{mr} \rightarrow \end{aligned}$$

$$\begin{aligned} \text{(a)} \quad \pm a_A &= \bar{a} + r\alpha = \frac{P}{m} + r \left( \frac{P}{mr} \right) = 2 \frac{P}{m} \\ a_A &= 2 \frac{3 \text{ N}}{2.4 \text{ kg}} = 2.5 \text{ m/s}^2 \end{aligned}$$

$$a_A = 2.5 \text{ m/s}^2 \rightarrow$$

$$\begin{aligned} \text{(b)} \quad \pm a_B &= \bar{a} - r\alpha = \frac{P}{m} - r \left( \frac{P}{mr} \right) = 0 \\ a_B &= 0 \end{aligned}$$

16.51 and 16.52



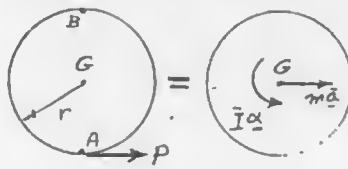
GIVEN:  $P = 3 \text{ N}$   
 $m = 2.4 \text{ kg}$

PROBLEM 16.51

FIND:  
 (a)  $a_A$   
 (b)  $a_B$

PROBLEM 16.52

SHOW THAT FOR  
 $360^\circ$  ROTATION DISK  
 WILL MOVE DISTANCE  $\pi r$ .



$$\text{DISK: } \bar{I} = \frac{1}{2} mr^2$$

$$\begin{aligned} \sum F_x &= \sum (F_x)_\text{eff}: \\ P &= m\bar{a} \\ \bar{a} &= \frac{P}{m} \rightarrow \end{aligned}$$

$$\begin{aligned} +\sum M_G &= \sum (M_G)_\text{eff} \\ Pr &= \bar{I} \alpha \\ Pr &= \frac{1}{2} mr^2 \alpha \\ \alpha &= \frac{2P}{mr} \rightarrow \end{aligned}$$

PROBLEM 16.51

$$\text{(a)} \quad \pm a_A = \bar{a} + r\alpha = \frac{P}{m} + r \cdot \frac{2P}{mr} = 3 \frac{P}{m}$$

$$a_A = 3 \frac{3 \text{ N}}{2.4 \text{ kg}} = 3.75 \text{ m/s}^2$$

$$a_A = 3.75 \text{ m/s}^2 \rightarrow$$

$$\text{(b)} \quad \pm a_B = \bar{a} - r\alpha = \frac{P}{m} - r \cdot \frac{2P}{mr} = -\frac{P}{m}$$

$$a_B = -\frac{3 \text{ N}}{2.4 \text{ kg}} = -1.25 \text{ m/s}^2$$

$$a_B = 1.25 \text{ m/s}^2 \leftarrow$$

PROBLEM 16.52

LET  $t_1$  = TIME REQUIRED FOR  $360^\circ$  ROTATION

$$\theta = \frac{1}{2} \alpha t_1^2; \quad 2\pi \text{ rad} = \frac{1}{2} \left( \frac{2P}{mr} \right) t_1^2$$

$$t_1^2 = \frac{2\pi mr}{P}$$

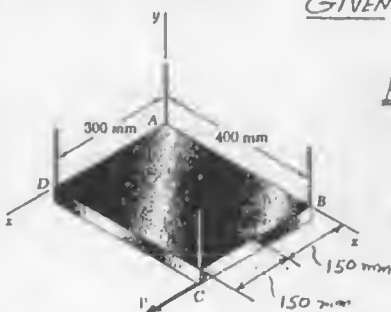
LET  $x_1$  = DISTANCE  $G$  MOVES  
 DURING  $360^\circ$  ROTATION

$$x_1 = \frac{1}{2} \bar{a} t_1^2 = \frac{1}{2} \frac{P}{m} \left( \frac{2\pi mr}{P} \right)$$

$$x_1 = \pi r \quad \text{Q.E.D.}$$



# 16.53 and 16.54



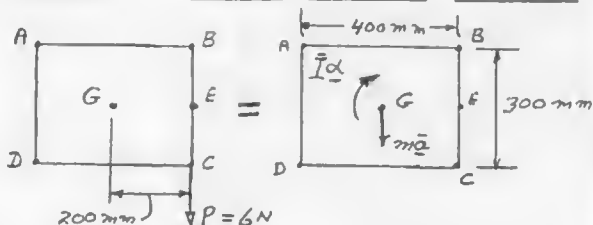
GIVEN:  $m = 5 \text{ kg}$   
 $P = 6 \text{ N}$

## PROBLEM 16.53

FIND: (a)  $\alpha_E$   
(b)  $\alpha_B$

## PROBLEM 16.54

FIND: (a) POINT OF ZERO ACCELERATION  
(b)  $\alpha$



$$\bar{I} = \frac{1}{12} m (b^2 + h^2) = \frac{1}{12} (5 \text{ kg}) [(0.4 \text{ m})^2 + (0.3 \text{ m})^2] = 0.10417 \text{ kg} \cdot \text{m}^2$$

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: P = m\bar{a}$$

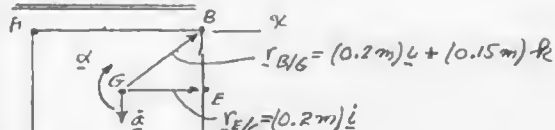
$$6 \text{ N} = (5 \text{ kg}) \bar{a} \quad \bar{a} = + (1.2 \text{ m/s}^2) \hat{j}$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: P(0.2 \text{ m}) = \bar{I} \alpha$$

$$(6 \text{ N})(0.2 \text{ m}) = (0.10417 \text{ kg} \cdot \text{m}^2) \alpha$$

$$\alpha = - (11.52 \text{ rad/s}^2) \hat{j}$$

## PROBLEM 16.53:



$$(a) \alpha_E = \bar{\alpha} + \alpha \times r_{E/G} = + (1.2 \text{ m/s}^2) \hat{j} - (11.52 \text{ rad/s}^2) \hat{j} \times (0.2 \text{ m}) \hat{i}$$

$$= + (1.2 \text{ m/s}^2) \hat{j} + (2.304 \text{ m/s}^2) \hat{j}$$

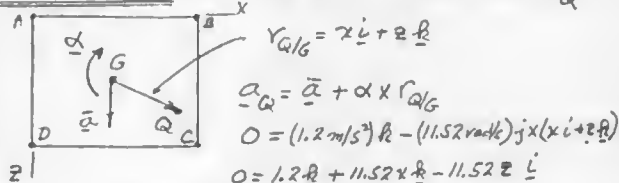
$$\alpha_E = (3.50 \text{ m/s}^2) \hat{j}$$

$$(b) \alpha_B = \bar{\alpha} + \alpha \times r_{B/G} = + (1.2 \text{ m/s}^2) \hat{j} - (11.52 \text{ rad/s}^2) \hat{j} \times [(0.2 \text{ m}) \hat{i} + (0.15 \text{ m}) \hat{j}]$$

$$= + (1.2 \text{ m/s}^2) \hat{j} + [2.304 \text{ m/s}^2] \hat{j} + (1.728 \text{ m/s}^2) \hat{i}$$

$$\alpha_B = (1.728 \text{ m/s}^2) \hat{i} + (3.5 \text{ m/s}^2) \hat{j}$$

## PROBLEM 16.54: FOR POINT Q WE SEEK $\alpha_Q = 0$



$$r_{Q/G} = x \hat{i} + z \hat{j}$$

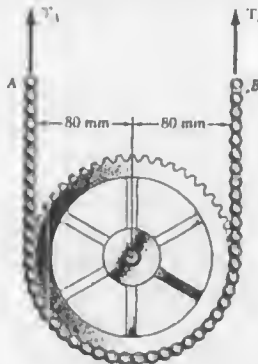
$$\alpha_Q = \bar{\alpha} + \alpha \times r_{Q/G}$$

$$0 = (1.2 \text{ m/s}^2) \hat{j} - (11.52 \text{ rad/s}^2) \hat{j} \times (x \hat{i} + z \hat{j})$$

$$0 = 1.2 \hat{j} + 11.52 x \hat{j} - 11.52 z \hat{i}$$

$x$  COMPONENTS:  $0 = 11.52 x$ ;  $x = 0$   
 $z$  COMPONENTS:  $0 = 1.2 + 11.52 z$ ;  $z = -0.1042$   
POINT OF ZERO ACCELERATION IS 104.2 mm TO LEFT OF G

# 16.55 and 16.56



GIVEN:  $m = 3 \text{ kg}$   
 $r = 70 \text{ mm}$

FIND:  $\alpha_A$  AND  $\alpha_B$

## PROBLEM 16.55

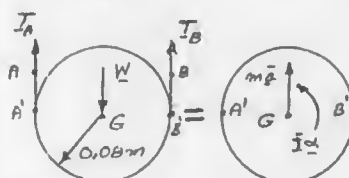
FOR  $T_A = 14 \text{ N}$   
 $T_B = 18 \text{ N}$

## PROBLEM 16.56

FOR  $T_A = 14 \text{ N}$   
 $T_B = 12 \text{ N}$

$$\bar{I} = m \bar{r}^2 = (3 \text{ kg})(0.07 \text{ m})^2 = 147 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$r = 0.08 \text{ m}$$



$$W = mg$$

$$= (3 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 29.43 \text{ N}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: T_A + T_B - W = m\bar{a}$$

$$T_A + T_B - 29.43 \text{ N} = (3 \text{ kg}) \bar{a}$$

$$+\uparrow \bar{a} = \frac{1}{3} (T_A + T_B - 29.43) \quad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: T_B(0.08 \text{ m}) - T_A(0.08 \text{ m}) = \bar{I} \alpha$$

$$(T_B - T_A)(0.08 \text{ m}) = (147 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \alpha$$

$$+\circlearrowleft \alpha = 5.442 (T_B - T_A) \quad (2)$$

## PROBLEM 16.55

$T_A = 14 \text{ N}$ ,  $T_B = 18 \text{ N}$

$$\text{EQ. (1): } +\uparrow \bar{a} = \frac{1}{3} (14 + 18 - 29.43) = 0.8567 \text{ m/s}^2$$

$$\text{EQ. (2): } +\circlearrowleft \alpha = 5.442 (18 - 14) = 21.769 \text{ rad/s}^2$$

$$+\uparrow \alpha_A = (\alpha_A)_t = \bar{a} + r\alpha = 0.8567 + (0.08)(21.769) = -0.885 \text{ m/s}^2$$

$$\alpha_A = 0.885 \text{ m/s}^2 \downarrow$$

$$+\uparrow \alpha_B = (\alpha_B)_t = \bar{a} + r\alpha = 0.8567 + (0.08)(21.769) = +2.60 \text{ m/s}^2$$

$$\alpha_B = 2.60 \text{ m/s}^2 \uparrow$$

## PROBLEM 16.56

$T_A = 14 \text{ N}$ ,  $T_B = 12 \text{ N}$

$$\text{EQ. (1): } +\uparrow \bar{a} = \frac{1}{3} (14 + 12 - 29.43) = -1.1433 \text{ m/s}^2$$

$$\bar{a} = 1.1433 \text{ m/s}^2 \downarrow$$

$$\text{EQ. (2): } +\circlearrowleft \alpha = 5.442 (12 - 14) = -10.884 \text{ rad/s}^2$$

$$\alpha = 10.884 \text{ rad/s}^2 \downarrow$$

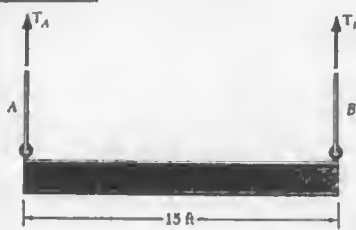
$$+\uparrow \alpha_A = (\alpha_A)_t = \bar{a} + r\alpha = -1.1433 + (0.08)(10.884) = -0.273 \text{ m/s}^2$$

$$\alpha_A = 0.273 \text{ m/s}^2 \downarrow$$

$$+\uparrow \alpha_B = (\alpha_B)_t = \bar{a} + r\alpha = -1.1433 + (0.08)(10.884) = -2.01 \text{ m/s}^2$$

$$\alpha_B = 2.01 \text{ m/s}^2 \downarrow$$

16.57

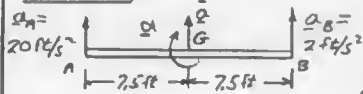
GIVEN:  $W = 500 \text{ lb}$ 

$$a_A = 20 \text{ ft/s}^2 \uparrow$$

$$a_B = 2 \text{ ft/s}^2 \uparrow$$

FIND:  $T_A$  AND  $T_B$ 

KINEMATICS:



$$a_B = a_A + (15 \text{ ft})\alpha$$

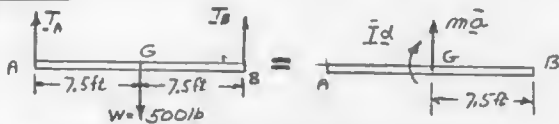
$$2 \uparrow = 20 \uparrow + 15 \alpha$$

$$\alpha = 1.2 \text{ rad/s}^2$$

$$\bar{a} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(2 + 20)$$

$$\bar{a} = 11 \text{ ft/s}^2 \uparrow$$

KINETICS:



$$\bar{I} = \frac{1}{2}mL^2 = \frac{1}{2} \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2} (15 \text{ ft})^2 = 291.15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: T_A(15 \text{ ft}) - W(7.5 \text{ ft}) = m\bar{a}(7.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(15 \text{ ft}) - (500 \text{ lb})(7.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2} (11 \text{ ft/s}^2)(7.5 \text{ ft}) + (291.15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(1.2 \text{ rad/s}^2)$$

$$15T_A - 3750 = 1281 + 349.3$$

$$T_A = 358.716$$

$$T_A = 359 \text{ lb}$$

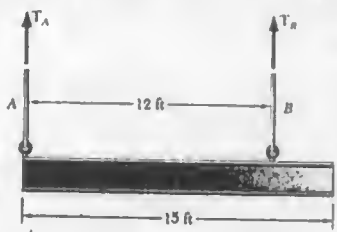
$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: T_A + T_B - W = m\bar{a}$$

$$358.716 + T_B - 500 \text{ lb} = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2} (11 \text{ ft/s}^2)$$

$$T_B = 312.216$$

$$T_B = 312 \text{ lb}$$

16.58



GIVEN:

$$W = 500 \text{ lb}$$

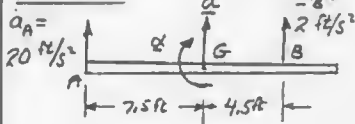
$$a_A = 20 \text{ ft/s}^2 \uparrow$$

$$a_B = 2 \text{ ft/s}^2 \uparrow$$

FIND:

 $T_A$  AND  $T_B$ 

KINEMATICS:



$$a_B = a_A + 12\alpha$$

$$2 \uparrow = 20 \uparrow + 12\alpha$$

$$\alpha = 1.5 \text{ rad/s}^2$$

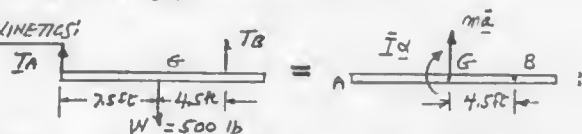
$$\bar{a} = a_A + 7.5\alpha$$

$$= 20 \uparrow + (7.5)(1.5) \downarrow$$

$$\bar{a} = 8.75 \text{ ft/s}^2 \uparrow$$

$$\bar{I} = \frac{1}{2}mL^2 = \frac{1}{2} \frac{500}{32.2} (15)^2 = 291.15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

KINETICS:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: T_A(12 \text{ ft}) + W(4.5 \text{ ft}) = m\bar{a}(4.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(12 \text{ ft}) - (500 \text{ lb})(4.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2} (8.75 \text{ ft/s}^2)(4.5 \text{ ft}) + (291.15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(1.5 \text{ rad/s}^2)$$

$$12T_A - 2250 = 611.4 + 437$$

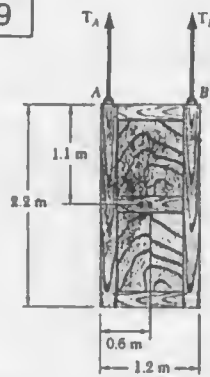
$$T_A = 275 \text{ lb}$$

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: T_A + T_B - W = m\bar{a}$$

$$275 \text{ lb} + T_B - 500 = \frac{500}{32.2} (8.75)$$

$$T_B = 361 \text{ lb}$$

16.59



GIVEN:

$$(a_A)_y = 9 \text{ m/s}^2 \uparrow$$

$$(a_B)_y = 3 \text{ m/s}^2 \uparrow$$

$$m = 180 \text{ kg}$$

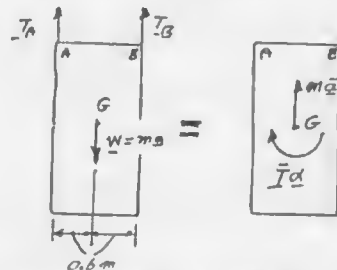
FIND:

 $T_A$  AND  $T_B$ 

KINETICS:

$$m = 180 \text{ kg}$$

$$\bar{I} = \frac{1}{2}m(b^2 + c^2) = \frac{1}{2}(180 \text{ kg})(2.2^2 + 1.2^2) = 94.2 \text{ kg} \cdot \text{m}^2$$



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: T_A + T_B - mg = m\bar{a}$$

$$T_A + T_B - (180)(9.81) = (180)\bar{a}$$

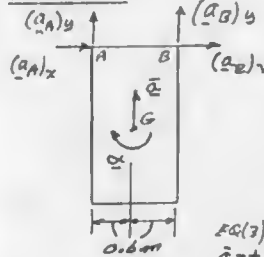
$$T_A + T_B = 1765.8 + 180\bar{a} \quad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: T_A(0.6 \text{ m}) - T_B(0.6 \text{ m}) = \bar{I}\alpha$$

$$0.6(T_A - T_B) = 94.2\alpha$$

$$T_A - T_B = 15.7\alpha \quad (2)$$

KINEMATICS:



$$(a = \bar{a} + r\alpha)$$

$$(a_A)_y = \bar{a} + 0.6\alpha$$

$$9 \text{ m/s}^2 = \bar{a} + 0.6\alpha \quad (3)$$

$$(a_B)_y = \bar{a} - 0.6\alpha$$

$$3 \text{ m/s}^2 = \bar{a} - 0.6\alpha \quad (4)$$

$$\text{Eq(3)} + \text{Eq(4)}: 12 = 2\bar{a}$$

$$\bar{a} = 6; \quad \bar{a} = 6 \text{ m/s}^2 \uparrow$$

$$\text{Eq(3)}: 9 = 6 + 0.6\alpha$$

$$\alpha = +5; \quad \alpha = 5 \text{ rad/s}^2 \uparrow$$

$$\text{Eq(1)}: T_A + T_B = 1765.8 + 180(6)$$

$$T_A + T_B = 2845.8 \quad (5)$$

$$\text{Eq(2)}: T_A - T_B = (15.7)(5 \text{ rad/s}^2)$$

$$T_A - T_B = +78.5 \quad (6)$$

$$\text{Eq(5)} + \text{Eq(6)}: 2T_A = 3630.3$$

$$T_A = 1815.4 \text{ N}$$

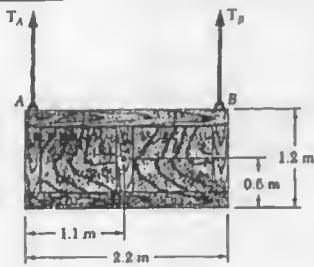
$$T_A = 1815 \text{ N}$$

$$\text{Eq(1)}: T_A + 1815.4 = 2845.8$$

$$T_B = 1030.4 \text{ N}$$

$$T_B = 1030 \text{ N}$$

16.60

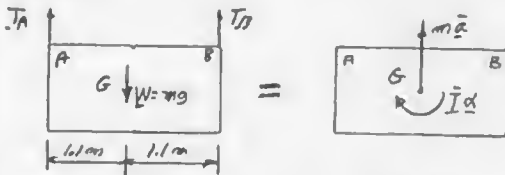


GIVEN:  
 $(a_A)_y = 9 \text{ m/s}^2 \uparrow$   
 $(a_B)_y = 3 \text{ m/s}^2 \uparrow$   
 $m = 180 \text{ kg}$

FIND:  
 $T_A$  AND  $T_B$

$$\bar{I} = \frac{1}{12} m(a^2 + b^2) = \frac{1}{12} (180 \text{ kg})(2.2^2 + 1.2^2) = 94.2 \text{ kg} \cdot \text{m}^2$$

KINETICS:



$$\begin{aligned} \uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T_A + T_B - mg &= m\bar{a} \\ T_A + T_B + (180 \text{ kg})(9.81 \text{ m/s}^2) &= (180 \text{ kg})\bar{a} \\ T_A + T_B &= 1765.8 + 180\bar{a} \quad (1) \end{aligned}$$

$$\begin{aligned} \curvearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad T_A(1.1 \text{ m}) - T_B(1.1 \text{ m}) &= \bar{I}\alpha \\ 1.1(T_A - T_B) &= (94.2 \text{ kg} \cdot \text{m}^2)\alpha \\ T_A - T_B &= 85.636\alpha \quad (2) \end{aligned}$$

KINEMATICS

$$\begin{aligned} (a_A)_y &= \bar{a} + r\alpha \\ (a_B)_y &= \bar{a} + 1.1\alpha \\ 9 \text{ m/s}^2 &= \bar{a} + 1.1\alpha \quad (3) \\ (a_B)_y &= \bar{a} - 1.1\alpha \\ 3 \text{ m/s}^2 &= \bar{a} - 1.1\alpha \quad (4) \end{aligned}$$

$$\begin{aligned} EQ(3) + EQ(4): \quad 12 &= 2\bar{a} \\ \bar{a} &= +6 \text{ m/s}^2 \quad \underline{\bar{a}} = 6 \text{ m/s}^2 \uparrow \end{aligned}$$

$$\begin{aligned} EQ(3) - EQ(4): \quad 6 &= 2.2\alpha \\ \alpha &= 2.727 \text{ rad/s}^2 \quad \underline{\alpha} = 2.73 \text{ rad/s}^2 \curvearrowright \end{aligned}$$

$$\begin{aligned} EQ(1): \quad T_A + T_B &= 1765.8 + 180(6) \\ T_A + T_B &= 2845.8 \quad (5) \end{aligned}$$

$$\begin{aligned} EQ(2): \quad T_A - T_B &= 85.636(2.727) \\ T_A - T_B &= 233.5 \quad (6) \end{aligned}$$

$$\begin{aligned} EQ(5) + EQ(6): \quad 2T_A &= 3079.3 \\ T_A &= 1539.7 \text{ N} \\ T_A &= 1540 \text{ N} \quad \blacktriangleleft \end{aligned}$$

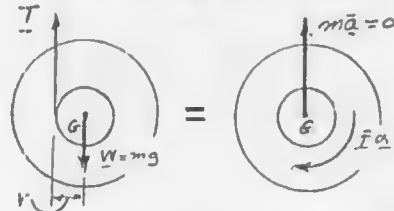
$$\begin{aligned} EQ(5) - EQ(6): \quad 2T_B &= 2612.3 \\ T_B &= 1306.2 \text{ N} \\ T_B &= 1306 \text{ N} \quad \blacktriangleleft \end{aligned}$$

16.61



GIVEN:  
 INNER AXLE:  $r$   
 CENTRICAL RADIUS OF GYRATION:  $\bar{r}$   
 MASS:  $m$

FIND:  $T$  FOR  $\bar{a} = 0$



$$\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T - mg = 0 \quad ; \quad T = mg$$

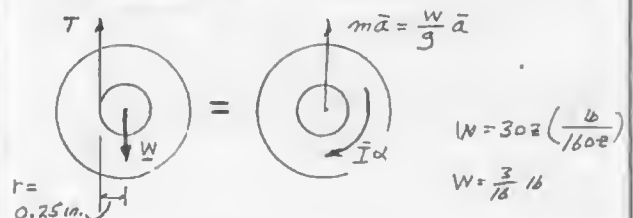
$$\begin{aligned} \curvearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Tr &= \bar{I}\alpha \\ mgr &= m\bar{r}^2\alpha \\ \alpha &= \frac{rg}{\bar{r}^2} \quad \alpha = \frac{rg}{\bar{r}^2} \quad \blacktriangleleft \end{aligned}$$

16.62



GIVEN:  $W = 30 \text{ lb}$   
 $\bar{r} = 1.25 \text{ in.}$   
 RADIUS OF AXLE:  $0.25 \text{ in.}$   
 $\bar{a} = 3 \text{ ft/s}^2 \uparrow$

FIND: (a)  $T$   
 (b)  $\alpha$

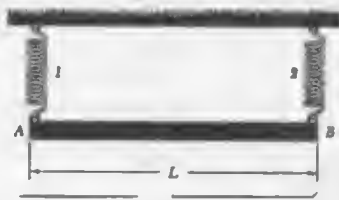


$$\begin{aligned} \uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T - W &= \frac{W}{g}\bar{a} \\ T - \frac{3}{16} \text{ lb} &= \left(\frac{3}{16} \text{ lb}\right) \frac{3 \text{ ft/s}^2}{32.2 \text{ ft/s}^2} \end{aligned}$$

$$T = 0.205 \text{ lb} \quad T = 0.205 \text{ lb} \quad \blacktriangleleft$$

$$\begin{aligned} \curvearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Tr &= \bar{I}\alpha \\ (0.205 \text{ lb}) \left(\frac{0.25}{12} \text{ ft}\right) &= m\bar{r}^2\alpha \\ 4.271 \times 10^{-3} \text{ lb} \cdot \text{ft} &= \frac{3/16 \text{ lb}}{32.2 \text{ ft/s}^2} \left(\frac{1.25}{12} \text{ ft}\right)^2 \alpha \\ \alpha &= 67.6 \text{ rad/s}^2 \quad \underline{\alpha} = 67.6 \text{ rad/s}^2 \quad \blacktriangleleft \end{aligned}$$

16.63



JUST AFTER  
SPRING 2  
BREAKS,  
FIND: (a)  $\alpha$   
(b)  $a_A$ , (c)  $a_B$

STATICS:  $T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

(a)  $T_1 = \frac{1}{2}mg$

$\sum M_G = \sum (M_G)_e$ :  $T(L/2) = \bar{I}\alpha$   
 $\frac{1}{2}mg(L/2) = \frac{1}{12}mL^2\alpha$   
 $\alpha = 3g/L$   $\alpha = \frac{3g}{L} \curvearrowright$

$\sum F_y = \sum (F_y)_e$ :  $W - T_1 = m\bar{a}$   
 $mg - \frac{1}{2}mg = m\bar{a}$   
 $\bar{a} = \frac{1}{2}g$   $\bar{a} = \frac{1}{2}g \downarrow$

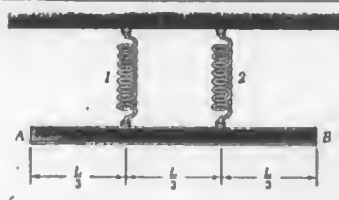
(b) ACCELERATION OF A:

$a_A = \bar{a}_G + a_{A/G}$   
 $\downarrow a_A = \frac{1}{2}g - \frac{1}{2}\alpha$   
 $= \frac{1}{2}g - \frac{1}{2}(\frac{3g}{L})$   
 $a_A = -g$ ;  $a_A = g \uparrow$

(c) ACCELERATION OF B:

$a_B = \bar{a}_G + a_{B/G}$   
 $\downarrow a_B = \bar{a} + \frac{1}{2}\alpha = \frac{1}{2}g + \frac{1}{2}(\frac{3g}{L}) = +2g$ ;  $a_B = 2g \downarrow$

16.64



JUST AFTER  
SPRING 2  
BREAKS,  
FIND: (a)  $\alpha$   
(b)  $a_A$ , (c)  $a_B$

STATICS:  $T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

(a)  $T_1 = \frac{1}{2}mg$

$\sum M_G = \sum (M_G)_e$ :  $T(L/3) = \bar{I}\alpha$   
 $\frac{1}{2}mg(L/3) = \frac{1}{12}mL^2\alpha$   
 $\alpha = g/L$   $\alpha = \frac{g}{L} \curvearrowright$

$\sum F_y = \sum (F_y)_e$ :  $W - T_1 = m\bar{a}$   
 $mg - \frac{1}{2}mg = m\bar{a}$   
 $\bar{a} = \frac{1}{2}g$   $\bar{a} = \frac{1}{2}g \downarrow$

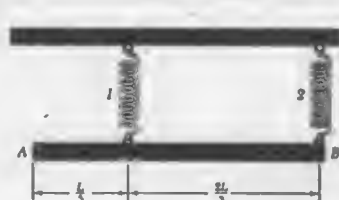
(b) ACCELERATION OF A:

$a_A = \bar{a}_G + a_{A/G}$   
 $\downarrow a_A = \frac{1}{2}g - \frac{1}{3}\alpha$   
 $= \frac{1}{2}g - \frac{1}{3}(\frac{g}{L}) = 0$   
 $a_A = 0$

(c) ACCELERATION OF B:

$a_B = \bar{a}_G + a_{B/G}$   
 $\downarrow a_B = \bar{a} + \frac{1}{3}\alpha = \frac{1}{2}g + \frac{1}{3}(\frac{g}{L}) = +g$   $a_B = g \downarrow$

16.65



JUST AFTER SPRING 2 BREAKS,  
FIND: (a)  $\alpha$ , (b)  $a_A$ , (c)  $a_B$

STATICS:

$\sum M_B = 0$   
 $T_1(\frac{2L}{3}) - mg(\frac{L}{3}) = 0$   
 $T_1 = \frac{3}{4}mg$

(a) KINETICS

$T_1 = \frac{3}{4}mg$

$\sum M_G = \sum (M_G)_e$ :  $T(L/3) = \bar{I}\alpha$   
 $\frac{3}{4}mg(L/3) = \frac{1}{12}mL^2\alpha$   
 $\alpha = \frac{3g}{2L}$   $\alpha = \frac{3g}{2L} \curvearrowright$

$\sum F_y = \sum (F_y)_e$ :  $W - T_1 = m\bar{a}$   
 $mg - \frac{3}{4}mg = m\bar{a}$   
 $\bar{a} = \frac{1}{4}g$   $\bar{a} = \frac{1}{4}g \downarrow$

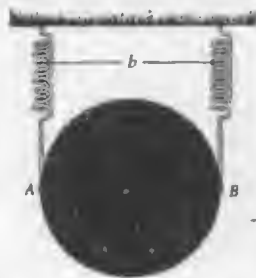
(b) ACCELERATION OF A:

$a_A = \bar{a}_G + a_{A/G}$   
 $\downarrow a_A = \frac{1}{4}g - \frac{1}{3}\alpha$   
 $= \frac{1}{4}g - \frac{1}{3}(\frac{3g}{2L})$   
 $a_A = -\frac{1}{2}g$ ;  $a_A = \frac{1}{2}g \uparrow$

(c) ACCELERATION OF B:

$a_B = \bar{a}_G + a_{B/G}$   
 $\downarrow a_B = \frac{1}{4}g + \frac{1}{3}(\alpha)$   
 $= \frac{1}{4}g + \frac{1}{3}(\frac{3g}{2L})$   
 $a_B = +g$   $a_B = g \downarrow$

16.66



JUST AFTER  
SPRING 2  
BREAKS,

FIND:

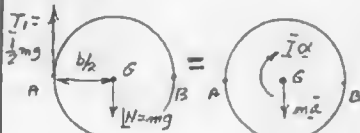
(a)  $a_A$

(b)  $a_B$

$$\bar{I} = \frac{1}{2} m \left( \frac{b}{2} \right)^2 = \frac{1}{8} m b^2$$

STATICS:  $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} m g$

KINETICS:



$\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: W - T_1 = m \bar{a}$   
 $m g - \frac{1}{2} m g = m \bar{a} \quad \bar{a} = \frac{1}{2} g \downarrow$

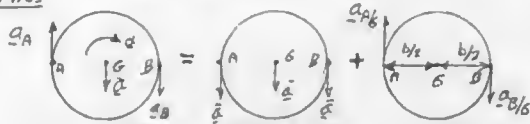
$\downarrow \Sigma M_G = \Sigma (M_G)_{eff}$

$T_1 \frac{b}{2} = \bar{I} \alpha$

$\frac{1}{2} m g \left( \frac{b}{2} \right) = \frac{1}{8} m b^2 \alpha$

$\alpha = 2 g / b \downarrow$

KINEMATICS:

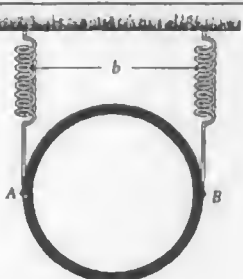


PLANE MOTION = TRANSLATION + ROTATION

(a)  $a_A = a_G + a_{A/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \uparrow = \frac{1}{2} g \downarrow + \frac{b}{2} \left( \frac{2g}{b} \right) \uparrow; a_A = \frac{1}{2} g \uparrow$

(b)  $a_B = a_G + a_{B/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \downarrow = \frac{1}{2} g \downarrow + \frac{b}{2} \left( \frac{2g}{b} \right) \downarrow; a_B = \frac{3}{2} g \downarrow$

16.67



JUST AFTER  
SPRING 2  
BREAKS,

FIND:

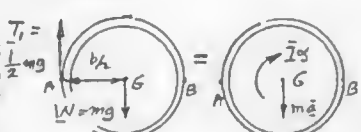
(a)  $a_A$

(b)  $a_B$

$$\bar{I} = m \left( \frac{b}{2} \right)^2 = \frac{1}{4} m b^2$$

STATICS:  $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} m g$

KINETICS:



$\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: W - T = m \bar{a}$   
 $m g - \frac{1}{2} m g = m \bar{a} \quad \bar{a} = \frac{1}{2} g \downarrow$

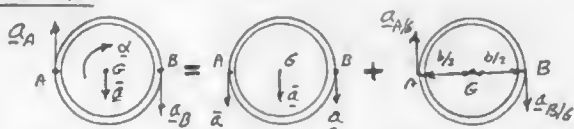
$\downarrow \Sigma M_G = \Sigma (M_G)_{eff}$

$T_1 \left( \frac{b}{2} \right) = \bar{I} \alpha$

$\frac{1}{2} m g \left( \frac{b}{2} \right) = \frac{1}{4} m b^2 \alpha$

$\alpha = g / b \downarrow$

KINEMATICS:

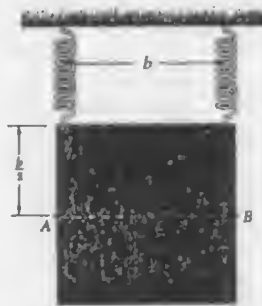


PLANE MOTION = TRANSLATION + ROTATION

(a)  $a_A = a_G + a_{A/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \uparrow = \frac{1}{2} g \downarrow + \frac{b}{2} \left( \frac{g}{b} \right) \uparrow; a_A = 0$

(b)  $a_B = a_G + a_{B/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \downarrow = \frac{1}{2} g \downarrow + \frac{b}{2} \left( \frac{g}{b} \right) \downarrow; a_B = g \downarrow$

16.68



JUST AFTER  
SPRING 2  
BREAKS

FIND: (a)  $a_A$

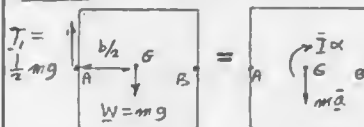
(b)  $a_B$

$$\bar{I} = \frac{1}{12} m (b^2 + b^2)$$

$$\bar{I} = \frac{1}{6} m b^2$$

STATICS:  $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} m g$

KINETICS:



$\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: W - T_1 = m \bar{a}$   
 $m g - \frac{1}{2} m g = m \bar{a} \quad \bar{a} = \frac{1}{2} g \downarrow$

$\downarrow \Sigma M_G = \Sigma (M_G)_{eff}$

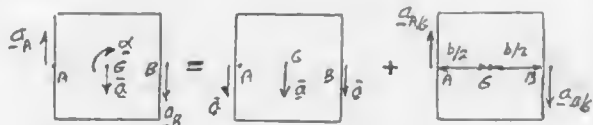
$T_1 \frac{b}{2} = \bar{I} \alpha$

$\frac{1}{2} m g \left( \frac{b}{2} \right) = \frac{1}{6} m b^2 \alpha$

$\alpha = \frac{3g}{2b} \downarrow$

$\bar{a} = \frac{1}{2} g \downarrow$

KINEMATICS:



PLANE MOTION = TRANSLATION + ROTATION

(a)  $a_A = a_G + a_{A/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \uparrow$

$a_A = \frac{g}{2} \downarrow + \frac{b}{2} \left( \frac{3g}{2b} \right) \uparrow = \frac{g}{4} \uparrow$

$a_A = \frac{1}{4} g \uparrow$

(b)  $a_B = a_G + a_{B/G} = \bar{a} \downarrow + \frac{b}{2} \alpha \downarrow$

$a_B = \frac{g}{2} \downarrow + \frac{b}{2} \left( \frac{3g}{2b} \right) \downarrow = \frac{5g}{4} \downarrow$

$a_B = \frac{5}{4} g \downarrow$

# 16.69 and 16.70

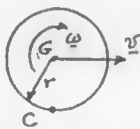


GIVEN:  $\vec{v}_0 = 15 \text{ ft/s} \rightarrow$   
 $v = 4 \text{ in.}, \mu_k = 0.10$   
 PROBLEM 16.69:  $\omega_0 = 9 \text{ rad/s}$   
 PROBLEM 16.70:  $\omega_0 = 18 \text{ rad/s}$   
 FIND: (a)  $t$ , WHEN ROLLING STARTS,  
 (b)  $v$  AT  $t_1$ ,  
 (c) DISTANCE TRAVELED AT  $t_1$

## KINETICS:

$$\begin{aligned} \sum \vec{F}_x &= \sum (F_x)_{\text{eff}} \\ -\mu_k mg &= m\vec{a} \\ \vec{a} &= -\mu_k g \rightarrow \\ +\sum \vec{M}_C &= \sum (M_C)_{\text{eff}}: Fr = I\alpha \\ (\mu_k mg)r &= \frac{2}{5}mr^2\alpha \\ \alpha &= \frac{5}{2}\frac{\mu_k g}{r} \end{aligned}$$

## KINEMATICS:



WHEN SPHERE ROLLS, INSTANT  
 CENTRAL OF ROTATION IS AT C  
 AND WHEN  $t = t_1$ ,  $v = r\omega$  (1)

$$\begin{aligned} \vec{v} &= \vec{v}_0 - \vec{a}t = \vec{v}_0 - \mu_k g t \\ \omega &= -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t \end{aligned} \quad (2)$$

WHEN  $t = t_1$ :  
 EQ(1):  $v = r\omega$

$$\begin{aligned} \vec{v}_0 - \mu_k g t_1 &= (-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t_1)r \\ \vec{v}_0 - \mu_k g t_1 &= -\omega_0 r + \frac{5}{2}\mu_k g t_1 \\ t_1 &= \frac{2(\vec{v}_0 + r\omega_0)}{7\mu_k g} \end{aligned} \quad (3)$$

PROBLEM 16.69:  $\vec{v}_0 = 15 \text{ ft/s}$ ,  $\omega_0 = 9 \text{ rad/s}$ ,  $r = 4 \text{ in.} = \frac{1}{3} \text{ ft}$

$$(a) \quad t_1 = \frac{2(15 + \frac{1}{3}(9))}{0.1(32.2)} = 1.59725 \quad t_1 = 1.5975 \quad \blacktriangleleft$$

$$(b) \text{ EQ(2): } \vec{v}_1 = \vec{v}_0 - \mu_k g t_1 = 15 - 0.1(32.2)(1.59725) \\ \vec{v}_1 = 15 - 5.1429 = 9.8571 \text{ ft/s} \\ \vec{v}_1 = 9.86 \text{ ft/s} \rightarrow \quad \blacktriangleleft$$

$$(c) \quad \vec{a} = -\mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \leftarrow \\ \pm s_1 = \vec{v}_0 t_1 - \frac{1}{2}\vec{a}t_1^2 \\ = (15 \text{ ft/s})(1.59725) - \frac{1}{2}(3.22 \text{ ft/s}^2)(1.59725)^2 \\ = 23.96 - 4.11 = 19.85 \text{ ft} \\ \underline{s_1 = 19.85 \text{ ft}} \rightarrow \quad \blacktriangleleft$$

PROBLEM 16.70:  $\vec{v}_0 = 15 \text{ ft/s}$ ,  $\omega_0 = 18 \text{ rad/s}$ ,  $r = \frac{1}{3} \text{ ft}$

$$(a) \text{ EQ(3): } t_1 = \frac{2(15 + \frac{1}{3}(18))}{0.1(32.2)} = 1.86345 \\ t_1 = 1.8635 \quad \blacktriangleleft$$

$$(b) \text{ EQ(2): } \vec{v}_1 = \vec{v}_0 - \mu_k g t_1 = 15 - 0.1(32.2)(1.8634) \\ \vec{v}_1 = 15 - 6.000 = 9 \text{ ft/s} \\ \underline{\vec{v}_1 = 9 \text{ ft/s}} \rightarrow \quad \blacktriangleleft$$

$$(c) \quad \vec{a} = -\mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \leftarrow \\ \pm s_1 = \vec{v}_0 t_1 - \frac{1}{2}\vec{a}t_1^2 \\ = (15 \text{ ft/s})(1.86345) - \frac{1}{2}(3.22 \text{ ft/s}^2)(1.86345)^2 \\ = 27.95 - 5.59 = 22.36 \text{ ft} \\ \underline{\vec{v}_1 = 22.4 \text{ ft}} \rightarrow \quad \blacktriangleleft$$

# 16.71 and 16.72



GIVEN:  
 $m = \text{MASS}$   
 $r = \text{RADIUS}$   
 $\mu_k = \text{COEFF. OF KINETIC FRICTION}$

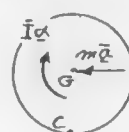
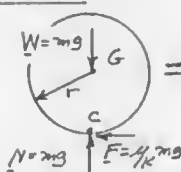
## SPHERE PROBLEM 16.71

## HOOP PROBLEM 16.72

FIND: FOR EACH PROBLEM

- (a)  $\omega_0$  SO FINAL VELOCITY IS ZERO  
 (b) TIME  $t$ , WHEN VELOCITY BECOMES ZERO  
 (c) DISTANCE  $s$ , MOVED BEFORE  $v$  BECOMES ZERO

## KINETICS:



$$\begin{aligned} \sum \vec{F}_x &= \sum (F_x)_{\text{eff}}: F = m\vec{a} \\ \mu_k mg &= m\vec{a} \quad \vec{a} = \mu_k g \leftarrow \\ +\sum \vec{M}_C &= \sum (M_C)_{\text{eff}}: Fr = I\alpha \\ (\mu_k mg)r &= m\vec{a}r^2\alpha \\ \alpha &= \frac{\mu_k g}{r} \end{aligned}$$

## KINEMATICS:

$$\begin{aligned} \pm \vec{v} &= \vec{v}_0 - \vec{a}t \\ \vec{v} &= \vec{v}_0 - \mu_k g t \\ \text{FOR } \vec{v} = 0 \text{ WHEN } t = t_1 \\ 0 &= \vec{v}_0 - \mu_k g t_1; \quad t_1 = \frac{v_0}{\mu_k g} \end{aligned} \quad (1)$$

$$\begin{aligned} +\omega &= \omega_0 - \alpha t \\ \omega &= \omega_0 - \frac{\mu_k g}{r}t \end{aligned}$$

$$\text{FOR } \omega = 0 \text{ WHEN } t = t_1 \\ 0 = \omega_0 - \frac{\mu_k g}{r}t_1; \quad t_1 = \frac{\vec{a}^2}{\mu_k g} \omega_0 \quad (2)$$

SET EQ(1) = EQ(2)

$$\frac{v_0}{\mu_k g} = \frac{\vec{a}^2}{\mu_k g} \omega_0; \quad \omega_0 = \frac{r}{\vec{a}^2} v_0 \quad (3)$$

$$\text{DISTANCE TRAVELED: } s_1 = \vec{v}_0 t_1 - \frac{1}{2}\vec{a}t_1^2 \\ s_1 = \vec{v}_0 \left( \frac{v_0}{\mu_k g} \right) - \frac{1}{2}(\mu_k g) \left( \frac{v_0}{\mu_k g} \right)^2; \quad s_1 = \frac{v_0^2}{2\mu_k g} \quad (4)$$

PROBLEM 16.71 SPHERE  $\vec{a} = \frac{5}{2}\frac{\mu_k g}{r}$

$$(a) \text{ EQ(3): } \omega_0 = \frac{r}{\frac{5}{2}\mu_k g} v_0 = \frac{2}{5}\frac{v_0}{r} \quad \omega_0 = \frac{5}{2}\frac{v_0}{r} \quad \blacktriangleleft$$

$$(b) \text{ EQ(1): } t_1 = \frac{v_0}{\mu_k g} \quad \blacktriangleleft$$

$$(c) \text{ EQ(4): } s_1 = \frac{v_0^2}{2\mu_k g} \quad \blacktriangleleft$$

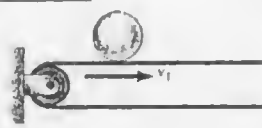
PROBLEM 16.72 HOOP  $\vec{a} = r$

$$(a) \text{ EQ(3): } \omega_0 = \frac{r}{r} v_0 = \frac{v_0}{r} \quad \omega_0 = \frac{v_0}{r} \quad \blacktriangleleft$$

$$(b) \text{ AND (c) SAME AS ABOVE: } t_1 = \frac{v_0}{\mu_k g} \quad \blacktriangleleft$$

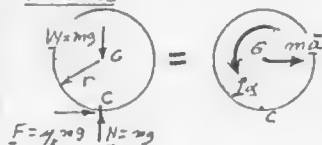
$$s_1 = \frac{v_0^2}{2\mu_k g} \quad \blacktriangleleft$$

## 16.73



GIVEN: SPHERE PLACED ON BELT WITH NO VELOCITY.  
 $r$  = RADIUS,  
 $\mu_k$  = COEF. KINETIC FRICTION  
FIND: (a)  $t_1$  WHEN SPHERE ROLLS  
 (b)  $\vec{v}$  AND  $\omega$  WHEN  $t = t_1$

KINETICS:



$$\begin{aligned} \rightarrow \Sigma F_x = \Sigma (F_x)_{\text{ext}}: F &= m\bar{a} \\ \mu_k mg &= m\bar{a} \\ \bar{a} &= \mu_k g \rightarrow \\ +) \Sigma M_G = \Sigma (M_G)_{\text{ext}}: Fr &= I\alpha \\ (\mu_k mg)r &= \frac{2}{5}mr^2\alpha \\ \alpha &= \frac{5}{2} \frac{\mu_k g}{r} \end{aligned}$$

KINEMATICS:  $\frac{1}{2}\bar{v} = \bar{a}t = \mu_k g t$

$$+ ) \omega = \alpha t = \frac{5}{2} \frac{\mu_k g}{r} t$$

$C$  = POINT OF CONTACT WITH BELT

$$\begin{aligned} \rightarrow v_C &= \bar{v} + \omega r = \mu_k g t + \left( \frac{5}{2} \frac{\mu_k g}{r} t \right) r \\ v_C &= \frac{7}{2} \mu_k g t \end{aligned}$$

(a) WHEN SPHERE STARTS ROLLING ( $t = t_1$ ), WE HAVE

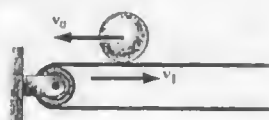
$$v_C = v_1; \quad v_1 = \frac{7}{2} \mu_k g t_1, \quad t_1 = \frac{2}{7} \frac{v_1}{\mu_k g}$$

(b) VELOCITIES WHEN  $t = t_1$

$$\text{EQ(1): } \bar{v} = \mu_k g \left( \frac{2}{7} \frac{v_1}{\mu_k g} \right) \quad \bar{v} = \frac{2}{7} v_1 \rightarrow$$

$$\text{EQ(2): } \omega = \left( \frac{5}{2} \frac{\mu_k g}{r} \right) \left( \frac{2}{7} \frac{v_1}{\mu_k g} \right) \quad \omega = \frac{5}{7} \frac{v_1}{r}$$

## 16.74



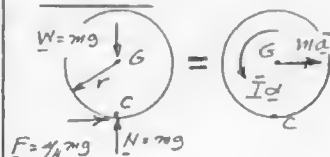
GIVEN: SPHERE WITH  $v_0 \leftarrow$  AND  $\omega_0 = 0$  PLACED ON BELT.  $r$  = RADIUS  
 $\mu_k$  = COEF. KINETIC FRICTION  
FIND: (a)  $v_0$  SO THAT SPHERE WILL HAVE NO

LINEAR VELOCITY AFTER IT STARTS ROLLING

ON BELT, (b)  $t_1$  WHEN SPHERE STARTS ROLLING

(c) DISTANCE SPHERE WILL HAVE MOVED WHEN  $t = t_1$

KINETICS:



$$\begin{aligned} \rightarrow \Sigma F_x = \Sigma (F_x)_{\text{ext}}: F &= m\bar{a} \\ \mu_k mg &= m\bar{a} \\ \bar{a} &= \mu_k g \rightarrow \\ +) \Sigma M_G = \Sigma (M_G)_{\text{ext}}: Fr &= I\alpha \\ (\mu_k mg)r &= \frac{2}{5}mr^2\alpha \\ \alpha &= \frac{5}{2} \frac{\mu_k g}{r} \end{aligned}$$

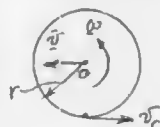
KINEMATICS:

$$\frac{1}{2}\bar{v} = v_0 - \bar{a}t = v_0 - \mu_k g t$$

$$+ ) \omega = \alpha t = \frac{5}{2} \frac{\mu_k g}{r} t$$

(CONTINUED)

## 16.74 continued



$C$  = POINT OF CONTACT WITH BELT

$$\begin{aligned} \rightarrow v_C &= -\bar{v} + r\omega \\ v_C &= -\bar{v} + r \left( \frac{5}{2} \frac{\mu_k g}{r} t \right) \\ v_C &= -\bar{v} + \frac{5}{2} \mu_k g t \end{aligned}$$

(3)

BUT, WHEN  $t = t_1$ ,  $\bar{v} = 0$  AND  $v_C = v_1$

$$\text{EQ(3): } v_1 = \frac{5}{2} \mu_k g t_1; \quad t_1 = \frac{2v_1}{5\mu_k g}$$

$$\text{EQ(1): } \bar{v} = v_0 - \mu_k g t$$

WHEN  $t = t_1$ ,  $\bar{v} = 0$ ,

$$0 = v_0 - \mu_k g \left( \frac{2v_1}{5\mu_k g} \right); \quad v_0 = \frac{2}{5} v_1$$

DISTANCE WHEN  $t = t_1$ :

$$\rightarrow s = v_0 t_1 - \frac{1}{2} \mu_k g t_1^2$$

$$s = \left( \frac{2}{5} v_1 \right) \left( \frac{2v_1}{5\mu_k g} \right) - \frac{1}{2} (\mu_k g) \left( \frac{2v_1}{5\mu_k g} \right)^2$$

$$s = \frac{v_1^2}{\mu_k g} \left( \frac{4}{25} - \frac{2}{25} \right); \quad s = \frac{2}{25} \frac{v_1^2}{\mu_k g}$$

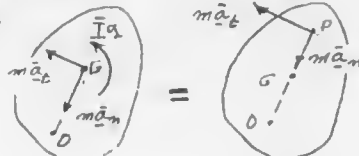
## 16.75



SHOW THAT  $I_G$  (FIG. 16.15) CAN BE ELIMINATED BY ATTACHING

$m\bar{a}_G$  AND  $m\bar{a}_m$  AT POINT P ON  $OG$  WHERE  $GP = \frac{I_G^2}{r^2}$

FIG. 16.15b



$$OG = \bar{r} \quad \bar{a}_G = \bar{r}\alpha$$

WE FIRST OBSERVE THAT THE SUM OF THE VECTORS IS THE SAME IN BOTH FIGURES

TO HAVE THE SAME SUM OF MOMENTS ABOUT G, WE MUST HAVE

$$+ ) \Sigma M_G = \Sigma M_G: \bar{I}\alpha = (m\bar{a}_G)(GP)$$

$$m\bar{r}^2\alpha = m\bar{r}\bar{a}_G(GP)$$

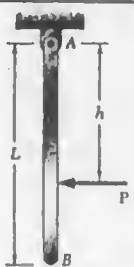
$$GP = \frac{\bar{r}^2}{\bar{r}} \quad (\text{Q.E.D.})$$

NOTE: THE CENTER OF ROTATION AND THE CENTER OF PERCUSSION ARE INTERCHANGEABLE. INDEED, SINCE  $OG = \bar{r}$ , WE MAY WRITE

$$GP = \frac{\bar{r}^2}{GO} \quad \text{OR} \quad GO = \frac{\bar{r}^2}{GP}$$

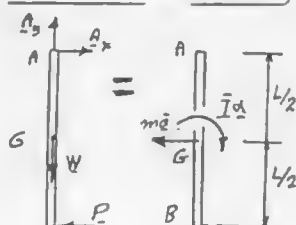
THUS IF POINT P IS SELECTED AS CENTER OF ROTATION, THEN POINT O IS THE CENTER OF PERCUSSION.

16.76



GIVEN:  $L = 36 \text{ in.}$   
 $W = 4 \text{ lb}$   
 $P = 1.5 \text{ lb}$   
 $h = L = 36 \text{ in.}$

FIND: (a)  $\alpha$   
 (b)  $A_x$  and  $A_y$



$$\bar{a} = \frac{L}{2} \alpha \quad \bar{I} = \frac{1}{12} mL^2$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}$$

$$PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

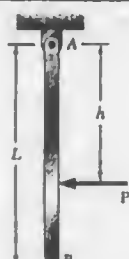
$$= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12} mL^2 \alpha$$

$$PL = \frac{1}{3} mL^2 \alpha$$

(a)  $\alpha = \frac{3P}{mL} = \frac{3(1.5 \text{ lb})}{(4 \text{ lb}/32.2 \text{ ft/s}^2)(3 \text{ ft})} = 12.08 \text{ rad/s}^2$   
 $\alpha = 12.08 \text{ rad/s}^2$

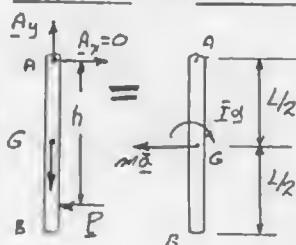
(b)  $+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A_y - W = 0$   
 $A_y = W = 4 \text{ lb} \quad A_y = 4 \text{ lb} \uparrow$   
 $+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: A_x - P = -m\bar{a}$   
 $A_x = P - m(\frac{L}{2}\alpha) = P - m\frac{L}{2}(\frac{3P}{mL}) = -\frac{P}{2}$   
 $A_x = -\frac{P}{2} = -\frac{1.5 \text{ lb}}{2} = -0.75 \text{ lb} \quad A_x = 0.75 \text{ lb} \leftarrow$

16.78



GIVEN:  $L = 36 \text{ in.}$   
 $W = 4 \text{ lb}$   
 $P = 1.5 \text{ lb}$

FIND: (a)  $h$  for  $A_x = 0$ .  
 (b) CORRESPONDING ANGULAR ACCEL.  $\alpha$ .



$$\bar{a} = \frac{L}{2} \alpha \quad \bar{I} = \frac{1}{12} mL^2$$

$$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$P = m\bar{a}$$

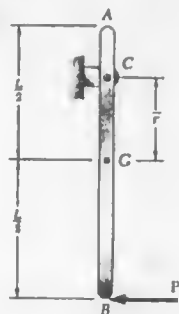
$$P = m(\frac{L}{2}\alpha)$$

$$\alpha = \frac{2P}{mL}$$

$\alpha = \frac{2(1.5 \text{ lb})}{(\frac{4 \text{ lb}}{32.2 \text{ ft/s}^2})(3 \text{ ft})}$   
 $\alpha = 8.05 \text{ rad/s}^2$

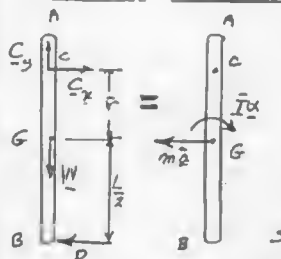
$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}:$   
 $P(h - \frac{L}{2}) = \bar{I}\alpha: P(h - \frac{L}{2}) = \frac{1}{12} mL^2(\frac{2P}{mL}) = \frac{PL}{6}$   
 $(h - \frac{L}{2}) = \frac{L}{6}; h = \frac{L}{2} + \frac{L}{6} = \frac{2}{3}L \quad h = 24 \text{ in.}$

16.77



GIVEN:  $L = 900 \text{ mm}$   
 $m = 4 \text{ kg}$   
 $P = 75 \text{ N}$   
 $\bar{r} = 225 \text{ mm}$

FIND: (a)  $\alpha$   
 (b)  $C_x$  and  $C_y$



$$\bar{a} = \bar{r}\alpha \quad \bar{I} = \frac{1}{12} mL^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}:$$

$$P(\bar{r} + \frac{L}{2}) = (m\bar{a})\bar{r} + \bar{I}\alpha$$

$$= (m\bar{r}\alpha)\bar{r} + \frac{1}{12} mL^2 \alpha$$

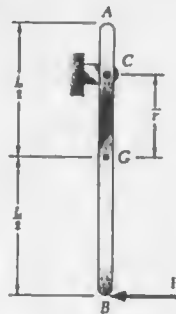
$$P(\bar{r} + \frac{L}{2}) = m(\bar{r}^2 + \frac{1}{12} L^2)\alpha$$

SUBSTITUTE DATA:

(a)  $(75 \text{ N})(0.225 \text{ m} + \frac{0.9 \text{ m}}{2}) = (4 \text{ kg})[(0.225 \text{ m})^2 + \frac{1}{12}(0.9 \text{ m})^2]\alpha$   
 $50.625 = 0.4725\alpha \quad \alpha = 107.14 \text{ rad/s}^2$   
 $\alpha = 107.1 \text{ rad/s}^2$

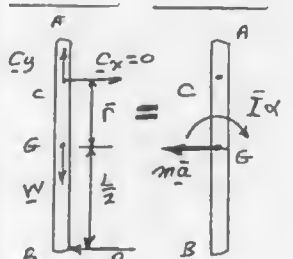
(b)  $+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A_y - W = 0$   
 $A_y = W = mg = (4 \text{ kg})(9.81 \text{ m/s}^2); A_y = 39.2 \text{ N} \uparrow$   
 $+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: A_x - P = -m\bar{a}$   
 $A_x = P - m\bar{a} = P - m(\bar{r}\alpha)$   
 $= 75 \text{ N} - (4 \text{ kg})(0.225 \text{ m})(107.14 \text{ rad/s}^2)$   
 $A_x = 75 \text{ N} - 96.4 \text{ N} = -21.4 \text{ N}; A_x = 21.4 \text{ N} \leftarrow$

16.79



GIVEN:  $L = 900 \text{ mm}$   
 $m = 4 \text{ kg}$   
 $P = 75 \text{ N}$

FIND: (a)  $\bar{r}$  for  $C_x = 0$   
 (b) CORRESPONDING ANGULAR ACCEL.  $\alpha$ .



$$\bar{a} = \bar{r}\alpha \quad \bar{I} = \frac{1}{12} mL^2$$

$$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$P = m\bar{a}$$

$$P = m(\bar{r}\alpha)$$

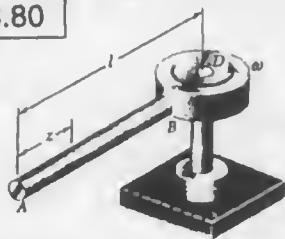
$$\alpha = \frac{P}{m\bar{r}} \quad (1)$$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: P\frac{L}{2} = \bar{I}\alpha$   
 $P\frac{L}{2} = \frac{1}{12} mL^2 \alpha$   
 $P\frac{L}{2} = \frac{1}{12} mL^2 (\frac{P}{m\bar{r}})$   
 $\frac{L}{2} = \frac{L^2}{12\bar{r}}; \bar{r} = \frac{1}{6}L \quad \bar{r} = \frac{900 \text{ mm}}{6}$   
 $\bar{r} = 150 \text{ mm}$

EQ(1):  $\alpha = \frac{P}{m\bar{r}} = \frac{P}{m(\frac{L}{6})} = \frac{6P}{mL}$   
 $\alpha = \frac{6(75 \text{ N})}{(4 \text{ kg})(0.9 \text{ m})} = 125 \text{ rad/s}^2; \alpha = 125 \text{ rad/s}^2$



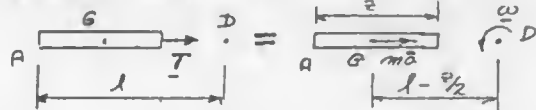
16.80



GIVEN:  $W = 0.25 \text{ lb/ft}$   
 $l = 1.2 \text{ ft}$   
 $\omega = 150 \text{ rpm}$   
 $z = 0.9 \text{ ft}$

FIND: TENSION IN ROD  
 (a) IN TERMS OF  $\omega, l, z$ , AND  $W$ .  
 (b) FOR GIVEN DATA

IN HORIZONTAL PLANE:  $\bar{a} = r\omega^2 = (l - \frac{z}{2})\omega^2$



$$\sum F = \sum F_{eff}: T = m\bar{a} = \left(\frac{W}{g}z\right)\left(l - \frac{z}{2}\right)\omega^2$$

$$T = \frac{W}{g}\left(lz - \frac{z^2}{2}\right)\omega^2$$

SUBSTITUTE DATA:

$$\omega = 150 \text{ rpm} \left(\frac{2\pi}{60}\right) = 5\pi \text{ rad/s}, z = 0.9 \text{ ft}$$

$$T = \frac{0.25 \text{ lb/ft}}{32.2 \text{ ft/s}^2} \left[ (1.2 \text{ ft})(0.9 \text{ ft}) - \frac{(0.9 \text{ ft})^2}{2} \right] (5\pi \text{ rad/s})^2$$

$$T = 1.293 \text{ lb}$$

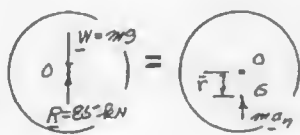
16.81



GIVEN: FLYWHEEL, CENTER OF ROTATION AT O, AND MASS CENTER AT G  
 $W = 1200 \text{ rpm}$ . MAXIMUM FORCE EXERTED ON SHAFT IS  $55 \text{ kN}$  AND  $85 \text{ kN}$ .  
 FIND: (a) MASS OF FLYWHEEL  
 (b) DISTANCE  $\bar{r}$

$$\omega = 1200 \text{ rpm} \left(\frac{2\pi}{60}\right) = 40\pi \text{ rad/s}$$

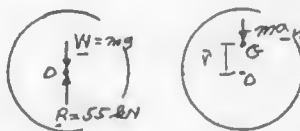
$$a_n = \bar{r}\omega^2$$



$$+\uparrow \sum F = \sum F_{eff}$$

$$85 \text{ kN} - mg = ma_n$$

$$85 - mg = m\bar{r}\omega^2 \quad (1)$$



$$+\uparrow \sum F = \sum F_{eff}$$

$$55 \text{ kN} + mg = ma_n$$

$$55 + mg = m\bar{r}\omega^2 \quad (2)$$

$$EQ(2) - EQ(1): 30 \text{ kN} - 2mg = 0$$

$$30 \times 10^3 \text{ N} = 2m(9.81 \text{ m/s}^2)$$

$$m = 1529 \text{ kg}$$

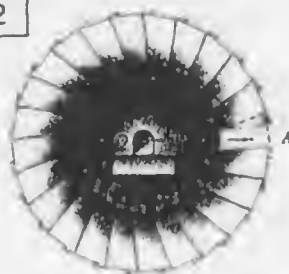
$$EQ(1) + EQ(2): 140 \text{ kN} = 2m\bar{r}\omega^2$$

$$140 \times 10^3 \text{ N} = 2(1529 \text{ kg})\bar{r}(40\pi)^2$$

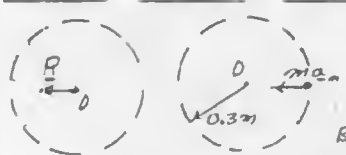
$$\bar{r} = 2.90 \times 10^{-2} \text{ m}$$

$$\bar{r} = 2.90 \text{ cm}$$

16.82



GIVEN: A 45-g VANE IS THROWN OFF FROM BALANCED TURBIN DISK.  
 $\omega = 9600 \text{ rpm}$   
 FIND: REACTION AT O



$$\omega = 9600 \text{ rpm} \left(\frac{2\pi}{60}\right)$$

$$\omega = 320\pi \text{ rad/s}$$

CONSIDER VANE BEFORE IT'S THROWN OFF

$$+\uparrow \sum F = \sum F_{eff}: R = ma_n = m\bar{r}\omega^2$$

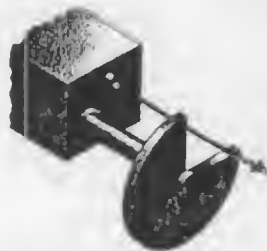
$$= (45 \times 10^{-3} \text{ kg})(0.3 \text{ m})(320\pi)^2$$

$$R = 13.64 \text{ kN}$$

BEFORE VANE WAS THROWN OFF DISK WAS BALANCED ( $R=0$ ). REMOVING VANE AT A ALSO REMOVES ITS REACTION, SO DISK IS UNBALANCED AND REACTION IS

$$R = 13.64 \text{ kN} \rightarrow$$

16.83



GIVEN: 0.125-lb SHUTTER OF RADIUS 0.75 in.  
 $W = 24 \text{ cycles per second}$

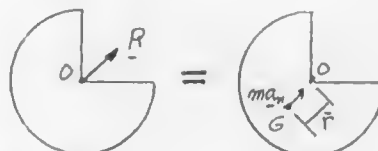
FIND: MAGNITUDE OF FORCE EXERTED ON SHAFT BY SHUTTER

SEE INSIDE FRONT COVER FOR CENTROID OF A CIRCULAR SECTOR

$$\bar{r} = \frac{2r \sin \alpha}{3\alpha}$$

$$\bar{r} = \frac{2(0.75 \text{ in.}) \sin(\frac{3}{4}\pi)}{3(\frac{3}{4}\pi)}$$

$$\bar{r} = 0.15005 \text{ in.}$$



$$a_n = \bar{r}\omega^2$$

$$\omega = 24 \frac{\text{rev}}{\text{s}}$$

$$= 24(2\pi) \frac{\text{rad}}{\text{s}}$$

$$\omega = 150.8 \text{ rad/s}$$

$$+\uparrow \sum F = \sum F_{eff}$$

$$R = ma_n = m\bar{r}\omega^2$$

$$= \frac{(0.125 \text{ lb})}{32.2 \text{ ft/s}^2} \left( \frac{0.15005 \text{ ft}}{12} \right) (150.8 \text{ rad/s})^2$$

$$R = 1.1038 \text{ lb}$$

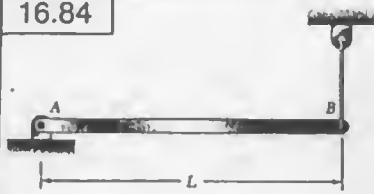
FORCE ON SHAFT IS  $R = 1.104 \text{ lb}$

MAGNITUDE:  $R = 1.104 \text{ lb}$

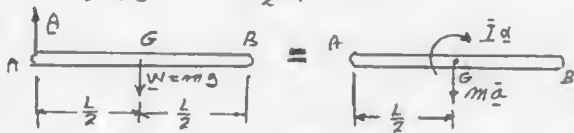
16.84

GIVEN: CABLE  
SUDDENLY BREAKS.

FIND: (a)  $a_B$   
(b) REACTION  
AT A



$$\omega = 0 \quad \ddot{\alpha} = \frac{L}{2} \alpha$$



$$\begin{aligned} +\sum M_A = \sum (M_A)_{\text{eff}}: \quad W \frac{L}{2} &= \bar{I} \alpha + m \ddot{\alpha} \frac{L}{2} \\ mg \frac{L}{2} &= \frac{1}{12} mL^2 \alpha + m \left( \frac{L}{2} \alpha \right) \frac{L}{2} \\ mg \frac{L}{2} &= \frac{1}{3} mL^2 \alpha \quad \alpha = \frac{3}{2} \frac{g}{L} \end{aligned}$$

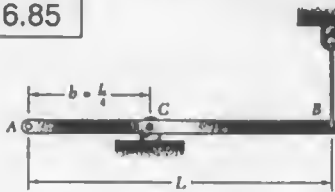
$$\begin{aligned} +\sum F_y = \sum (F_y)_{\text{eff}}: \quad A - mg &= -m \ddot{\alpha} = -m \frac{L}{2} \alpha \\ A - mg &= -m \left( \frac{L}{2} \times \frac{3}{2} \frac{g}{L} \right) \\ A - mg &= -\frac{3}{4} mg \\ A &= \frac{1}{4} mg \uparrow \quad A = \frac{1}{4} mg \uparrow \end{aligned}$$

$$\begin{aligned} a_B &= a_A + a_{B/A} = 0 + L \alpha \downarrow \\ a_B &= L \left( \frac{3}{2} \frac{g}{L} \right) = \frac{3}{2} g \downarrow \quad a_B = \frac{3}{2} g \downarrow \end{aligned}$$

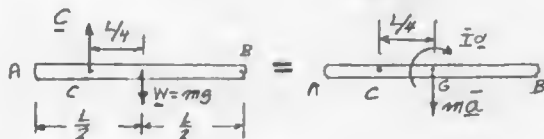
16.85

GIVEN: CABLE  
SUDDENLY BREAKS.

FIND: (a)  $a_B$   
(b) REACTION  
AT C.



$$\omega = 0 \quad \ddot{\alpha} = \frac{L}{4} \alpha$$



$$\begin{aligned} +\sum M_C = \sum (M_C)_{\text{eff}}: \quad W \frac{L}{4} &= \bar{I} \alpha + m \ddot{\alpha} \frac{L}{4} \\ mg \frac{L}{4} &= \frac{1}{12} mL^2 \alpha + m \left( \frac{L}{4} \alpha \right) \frac{L}{4} \\ mg \frac{L}{4} &= \frac{7}{48} mL^2 \alpha \\ \alpha &= \frac{12g}{7L} \end{aligned}$$

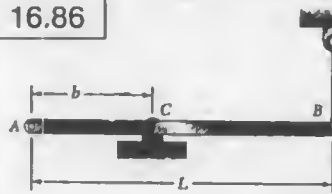
$$\begin{aligned} +\sum F_y = \sum (F_y)_{\text{eff}}: \quad C - mg &= -m \ddot{\alpha} = -m \frac{L}{4} \alpha \\ C - mg &= -m \left( \frac{L}{4} \times \frac{12g}{7L} \right) \\ C - mg &= -\frac{3}{7} mg \\ C &= \frac{4}{7} mg \uparrow \quad C = \frac{4}{7} mg \uparrow \end{aligned}$$

$$a_B = a_C + a_{B/C} = 0 + \frac{3L}{4} \alpha$$

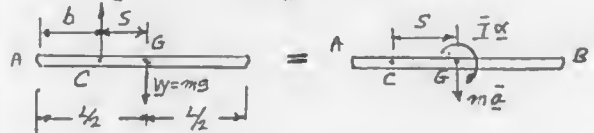
$$a_B = \frac{3L}{4} \left( \frac{12g}{7L} \right) = \frac{9}{7} g \downarrow \quad a_B = \frac{9}{7} g \downarrow$$

16.86

GIVEN: CABLE  
SUDDENLY BREAKS  
FIND: (a) DISTANCE  $b$   
FOR WHICH  $a_A$  IS  
MAXIMUM.  
(b) CORRESPONDING  
 $a_A$  AND REACTION AT C.



$$\text{LET } S = \frac{L}{2} - b, \quad \ddot{\alpha} = S \alpha$$



$$\begin{aligned} +\sum M_C = \sum (M_C)_{\text{eff}}: \quad W S &= \bar{I} \alpha + m a_S \\ mg S &= \frac{1}{12} mL^2 \alpha + m (S \alpha) S \\ mg S &= m \left( \frac{1}{12} L^2 + S^2 \right) \alpha \\ \alpha &= \frac{5g}{\frac{L^2}{12} + S^2} \end{aligned}$$

FOR ROTATION ABOUT C:  $a_A = \frac{L}{2} - S$

$$a_A = \frac{(\frac{L}{2} - S) S g}{\frac{L^2}{12} + S^2} = \frac{\frac{L}{2} S - S^2}{\frac{L^2}{12} + S^2} g$$

$$a_A = \frac{LS - 2S^2}{L^2 + 12S^2} (6g)$$

DIFFERENTIATE WITH RESPECT TO  $S$ .

$$\frac{d a_A}{d S} = \frac{(L^2 + 12S^2)(L - 4S) - (LS - 2S^2)(24S)}{(L^2 + 12S^2)^2} (6g)$$

SET NUMERATOR EQUAL TO ZERO

$$L^2 - 4SL^2 + 12S^2L - 48S^3 - 24S^2L + 48S^3 = 0$$

$$L^2 - 4SL^2 - 12S^2L = 0$$

$$L(L^2 - 4SL - 12S^2) = 0$$

$$L(L - 6S)(L + 2S) = 0$$

$$S = -\frac{L}{2} \quad \text{AND } S = \frac{L}{6}$$

(a)

FOR  $S = -\frac{L}{2}$ ,  $b = L$  AND SUPPORT WAS AT B, IMPOSSIBLE

FOR  $S = \frac{L}{6}$ ,  $b = \frac{L}{3}$  THIS RESULTS IN MAX  $a_A$

$$b = \frac{L}{3}$$

(b) EQ.1 WITH  $S = \frac{L}{6}$

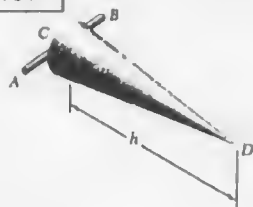
$$\alpha = \frac{\frac{L}{6} g}{\frac{L^2}{12} + (\frac{L}{6})^2} = \frac{\frac{1}{6} g}{\frac{1}{9} L} \quad \alpha = \frac{3}{2} \frac{g}{L}$$

$$a_A = S \alpha = \frac{L}{6} \left( \frac{3}{2} \frac{g}{L} \right) = \frac{1}{4} g \quad \text{MAX: } a_A = \frac{1}{4} g \uparrow$$

$$\begin{aligned} +\sum F_y = \sum (F_y)_{\text{eff}}: \quad C - mg &= -m \ddot{\alpha} \\ C - mg &= -m S \alpha \\ C - mg &= -m \left( \frac{L}{6} \right) \left( \frac{3}{2} \frac{g}{L} \right) \\ C - mg &= -\frac{1}{4} mg \end{aligned}$$

$$C = \frac{3}{4} mg \uparrow$$

16.87



GIVEN:  $m = \text{MASS OF CONE}$   
IMMEDIATELY AFTER  
CONE IS RELEASED  
FIND: (a)  $a_D$   
(b) REACTION AT C

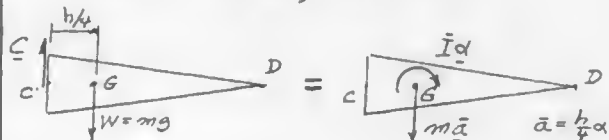
FROM INSIDE COVER  
 $I_D = \frac{3}{5} m (\frac{1}{4} h^2 + h^2)$   
 FOR SLIMMER CONE, NEGLECT  $\frac{1}{4} h^2$   
 $I_D = \frac{3}{5} m h^2$

PARALLEL AXIS THEOREM

$$I_D = \bar{I} + m(\frac{3}{4}h)^2; \quad \frac{3}{5}mh^2 = \bar{I} + \frac{9}{16}mh^2$$

$$\bar{I} = (\frac{3}{5} - \frac{9}{16})mh^2 \quad \bar{I} = \frac{3}{80}mh^2$$

SAME RESULT CAN BE OBTAINED FROM  
SAMPLE PROB. 9.11, PAGE 503.



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}: \quad 1N \frac{h}{4} = \bar{I} \alpha + m \bar{a} \frac{h}{4}$$

$$mg \frac{h}{4} = \frac{3}{80} m h^2 \alpha + m (\frac{h}{4} \alpha) \frac{h}{4}$$

$$mg \frac{h}{4} = (\frac{3}{80} + \frac{1}{16}) m h^2 \alpha$$

$$mg \frac{h}{4} = \frac{1}{10} m h^2 \alpha$$

$$\alpha = \frac{5}{2} \frac{g}{h}$$

$$(a) \quad a_D = h \alpha = h (\frac{5}{2} \frac{g}{h}) \quad a_D = 2.50g \downarrow$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad C - mg = -m \bar{a}$$

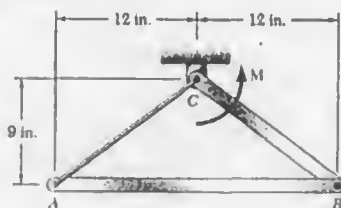
$$C - mg = -m \frac{h}{4} \alpha$$

$$C - mg = -m \frac{h}{4} (\frac{5}{2} \frac{g}{h})$$

$$C - mg = -\frac{5}{8} mg$$

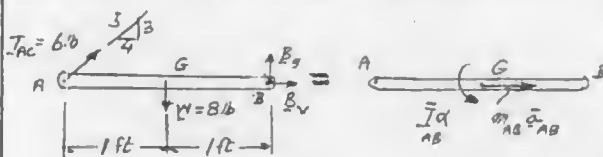
$$C = \frac{3}{8} mg \uparrow$$

16.38



GIVEN:  $W_{AB} = 8 \text{ lb}$   
 $W_{BC} = 5 \text{ lb}$   
 $\omega = 0$   
 $T_{AC} = 6 \text{ lb}$   
FIND:  
(a)  $\alpha$   
(b)  $M$

ROD AB:  $\bar{I}_{AB} = \frac{1}{12} \frac{W}{g} L_{AB}^2 = \frac{1}{12} \frac{8 \text{ lb}}{32.2} (2 \text{ ft})^2 = 0.8282 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}: \quad \frac{3}{5} (6 \text{ lb}) (2 \text{ ft}) - (8 \text{ lb}) (1 \text{ ft}) = \bar{I}_{AB} \alpha$$

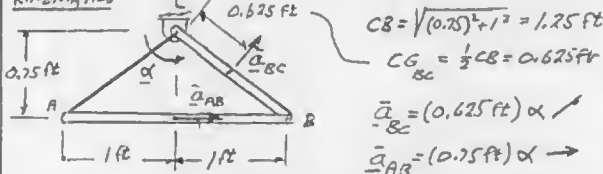
$$\bar{I}_{AB} \alpha = 0.8 \text{ lb} \cdot \text{ft}$$

$$(0.8282 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) \alpha = 0.8 \text{ lb} \cdot \text{ft}$$

$$\alpha = 9.66 \text{ rad/s}^2 \uparrow$$

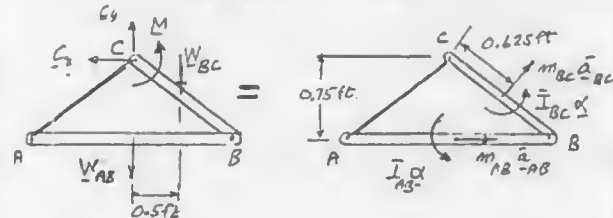
ENTIRE ASSEMBLY: SINCE AC IS TAUT, ASSEMBLY  
ROTATES ABOUT C AS A RIGID BODY.

KINEMATICS



KINETICS

$$\bar{I}_{BC} = \frac{1}{12} \frac{W_{BC}}{g} (C B)^2 = \frac{1}{12} \frac{5 \text{ lb}}{32.2} (1.25 \text{ ft})^2 = 0.0202 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}: \quad$$

$$M - W_{BC} (0.5 \text{ ft}) = m_{BC} \bar{a}_{BC} (0.625 \text{ ft}) + \bar{I}_{BC} \alpha + m_{AB} \bar{a}_{AB} (1 \text{ ft}) + \bar{I}_{AB} \alpha$$

$$M - (5 \text{ lb}) (0.5 \text{ ft}) = \frac{5 \text{ lb}}{32.2} (0.625 \text{ ft})^2 \alpha + (0.0202 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) \alpha + \frac{8 \text{ lb}}{32.2} (1 \text{ ft})^2 \alpha + \bar{I}_{AB} \alpha$$

SUBSTITUTE  $\alpha = 9.66 \text{ rad/s}^2$  AND FROM EQ (1),  $\bar{I}_{AB} \alpha = 0.8 \text{ lb} \cdot \text{ft}$

$$M - 2.5 = (0.0606) (9.66) + (0.0202) (9.66) + (0.1399) (9.66) + 0.8$$

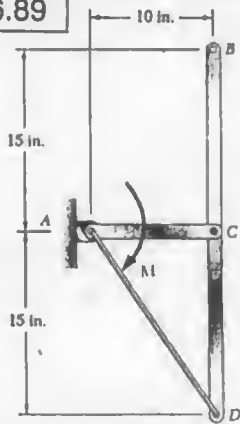
$$M - 2.5 = 0.5860 + 0.1950 + 1.35 + 0.8$$

$$M - 2.5 = 2.93$$

$$M = 5.43 \text{ lb} \cdot \text{ft}$$

$$M = 5.43 \text{ lb} \cdot \text{ft} \uparrow$$

16.89

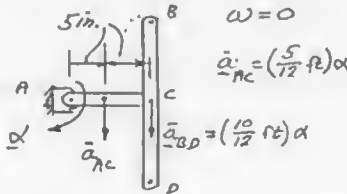


GIVEN:  $W_{AC} = 8 \text{ lb}$   
 $W_{BD} = 20 \text{ lb}$   
 $M = 6 \text{ lb}\cdot\text{ft}$

FIND: (a)  $\alpha$   
 (b)  $T$  in cord AD

ENTIRE ASSEMBLY:

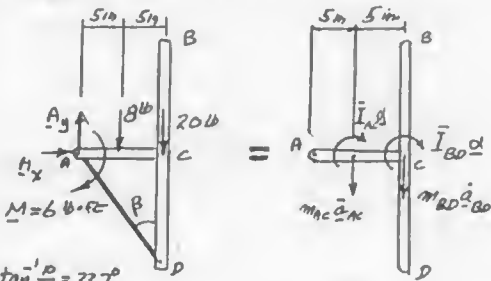
KINEMATICS



KINETICS

$$\bar{I}_{AC} = \frac{1}{12} \frac{W_{AC}}{g} (AC)^2 = \frac{1}{12} \cdot \frac{8 \text{ lb}}{32.2} \left( \frac{10}{12} \text{ ft} \right)^2 = 0.014378 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\bar{I}_{BD} = \frac{1}{12} \frac{W_{BD}}{g} (BD)^2 = \frac{1}{12} \cdot \frac{20 \text{ lb}}{32.2} \left( \frac{30}{12} \text{ ft} \right)^2 = 0.3235 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$



$$\beta = \tan^{-1} \frac{10}{15} = 33.7^\circ$$

$$\sum \mathcal{M}_A = \sum (M_A)_{eff}:$$

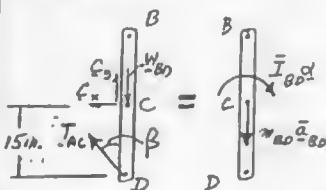
$$(8 \text{ lb}) \left( \frac{5}{12} \text{ ft} \right) + (20 \text{ lb}) \left( \frac{10}{12} \text{ ft} \right) + 6 \text{ lb}\cdot\text{ft} = \bar{I}_{AC} \alpha + \bar{I}_{BD} \alpha + m_{BD} \bar{a}_{BD} \left( \frac{10}{12} \text{ ft} \right) + \bar{I}_{BD} \alpha$$

$$26 \text{ ft}\cdot\text{lb} = \frac{8 \text{ lb}}{32.2} \left( \frac{5}{12} \text{ ft} \right)^2 \alpha + \bar{I}_{AC} \alpha + \frac{20 \text{ lb}}{32.2} \left( \frac{10}{12} \text{ ft} \right)^2 \alpha + \bar{I}_{BD} \alpha$$

$$26 = (0.04313 + 0.014378 + 0.4313 + 0.3235) \alpha$$

$$26 = 0.8123 \alpha; \quad \alpha = 32.01 \text{ rad/s}^2 \quad \alpha = 32.0 \text{ rad/s}^2$$

ROD BD:



$$\sum \mathcal{M}_C = \sum (M_C)_{eff}:$$

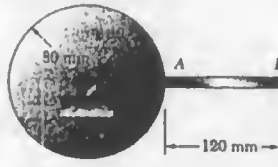
$$(T_{AC} \sin \beta) \left( \frac{15}{12} \text{ ft} \right) = \bar{I}_{BD} \alpha$$

$$(T_{AC} \sin 33.7^\circ) \left( \frac{15}{12} \text{ ft} \right) = (0.3235 \text{ lb}\cdot\text{ft}\cdot\text{s}^2) (32.01 \text{ rad/s}^2)$$

$$T_{AC} = 14.93 \text{ lb}$$

$$T_{AC} = 14.93 \text{ lb}$$

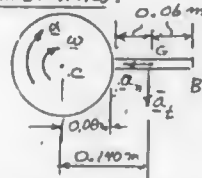
16.90



GIVEN:  $m_{AB} = 1.5 \text{ kg}$   
 $m_{DISK} = 5 \text{ kg}$   
 $\omega = 10 \text{ rad/s}$

FIND: (a)  $\alpha$   
 (b) COMPONENTS OF REACTION AT C

KINEMATICS:



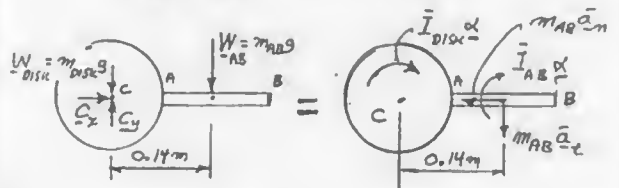
$$\bar{a}_G = (CG) \omega^2 = (0.14 \text{ m}) (10 \text{ rad/s})^2$$

$$\bar{a}_G = 14 \text{ m/s}^2$$

$$\bar{a}_B = (CB) \alpha = (0.14 \text{ m}) \alpha$$

$$\text{KINETICS: } \bar{I}_{DISK} = \frac{1}{2} m_{DISK} (CG)^2 = \frac{1}{2} (5 \text{ kg}) (0.09 \text{ m})^2 = 16 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} (AB)^2 = \frac{1}{12} (1.5 \text{ kg}) (0.12 \text{ m})^2 = 1.8 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$



(a)

$$\sum \mathcal{M}_C = \sum (M_C)_{eff}:$$

$$W_{AB} (0.14 \text{ m}) = \bar{I}_{DISK} \alpha + m_{AB} \bar{a}_B (0.14 \text{ m}) + \bar{I}_{AB} \alpha$$

$$(1.5 \text{ kg}) (9.81 \text{ m/s}^2) (0.14 \text{ m}) = \bar{I}_{DISK} \alpha + (1.5 \text{ kg}) (0.14 \text{ m}) \alpha + \bar{I}_{AB} \alpha$$

$$2.060 \text{ N}\cdot\text{m} = (16 \times 10^{-3} + 224 \times 10^{-3} + 1.8 \times 10^{-3}) \alpha$$

$$2.060 \text{ N}\cdot\text{m} = (422 \times 10^{-6} \text{ kg}\cdot\text{m}^2) \alpha$$

$$\alpha = 43.64 \text{ rad/s}^2$$

$$\alpha = 43.6 \text{ rad/s}^2$$

(b)

$$\sum F_x = \sum (F_x)_{eff}$$

$$C_x = -m_{AB} \bar{a}_B = -(1.5 \text{ kg}) (14 \text{ m/s}^2)$$

$$C_y = -21.0 \text{ N}$$

$$C_y = 21.0 \text{ N}$$

$$\sum F_y = \sum (F_y)_{eff}:$$

$$a_B = (0.14 \text{ m}) \alpha$$

$$C_y - m_{DISK} g - m_{AB} g = -m_{AB} \bar{a}_B$$

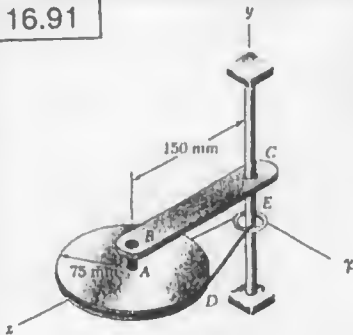
$$C_y - (5 \text{ kg}) (9.81) - (1.5 \text{ kg}) (9.81) = -(1.5 \text{ kg}) (0.14 \text{ m}) (43.64 \text{ rad/s}^2)$$

$$C_y - 47.05 \text{ N} - 14.715 \text{ N} = -9.164 \text{ N}$$

$$C_y = 54.6 \text{ N}$$

$$C_y = 54.6 \text{ N}$$

16.91



GIVEN:

$$m_{\text{disk}} = 7.5 \text{ kg}$$

$$m_{\text{BC}} = 3.8 \text{ kg}$$

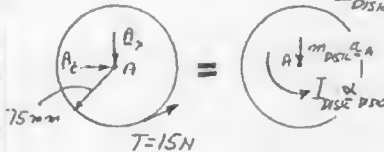
$$\text{CORD: } T = 15 \text{ N}$$

IMMEDIATELY AFTER  
RELEASE FROM REST,FIND: (a)  $\alpha_{\text{disk}}$ (b)  $a_A$ 

DISK:

$$\bar{I}_{\text{disk}} = \frac{1}{2} m_{\text{disk}} r^2 = \frac{1}{2} (7.5 \text{ kg}) (0.075 \text{ m})^2$$

$$\bar{I}_{\text{disk}} = 14.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: (15 \text{ N}) (0.075 \text{ m}) = \bar{I}_{\text{disk}} \alpha_{\text{disk}}$$

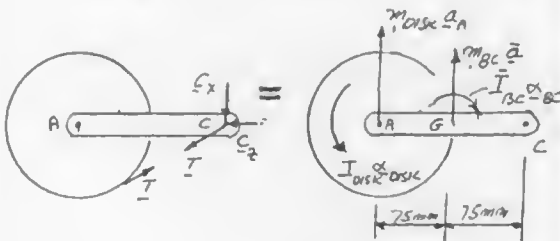
$$1.125 \text{ N} \cdot \text{m} = (14.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \alpha_{\text{disk}}$$

$$\alpha_{\text{disk}} = 80.0 \text{ rad/s}^2$$

$$\alpha_{\text{disk}} = 80.0 \text{ rad/s}^2$$

ENTIRE ASSEMBLY

$$\bar{I}_{\text{BC}} = \frac{1}{12} m_{\text{BC}} (BC)^2 = \frac{1}{12} (3.8 \text{ kg}) (0.15 \text{ m})^2 = 5.625 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

ASSUME  $\alpha_{\text{BC}} \downarrow$ 

$$a_A = (0.15 \text{ m}) \alpha_{\text{BC}} \uparrow$$

$$a = (0.075 \text{ m}) \alpha_{\text{BC}}$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}:$$

$$0 = I_{\text{disk}} \alpha_{\text{disk}} - m_{\text{disk}} a_A (0.15 \text{ m}) - m_{\text{BC}} a (0.075 \text{ m}) - \bar{I}_{\text{BC}} \alpha_{\text{BC}}$$

$$0 = (14.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2) (80 \text{ rad/s}^2) - (7.5 \text{ kg}) (0.15 \text{ m}) \alpha_{\text{BC}} - (3.8 \text{ kg}) (0.075 \text{ m}) \alpha_{\text{BC}} - (5.625 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \alpha_{\text{BC}}$$

$$0 = 1.125 - 0.1125 \alpha_{\text{BC}} - 0.285 \alpha_{\text{BC}} - 5.625 \times 10^{-3} \alpha_{\text{BC}}$$

$$0 = 1.125 - 0.135 \alpha_{\text{BC}}$$

$$\alpha_{\text{BC}} = 8.333 \text{ rad/s}^2 \quad \alpha_{\text{BC}} = 8.33 \text{ rad/s}^2$$

$$a_A = (AC) \alpha_{\text{BC}} = (0.15 \text{ m}) (8.333 \text{ rad/s}^2)$$

$$a_A = +1.25 \text{ m/s}^2$$

$$a_A = 1.25 \text{ m/s}^2 \uparrow$$

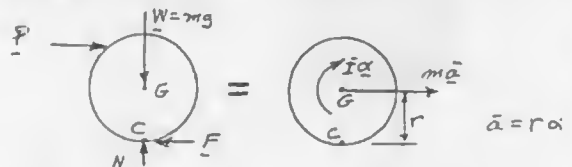
NOTE: ANSWERS CAN ALSO BE WRITTEN:

$$\alpha_{\text{disk}} = (80 \text{ rad/s}^2) \underline{j} \quad a_A = -(1.25 \text{ m/s}^2) \underline{j}$$

16.92

DERIVE  $\Sigma M_C = I_C \alpha$  FOR THE ROLLING

DISK OF FIG. 16.17.



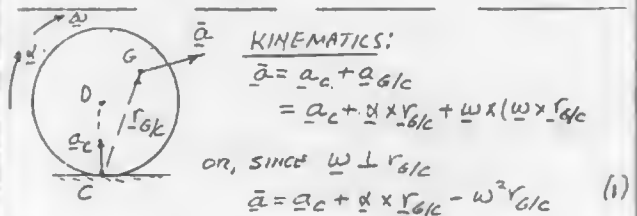
$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \Sigma M_C = (m \bar{a}) r + \bar{I} \alpha$$

$$= (m r \alpha) r + \bar{I} \alpha$$

$$\Sigma M_C = (m r^2 + \bar{I}) \alpha$$

BUT, WE KNOW THAT  $I_C = m r^2 + \bar{I}$ THUS:  $\Sigma M_C = I_C \alpha$  (Q.E.D.)

16.93

FOR ALL UNBALANCED DISK SHOW  
THAT  $\Sigma M = I_C \alpha$  IS VALID ONLY WHEN THE  
MASS CENTER G, THE GEOMETRIC CENTER O, AND  
THE INSTANTANEOUS CENTER C HAPPEN TO LIE  
IN A STRAIGHT LINE.

KINEMATICS:

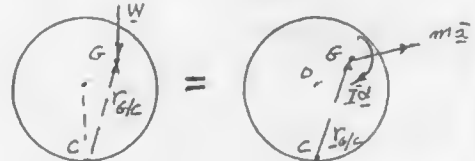
$$\bar{a} = a_O + a_{G/O}$$

$$= a_O + \alpha \times r_{G/O} + \omega \times (\omega \times r_{G/O})$$

OR, SINCE  $\omega \perp r_{G/O}$ 

$$\bar{a} = a_O + \alpha \times r_{G/O} - \omega^2 r_{G/O} \quad (1)$$

KINETICS



$$\Sigma M_C = \Sigma (M_C)_{\text{eff}}: \Sigma M_C = \bar{I} \alpha + r_{G/C} \times m \bar{a}$$

$$\text{RECALL EQ (1): } \Sigma M_C = \bar{I} \alpha + r_{G/C} \times m (a_O + \alpha \times r_{G/O} - \omega^2 r_{G/O})$$

$$\Sigma M_C = \bar{I} \alpha + r_{G/C} \times m a_O + m r_{G/C} \times (\alpha \times r_{G/O}) - m \omega^2 r_{G/C} \times r_{G/O}$$

BUT  $r_{G/C} \times r_{G/O} = 0$  AND  $\alpha \perp r_{G/O}$ 

$$r_{G/C} \times m (\alpha \times r_{G/O}) = m r_{G/C}^2 \alpha$$

$$\text{THUS: } \Sigma M_C = (\bar{I} + m r_{G/C}^2) \alpha + r_{G/C} \times m a_O$$

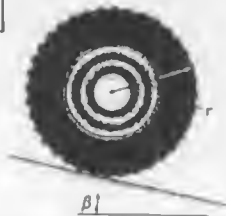
SINCE  $I_C = \bar{I} + m r_{G/C}^2$ 

$$\Sigma M_C = I_C \alpha + r_{G/C} \times m a_O \quad (2)$$

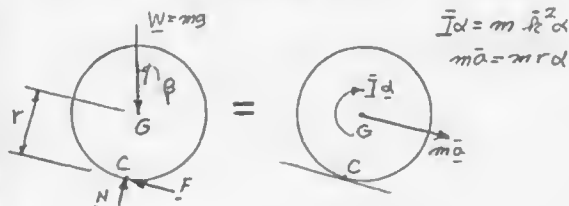
EQ (2) REDUCES TO  $\Sigma M_C = I_C \alpha$  WHEN  $r_{G/C} \times m a_O = 0$   
THAT IS, WHEN  $r_{G/C}$  AND  $a_O$  ARE COLLINEAR.REFERRING TO THE FIRST DIAGRAM, WE NOTE  
THAT THIS WILL OCCUR ONLY WHEN  
POINTS G, O, AND C LIE IN A STRAIGHT LINE.

(Q.E.D.)

16.94



GIVEN: ROLLING WHEEL

FIND:  $\bar{a}$  IN TERMS OF  $r$ ,  $\bar{r}_c$ ,  $\beta$ , AND  $g$ .

$$\bar{I} \alpha = m \bar{r}_c^2 \alpha$$

$$m \bar{a} = m r \alpha$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (W \sin \beta) r = (m \bar{a}) r + \bar{I} \alpha$$

$$(mg \sin \beta) r = (m r \alpha) r + m \bar{r}_c^2 \alpha$$

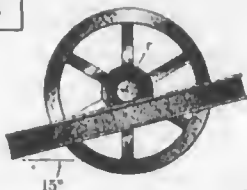
$$r g \sin \beta = (r^2 + \bar{r}_c^2) \alpha$$

$$\alpha = \frac{r g \sin \beta}{r^2 + \bar{r}_c^2}$$

$$\bar{a} = r \alpha = r \frac{r g \sin \beta}{r^2 + \bar{r}_c^2}$$

$$\bar{a} = \frac{r^2}{r^2 + \bar{r}_c^2} g \sin \beta$$

16.95

GIVEN: STARTING FROM REST, FLYWHEEL MOVES 16 ft IN 40 s  
 $r = 1.5 \text{ in.}$ FIND:  $\bar{I}$ 

KINEMATICS:  $s = v_0 t + \frac{1}{2} \bar{a} t^2$

$$16 \text{ ft} = 0 + \frac{1}{2} \bar{a} (40 \text{ s})^2$$

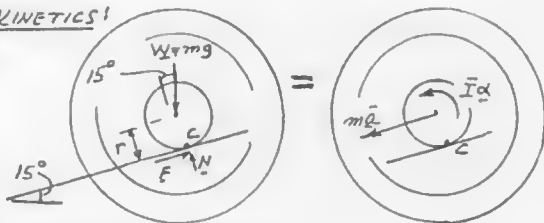
$$\bar{a} = 0.02 \text{ ft/s}^2$$

SINCE  $r = 1.5 \text{ in} = 0.125 \text{ ft}$

$$\bar{a} = r \alpha; 0.02 \text{ ft/s}^2 = (0.125 \text{ ft}) \alpha$$

$$\alpha = 0.16 \text{ rad/s}^2$$

KINETICS:



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}:$$

$$(mg \sin 15^\circ) r = \bar{I} \alpha + (m \bar{a}) r$$

$$(mg \sin 15^\circ) r = \bar{I} \alpha + (m r \alpha) r$$

$$g r \sin 15^\circ = (\bar{r}_c^2 + r^2) \alpha$$

DATA:  $r = 0.125 \text{ ft}$ ,  $\alpha = 0.16 \text{ rad/s}^2$

$$(32.2 \text{ ft/s}^2)(0.125 \text{ ft}) \sin 15^\circ = (\bar{r}_c^2 + r^2)(0.16 \text{ rad/s}^2)$$

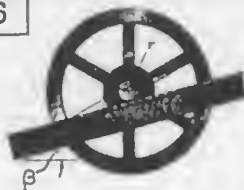
$$\bar{r}_c^2 + r^2 = 6.511 \text{ ft}^2$$

$$\bar{r}_c^2 + (0.125 \text{ ft})^2 = 6.511 \text{ ft}^2$$

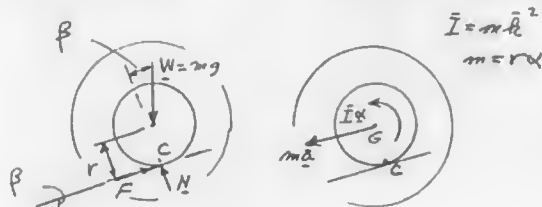
$$\bar{r}_c^2 = 6.4953$$

$$\bar{r}_c = 2.55 \text{ ft}$$

16.96



GIVEN:

 $\bar{r}_c$  = CENTROIDAL RADIUS OF GYRATION  
 $\mu_s$  = COEF. STATIC FRICTIONFIND: LARGEST  $\beta$  FOR ROLLING WITHOUT SLIPPING

$$\bar{I} = m \bar{r}_c^2$$

$$m = r \alpha$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (mg \sin \beta) r = \bar{I} \alpha + (m \bar{a}) r$$

$$mg \sin \beta r = m \bar{r}_c^2 \alpha + m r^2 \alpha$$

$$\alpha = \frac{g r}{r^2 + \bar{r}_c^2} \sin \beta \quad (1)$$

$$+\rightarrow \Sigma F = \Sigma F_{\text{eff}}: F - mg \sin \beta = -m \bar{a}$$

$$F - mg \sin \beta = -m r \alpha$$

$$F = mg \sin \beta - m r \alpha$$

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: N - mg \cos \beta = 0$$

$$N = mg \cos \beta$$

IF SLIPPING IMPENDS  $F = \mu_s N$  OR  $\mu_s = \frac{F}{N}$

$$\mu_s = \frac{F}{N} = \frac{mg \sin \beta - m r \alpha}{mg \cos \beta} = \frac{\sin \beta - \frac{r}{g} \alpha}{\cos \beta}$$

SUBSTITUTING FOR  $\alpha$  FROM EQ.(1)

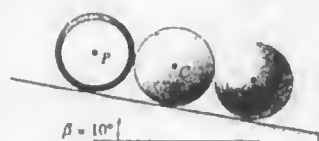
$$\mu_s = \frac{\sin \beta - \frac{r}{g} \cdot \frac{g r}{r^2 + \bar{r}_c^2} \sin \beta}{\cos \beta}$$

$$\mu_s = \tan \beta \left[ 1 - \frac{r^2}{r^2 + \bar{r}_c^2} \right] = \tan \beta \left[ \frac{\bar{r}_c^2}{r^2 + \bar{r}_c^2} \right]$$

$$\tan \beta = \mu_s \frac{r^2 + \bar{r}_c^2}{\bar{r}_c^2}$$

$$\tan \beta = \mu_s \left[ 1 + \left( \frac{r}{\bar{r}_c} \right)^2 \right]$$

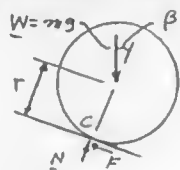
16.97



GIVEN: PIPE, CYLINDER  
AND SPHERE ARE  
RELEASED FROM REST.

AFTER 4 S, FIND  
DISTANCE BETWEEN  
(a) PIPE AND CYLINDER  
(b) CYLINDER AND SPHERE.

GENERAL CASE:  $\vec{I} = m\vec{R}^2$   $\vec{a} = r\alpha$



$$+ \sum M_C = \sum (M_C)_{eff} \\ (W \sin \beta) r = \vec{I} \alpha + m \vec{a} r \\ m g \sin \beta r = m \vec{R}^2 \alpha + m r^2 \alpha \\ \alpha = \frac{r g \sin \beta}{r^2 + \vec{R}^2}$$

$$\vec{a} = r \alpha = \frac{r g \sin \beta}{r^2 + \vec{R}^2}$$

$$\vec{a} = \frac{r^2}{r^2 + \vec{R}^2} g \sin \beta \quad (1)$$

FOR PIPE:  $\vec{R} = r$   $\vec{a}_p = \frac{r^2}{r^2 + r^2} g \sin \beta = \frac{1}{2} g \sin \beta$

FOR CYLINDER:  $\vec{R}^2 = \frac{1}{2} r^2$   $\vec{a}_c = \frac{r^2}{r^2 + \frac{1}{2} r^2} g \sin \beta = \frac{2}{3} g \sin \beta$

FOR SPHERE:  $\vec{R}^2 = \frac{2}{5} r^2$   $\vec{a}_s = \frac{r^2}{r^2 + \frac{2}{5} r^2} g \sin \beta = \frac{5}{7} g \sin \beta$

(a) BETWEEN PIPE AND CYLINDER

$$a_{c/p} = a_c - a_p = \left( \frac{2}{3} - \frac{1}{2} \right) g \sin \beta = \frac{1}{6} g \sin \beta$$

$$x_{c/p} = \frac{1}{2} a_{c/p} t^2 = \frac{1}{2} \left( \frac{1}{6} g \sin \beta \right) t^2$$

SI UNITS:  $x_{c/p} = \frac{1}{2} \left( \frac{1}{6} 9.81 \text{ m/s}^2 \right) \sin 10^\circ (4 \text{ s})^2 = 2.27 \text{ m}$

US UNITS:  $x_{c/p} = \frac{1}{2} \left( \frac{1}{6} 32.2 \text{ ft/s}^2 \right) \sin 10^\circ (4 \text{ s})^2 = 7.46 \text{ ft}$

(b) BETWEEN SPHERE AND CYLINDER

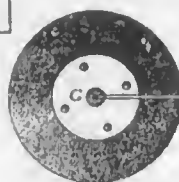
$$a_{s/c} = a_s - a_c = \left( \frac{5}{7} - \frac{2}{3} \right) g \sin \beta = \frac{1}{21} g \sin \beta$$

$$x_{s/c} = \frac{1}{2} a_{s/c} t^2 = \frac{1}{2} \left( \frac{1}{21} g \sin \beta \right) t^2$$

SI UNITS:  $x_{s/c} = \frac{1}{2} \left( \frac{1}{21} 9.81 \text{ m/s}^2 \right) \sin 10^\circ (4 \text{ s})^2 = 0.647 \text{ m}$

US UNITS:  $x_{s/c} = \frac{1}{2} \left( \frac{1}{21} 32.2 \text{ ft/s}^2 \right) \sin 10^\circ (4 \text{ s})^2 = 2.13 \text{ ft}$

16.98



GIVEN:  $W = 10 \text{ lb}$   
 $R_o = 8 \text{ in}$ ,  $R_i = 4 \text{ in}$   
 $\vec{R} = 6 \text{ in}$   
 $P = 5 \text{ lb}$

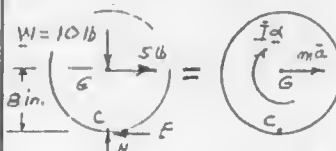
$\mu_s = 0.25$ ,  $\mu_k = 0.20$

FIND: (a) DOES DISK SLIDE,  
(b)  $\alpha$  AND  $\vec{a}$ .

ASSUME DISK ROLLS:  $\vec{a} = r \alpha = \left( \frac{R}{\sqrt{2}} \text{ ft} \right) \alpha$

$$\vec{I} = m \vec{R}^2 = \frac{10 \text{ lb}}{32.2} \left( \frac{6}{12} \text{ ft} \right)^2$$

$$\vec{I} = 0.07764 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$+ \sum M_C = \sum (M_C)_{eff}: (5 \text{ lb}) \left( \frac{8}{12} \text{ ft} \right) = (m \vec{a}) r + \vec{I} \alpha \\ 3.333 \text{ lb} \cdot \text{ft} = \frac{10 \text{ lb}}{32.2} \left( \frac{8}{12} \text{ ft} \right) \alpha + 0.07764 \alpha$$

$$3.333 = 0.21566 \alpha$$

$$\alpha = 15.456 \text{ rad/s}^2$$

$$\alpha = 15.46 \text{ rad/s}^2$$

$$\vec{a} = r \alpha = \left( \frac{8}{12} \text{ ft} \right) (15.456 \text{ rad/s}^2)$$

$$\vec{a} = 10.30 \text{ ft/s}^2 \rightarrow$$

$$+ \sum F_x = \sum (F_x)_{eff}: -F + 5 \text{ lb} = m \vec{a} \\ -F + 5 \text{ lb} = \frac{10 \text{ lb}}{32.2} (10.30 \text{ ft/s}^2); F = 1.80 \text{ lb}$$

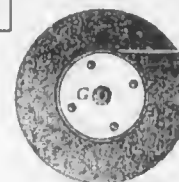
$$+ \uparrow \sum F_y = \sum (F_y)_{eff}: N - 10 \text{ lb} = 0$$

$$N = 10 \text{ lb}$$

$$F_m = \mu_s N = 0.25 (10 \text{ lb}) = 2.5 \text{ lb}$$

SINCE  $F < F_m$ , DISK ROLLS WITH NO SLIDING

16.99



GIVEN:  $W = 10 \text{ lb}$   
 $R_o = 8 \text{ in}$ ,  $R_i = 4 \text{ in}$   
 $\vec{R} = 6 \text{ in}$ ,  $P = 10 \text{ lb}$

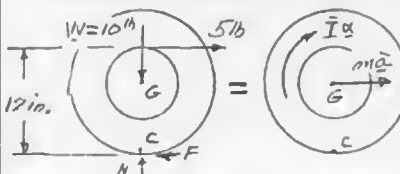
$\mu_s = 0.25$ ,  $\mu_k = 0.20$   
FIND: (a) DOES DISK SLIDE,  
(b)  $\alpha$  AND  $\vec{a}$ .

ASSUME DISK ROLLS:  $\vec{a} = r \alpha = \left( \frac{R}{\sqrt{2}} \text{ ft} \right) \alpha$

$$\vec{I} = m \vec{R}^2$$

$$= \frac{10 \text{ lb}}{32.2} \left( \frac{6}{12} \text{ ft} \right)^2$$

$$\vec{I} = 0.07764 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$+ \sum M_C = \sum (M_C)_{eff}: (10 \text{ lb}) \left( \frac{8}{12} \text{ ft} \right) = (m \vec{a}) r + \vec{I} \alpha \\ 5 \text{ lb} \cdot \text{ft} = \frac{10 \text{ lb}}{32.2} \left( \frac{8}{12} \text{ ft} \right) \alpha + 0.07764 \alpha$$

$$5 = 0.21566 \alpha$$

$$\alpha = 23.184 \text{ rad/s}^2$$

$$\alpha = 23.2 \text{ rad/s}^2$$

$$\vec{a} = r \alpha = \left( \frac{8}{12} \text{ ft} \right) (23.184 \text{ rad/s}^2)$$

$$\vec{a} = 15.46 \text{ ft/s}^2 \rightarrow$$

$$+ \sum F_x = \sum (F_x)_{eff}: -F + 10 \text{ lb} = m \vec{a} \\ -F + 10 \text{ lb} = \frac{10 \text{ lb}}{32.2} (15.46 \text{ ft/s}^2); F = 0.20 \text{ lb}$$

$$+ \uparrow \sum F_y = \sum (F_y)_{eff}: N - 10 \text{ lb} = 0$$

$$N = 10 \text{ lb}$$

$$F_m = \mu_s N = 0.25 (10 \text{ lb}) = 2.5 \text{ lb}$$

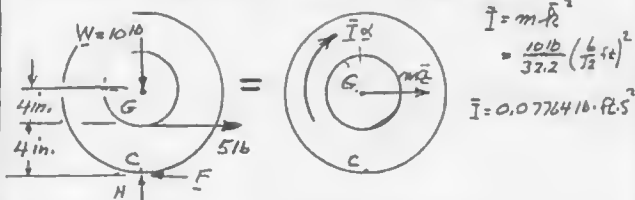
SINCE  $F < F_m$ , DISK ROLLS WITH NO SLIDING

16.100



GIVEN:  $W = 10 \text{ lb}$   
 $r_o = 8 \text{ in.}$ ,  $r_c = 4 \text{ in.}$   
 $r_k = 6 \text{ in.}$ ,  $P = 5 \text{ lb}$   
 $\mu_s = 0.25$ ,  $\mu_k = 0.20$   
 FIND:  
 (a) DOES DISK SLIDE  
 (b)  $\alpha$  AND  $\bar{a}$ .

ASSUME DISK ROLLS:  $\bar{a} = r_c \alpha = \left(\frac{8}{12} \text{ ft}\right) \alpha$



$$+\uparrow \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (5 \text{ lb}) \left(\frac{4}{12} \text{ ft}\right) = (m \bar{a}) r + \bar{I} \alpha$$

$$1.6667 \text{ lb} \cdot \text{ft} = \frac{10 \text{ lb}}{32.2} \left(\frac{8}{12} \text{ ft}\right) \alpha + 0.07764 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \alpha$$

$$1.6667 = 0.21586 \alpha$$

$$\alpha = 7.728 \text{ rad/s}^2$$

$$\bar{a} = r_c \alpha = \left(\frac{8}{12} \text{ ft}\right) (7.728 \text{ rad/s}^2) = 5.153 \text{ ft/s}^2$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: -F + 5 \text{ lb} = m \bar{a}$$

$$-F + 5 \text{ lb} = \frac{10 \text{ lb}}{32.2} (5.153 \text{ ft/s}^2)$$

$$F = 3.40 \text{ lb}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N - 10 \text{ lb} = 0 \quad N = 10 \text{ lb}$$

$$F_m = \mu_s N = 0.25(10 \text{ lb}) = 2.5 \text{ lb}$$

SINCE  $F > F_m$ , DISK SLIDES

KNOWING THAT DISK SLIDES

$$F = \mu_k N = 0.20(10 \text{ lb}) = 2 \text{ lb}$$

$$+\rightarrow \Sigma M_G = \Sigma (M_G)_{\text{eff}}:$$

$$F \left(\frac{8}{12} \text{ ft}\right) - (5 \text{ lb}) \left(\frac{4}{12} \text{ ft}\right) = \bar{I} \alpha$$

$$(2 \text{ lb}) \left(\frac{8}{12} \text{ ft}\right) - 1.6667 \text{ lb} \cdot \text{ft} = (0.07764 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) \alpha$$

$$-0.8333 = 0.07764 \alpha$$

$$\alpha = -4.29 \text{ rad/s}^2 \quad \underline{\alpha = 4.29 \text{ rad/s}^2}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$-F + 5 \text{ lb} = m \bar{a}$$

$$-2 \text{ lb} + 5 \text{ lb} = \frac{10 \text{ lb}}{32.2} \bar{a}$$

$$\bar{a} = 9.66 \text{ ft/s}^2 \quad \underline{\bar{a} = 9.66 \text{ ft/s}^2}$$

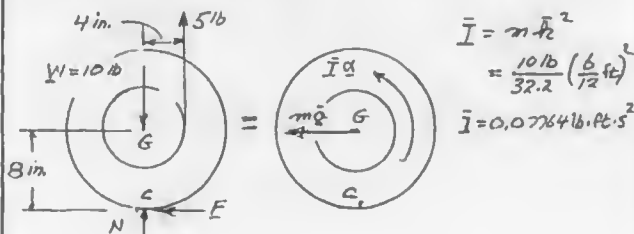
16.101



GIVEN:  $W = 10 \text{ lb}$   
 $r_o = 8 \text{ in.}$ ,  $r_c = 4 \text{ in.}$   
 $r_k = 6 \text{ in.}$ ,  $P = 5 \text{ lb}$   
 $\mu_s = 0.25$ ,  $\mu_k = 0.20$

FIND:  
 (a) DOES DISK SLIDE  
 (b)  $\alpha$  AND  $\bar{a}$

ASSUME DISK ROLLS:  $\bar{a} = r_c \alpha = \left(\frac{8}{12} \text{ ft}\right) \alpha$



$$+\uparrow \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (5 \text{ lb}) \left(\frac{4}{12} \text{ ft}\right) = (m \bar{a}) r + \bar{I} \alpha$$

$$1.6667 \text{ lb} \cdot \text{ft} = \frac{10 \text{ lb}}{32.2} \left(\frac{8}{12} \text{ ft}\right) \alpha + 0.07764 \alpha$$

$$1.6667 = 0.21586 \alpha$$

$$\alpha = 7.728 \text{ rad/s}^2$$

$$\bar{a} = r_c \alpha = \left(\frac{8}{12} \text{ ft}\right) (7.728 \text{ rad/s}^2) = 5.153 \text{ ft/s}^2$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F = m \bar{a}$$

$$F = \frac{10 \text{ lb}}{32.2} (5.153 \text{ ft/s}^2); F = 1.60 \text{ lb}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N - 10 \text{ lb} + 5 \text{ lb} = 0 \quad N = 5 \text{ lb}$$

$$F_m = \mu_s N = 0.25(5 \text{ lb}) = 1.25 \text{ lb}$$

SINCE  $F > F_m$ , DISK SLIDES

KNOWING THAT DISK SLIDES  $F = \mu_k N = 0.2(5)$   
 $F = 1.00 \text{ lb}$

$$+\rightarrow \Sigma M_G = \Sigma (M_G)_{\text{eff}}:$$

$$(5 \text{ lb}) \left(\frac{4}{12} \text{ ft}\right) - F \left(\frac{8}{12} \text{ ft}\right) = \bar{I} \alpha$$

$$(5 \text{ lb}) \left(\frac{4}{12} \text{ ft}\right) - (1.00 \text{ lb}) \left(\frac{8}{12} \text{ ft}\right) = 0.07764 \alpha$$

$$1.000 = 0.07764 \alpha$$

$$\alpha = 12.88 \text{ rad/s}^2 \quad \underline{\alpha = 12.88 \text{ rad/s}^2}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F = m \bar{a}$$

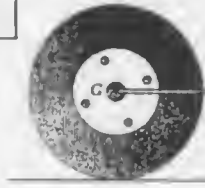
$$1.00 \text{ lb} = \frac{10 \text{ lb}}{32.2} \bar{a}$$

$$\bar{a} = 3.22 \text{ ft/s}^2$$

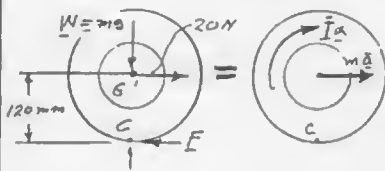
$$\underline{\bar{a} = 3.22 \text{ ft/s}^2}$$



16.102



GIVEN:  $m = 6 \text{ kg}$   
 $r_o = 120 \text{ mm}$ ,  $r_i = 60 \text{ mm}$   
 $\bar{r}_G = 90 \text{ mm}$   
 $P = 20 \text{ N}$   
 DISK ROLLS  
 FIND: (a)  $\alpha$  AND  $\bar{a}$   
 (b) MINIMUM  $\mu_s$



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{r}_G^2 = (6 \text{ kg})(0.09 \text{ m})^2 = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$F = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (20 \text{ N})(0.12 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$2.4 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \alpha$$

$$2.4 = 135.0 \times 10^{-3} \alpha$$

$$(a) \quad \alpha = 17.778 \text{ rad/s}^2; \quad \alpha = 17.78 \text{ rad/s}^2 \rightarrow$$

$$\bar{a} = r\alpha = (0.12 \text{ m})(17.778 \text{ rad/s}^2) = 2.133 \text{ m/s}^2; \quad \bar{a} = 2.13 \text{ m/s}^2 \rightarrow$$

$$(b) +\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N - mg = 0$$

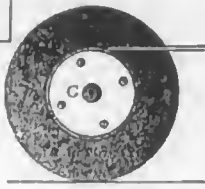
$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \quad N = 58.86 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: 20 \text{ N} - F = m\bar{a}$$

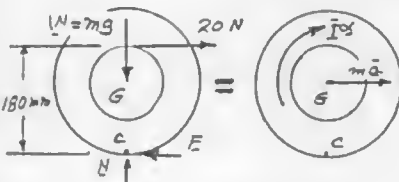
$$20 \text{ N} - F = (6 \text{ kg})(2.133 \text{ m/s}^2) \quad F = 7.20 \text{ N} \leftarrow$$

$$(\mu_s)_{\text{min}} = \frac{F}{N} = \frac{7.20 \text{ N}}{58.86 \text{ N}} \quad (\mu_s)_{\text{min}} = 0.122$$

16.103



GIVEN:  $m = 6 \text{ kg}$   
 $r_o = 120 \text{ mm}$ ,  $r_i = 60 \text{ mm}$   
 $\bar{r}_G = 90 \text{ mm}$   
 $P = 20 \text{ N}$   
 DISK ROLLS  
 FIND: (a)  $\alpha$  AND  $\bar{a}$   
 (b) MINIMUM  $\mu_s$



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{r}_G^2 = (6 \text{ kg})(0.09 \text{ m})^2 = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\bar{I} = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (20 \text{ N})(0.18 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$3.6 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \alpha$$

$$3.6 = 135 \times 10^{-3} \alpha$$

$$\alpha = 26.667 \text{ rad/s}^2 \quad \alpha = 26.7 \text{ rad/s}^2 \rightarrow$$

$$(a) \quad \bar{a} = r\alpha = (0.12 \text{ m})(26.667 \text{ rad/s}^2) = 3.2 \text{ m/s}^2$$

$$\bar{a} = 3.2 \text{ m/s}^2 \rightarrow$$

$$(b) +\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N - mg = 0$$

$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \quad N = 58.86 \text{ N} \uparrow$$

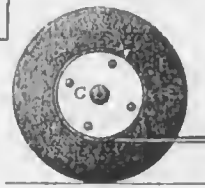
$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: 20 \text{ N} - F = m\bar{a}$$

$$20 \text{ N} - F = (6 \text{ kg})(3.2 \text{ m/s}^2)$$

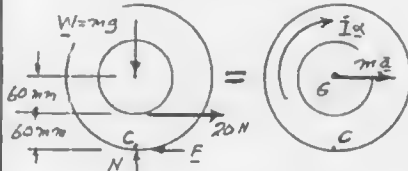
$$F = 0.8 \text{ N} \leftarrow$$

$$(\mu_s)_{\text{min}} = \frac{F}{N} = \frac{0.8 \text{ N}}{58.86 \text{ N}} \quad (\mu_s)_{\text{min}} = 0.0136$$

16.104



GIVEN:  $m = 6 \text{ kg}$   
 $r_o = 120 \text{ mm}$ ,  $r_i = 60 \text{ mm}$   
 $\bar{r}_G = 90 \text{ mm}$   
 $P = 20 \text{ N}$   
 DISK ROLLS  
 FIND: (a)  $\alpha$  AND  $\bar{a}$   
 (b) MINIMUM  $\mu_s$



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{r}_G^2 = (6 \text{ kg})(0.09 \text{ m})^2 = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\bar{I} = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (20 \text{ N})(0.06 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$1.2 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \alpha$$

$$1.2 = 135 \times 10^{-3} \alpha$$

$$(a) \quad \alpha = 8.889 \text{ rad/s}^2 \quad \alpha = 8.89 \text{ rad/s}^2 \rightarrow$$

$$\bar{a} = r\alpha = (0.12 \text{ m})(8.889 \text{ rad/s}^2) = 1.067 \text{ m/s}^2$$

$$\bar{a} = 1.067 \text{ m/s}^2 \rightarrow$$

$$(b) +\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N - mg = 0$$

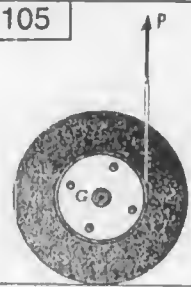
$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \quad N = 58.86 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: 20 \text{ N} - F = m\bar{a}$$

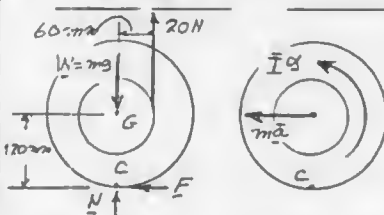
$$(20 \text{ N}) - F = (6 \text{ kg})(1.067 \text{ m/s}^2); \quad F = 13.6 \text{ N} \leftarrow$$

$$(\mu_s)_{\text{min}} = \frac{F}{N} = \frac{13.6 \text{ N}}{58.86 \text{ N}} \quad (\mu_s)_{\text{min}} = 0.231$$

16.105



GIVEN:  $m = 6 \text{ kg}$   
 $r_o = 120 \text{ mm}$ ,  $r_i = 60 \text{ mm}$   
 $\bar{r}_G = 90 \text{ mm}$   
 $P = 20 \text{ N}$   
 DISK ROLLS  
 FIND: (a)  $\alpha$  AND  $\bar{a}$   
 (b) MINIMUM  $\mu_s$



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{r}_G^2 = (6 \text{ kg})(0.09 \text{ m})^2 = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\bar{I} = 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (20 \text{ N})(0.06 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$1.2 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 40.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \alpha$$

$$1.2 = 135 \times 10^{-3} \alpha$$

$$(a) \quad \alpha = 8.889 \text{ rad/s}^2 \quad \alpha = 8.89 \text{ rad/s}^2 \rightarrow$$

$$\bar{a} = r\alpha = (0.12 \text{ m})(8.889 \text{ rad/s}^2) = 1.067 \text{ m/s}^2$$

$$\bar{a} = 1.067 \text{ m/s}^2 \leftarrow$$

$$(b) +\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N + 20 \text{ N} - mg = 0$$

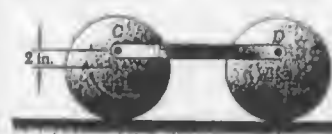
$$N + 20 \text{ N} - (6 \text{ kg})(9.81 \text{ m/s}^2) \quad N = 38.86 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F = m\bar{a}$$

$$F = (6 \text{ kg})(1.067 \text{ m/s}^2) \quad F = 6.4 \text{ N} \leftarrow$$

$$(\mu_s)_{\text{min}} = \frac{F}{N} = \frac{6.4 \text{ N}}{38.86 \text{ N}} \quad (\mu_s)_{\text{min}} = 0.165$$

16.106



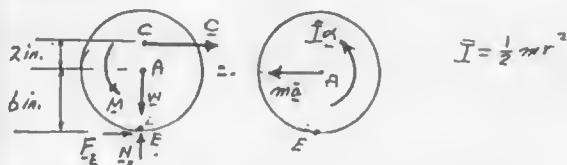
GIVEN: 4-16 DISKS  
3-16 ROD  
 $M = 1.516 \text{ lb} \cdot \text{ft}^2$   
DISKS ROLL  
FIND: (a)  $\bar{a}$  OF DISKS  
(b) HORIZ. COMP. OF  $\bar{D}$   
ACTING ON DISK B

$$\alpha_A = \alpha_B = \alpha$$

$$\bar{a}_A = \bar{a}_B = \bar{a} = r\alpha$$

DISK A:

$$W = 4/16$$



$$+\sum M_E = \Sigma (M_E)_{\text{eff}}: M - C\left(\frac{6}{12} \text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$$

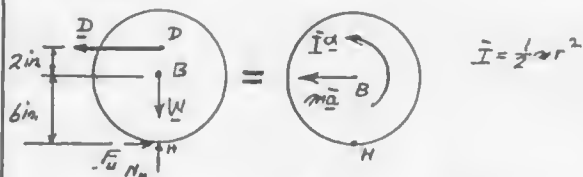
$$1.516 \text{ lb} \cdot \text{ft} - \frac{6}{12} C = (\pi r d)r + \frac{1}{2} \pi r^2 \alpha$$

$$= \frac{3}{2} \pi r^2 \alpha$$

$$1.516 \text{ lb} \cdot \text{ft} - \frac{6}{12} C = \frac{3}{2} \frac{4/16}{32.2} \left(\frac{6}{12} \text{ ft}\right)^2 \alpha$$

$$1.5 - \frac{1}{2} C = 0.046584 \alpha \quad (1)$$

DISK B:



$$+\sum M_H = \Sigma (M_H)_{\text{eff}}: D\left(\frac{6}{12} \text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha = \pi r^2 \alpha + \frac{1}{2} \pi r^2 \alpha$$

$$D\left(\frac{6}{12}\right) = \frac{3}{2} \frac{4/16}{32.2} \left(\frac{6}{12} \text{ ft}\right)^2 \alpha$$

$$\frac{2}{3} D = 0.046584 \alpha \quad (2)$$

ROD CD:

$$\bar{a}_R = \bar{a}_C = \bar{a}_A + \bar{a}_{C/A}$$

$$\bar{a}_R = \frac{6}{12} \alpha + \frac{2}{12} \alpha = \frac{8}{12} \alpha$$

$$+\sum F_x = \Sigma (F_x)_{\text{eff}}: C - D = m \bar{a}_R$$

$$C - D = \frac{3/16}{32.2} \left(\frac{8}{12} \alpha\right)$$

MULTIPLY

$$\text{BY } \frac{3}{2}: \frac{2}{3} C - \frac{6}{3} D = 0.041408 \alpha \quad (3)$$

$$\text{ADD (1), (2), (3): } 1.5 - \frac{1}{2} C + \frac{2}{3} D + \frac{2}{3} C - \frac{2}{3} D = 0.13458 \alpha$$

$$\alpha = 11.146 \text{ rad/s}^2$$

$$(a) \bar{a}_A = \bar{a}_B = r\alpha = \left(\frac{6}{12} \text{ ft}\right)(11.146 \text{ rad/s}^2) = 5.573 \text{ ft/s}^2$$

$$\bar{a}_A = \bar{a}_B = 5.57 \text{ ft/s}^2 \leftarrow$$

(b) SUBSTITUTE FOR  $\alpha$  IN (2)

$$\frac{2}{3} D = 0.046584(11.146)$$

$$D = 0.77916 \text{ lb} \leftarrow$$

16.107



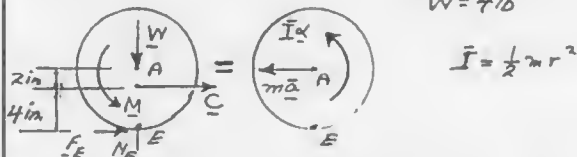
GIVEN: 4-16 DISK  
3-16 ROD  
 $M = 1.516 \text{ lb} \cdot \text{ft}^2$   
DISKS ROLL  
FIND: (a)  $\bar{a}$  OF DISKS  
(b) HORIZ. COMP. OF  $\bar{D}$   
ACTING ON DISK B

$$\alpha_A = \alpha_B = \alpha$$

$$\bar{a}_A = \bar{a}_B = \bar{a} = r\alpha$$

DISK A:

$$W = 4/16$$



$$+\sum M_E = \Sigma (M_E)_{\text{eff}}: M - C\left(\frac{4}{12} \text{ in.}\right) = (m\bar{a})r + \bar{I}\alpha$$

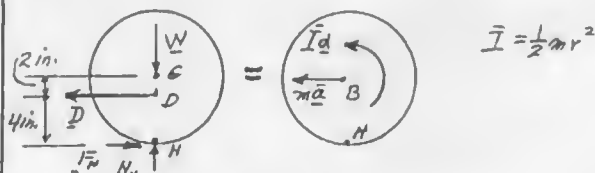
$$1.516 \text{ lb} \cdot \text{ft} - \frac{4}{12} C = (\pi r d)r + \frac{1}{2} \pi r^2 \alpha$$

$$= \frac{3}{2} \pi r^2 \alpha$$

$$1.516 \text{ lb} \cdot \text{ft} - \frac{4}{12} C = \frac{3}{2} \frac{4/16}{32.2} \left(\frac{4}{12} \text{ ft}\right)^2 \alpha$$

$$1.5 - \frac{1}{3} C = 0.046584 \alpha \quad (1)$$

DISK B:



$$+\sum M_H = \Sigma (M_H)_{\text{eff}}: D\left(\frac{4}{12} \text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha = \pi r^2 \alpha + \frac{1}{2} \pi r^2 \alpha$$

$$D\left(\frac{4}{12}\right) = \frac{3}{2} \frac{4/16}{32.2} \left(\frac{4}{12} \text{ ft}\right)^2 \alpha$$

$$\frac{1}{3} D = 0.046584 \alpha \quad (2)$$

ROD CD

$$\bar{a}_R = \bar{a}_C = \bar{a}_A + \bar{a}_{C/A}$$

$$\bar{a}_R = \frac{4}{12} \alpha + \frac{2}{12} \alpha = \frac{6}{12} \alpha$$

$$+\sum F_x = \Sigma (F_x)_{\text{eff}}: C - D = m \bar{a}_R$$

$$C - D = \frac{3/16}{32.2} \left(\frac{6}{12} \alpha\right)$$

MULTIPLY

$$\text{BY } \frac{1}{2}: \frac{1}{2} C - \frac{1}{2} D = 0.010352 \alpha \quad (3)$$

$$\text{ADD (1), (2), (3): } 1.5 - \frac{1}{3} C + \frac{1}{3} D + \frac{1}{2} C - \frac{1}{2} D = 0.10352 \alpha$$

$$\alpha = 14.490 \text{ rad/s}^2$$

$$(a) \bar{a}_A = \bar{a}_B = r\alpha = \left(\frac{6}{12} \text{ ft}\right)(14.490 \text{ rad/s}^2) = 7.245 \text{ ft/s}^2$$

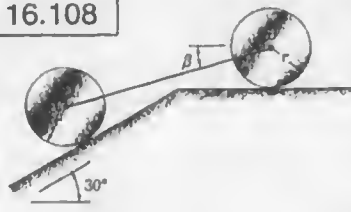
$$\bar{a}_A = \bar{a}_B = 7.24 \text{ ft/s}^2 \leftarrow$$

(b) SUBSTITUTE FOR  $\alpha$  IN (2)

$$\frac{1}{3} D = 0.046584(14.490); \quad D = 2.015 \text{ lb}$$

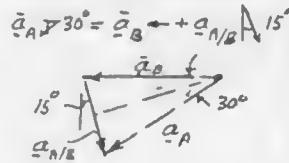
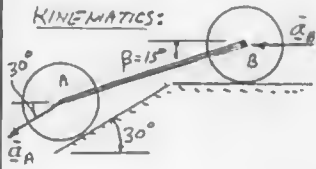
$$D = 2.02 \text{ lb} \leftarrow$$

16.108



GIVEN: DISKS OF MASS  $m$   
AND ROLL ON SURFACES.  
RELEASE FROM REST  
WHEN  $\beta = 15^\circ$ .  
FIND: (a)  $\bar{a}_A$ , (b)  $\bar{a}_B$

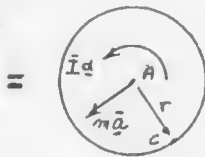
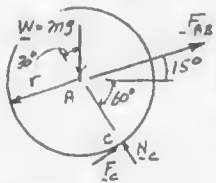
KINEMATICS:



ISOSCELES TRIANGLE  $\therefore \bar{a}_A = \bar{a}_B$

Denote by  $\bar{a} = \bar{a}_A = \bar{a}_B$

KINETICS: DISK A:



$$\bar{a} = r\alpha$$

$$I = \frac{1}{2}mr^2$$

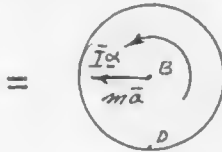
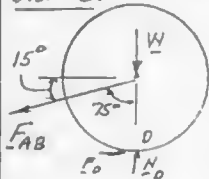
$$+\uparrow \Sigma M_C = \Sigma (M_C)_{eff}:$$

$$(mg \sin 30^\circ)r - (F_{AB} \sin 75^\circ)r = (m\bar{a})r + \bar{I}\alpha$$

$$= (mr\alpha)r + \frac{1}{2}mr^2\alpha$$

$$mgr \sin 30^\circ - F_{AB}r \sin 75^\circ = \frac{3}{2}mr^2\alpha \quad (1)$$

DISK B:



$$\bar{I} = \frac{1}{2}mr^2$$

$$+\uparrow \Sigma M_D = \Sigma (M_D)_{eff}: (F_{AB} \sin 75^\circ)r = (m\bar{a})r + \bar{I}\alpha$$

$$= (mr\alpha)r + \frac{1}{2}mr^2\alpha$$

$$F_{AB}r \sin 75^\circ = \frac{3}{2}mr^2\alpha \quad (2)$$

$$EQ(1) + EQ(2): mgr \sin 30^\circ = 3mr^2\alpha$$

$$\alpha = \frac{g}{3r} \sin 30^\circ = \frac{1}{6} \frac{g}{r}$$

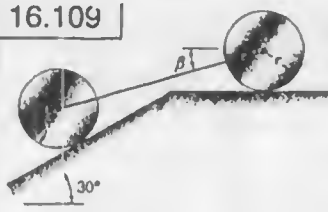
$$\bar{a} = r\alpha = r \left( \frac{1}{6} \frac{g}{r} \right) = \frac{1}{6}g$$

RECALL  $\bar{a}_A = \bar{a}_B = \bar{a}$ 

$$\bar{a}_A = \frac{1}{6}g \nearrow 30^\circ$$

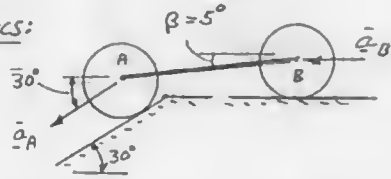
$$\bar{a}_B = \frac{1}{6}g \leftarrow$$

16.109

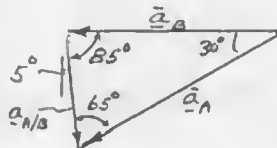


GIVEN: DISKS OF MASS  $m$   
AND ROLL ON SURFACES.  
RELEASE FROM REST  
WHEN  $\beta = 5^\circ$ .  
FIND: (a)  $\bar{a}_A$ , (b)  $\bar{a}_B$

KINEMATICS:



$$\bar{a}_A \nearrow 30^\circ = \bar{a}_B \leftarrow + a_{A/B} \nearrow 5^\circ$$



LAW OF SINES

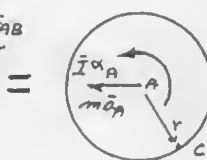
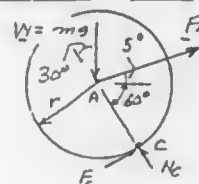
$$\frac{\bar{a}_B}{\sin 65^\circ} = \frac{\bar{a}_A}{\sin 85^\circ}$$

$$\bar{a}_B = 0.90852 \bar{a}_A$$

$$\text{SINCE } \bar{a}_B = r\alpha_B \text{ AND } \bar{a}_A = r\alpha_A$$

$$\alpha_B = 0.90852 \alpha_A \quad (1)$$

KINETICS: DISK A



$$\bar{a}_A = r\alpha_A$$

$$I = \frac{1}{2}mr^2$$

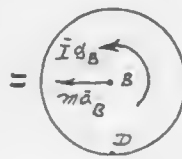
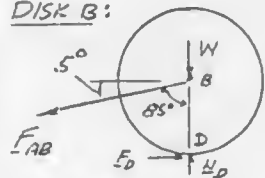
$$+\uparrow \Sigma M_C = \Sigma (M_C)_{eff}:$$

$$(mg \sin 30^\circ)r - (F_{AB} \sin 65^\circ)r = (m\bar{a})r + \bar{I}\alpha_A$$

$$= (mr\alpha_A)r + \frac{1}{2}mr^2\alpha_A$$

$$mg \sin 30^\circ - F_{AB} \sin 65^\circ = \frac{3}{2}mr\alpha_A \quad (2)$$

DISK B:



$$I = \frac{1}{2}mr^2$$

$$\bar{a}_B = r\alpha_B$$

$$+\uparrow \Sigma M_D = \Sigma (M_D)_{eff}: (F_{AB} \sin 85^\circ)r = (m\bar{a}_B)r + \bar{I}\alpha_B$$

$$= (mr\alpha_B)r + \frac{1}{2}mr^2\alpha_B$$

$$F_{AB} = \frac{3}{2} \frac{mr}{\sin 85^\circ} \alpha_B \quad (3)$$

SUBSTITUTE FOR  $\bar{a}_B$  FROM EQ(1) AND  $F_{AB}$  FROM EQ(3) INTO EQ(2)

$$mg \sin 30^\circ - \frac{3}{2}mr \frac{\sin 65^\circ}{\sin 85^\circ} (0.90852 \alpha_A) = \frac{3}{2}mr\alpha_A$$

$$0.5 \frac{g}{r} = \frac{3}{2} (0.82654 + 1) \alpha_A$$

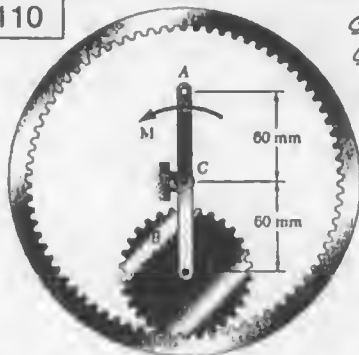
$$\alpha_A = 0.1825 \frac{g}{r} : \bar{a}_A = r\alpha_A = 0.1825g$$

$$\bar{a}_A = 0.1825g \nearrow 30^\circ$$

$$EQ(1) \quad \bar{a}_B = 0.90852 \bar{a}_A = (0.90852)(0.1825g) = 0.1659g$$

$$\bar{a}_B = 0.1659g \leftarrow$$

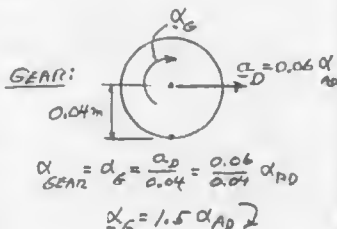
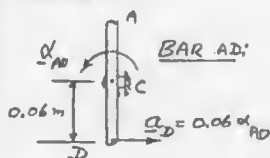
16.110

GIVEN:  $M = 1.25 \text{ N} \cdot \text{m}$ GEAR,  $m = 1.8 \text{ kg}$  $r = 32 \text{ mm}$ BAR AD:  $m_{AD} = 2.5 \text{ kg}$ 

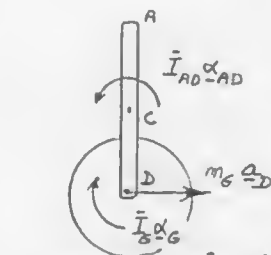
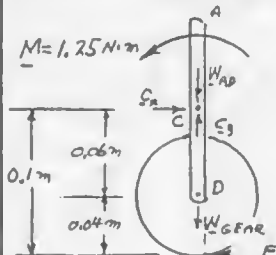
FIND:

(a)  $\alpha_{AD}$ (b)  $a_D$ 

KINEMATICS:



KINETICS: BAR AND GEAR

 $+\sum M_C = \sum (M_C)_{eff}$ 

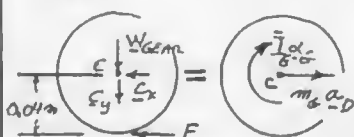
$$M - F(0.1 \text{ m}) = \bar{I}_{AD} \alpha_{AD} + (m_G a_D)(0.06 \text{ m}) - \bar{I}_G \alpha_G$$

$$M - 0.1 F = (3 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \alpha_{AD} + [(1.8 \text{ kg})(0.06 \text{ m})](0.06 \text{ m}) - [(1.843 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(1.5 \alpha_{AD})]$$

$$M - 0.1 F = 3 \times 10^{-3} \alpha_{AD} + 6.48 \times 10^{-3} \alpha_{AD} - 2.765 \times 10^{-3} \alpha_{AD}$$

$$1.25 \text{ N} \cdot \text{m} - 0.1 F = 6.715 \times 10^{-3} \alpha_{AD} \quad (1)$$

GEAR



$$+\sum M_C = \sum (M_C)_{eff}$$

$$F(0.04 \text{ m}) = \bar{I}_G \alpha_G$$

$$0.04 F = (1.843 \times 10^{-3}) \alpha_G$$

$$0.04 F = (1.843 \times 10^{-3}) 1.5 \alpha_{AD}$$

$$F = 69.12 \times 10^{-3} \alpha_{AD}$$

SUBSTITUTE FOR F IN EQ(1)

$$1.25 - (0.1)(69.12 \times 10^{-3}) \alpha_{AD} = 6.715 \times 10^{-3} \alpha_{AD}$$

$$1.25 = 13.627 \times 10^{-3} \alpha_{AD}$$

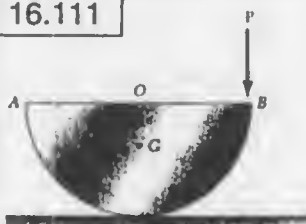
$$\alpha_{AD} = 91.73 \text{ rad/s}^2$$

$$\alpha_{AD} = 91.73 \text{ rad/s}^2$$

$$a_D = (CD) \alpha_{AD} = (0.06 \text{ m})(91.73 \text{ rad/s}^2) = 5.50 \text{ m/s}^2$$

$$a_D = 5.50 \text{ m/s}^2$$

16.111



GIVEN: HALF CYLINDER

MASS = m

ROLLING WITH NO SLIPPING

FIND: (a)  $\alpha$ (b)  $(\dot{\theta}_s)_{min}$ 

KINEMATICS:

ASSUME  $\alpha$ ,  $a_G = r\alpha \rightarrow$ 

FROM INSIDE CONTACT OF TEST

 $OG = 4r/3\pi$ 

$$\vec{a} = \vec{a}_G + \vec{a}_{G/O} = [a_G \rightarrow] + [(OG)\alpha \leftarrow]$$

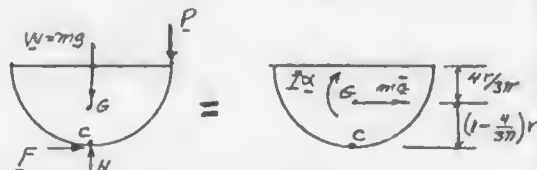
$$\vec{a} = [r\alpha \rightarrow] + [\frac{4r}{3\pi}\alpha \leftarrow] = r(1 - \frac{4}{3\pi})\alpha \rightarrow$$

KINETICS:  $m = \text{MASS OF HALF CYLINDER}; I_O = \frac{1}{2}mr^2$ 

$$I_O = \bar{I} + m(OG)^2; \frac{1}{2}mr^2 = \bar{I} + (\frac{4r}{3\pi})^2 m$$

$$\bar{I} = mr^2(\frac{1}{2} - \frac{16}{9\pi^2})$$

(a)



$$+\sum M_C = \sum (M_C)_{eff}; Pr = \bar{I}\alpha + (m\bar{a})(1 - \frac{4}{3\pi})r$$

$$Pr = mr^2(\frac{1}{2} - \frac{16}{9\pi^2})\alpha + mr(1 - \frac{4}{3\pi})\alpha(1 - \frac{4}{3\pi})r$$

$$Pr = mr^2(\frac{1}{2} - \frac{16}{9\pi^2})\alpha + mr^2(1 - \frac{8}{3\pi} + \frac{16}{9\pi^2})\alpha$$

$$Pr = mr^2(\frac{1}{2} - \frac{8}{3\pi})\alpha$$

$$Pr = mr^2(0.6517)\alpha$$

$$\alpha = 1.5357 \frac{P}{mr} \quad \alpha = 1.536 \frac{P}{mr}$$

(b)  $+\sum F_x = \sum (F_x)_{eff}; F = m\bar{a}$ 

$$F = mr(1 - \frac{4}{3\pi})\alpha = mr(0.57559)\alpha$$

$$F = mr(0.57559)(1.5357 \frac{P}{mr}) = 0.8839 P$$

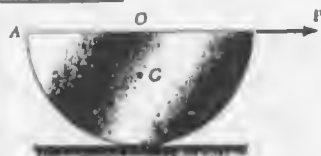
$$+\sum F_y = \sum (F_y)_{eff}; N - P - mg = 0$$

$$N = mg + P$$

$$(\dot{\theta}_s)_{min} = \frac{F}{N} = \frac{0.8839 P}{mg + P}$$

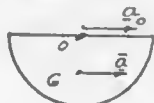
$$(\dot{\theta}_s)_{min} = 0.884 \frac{P}{mg + P}$$

16.112



GIVEN: HALF CYLINDER  
MASS =  $m$   
ROLLING WITH  
NO SLIPPING  
FIND: (a)  $\alpha$   
(b)  $(4s)_{min}$

KINEMATICS: ASSUME  $\alpha \downarrow$ ,  $a_o = r\alpha \rightarrow$   
FROM INSIDE COVER OF TEXT  
 $OG = 4r/3\pi$

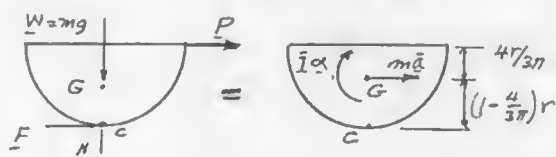


$$\vec{a} = \vec{a}_o + \vec{a}_{G/o} = [a_o \rightarrow] + [(OG)\alpha \leftarrow]$$

$$\vec{a} = [r\alpha \rightarrow] + \left[\frac{4r}{3\pi}\alpha \leftarrow\right] = r\left(1 - \frac{4}{3\pi}\right)\alpha \rightarrow$$

KINETICS:  $m$  = MASS OF HALF CYLINDER;  $I_o = \frac{1}{2}mr^2$   
 $I_o = \bar{I} + m(OG)^2$ ;  $\frac{1}{2}mr^2 = \bar{I} + \left(\frac{4r}{3\pi}\right)^2 m$   
 $\bar{I} = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$

(a)



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}: Pr = \bar{I}\alpha + (m\bar{a})\left(1 - \frac{4}{3\pi}\right)r$$

$$Pr = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\alpha + mr\left(1 - \frac{4}{3\pi}\right)\alpha\left(1 - \frac{4}{3\pi}\right)r$$

$$Pr = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\alpha + mr^2\left(1 - \frac{8}{3\pi} + \frac{16}{9\pi^2}\right)\alpha$$

$$Pr = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\alpha$$

$$Pr = mr^2(0.65117)\alpha$$

$$\alpha = 1.5367 \frac{P}{mr} \quad \alpha = 1.536 \frac{P}{mr}$$

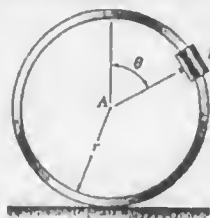
(b)  $+\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$   
 $P - F = m\bar{a} = mr\left(1 - \frac{4}{3\pi}\right)\alpha$   
 $P - F = mr\left(1 - \frac{4}{3\pi}\right)\left(1.5367 \frac{P}{mr}\right) = 0.8839 P$   
 $F = P - 0.8839 P = 0.1161 P$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$   $N - W = 0$   
 $N = W$

$$(4s)_{min} = \frac{F}{N} = \frac{0.1161 P}{W}$$

$$(4s)_{min} = 0.116 \frac{P}{W}$$

16.113



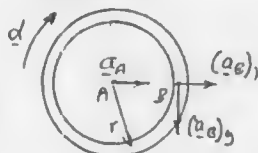
GIVEN:  
 $m_h$  = MASS OF CLAMP B  
 $m_h$  = MASS OF HOOD  
 $m_h = 3m_B$   
 $\theta = 90^\circ$

SYSTEM IS RELEASED  
AND ROLLS WITHOUT SLIDING

FIND: (a)  $\alpha$   
(b)  $(a_B)_x$  AND  $(a_B)_y$

KINEMATICS:

$$a_A = r\alpha \rightarrow, \quad a_{B/A} = r\alpha \downarrow$$



$$a_B = a_A \rightarrow + a_{B/A} \downarrow$$

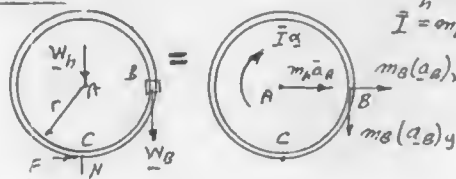
$$a_B = r\alpha \rightarrow + r\alpha \downarrow$$

$$(a_B)_x = r\alpha \rightarrow \quad (a_B)_y = r\alpha \downarrow$$

KINETICS:

$$m_h = 3m_B$$

$$\bar{I} = m_h r^2 = 3m_B r^2$$



(a)

$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$

$$W_h r = \bar{I}\alpha + m_h \bar{a}_A r + m_B (a_B)_x r + m_B (a_B)_y r$$

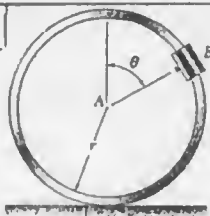
$$m_B g r = 3m_B r^2 \alpha + (3m_B) r^2 \alpha + m_B r^2 \alpha + m_B r^2 \alpha$$

$$g r = 8 r^2 \alpha$$

$$\alpha = \frac{1}{8} \frac{g}{r}$$

(b)  $(a_B)_x = r\alpha = \frac{1}{8} g \rightarrow$ ,  $(a_B)_y = r\alpha = \frac{1}{8} g \downarrow$

16.114



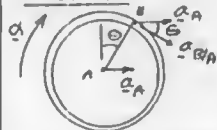
GIVEN:  $m_h$  = MASS OF HOOD  
 $m_B$  = MASS OF CLAMP  
SYSTEM IS RELEASED  
AND ROLLS WITHOUT  
SLIDING.  
FIND:  $\alpha$  IN TERMS  
OF  $m_B$ ,  $m_h$ ,  $r$ , AND  $\theta$

KINEMATICS:

$$a_A = r\alpha \rightarrow \quad a_{B/A} = r\alpha \swarrow \searrow$$

$$a_B = a_A \rightarrow + a_{B/A} \swarrow \searrow$$

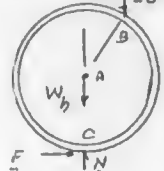
$$a_B = r\alpha \rightarrow + r\alpha \swarrow \searrow$$



KINETICS:

$$r \sin \theta$$

$$W_B = m_B g$$



$$\bar{I} = m_h r^2$$

$$m_h r \alpha$$

$$m_B r \alpha$$

$$m_B r \alpha$$

$$r \cos \theta$$

$$r$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}: W_B r \sin \theta = \bar{I}\alpha + m_h a_A r + m_B r \alpha (r + r \cos \theta) + m_B r \alpha \sin \theta (r \sin \theta) + m_B r \alpha \cos \theta (r + r \cos \theta)$$

(CONTINUED)

# 16.114 continued

$$m_B g r \sin \theta = m_B r^2 \alpha + m_B (r \alpha) r + m_B r \alpha (1 + \cos \theta) (r + r \cos \theta) + m_B r \alpha \sin \theta (r \sin \theta)$$

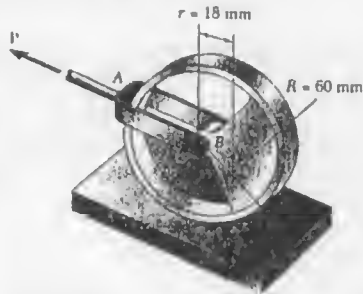
$$m_B g r \sin \theta = 2 m_B r^2 \alpha + m_B r^2 \alpha [(1 + \cos \theta)^2 + \sin^2 \theta]$$

$$= 2 m_B r^2 \alpha + m_B r^2 \alpha [1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta]$$

$$m_B g r \sin \theta = r^2 \alpha [2 m_B + m_B (2 + 2 \cos \theta)]$$

$$\alpha = \frac{g}{2r} \frac{m_B \sin \theta}{m_B (1 + \cos \theta)}$$

# 16.115 and 16.116



GIVEN:  $m = 1.5 \text{ kg}$   
 $R = 60 \text{ mm}$

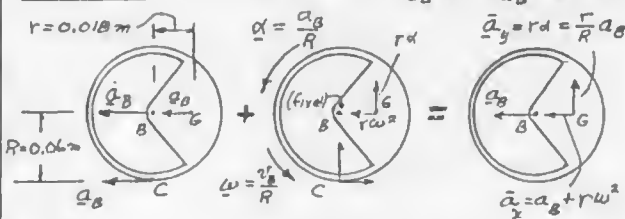
## PROBLEM 16.115:

FIND:  $P$  WHEN  
 $v_B = 0.35 \text{ m/s} \leftarrow$   
 $a_B = 1.2 \text{ m/s}^2 \leftarrow$

## PROBLEM 16.116:

FIND:  $P$  WHEN  
 $v_B = 0.35 \text{ m/s} \leftarrow$   
 $a_B = 1.2 \text{ m/s}^2 \leftarrow$

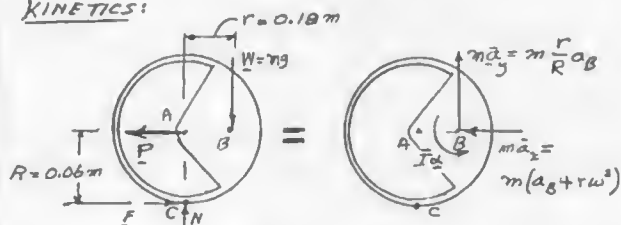
KINEMATICS: CHOOSE POSITIVE  $v_B$  AND  $a_B$  TO LEFT



TRANS. WITH B + ROTATION ABOUT B = ROLLING MOTION

$$\bar{a} = [a_B + r \omega^2] \leftarrow + \left[ \frac{r}{R} a_B \right] \uparrow$$

KINETICS:



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}:$$

$$PR - Wr = (m \bar{a}_y) r + (m \bar{a}_x) R + \bar{I} \alpha$$

$$PR - mg r = m \left( \frac{r}{R} a_B \right) r + m (a_B + r \omega^2) R + m \bar{I} \frac{a_B}{R}$$

$$= m a_B \left( \frac{r^2}{R} + R + \frac{R^2}{R} \right) + m r \left( \frac{v_B^2}{R} \right) R$$

$$P = mg \left( \frac{r}{R} \right) + m a_B \left( 1 + \frac{r^2 + R^2}{R^2} \right) + m \frac{r}{R^2} v_B^2 \quad (1)$$

(CONTINUED)

# 16.115 and 16.116 continued

SUBSTITUTE:  $m = 1.5 \text{ kg}$ ,  $r = 0.018 \text{ m}$ ,  $R = 0.06 \text{ m}$ ,

$\bar{I}_C = 0.044 \text{ kg} \cdot \text{m}^2$  AND  $g = 9.81 \text{ m/s}^2$  IN EQ (1)

$$P = 1.5(9.81) \frac{0.018}{0.06} + 1.5(a_B) \left( 1 + \frac{0.018^2 + 0.06^2}{0.06^2} \right) + 1.5 \frac{0.018}{0.06^2} v_B^2$$

$$P = 4.4145 + 2.4417 a_B + 7.5 v_B^2 \quad (2)$$

PROBLEM 16.115:  $v_B = 0.35 \text{ m/s} \leftarrow$ ;  $v_B = +0.35 \text{ m/s}$   
 $a_B = 1.2 \text{ m/s}^2 \leftarrow$ ;  $a_B = +1.2 \text{ m/s}^2$

SUBSTITUTE IN EQ (2):

$$P = 4.4145 + 2.4417(+1.2) + 7.5(+0.35)^2$$

$$= 4.4145 + 2.9300 + 0.9188 = 8.263 \text{ N}$$

$$P = 8.26 \text{ N} \leftarrow$$

PROBLEM 16.116: RECALL WE ASSUMED POSITIVE TO LEFT

$$v_B = 0.35 \text{ m/s} \rightarrow$$

$$a_B = 1.2 \text{ m/s} \rightarrow$$

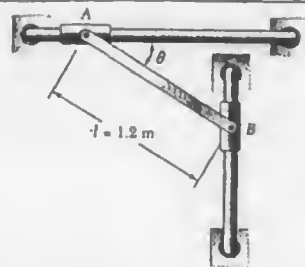
SUBSTITUTE INTO EQ (2):

$$P = 4.4145 + 2.4417(-1.2) + 7.5(-0.35)^2$$

$$= 4.4145 - 2.9300 + 0.9188 = 2.403 \text{ N}$$

$$P = 2.40 \text{ N} \leftarrow$$

# 16.117



GIVEN:

$$m = 10 \text{ kg}$$

$$\theta = 25^\circ$$

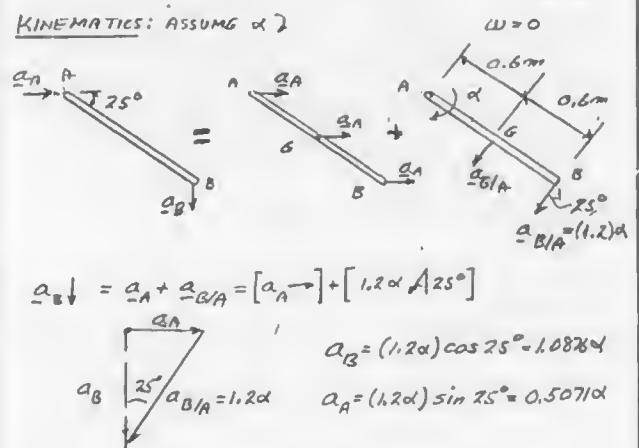
RELEASE FROM REST

FIND:

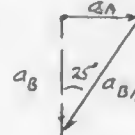
$$(a) \bar{A}$$

$$(b) \bar{B}$$

KINEMATICS: ASSUME  $\alpha$



$$\bar{a}_B \downarrow = \bar{a}_A + \bar{a}_{B/A} = [\bar{a}_A \rightarrow] + [1.2 \alpha \uparrow 25^\circ]$$



$$a_B = (1.2 \alpha) \cos 25^\circ = 1.087 \alpha$$

$$a_A = (1.2 \alpha) \sin 25^\circ = 0.5071 \alpha$$

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A} = [\bar{a}_A \rightarrow] + [0.6 \alpha \uparrow 25^\circ]$$

$$\bar{a}_B = [0.5071 \alpha \rightarrow] + [0.6 \alpha \uparrow 25^\circ]$$

$$\bar{a}_x = (a_B)_x = [0.5071 \alpha \rightarrow] + [0.2536 \alpha \rightarrow]$$

$$\bar{a}_x = 0.2535 \alpha \rightarrow$$

$$\bar{a}_y = [0.6 \alpha \cos 25^\circ \downarrow] = 0.5438 \alpha \downarrow$$

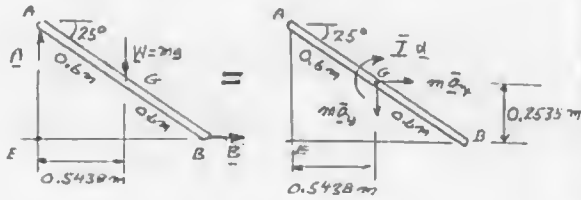
(CONTINUED)

# 16.117 continued

WE HAVE FOUND FOR  $\alpha$ :

$$\bar{a}_x = 0.2535\alpha \rightarrow; \bar{a}_y = 0.5438\alpha \downarrow$$

KINETICS:  $\bar{I} = \frac{1}{12} mL^2 = \frac{1}{12} m(1.2\text{ m})^2$



$$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}: mg(0.5438\text{ m}) = \bar{I}\alpha + m\bar{a}_x(0.2535\text{ m}) + m\bar{a}_y(0.5438\text{ m})$$

$$mg(0.5438) = \frac{1}{12} m(1.2)^2 \alpha + m(0.2535)^2 \alpha + m(0.5438)^2 \alpha$$

$$g(0.5438) = 0.48\alpha \quad \alpha = 1.133g = 11.11 \text{ rad/s}^2 \quad \leftarrow$$

$$(a) +\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: A - mg = -m\bar{a}_y = -m(0.5438\alpha)$$

$$A - 10(9.81) = -(10)(0.5438)(11.11)$$

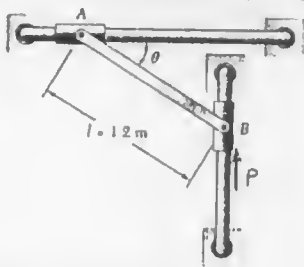
$$A = 98.1 - 60.44 = 37.66 \text{ N} \quad \underline{A = 37.7 \text{ N} \uparrow} \quad \leftarrow$$

$$(b) +\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: B = m\bar{a}_x = m(0.2535\alpha)$$

$$B = 10(0.2535)(11.11)$$

$$B = 28.18 \text{ N} \quad \underline{B = 28.2 \text{ N} \rightarrow} \quad \leftarrow$$

# 16.118



GIVEN:

$$m = 10 \text{ kg}$$

$$\theta = 25^\circ$$

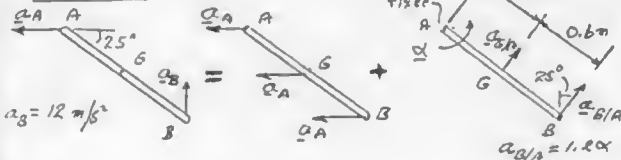
$$a_B = 12 \text{ m/s}^2 \uparrow$$

$$\omega = 0$$

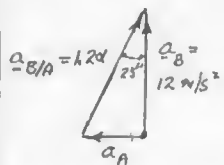
FIND: (a)  $\underline{P}$   
(b)  $\underline{A}$

KINEMATICS:

$$\omega = 0$$



$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}; [12 \text{ m/s}^2 \uparrow] = [\underline{a}_A \rightarrow] + [1.2\alpha \uparrow 25^\circ]$$



$$a_B = a_A + a_{B/A} \cos 25^\circ$$

$$12 \text{ m/s}^2 = (1.2\alpha) \cos 25^\circ$$

$$\alpha = 11.034 \text{ rad/s}^2 \quad \leftarrow$$

$$\underline{a}_A = 12 \tan 25^\circ = 5.596 \text{ m/s}^2 \quad \leftarrow$$

$$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}: \underline{a}_G = [5.596 \leftarrow] + [0.6\alpha \uparrow 25^\circ]$$

$$\underline{a}_G = [5.596 \leftarrow] + [0.6(11.034) \uparrow 25^\circ]$$

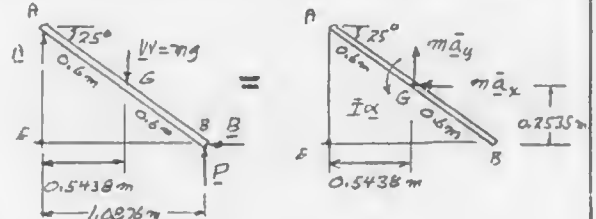
$$\underline{a}_x = (a_G)_x = 7.798 \text{ m/s}^2 \quad \leftarrow$$

$$\underline{a}_y = (a_G)_y = 6.00 \text{ m/s}^2 \uparrow$$

(CONTINUED)

# 16.118 continued

KINETICS:  $\bar{I} = \frac{1}{12} mL^2 = \frac{1}{12} m(1.2)^2$



(a)

$$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}: P(1.0876) - W(0.5438) = \bar{I}\alpha + m\bar{a}_x(0.2535) + m\bar{a}_y(0.5438)$$

$$W = mg = 10(9.81) = 98.1 \text{ N}$$

$$\bar{I}\alpha = \frac{1}{12} mL^2 \alpha = \frac{1}{12} (10)(1.2)^2 (11.034) = 13.24 \text{ N}\cdot\text{m}$$

$$m\bar{a}_x = (10)(7.798) = 77.98 \text{ N}$$

$$m\bar{a}_y = (10)(6.00) = 60 \text{ N}$$

$$P(1.0876) - (98.1)(0.5438) = 13.24 + (77.98)(0.2535) + (60)(0.5438)$$

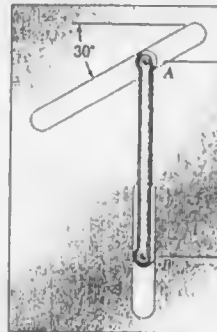
$$P(1.0876) - 53.347 = 13.24 + 7.096 + 32.628$$

$$P(1.0876) = 106.311; \quad P = 97.748 \text{ N} \quad \underline{P = 97.7 \text{ N} \uparrow} \quad \leftarrow$$

$$(b) +\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: A - W + P = m\bar{a}_y$$

$$A - 98.1 + 97.748 = 60 \text{ N}; \quad A = 60.4 \text{ N} \uparrow \quad \leftarrow$$

# 16.119



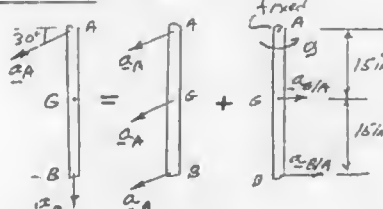
GIVEN:

$$W = 8 \text{ lb}$$

ROD RELEASED FROM REST

FIND: (a)  $\underline{\alpha}$   
(b)  $\underline{\beta}$

KINEMATICS  $\omega = 0$



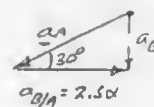
$$a_{B/A} = \left(\frac{15}{12} \text{ ft}\right) \alpha \rightarrow$$

$$= 1.25\alpha \rightarrow$$

$$a_{B/A} = \left(\frac{30}{12} \text{ ft}\right) \alpha \rightarrow$$

$$= 2.5\alpha \rightarrow$$

$$a_B = a_A + a_{B/A}: [a_B \uparrow] = [a_A \uparrow 30^\circ] + [1.25\alpha \rightarrow]$$



$$a_A = \frac{2.5\alpha}{\cos 30^\circ} = 2.887\alpha \uparrow 30^\circ$$

$$a_B = (2.5\alpha) \tan 30^\circ = 1.443\alpha \downarrow$$

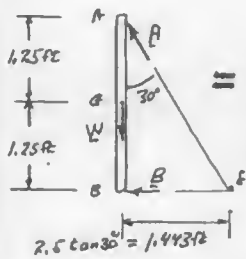
$$\underline{\ddot{a}} = \underline{a}_G = \underline{a}_A + \underline{a}_{G/A}; \underline{\ddot{a}} = [2.887\alpha \uparrow 30^\circ] + [1.25\alpha \rightarrow]$$

$$\underline{\ddot{a}}_x = [2.5\alpha \leftarrow] + [1.25\alpha \rightarrow] = 1.25\alpha \leftarrow$$

$$\underline{\ddot{a}}_y = [1.443\alpha \downarrow] = 1.443\alpha \downarrow$$

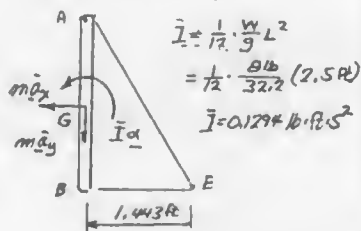
(CONTINUED)

# 16.119 continued



WE HAVE:

$$\bar{a}_x = 1.25\alpha \leftarrow ; \bar{a}_y = 1.443\alpha \downarrow$$



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff} : W(1.443 \text{ ft}) = \bar{I}\alpha + m\bar{a}_x(1.25 \text{ ft}) + m\bar{a}_y(1.443 \text{ ft})$$

$$B(1.443) = 0.1294\alpha + \frac{9}{32.2}(1.25)^2\alpha + \frac{9}{32.2}(1.443)^2\alpha$$

$$11.544 = 1.035\alpha$$

$$\alpha = 11.154 \text{ rad/s}^2$$

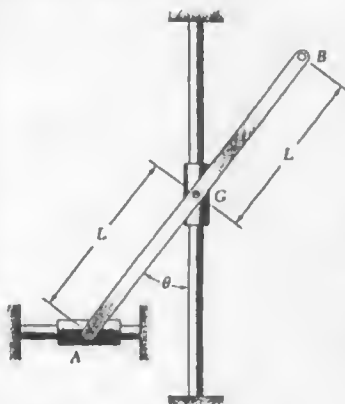
$$+2\Sigma M_A = \Sigma (M_A)_{eff} : B(2.5 \text{ ft}) = \bar{I}\alpha + m\bar{a}_x(1.25 \text{ ft})$$

$$2.5B = -(0.1294)(11.154) + \frac{9}{32.2}(1.25)(11.154)(1.25)$$

$$2.5B = -1.443 + 4.330$$

$$B = 1.155 \text{ lb}$$

# 16.120 and 16.121



GIVEN:

$$W = 1 \text{ NEWTON ON AB}$$

ROD IS RELEASED FROM REST.

FIND: (a)  $\alpha$   
(b)  $A$

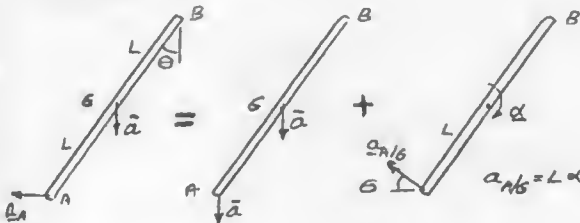
PROBLEM 16.120

SOLVE IN TERMS OF  $W, L$ , AND  $\theta$

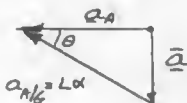
PROBLEM 16.121

SOLVE FOR  $W = 14 \text{ lb}$   
 $L = 15 \text{ in.}$  AND  $\theta = 30^\circ$

KINEMATICS:  $\omega = 0$



$$\bar{a}_A = \bar{a}_G + \bar{a}_{G/A} : [\alpha \leftarrow] = [\bar{a} \downarrow] + [L\alpha \nearrow \theta]$$

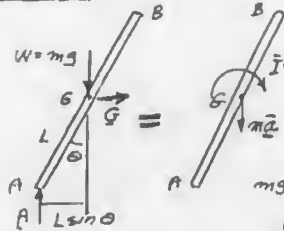


$$\bar{a} = L\alpha \sin \theta \downarrow$$

(CONTINUED)

# 16.120 and 16.121 continued

KINETICS:



$$\bar{I} = \frac{1}{12}m(2L)^2 = \frac{1}{3}mL^2$$

$$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{eff}$$

$$G = 0$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$$

$$mg(L \sin \theta) = \bar{I}\alpha + m\bar{a}(L \sin \theta)$$

$$mgL \sin \theta = \frac{1}{3}mL^2\alpha + m(L\alpha \sin \theta)(L \sin \theta)$$

$$\alpha = \frac{g}{L} \left[ \frac{\sin \theta}{\frac{1}{3} + \sin^2 \theta} \right]$$

$$+\circlearrowleft \Sigma F_y = \Sigma (F_y)_{eff} : A - mg = -m\bar{a} = -mL\alpha \sin \theta$$

$$A = mg - mL \left[ \frac{\sin \theta}{\frac{1}{3} + \sin^2 \theta} \right] \sin \theta$$

$$A = mg \frac{1 + \sin^2 \theta - \sin^2 \theta}{\frac{1}{3} + \sin^2 \theta}$$

$$A = \frac{mg}{1 + 3 \sin^2 \theta}$$

PROBLEM 16.121:  $W = mg = 14 \text{ lb}$ ,  $L = 15 \text{ in.} = 1.25 \text{ ft}$ ,  $\theta = 30^\circ$

$$\alpha = \frac{32.2}{1.25} \left[ \frac{\sin 30^\circ}{\frac{1}{3} + \sin^2 30^\circ} \right]$$

$$\alpha = 22.1 \text{ rad/s}^2$$

$$A = \frac{14 \text{ lb}}{1 + 3 \sin^2 30^\circ} = \frac{14}{1 + \frac{3}{4}}$$

$$A = 8 \text{ lb} \uparrow$$

# 16.122



GIVEN:

$$m = 5 \text{ kg}$$

$$L = 750 \text{ mm}$$

$$A = 20^\circ$$

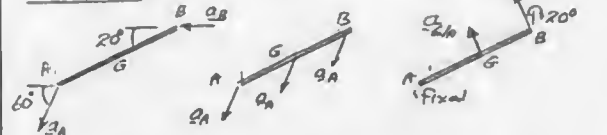
RELEASE FROM REST

FIND:

(a)  $\alpha$

(b)  $A$

KINEMATICS



$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$$

$$[\alpha \leftarrow] = [\alpha \nearrow 60^\circ] + [L\alpha \nearrow 20^\circ]$$

LAW OF SINES

$$\frac{\alpha_A}{\sin 20^\circ} = \frac{\alpha_B}{\sin 50^\circ} = \frac{L\alpha}{\sin 60^\circ}$$

$$\alpha_A = 1.0851 L\alpha \nearrow 60^\circ$$

$$\alpha_B = 0.88455 L\alpha \leftarrow$$

$$\alpha_{B/A} = \frac{1}{2} \alpha \nearrow 20^\circ$$

$$\alpha_G = \bar{a} = \alpha_A + \alpha_{G/A} = [1.0851 L\alpha \nearrow 60^\circ] + [\frac{1}{2} \alpha \nearrow 20^\circ]$$

$$+\circlearrowleft \bar{a}_x = (1.0851 L\alpha) \cos 60^\circ + (0.5 L\alpha) \sin 20^\circ$$

$$= 0.54254 L\alpha + 0.1701 L\alpha ; \bar{a}_y = 0.71355 L\alpha \leftarrow$$

$$+\downarrow \bar{a}_y = (1.0851 L\alpha) \sin 60^\circ - (0.5 L\alpha) \cos 20^\circ$$

$$= 0.93972 L\alpha - 0.46985 L\alpha ; \bar{a}_y = 0.46985 L\alpha \downarrow$$

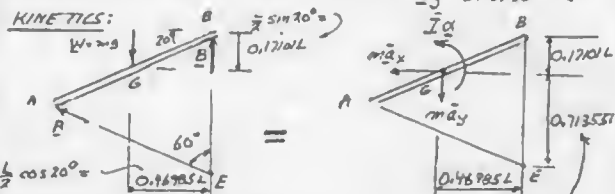
(CONTINUED)



# 16.122 continued

WE HAVE:  $\ddot{a}_x = 0.71355L\alpha$   
 $\ddot{a}_y = 0.46985L\alpha$

KINETICS:



TRIANGLE ABE:  $\angle ABE = 70^\circ$   
 $\angle ABE = 70^\circ$ ,  $\angle BAE = 50^\circ$

LAW OF SINES  $AB = L$

$$\frac{BE}{\sin 50^\circ} = \frac{L}{\sin 60^\circ}; BE = 0.88955L$$

$\sum M_A = \sum (M_A)_{eff}$

$$mg(0.46985L) = \bar{I}\alpha + m\ddot{a}_x(0.71355L) + m\ddot{a}_y(0.46985L)$$

$$0.46985mgL = \frac{1}{2}mL^2\alpha + m(0.71355L\alpha)(0.71355L) + m(0.46985L\alpha)(0.46985L)$$

$$0.46985mgL = mL^2(0.81325)\alpha$$

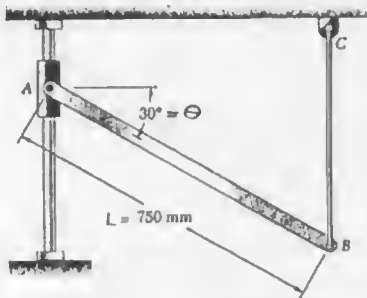
$$\alpha = 0.57775 \frac{g}{L} = 0.57775 \frac{9.81 \text{ m/s}^2}{0.75 \text{ m}} = 7.557 \text{ rad/s}^2$$

$\sum F_y = \sum (F_y)_{eff}$ :  $A \sin 60^\circ = m\ddot{a}_y = m(0.71355L\alpha)$

$$A \sin 60^\circ = (5.89)(0.71355)(0.75 \text{ m})(7.557 \text{ rad/s}^2)$$

$$A = 23.3 \text{ N} \angle 30^\circ$$

# 16.123



GIVEN:  $m = 8 \text{ kg}$

RELEASE FROM REST.

FIND:  
 (a)  $\alpha$   
 (b)  $A$

KINEMATICS:  $\omega = 0$

$$\ddot{a}_B = \ddot{a}_A + \ddot{a}_{B/A}; [\ddot{a}_B] = [\ddot{a}_A] + [L\ddot{\theta}]$$

$$\ddot{a}_B = \ddot{a}_A + L\ddot{\theta} \cos \theta$$

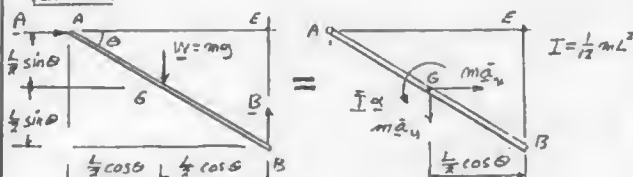
$$\ddot{a}_B = L\ddot{\theta} \cos \theta$$

$$\ddot{a}_B = \ddot{a}_A + \ddot{a}_{B/A} = \ddot{a}_A + \frac{1}{2}\ddot{\theta}$$

$$\ddot{a}_B = [L\ddot{\theta} \cos \theta] + [\frac{1}{2}\ddot{\theta}]$$

$$\ddot{a}_B = \frac{1}{2}\ddot{\theta} \sin \theta \rightarrow; \ddot{a}_B = \frac{1}{2}\ddot{\theta} \cos \theta \uparrow$$

KINETICS



$\sum M_A = \sum (M_A)_{eff}$ :  $mg \frac{1}{2} \cos \theta = \bar{I}\alpha + m\ddot{a}_x(\frac{1}{2} \sin \theta) + m\ddot{a}_y(\frac{1}{2} \cos \theta)$

$$mg \frac{1}{2} \cos \theta = \frac{1}{2}mL^2\alpha + m(\frac{1}{2} \sin \theta)\ddot{a}_x + m(\frac{1}{2} \cos \theta)\ddot{a}_y$$

$$mg \frac{1}{2} \cos \theta = \frac{1}{2}mL^2\alpha \quad \alpha = \frac{3}{2} \frac{g}{L} \cos \theta$$

(CONTINUED)

# 16.123 continued

$$\alpha = \frac{3}{2} \frac{g}{L} \cos \theta$$

$\sum F_x = \sum (F_x)_{eff}$ :  $A = m\ddot{a}_x = m \frac{1}{2} \alpha \sin \theta$

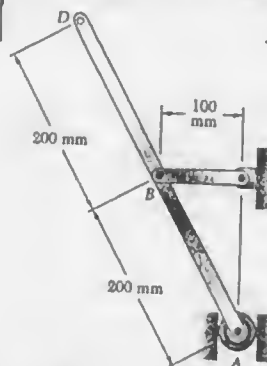
$$A = m \frac{1}{2} \left( \frac{3}{2} \frac{g}{L} \cos \theta \right) \sin \theta; A = \frac{3}{4} mg \sin \theta \cos \theta \rightarrow$$

DATA:  $m = 8 \text{ kg}$ ,  $\theta = 30^\circ$ ,  $L = 0.75 \text{ m}$

$$\alpha = \frac{3}{2} \frac{9.81 \text{ m/s}^2}{0.75 \text{ m}} \cos 30^\circ; \alpha = 16.99 \text{ rad/s}^2$$

$$A = \frac{3}{4} (8 \text{ kg}) (9.81 \text{ m/s}^2) \sin 30^\circ \cos 30^\circ; A = 25.5 \text{ N} \rightarrow$$

# 16.124



GIVEN:  $m_{AB} = 4 \text{ kg}$

$$\omega_{BC} = 6 \text{ rad/s}$$

$$\alpha_{BC} = 15 \text{ rad/s}^2$$

FIND:  
 REACTION AT A.

CRANK BC:

$$BC = 0.1 \text{ m}$$

$$\omega = 6 \text{ rad/s}$$

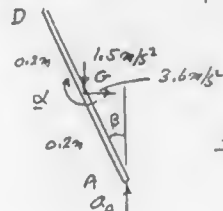
$$\alpha = 15 \text{ rad/s}^2$$

$$(a_B)_t = (BC)\omega = (0.1 \text{ m})(15 \text{ rad/s}^2) = 1.5 \text{ m/s}^2$$

$$(a_B)_n = (BC)\omega^2 = (0.1 \text{ m})(6 \text{ rad/s})^2 = 3.6 \text{ m/s}^2 \rightarrow$$

ROD ABD:

$$\beta = \sin^{-1} \frac{BC}{AB} = \sin^{-1} \frac{0.1 \text{ m}}{0.2 \text{ m}} = 30^\circ$$



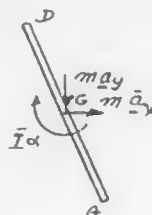
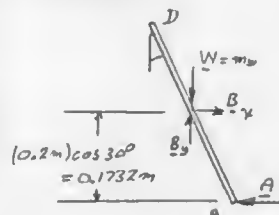
$$a_A = a_G + a_{A/G}$$

$$[a_A] = [1.5] + [3.6] + [0.2\alpha] \beta$$

$$0 = 3.6 - (0.2\alpha) \cos \beta$$

$$\alpha = \frac{3.6}{0.2 \cos \beta} = \frac{18}{\cos 30^\circ} = 20.78 \text{ rad/s}^2$$

KINETICS:



$\sum M_A = \sum (M_A)_{eff}$ :

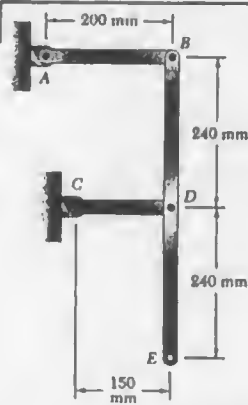
$$A(0.1732 \text{ m}) = \bar{I}\alpha = \frac{1}{2}mL^2\alpha$$

$$= \frac{1}{2}(4 \text{ kg})(0.4 \text{ m})^2(20.78 \text{ rad/s}^2)$$

$$A = 6.39 \text{ N}$$

$$A = 6.40 \text{ N} \rightarrow$$

16.125

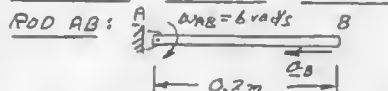
GIVEN:  $m_{BDE} = 5 \text{ kg}$ 

$$\omega_{AB} = 6 \text{ rad/s} \downarrow$$

$$\alpha_{AB} = 0$$

FIND: HORIZONTAL COMPONENT OF REACTION AT

- (a) B  
(b) D

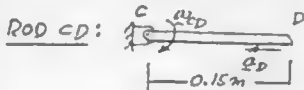


$$v_B = r\omega = (0.2 \text{ m})(6 \text{ rad/s})$$

$$v_B = 1.2 \text{ m/s} \downarrow$$

$$a_B = (0.2 \text{ m})(6 \text{ rad/s})^2$$

$$a_B = 7.2 \text{ m/s}^2 \leftarrow$$



$$v_D = v_B = 1.2 \text{ m/s} \downarrow$$

$$v_D = (0.15 \text{ m})\omega_{CD}$$

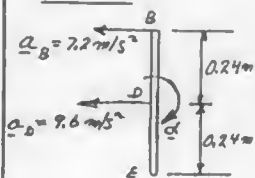
$$1.2 \text{ m/s} = (0.15 \text{ m})\omega_{CD}$$

$$\omega_{CD} = 8 \text{ rad/s}$$

$$a_D = (0.15 \text{ m})(8 \text{ rad/s})^2$$

$$a_D = 9.6 \text{ m/s}^2 \leftarrow$$

ROD BDE:



$$a_D = a_B + a_{D/B}$$

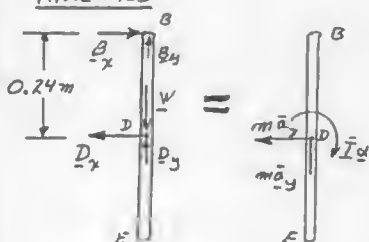
$$[a_D \leftarrow] = [a_B \leftarrow] + [(BD)\alpha \rightarrow]$$

$$[2.2 \leftarrow] = [9.6 \leftarrow] + [(0.24)\alpha \rightarrow]$$

$$2.2 = 9.6 - 0.24\alpha$$

$$\alpha = 10 \text{ rad/s}^2 \leftarrow$$

KINETICS:



$$m\bar{a}_x = \sum F_x$$

$$\bar{I} = \frac{1}{12} m(BE)^2$$

$$= \frac{1}{12} (5 \text{ kg})(0.48 \text{ m})^2$$

$$\bar{I} = 96 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$+\sum M_D = \sum (M_D)_{eff}: B_x(0.24 \text{ m}) = \bar{I}\alpha$$

$$B_x(0.24 \text{ m}) = (96 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(10 \text{ rad/s}^2)$$

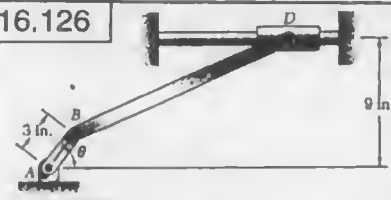
$$B_x = +4 \text{ N} \quad B_x = 4 \text{ N} \rightarrow$$

$$+\sum F_x = \sum (F_x)_{eff}: D_x - B_x = m\bar{a}_x$$

$$D_x - 4 \text{ N} = (5 \text{ kg})(9.6 \text{ m/s}^2)$$

$$D_x = +52 \text{ N} \quad D_x = 52 \text{ N} \leftarrow$$

16.126



GIVEN:

$$BD = 15 \text{ in}, W = 8 \text{ lb}$$

$$\omega_{AB} = 300 \text{ rpm} \downarrow$$

$$\alpha_{AB} = 0$$

$$\theta = 0$$

FIND: D

CRANK AB:

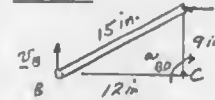
$$AB = 3 \text{ m} = 0.25 \text{ ft}$$

$$\omega_{AB} = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$$

$$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 2.5\pi \text{ ft/s} \uparrow$$

$$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74 \text{ ft/s}^2 \leftarrow$$

ROD BD:



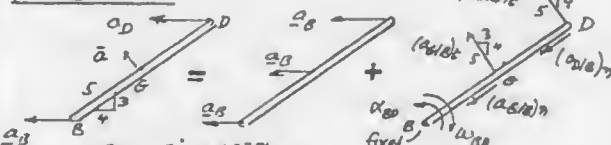
VELOCITY: INSTANT CTR. A-C

$$v_B = (BC)\omega_{BD}$$

$$2.5\pi \text{ ft/s} = (1 \text{ ft})\omega_{BD}$$

$$\omega_{BD} = 2.5\pi \text{ rad/s} \downarrow$$

ACCELERATION



$$BD = 15 \text{ in} = 1.25 \text{ ft}$$

$$(a_{D/B})_t = (BD)\alpha_{BD} = 1.25\alpha_{BD}$$

$$(a_{D/B})_n = (BD)\omega_{BD}^2 = (1.25)(2.5\pi)^2 = 77.11 \text{ ft/s}^2 \leftarrow$$

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$[a_D \leftarrow] = [246.74 \text{ ft/s}^2 \leftarrow] + [1.25\alpha_{BD} \uparrow] + [77.11 \text{ ft/s}^2 \leftarrow]$$

$$+ \uparrow 0 = (1.25\alpha_{BD}) \frac{4}{5} - (77.11) \frac{3}{4}$$

$$\alpha_{BD} = 46.26 \text{ rad/s}^2 \uparrow$$

$$(a_{D/B})_t = (BD)\alpha_{BD} = (1.25 \text{ ft})(46.26 \text{ rad/s}^2) = 57.82 \text{ ft/s}^2 \uparrow$$

$$(a_{D/B})_n = (BD)\omega_{BD}^2 = (1.25 \text{ ft})(2.5\pi \text{ rad/s})^2 = 30.55 \text{ ft/s}^2 \leftarrow$$

$$\bar{a} = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

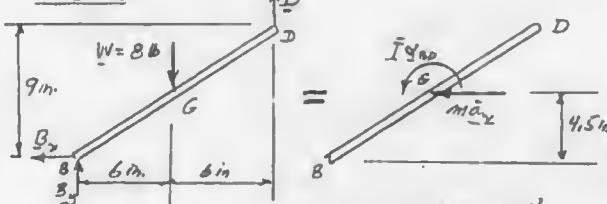
$$\bar{a} = [246.74 \text{ ft/s}^2 \leftarrow] + [57.82 \text{ ft/s}^2 \uparrow] + [30.55 \text{ ft/s}^2 \leftarrow]$$

$$+ \uparrow \bar{a}_x = 246.74 + (20.97) \frac{3}{5} + (30.55) \frac{4}{5}$$

$$\bar{a}_x = 246.74 + 12.48 + 24.44 = 283.66 \text{ ft/s}^2 \leftarrow$$

$$+ \uparrow \bar{a}_y = (20.97) \frac{4}{5} - (30.55) \frac{3}{5} = 0 \quad \bar{a}_y = 0$$

KINETICS



$$\bar{I} = \frac{1}{12} \frac{W}{g} l^2 = \frac{1}{12} \frac{8 \text{ lb}}{32.2} \left( \frac{15}{12} \text{ ft} \right)^2$$

$$\bar{I} = 0.03235 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$+\sum M_B = \sum (M_B)_{eff}:$$

$$D \left( \frac{12}{12} \text{ ft} \right) - W \left( \frac{6}{12} \text{ ft} \right) = \bar{I}\alpha_{BD} + m\bar{a}_x \left( \frac{9.5}{12} \text{ ft} \right)$$

$$D - (8 \text{ lb}) \frac{6}{12} = (0.03235 \text{ lb} \cdot \text{s}^2)(46.26 \text{ rad/s}^2)$$

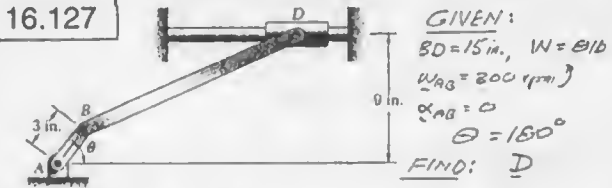
$$+ \frac{8 \text{ lb}}{32.2} (283.66 \text{ ft/s}^2) \left( \frac{4.5}{12} \text{ ft} \right)$$

$$D - 4 = 1.4967 + 27.478$$

$$D = 32.97 \text{ lb}$$

$$D = 33.0 \text{ lb} \uparrow$$

16.127



GIVEN:

$$BD = 15 \text{ in.}, W = 8 \text{ lb}$$

$$\omega_{AB} = 300 \text{ rpm}$$

$$\angle_{AB} = 0$$

$$\theta = 150^\circ$$

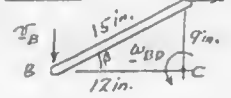
FIND:  $\underline{D}$ 

CRANK AB:  $\omega = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$

$$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 7.854 \text{ ft/s} \downarrow$$

$$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74 \text{ ft/s}^2 \rightarrow$$

ROD BD:



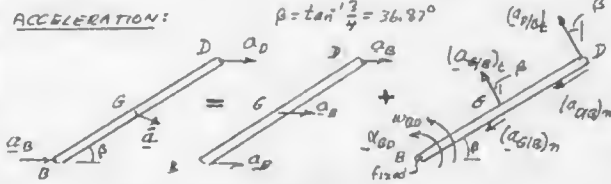
VELOCITY: INSTANT CTR. AT C.

$$v_B = (BC)\omega_{BD}$$

$$7.854 \text{ ft/s} = (14\text{ in.})\omega_{BD}$$

$$\omega_{BD} = 7.854 \text{ rad/s}$$

ACCELERATION:



$$BD = 15 \text{ in.} = 1.25 \text{ ft}$$

$$(a_{D/B})_t = (BD)\alpha_{BD} = (1.25 \text{ ft})\alpha_{BD}$$

$$(a_{D/B})_n = (BD)\omega_{BD}^2 = (1.25)(7.854)^2 = 77.11 \text{ ft/s}^2 \rightarrow \beta$$

$$a_D = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$[a_D \rightarrow] = [246.74 \text{ ft/s}^2 \rightarrow] + [1.25 \alpha_{BD} \rightarrow \beta] + [77.11 \text{ ft/s}^2 \rightarrow \beta]$$

$$\uparrow \uparrow 0 = 1.25 \alpha_{BD} \cos \beta = 77.11 \sin \beta$$

$$\alpha_{BD} = \frac{77.11}{1.25} \cdot \frac{\sin \beta}{\cos \beta} = 61.68 \tan \beta = 61.68 \left( \frac{3}{4} \right)$$

$$\alpha_{BD} = 46.266 \text{ rad/s}^2$$

$$(a_{G/B})_t = (BG)\alpha_{BD} = \left( \frac{1.25}{2} \text{ ft} \right) (46.266 \text{ rad/s}^2) = 28.92 \text{ ft/s}^2 \nearrow \beta$$

$$(a_{G/B})_n = (BG)\omega_{BD}^2 = \left( \frac{1.25}{2} \text{ ft} \right) (7.854 \text{ rad/s})^2 = 38.55 \text{ ft/s}^2 \rightarrow \beta$$

$$\bar{a} = a_B + (a_{G/B})_t + (a_{G/B})_n$$

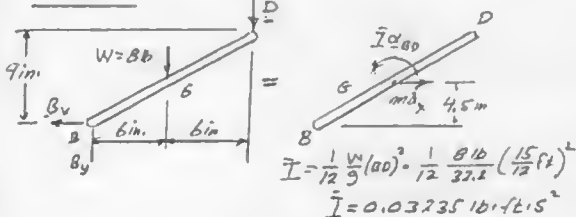
$$\bar{a} = [246.74 \text{ ft/s}^2 \rightarrow] + [28.92 \text{ ft/s}^2 \nearrow \beta] + [38.55 \text{ ft/s}^2 \rightarrow \beta]$$

$$\rightarrow \bar{a}_x = 246.74 - (28.92) \sin \beta - (38.55) \cos \beta$$

$$\bar{a}_x = 246.74 - 17.346 - 30.84 = 198.55 \text{ ft/s}^2 \rightarrow$$

$$\uparrow \bar{a}_y = (28.92) \cos \beta - (38.55) \sin \beta = 0; \bar{a}_y = 0$$

KINETICS



$$\uparrow \sum M_G = \sum (M_G)_{eff}: D(1 \text{ ft}) + W(0.5 \text{ ft}) = \bar{I} \alpha_{BD} + m \bar{a}_x \left( \frac{4.5}{12} \text{ ft} \right)$$

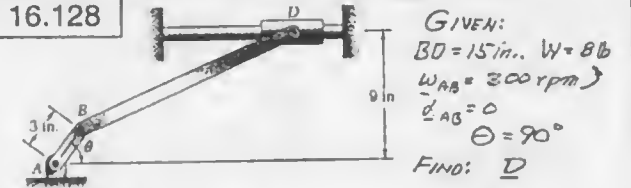
$$D + (8 \text{ lb})(0.5 \text{ ft}) = (0.03235 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (46.266 \text{ rad/s}^2) + \frac{8 \text{ lb}}{32.2} (198.55 \text{ ft/s}^2) \left( \frac{4.5}{12} \text{ ft} \right)$$

$$D + 4 = -1.4767 + 18.498$$

$$D = 13.00 \text{ lb}$$

$$\underline{D} = 13.00 \text{ lb} \downarrow$$

16.128



GIVEN:

$$BD = 15 \text{ in.}, W = 8 \text{ lb}$$

$$\omega_{AB} = 300 \text{ rpm}$$

$$\angle_{AB} = 0$$

$$\theta = 90^\circ$$

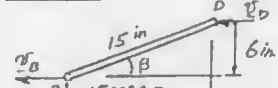
FIND:  $\underline{D}$ 

CRANK AB:  $\omega = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$

$$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 7.854 \text{ ft/s} \downarrow$$

$$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74 \text{ ft/s}^2 \downarrow$$

ROD BD:

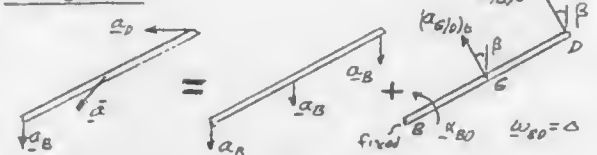


INSTANT CENTER AT C

$$\therefore \omega_{BD} = 0$$

$$\beta = \sin^{-1} \frac{6 \text{ in.}}{15 \text{ in.}} = 23.58^\circ$$

ACCELERATION:



$$BD = 15 \text{ in.} = 1.25 \text{ ft}; (a_{D/B})_t = (BD)\alpha_{BD} = (1.25 \text{ ft})\alpha_{BD}$$

$$a_D = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$$[a_D \rightarrow] = [246.74 \text{ ft/s}^2 \downarrow] + [1.25 \alpha_{BD} \nearrow \beta] + 0$$

$$\uparrow \uparrow 0 = -246.74 + (1.25 \alpha_{BD}) \cos \beta$$

$$\alpha_{BD} = \frac{246.74}{1.25 \cos 23.57^\circ} \quad \alpha_{BD} = 215.36 \text{ rad/s}^2$$

$$(a_{G/B})_t = (BG)\alpha_{BD} = \left( \frac{1.25}{2} \text{ ft} \right) (215.36) = 134.6 \text{ ft/s}^2$$

$$\bar{a} = a_B + (a_{G/B})_t + (a_{G/B})_n$$

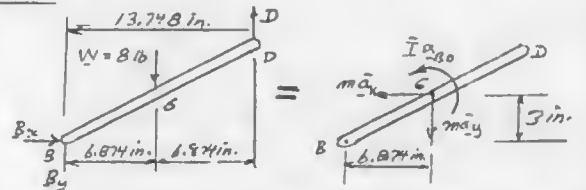
$$\bar{a} = [246.74 \text{ ft/s}^2 \downarrow] + [134.6 \text{ ft/s}^2 \nearrow \beta] + 0$$

$$\rightarrow \bar{a}_x = [134.6] \sin 23.57^\circ = 53.92 \text{ ft/s}^2; \bar{a}_x = 53.92 \text{ ft/s}^2 \rightarrow$$

$$\uparrow \bar{a}_y = 246.74 - [134.6] \cos 23.57^\circ = 246.74 - 123.37$$

$$\bar{a}_y = 123.37 \text{ ft/s}^2 \quad \bar{a}_y = 123.37 \text{ ft/s}^2 \downarrow$$

KINETICS



$$\bar{I} = \frac{1}{12} \frac{W}{g} (BD)^2 = \frac{1}{12} \frac{8 \text{ lb}}{32.2} \left( \frac{15}{12} \text{ ft} \right)^2 = 0.03235 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\uparrow \sum M_G = \sum (M_G)_{eff}:$$

$$\alpha \left( \frac{13.748}{12} \text{ ft} \right) - W \left( \frac{4.5}{12} \text{ ft} \right) = \bar{I} \alpha + m \bar{a}_x \left( \frac{3}{12} \text{ ft} \right) - m \bar{a}_y \left( \frac{6.874}{12} \text{ ft} \right)$$

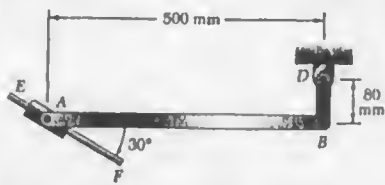
$$1.1456 D - (8 \text{ lb})(0.5728) = (0.03235) (215.36) + \left( \frac{8}{32.2} \right) (53.92)(0.25) - \left( \frac{8}{32.2} \right) (123.37)(0.5728)$$

$$1.1456 D - 4.583 = 6.967 + 3.243 - 17.557$$

$$D = -2.325 \text{ lb}$$

$$\underline{D} = 2.32 \text{ lb} \downarrow$$

16.129



GIVEN:

$$\begin{aligned} \text{ROD } AB &= 3 \text{ kg} \\ \omega_{BD} &= 15 \text{ rad/s} \\ \alpha_{BD} &= 60 \text{ rad/s}^2 \end{aligned}$$

FIND:  $A$ 

CRANK BD:  $\omega_{BD} = 15 \text{ rad/s}$ ,  $v_B = (0.08 \text{ m})(15 \text{ rad/s}) = 1.2 \text{ m/s}$   
 $\alpha_{BD} = 60 \text{ rad/s}^2$   
 $(a_B)_x = (0.08 \text{ m})(60 \text{ rad/s}^2) = 4.8 \text{ m/s}^2$   
 $(a_B)_y = (0.08 \text{ m})(15 \text{ rad/s})^2 = 18 \text{ m/s}^2$

ROD AB:

VELOCITY: INSTANT. CTR. AT C.  
 $CB = (0.5 \text{ m}) \tan 30^\circ = 0.86603 \text{ m}$   
 $\omega_{AB} = \frac{v_B}{CB} = \frac{1.2 \text{ m/s}}{0.86603 \text{ m}} = 1.3856 \text{ rad/s}$

ACCELERATION:

$(a_B)_x = 4.8 \text{ m/s}^2$   
 $(a_B)_y = 18 \text{ m/s}^2$   
 $(a_{B/A})_x = \omega_{AB}^2 CB = (1.3856)^2 (0.86603) = 1.651 \text{ m/s}^2$   
 $(a_{B/A})_y = \alpha_{AB} CB = (60) (0.86603) = 51.962 \text{ m/s}^2$

$$\begin{aligned} (a_{A/B})_x &= (AB) \alpha_{AB} = 0.5 \alpha_{AB} \\ (a_{A/B})_y &= (AB) \omega_{AB}^2 = (0.5)(1.3856)^2 = 0.478 \text{ m/s}^2 \\ (a_{G/B})_x &= (GB) \alpha_{AB} = 0.25 \alpha_{AB} \\ (a_{G/B})_y &= (GB) \omega_{AB}^2 = (0.25)(1.3856)^2 = 0.239 \text{ m/s}^2 \end{aligned}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{B/A} = \vec{a}_B + (\vec{a}_{B/A})_x + (\vec{a}_{B/A})_y$$

$$[a_A \angle 30^\circ] = [4.8 \rightarrow] + [18 \uparrow] + [0.5 \alpha_{AB} \downarrow] + [0.96 \rightarrow]$$

$$\begin{aligned} \pm a_A \cos 30^\circ &= 4.8 - 0.96; & a_A &= 4.434 \text{ m/s}^2 \angle 30^\circ \\ + (4.434) \sin 30^\circ &= 18 - 0.5 \alpha_{AB}; & \alpha_{AB} &= 31.566 \text{ rad/s}^2 \end{aligned}$$

$$\vec{a} = \vec{a}_B + \vec{a}_{G/B} = \vec{a}_B + (\vec{a}_{G/B})_x + (\vec{a}_{G/B})_y$$

$$\vec{a} = [4.8 \rightarrow] + [18 \uparrow] + [0.25(31.566) \downarrow] + [0.48 \rightarrow]$$

$$\begin{aligned} \pm \vec{a}_x &= 4.8 - 0.48 = 4.32; & \vec{a}_x &= 4.32 \text{ m/s}^2 \\ + \vec{a}_y &= 18 - 7.992 = 10.108; & \vec{a}_y &= 10.108 \text{ m/s}^2 \end{aligned}$$

KINETICS:

$$\vec{I} = \frac{1}{12} m (AB)^2 = \frac{3 \text{ kg}}{12} (0.5 \text{ m})^2 = 0.0625 \text{ kg} \cdot \text{m}^2$$

$\vec{I} = \frac{1}{12} m (AB)^2 = \frac{3 \text{ kg}}{12} (0.5 \text{ m})^2 = 0.0625 \text{ kg} \cdot \text{m}^2$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$$

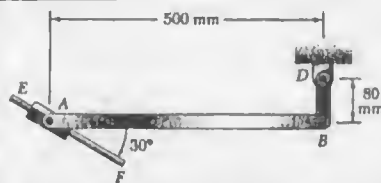
$$\begin{aligned} (A \sin 60^\circ)(0.5 \text{ m}) - mg(0.25 \text{ m}) &= -\vec{I} \alpha_{AB} + m \vec{a}_y (0.25 \text{ m}) \\ 0.433 A - (3 \text{ kg})(9.81 \text{ m/s}^2)(0.25 \text{ m}) &= -(0.0625 \text{ kg} \cdot \text{m}^2)(31.566 \text{ rad/s}^2) \\ &\quad + (3 \text{ kg})(10.108 \text{ m/s}^2)(0.25 \text{ m}) \end{aligned}$$

$$0.433 A - 7.358 = -1.973 + 7.581$$

$$A = 29.94 \text{ N}$$

$$A = 29.9 \text{ N} \angle 60^\circ$$

16.130



GIVEN:

$$\begin{aligned} \text{ROD } AB &= 3 \text{ kg} \\ \omega_{BD} &= 15 \text{ rad/s} \\ \alpha_{BD} &= 60 \text{ rad/s}^2 \end{aligned}$$

FIND:  $A$ 

CRANK BD:  $\omega_{BD} = 15 \text{ rad/s}$ ,  $v_B = (0.08 \text{ m})(15 \text{ rad/s}) = 1.2 \text{ m/s}$   
 $\alpha_{BD} = 60 \text{ rad/s}^2$   
 $(a_B)_x = (0.08 \text{ m})(60 \text{ rad/s}^2) = 4.8 \text{ m/s}^2$   
 $(a_B)_y = (0.08 \text{ m})(15 \text{ rad/s})^2 = 18 \text{ m/s}^2$

ROD AB:

VELOCITY: INSTANT. CTR. AT C  
 $CB = (0.5 \text{ m}) \tan 30^\circ = 0.86603 \text{ m}$   
 $\omega_{AB} = \frac{v_B}{CB} = \frac{1.2 \text{ m/s}}{0.86603 \text{ m}} = 1.3856 \text{ rad/s}$

ACCELERATION:

$(a_B)_x = 4.8 \text{ m/s}^2$   
 $(a_B)_y = 18 \text{ m/s}^2$   
 $(a_{B/A})_x = \omega_{AB}^2 CB = (1.3856)^2 (0.86603) = 1.651 \text{ m/s}^2$   
 $(a_{B/A})_y = \alpha_{AB} CB = (60) (0.86603) = 51.962 \text{ m/s}^2$

$$(a_{A/B})_x = (AB) \alpha_{AB} = 0.5 \alpha_{AB}$$

$$(a_{A/B})_y = (AB) \omega_{AB}^2 = (0.5)(1.3856)^2 = 0.478 \text{ m/s}^2$$

$$(a_{G/B})_x = (GB) \alpha_{AB} = 0.25 \alpha_{AB}$$

$$(a_{G/B})_y = (GB) \omega_{AB}^2 = (0.25)(1.3856)^2 = 0.239 \text{ m/s}^2$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{B/A} = \vec{a}_B + (\vec{a}_{B/A})_x + (\vec{a}_{B/A})_y$$

$$[a_A \angle 30^\circ] = [4.8 \rightarrow] + [18 \uparrow] + [0.5 \alpha_{AB} \downarrow] + [0.96 \rightarrow]$$

$$\begin{aligned} \pm a_A \cos 30^\circ &= 4.8 + 0.96; & a_A &= 6.651 \text{ m/s}^2 \angle 30^\circ \\ + (6.651) \sin 30^\circ &= 18 - 0.5 \alpha_{AB}; & \alpha_{AB} &= 42.65 \text{ rad/s}^2 \end{aligned}$$

$$\vec{a} = \vec{a}_B + \vec{a}_{G/B} = \vec{a}_B + (\vec{a}_{G/B})_x + (\vec{a}_{G/B})_y$$

$$\vec{a} = [4.8 \rightarrow] + [18 \uparrow] + [0.25(42.65) \downarrow] + [0.48 \rightarrow]$$

$$\pm \vec{a}_x = 4.8 + 0.48 = 5.28; & \vec{a}_x &= 5.28 \text{ m/s}^2$$

$$+ \vec{a}_y = 18 - 10.663 = 7.337; & \vec{a}_y &= 7.337 \text{ m/s}^2$$

KINETICS:  $\vec{I} = \frac{1}{12} m (AB)^2 = \frac{3 \text{ kg}}{12} (0.5 \text{ m})^2 = 0.0625 \text{ kg} \cdot \text{m}^2$

$\vec{I} = \frac{1}{12} m (AB)^2 = \frac{3 \text{ kg}}{12} (0.5 \text{ m})^2 = 0.0625 \text{ kg} \cdot \text{m}^2$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$$

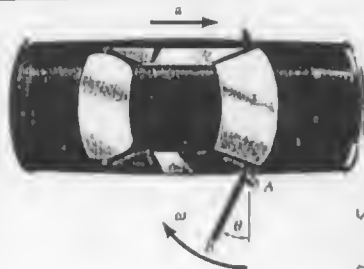
$$\begin{aligned} (A \sin 60^\circ)(0.5 \text{ m}) - mg(0.25 \text{ m}) &= -\vec{I} \alpha_{AB} + m \vec{a}_y (0.25 \text{ m}) \\ 0.433 A - (3 \text{ kg})(9.81 \text{ m/s}^2)(0.25 \text{ m}) &= -(0.0625 \text{ kg} \cdot \text{m}^2)(42.65 \text{ rad/s}^2) \\ &\quad + (3 \text{ kg})(7.337 \text{ m/s}^2)(0.25 \text{ m}) \end{aligned}$$

$$0.433 A - 7.358 = -2.666 + 5.503$$

$$A = 23.55 \text{ N}$$

$$A = 23.5 \text{ N} \angle 60^\circ$$

# 16.131 and 16.132



GIVEN: 80-lb DOOR  
WITH MASS CENTER  
 $\bar{r} = 22$  in. FROM A  
AND  $\bar{R} = 12.5$  in.  
INITIALLY  $\theta = 0$ .

PROBLEM 16.131

$\omega = 6 \text{ rad/s} \rightarrow$

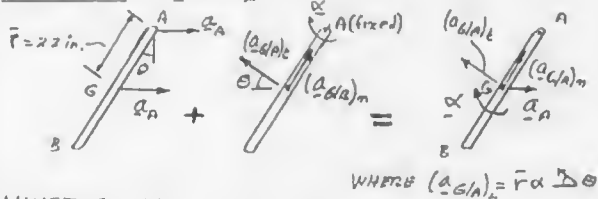
FIND: ANGULAR  
VELOCITY  $\omega$  WHEN  $\theta = 90^\circ$

PROBLEM 16.132

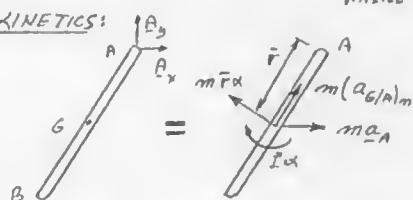
FIND:  $\omega$  SO THAT

$\omega = 2 \text{ rad/s}$  WHEN  $\theta = 90^\circ$

KINEMATICS:  $\omega = \omega_A$



KINETICS:



$$+\sum \bar{M}_A = \sum (\bar{M}_A)_{\text{eff}}: 0 = \bar{I} \alpha + (m \bar{r} \alpha) \bar{r} - m \bar{a}_A (\bar{r} \cos \theta)$$

$$m \bar{r}^2 \alpha + m \bar{r}^2 \alpha = m \bar{a}_A \bar{r} \cos \theta$$

$$\alpha = \frac{\bar{a}_A \bar{r}}{\bar{r}^2 + \bar{r}^2} \cos \theta$$

SETTING  $\alpha = \omega \frac{d\omega}{d\theta}$ , AND USING  $\bar{r} = \frac{22}{12}$  ft,  $\bar{R} = \frac{12.5}{12}$  ft

$$\omega \frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\right)^2 \alpha_A}{\left[\left(\frac{12.5}{12}\right)^2 + \left(\frac{22}{12}\right)^2\right]} \cos \theta = 0.41234 \alpha_A \cos \theta$$

$$\omega d\omega = 0.41234 \alpha_A \cos \theta d\theta$$

$$\int_0^{\omega_f} \omega d\omega = \int_0^{\pi/2} (0.41234 \alpha_A) \cos \theta d\theta$$

$$\left[\frac{1}{2} \omega^2\right]_0^{\omega_f} = 0.41234 \alpha_A \left[\sin \theta\right]_0^{\pi/2}$$

$$\omega_f^2 = 0.82468 \alpha_A \quad (1)$$

PROBLEM 16.131

GIVEN DATA:  $\alpha_A = 6 \text{ ft/s}^2 \rightarrow$

$$\omega_f^2 = 0.82468 (6) = 4.948$$

$$\omega_f = 2.22 \text{ rad/s}$$

PROBLEM 16.132

GIVEN DATA:  $\omega_f = 2 \text{ rad/s}$

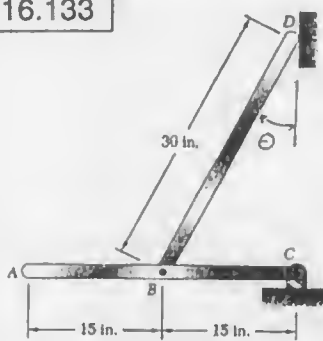
$$\text{EQ (1): } \omega_f^2 = 0.82468 \alpha_A$$

$$(2)^2 = 0.82468 \alpha_A$$

$$\alpha_A = 4.85 \text{ ft/s}^2$$

$$\alpha_A = 4.85 \text{ ft/s}^2 \rightarrow$$

# 16.133



GIVEN:

$W_{AC} = W_{BD} = 8 \text{ lb}$   
IMMEDIATELY AFTER  
SYSTEM IS RELEASED  
FROM REST.

FIND:  $D$

NOTE:  $\theta = \sin^{-1} \frac{15 \text{ in.}}{30 \text{ in.}}$

$$\theta = 30^\circ$$

KINEMATICS: BAR AC: ROTATION ABOUT C

$$\bar{a} = (BC) \alpha = \left(\frac{15}{12}\right) \alpha$$

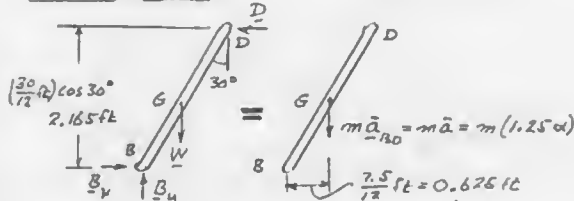
$$\bar{a} = 1.25 \alpha \downarrow$$

BAR BC:

$$\bar{a}_{D/B} = L \alpha \quad \text{MUST BE ZERO SINCE } \bar{a}_D = \bar{a}_B$$

$$\therefore \alpha_{D/B} = 0 \text{ AND } \bar{a}_{D/B} = \bar{a}$$

KINETICS: BAR BD



$$+\uparrow \sum F_y = \sum (F_y)_{\text{eff}}: B_y - W = -m \bar{a}$$

$$B_y - 8 \text{ lb} = -\frac{8 \text{ lb}}{32.2} (1.25 \alpha)$$

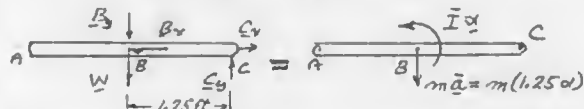
$$B_y = 8 - 0.3105 \alpha \quad (1)$$

$$+\sum \bar{M}_B = \sum (\bar{M}_B)_{\text{eff}}: D(2.165 \text{ ft}) - W(0.625 \text{ ft}) = -m \bar{a}(0.625 \text{ ft})$$

$$D(2.165 \text{ ft}) - (8 \text{ lb})(0.625 \text{ ft}) = -\frac{8 \text{ lb}}{32.2} (1.25 \alpha)(0.625 \text{ ft})$$

$$D = 2.309 - 0.08965 \alpha \quad (2)$$

$$\text{BAR AC: } \bar{I} = \frac{1}{2} m (AC)^2 = \frac{1}{2} \frac{8 \text{ lb}}{32.2} (2.5 \text{ ft})^2 = 0.1294 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$+\sum \bar{M}_C = \sum (\bar{M}_C)_{\text{eff}}: W(1.25 \text{ ft}) + B_y(1.25 \text{ ft}) = \bar{I} \alpha + m(1.25 \alpha)(1.25 \text{ ft})$$

SUBSTITUTE FROM EQ (1) FOR  $B_y$

$$8(1.25) + (8 - 0.3105 \alpha)(1.25) = (0.1294) \alpha + \frac{8}{32.2} (1.25)^2 \alpha$$

$$10 + 10 - 0.3881 \alpha = 0.1294 \alpha + 0.3782 \alpha$$

$$20 = 0.9057 \alpha$$

$$\alpha = 22.08 \text{ rad/s}^2$$

EQ (2):

$$D = 2.309 - 0.08965 \alpha$$

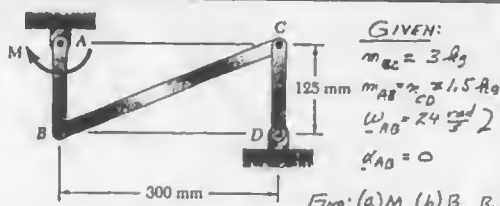
$$= 2.309 - 0.08965 (22.08)$$

$$= 2.309 - 1.961$$

$$D = 0.330 \text{ lb}$$

$$D = 0.330 \text{ lb} \leftarrow$$

16.134



KINEMATICS:  $\omega_{AB} = 24 \text{ rad/s}$ ,  $\omega_{CD} = 480 \text{ rad/s}^2$ ,  $\omega_{BC} = 0$ ,  $\alpha_{BC} = 0$

BAR AB:

$$\omega_{AB} = 24 \text{ rad/s}$$

$$\alpha_{AB} = 0$$

$$a_B = (\omega_{AB})^2 r_{AB} = (24)^2 (0.125) = 72 \text{ m/s}^2 \uparrow$$

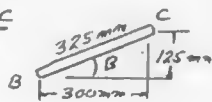
BAR CD:

$$\omega_{CD} = 480 \text{ rad/s}^2$$

$$\alpha_{CD} = 480 \text{ rad/s}^2$$

$$a_C = (\omega_{CD})^2 r_{CD} = (480)^2 (0.125) = 28800 \text{ m/s}^2$$

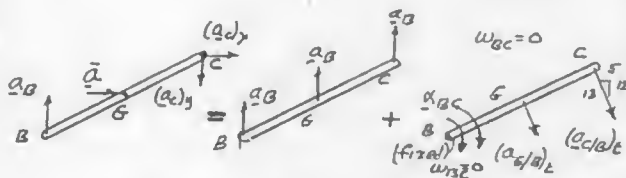
BAR BC



Geometry of BC:

$$\tan \beta = \frac{0.125}{0.3} = \frac{5}{12}$$

$$\beta = 22.62^\circ$$



$$(a_G)_x = (a_B)_x + (a_{G/B})_x = 0 + (a_{G/B})_x = 0.125 \alpha_{BC}$$

$$(a_G)_y = (a_B)_y + (a_{G/B})_y = 72 + (a_{G/B})_y = 72 + 0.325 \alpha_{BC}$$

$$a_G = a_B + (a_{G/B})$$

$$(a_G)_x + (a_G)_y = a_B + (a_{G/B})$$

$$[0.125 \alpha_{BC}] + [72 + 0.325 \alpha_{BC}] = [72] + [0.325 \alpha_{BC}]$$

$$+ \downarrow 72 = -72 + (0.325 \alpha_{BC}) \left( \frac{12}{13} \right); \alpha_{BC} = 480 \text{ rad/s}^2$$

$$+ \nearrow 0.125 \alpha_{CD} = (0.325)(480) \left( \frac{5}{13} \right); \alpha_{CD} = 480 \text{ rad/s}^2$$

$$\bar{a} = \bar{a}_B + \bar{a}_{G/B} = \bar{a}_B + (\bar{a}_{G/B})_t$$

$$= [72 \text{ m/s}^2 \uparrow] + [0.125 \alpha_{BC} \left( \frac{5}{13} \right)]$$

$$= [72 \text{ m/s}^2 \uparrow] + [0.125 (480) \left( \frac{5}{13} \right)]$$

$$\bar{a} = [72 \text{ m/s}^2 \uparrow] + [78 \text{ m/s}^2 \left( \frac{5}{13} \right)]$$

$$\pm \bar{a}_x = 78 \left( \frac{5}{13} \right) = 30 \text{ m/s}^2 \rightarrow$$

$$\uparrow \bar{a}_y = 72 - 78 \left( \frac{12}{13} \right) = 72 - 72 = 0$$

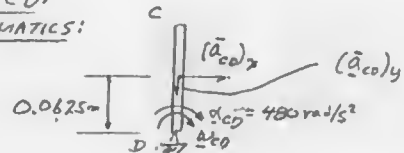
TOTAL ACCELERATION OF G IS  $30 \text{ m/s}^2 \rightarrow$

(CONTINUED)

16.134 continued

BAR CD:

KINEMATICS:



$$(\bar{a}_C)_x = (0.0625 \text{ m})(480 \text{ rad/s}^2) = 30 \text{ m/s}^2 \rightarrow$$

KINETICS

$$W_{CD} = I_{CD} \alpha_{CD} = m(\bar{a}_C)_x$$

$$R_D = D_y$$

$$\bar{I}_{CD} = \frac{1}{12} m(CD)^2 = \frac{1}{12} (1.5 \text{ kg})(0.125 \text{ m})^2 = 1.953 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\pm \Sigma M_D = \Sigma (M_n)_{\text{eff}}:$$

$$C_x(0.125 \text{ m}) = \bar{I}_{CD} \alpha_{CD} + m(\bar{a}_C)_x(0.0625 \text{ m})$$

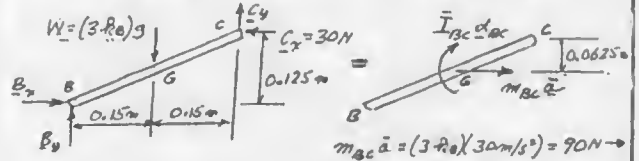
$$0.125 C_x = (1.953 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(480 \text{ rad/s}^2) + (1.5 \text{ kg})(30 \text{ m/s}^2)(0.0625 \text{ m})$$

$$0.125 C_x = 3.75$$

$$C_x = 30 \text{ N}$$

BAR BC:

$$\bar{I}_{BC} = \frac{1}{12} m(BC)^2 = \frac{1}{12} (3 \text{ kg})(0.325 \text{ m})^2 = 26.406 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$



$$\pm \Sigma F_x = \Sigma (F_x)_{\text{eff}}: B_x - C_x = m_{BC} \bar{a}_x$$

$$B_x - 30 \text{ N} = 90 \text{ N}$$

$$B_x = 120 \text{ N}$$

$$B_x = 120 \text{ N} \rightarrow$$

$$\pm \Sigma M_G = \Sigma (M_n)_{\text{eff}}:$$

$$B_y(0.3 \text{ m}) - B_x(0.125 \text{ m}) - W(0.15 \text{ m}) = \bar{I}_{BC} \alpha_{BC} - (m_{BC} \bar{a}) (0.0625 \text{ m})$$

$$0.3 B_y - (120)(0.125) - (3)(9.81)(0.15) = (26.406 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(480 \text{ rad/s}^2) - (90 \text{ N})(0.0625)$$

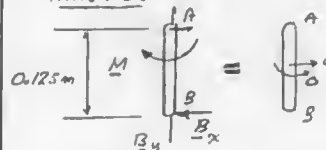
$$0.3 B_y - 15 - 4.4145 = 12.675 - 5.625$$

$$0.3 B_y = 26.465; B_y = 88.22 \text{ N}; B_y = 88.2 \text{ N} \uparrow$$

BAR AB:



KINETICS



$$\pm \Sigma M_A = \Sigma (M_n)_{\text{eff}}:$$

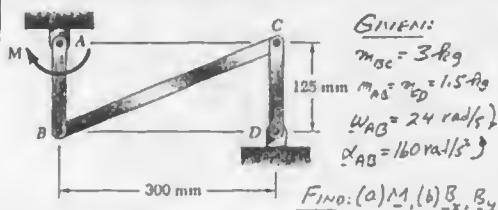
$$M + B_x(0.125 \text{ m}) = 0$$

$$M + (120 \text{ N})(0.125 \text{ m}) = 0$$

$$M = -15 \text{ N} \cdot \text{m}$$

$$M = 15 \text{ N} \cdot \text{m}$$

16.135



GIVEN:

$$m_{BC} = 3 \text{ kg}$$

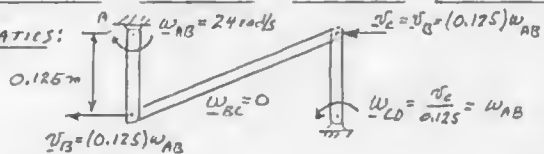
$$m_{AB} = m_{CD} = 1.5 \text{ kg}$$

$$\omega_{AB} = 24 \text{ rad/s}$$

$$\alpha_{AB} = 160 \text{ rad/s}^2$$

FIND: (a)  $M$ , (b)  $B_x$ ,  $B_y$ 

KINEMATICS:



BAR AB:

$$\omega_{AB} = 24 \text{ rad/s}$$

$$\alpha_{AB} = 160 \text{ rad/s}^2$$

$$(a_B)_x = (AB)\alpha_{AB} = (0.125)(160)$$

$$(a_B)_x = 20 \text{ m/s}^2$$

$$(a_B)_y = (AB)\omega_{AB}^2 = (0.125)(24)^2$$

$$(a_B)_y = 72 \text{ m/s}^2$$

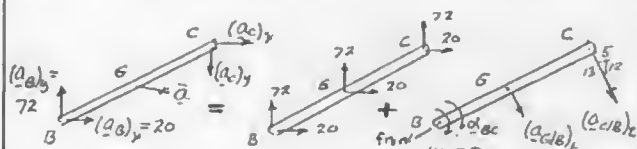
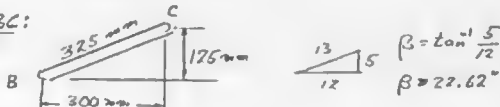
BAR CD:

$$(a_C)_x = 0.125\alpha_{CD}$$

$$(a_C)_y = (CD)\omega_{CD}^2 = (0.125)(24)^2$$

$$(a_C)_y = 72 \text{ m/s}^2$$

BAR BC:



$$(a_C/B)_x = (BC)\alpha_{BC} = (0.325)\alpha_{BC} = 0.325\alpha_{BC}$$

$$(a_C/B)_y = (BC)\omega_{BC}^2 = \left(\frac{0.325}{2}\right)\alpha_{BC} = 0.1625\alpha_{BC}$$

$$a_C = a_B + a_{C/B}: (a_C)_x + (a_C)_y = (a_B)_x + (a_B)_y + (a_{C/B})_x + (a_{C/B})_y$$

$$[0.125\alpha_{CD} \rightarrow] + [72 \text{ m/s}^2 \downarrow] = [20 \text{ m/s}^2 \rightarrow] + [72 \text{ m/s}^2 \uparrow] + [0.325\alpha_{BC} \downarrow] + [0.1625\alpha_{BC} \uparrow]$$

$$+ \downarrow 72 = -72 + (0.325\alpha_{BC})\left(\frac{12}{13}\right); \alpha_{BC} = 480 \text{ rad/s}^2$$

$$\rightarrow 0.125\alpha_{CD} = 20 + (0.325)(480)\left(\frac{5}{13}\right); \alpha_{CD} = 640 \text{ rad/s}^2$$

$$\bar{a} = a_B + a_{G/B}: (a_B)_x + (a_B)_y + (a_{G/B})_x + (a_{G/B})_y$$

$$\bar{a} = [20 \text{ m/s}^2 \rightarrow] + [72 \text{ m/s}^2 \uparrow] + [0.1625\alpha_{BC} \downarrow] + [0.1625\alpha_{BC} \uparrow]$$

$$\bar{a} = [20 \text{ m/s}^2 \rightarrow] + [72 \text{ m/s}^2 \uparrow] + [78 \text{ m/s}^2 \downarrow] + [78 \text{ m/s}^2 \uparrow]$$

$$\rightarrow \bar{a}_x = 20 + 78\left(\frac{5}{13}\right) = 20 + 30 = 50 \text{ m/s}^2$$

$$\uparrow \bar{a}_y = 72 - 78\left(\frac{12}{13}\right) = 0$$

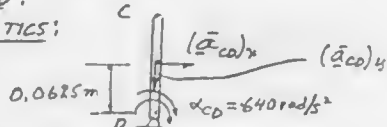
TOTAL ACCELERATION OF G IS  $50 \text{ m/s}^2 \rightarrow$ 

(CONTINUED)

16.135 continued

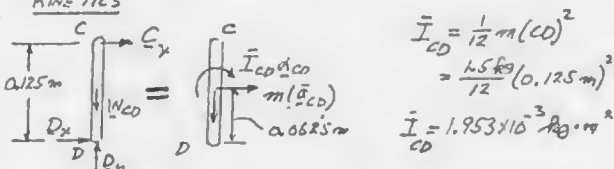
BAR CD:

KINEMATICS:



$$(\bar{a}_{CD})_x = (0.0625 \text{ m})(640 \text{ rad/s}^2) = 40 \text{ m/s}^2 \rightarrow$$

KINETICS



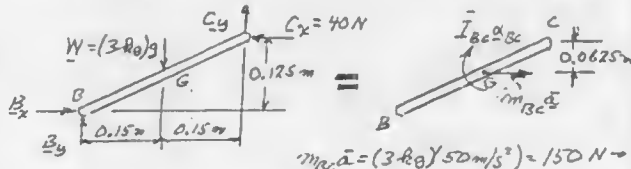
$$\rightarrow \Sigma M_D = \Sigma (M_D)_{eff}: C_x(0.125 \text{ m}) = \bar{I}_{CD}\alpha_{CD} + m(\bar{a}_{CD})_x(0.0625 \text{ m})$$

$$0.125C_x = (1.953 \times 10^{-3} \text{ kg})(640 \text{ rad/s}^2)$$

$$+ (1.5 \text{ kg})(40 \text{ m/s}^2)(0.0625 \text{ m})$$

$$0.125C_x = 5.00 \quad C_x = 40 \text{ N}$$

$$\text{BAR BC: } \bar{I}_{BC} = \frac{1}{2}m(BC)^2 = \frac{3.89}{12}(0.325 \text{ m})^2 = 26.406 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$



$$\rightarrow \Sigma F_y = \Sigma (F_y)_{eff}: B_y - C_y = m_{BC}\bar{a}_y$$

$$B_y - 40 \text{ N} = 150 \text{ N}$$

$$B_y = 190 \text{ N} \quad B_x = 190 \text{ N} \rightarrow$$

$$\rightarrow \Sigma M_C = \Sigma (M_C)_{eff}:$$

$$B_y(0.3 \text{ m}) - B_x(0.125 \text{ m}) - W(0.15 \text{ m}) = \bar{I}_{BC}\alpha_{BC} - (m_{BC}\bar{a}_x)(0.0625 \text{ m})$$

$$0.3B_y - (190)(0.125) - (3.89)(9.81)(0.15 \text{ m}) = (26.406 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(480 \text{ rad/s}^2)$$

$$- (150 \text{ N})(0.0625 \text{ m})$$

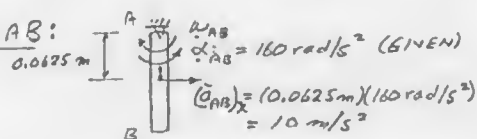
$$0.3B_y - 23.75 - 4.4145 = 12.675 - 9.375$$

$$0.3B_y = 31.465$$

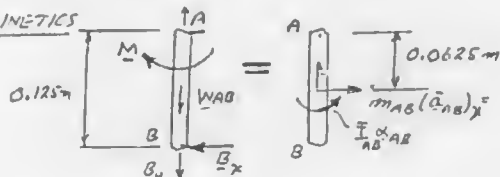
$$B_y = 104.88 \text{ N}$$

$$B_y = 104.9 \text{ N} \uparrow$$

BAR AB:



KINETICS



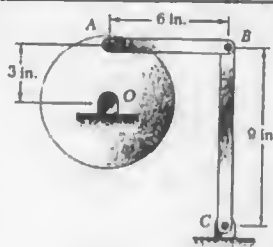
$$\rightarrow \Sigma M_A = \Sigma (M_A)_{eff}: M - B_x(0.125 \text{ m}) = -\bar{I}_{AB}\alpha_{AB} - m_{AB}(a_{AB})_x(0.0625 \text{ m})$$

$$M + (190)(0.125) = -\frac{1}{12}(1.5)(0.125)^2(160) - (1.5)(10)(0.0625)$$

$$M + 23.75 = -0.3125 - 0.9375$$

$$M = -25.0 \text{ N} \cdot \text{m}$$

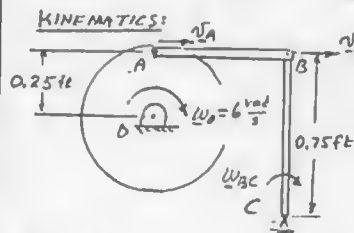
16.136



GIVEN:  
 $W_{AB} = 4 \text{ lb}$   
 $W_{BC} = 6 \text{ lb}$   
 Disk:  $\omega_0 = 6 \text{ rad/s}$   
 $\dot{\omega}_0 = 0$

FIND: FORCES  
 ON AB AT A AND B

KINEMATICS:



VELOCITY

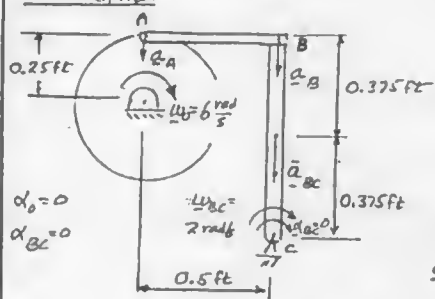
$$W_{AB} = 0$$

$$V_B = V_A = (0.25 \text{ ft})(6 \text{ rad/s}) = 1.5 \text{ ft/s}$$

$$\omega_{BC} = \frac{V_B}{0.75 \text{ ft}} = \frac{1.5 \text{ ft/s}}{0.75 \text{ ft}} = 2 \text{ rad/s}$$

$$\omega_{BC} = 2 \text{ rad/s}$$

ACCELERATION



$$a_A = (0.25 \text{ ft})(6 \text{ rad/s}^2) = 1.5 \text{ ft/s}^2$$

$$a_A = 1.5 \text{ ft/s}^2$$

$$a_B = (0.75 \text{ ft})(2 \text{ rad/s}^2) = 1.5 \text{ ft/s}^2$$

$$\bar{a}_{BC} = (0.375 \text{ ft})(2 \text{ rad/s}^2) = 0.75 \text{ ft/s}^2$$

$$\bar{a}_{BC} = 0.75 \text{ ft/s}^2$$

$$\bar{a}_{AB} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(1.5 + 1.5) = 1.5 \text{ ft/s}^2$$

$$\bar{a}_{AB} = 1.5 \text{ ft/s}^2$$

$$a_A = a_B + (0.5 \text{ ft})\alpha_{AB}$$

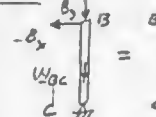
$$1.5 \text{ ft/s}^2 = 1.5 \text{ ft/s}^2 + (0.5 \text{ ft})\alpha_{AB}$$

$$\alpha_{AB} = 0 \text{ rad/s}^2$$

KINETICS:  $\bar{I}_{AB} = \frac{1}{12} m_{AB} (AB)^2 = \frac{1}{12} \frac{4 \text{ lb}}{32.2} (0.5 \text{ ft})^2$

$$\bar{I}_{AB} = 2.588 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

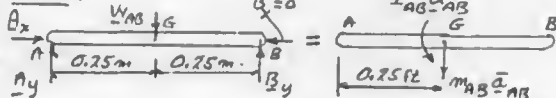
ROD BC:



$$\text{SINCE } \alpha_{BC} = 0, \bar{a} = 0$$

$$\Sigma M_C = 0 \text{ YIELDS } B_x = 0$$

ROD AB:



$$\Sigma F_x = \Sigma (F_x)_{\text{eff}}: A_x = 0$$

$$\Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$B_y(0.5 \text{ ft}) - W_{AB}(0.25 \text{ ft}) = \bar{I}_{AB}\alpha_{AB} - m_{AB}\bar{a}_{AB}(0.25 \text{ ft})$$

$$0.5 B_y - (4 \text{ lb})(0.25 \text{ ft}) = (2.588 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(0) - \frac{4 \text{ lb}}{32.2} (1.5 \text{ ft/s}^2)(0.25 \text{ ft})$$

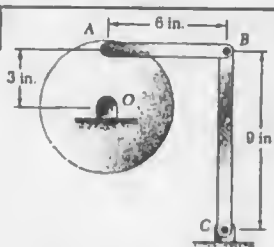
$$0.5 B_y - 1 = 0.03106 - 0.1863$$

$$0.5 B_y = 0.8447 \quad B_y = 1.689 \text{ lb} \quad B = 1.689 \text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A_y - W_{AB} + B_y = -m_{AB}\bar{a}_{AB}$$

$$A_y - 4 \text{ lb} + 1.689 \text{ lb} = -\frac{4 \text{ lb}}{32.2} (1.5 \text{ ft/s}^2); \quad A_y = 1.515 \text{ lb}; \quad A = 1.515 \text{ lb} \uparrow$$

16.137



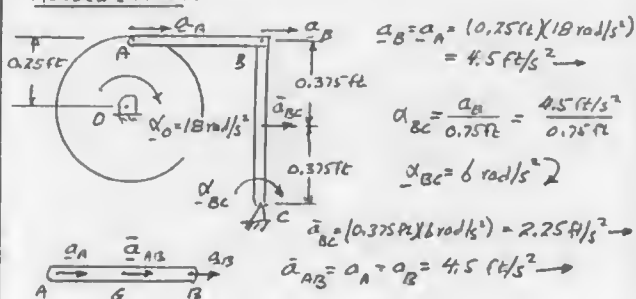
GIVEN:  
 $W_{AB} = 4 \text{ lb}$   
 $W_{BC} = 6 \text{ lb}$   
 Disk:  $\omega_0 = 18 \text{ rad/s}$   
 $\dot{\omega}_0 = 18 \text{ rad/s}^2$

FIND: FORCES ON  
 AB AT A AND B

KINEMATICS:

VELOCITY OF ALL ELEMENTS = C

ACCELERATION:



$$a_B = a_A = (0.25 \text{ ft})(18 \text{ rad/s}^2) = 4.5 \text{ ft/s}^2$$

$$\alpha_{BC} = \frac{a_B}{0.75 \text{ ft}} = \frac{4.5 \text{ ft/s}^2}{0.75 \text{ ft}} = 6 \text{ rad/s}^2$$

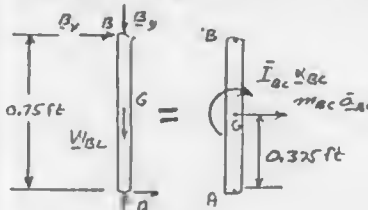
$$\alpha_{BC} = 6 \text{ rad/s}^2$$

$$\bar{a}_{BC} = (0.375 \text{ ft})(6 \text{ rad/s}^2) = 2.25 \text{ ft/s}^2$$

$$\bar{a}_{AB} = a_A = a_B = 4.5 \text{ ft/s}^2$$

KINETICS:  $\bar{I}_{BC} = \frac{1}{12} m_{BC} (BC)^2 = \frac{1}{12} \frac{6 \text{ lb}}{32.2} (0.75 \text{ ft})^2$

$$\bar{I}_{BC} = 8.734 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$



$$\Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$B_x(0.75 \text{ ft}) = \bar{I}_{BC}\alpha_{BC} + m_{BC}\bar{a}_{BC}(0.375 \text{ ft})$$

$$0.75 B_x = (8.734 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(6 \text{ rad/s}^2)$$

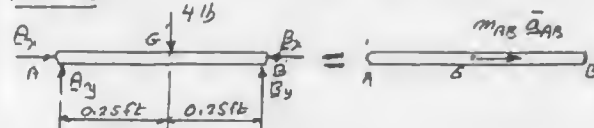
$$+ (\frac{6 \text{ lb}}{32.2})(2.25 \text{ ft/s}^2)(0.375 \text{ ft})$$

$$0.75 B_x = 0.0524 + 0.1572$$

$$B_x = 0.2795 \text{ lb}$$

$$(\text{ON AB}) \quad B_x = 0.280 \text{ lb} \leftarrow$$

ROD AB:



$$\Sigma F_x = \Sigma (F_x)_{\text{eff}}: A_x + B_x = m_{AB}\bar{a}_{AB}$$

$$A_x - 0.2795 \text{ lb} = (\frac{4 \text{ lb}}{32.2})(4.5 \text{ ft/s}^2)$$

$$A_x - 0.2795 \text{ lb} = 0.5590 \text{ lb}$$

$$A_x = 0.8385 \text{ lb}$$

$$A_x = 0.839 \text{ lb} \rightarrow$$

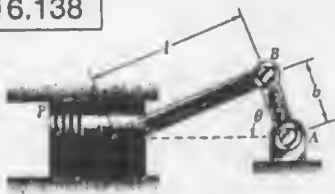
$$\Sigma M_A: B_y = 2 \text{ lb} \uparrow$$

$$\Sigma M_B: A_y = 2 \text{ lb} \uparrow$$





16.138



GIVEN:  $\omega_{AB} = 600 \text{ rpm}$   
 $l = 250 \text{ mm}$ ,  $b = 100 \text{ mm}$   
 $m_{BD} = 1.2 \text{ kg}$ ,  $m_P = 1.8 \text{ kg}$   
 $\theta = 180^\circ$   
 FIND: FORCES ON  
 BD AT B AND D

KINEMATICS: CRANK AB:

$\omega_{AB} = 600 \text{ rpm} \left( \frac{2\pi}{60} \right) = 62.832 \text{ rad/s}$   
 $a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s})^2$   
 $a_B = 394.78 \text{ m/s}^2 \leftarrow$

Also:  $v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \downarrow$

CONNECTING ROD BD:

VELOCITY

INSTANT CENTER AT D:

$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s} \uparrow$

ACCELERATION:

$a_D = a_B + a_{D/B} = [a_B \leftarrow] + [(BD)\omega_{BD}^2 \rightarrow]$   
 $a_D = [394.78 \text{ m/s}^2 \leftarrow] + [0.25 \text{ m} \times 25.133 \text{ rad/s}^2 \rightarrow]$   
 $a_D = [394.78 \text{ m/s}^2 \leftarrow] + [157.92 \text{ m/s}^2 \rightarrow] = 236.86 \text{ m/s}^2 \leftarrow$   
 $\bar{a}_{BD} = \frac{1}{2}(a_B + a_D) = \frac{1}{2}(394.78 \leftarrow + 236.86 \leftarrow) = 315.82 \text{ m/s}^2 \leftarrow$

KINETICS OF PISTON

$\boxed{P \leftarrow D} = \boxed{P \leftarrow} \rightarrow m_P \bar{a}_D = (1.8 \text{ kg})(236.86 \text{ m/s}^2)$   
 $D = 426.35 \text{ N} \leftarrow$

FORCE EXERTED ON CONNECTING ROD AT D IS:  
 $D = 426.35 \text{ N} \leftarrow$

KINETICS OF CONNECTING ROD: (NEGLECT WEIGHT)

$D = 426.35 \text{ N}$   
 $\rightarrow \boxed{D \leftarrow B} = \boxed{D \leftarrow B} \rightarrow m_{BD} \bar{a}_{BD}$

$\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}:$

$B - D = m_{BD} \bar{a}_{BD}$

$B - 426.35 \text{ N} = (1.2 \text{ kg})(315.82 \text{ m/s}^2)$

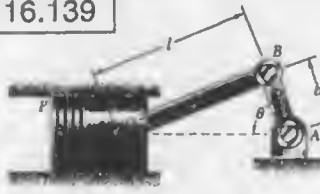
$B = 426.35 \text{ N} + 378.98 \text{ N} = 805.33 \text{ N}$

FORCES EXERTED ON CONNECTING ROD

$B = 805 \text{ N} \leftarrow$

$D = 426 \text{ N} \rightarrow$

16.139



GIVEN:  $\omega_{AB} = 600 \text{ rpm}$   
 $l = 250 \text{ mm}$ ,  $b = 150 \text{ mm}$   
 $m_{BD} = 1.2 \text{ kg}$ ,  $m_P = 1.8 \text{ kg}$   
 $\theta = 0$   
 FIND: FORCES ON  
 BD AT B AND D

KINEMATICS: CRANK AB:

$\omega_{AB} = 600 \text{ rpm} \left( \frac{2\pi}{60} \right) = 62.832 \text{ rad/s}$   
 $a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s})^2$   
 $a_B = 394.78 \text{ m/s}^2 \rightarrow$

Also:  $v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \uparrow$

CONNECTING ROD BD:

VELOCITY

INSTANT CENTER AT D:

$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s} \uparrow$

ACCELERATION:

$a_D = a_B + a_{D/B} = [a_B \rightarrow] + [(BD)\omega_{BD}^2 \rightarrow]$   
 $a_D = [394.78 \text{ m/s}^2 \rightarrow] + [0.25 \text{ m} \times 25.133 \text{ rad/s}^2 \rightarrow]$   
 $a_D = [394.78 \text{ m/s}^2 \rightarrow] + [157.92 \text{ m/s}^2 \rightarrow] = 552.70 \text{ m/s}^2 \rightarrow$   
 $\bar{a}_{BD} = \frac{1}{2}(a_B + a_D) = \frac{1}{2}(394.78 \rightarrow + 552.70 \rightarrow) = 473.74 \text{ m/s}^2 \rightarrow$

KINETICS OF PISTON

$\boxed{P \rightarrow D} = \boxed{P \rightarrow} \rightarrow m_D \bar{a}_D = (1.8 \text{ kg})(552.70 \text{ m/s}^2)$   
 $D = 994.86 \text{ N} \rightarrow$

FORCE EXERTED ON CONNECTING ROD AT D IS:  
 $D = 994.86 \text{ N} \leftarrow$

KINETICS OF CONNECTING ROD (NEGLECT WEIGHT)

$D = 994.86 \text{ N}$   
 $\leftarrow \boxed{D \rightarrow B} = \boxed{D \rightarrow B} \rightarrow m_{BD} \bar{a}_{BD}$

$\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}:$

$B - D = m_{BD} \bar{a}_{BD}$

$B - 994.86 \text{ N} = (1.2 \text{ kg})(473.74 \text{ m/s}^2)$

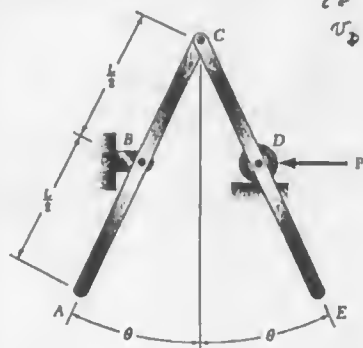
$B = 994.86 \text{ N} + 568.44 \text{ N} = 1563.3 \text{ N}$

FORCES ACTING ON CONNECTING ROD

$B = 1563 \text{ N} \rightarrow$

$D = 995 \text{ N} \leftarrow$

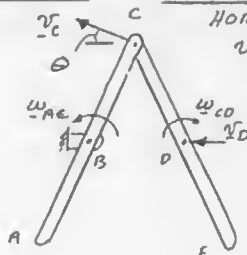
# 16.140 and 16.141



**GIVEN:** RODS AC AND CE EACH 1/2 INCH W.  
 $v_D = \text{CONSTANT TO LEFT}$   
**PROBLEM 16.140**  
**FIND:** P IN TERMS OF L, W,  $v_D$  AND  $\theta$   
**PROBLEM 16.141**  
**FIND:** P IF  $L = 3 \text{ ft}$ ,  $W = 24 \text{ lb}$ ,  $v_D = 6 \text{ ft/s}$ ,  $\theta = 55^\circ$

**KINEMATICS:**

**VELOCITY:**



HORIZ. COMPONENT OF  $v_C$  EQUALS  $v_D/2$

$$v_C \cos \theta = \frac{1}{2} v_D$$

$$v_C = \frac{v_D}{2 \cos \theta}$$

ROD AC:

$$v_C = \frac{L}{2} \omega_{AC}$$

$$\frac{v_D}{2 \cos \theta} = \frac{L}{2} \omega_{AC}$$

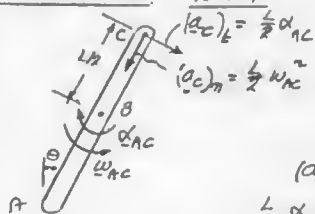
$$\omega_{AC} = \frac{v_D}{L \cos \theta}$$

BY SYMMETRY:  $|\omega_{CE}| = |\omega_{AC}|$

ALSO SINCE  $a_D = 0$ , HORIZONTAL COMPONENT OF  $a_C$  IS ZERO. THUS  $a_C$  IS VERTICAL

**ACCELERATION:**

**ROD AC**



$$(a_c)_t = (a_c)_n \tan \theta$$

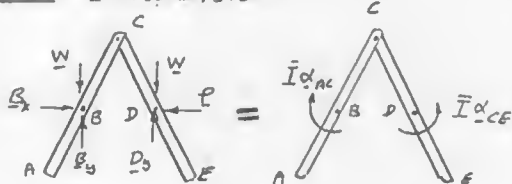
$$\frac{1}{2} \alpha_{AC} = \frac{1}{2} \omega_{AC}^2 \tan \theta$$

$$\alpha_{AC} = \omega_{AC}^2 \tan \theta = \left( \frac{v_D}{L \cos \theta} \right)^2 \tan \theta = \frac{v_D^2 \tan \theta}{L^2 \cos^2 \theta}$$

BY SYMMETRY:  $|\alpha_{AC}| = |\alpha_{CE}|$

$$\alpha_{CE} = \frac{v_D^2 \tan \theta}{L^2 \cos^2 \theta}$$

**KINETICS: ENTIRE SYSTEM**



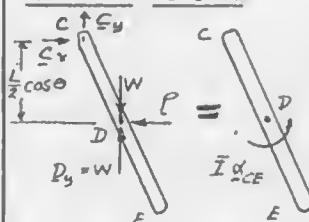
$$\sum M_B = \sum (M_B)_{eff}: (W - D_y)(30) = \bar{I}(\alpha_{CE} - \alpha_{AC})$$

SINCE  $\alpha_{CE} = \alpha_{AC}$ , WE FIND  $D_y = W \uparrow$

(CONTINUED)

# 16.140 and 16.141 continued

**KINETICS: ROD CE**



$$+\sum M_C = \sum (M_C)_{eff}: P \frac{L}{2} \cos \theta = -\bar{I} \alpha_{CE}$$

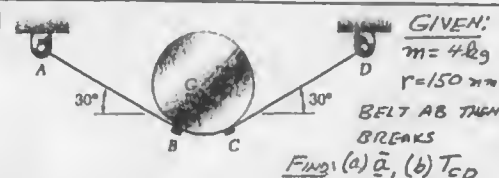
$$P \frac{L}{2} \cos \theta = -\frac{m}{12} L^2 \left( \frac{v_D^2 \tan \theta}{L^2 \cos^2 \theta} \right)$$

$$P = \frac{m v_D^2 \tan \theta}{6 L \cos^2 \theta}$$

**DATA:**  $L = 3 \text{ ft}$ ,  $W = 24 \text{ lb}$ ,  $v_D = 6 \text{ ft/s}$ ,  $\theta = 55^\circ$

$$P = \frac{24 \text{ lb}}{3 \times 2 \text{ ft/s}^2} \frac{(6 \text{ ft/s})^2 \tan 55^\circ}{6 (3 \text{ ft}) \cos^2 55^\circ} = 11.28 \text{ lb}$$

## \*16.142



**GIVEN:**

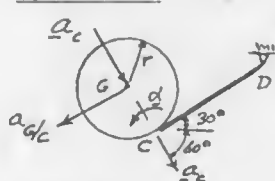
$m = 4 \text{ kg}$

$r = 150 \text{ mm}$

BELT AB THEN BREAKS

**FIND:** (a)  $\bar{a}$ , (b)  $T_{CD}$

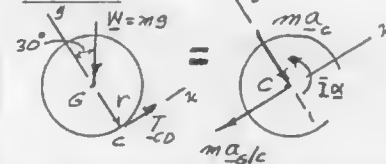
**KINEMATICS:**  $v = 0$



$$\bar{a} = a_c = a_c + a_{G/c}$$

WHERE  $a_{G/c} = r \alpha$

**KINETICS**



$$+\uparrow \sum F_y = \sum (F_y)_{eff}: -W \cos 30^\circ = -m a_c$$

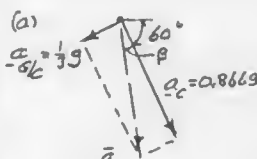
$$a_c = 0.866g \nabla 60^\circ$$

$$+\sum M_C = \sum (M_C)_{eff}: (W \sin 30^\circ) r = \bar{I} \alpha + (m a_{G/c}) r$$

$$m g r \sin 30^\circ = \left( \frac{1}{2} m r^2 \right) \alpha + (m r \alpha) r$$

$$\frac{1}{2} g = \frac{3}{2} r \alpha \quad \alpha = \frac{1}{3} \frac{g}{r}$$

$$a_{G/c} = r \alpha = \frac{1}{3} g \quad a_{G/c} = \frac{1}{3} g \nabla 30^\circ$$



$$\beta = \tan^{-1} \frac{1/3}{0.866} = 21.052^\circ$$

$$\bar{a} = \frac{0.866g}{\cos \beta} = \frac{0.866g}{\cos 21.052^\circ}$$

$$\bar{a} = 0.92779g = 0.92779(9.81)$$

$$\bar{a} = 9.10 \text{ m/s}^2 \nabla 81.1^\circ$$

(b)  $+\sum F_x = \sum (F_x)_{eff}: T - W \sin 30^\circ = -m a_{G/c}$

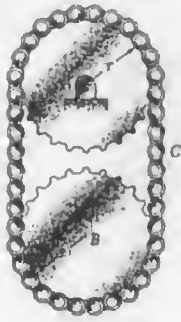
$$T - W \sin 30^\circ = -m a_{G/c}$$

$$T = 0.5 mg - m \left( \frac{g}{3} \right) = \frac{1}{6} mg$$

$$T = \frac{1}{6} (4 \text{ kg}) (9.81 \text{ m/s}^2) = 6.54 \text{ N}$$

$$T = 6.54 \text{ N}$$

\*16.143



GIVEN:

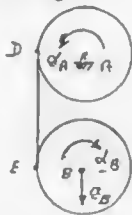
DISK OF MASS  $m$  AND  
RADIUS  $r$   
PIN AT C IS REMOVED

FIND:

- (a)  $\underline{a}_A$  AND  $\underline{a}_B$   
(b) TENSION IN CHAIN  
(c)  $\underline{a}_B$

KINEMATICS:

$$\omega_A = \omega_B = 0$$

ASSUME  $\underline{a}_A$  AND  $\underline{a}_B$ 

$$\underline{a}_D = r\alpha \downarrow$$

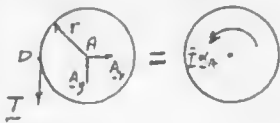
$$\underline{a}_E = \underline{a}_D = r\alpha \downarrow$$

$$\underline{a}_B = \underline{a}_E + \underline{a}_{B/E}$$

$$= (r\alpha + r\alpha)$$

$$\underline{a}_B = r(\alpha + \alpha) \downarrow$$

KINETICS: DISK A:



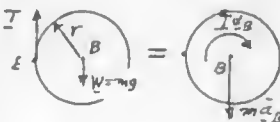
$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}:$$

$$Tr = \bar{I} \alpha_A$$

$$Tr = \frac{1}{2} m r^2 \alpha_A$$

$$\alpha_A = \frac{2T}{mr} \quad (1)$$

DISK B:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}:$$

$$Tr = \bar{I} \alpha_B$$

$$Tr = \frac{1}{2} m r^2 \alpha_B$$

$$\alpha_B = \frac{2T}{mr} \quad (2)$$

FROM (1) AND (2) WE NOTE THAT  $\alpha_A = \alpha_B$ 

$$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}:$$

$$Wr = \frac{1}{2} m r^2 \alpha_B + m r(\alpha_A + \alpha_B) r$$

$$\alpha_A = \alpha_B: \quad Wr = \frac{5}{2} m r^2 \alpha_A \quad \alpha_A = \frac{2}{5} \frac{g}{r}$$

$$\alpha_B = \frac{2}{5} \frac{g}{r}$$

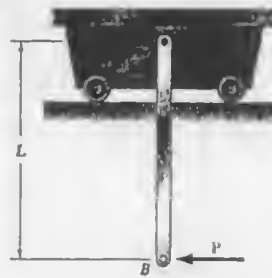
SUBSTITUTE FOR  $\alpha_A$  INTO (1):

$$\frac{2}{5} \frac{g}{r} = \frac{2T}{mr} \quad T = \frac{1}{5} mg$$

$$\underline{a}_B = r(\alpha_A + \alpha_B) = r(2\alpha_A) = 2r\left(\frac{2}{5} \frac{g}{r}\right)$$

$$\underline{a}_B = \frac{4}{5} g \downarrow$$

\*16.144



GIVEN:

CART OF MASS  $m$   
ROD OF MASS  $m$   
CART AT REST  
WHEN  $\underline{P}$  IS  
APPLIEDFIND:  $\underline{a}_A$   
 $\underline{a}_B$ 

KINEMATICS:

$$\underline{a}_{CART} = \underline{a}_A$$

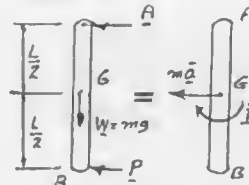
$$\underline{a}_A = \frac{L}{2} \alpha - \bar{a}$$

KINETICS: CART

$$\underline{a}_A = \underline{a}_A \quad m \underline{a}_A = m\left(\frac{L}{2} \alpha - \bar{a}\right)$$

$$\pm \Sigma F_x = \Sigma (F_x)_{eff} \quad A = m\left(\frac{L}{2} \alpha - \bar{a}\right) \quad (1)$$

ROD AB:



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}:$$

$$PL = m \bar{a} \frac{L}{2} + \bar{I} \alpha$$

$$PL = \frac{1}{2} m L \bar{a} + \frac{1}{12} m L^2 \alpha \quad (2)$$

$$\pm \Sigma F_y = \Sigma (F_y)_{eff}:$$

$$P + A = m \bar{a}$$

$$P + m\left(\frac{L}{2} \alpha - \bar{a}\right) = m \bar{a}$$

$$P = 2m \bar{a} - \frac{1}{2} m L \alpha \quad (3)$$

$$\text{MULTIPLY (3) BY } \frac{1}{6}: \quad \frac{1}{6} PL = \frac{1}{3} m L \bar{a} - \frac{1}{12} m L^2 \alpha \quad (4)$$

ADD (2) AND (4):

$$\frac{7}{6} PL = \frac{5}{6} m L \bar{a}$$

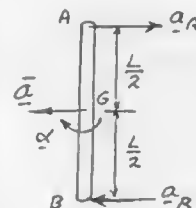
$$\bar{a} = \frac{7}{5} \frac{P}{m}$$

SUBSTITUTE (5) INTO (3):

$$P = 2m\left(\frac{7}{5} \frac{P}{m}\right) - \frac{1}{2} m L \alpha$$

$$P = \frac{14}{5} P - \frac{1}{2} m L \alpha$$

$$\alpha = \frac{18}{5} \frac{P}{mL}$$



$$\underline{a}_A = \frac{L}{2} \alpha - \bar{a}$$

$$= \frac{L}{2} \left( \frac{18}{5} \frac{P}{mL} \right) - \frac{7}{5} \frac{P}{m}$$

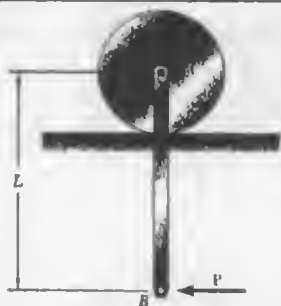
$$\underline{a}_A = \frac{2}{5} \frac{P}{m} \rightarrow$$

$$\underline{a}_B = \frac{L}{2} \alpha + \bar{a}$$

$$= \frac{L}{2} \left( \frac{18}{5} \frac{P}{mL} \right) + \frac{7}{5} \frac{P}{m}$$

$$\underline{a}_B = \frac{16}{5} \frac{P}{m} \leftarrow$$

16.145

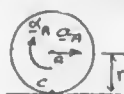


GIVEN:  
BAR AB OF MASS  $m$   
CYLINDER OF MASS  $m$   
SYSTEM AT REST  
WHEN  $P$  IS APPLIED

FIND:  $\alpha_A$   
 $\alpha_B$

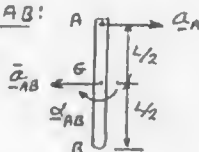
KINEMATICS:

ROLLING  
WITHOUT  
SLIDING?  
 $(a_c)_x = 0$



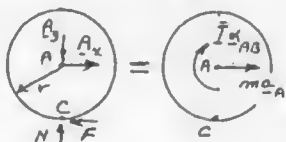
CYLINDER:  $\omega = 0$   
 $\pm \alpha_A = (a_c)_x + \alpha_A r$   
 $= 0 + r \alpha_A$   
 $\alpha_A = r \alpha_A \rightarrow \alpha_A = \frac{\alpha_A}{r}$

ROD AB:



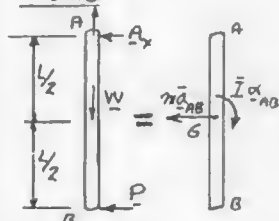
$\pm \alpha_A = \frac{L}{2} \alpha_{AB} - \bar{a}_{AB}$

KINETICS: CYLINDER:



$\rightarrow \Sigma M_C = \Sigma (M_C)_{eff}$   
 $A_y r = m a_A r + \bar{I} \alpha_A$   
 $A_y = m a_A + \frac{1}{2} m r^2 \left( \frac{a_A}{r} \right)$   
 $A_y = \frac{3}{2} m a_A$   
 $A_y = \frac{3}{2} m \left( \frac{L}{2} \alpha_{AB} - \bar{a}_{AB} \right) \quad (1)$

ROD AB:



$\rightarrow \Sigma M_A = \Sigma (M_A)_{eff}$   
 $PL = m \bar{a}_{AB} \frac{L}{2} + \bar{I} \alpha_{AB}$   
 $PL = m \bar{a}_{AB} \frac{L}{2} + \frac{m}{12} L^2 \alpha_{AB} \quad (2)$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$ :  $P + A_x = m \bar{a}_{AB}$   
SUBSTITUTE FROM (1):  $P + \frac{3}{2} m \left( \frac{L}{2} \alpha_{AB} - \bar{a}_{AB} \right) = m \bar{a}_{AB}$

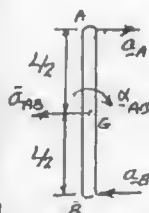
$P = \frac{5}{2} m \bar{a}_{AB} - \frac{3}{4} m L \alpha_{AB} \quad (3)$

MULTIPLY BY  $\frac{L}{9}$ :  $\frac{1}{9} P = \frac{5L}{18} m \bar{a}_{AB} - \frac{1}{12} m L^2 \alpha_{AB} \quad (4)$

(4) + (2):  $\frac{10}{9} PL = \left( \frac{1}{2} + \frac{5}{18} \right) m L \bar{a}_{AB} = \frac{7}{9} m L \bar{a}_{AB}$   
 $\bar{a}_{AB} = \frac{10}{7} \frac{P}{m} \quad (5)$

(5) -> (3):  $P = \frac{5}{2} m \left( \frac{10}{7} \frac{P}{m} \right) - \frac{3}{4} m L \alpha_{AB}$

$P = \frac{25}{7} P - \frac{3}{4} m L \alpha_{AB}$   
 $-\frac{18}{7} P = -\frac{3}{4} m L \alpha_{AB} \quad \alpha_{AB} = \frac{24}{7} \frac{P}{mL}$



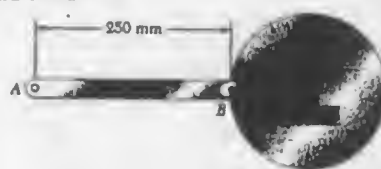
$\pm \alpha_A = \frac{L}{2} \alpha_{AB} - \bar{a}_{AB} = \frac{L}{2} \left( \frac{24}{7} \frac{P}{mL} \right) - \frac{10}{7} \frac{P}{m}$

$\alpha_A = \left( \frac{12}{7} - \frac{10}{7} \right) \frac{P}{m} = \frac{2}{7} \frac{P}{m}$

$\pm \alpha_B = \frac{1}{2} \alpha_{AB} + \bar{a}_{AB} = \frac{1}{2} \left( \frac{24}{7} \frac{P}{mL} \right) + \frac{10}{7} \frac{P}{m}$

$\alpha_B = \left( \frac{12}{7} + \frac{10}{7} \right) \frac{P}{m} = \frac{22}{7} \frac{P}{m}$

16.146

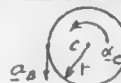


GIVEN:  $m_{AB} = 5 \text{ kg}$   
 $m_C = 8 \text{ kg}$

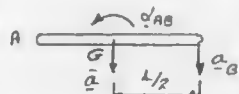
RELEASE FROM REST  
FIND: (a)  $\alpha_A$   
(b)  $\alpha_C$

KINEMATICS:

$\omega = 0$

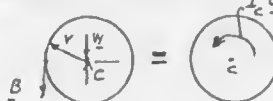


$\alpha_B = r \alpha_C$



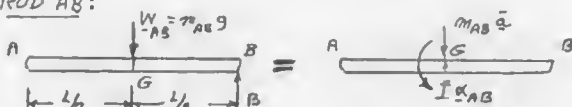
$\pm \bar{a} = a_B + a_{B/A}$   
 $\bar{a} = r \alpha_C + \frac{L}{2} \alpha_{AB}$

KINETICS: DISK



$\rightarrow \Sigma M_C = \Sigma (M_C)_{eff}$   
 $B r = \bar{I} \alpha_C$   
 $B r = \frac{1}{2} m_C r^2 \alpha_C$   
 $B = \frac{1}{2} m_C r \alpha_C$

ROD AB:



$\rightarrow \Sigma M_C = \Sigma (M_C)_{eff}$ :  $B \frac{L}{2} = \bar{I} \alpha_{AB}$   
 $\left( \frac{1}{2} m_C r \alpha_C \right) \frac{L}{2} = \frac{1}{12} m_{AB} L^2 \alpha_{AB}$   
 $\alpha_C = \frac{1}{3} \frac{m_{AB}}{m_C} \cdot \frac{L}{r} \alpha_{AB} \quad (1)$

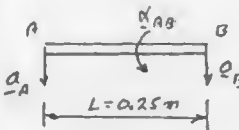
$\rightarrow \Sigma F_y = \Sigma (F_y)_{eff}$ :  
 $m_{AB} g - B = m_{AB} \bar{a}$   
 $m_{AB} g - \frac{1}{2} m_C r \alpha_C = m_{AB} \left( r \alpha_C + \frac{L}{2} \alpha_{AB} \right)$   
 $g = \frac{1}{2} \alpha_{AB} + \left( \frac{1}{2} \frac{m_C}{m_{AB}} + 1 \right) r \alpha_C$   
 $\frac{g}{L} = \frac{1}{2} \alpha_{AB} + \left( \frac{1}{2} \frac{m_C}{m_{AB}} + 1 \right) \frac{r}{L} \cdot \left( \frac{1}{3} \frac{m_{AB}}{m_C} \cdot \frac{L}{r} \right) \alpha_{AB}$   
 $\frac{g}{L} = \left( \frac{1}{2} + \frac{1}{6} + \frac{1}{3} \frac{m_{AB}}{m_C} \right) \alpha_{AB} = \frac{1}{3} \left( 2 + \frac{m_{AB}}{m_C} \right) \alpha_{AB}$   
 $\alpha_{AB} = \frac{3g}{L} \frac{1}{\left( 2 + \frac{m_{AB}}{m_C} \right)} \quad (2)$

$m_{AB} = 5 \text{ kg}, m_C = 8 \text{ kg}, r = 0.1 \text{ m}, L = 0.25 \text{ m}$

EG(1):  $\alpha_{AB} = \frac{3(9.81 \text{ m/s}^2)}{0.25 \text{ m}} \cdot \frac{1}{2 + \frac{5 \text{ kg}}{8 \text{ kg}}} = 44.846 \text{ rad/s}^2$

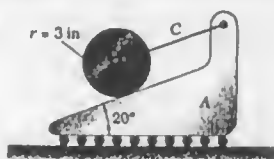
EG(2):  $\alpha_C = \frac{1}{3} \frac{5 \text{ kg}}{8 \text{ kg}} \cdot \frac{0.25 \text{ m}}{0.1 \text{ m}} (44.846 \text{ rad/s}^2) = 23.357 \text{ rad/s}^2$

$\alpha_B = r \alpha_C = (0.1 \text{ m}) (23.357 \text{ rad/s}^2) = 2.336 \text{ m/s}^2$   
 $\alpha_B = 2.34 \text{ m/s}^2$



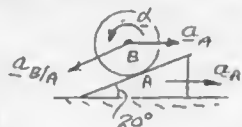
$\pm \alpha_A = \alpha_B + a_{B/A}$   
 $\alpha_A = \alpha_B + L \alpha_{AB}$   
 $\alpha_A = 2.336 \text{ m/s}^2 + (0.25 \text{ m}) (44.846 \text{ rad/s}^2)$   
 $\alpha_A = 2.336 + 11.212$   
 $\alpha_A = 13.55 \text{ m/s}^2$

\*16.147



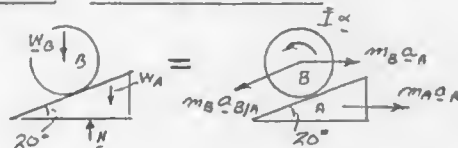
GIVEN:  $W_B = 6 \text{ lb}$   
 $W_A = 4 \text{ lb}$   
 AFTER CORD IS CUT  
 CYLINDER ROLLS.  
 FIND: (a)  $a_A$   
 (b)  $\alpha$

KINEMATICS: WE RESOLVE  $a_B$  INTO  $a_A$  AND A COMPONENT PARALLEL TO THE INCLINE



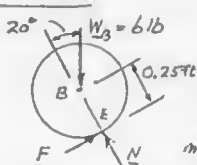
$a_B = a_A + a_{B/A}$   
 WHERE  $a_{B/A} = r\alpha$ , SINCE THE CYLINDER ROLLS ON WEDGE A.  
 $a_{B/A} = (0.25 \text{ ft})\alpha$

KINETICS: CYLINDER AND WEDGE



$$\begin{aligned} \sum F_x = \sum (F_x)_{ef}: 0 &= m_A a_A + m_B a_A - m_B a_{B/A} \cos 20^\circ \\ 0 &= \frac{(4+6) \text{ lb}}{g} a_A - \frac{6 \text{ lb}}{g} \left( \frac{3}{12} \text{ ft} \right) \alpha \cos 20^\circ \\ a_A &= (0.15 \cos 20^\circ) \alpha \quad (1) \end{aligned}$$

CYLINDER



$$\begin{aligned} I &= \frac{1}{2} W_B r^2 = \frac{1}{2} \frac{6 \text{ lb}}{g} (0.25 \text{ ft})^2 \\ I &= \frac{3}{16g} \end{aligned}$$

$$\begin{aligned} \sum M_E = \sum (M_E)_{ef}: \\ (6 \text{ lb}) \sin 20^\circ (0.25 \text{ ft}) &= I \alpha + (m_B a_{B/A}) (0.25 \text{ ft}) \\ &\quad - m_B a_A \cos 20^\circ (0.25 \text{ ft}) \end{aligned}$$

$$1.5 \sin 20^\circ = \frac{3}{16(32.2)} \alpha + \frac{6 \text{ lb}}{32.2} (0.25 \alpha) (0.25) - \frac{6 \text{ lb}}{32.2} a_A \cos 20^\circ (0.25)$$

$$0.51303 = 0.00582 \alpha + 0.01165 \alpha - 0.04378 a_A$$

SUBSTITUTE FROM (1):

$$0.51303 = 0.01747 \alpha - 0.04378 (0.15 \cos 20^\circ) \alpha$$

$$0.51303 = (0.01747 - 0.00617) \alpha$$

$$\alpha = 45.41 \text{ rad/s}^2$$

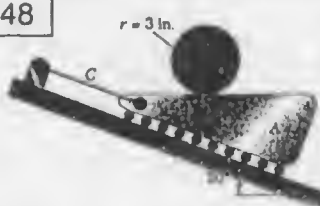
$$\alpha = 45.41 \text{ rad/s}^2$$

$$\begin{aligned} \text{EQ (1): } a_A &= (0.15 \cos 20^\circ) \alpha \\ &= (0.15 \cos 20^\circ) (45.41) \end{aligned}$$

$$a_A = 6.401 \text{ ft/s}^2$$

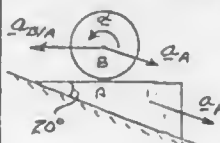
$$a_A = 6.40 \text{ ft/s}^2 \rightarrow$$

\*16.148



GIVEN:  $W_B = 6 \text{ lb}$   
 $W_A = 4 \text{ lb}$   
 AFTER CORD IS CUT  
 CYLINDER ROLLS  
 FIND: (a)  $a_A$   
 (b)  $\alpha$

KINEMATICS: WE RESOLVE  $a_B$  INTO  $a_A$  AND A HORIZONTAL COMPONENT  $a_{B/A}$

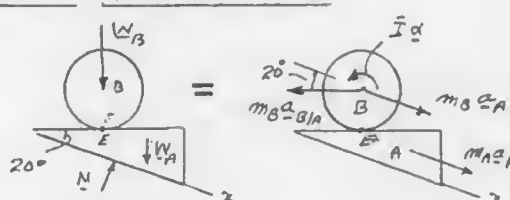


$$a_B = a_A + a_{B/A}$$

WHERE  $a_{B/A} = r\alpha$ , SINCE THE CYLINDER ROLLS ON WEDGE A.

$$a_{B/A} = (0.25 \text{ ft}) \alpha$$

KINETICS: CYLINDER AND WEDGE:



$$\begin{aligned} \sum F_x = \sum (F_x)_{ef}: \\ (W_A + W_B) \sin 20^\circ &= (m_A + m_B) a_A - m_B a_{B/A} \cos 20^\circ \\ (10 \text{ lb}) \sin 20^\circ &= \left( \frac{10}{g} \right) a_A - \left( \frac{6}{g} \right) (0.25 \alpha) \cos 20^\circ \end{aligned}$$

$$\begin{aligned} a_A &= g \sin 20^\circ + \frac{6}{10} (0.25) \cos 20^\circ \alpha \\ a_A &= g \sin 20^\circ + 0.15 \cos 20^\circ \alpha \quad (1) \end{aligned}$$

CYLINDER:  $\sum M_E = \sum (M_E)_{ef}$

$$0 = I \alpha + (m_B a_{B/A}) (0.25 \text{ ft}) - (m_B a_A \cos 20^\circ) (0.25 \text{ ft})$$

$$0 = \frac{1}{2} \frac{6 \text{ lb}}{g} (0.25 \text{ ft})^2 \alpha + \frac{6 \text{ lb}}{g} (0.25 \alpha) (0.25) - \frac{6 \text{ lb}}{g} a_A \cos 20^\circ (0.25)$$

$$0 = \frac{1}{g} [0.1875 \alpha + 0.375 \alpha - 1.4095 a_A]$$

$$0 = 0.5625 \alpha - 1.4095 a_A; \quad \alpha = 2.506 a_A \quad (2)$$

SUBSTITUTE FOR  $\alpha$  FROM (2) INTO (1):

$$a_A = g \sin 20^\circ + 0.15 \cos 20^\circ (2.506 a_A)$$

$$a_A = 11.013 + 0.3532 a_A$$

$$(1 - 0.3532) a_A = 11.013$$

$$a_A = 17.027 \text{ ft/s}^2$$

$$a_A = 17.03 \text{ ft/s}^2 \angle 20^\circ$$

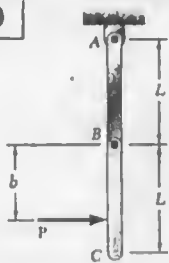
EQ (2)

$$\begin{aligned} \alpha &= 2.506 a_A \\ &= 2.506 (17.027) \end{aligned}$$

$$\alpha = 42.7 \text{ rad/s}^2$$

$$\alpha = 42.7 \text{ rad/s}^2$$

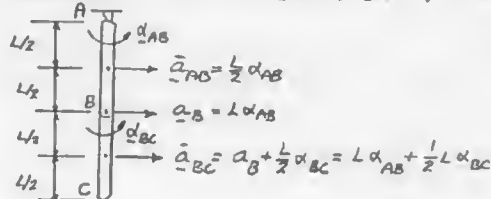
\* 16.149



GIVEN:  $P = 20 \text{ N}$   
 $m_{AB} = m_{BC} = m = 3 \text{ kg}$   
 $L = 500 \text{ mm}$   
 $b = L = 500 \text{ mm}$

FIND:  $\alpha_{AB}$  AND  $\alpha_{BC}$

KINEMATICS: ASSUME  $\alpha_{AB}$ ,  $\alpha_{BC}$ , AND  $\omega_{AB} = \omega_{BC} = 0$



KINETICS: BAR BC

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$PL = \bar{I}_{BC} \alpha_{BC} + (m \bar{a}_{BC}) \frac{L}{2}$$

$$= \frac{m}{12} L^2 \alpha_{BC} + m(L \alpha_{AB} + \frac{L}{2} \alpha_{BC}) \frac{L}{2}$$

$$P = \frac{1}{2} mL \alpha_{AB} + \frac{1}{3} mL \alpha_{BC} \quad (1)$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: P - B_x = m \bar{a}_{BC}$$

$$P - B_x = m(L \alpha_{AB} + \frac{1}{2} L \alpha_{BC}) \quad (2)$$

BAR AB:

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$B_x L = \bar{I}_{AB} \alpha_{AB} + (m \bar{a}_{AB}) \frac{L}{2}$$

$$= \frac{m}{12} L^2 \alpha_{AB} + m(\frac{L}{2} \alpha_{AB}) \frac{L}{2}$$

$$B_x = \frac{1}{3} mL \alpha_{AB} \quad (3)$$

$$\text{ADD (2) AND (3): } P = \frac{4}{3} mL \alpha_{AB} + \frac{1}{2} mL \alpha_{BC} \quad (4)$$

SUBTRACT (1) FROM (4)

$$0 = \frac{5}{6} mL \alpha_{AB} + \frac{1}{6} mL \alpha_{BC}$$

$$\alpha_{BC} = -5 \alpha_{AB} \quad (5)$$

SUBSTITUTE FOR  $\alpha_{BC}$  IN (1):

$$P = \frac{1}{2} mL \alpha_{AB} + \frac{1}{3} mL (-5 \alpha_{AB}) = -\frac{7}{6} mL \alpha_{AB}$$

$$\alpha_{AB} = -\frac{6}{7} \frac{P}{mL} \quad (6)$$

$$\text{EQ (5): } \alpha_{BC} = -5 \left( -\frac{6}{7} \frac{P}{mL} \right) \quad \alpha_{BC} = \frac{30}{7} \frac{P}{mL} \quad (7)$$

DATA:  $L = 0.5 \text{ m}$ ,  $m = 3 \text{ kg}$ ,  $P = 20 \text{ N}$

$$\text{EQ (6): } \alpha_{AB} = -\frac{6}{7} \frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})} = -11.43 \text{ rad/s}^2$$

$$\alpha_{AB} = 11.43 \text{ rad/s}^2$$

$$\text{EQ (7): } \alpha_{BC} = \frac{30}{7} \frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})} = 57.14 \text{ rad/s}^2$$

$$\alpha_{BC} = 57.1 \text{ rad/s}^2$$

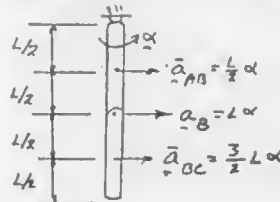
\* 16.150



GIVEN:  $P = 20 \text{ N}$   
 $m_{AB} = m_{BC} = m = 3 \text{ kg}$   
 $L = 500 \text{ mm}$

FIND: (a) DISTANCE  $b$  FOR WHICH BARS MOVE AS A SINGLE RIGID BODY  
 (b)  $\alpha$  OF BARS

KINEMATICS: WE CHOOSE  $\alpha = \alpha_{AB} = \alpha_{BC}$



KINETICS: BARS AB AND BC (ACTING AS RIGID BODY)

$$m_{ABC} = 2m$$

$$\bar{I} = \frac{1}{12} (2m)(2L)^2$$

$$I = \frac{2}{3} mL^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: P(L+b) = \bar{I}_{ABC} \alpha + m_{ABC} a_B L$$

$$P(L+b) = \frac{2}{3} mL^2 \alpha + (2m)(L\alpha)L$$

$$P(L+b) = \frac{8}{3} mL^2 \alpha \quad (1)$$

BAR BC

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$Pb = \bar{I}_{BC} \alpha + (m \bar{a}_{BC}) \frac{L}{2}$$

$$= \frac{m}{12} L^2 \alpha + m(\frac{3}{2} L\alpha) \frac{L}{2}$$

$$Pb = \frac{5}{6} mL^2 \alpha$$

$$\alpha = \frac{6}{5} \frac{Pb}{mL^2} \quad (2)$$

SUBSTITUTE FOR  $\alpha$  INTO (1)

$$P(L+b) = \frac{8}{3} mL^2 \left( \frac{6}{5} \frac{Pb}{mL^2} \right)$$

$$PL + Pb = \frac{16}{5} Pb \quad ; \quad L = \left( \frac{16}{5} - 1 \right) b = \frac{11}{5} b$$

$$b = \frac{5}{11} L$$

$$\text{EQ (2): } \alpha = \frac{6}{5} \frac{P}{mL} \left( \frac{5}{11} L \right) \quad \alpha = \frac{6}{11} \frac{P}{mL}$$

DATA:  $L = 0.5 \text{ m}$ ,  $m = 3 \text{ kg}$ ,  $P = 20 \text{ N}$

$$(a) \quad b = \frac{5}{11} L = \frac{5}{11} (500 \text{ mm}) \quad b = 227 \text{ mm}$$

$$(b) \quad \alpha = \frac{6}{11} \frac{P}{mL} = \frac{6}{11} \frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})} = 7.273 \text{ rad/s}^2$$

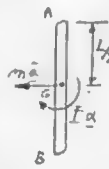
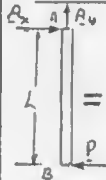
$$\alpha = 7.27 \text{ rad/s}^2$$

\*16.151



GIVEN:  $L = 36 \text{ in.}$   
 $W = 4 \text{ lb.}$   
 $P = 1.5 \text{ lb}$   
 $b = L = 36 \text{ in.}$

FIND:  $M_{\max}$  AND  
 SHOW THAT  $M_{\max}$  IS  
 INDEPENDENT OF  $W$



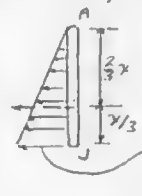
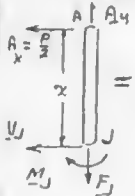
ROD AB:  $\ddot{a} = \frac{L}{2} \alpha$   
 $+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}$   
 $PL = (m\ddot{a})\frac{L}{2} + I\alpha$   
 $= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12}mL^2\alpha$   
 $\alpha = \frac{3P}{mL} \quad (1)$

$+\circlearrowleft \Sigma F_x = \Sigma (F_x)_{\text{eff}}: A_x - P = -m\ddot{a}$   
 $A_x = P - m\frac{L}{2}\alpha = P - m\frac{L}{2}(\frac{3P}{mL}) = -\frac{P}{2}; \quad A_x = \frac{1}{2}P \leftarrow$

PORTION AJ OF ROD:

EXTERNAL FORCES:  $A_x$ ,  $W_J$ , AXIAL FORCE  $F_J$ ,  
 SHEAR  $V_J$ , AND BENDING MOMENT  $M_J$

EFFECTIVE FORCES: SINCE ACCELERATION AT ANY  
 POINT IS PROPORTIONAL TO DISTANCE FROM A, EFFECTIVE  
 FORCES ARE LINEARLY DISTRIBUTED. SINCE MASS PER  
 UNIT LENGTH IS  $m/L$ , AT POINT J WE FIND



$(\frac{m}{L})\alpha_J = \frac{m}{L}(x\alpha)$   
 USING (1):  $\frac{m}{L}\alpha_J = \frac{m}{L}(\frac{3P}{mL})$   
 $\frac{m}{L}\alpha_J = \frac{3Px}{L^2}$

$+\circlearrowleft \Sigma M_J = \Sigma (M_J)_{\text{eff}}: M_J - A_x x = -\frac{1}{2}(\frac{3Px}{L^2})x(\frac{2x}{3})$   
 $M_J = \frac{1}{2}Px - \frac{1}{2}\frac{P}{L^2}x^3 \quad (2)$

For  $M_{\max}$ :  $\frac{dM_J}{dx} = \frac{P}{2} - \frac{3}{2}\frac{P}{L^2}x^2 = 0$   
 $x = \frac{L}{\sqrt{3}} \quad (3)$

SUBSTITUTING INTO (2)

$(M_J)_{\max} = \frac{1}{2}\frac{PL}{\sqrt{3}} - \frac{1}{2}\frac{P}{L^2}(\frac{L}{\sqrt{3}})^3 = \frac{1}{2}\frac{PL}{\sqrt{3}}(\frac{2}{3})$

$(M_J)_{\max} = \frac{PL}{3\sqrt{3}} \quad (4)$

NOTE: Eqs. (3) AND (4) ARE INDEPENDENT OF  $W$

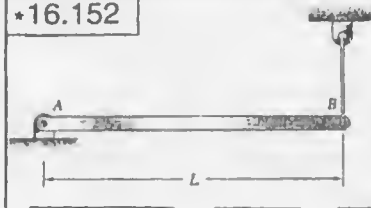
DATA:  $L = 36 \text{ in.}$ ,  $P = 1.5 \text{ lb}$

EG (3):  $x = \frac{L}{\sqrt{3}} = \frac{36 \text{ in.}}{\sqrt{3}} = 20.78 \text{ in.}$

EG (4):  $(M_J)_{\max} = \frac{(1.5 \text{ lb})(36 \text{ in.})}{3\sqrt{3}} = 10.392 \text{ lb}\cdot\text{in.}$

$M_{\max} = 10.37 \text{ lb}\cdot\text{in.}$  20.8 in. BELOW A

\*16.152



GIVEN:  
 $m = \text{MASS OF AB}$   
 CORD BREAKS  
 DRAW:  
 $V$  AND  $M$  DIAGRAMS

FROM ANSWERS TO PROBLEM 16.84:

$a_B = \frac{3}{2}g \quad A = \frac{1}{4}mg \uparrow$

WE NOW FIND

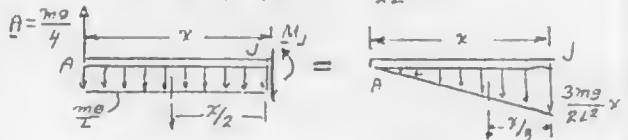
$\alpha = \frac{a_B}{L} = \frac{3g}{2L}$   
 $a_J = x\alpha = \frac{3g}{2L}x \downarrow$

PORTION AJ OF ROD:

EXTERNAL FORCES: REACTION  $A$ , DISTRIBUTED LOAD  
 PER UNIT LENGTH  $mg/L$ , SHEAR  $V_J$ , BENDING MOMENT  $M_J$ .

EFFECTIVE FORCES: SINCE  $a \sim x$ , THE EFFECTIVE  
 FORCES ARE LINEARLY DISTRIBUTED. THE EFFECTIVE  
 FORCE PER UNIT LENGTH AT J IS:

$\frac{m}{L}a_J = \frac{m}{L} \cdot \frac{3g}{2L}x = \frac{3mg}{2L^2}x$



$+\circlearrowleft \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \frac{mg}{L}x - \frac{mg}{4} + V_J = \frac{1}{2}(\frac{3mg}{2L^2}x)x$   
 $V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L^2}x^2$

$+\circlearrowleft \Sigma M_J = \Sigma (M_J)_{\text{eff}}: (\frac{mg}{L}x)\frac{x}{2} - \frac{mg}{4}x + M_J = \frac{1}{2}(\frac{3mg}{2L^2}x)x(\frac{x}{3})$   
 $M_J = \frac{mg}{4}x - \frac{1}{2}\frac{mg}{L}x^2 + \frac{1}{4}\frac{mg}{L^2}x^3$

FIND  $V_{\min}$ :  $\frac{dV_J}{dx} = -\frac{mg}{L} + \frac{3}{2}\frac{mg}{L^2}x = 0; \quad x = \frac{2}{3}L$

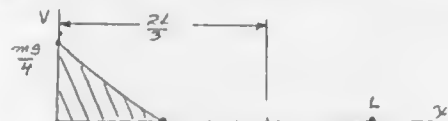
$V_{\min} = \frac{mg}{4} - \frac{mg}{L}(\frac{2}{3}L) + \frac{3}{4}\frac{mg}{L^2}(\frac{2}{3}L)^2; \quad V_{\min} = -\frac{mg}{12}$

FIND  $M_{\max}$  WHERE  $V_J = 0$ :  $V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L^2}x^2 = 0$

$3x^2 - 4Lx + L^2 = 0$   
 $(3x - L)(x - L) = 0 \quad x = \frac{L}{3} \text{ AND } x = L$

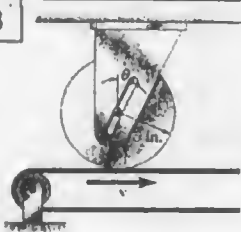
$M_{\max} = \frac{mg}{4}(\frac{L}{3}) - \frac{1}{2}\frac{mg}{L}(\frac{L}{3})^2 + \frac{1}{4}\frac{mg}{L^2}(\frac{L}{3})^3 = \frac{mgL}{27}$

$M_{\min} = \frac{mg}{4}L - \frac{1}{2}\frac{mg}{L}L^2 + \frac{1}{4}\frac{mg}{L^2}L^3 = 0$



$M_{\max} = \frac{mgL}{27}$  AT  $\frac{L}{3}$  FROM A

16.153



GIVEN:

$$\theta = 30^\circ$$

$$\mu_k = 0.20$$

FIND:  $\alpha$  WHILE  
SLIPPING OCCURS

$$r = \frac{5}{12} \text{ ft}$$

$$\begin{aligned} \sum F_x &= \sum (F_x)_{\text{eff}} \\ \mu_k N - R \cos \theta &= 0 \\ R \cos \theta &= \mu_k N \quad (1) \\ + \sum F_y &= \sum (F_y)_{\text{eff}} \\ R \sin \theta + N - mg &= 0 \\ R \sin \theta &= mg - N \quad (2) \end{aligned}$$

$$\text{DIVIDE (2) BY (1): } \tan \theta = \frac{mg - N}{\mu_k N}; 0.5774 = \frac{mg - N}{0.2N}$$

$$0.1155N = mg - N; N = \frac{mg}{1.1155} = 0.8965 mg$$

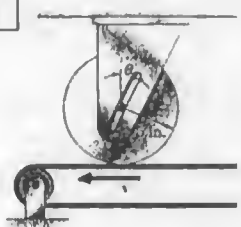
$$+ \sum M_O = \sum (M_O)_{\text{eff}}: -\mu_k N r = \bar{I} \alpha$$

$$(0.2)(0.8965 mg) r = \frac{1}{2} m r^2 \alpha$$

$$\alpha = 0.35858 \frac{g}{r} = 0.35858 \frac{32.2 \text{ ft/s}^2}{(5/12) \text{ ft}} = 27.71 \text{ rad/s}^2$$

$$\alpha = 27.7 \text{ rad/s}^2$$

16.154



GIVEN:

$$\theta = 30^\circ$$

$$\mu_k = 0.20$$

FIND:  $\alpha$  WHILE  
SLIPPING OCCURS

$$r = \frac{5}{12} \text{ ft}$$

$$\begin{aligned} \sum F_x &= \sum (F_x)_{\text{eff}}: \\ R \cos \theta - \mu_k N &= 0 \\ R \cos \theta &= \mu_k N \quad (1) \\ + \sum F_y &= \sum (F_y)_{\text{eff}}: \\ R \sin \theta + mg - N &= 0 \\ R \sin \theta &= N - mg \quad (2) \end{aligned}$$

$$\text{DIVIDE (2) BY (1): } \tan \theta = \frac{N - mg}{\mu_k N}; 0.5774 = \frac{N - mg}{0.2N}$$

$$0.1155N = N - mg; N = \frac{mg}{0.8845} = 1.1306 mg$$

$$+ \sum M_O = \sum (M_O)_{\text{eff}}: \mu_k N r = \bar{I} \alpha$$

$$(0.2)(1.1306 mg) r = \frac{1}{2} m r^2 \alpha$$

$$\alpha = 0.4522 \frac{g}{r} = 0.4522 \frac{32.2 \text{ ft/s}^2}{(5/12) \text{ ft}}$$

$$\alpha = 34.948 \text{ rad/s}^2$$

$$\alpha = 34.9 \text{ rad/s}^2$$

16.155



GIVEN: CYLINDERS

FIND: (a) MAXIMUM  $\alpha$   
FOR ROLLING WITH  
NO SLIDING  
(b) MINIMUM  $\alpha$   
FOR CYLINDER TO  
MOVE  $\rightarrow$  WITH NO ROTATING(a) CYLINDER ROLLS WITHOUT SLIDING:  $\alpha = r\alpha$  OR  $\alpha = \frac{a}{r}$ 

$$\begin{aligned} B_y &= 4P \\ B_x &= P \end{aligned}$$

$$\begin{aligned} \sum F_x &= \sum (F_x)_{\text{eff}}: P - \mu_k N = m\bar{a} \\ \sum F_y &= \sum (F_y)_{\text{eff}}: N - 4P - mg = 0 \end{aligned}$$

$$+ \sum M_O = \sum (M_O)_{\text{eff}}: Pr - (\mu_k N)r = \bar{I} \alpha + (m\bar{a})r$$

$$P(1 - \mu_k)r = \frac{1}{2} m r^2 \left(\frac{\bar{a}}{r}\right) + (m\bar{a})r$$

$$P = \frac{3}{2} \frac{m\bar{a}}{(1 - \mu_k)} \quad (1)$$

$$+ \sum F_y = 0: N - \mu_k P - mg = 0 \quad (2)$$

$$+ \sum F_x = \sum (F_x)_{\text{eff}}: P - \mu_k N = m\bar{a} \quad (3)$$

SOLVE (2) FOR N AND SUBSTITUTE FOR N INTO (3).

$$P - \mu_k^2 P - \mu_k mg = m\bar{a}$$

$$\text{SUBSTITUTE P FROM (1): } (1 - \mu_k^2) \frac{3}{2} \frac{m\bar{a}}{(1 - \mu_k)} - \mu_k mg = m\bar{a}$$

$$3(1 + \mu_k)\bar{a} - 2\mu_k g = 2\bar{a}$$

$$\bar{a}(1 + 3\mu_k) - 2\mu_k g = 0$$

$$\bar{a} = \frac{2\mu_k}{1 + 3\mu_k} g$$

(b) CYLINDER TRANSLATES:  $\alpha = 0$ SLIDING OCCURS AT A:  $A_x = \mu_k N$ ASSUME SLIDING IMPEDS AT B:  $B_y = \mu_k P$ 

$$\begin{aligned} B_y &= \mu_k P \\ B_x &= P \end{aligned}$$

$$\begin{aligned} \sum F_x &= \sum (F_x)_{\text{eff}}: P - \mu_k N = m\bar{a} \\ \sum F_y &= \sum (F_y)_{\text{eff}}: N - \mu_k P - mg = 0 \end{aligned}$$

$$+ \sum M_O = \sum (M_O)_{\text{eff}}: Pr - \mu_k Pr = (m\bar{a})r$$

$$P(1 - \mu_k)r = m\bar{a}r$$

$$P = \frac{m\bar{a}}{1 - \mu_k} \quad (4)$$

$$+ \sum F_x = \sum (F_x)_{\text{eff}}: P - \mu_k N = m\bar{a} \quad (5)$$

$$+ \sum F_y = \sum (F_y)_{\text{eff}}: N - \mu_k P - mg = 0 \quad (6)$$

SOLVE (5) FOR N AND SUBSTITUTE FOR N INTO (6).

$$P - \mu_k^2 P - \mu_k mg = m\bar{a}$$

SUBSTITUTE FOR P FROM (4):

$$\frac{m\bar{a}}{1 - \mu_k} (1 - \mu_k^2) - \mu_k mg = m\bar{a}$$

$$\bar{a}(1 + \mu_k) - \mu_k g = \bar{a}$$

$$\bar{a} - \mu_k g = 0$$

$$\bar{a} = g$$

SUMMARY:  $\alpha < \frac{2\mu_k}{1 + 3\mu_k} g$ ; ROLLING

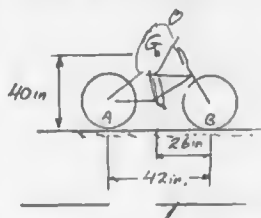
$$\frac{2\mu_k}{1 + 3\mu_k} g < \alpha < g$$
 : ROTATING AND SLIDING

$$\alpha > g$$
 : TRANSLATION



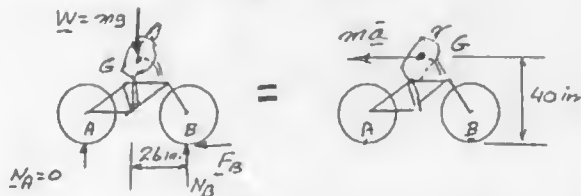
16.156

$$v_0 = 20 \text{ mph}$$



FIND: SHORTEST  
STOPPING DISTANCE IF  
CYCLIST IS NOT TO BE  
THROWN OVER FRONT  
WHEEL

WHEN CYCLIST IS ABOUT TO BE THROWN OVER  
THE FRONT WHEEL,  $N_A = 0$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: mg(26 \text{ in.}) = m\bar{a}(40 \text{ in.})$$

$$\bar{a} = \frac{26}{40}g = \frac{26}{40}(32.2 \text{ ft/s}^2) = 20.93 \text{ ft/s}^2$$

UNIFORMLY ACCELERATED MOTION:

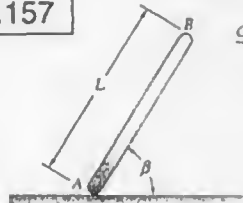
$$v_0 = 20 \text{ mph} = 29.333 \text{ ft/s}$$

$$v^2 - v_0^2 = 2a\Delta x: 0 - (29.333 \text{ ft/s})^2 = 2(-20.93 \text{ ft/s}^2)\Delta x$$

$$S = 20.555 \text{ ft}$$

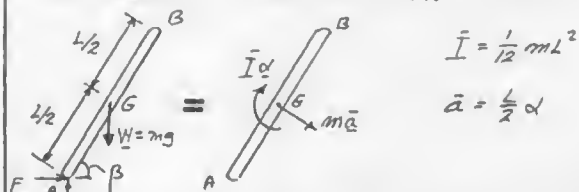
$$S = 20.6 \text{ ft}$$

16.157



GIVEN:  $\beta = 70^\circ$ . UNIFORM ROD  
RELEASED FROM REST.  
FRICTION IS SUFFICIENT  
TO PREVENT SLIDING AT A.  
FIND: (a)  $\alpha$ , (b)  $N_A$ , (c)  $F_A$

WE NOTE ROD ROTATES ABOUT A.  $\omega = 0$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + (m\bar{a})\frac{L}{2}$$

$$\frac{1}{2}mgL\cos\beta = \frac{1}{12}mL^2\alpha + (m\frac{L}{2}\alpha)\frac{L}{2}$$

$$= \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\beta}{L} \quad (1)$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: F_A = m\bar{a}\sin\beta$$

$$F_A = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\sin\beta$$

$$F_A = \frac{3}{4}mg\sin\beta\cos\beta \quad (2)$$

(CONTINUED)

16.157 continued

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: N_A - mg = -m\bar{a}\cos\beta = -m\left(\frac{L}{2}\alpha\right)\cos\beta$$

$$N_A - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\cos\beta$$

$$N_A = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \quad (3)$$

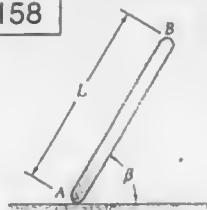
FOR  $\beta = 70^\circ$ :

$$(a) \text{ Eq. (1): } \alpha = \frac{3}{2}\frac{g\cos 70^\circ}{L}; \quad \alpha = 0.513 \frac{g}{L}$$

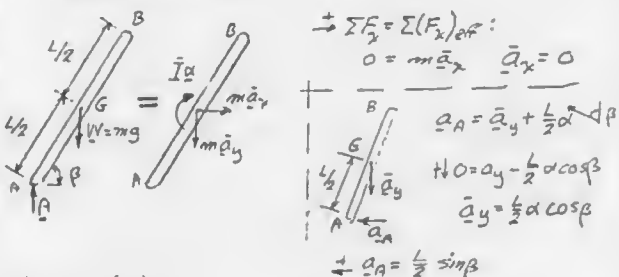
$$(b) \text{ Eq. (3): } N_A = mg\left(1 - \frac{3}{4}\cos^2 70^\circ\right); \quad N_A = 0.912 mg \uparrow$$

$$(c) \text{ Eq. (2): } F_A = \frac{3}{4}mg\sin 70^\circ\cos 70^\circ; \quad F_A = 0.241 mg \rightarrow$$

16.158



GIVEN:  $\beta = 70^\circ$ . UNIFORM  
ROD RELEASED FROM REST.  
FRICTION AT SURFACE  
EQUALS ZERO.  
FIND: (a)  $\alpha$ , (b)  $\bar{a}$ ,  
(c) REACTION AT A.



$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$$

$$0 = m\bar{a}_x \quad \bar{a}_x = 0$$

$$\bar{a}_A = \bar{a}_y + \frac{L}{2}\alpha \sin\beta$$

$$0 = \bar{a}_y - \frac{L}{2}\alpha \cos\beta$$

$$\bar{a}_y = \frac{L}{2}\alpha \cos\beta$$

$$\bar{a}_A = \frac{L}{2}\sin\beta$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$mg - A = m\bar{a}_y = m\left(\frac{L}{2}\alpha \cos\beta\right) \quad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}:$$

$$A\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$A = \frac{mL}{6}\frac{\alpha}{\cos\beta} \quad (2)$$

SUBSTITUTE (2) INTO (1):

$$mg - \frac{mL}{6}\frac{\alpha}{\cos\beta} = m\frac{L}{2}\alpha \cos\beta$$

$$g = \left(\frac{L}{2}\cos\beta + \frac{L}{6\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(3\cos\beta + \frac{1}{\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(\frac{3\cos^2\beta + 1}{\cos\beta}\right)\alpha$$

$$\alpha = \frac{6g}{L}\left(\frac{\cos\beta}{1 + 3\cos^2\beta}\right)$$

$$\bar{a}_A = \frac{L}{2}\alpha \cos\beta = \frac{L}{2}\left(\frac{6g}{L}\frac{\cos\beta}{1 + 3\cos^2\beta}\right)\cos\beta = 3g\left(\frac{\cos^2\beta}{1 + 3\cos^2\beta}\right)$$

$$A = \frac{mL}{6}\frac{\alpha}{\cos\beta} = \frac{mL}{6}\left(\frac{6g}{L}\frac{\cos\beta}{1 + 3\cos^2\beta}\right)\frac{1}{\cos\beta} = mg\frac{1}{1 + 3\cos^2\beta} \uparrow$$

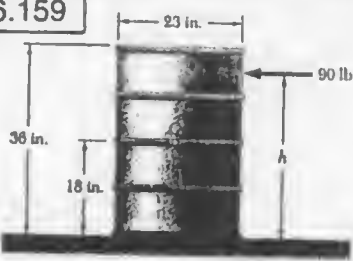
FOR  $\beta = 70^\circ$ :

$$(a) \alpha = \frac{6g}{L}\frac{\cos 70^\circ}{1 + 3\cos^2 70^\circ}; \quad \alpha = 1.519 \frac{g}{L}$$

$$(b) \bar{a}_A = 3g\frac{\cos^2 70^\circ}{1 + 3\cos^2 70^\circ}; \quad \bar{a}_A = 0.260 g \downarrow$$

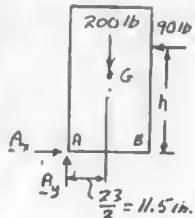
$$(c) A = mg\frac{1}{1 + 3\cos^2 70^\circ}; \quad A = 0.740 mg \uparrow$$

16.159



GIVEN:  $W = 200\text{ lb}$   
 $\mu_s = 0.40, \mu_k = 0.35$

FIND: (a)  $\bar{a}$   
 (b) RANGE OF  $h$   
 FOR WHICH BARREL  
 WILL NOT TIP.



WEIGHT = 200 lb

FOR TIPPING.  
 ABOUT A IMPENDING  
 REACTION IS AT A

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A_y - 200\text{ lb} = 0; A_y = 200\text{ lb} \uparrow$$

FOR SLIDING:  $A_x = \mu_k A_y = 0.35(200) = 70\text{ lb} \rightarrow$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: 90\text{ lb} - A_x = m\bar{a}$$

$$90\text{ lb} - 70\text{ lb} = \frac{200\text{ lb}}{g} \bar{a}$$

$$\bar{a} = \frac{20\text{ lb}}{200\text{ lb}} g = 0.1g = 0.1(32.2) \quad \bar{a} = 3.22\text{ ft/s}^2 \leftarrow$$

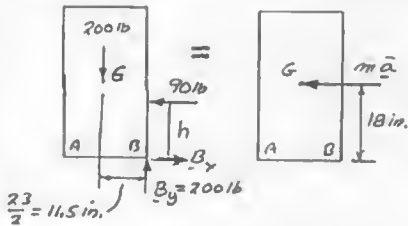
$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: (90\text{ lb})h - (200\text{ lb})\left(\frac{11.5}{12}\text{ ft}\right) = m\bar{a}\left(\frac{18}{12}\text{ ft}\right)$$

$$90h - 191.67 = \frac{200\text{ lb}}{32.2} (3.22\text{ ft/s}^2)(1.5\text{ ft})$$

$$90h - 191.67 = 30; 90h = 221.67$$

$$h = 2.463\text{ ft} \quad h = 29.6\text{ in.}$$

FOR TIPPING IMPENDING ABOUT B, REACTION IS AT B



$$B_x = \mu_k B_y = 0.35(200) = 70\text{ lb}$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \text{SAME AS ABOVE: } \bar{a} = 3.22\text{ ft/s}^2 \leftarrow$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: (90\text{ lb})h + (200\text{ lb})\left(\frac{11.5}{12}\text{ ft}\right) = m\bar{a}\left(\frac{18}{12}\text{ ft}\right)$$

$$90h + 191.67 = \frac{200\text{ lb}}{32.2} (3.22\text{ ft/s}^2)(1.5\text{ ft})$$

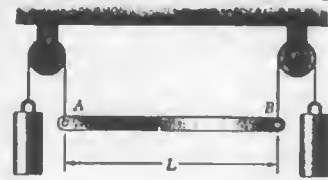
$$90h + 191.67 = 30$$

$$90h = -161.67$$

$$h < 0 \quad \text{IMPOSSIBLE}$$

THUS RANGE FOR NO TIPPING IS  
 $h < 29.6\text{ in}$

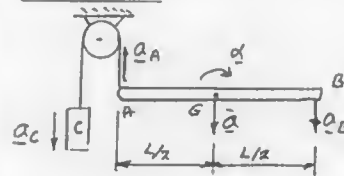
16.160



GIVEN: WEIGHTS  
 BAR AB:  $W$   
 COUNTERWEIGHTS  
 $= \frac{1}{2}W$ .  
 IMMEDIATELY  
 AFTER WIRE  
 AT B IS CUT.

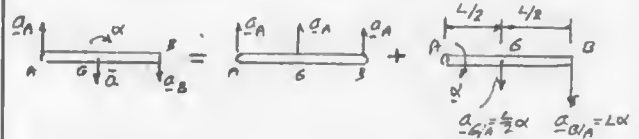
FIND: (a)  $a_A$ , (b)  $a_B$ .

KINEMATICS:



$$W = 0$$

$$a_C = a_A$$



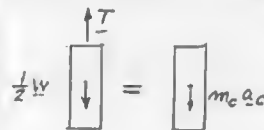
$$[\bar{a}] = [a_A] + \left[\frac{L}{2}\alpha\right]$$

$$[a_B] = [a_A] + [L\alpha]$$

$$\bar{a} = \left(\frac{L}{2}\alpha - a_A\right)$$

$$a_B = (L\alpha - a_A)$$

KINETICS: COUNTERWEIGHT  $m = \text{MASS OF BAR AB}$



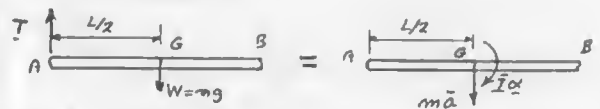
$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$\frac{1}{2}W - T = m_c a_C = \frac{1}{2}m a_A$$

$$\frac{1}{2}mg - T = \frac{1}{2}m a_A$$

$$T = \frac{1}{2}m(g - a_A) \quad (1)$$

KINETICS BAR AB



$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: mg - T = m\bar{a}$$

$$mg - \frac{1}{2}m(g - a_A) = m\left(\frac{L}{2}\alpha - a_A\right)$$

$$2g - g + a_A = L\alpha - 2a_A \quad g + 3a_A = L\alpha \quad (2)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: T\frac{L}{2} = I\alpha$$

$$\frac{1}{2}m(g - a_A)\frac{L}{2} = \frac{1}{12}mL^2\alpha$$

$$3g - 3a_A = L\alpha \quad (3)$$

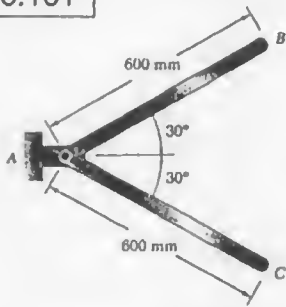
$$\text{ADD EGS. (2) AND (3): } 4g = 2L\alpha \quad \alpha = \frac{2g}{L}$$

$$\text{SUBSTITUTE INTO (2): } g + 3a_A = L\left(\frac{2g}{L}\right); a_A = \frac{1}{3}g \uparrow$$

$$\bar{a} = \left(\frac{L}{2}\alpha - a_A\right) = \frac{L}{2}\left(\frac{2g}{L}\right) - \frac{1}{3}g; \quad \bar{a} = \frac{2}{3}g \downarrow$$

$$a_B = (L\alpha - a_A) = L\left(\frac{2g}{L}\right) - \frac{1}{3}g; \quad a_B = \frac{5}{3}g \downarrow$$

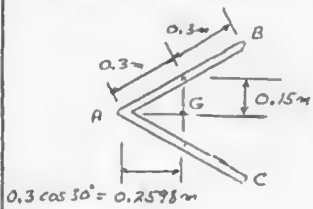
16.161



GIVEN:

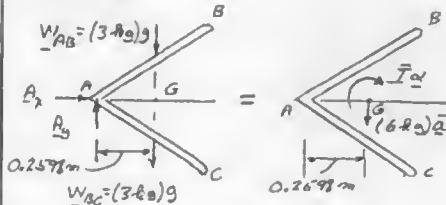
$$m_{AB} = m_{AC} = m = 3 \text{ kg}$$

SYSTEM RELEASED FROM REST

FIND: (a)  $a_B$   
(b)  $A$ CENTER OF MASS AND  $\bar{I}$ 

$$\begin{aligned}\bar{I} &= 2 \left[ \bar{I}_{AB} + m_{AB} (0.15 \text{ m})^2 \right] \\ \bar{I} &= 2 \left[ \frac{1}{12} (3 \text{ kg}) (0.6 \text{ m})^2 + (3 \text{ kg}) (0.15 \text{ m})^2 \right] \\ \bar{I} &= 2 \left[ 0.09 \text{ kg} \cdot \text{m}^2 + 0.0675 \text{ kg} \cdot \text{m}^2 \right] \\ \bar{I} &= 0.315 \text{ kg} \cdot \text{m}^2\end{aligned}$$

KINETICS



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$\begin{aligned}2(3 \text{ kg})(9.81 \text{ m/s}^2)(0.2598 \text{ m}) &= \bar{I} \alpha + (1 \text{ m} \bar{a})(0.2598 \text{ m}) \\ 15.292 &= (0.315 \text{ kg} \cdot \text{m}^2) \alpha + (6 \text{ kg})(0.2598 \text{ m}) \alpha \\ 15.292 &= 0.720 \alpha\end{aligned}$$

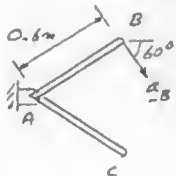
$$\alpha = 21.24 \text{ rad/s}^2$$

$$\bar{a} = 0.2598(21.24) = 5.518 \text{ m/s}^2$$

$$+\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\begin{aligned}+\uparrow \Sigma F_y &= \Sigma (F_y)_{\text{eff}}: A_y - 2(3 \text{ kg})(9.81 \text{ m/s}^2) = -(6 \text{ kg}) \bar{a} \\ A_y - 58.86 &= -(6 \text{ kg})(5.518 \text{ m/s}^2) \\ A_y - 58.86 &= -33.11 \quad A_y = 25.75 \text{ N}\end{aligned}$$

$$\text{SINCE } A_x = 0, \quad A = 25.75 \text{ N} \uparrow$$

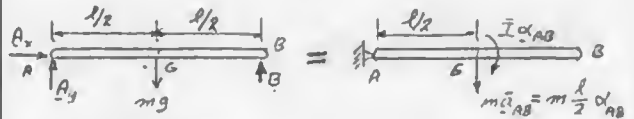


$$\begin{aligned}a_B &= (0.6 \text{ m}) \alpha \\ &= (0.6 \text{ m})(21.24 \text{ rad/s}^2)\end{aligned}$$

$$a_B = 12.74 \text{ m/s}^2 \angle 60^\circ$$

16.162

GIVEN:

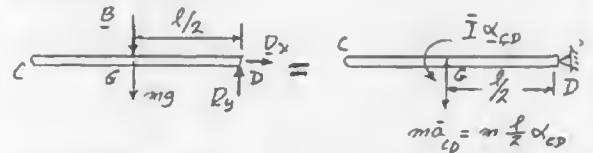
RODS OF MASS  $m$   
RELEASED FROM RESTFIND: (a)  $a_C$ , (b)  $B$ ROD AB: ( $\omega = 0$ )

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$mg\left(\frac{l}{2}\right) - B(l) = \bar{I} \alpha + m \bar{a}_{AB}\left(\frac{l}{2}\right)$$

$$\frac{1}{2} mg l - B l = \frac{1}{12} m l^2 \alpha + m \left(\frac{l}{2} \alpha\right) \frac{l}{2}$$

$$\frac{1}{2} mg l - B l = \frac{1}{3} m l^2 \alpha \quad (1)$$

ROD CD: ( $\omega = 0$ )

$$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}:$$

$$mg\left(\frac{l}{2}\right) + B\left(\frac{l}{2}\right) = \bar{I} \alpha_C + m a_{CD} \frac{l}{2}$$

$$\frac{1}{2} mg l + \frac{1}{2} B l = \frac{1}{12} m l^2 \alpha_C + m \left(\frac{l}{2} \alpha_C\right) \frac{l}{2}$$

MULTIPLY BY 2:

$$mg l + B l = \frac{2}{3} m l^2 \alpha_C \quad (2)$$

$$\text{ADD (1) AND (2): } \frac{3}{2} mg l = m l^2 \left( \frac{1}{3} \alpha_{AB} + \frac{2}{3} \alpha_C \right) \quad (3)$$

$$\text{MULTIPLY BY 3: } \alpha_{AB} + 2 \alpha_C = \frac{9}{2} \frac{g}{l} \quad (4)$$

KINEMATICS:

WE MUST HAVE

$$\begin{aligned}a_C &= l \alpha_C \\ a_{AB} &= \frac{l}{2} \alpha_{AB} \\ a_C &= \frac{1}{2} a_{AB}\end{aligned} \quad (5)$$

SUBSTITUTE FOR  $\alpha_{AB}$  FROM (5) INTO (4)

$$\frac{1}{2} \alpha_C + 2 \alpha_C = \frac{9}{2} \frac{g}{l}$$

$$\frac{5}{2} \alpha_C = \frac{9}{2} \frac{g}{l}; \quad \alpha_C = 1.8 \frac{g}{l} \quad (6)$$

(a) ACCELERATION OF C:

$$a_C = l \alpha_C = l \left( 1.8 \frac{g}{l} \right); \quad a_C = 1.8 g \downarrow$$

(b) FORCE ON KNOB B:

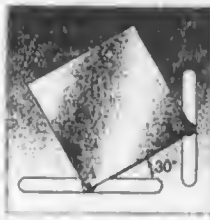
SUBSTITUTE FOR  $\alpha_C$  FROM (6) INTO (2)

$$mg l + B l = \frac{2}{3} m l^2 \left( 1.8 \frac{g}{l} \right)$$

$$B = 1.2 mg - mg$$

$$(\text{ON ROD AB}): B = 0.2 mg \uparrow$$

16.163

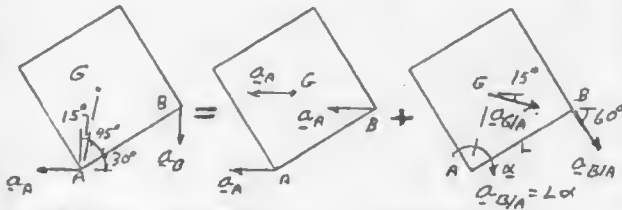


GIVEN: SQUARE PLATE  
OF SIDE  $L = 150 \text{ mm}$   
AND  $m = 2.5 \text{ kg}$  IS  
RELEASED FROM REST

FIND: (a)  $\alpha$   
(b)  $A$

KINEMATICS:

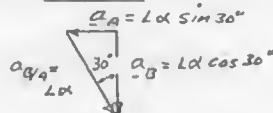
$$AG = \frac{1}{2} \sqrt{2} L = \frac{L}{\sqrt{2}} \quad a_{G/A} = (AG)\alpha = \frac{L\alpha}{\sqrt{2}}$$



PLANE MOTION = TRANSLATION + ROTATION

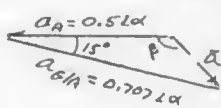
$$a_B = a_A + a_{B/A} \angle 60^\circ$$

$$a_B = a_A + L\alpha \angle 60^\circ$$



$$\bar{a} = a_A + a_{G/A} = 15^\circ$$

$$\bar{a} = L\alpha \sin 30^\circ + \frac{L\alpha}{\sqrt{2}} \angle 15^\circ = 0.5L\alpha + 0.707L\alpha \angle 15^\circ$$



$$\begin{aligned} \bar{a}^2 &= a_A^2 + a_{G/A}^2 - 2a_A a_{G/A} \cos 15^\circ \\ \bar{a}^2 &= (0.5L\alpha)^2 + (0.707L\alpha)^2 \\ &\quad - 2(0.5L\alpha)(0.707L\alpha) \cos 15^\circ \\ \bar{a}^2 &= L^2 \alpha^2 (0.25 + 0.5 - 0.6830) \\ \bar{a} &= L^2 \alpha^2 (0.06699); \quad \bar{a} = 0.25882L\alpha \end{aligned}$$

LAW OF SINES

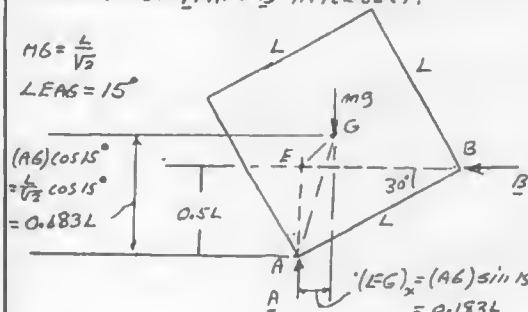
$$\frac{\bar{a}}{\sin 15^\circ} = \frac{a_{G/A}}{\sin \beta}; \quad \sin \beta = \frac{a_{G/A} \sin 15^\circ}{\bar{a}} = \frac{0.707L\alpha}{0.25882L\alpha} \sin 15^\circ$$

$$\sin \beta = 0.707; \quad \beta = 45^\circ$$

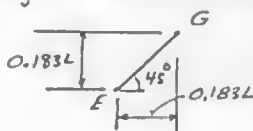
KINETICS ( $\omega = 0$ )

WE FIND THE LOCATION OF POINT E WHERE LINES  
OF ACTION OF  $A$  AND  $B$  INTERSECT.

$$MG = \frac{L}{\sqrt{2}} \\ LEAG = 15^\circ$$



$$(EG)_y = 0.6829L - 0.5L = 0.183L$$

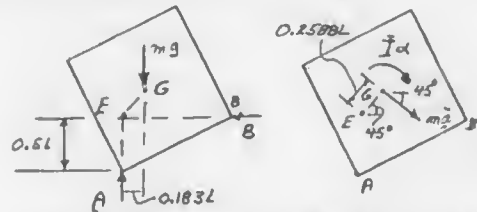


$$EG = (0.183L)\sqrt{2} = 0.2588L$$

(CONTINUED)

16.163 continued

$$\bar{I} = \frac{1}{6} mL^2$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: mg(0.183L) = \bar{I}\alpha + (m\bar{a})(0.2588L)$$

$$0.183mgL = \frac{1}{6} mL^2 \alpha + m(0.2588L)(0.2588L)$$

$$0.183gL = L^2 \alpha \left( \frac{1}{6} + 0.06698 \right)$$

$$0.183 \frac{g}{L} = 0.2336\alpha; \quad \alpha = 0.7834 \frac{g}{L}$$

$$\alpha = 0.7834 \frac{9.81 \text{ m/s}^2}{0.15 \text{ m}} \quad \alpha = 51.2 \text{ rad/s}^2$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A - mg = -m\bar{a} \sin 45^\circ \\ = -m(0.2588L\alpha) \sin 45^\circ \\ = -m(0.2588L)(0.7834 \frac{g}{L}) \sin 45^\circ$$

$$A - mg = -0.1434mg$$

$$A = 0.8566mg = 0.8566(2.5 \text{ kg})(9.81 \text{ m/s}^2) = 21.01 \text{ N}$$

$$A = 21.0 \text{ N} \uparrow$$

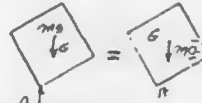
16.164



GIVEN: SQUARE PLATE  
OF SIDE  $L = 150 \text{ mm}$   
AND  $m = 2.5 \text{ kg}$  IS  
RELEASED FROM REST.

FIND: (a)  $\alpha$   
(b)  $A$

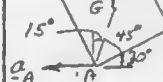
SINCE BOTH  $A$  AND  $mg$  ARE  
VERTICAL,  $\bar{a}_x = 0$  AND  $\bar{a}$  IS  $\downarrow$



KINEMATICS

$$AG = \frac{L}{\sqrt{2}} \angle 15^\circ \quad a_{G/A} = (AG)\alpha \angle 15^\circ$$

$$\bar{a} = a_A + a_{G/A} \angle 15^\circ$$

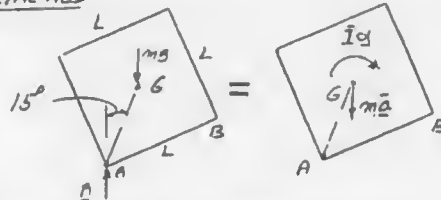


$$\bar{a} = \frac{L\alpha}{\sqrt{2}} \sin 15^\circ$$

$$\bar{a} = 0.183L\alpha \angle 15^\circ$$

KINETICS

$$\bar{I} = \frac{1}{6} mL^2$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: mg(AG) \sin 15^\circ = \bar{I}\alpha + m\bar{a}(AG) \sin 15^\circ$$

$$mg\left(\frac{L}{\sqrt{2}}\right) \sin 15^\circ = \frac{1}{6} mL^2 \alpha + m(0.183L\alpha)\left(\frac{L}{\sqrt{2}}\right) \sin 15^\circ$$

$$0.183 \frac{g}{L} = \left( \frac{1}{6} + 0.033494 \right) \alpha$$

$$0.183 \frac{g}{L} = 0.2002\alpha; \quad \alpha = 0.9143 \frac{g}{L} = 0.9143 \frac{9.81 \text{ m/s}^2}{0.15 \text{ m}}$$

$$\alpha = 59.8 \text{ rad/s}^2$$

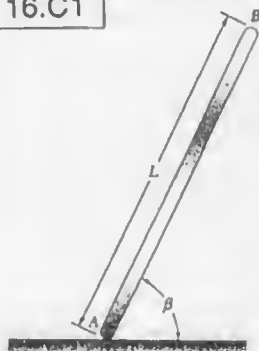
$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: A - mg = -m\bar{a}$$

$$A - mg = -m(0.183L\alpha) = -m(0.183L)(0.9143 \frac{g}{L})$$

$$A - mg = -0.1673mg; \quad A = 0.8326mg$$

$$A = 0.8326(2.5 \text{ kg})(9.81 \text{ m/s}^2); \quad A = 20.4 \text{ N} \uparrow$$

## 16.C1



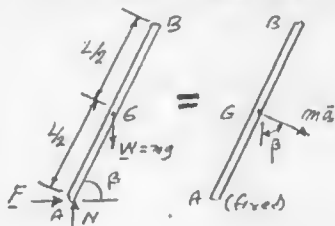
GIVEN:  $W = 516$

ROD AB RELEASED FROM REST.

(a) FOR NO SLIPPING AT A, FIND  $N_A$  AND  $F_A$  IMMEDIATELY AFTER RELEASE FOR  $\beta = 0$  TO  $85^\circ$  USING  $5^\circ$  INCREMENTS.

(b) FOR  $\mu_s = 0.50$ , FIND RANGE OF VALUES OF  $\beta$  FOR WHICH ROD WILL SLIP IMMEDIATELY AFTER RELEASE.

WE NOTE THAT ROD ROTATES ABOUT A AND THAT IMMEDIATELY AFTER RELEASE  $\omega = 0$ .



$$\bar{I} = \frac{1}{12} mL^2$$

$$\bar{\alpha} = \frac{1}{2} \alpha$$

$$+\circlearrowleft \Sigma (M_A) = \Sigma (M_A)_{eff}: mg\left(\frac{L}{2} \cos \beta\right) = \bar{I} \alpha + m \bar{a} \left(\frac{L}{2}\right)$$

$$\frac{1}{2} m g L \cos \beta = \frac{1}{12} mL^2 \alpha + m \left(\frac{L}{2} \alpha\right) \frac{L}{2}$$

$$= \frac{1}{3} mL^2 \alpha$$

$$\alpha = \frac{3}{2} \frac{g}{L} \cos \beta \quad (1)$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: F = m \bar{a} \sin \beta$$

$$F = m \frac{L}{2} \alpha \sin \beta = m \frac{L}{2} \left(\frac{3}{2} \frac{g}{L} \cos \beta\right) \sin \beta$$

$$F = \frac{3}{4} m g \sin \beta \cos \beta \quad (2)$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: N - mg = m \bar{a} \cos \beta$$

$$= m \frac{L}{2} \alpha \cos \beta = m \frac{L}{2} \left(\frac{3}{2} \frac{g}{L} \cos \beta\right) \cos \beta$$

$$N = mg \left(1 - \frac{3}{4} \cos^2 \beta\right) \quad (3)$$

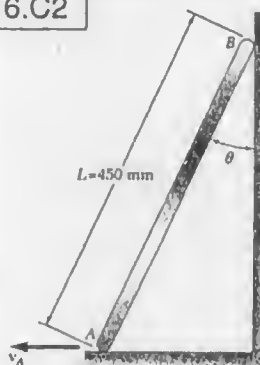
OUTLINE OF PROGRAM:

(a) FOR  $\beta = 0$  TO  $85^\circ$  AT  $5^\circ$  INCREMENTS, DETERMINE  $F$  (FROM EQ.(2)) AND  $N$  (FROM EQ.(3)). ALSO DETERMINE REQUIRED VALUE OF  $\mu = F/N$

(b) USE SMALLER INCREMENTS TO FIND TWO VALUES OF  $\beta$  CORRESPONDING TO  $\mu_s = 0.50$ .

beta	F	N	mu	Result
0.000	0.000	1.250	0.000	no slip
5.000	0.328	1.278	0.255	no slip
10.000	0.641	1.303	0.470	no slip
15.000	0.938	1.501	0.624	slip
20.000	1.205	1.689	0.714	slip
25.000	1.436	1.920	0.748	slip
30.000	1.624	2.186	0.742	slip
35.000	1.762	2.484	0.709	slip
40.000	1.847	2.799	0.660	slip
45.000	1.875	3.125	0.600	slip
50.000	1.847	3.451	0.535	slip
55.000	1.762	3.768	0.468	no slip
60.000	1.624	4.063	0.400	no slip
65.000	1.436	4.330	0.332	no slip
70.000	1.205	4.561	0.264	no slip
75.000	0.938	4.749	0.197	no slip
80.000	0.641	4.887	0.131	no slip
85.000	0.328	4.972	0.065	no slip
----- Seek start of range -----				
10.810	0.891	1.382	0.500	no slip
10.620	0.891	1.382	0.500	slip
----- Seek end of range -----				
52.820	1.809	3.618	0.500	slip
52.830	1.809	3.618	0.500	no slip
52.840	1.809	3.619	0.500	no slip

## 16.C2



GIVEN:  $m = 5.8g$

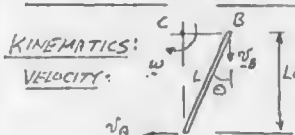
$N_A = 1.5 m/s \leftarrow$

$\dot{\alpha} = 0$

FIND:

NORMAL REACTIONS AT A AND B FOR  $\theta = 0$  TO  $50^\circ$  USING  $5^\circ$  INCREMENTS.

VALUE OF  $\theta$  AT WHICH END B LOSES CONTACT WITH WALL



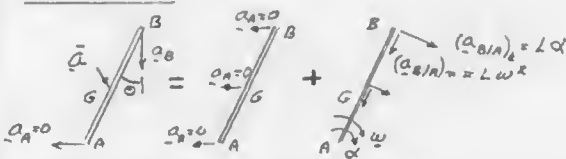
KINEMATICS:

VELOCITY:

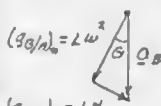
INSTANT CTR. AT C.

$$\omega = \frac{v_A}{L \cos \theta} \quad (1)$$

ACCELERATION



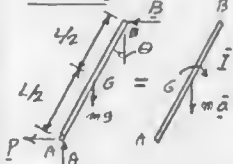
$$[a_B] = a_A + [a_{B/A}]_t + [a_{B/A}]_n$$



$$a_B = \frac{L \omega^2}{\cos \theta}$$

$$\bar{a} = \frac{1}{2} (a_A + a_B) = \frac{1}{2} a_B; \quad \bar{a} = \frac{L \omega^2}{2 \cos \theta} \quad (2)$$

KINETICS:



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: A - mg = m \bar{a}$$

$$A = m(g - \bar{a})$$

$$\bar{I} = \frac{1}{12} mL^2 \quad (3)$$

$$\bar{I} = \frac{1}{12} mL^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}: B(L \cos \theta) - mg\left(\frac{L}{2} \sin \theta\right) = -\bar{I} \alpha - m \bar{a} \left(\frac{L}{2} \sin \theta\right)$$

$$B = \frac{m(g - \bar{a}) \frac{L}{2} \sin \theta + \frac{1}{2} mL^2 \alpha}{L \cos \theta} \quad (4)$$

OUTLINE OF PROGRAM: DATA  $m = 5.8g$ ,  $L = 0.45m$ .

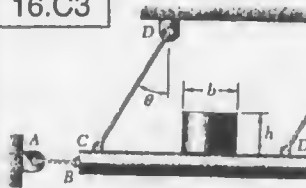
FOR EACH VALUE OF  $\theta$  EVALUATE  $\omega$  AND  $\bar{a}$ .

THEN USE  $\omega$  AND  $\bar{a}$  TO EVALUATE A AND B.

USING SMALLER INCREMENTS FIND VALUE OF  $\theta$  FOR WHICH  $B = 0$ .

theta deg.	omega rad/s	alpha rad/s^2	$\bar{a}$ m/s^2	A N	B N
0.000	3.333	0.000	2.500	36.550	0.000
5.000	3.346	0.980	2.529	36.406	1.408
10.000	3.385	2.020	2.617	35.963	2.786
15.000	3.451	3.191	2.774	35.180	4.094
20.000	3.547	4.580	3.013	33.986	5.271
25.000	3.678	6.308	3.358	32.259	6.218
30.000	3.849	8.553	3.848	29.805	6.752
35.000	4.089	11.595	4.548	26.309	6.557
40.000	4.351	15.888	5.561	21.243	5.024
45.000	4.714	22.222	7.071	13.695	0.955
50.000	5.186	32.048	9.413	1.884	-8.168
----- Find theta for B = 0 -----					
45.747	4.777	23.420	7.357	12.265	0.002
45.748	4.777	23.422	7.367	12.264	0.001
45.749	4.777	23.423	7.358	12.262	-0.001

## 16.C3



GIVEN:  $b = 8 \text{ in}$ ,  $h = 6 \text{ in}$ .  
3D-16 CYLINDER  
10-16 PLATFORM  
AFTER AB IS CUT,  
FIND  $\mu_s$  FOR WHICH  
CYLINDER DOES NOT SLIP  
FOR  $\theta = 0$  TO  $30^\circ$  USING

$5^\circ$  INCREMENTS. THEN FOR  $\mu_s = 0.60$ , FIND  $\theta$  FOR  
WHICH SLIPPING IMPENDS. IN ALL CASES, CHECK  
WHETHER CYLINDER TIPS.

$$I_C = \frac{1}{12} m_c b^2$$

$$W_C = m_c g$$

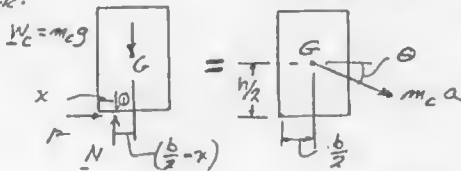
$$I_D = \frac{1}{12} m_c b^2$$

$$W_D = m_p g$$

$$+ \uparrow \Sigma F = \Sigma F_{\text{ext}}: (m_c + m_p) g \sin \theta = (m_c + m_p) a$$

$$a = g \sin \theta$$

CYLINDER:



RESULTANT OF FORCES EXERTED BY PLATFORM  
ON TO CYLINDER ACTS AT DISTANCE  $x$  FROM CORNER.

$$+ \uparrow \Sigma F_x = \Sigma F_{x, \text{ext}}: F = m_c a \cos \theta$$

$$+ \uparrow \Sigma F_y = \Sigma F_{y, \text{ext}}: N - m_c g = -m_c a \sin \theta$$

$$N = m_c (g - a \sin \theta)$$

$$(\mu_s) = \frac{F}{N} = \frac{m_c a \cos \theta}{m_c (g - a \sin \theta)} = \frac{(g \sin \theta) \cos \theta}{g - (g \sin \theta) \sin \theta}$$

$$\mu_s = \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta}$$

$$+ \uparrow \Sigma M_D = \Sigma (M_D)_{\text{ext}}: m_c g \left( \frac{b}{2} - x \right) = m_c a \cos \theta \left( \frac{h}{2} \right) + m_c a \sin \theta \left( \frac{b}{2} - x \right)$$

$$x \left( \frac{b}{2} - x \right) = (g \sin \theta) \cos \theta \left( \frac{h}{2} \right) + (g \sin \theta) \sin \theta \left( \frac{b}{2} - x \right)$$

$$\left( \frac{b}{2} - x \right) (1 - \sin^2 \theta) = \frac{1}{2} \sin \theta \cos \theta h; \left( \frac{b}{2} - x \right) \cos^2 \theta = \frac{1}{2} \sin \theta \cos \theta h$$

$$\frac{b}{2} - x = \frac{h}{2} \frac{\sin \theta}{\cos \theta}; x = \frac{1}{2} (b - h \tan \theta)$$

CYLINDER TIPS IF  $x \leq 0$ ;  $\tan \theta \geq \frac{b}{h} = \frac{8 \text{ in}}{6 \text{ in}}$ ;  $\theta \geq 53.1^\circ$

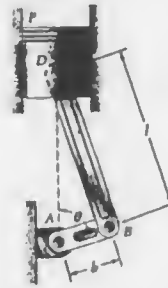
OUTLINE OF PROGRAM FOR  $b = 8 \text{ in}$  AND  $h = 6 \text{ in}$ .  
EVALUATE  $\mu_s$  AND  $x$  FOR EACH VALUE OF  $\theta$ .  
PRINT  $\mu_s$  AS MINIMUM VALUE OF  $\mu_s$  FOR  
NO SLIDING.

theta	x	mu req.	?slip?	?tip?
0.000	4.000	0.000	no slip	no tip
5.000	3.738	0.087	no slip	no tip
10.000	3.471	0.176	no slip	no tip
15.000	3.196	0.268	no slip	no tip
20.000	2.908	0.364	no slip	no tip
25.000	2.601	0.466	no slip	no tip
30.000	2.268	0.577	no slip	no tip
35.000	1.899	0.700	slips	no tip

-- Find theta for  $\mu = 0.60$  --

30.960	2.200	0.5999
30.980	2.199	0.6004

## 16.C4



GIVEN: ENGINE SYSTEM OF  
PROB 15.C3.

$$W_{AB} = 1000 \text{ lb}$$

$$l = 160 \text{ mm}, b = 60 \text{ mm}$$

$$m_P = 2.5 \text{ kg}, m_{BD} = 3 \text{ kg}$$

FIND: COMPONENTS OF  
DYNAMIC REACTIONS ON BD  
AT POINTS B AND D FOR  
 $\theta = 0$  TO  $180^\circ$  USING  $10^\circ$   
INCREMENTS.

VELOCITY:

$$\beta = \sin^{-1} \frac{b \sin \theta}{l} \quad (1)$$

FROM SOLUTION OF PROB 16.C3

$$v_B = b \omega_{AB} \quad (2)$$

$$\omega_{BD} = \frac{v_B \cos \theta}{l \cos \beta} \quad (3)$$

ACCELERATION

$$a_B = b \omega_{AB}^2 \quad (4)$$

$$a = \frac{l \omega_{BD}^2 \sin \beta - a_B \cos \theta}{l \cos \beta} \quad (5)$$

$$+ \uparrow a_n = a_B \cos \theta + l \omega_{BD}^2 \cos \beta + l \omega_{BD}^2 \sin \beta \quad (6)$$

KINETICS: WE FIRST FIND  $\bar{a}_x$  AND  $\bar{a}_y$ .

$$+ \downarrow (\bar{a}_B)_x = -a_B \sin \theta \quad \text{AND} \quad + \downarrow (\bar{a}_B)_y = a_B \cos \theta$$

SINCE G IS AT THE MIDDLE OF BD

$$+ \downarrow \bar{a}_x = \frac{1}{2} (\bar{a}_B)_x \quad (7)$$

$$+ \downarrow \bar{a}_y = \frac{1}{2} [(\bar{a}_B)_y + a_D] \quad (8)$$

PISTON 
$$P_y \rightarrow \square = \square$$

$$+ \downarrow \Sigma F_y = \Sigma (F_y)_{\text{ext}}$$

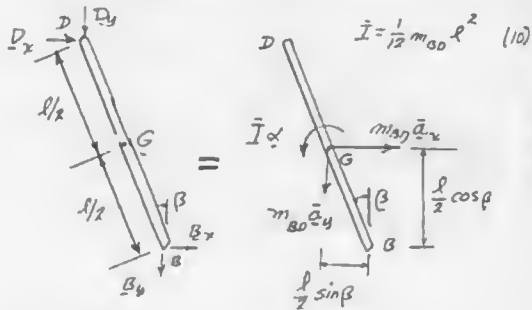
$$D_y = m_P a_D \quad (9)$$

NOTE: SINCE WE SEEK THE DYNAMIC  
REACTIONS, WE OMIT THE WEIGHT  
OF THE PISTON AND CONNECTING  
ROD

(CONTINUED)

## 16.C4 continued

## KINETICS: CONNECTING ROD



$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}: D_x l \cos \beta - D_y l \sin \beta = -I_G \alpha + m_{BD} a_x \left( \frac{l}{2} \cos \beta \right) - m_{BD} \left( \frac{l}{2} \sin \beta \right) \ddot{a}_y$$

Divide by  $l$  solve for  $D_x$ 

$$D_x = D_y \frac{\sin \beta}{\cos \beta} - \frac{I_G \alpha}{l \cos \beta} + \frac{m_{BD}}{2} \ddot{a}_x - \frac{m_{BD}}{2} \ddot{a}_y \frac{\sin \beta}{\cos \beta}$$

$$D_x = D_y \tan \beta - \frac{I_G \alpha}{l \cos \beta} + \frac{m_{BD}}{2} \ddot{a}_x - \frac{m_{BD}}{2} \ddot{a}_y \tan \beta \quad (11)$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: B_x + D_x = m_{BD} \ddot{a}_x$$

$$B_x = m_{BD} \ddot{a}_x - D_x \quad (12)$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: B_y + D_y = m_{BD} \ddot{a}_y$$

$$B_y = m_{BD} \ddot{a}_y - D_y \quad (13)$$

## OUTLINE OF PROGRAM:

ENTER DATA:  $\omega_{AB} = 1000 \text{ rpm} \left( \frac{2\pi}{60} \right) = \frac{100}{3} \pi \text{ rad/s}$   
 $m_A = 2.5 \text{ kg}, m_B = 3 \text{ kg}$   
 $l = 0.1 \text{ m}, \delta = 0.06 \text{ m}$

PROGRAM, IN SEQUENCE, EQS. (1) THROUGH (12).

EVALUATE AND PRINT  $B_x, B_y, D_x$ , AND  $D_y$  FOR VALUES OF  $\theta$  FROM 0 TO 180° AT 5° INCREMENTS:

Positive directions of force components are:  
DOWN and TO THE RIGHT

theta deg	Bx N	By N	Dx N	Dy N
0	0.00	4605.82	0.00	-2261.78
10	108.19	4497.37	-279.57	-2203.38
20	182.74	4177.66	-520.30	-2031.39
30	194.59	3663.87	-688.07	-1755.71
40	124.19	2985.61	-758.59	-1393.47
50	-33.57	2185.52	-722.48	-969.45
60	-265.48	1318.43	-589.25	-515.59
70	-539.76	447.34	-387.68	-68.61
80	-811.62	-364.52	-160.35	334.94
90	-1034.81	-1064.65	47.85	665.41
100	-1174.86	-1621.34	202.89	906.22
110	-1217.65	-2028.11	290.21	1056.59
120	-1169.20	-2300.43	314.47	1129.34
130	-1048.31	-2466.79	292.25	1145.24
140	-877.31	-2558.80	242.90	1126.72
150	-675.57	-2604.17	182.09	1093.40
160	-456.95	-2623.56	119.39	1060.08
170	-230.09	-2630.38	58.71	1036.51
180	-0.00	-2631.89	0.00	1028.08

## 16.C5

## GIVEN: UNIFORM

BAR OF MASS  $m$   
SUPPORTED BY  
SPRINGS OF  
CONSTANT  $k$ .

IMMEDIATELY AFTER  
CUTTING AC BREAKS

FIND  $\alpha$  AND  $\ddot{a}_B$   
FOR VALUES OF  $\theta$  FROM 0 TO 90°, USING 10° INCREMENTS.

STATICS: INITIAL SPRING TENSIONS

$$+\uparrow \Sigma F_y = 0$$

$$2T \cos \theta - mg = 0$$

$$T = \frac{mg}{2 \cos \theta}$$

KINETICS: JUST AFTER AC BREAKS

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: T \sin \theta = m \ddot{a}_x$$

$$\frac{mg}{2 \cos \theta} \sin \theta = m \ddot{a}_x$$

$$\ddot{a}_x = \frac{1}{2} g \tan \theta \quad (1)$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: mg - T \cos \theta = m \ddot{a}_y$$

$$mg - \frac{mg}{2 \cos \theta} \cos \theta = m \ddot{a}_y$$

$$\ddot{a}_y = \frac{1}{2} g \downarrow \quad (2)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}: (T \cos \theta) \frac{l}{2} = I_G \alpha$$

$$\frac{mg}{2 \cos \theta} \cos \theta \frac{l}{2} = \frac{1}{12} m l^2 \alpha$$

$$\alpha = \frac{3g}{l} \quad (3)$$

KINEMATICS:

$$(a_A)_x = \ddot{a}_x = \frac{1}{2} g \tan \theta \quad (4)$$

$$(a_B)_x = \ddot{a}_x = \frac{1}{2} g \tan \theta \quad (5)$$

$$+\downarrow (a_A)_y = \ddot{a}_y + \frac{1}{2} \alpha = \frac{1}{2} g + \frac{1}{2} \left( \frac{3g}{l} \right) = 2g \downarrow \quad (6)$$

$$+\uparrow (a_B)_y = -\ddot{a}_y + \frac{1}{2} \alpha = -\frac{1}{2} g + \frac{1}{2} \left( \frac{3g}{l} \right) = g \uparrow \quad (7)$$

$$\text{END A: } \beta = \tan^{-1} \frac{(a_A)_y}{(a_A)_x}; a_A = \frac{(a_A)_x}{\cos \beta} \quad (8,9)$$

$$\text{END B: } \gamma = \tan^{-1} \frac{(a_B)_y}{(a_B)_x}; a_B = \frac{(a_B)_x}{\cos \gamma} \quad (10,11)$$

## OUTLINE OF PROGRAM:

PROGRAM, IN SEQUENCE, EQS. (1) THROUGH (11).

EVALUATE AND PRINT  $a_A, \beta, a_B, \gamma$  FOR VALUES OF  $\theta$  FROM 0 TO 90° USING 10° INCREMENTS.

theta	[ $\frac{a_A}{g}$ ]	[ $\beta$ ]	[ $\frac{a_B}{g}$ ]	gsmms
0.000	2.000	90.000	1.000	90.000
10.000	2.002	87.476	1.004	84.962
20.000	2.008	84.801	1.016	79.686
30.000	2.021	81.787	1.041	73.898
40.000	2.044	78.153	1.084	67.240
50.000	2.087	73.409	1.164	59.210
60.000	2.179	66.587	1.323	49.107
70.000	2.426	55.516	1.699	36.052
80.000	3.470	35.196	3.007	19.425
90.000	infinite	0.000	infinite	0.000

17.1

GIVEN: 6000-lb FLYWHEEL,  $\bar{R} = 36$  in.,  $\omega_0 = 300$  rpm.

FIND: MAGNITUDE OF COUPLE DUE TO FRICTION KNOWING FLYWHEEL ROTATES 1500 REVOLUTIONS WHILE COASTING TO REST.

$$\omega_0 = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$$

$$\bar{I} = m \bar{R}^2 = \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2} (3 \text{ ft})^2 = 1677 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$T_1 = \frac{1}{2} \bar{I} \omega_0^2 = \frac{1}{2} (1677) (10\pi)^2 = 827,600 \text{ ft} \cdot \text{lb}, T_2 = 0$$

$$U_{1 \rightarrow 2} = -M\theta = -M(1500 \text{ rev}) \left( 2\pi \frac{\text{rad}}{\text{rev}} \right) = -9424.7 M$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 827,600 - 9424.7 M = 0$$

$$M = 87.81 \text{ lb} \cdot \text{ft} \quad M = 87.81 \text{ lb} \cdot \text{ft}$$

17.2

GIVEN: 50-Rg ROTOR,  $\bar{R} = 180$  mm,  $\omega_0 = 3600$  rpm,  $M_f = 3.5$  N·m

FIND: NUMBER OF REVOLUTIONS AS ROTOR COASTS TO REST

$$\omega_0 = 3600 \text{ rpm} \left( \frac{2\pi}{60} \right) = 120\pi \text{ rad/s}$$

$$\bar{I} = m \bar{R}^2 = (50 \text{ Rg}) (0.180 \text{ m})^2 = 1.620 \text{ Rg} \cdot \text{m}^2$$

$$T_1 = \frac{1}{2} \bar{I} \omega_0^2 = \frac{1}{2} (1.620) (120\pi)^2 = 115.12 \text{ Rg}, T_2 = 0$$

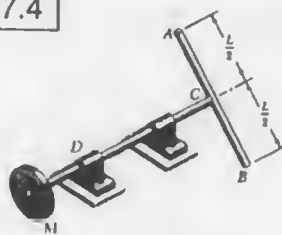
$$U_{1 \rightarrow 2} = -M\theta = -(3.5 \text{ N} \cdot \text{m}) \theta$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 115.12 \text{ Rg} - (3.5 \text{ N} \cdot \text{m}) \theta = 0$$

$$\theta = 32.891 \times 10^3 \text{ rad}$$

$$\theta = 5230 \text{ rev}$$

17.4



GIVEN:  $\omega_0 = 0$

$$\bar{I}_{\text{DISK}} = I_0$$

$W = \text{WEIGHT/UNIT LENGTH OF ROD}$

FIND: LENGTH  $L$  FOR MAXIMUM  $v_A$  AFTER COUPLE  $M$  IS APPLIED FOR ONE REVOLUTION

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \left( I_0 + \frac{1}{12} W L^2 \right) \omega_2^2$$

$$U_{1 \rightarrow 2} = M\theta = M(2\pi \text{ rad})$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 0 + 2\pi M = \frac{1}{2} \left( I_0 + \frac{W L^2}{12} \right) \omega_2^2$$

$$\omega_2^2 = \frac{4\pi M}{I_0 + \frac{W L^2}{12}}$$

$$v_A = \frac{L}{2} \omega_2: v_A^2 = \frac{L^2}{4} \omega_2^2 = \frac{\pi M L^2}{I_0 + \frac{W L^2}{12}}$$

DIFFERENTIATING WITH RESPECT TO  $L$ ,

$$2 v_A \left( \frac{dv_A}{dL} \right) = \left[ 2L \left( I_0 + \frac{W L^2}{12} \right) - L^2 \left( \frac{3W L^2}{12} \right) \right] \left[ \frac{\pi M}{\left( I_0 + \frac{W L^2}{12} \right)^2} \right]$$

$$\frac{dv_A}{dL} = 0: 2L \left( I_0 + \frac{W L^2}{12} \right) - L^2 \left( \frac{3W L^2}{12} \right) = 0$$

$$2I_0 L - \frac{W L^4}{12} = 0: L^3 = \frac{249 I_0}{W}$$

17.5

GIVEN: 300-Rg PUNCHING MACHINE FLYWHEEL,  $\bar{R} = 600$  mm,  $\omega_1 = 300$  rpm. EACH PUNCH REQUIRES 2500 J.

FIND: (a)  $\omega_2$  IMMEDIATELY AFTER  $M$  PUNCH  
(b) IF  $M = 25$  N·m, FIND REVOLUTIONS BEFORE  $\omega$  IS AGAIN 300 rpm.

$$\bar{I} = m \bar{R}^2 = (300 \text{ Rg}) (0.6 \text{ m})^2 = 108 \text{ Rg} \cdot \text{m}^2$$

$$(a) \omega_1 = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$$

$$T_1 = \frac{1}{2} m \omega_1^2 = \frac{1}{2} (108 \text{ Rg} \cdot \text{m}^2) (10\pi \text{ rad/s})^2$$

$$T_1 = 53.296 \text{ Rg}$$

$$U_{1 \rightarrow 2} = -2500 \text{ J} = -2.5 \text{ Rg}$$

$$T_2 = \frac{1}{2} m \omega_2^2 = \frac{1}{2} (108 \text{ Rg} \cdot \text{m}^2) \omega_2^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 53.296 \text{ Rg} - 2.5 \text{ Rg} = \frac{1}{2} (108 \text{ Rg} \cdot \text{m}^2) \omega_2^2$$

$$\omega_2 = 30.67 \text{ rad/s} \left( \frac{60}{2\pi} \right) = 292.9 \text{ rpm}$$

$$\omega_2 = 293 \text{ rpm}$$

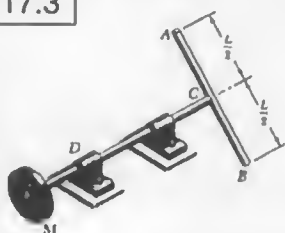
$$(b) U_{2 \rightarrow 1} = M\theta$$

$$2500 \text{ J} = (25 \text{ N} \cdot \text{m}) \theta$$

$$\theta = 100 \text{ rad} \left( \frac{\text{rev}}{2\pi \text{ rad}} \right) = 15.9155 \text{ rev}$$

$$\theta = 15.92 \text{ rev}$$

17.3



GIVEN: 8-lb DISK OF 9-in. DIAMETER  
ROD AB WEIGHS 3 lb/ft  
 $M = 416$  ft

FIND: LENGTH  $L$  IF  $\omega$  IS 300 rpm AFTER 2 REVOLUTIONS

$$r = 4.5 \text{ in.} = \frac{3}{8} \text{ ft}; \quad \omega = 300 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10\pi \text{ rad/s}$$

$$W_{\text{DISK}} = 8 \text{ lb}, \quad W_{\text{ROD}} = (3 \text{ lb/ft}) L$$

$$\bar{I} = \frac{1}{2} m_{\text{DISK}} r^2 + \frac{1}{12} m_{\text{ROD}} L^2$$

$$= \frac{1}{2} \frac{8}{g} \left( \frac{3}{8} \right)^2 + \frac{1}{12} \frac{3L}{g} L^2 = \frac{1}{g} \left( \frac{9}{16} + \frac{L^3}{4} \right)$$

$$T_1 = 0, \quad T_2 = \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2g} \left( \frac{9}{16} + \frac{L^3}{4} \right) (10\pi)^2$$

$$U_{1 \rightarrow 2} = M\theta = (416 \text{ ft}) (2 \text{ rev}) \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 16\pi$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 0 + 16\pi = \frac{1}{2g} \left( \frac{9}{16} + \frac{L^3}{4} \right) (10\pi)^2$$

$$\frac{16\pi(2g)}{(10\pi)^2} = \frac{9}{16} + \frac{L^3}{4}$$

$$3.2779 = \frac{9}{16} + \frac{L^3}{4}; \quad \frac{L^3}{4} = 2.717$$

$$L^3 = 10.869 \text{ ft}^3$$

$$L = 2.22 \text{ ft}$$



17.6

GIVEN:  $\omega_1 = 360 \text{ rpm}$  OF PUNCHING MACHINE FLYWHEEL. EACH PUNCH REQUIRES 1500 ft·lb. AFTER EACH PUNCH  $\omega_2 = 0.95 \omega_1$ .

FIND: (a)  $\bar{I}$  OF FLYWHEEL

(b) REVOLUTIONS REQUIRED FOR ANGULAR VELOCITY TO AGAIN BE 360 rpm IF CONSTANT 18 lb·ft COUPLE IS APPLIED

(a)

$$\omega_1 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right) = 12\pi \text{ rad/s}$$

$$\omega_2 = 0.95 \omega_1 = 0.94(12\pi \text{ rad/s}) = 11.4\pi \text{ rad/s}$$

$$T_1 = \frac{1}{2} \bar{I} \omega_1^2 = \frac{1}{2} \bar{I} (12\pi)^2$$

$$T_2 = \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} \bar{I} (11.4\pi)^2$$

$$U_{1 \rightarrow 2} = -1500 \text{ ft·lb}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad \frac{1}{2} \bar{I} (12\pi)^2 - 1500 = \frac{1}{2} \bar{I} (11.4\pi)^2$$

$$\bar{I} = \frac{2(1500)}{\pi^2(12^2 - 11.4^2)} = \frac{3000}{138.57} = 21.649 \text{ lb·ft·s}^2$$

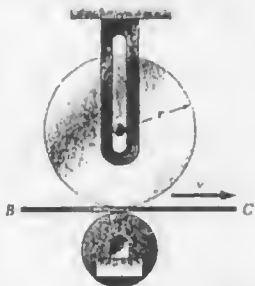
$$\bar{I} = 21.6 \text{ lb·ft·s}^2$$

(b)  $U_{2 \rightarrow 1} = M\theta; \quad 1500 \text{ ft·lb} = (18 \text{ ft·lb})\theta$

$$\theta = 83.33 \text{ rad} \left( \frac{rev}{2\pi \text{ rad}} \right) = 13.263 \text{ rev}$$

$$\theta = 13.26 \text{ rev}$$

17.7 and 17.8



GIVEN: DISK PLACED ON SURF WHEN  $\omega_0 = 0$ . COEFFICIENT OF KINETIC FRICTION =  $\mu_k$ .

FIND: REVOLUTIONS BEFORE  $\omega = \text{CONSTANT}$ .

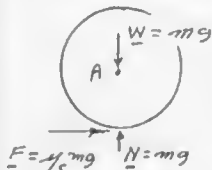
PROBLEM 17.7:

IN TERMS OF  $v$ ,  $r$ , AND  $\mu_k$ .

PROBLEM 17.8:

FOR  $r = 6 \text{ in}$ ,  $v = 40 \text{ ft/s}$ , AND  $\mu_k = 0.20$ .

ONLY FORCE DOING WORK IS  $F$ . SINCE ITS MOMENT ABOUT A IS  $M = rF$ , WE HAVE



$$U_{1 \rightarrow 2} = M\theta = rF\theta = r(\mu_k mg)\theta$$

ANGULAR VELOCITY BECOMES CONSTANT WHEN  $\omega_2 = \frac{v}{r}$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} \left( \frac{1}{2} mr^2 \right) \left( \frac{v}{r} \right)^2 = \frac{mv^2}{4}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + r\mu_k mg\theta = \frac{mv^2}{4}$$

$$\theta = \frac{v^2}{4r\mu_k g} \text{ rad}$$

$$\theta = \frac{v^2}{8\pi r\mu_k g} \text{ rev}$$

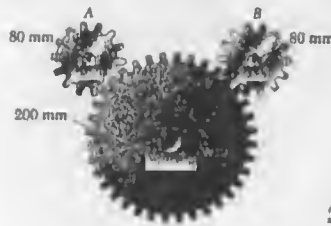
PROBLEM 17.8:  $r = 0.5 \text{ ft}$ ,  $\mu_k = 0.20$ ,  $v = 40 \text{ ft/s}$

$$\theta = \frac{(40 \text{ ft/s})^2}{8\pi(0.5 \text{ ft})(0.20)(32.2 \text{ ft/s}^2)}$$

$$\theta = 19.77 \text{ rev}$$

NOTE: RESULT IS INDEPENDENT OF  $m$ .

17.9 and 17.10



GIVEN:  $m_C = 12 \text{ kg}$ ,  $\bar{r}_C = 150 \text{ mm}$

$$m_A = m_B = 2.4 \text{ kg}$$

$$\bar{r}_A = \bar{r}_B = 60 \text{ mm}$$

$$M = 10 \text{ N·m}$$

FIND: (a) REVOLUTIONS OF C AS  $\omega_C$  INCREASES FROM 100 rpm TO 450 rpm  
(b) TANGENTIAL FORCE ON A

PROBLEM 17.9

M IS APPLIED TO GEAR C

PROBLEM 17.10

M IS APPLIED TO GEAR B

KINEMATICS:

$$\omega_A = \omega_B = \frac{200 \text{ mm}}{80 \text{ mm}} \omega_C = 2.5 \omega_C$$

$$(\omega_C)_1 = 100 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10.472 \text{ rad/s}$$

$$(\omega_C)_2 = 450 \text{ rpm} \left( \frac{2\pi}{60} \right) = 47.124 \text{ rad/s}$$

WORK AND ENERGY

$$\bar{I}_A = \bar{I}_B = m\bar{r}^2 = (2.4 \text{ kg})(0.06 \text{ m})^2 = 8.64 \times 10^{-3} \text{ kg·m}^2$$

$$\bar{I}_C = m\bar{r}^2 = (12 \text{ kg})(0.150 \text{ m})^2 = 0.270 \text{ kg·m}^2$$

POSITION 1:  $(\omega_C)_1 = 10.472 \text{ rad/s}$ ;

$$(\omega_A)_1 = (\omega_B)_1 = 2.5(\omega_C)_1 = 26.18 \text{ rad/s}$$

$$T_1 = \frac{1}{2} \bar{I}_A (\omega_A)_1^2 + \frac{1}{2} \bar{I}_C (\omega_C)_1^2$$

$$= \frac{1}{2} \left[ (8.64 \times 10^{-3})(26.18)^2 \right] + \frac{1}{2} (0.270)(10.472)^2 = 20.726 \text{ J}$$

POSITION 2:  $(\omega_C)_2 = 47.124 \text{ rad/s}$

$$(\omega_A)_2 = (\omega_B)_2 = 2.5(\omega_C)_2 = 117.81 \text{ rad/s}$$

$$T_2 = \frac{1}{2} \left[ (8.64 \times 10^{-3})(117.81)^2 \right] + \frac{1}{2} (0.270)(47.124)^2 = 419.71 \text{ J}$$

PROBLEM 17.9:  $M = 10 \text{ N·m}$  APPLIED TO GEAR C

$$U_{1 \rightarrow 2} = M\theta = 10\theta$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 20.726 \text{ J} + 10\theta = 419.71 \text{ J}$$

$$\theta = 39.90 \text{ rad} \quad \theta = 6.35 \text{ rev}$$

GEAR A:  $\theta_A = 2.5\theta = 2.5(39.90) = 99.75 \text{ rad}$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad \frac{1}{2} m_A (\omega_A)_1^2 + F(0.08)\theta_A = \frac{1}{2} m_A (\omega_A)_2^2$$

$$0.08 \text{ m} \left[ \frac{1}{2} (8.64 \times 10^{-3})(26.18)^2 + F(0.08)(99.75) \right] = \frac{1}{2} (8.64 \times 10^{-3})(117.81)^2$$

$$2.961 + 7.98 F = 59.96$$

$$F = 7.14 \text{ N} \quad \underline{F = 7.14 \text{ N} \uparrow}$$

PROBLEM 17.10:  $M = 10 \text{ N·m}$  APPLIED TO GEAR B

NOTE: ANGULAR SPEEDS ARE SAME AS IN PROB 17.9, THUS  $T_1$  AND  $T_2$  ARE ALSO THE SAME

$$T_1 = 20.726 \text{ J} \quad T_2 = 419.71 \text{ J}$$

WE HAVE  $U_{1 \rightarrow 2} = M\theta = 10\theta$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 20.726 \text{ J} + 10\theta = 419.71 \text{ J}$$

$$\theta = 39.90 \text{ rad}$$

$$\theta_B = 2.5\theta; \quad 39.90 \text{ rad} = 2.5\theta; \quad \theta = 15.96 \text{ rad}$$

$$\theta = 2.54 \text{ rev}$$

GEAR A:  $\theta_A = \theta_B = 39.90 \text{ rad}$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad \frac{1}{2} m_A (\omega_A)_1^2 + F(0.08)\theta_A = \frac{1}{2} m_A (\omega_A)_2^2$$

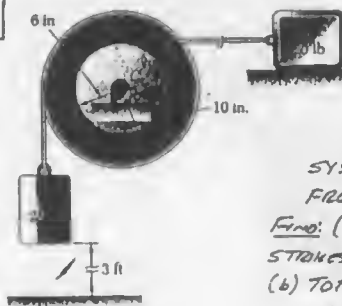
$$0.08 \text{ m} \left[ \frac{1}{2} (8.64 \times 10^{-3})(26.18)^2 + F(0.08)(39.90) \right] + \frac{1}{2} (8.64 \times 10^{-3})(117.81)^2$$

$$2.961 + 3.192 F = 59.96$$

$$F = 17.86 \text{ N}$$

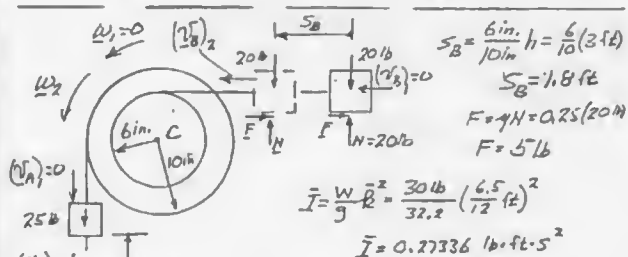
$$\underline{F = 17.86 \text{ N} \downarrow}$$

17.11



GIVEN:  
30-lb PULLEY  
 $\bar{R} = 6 \text{ in.}$   
 $\mu_k = 0.25$   
SYSTEM IS RELEASED FROM REST

FIND: (a)  $\bar{v}_A$  AS IT STRIKES THE GROUND  
(b) TOTAL DISTANCE THAT BLOCK B MOVES



(a)

$$T_1 = 0; \quad T_2 = \frac{1}{2} m_A (\bar{v}_A)_2^2 + \frac{1}{2} \bar{I} \omega_2^2 + \frac{1}{2} m_B (\bar{v}_B)_2^2$$

$$= \frac{1}{2} \frac{25 \text{ lb}}{32.2} \left( \frac{5}{8} \omega_2 \right)^2 + \frac{1}{2} (0.27336) \omega_2^2 + \frac{1}{2} \frac{20 \text{ lb}}{32.2} \left( \frac{1}{2} \omega_2 \right)^2$$

$$= 0.26958 \omega_2^2 + 0.13665 \omega_2^2 + 0.07764 \omega_2^2$$

$$T_2 = 0.48387 \omega_2^2$$

$$U_{1 \rightarrow 2} = W_A h - F(s_B) = (25 \text{ lb})(3 \text{ ft}) - (5 \text{ lb})(1.8 \text{ ft})$$

$$U_{1 \rightarrow 2} = 66 \text{ ft} \cdot \text{lb}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + 66 \text{ ft} \cdot \text{lb} = 0.48387 \omega_2^2$$

$$\omega_2^2 = 136.39 \quad \omega_2 = 11.679 \text{ rad/s}$$

$$(\bar{v}_A)_2 = \frac{5}{8} \omega_2 = \frac{5}{8} (11.679) \quad (\bar{v}_B)_2 = 9.73 \text{ ft/s} \quad \leftarrow$$

(b) BLOCK B COASTS TO REST

TOTAL ENERGY OF BLOCKS AND PULLEY JUST BEFORE IMPACT = 66 ft·lb

KINETIC ENERGY OF BLOCK A JUST BEFORE IMPACT

$$T_A = \frac{1}{2} \frac{W_A}{g} (\bar{v}_A)_2^2 = \frac{1}{2} \frac{25 \text{ lb}}{32.2} (9.73 \text{ ft/s})^2 = 36.75 \text{ ft} \cdot \text{lb}$$

AFTER BLOCK A STRIKES THE GROUND, WE FIND THAT THE KINETIC ENERGY OF THE PULLEY C AND BLOCK B IS

$$T_{C+B} = 66 \text{ ft} \cdot \text{lb} - 36.75 \text{ ft} \cdot \text{lb} = 29.25 \text{ ft} \cdot \text{lb}$$

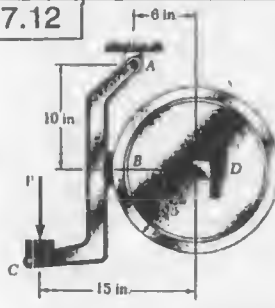
FOR SYSTEM TO STOP, 29.25 ft·lb OF ENERGY MUST BE DISSIPATED BY THE FRICTION FORCE,  $F = 5 \text{ lb}$ .

$$29.25 \text{ ft} \cdot \text{lb} = (5 \text{ lb})d$$

$$d = 5.85 \text{ ft}$$

TO FIND TOTAL DISTANCE MOVED BY B, WE ADD  $s_B = 1.8 \text{ ft}$ ; TOTAL DISTANCE = 1.8 + 5.85 = 7.65 ft

17.12



GIVEN:  $\bar{I} = 14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
 $\omega_1 = 360 \text{ rpm}$   
 $\mu_k = 0.35$

FIND: P SO THAT FLYWHEEL STOPS IN 100 REVOLUTIONS.

$$\omega_1 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right) = 12\pi \text{ rad/s}$$

$$T_1 = \frac{1}{2} \bar{I} \omega_1^2 = \frac{1}{2} (14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (12\pi \text{ rad/s})^2$$

$$T_1 = 9948.6 \text{ ft} \cdot \text{lb} \quad T_2 = 0$$

$$\Theta = 100 \text{ rev} \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 628.32 \text{ rad}$$

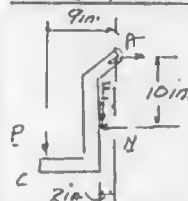
$$U_{1 \rightarrow 2} = -M\Theta = -F\bar{r}\Theta = -F \left( \frac{2}{3} \text{ ft} \right) (628.32 \text{ rad})$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 9948.6 - F \left( \frac{2}{3} \right) (628.32) = 0$$

$$F = 23.75 \text{ lb}$$

$$F = \mu_k N; \quad 23.75 \text{ lb} = (0.35)N; \quad N = 67.86 \text{ lb}$$

FREE BODY: BRAKE AC:



$$+\circlearrowleft \Sigma M_A = 0$$

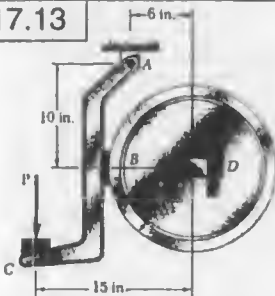
$$P(9 \text{ in.}) + F(2 \text{ in.}) - N(10 \text{ in.}) = 0$$

$$9P + (23.75)(2) - (67.86)(10) = 0$$

$$9P - 631.1 = 0$$

$$P = 70.12 \text{ lb} \quad \underline{P = 70.1 \text{ lb}} \quad \leftarrow$$

17.13



GIVEN:  $\bar{I} = 14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
 $\omega_1 = 360 \text{ rpm}$   
 $\mu_k = 0.35$

FIND: P SO THAT FLYWHEEL STOPS IN 100 REVOLUTIONS

$$\omega_1 = 360 \text{ rpm} \left( \frac{2\pi}{60} \right) = 12\pi \text{ rad/s}$$

$$T_1 = \frac{1}{2} \bar{I} \omega_1^2 = \frac{1}{2} (14 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (12\pi \text{ rad/s})^2$$

$$T_1 = 9948.6 \text{ ft} \cdot \text{lb} \quad T_2 = 0$$

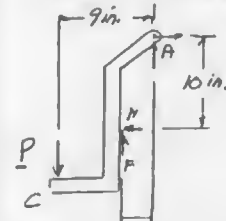
$$\Theta = 100 \text{ rev} \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 628.32 \text{ rad}$$

$$U_{1 \rightarrow 2} = -M\Theta = -F\bar{r}\Theta = -F \left( \frac{2}{3} \text{ ft} \right) (628.32 \text{ rad})$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 9948.6 - F \left( \frac{2}{3} \right) (628.32) = 0$$

$$F = 23.75 \text{ lb}$$

$$F = \mu_k N; \quad 23.75 \text{ lb} = 0.35N; \quad N = 67.86 \text{ lb}$$



FREE BODY: BRAKE AC

$$+\circlearrowleft \Sigma M_A = 0$$

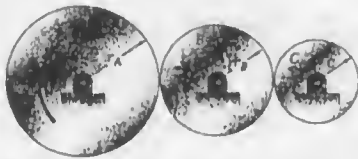
$$P(9 \text{ in.}) - F(2 \text{ in.}) - N(10 \text{ in.}) = 0$$

$$9P - (23.75)(2) - (67.86)(10) = 0$$

$$9P - 726.1 = 0$$

$$P = 80.68 \text{ lb} \quad \underline{P = 80.7 \text{ lb}} \quad \leftarrow$$

# 17.14 and 17.15



GIVEN: FRICTION  
DISKS A, B, AND C  
ARE MADE OF SAME  
MATERIAL AND  
HAVE SAME  
THICKNESS

PROBLEM 17.14: FIND: EXPRESSION FOR  $\omega_A$  AFTER

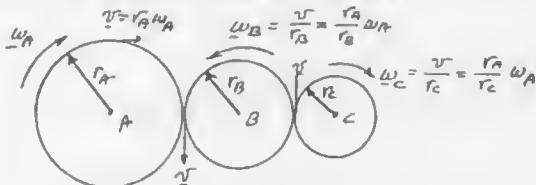
THE COUPLE  $M$  IS APPLIED FOR ONE REVOLUTION

PROBLEM 17.15: FIND: REVOLUTIONS OF A REQUIRED

FOR  $\omega_A = 150 \text{ rpm}$  WHEN  $M = 60 \text{ lb}\cdot\text{in}$ ,

$r_A = 8 \text{ in}$ ,  $r_B = 6 \text{ in}$ ,  $r_C = 4 \text{ in}$ , AND  $\omega_A = 12 \text{ lb}$ .

DEFINITE VELOCITY OF PERIMETER BY  $v$ :



DEFINITE MASS DENSITY OF MATERIAL BY  $\rho$  AND

THICKNESS OF DISKS BY  $t$ .

THEN MASS OF A DISK IS  $m = (\text{VOLUME})\rho = (\pi r^2 t)\rho$

AND  $\bar{I} = \frac{1}{2} m r^2 = \frac{\pi \rho t}{2} r^4$

KINETIC ENERGY:  $T = \sum \frac{1}{2} \bar{I} \omega^2$

$$T = \frac{1}{2} \left( \frac{\pi \rho t}{2} \right) \left[ r_A^4 \omega_A^2 + r_B^4 \omega_B^2 + r_C^4 \omega_C^2 \right]$$

$$\frac{1}{2} \left( \frac{\pi \rho t}{2} \right) \left[ r_A^4 \omega_A^2 + r_B^4 \left( \frac{r_A}{r_B} \right)^2 \omega_A^2 + r_C^4 \left( \frac{r_A}{r_C} \right)^2 \omega_A^2 \right]$$

$$T = \frac{1}{2} \left( \frac{\pi \rho t}{2} \right) r_A^4 \left[ \omega_A^2 + \left( \frac{r_B}{r_A} \right)^2 \omega_A^2 + \left( \frac{r_C}{r_A} \right)^2 \omega_A^2 \right] \quad (1)$$

WORK:  $U_1 \rightarrow U_2 = M\theta$

$\omega_1 = 0$ ;  $\omega_2 = \omega_A$

$$T_1 + U_{1 \rightarrow 2} = T_2: 0 + M\theta = \frac{\pi \rho t}{4} \omega_A^2 r_A^4 \left[ r_A^2 + r_B^2 + r_C^2 \right]$$

PROBLEM 17.14 For  $\theta = 2\pi$ :  $M(2\pi) = \frac{\pi \rho t}{4} \omega_A^2 r_A^4 \left[ 1 + \left( \frac{r_B}{r_A} \right)^2 + \left( \frac{r_C}{r_A} \right)^2 \right]$

$$\omega_A^2 = \frac{8 M \theta}{\rho t r_A^4 \left[ 1 + \left( \frac{r_B}{r_A} \right)^2 + \left( \frac{r_C}{r_A} \right)^2 \right]}$$

PROBLEM 17.15: RECALL THAT  $m_A = \pi r_A^2 t \rho$  AND WRITE

EQ.(1) AS:

$$T = \frac{1}{4} (\pi r_A^2 t \rho) (r_A^2 + r_B^2 + r_C^2)$$

$$T = \frac{1}{4} \left( \frac{m_A}{\pi} \right) r_A^2 \left[ 1 + \left( \frac{r_B}{r_A} \right)^2 + \left( \frac{r_C}{r_A} \right)^2 \right] \omega_A^2$$

DATA:  $\omega_A = 150 \text{ rpm} \left( \frac{2\pi}{60} \right) = 5\pi \text{ rad/s}$

$\omega_A = 12 \text{ lb}$ ,  $r_A = 8 \text{ in}$ ,  $r_B = 6 \text{ in}$ ,  $r_C = 4 \text{ in}$ .

$M = 60 \text{ lb}\cdot\text{in} = 5 \text{ ft}\cdot\text{lb}$

$$U_{1 \rightarrow 2} = M\theta = (5 \text{ ft}\cdot\text{lb})\theta$$

$$T_1 + U_{1 \rightarrow 2} = T_2:$$

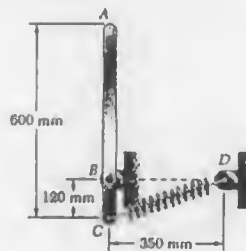
$$0 + 5\theta = \frac{1}{4} \left( \frac{12 \text{ lb}}{32.2} \right) \left( \frac{8 \text{ in}}{12} \right)^2 \left[ 1 + \left( \frac{6 \text{ in}}{8 \text{ in}} \right)^2 + \left( \frac{4 \text{ in}}{8 \text{ in}} \right)^2 \right] (5\pi)^2$$

$$5\theta = 0.041408 \left[ 1 + \frac{9}{4} + \frac{1}{4} \right] (5\pi)^2$$

$$5\theta = 18.518; \quad \theta = 3.704 \text{ rad} \left( \frac{\text{rev}}{2\pi \text{ rad}} \right) = 0.5894 \text{ rev}$$

$$\theta = 0.589 \text{ rev}$$

# 17.16



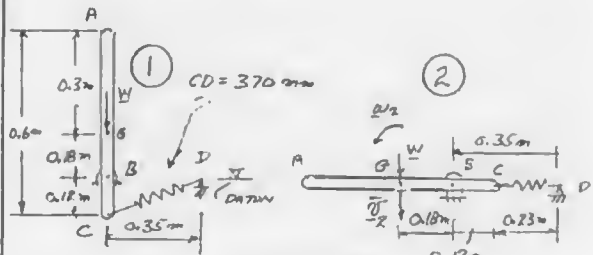
GIVEN: 4-Rd 1200 AC

SPRING:  $k = 400 \text{ N/m}$

UNSTRETCHED LENGTH  
= 150 mm.

ROD IS RELEASED FROM  
HORIZONTAL.

FIND:  $\omega$  AFTER ROD  
HAS ROTATED  $90^\circ$



POSITION 1: UNSTRETCHED LENGTH

SPRING:  $x_1 = CD - (150 \text{ mm}) = 370 - 150 = 220 \text{ mm} = 0.22 \text{ m}$

$$V_1 = \frac{1}{2} k x_1^2 = \frac{1}{2} (400 \text{ N/m}) (0.22 \text{ m})^2 = 9.68 \text{ J}$$

GRAVITY:  $V_1 = W h = m g h = (4 \text{ kg}) (9.81 \text{ m/s}^2) (0.15 \text{ m}) = 7.063 \text{ J}$

$$V_1 = V_2 + V_g = 9.68 \text{ J} + 7.063 \text{ J} = 16.743 \text{ J}$$

KINETIC ENERGY:  $T_1 = 0$

POSITION 2:

SPRING:  $x_2 = 230 \text{ mm} - 150 \text{ mm} = 80 \text{ mm} = 0.08 \text{ m}$

$$V_2 = \frac{1}{2} k x_2^2 = \frac{1}{2} (400 \text{ N/m}) (0.08 \text{ m})^2 = 1.28 \text{ J}$$

GRAVITY:  $V_2 = W h = 0$

$$V_2 = V_1 + V_g = 1.28 \text{ J}$$

KINETIC ENERGY:  $T_2 = \frac{1}{2} I \omega_2^2 = (0.18 \text{ kg}) \omega_2^2$

$$\bar{I} = \frac{1}{2} m L^2 = \frac{1}{2} (4 \text{ kg}) (0.6 \text{ m})^2 = 0.72 \text{ kg}\cdot\text{m}^2$$

$$T_2 = \frac{1}{2} m \bar{I} \omega_2^2 + \frac{1}{2} \bar{I} \omega_2^2$$

$$= \frac{1}{2} (4 \text{ kg}) (0.18 \omega_2^2) + \frac{1}{2} (0.72) \omega_2^2$$

$$T_2 = 0.72 \omega_2^2$$

CONSERVATION OF ENERGY:

$$T_1 + V_1 = T_2 + V_2$$

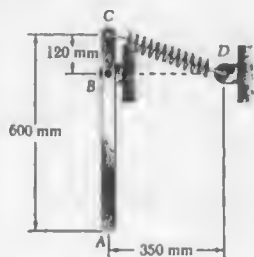
$$0 + 16.743 \text{ J} = 0.72 \omega_2^2 + 1.28 \text{ J}$$

$$\omega_2^2 = 123.9$$

$$\omega_2 = 11.131 \text{ rad/s}$$

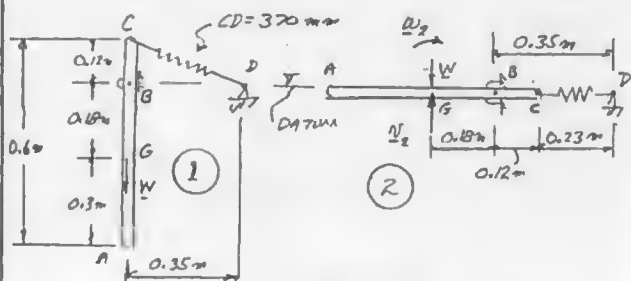
$$\omega_2 = 11.13 \text{ rad/s}$$

17.17



GIVEN: 4-lb rod AC  
 SPRING:  $k = 400 \text{ N/m}$   
 UNSTRETCHED LENGTH  
 $= 150 \text{ mm}$   
 ROD IS RELEASED  
 FROM REST

FIND:  $\omega$  AFTER ROD  
 HAS ROTATED  $90^\circ$



POSITION 1:

SPRING:  $x_1 = CD - (150 \text{ mm}) = 370 - 150 = 220 \text{ mm} = 0.22 \text{ m}$

$$V_e = \frac{1}{2} k x_1^2 = \frac{1}{2} (400 \text{ N/m}) (0.22 \text{ m})^2 = 9.68 \text{ J}$$

GRAVITY:  $V_g = Wh = mgh = (4 \text{ kg}) (9.81 \text{ m/s}^2) (0.12 \text{ m}) = -7.063 \text{ J}$

$$V_1 = V_e + V_g = 9.68 \text{ J} - 7.063 \text{ J} = 2.617 \text{ J}$$

KINETIC ENERGY  $T_1 = 0$

POSITION 2:

SPRING:  $x_2 = 230 \text{ mm} - 150 \text{ mm} = 80 \text{ mm} = 0.08 \text{ m}$

$$V_e = \frac{1}{2} k x_2^2 = \frac{1}{2} (400 \text{ N/m}) (0.08 \text{ m})^2 = 1.28 \text{ J}$$

GRAVITY:  $V_g = Wh = 0$

$$V_2 = V_e + V_g = 1.28 \text{ J}$$

KINETIC ENERGY:  $T_2 = V\omega_2 = (0.18 \text{ m})\omega_2$

$$\bar{I} = \frac{1}{12} m L^2 = \frac{1}{12} (4 \text{ kg}) (0.6 \text{ m})^2 = 0.12 \text{ kg} \cdot \text{m}^2$$

$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$$

$$= \frac{1}{2} (4 \text{ kg}) (0.18 \omega_2)^2 + \frac{1}{2} (0.12) \omega_2^2$$

$$T_2 = 0.1248 \omega_2^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.617 \text{ J} = 0.1248 \omega_2^2 + 1.28 \text{ J}$$

$$\omega_2^2 = 10.713$$

$$\omega_2 = 3.273 \text{ rad/s}$$

$$\omega_2 = 3.27 \text{ rad/s}$$

17.18



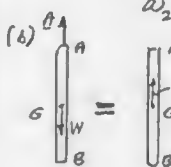
GIVEN: ROD OF WEIGHT  $W$  IS  
 RELEASED FROM REST.  
 FIND: (a)  $\omega$  AND  $A$  AS ROD  
 PASSES THROUGH THE VERTICAL  
 (b) SOLVE PART (a) FOR  
 $W = 1.8 \text{ lb}$ ,  $l = 3 \text{ ft}$

(1)  $\bar{v}_2 = \frac{l}{2} \omega_2$   
 $T_1 = 0$   
 $T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$   
 $= \frac{1}{2} m \left( \frac{l}{2} \omega_2 \right)^2 + \frac{1}{2} \left( \frac{1}{12} m l^2 \right) \omega_2^2$   
 $T_2 = \frac{1}{6} m l^2 \omega_2^2$   
 $U_{1 \rightarrow 2} = mg \frac{l}{2}$

(a)  $T_1 + U_{1 \rightarrow 2} = T_2$ :  $0 + mg \frac{l}{2} = \frac{1}{6} m l^2 \omega_2^2$

$$\omega_2^2 = \frac{3g}{l}$$

$$\omega_2 = \sqrt{\frac{3g}{l}}$$



$$\bar{a} = \frac{l}{2} \omega_2^2 = \frac{l}{2} \cdot \frac{3g}{l} = \frac{3}{2} g$$

$$+\uparrow \Sigma F = \Sigma (F)_m: A - W = m \bar{a}$$

$$A - mg = m \frac{3}{2} g$$

$$A = \frac{5}{2} mg$$

$$A = \frac{5}{2} W \uparrow$$

(b)  $W = 1.8 \text{ lb}$ ,  $l = 3 \text{ ft}$

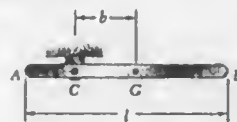
$$\omega_2^2 = \frac{3g}{l} = \frac{3 \cdot 32}{3} = 32.2$$

$$\omega_2 = 5.67 \text{ rad/s}$$

$$A = \frac{5}{2} W = \frac{5}{2} (1.8 \text{ lb})$$

$$A = 4.5 \text{ lb} \uparrow$$

17.19



GIVEN: ROD AB RELEASED  
 FROM REST. AS ROD PASSES  
 THROUGH VERTICAL,  
 FIND: (a) DISTANCE  $b$  FOR  
 WHICH  $\omega$  IS MAXIMUM.  
 (b) CORRESPONDING  $\omega$  AND  $C$

(1)  $\bar{v}_2 = b \omega_2$   
 $U_{1 \rightarrow 2} = Wb = mgb$   
 $T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} m (b \omega_2)^2 + \frac{1}{2} \left( \frac{1}{12} m l^2 \right) \omega_2^2 = \frac{1}{2} m \left[ b^2 + \frac{l^2}{12} \right] \omega_2^2$   
 $T_1 + U_{1 \rightarrow 2} = T_2$ :  $0 + mgb = \frac{1}{2} m \left[ b^2 + \frac{l^2}{12} \right] \omega_2^2$   

$$\omega_2^2 = 2g \left[ \frac{b}{b^2 + \frac{l^2}{12}} \right] \quad (1)$$

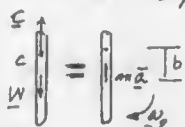
(a) MAXIMUM  $\omega_2$ :

$$\frac{d}{db} (\omega_2^2) = \frac{2g}{\left[ b^2 + \frac{l^2}{12} \right]^2} \left[ (b^2 + \frac{l^2}{12}) - b(2b) \right] = 0$$

$$\left[ -b^2 + \frac{l^2}{12} \right] = 0$$

$$b = \frac{l}{\sqrt{12}}$$

(b) EQ. 1:  $\omega_2^2 = 2g \left[ \frac{l/\sqrt{12}}{\frac{l^2}{12} + \frac{l^2}{12}} \right] = \sqrt{12} \frac{g}{l}$ ;  $(\omega_2)_{\max} = 1.86 \sqrt{\frac{g}{l}}$



$$\bar{a} = b \omega_2^2 = \frac{l}{\sqrt{12}} \cdot \sqrt{12} \frac{g}{l} = g \uparrow$$

$$+\uparrow \Sigma F = \Sigma (F)_m: C - W = m \bar{a}$$

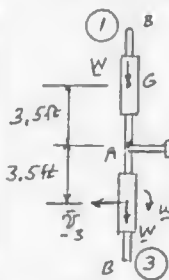
$$C = 2mg$$

$$C = 2W \uparrow$$

17.20



**GIVEN:**  
160-lb GYMNAST WITH  
 $\bar{r}_G = 1.5 \text{ ft}$   
HE IS ROTATING  
VERY SLOWLY ( $\omega_1 = 0$ )  
IN POSITION SHOWN.  
**FIND:**  $\omega$  AND  
FORCE EXERTED  
ON HIS HANDS  
AFTER HE HAS  
ROTATED THROUGH  
(a)  $90^\circ$ , (b)  $180^\circ$



$$\bar{I} = \frac{W}{g} \bar{r}_G^2 = \frac{160 \text{ lb}}{32.2} (1.5 \text{ ft})^2$$

$$\bar{I} = 11.18 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\bar{v}_2 = (3.5 \text{ ft}) \omega_2$$

$$\bar{v}_3 = (3.5 \text{ ft}) \omega_3$$

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$$

$$= \frac{1}{2} \frac{160}{32.2} (3.5 \omega)^2 + \frac{1}{2} (11.18) \omega^2$$

$$T = (30.435 + 5.59) \omega^2 = 36.025 \omega^2$$

(a)  $\theta = 90^\circ$ :

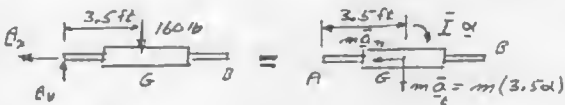
$$T_1 = 0; T_2 = 36.025 \omega_2^2$$

$$U_{1 \rightarrow 2} = W(3.5 \text{ ft}) = (160 \text{ lb})(3.5 \text{ ft}) = 560 \text{ ft} \cdot \text{lb}$$

$$T_1 + U_{1 \rightarrow 2} = T_2: 0 + 560 = 36.025 \omega_2^2$$

$$\omega_2^2 = 15.545$$

$$\omega_2 = 3.94 \text{ rad/s}$$



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}: (160)(3.5) = \frac{W}{g} (3.5 \alpha)(3.5) + \bar{I} \alpha$$

$$560 = \frac{160}{32.2} 3.5^2 \alpha + 11.18 \alpha$$

$$560 = 72.05 \alpha \quad \alpha = 7.772 \text{ rad/s}^2$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: A_y - 160 \text{ lb} = -m(3.5 \alpha)$$

$$A_y - 160 = -\frac{160}{32.2} (3.5)(7.772)$$

$$A_y - 160 = -135.17 \quad A_y = 24.837 \text{ lb} \uparrow$$

$$+\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: A_x = m \bar{a}_m = m(3.5 \text{ ft}) \omega_2^2$$

$$A_x = \frac{160}{32.2} (3.5)(15.545); \quad A_x = 270.3 \text{ lb} \leftarrow$$

$$A = 271 \text{ lb} \nearrow 5.2^\circ$$

$$\beta = \tan^{-1} \frac{24.837}{270.3} = 5.24^\circ$$

$$A = \frac{A_y}{\cos \beta} = \frac{270.3}{\cos 5.24^\circ} = 271.48 \text{ lb}$$

$$A = 271 \text{ lb} \nearrow 5.2^\circ$$

(CONTINUED)

17.20 continued

(b)  $\theta = 180^\circ$ :

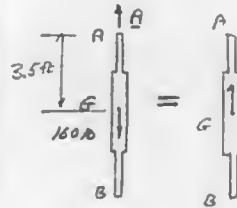
$$T_1 = 0; T_3 = 36.025 \omega_3^2$$

$$U_{1 \rightarrow 3} = W(2 \times 3.5 \text{ ft}) = (160 \text{ lb})(7 \text{ ft}) = 1120 \text{ ft} \cdot \text{lb}$$

$$T_1 + U_{1 \rightarrow 3} = T_3: 0 + 1120 = 36.025 \omega_3^2$$

$$\omega_3^2 = 31.09$$

$$\omega_3 = 5.58 \text{ rad/s}$$



$$m \bar{a}_m = m(3.5 \text{ ft}) \omega_3^2$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$$

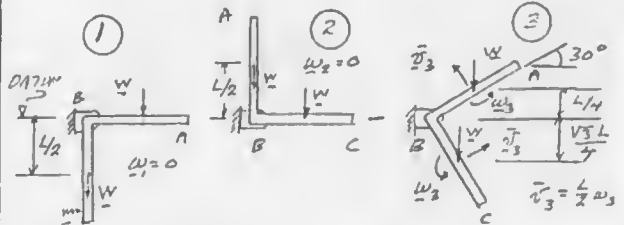
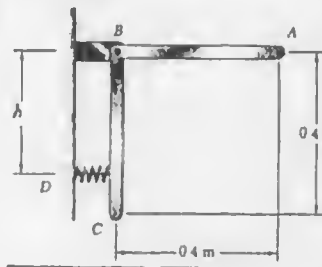
$$A - 160 \text{ lb} = \frac{160}{32.2} (3.5)(31.09)$$

$$A - 160 = 540.7$$

$$A = 700.7 \text{ lb} \quad A = 701 \text{ lb} \uparrow$$

17.21

**GIVEN:** TWO RODS EACH  
OF MASS  $m$  ARE WELDED  
TOGETHER AND PRESSED  
AGAINST SPRING AT D.  
AFTER RELEASE RODS  
ROTATE THROUGH MAX.  
ANGLE OF  $90^\circ$   
**FIND:** ANGULAR VELOCITY  
WHEN AB FORMS  $30^\circ$   
WITH HORIZONTAL.



$$\text{POSITION (1): } T_1 = 0, (V_G)_1, (V_G)_1 = -W \frac{L}{2}$$

$$\text{POSITION (2): } T_2 = 0, (V_G)_2 = 0, (V_G)_2 = +W \frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: 0 + (V_G)_1 - W \frac{L}{2} = 0 + W \frac{L}{2}$$

$$(V_G)_1 = WL$$

$$\text{POSITION (3): } (V_G)_3 = 0; (V_G)_3 = W \left( \frac{L}{4} \right) - W \left( \frac{\sqrt{3}L}{4} \right) = -0.183 WL$$

$$T_3 = 2 \left\{ \frac{1}{2} m \bar{v}_3^2 + \frac{1}{2} \bar{I} \omega_3^2 \right\}$$

$$= 2 \left\{ \frac{1}{2} \frac{W}{g} \left( \frac{1}{2} \omega_3 \right)^2 + \frac{1}{2} \left( \frac{1}{2} \frac{W}{g} L^2 \right) \omega_3^2 \right\} = \frac{1}{3} \frac{W}{g} L^2 \omega_3^2$$

$$T_1 + V_1 = T_3 + V_3:$$

$$0 + (V_G)_1 + (V_G)_1 = T_3 + (V_G)_3$$

$$0 + WL - \frac{1}{2} WL = \frac{1}{3} \frac{W}{g} L^2 \omega_3^2 - 0.183 WL$$

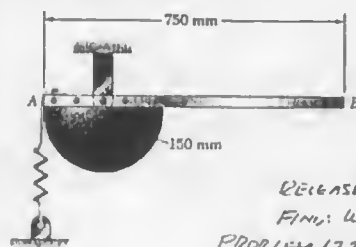
$$WL \left( 1 - \frac{1}{2} + 0.183 \right) = \frac{1}{3} \frac{W}{g} L^2 \omega_3^2$$

$$\omega_3^2 = 3(0.683) \frac{g}{L} = 2.049 \frac{g}{L} \quad \omega_3 = 1.431 \sqrt{\frac{g}{L}}$$

$$\text{For } L = 0.4 \text{ m; } \omega_3 = 1.431 \sqrt{\frac{9.81 \text{ m/s}^2}{0.4 \text{ m}}} = 7.086 \text{ rad/s}$$

$$\omega_3 = 7.09 \text{ rad/s}$$

# 17.22 and 17.23



GIVEN:  $m_{AB} = 6 \text{ kg}$   
1.8-kg SEMICIRCULAR DISK,  
SPRING OF  $k = 160 \text{ N/m}$   
UNSTRETCHED WHEN AB IS HORIZONTAL  
IF SYSTEM IS

RELEASED FROM REST,  
FIND:  $\omega$  AFTER  $90^\circ$  ROTATION

PROBLEM 17.22: WITH SPRING ATTACHED  
PROBLEM 17.23: SPRING REMOVED

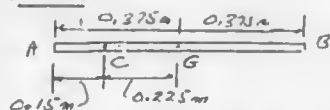
MOMENT OF INERTIA ABOUT C.

DISK:  $\bar{I}_C = \frac{4r}{3\pi} = \frac{4(0.15)}{3\pi} = 0.06366 \text{ m}$



$$I_C = \frac{1}{2} \pi r^2 = \frac{1}{2} (1.8 \text{ kg}) (0.15 \text{ m})^2 = 0.02025 \text{ kg} \cdot \text{m}^2$$

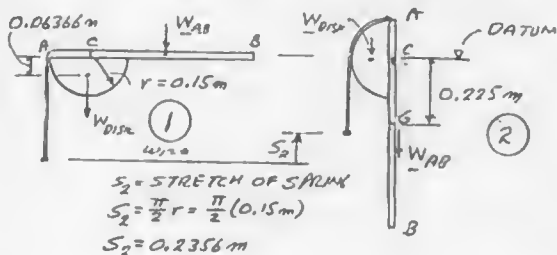
ROD AB:



$$I_C = \bar{I} + m \bar{r}^2 = \frac{1}{12} (6 \text{ kg}) (0.75 \text{ m})^2 + (6 \text{ kg}) (0.225 \text{ m})^2 = 0.28125 + 0.30375 = 0.585 \text{ kg} \cdot \text{m}^2$$

TOTAL  $I_C$  OF ASSEMBLY:

$$I_C = 0.02025 + 0.585 = 0.60525 \text{ kg} \cdot \text{m}^2$$



$S_2 = \text{STRETCH OF SPRING}$

$$S_2 = \frac{\pi}{2} r = \frac{\pi}{2} (0.15 \text{ m})$$

$$S_2 = 0.2356 \text{ m}$$

POSITION 1:  $T_1 = 0$ ,  $V_1 = W_{\text{DISK}}(-0.06366 \text{ m})$

$$V_1 = (1.8 \text{ kg})(9.81)(-0.06366) = -1.1188 \text{ J}$$

POSITION 2:  $(V_C)_2 = \frac{1}{2} k S_2^2 = \frac{1}{2} (160 \text{ N/m}) (0.2356 \text{ m})^2 = 4.44 \text{ J}$

$$(V_G)_2 = W_{AB}(-0.225 \text{ m}) = (6 \text{ kg})(9.81)(-0.225) = -13.24 \text{ J}$$

FOR NON CENTROIDAL ROTATION WE USE EQ. (17.10)

$$T_2 = \frac{1}{2} I_C \omega_2^2 = \frac{1}{2} (0.60525) \omega_2^2 = 0.3026 \omega_2^2$$

PROBLEM 17.22:  $T_1 + V_1 = T_2 + V_2$

$$0 - 1.1188 \text{ J} = 0.3026 \omega_2^2 + 4.44 \text{ J} - 13.24 \text{ J}$$

$$7.681 = 0.3026 \omega_2^2$$

$$\omega_2^2 = 25.38$$

$$\omega_2 = 5.04 \text{ rad/s}$$

PROBLEM 17.23: SPRING IS REMOVED, THUS

$(V_C)_2 = 4.44 \text{ J}$  IS REMOVED FROM POTENTIAL ENERGY IN POSITION 2. WE NOW WRITE

$$T_1 + V_1 = T_2 + V_2$$

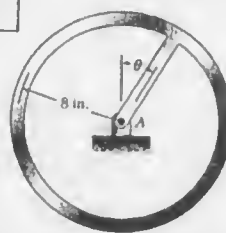
$$0 - 1.1188 \text{ J} = 0.3026 \omega_2^2 - 13.24 \text{ J}$$

$$12.121 = 0.3026 \omega_2^2$$

$$\omega_2^2 = 40.05$$

$$\omega_2 = 6.33 \text{ rad/s}$$

# 17.24



GIVEN: ASSEMBLY MADE OF  $\omega = 0.25 \text{ lb/ft}$  ROD. KNOWING THAT

$$\omega_{\min} = 0.8 \omega_{\max}$$

FIND: (a)  $\omega_{\max}$

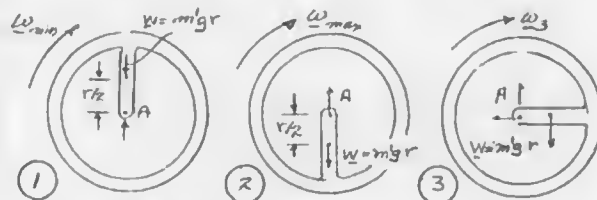
(b)  $\omega$  WHEN

$$G = 90^\circ$$

Denote mass per unit length by  $m'$  and radius by  $r$

$$I_A = I_{\text{rod}} + I_{\text{ring}} = \frac{1}{3} (m' l) r^2 + (2\pi r m') r^2 = 6.6165 m' r^3$$

FOR NON CENTROIDAL ROTATION THE KINETIC ENERGY OF THE ASSEMBLY IS  $\frac{1}{2} I_A \omega^2$



$$(a) \omega_{\min} = 0.8 \omega_{\max}$$

$$T_1 + V_1 = T_2 + V_2: \frac{1}{2} I_A \omega_{\min}^2 + m' g r \frac{r}{2} = \frac{1}{2} I_A \omega_{\max}^2 - m' g r \frac{r}{2}$$

$$\frac{1}{2} I_A (\omega_{\max}^2 - \omega_{\min}^2) = m' g r^2$$

$$\frac{1}{2} 6.6165 m' r^3 (1 - 0.8^2) \omega_{\max}^2 = m' g r^2$$

$$\omega_{\max}^2 = 0.83865 \frac{g}{r} = 0.83865 \frac{32.2 \text{ ft/s}^2}{(8/12 \text{ ft})} = 40.555$$

$$\omega_{\max} = 6.37 \text{ rad/s}$$

$$(b) T_2 + V_2 = T_3 + V_3:$$

$$\frac{1}{2} I_A \omega_{\max}^2 - m' g r \left(\frac{r}{2}\right) = \frac{1}{2} I_A \omega_3^2$$

$$\frac{1}{2} (6.6165 m' r^3) (0.83865 \frac{g}{r}) - \frac{m' g r^2}{2} = \frac{1}{2} (6.6165 m' r^3) \omega_3^2$$

$$2.7736 m' g r^2 - 0.5 m' g r^2 = 3.3162 m' r^3 \omega_3^2$$

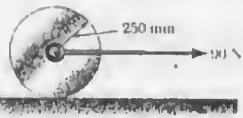
$$\omega_3^2 = \frac{2.2736}{3.3162} \frac{g}{r} = 0.6885 \frac{g}{r}$$

$$\omega_3^2 = 0.6885 \frac{32.2 \text{ ft/s}^2}{(8/12 \text{ ft})} = 33.26$$

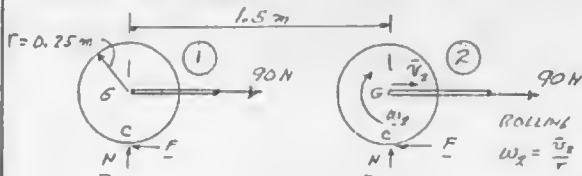
$$\omega_3 = 5.77 \text{ rad/s}$$

NOTE: RESULTS ARE INDEPENDENT OF WEIGHT PER UNIT LENGTH OF THE ROD USED TO MAKE THE ASSEMBLY.

17.25



GIVEN: 20-lb ROLLER  
ROLLS WITHOUT SLIPPING  
FIND: (a)  $\vec{v}$  AFTER 1.5 m motion.  
(b) FRICTION FORCE REQUIRED TO PREVENT SLIPPING.



INSTANT CENTER AT C; THUS  $F$  DOES NO WORK

$$T_1 = 0 \quad U_{1 \rightarrow 2} = (90 \text{ N})(1.5 \text{ m}) = 135 \text{ J}$$

$$T_2 = \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \left( \frac{\vec{v}_2}{r} \right)^2$$

$$(1) \quad T_2 = \frac{3}{4} m \vec{v}_2^2 = \frac{3}{4} (20 \text{ kg}) (\vec{v}_2^2) = 15 \vec{v}_2^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2: \quad 0 + 135 \text{ J} = 15 \vec{v}_2^2$$

$$\vec{v}_2^2 = 9 \quad \vec{v}_2 = 3 \text{ m/s} \rightarrow$$

(b) CONSIDER MOTION ABOUT MASS CENTER,

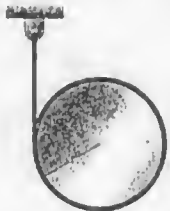
$$T_1 = 0 \quad T_2 = \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \left( \frac{\vec{v}_2}{r} \right)^2 = \frac{1}{4} m \vec{v}_2^2$$

$$U_{1 \rightarrow 2} = F(1.5 \text{ m})$$

$$T_1 + U_{1 \rightarrow 2} = T_2: \quad 0 + 1.5F = \frac{1}{4} m \vec{v}_2^2$$

$$1.5F = \frac{1}{2} (20 \text{ kg}) (3 \text{ m/s})^2: \quad F = 30 \text{ lb} \leftarrow$$

17.26 and 17.27



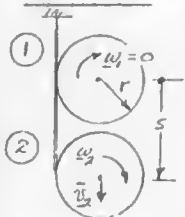
GIVEN: OBJECT SHOWN  
IS RELEASED FROM REST  
FIND:  $\vec{v}$  AFTER DOWNWARD  
MOVEMENT  $S$

PROBLEM 17.26

FOR A CYLINDER

PROBLEM 17.27

FOR A THIN-WALLED PIPE



$\bar{R}$  = RADIUS OF GYRATION

$$\vec{v} = r\omega \quad \omega = \frac{\vec{v}}{r}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$$

$$= \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} (m \bar{R}^2) \left( \frac{\vec{v}_2}{r} \right)^2$$

$$T_2 = \frac{1}{2} m \left( 1 + \frac{\bar{R}^2}{r^2} \right) \vec{v}_2^2 \quad U_{1 \rightarrow 2} = mgS$$

$$T_1 + U_{1 \rightarrow 2} = T_2: \quad 0 + mgS = \frac{1}{2} m \left( 1 + \frac{\bar{R}^2}{r^2} \right) \vec{v}_2^2$$

$$\vec{v}_2^2 = \frac{2gS}{1 + \frac{\bar{R}^2}{r^2}} \quad (1)$$

PROBLEM 17.26: CYLINDER

$$\bar{R}^2 = \frac{1}{2} r^2$$

$$\vec{v}_2^2 = \frac{2gS}{1 + \frac{1}{2}} = \frac{4gS}{3}$$

$$\vec{v}_2 = \sqrt{\frac{4gS}{3}} \downarrow$$

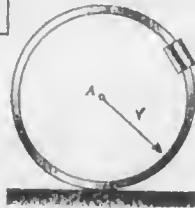
PROBLEM 17.27: THIN-WALLED PIPE

$$\bar{R}^2 = r^2$$

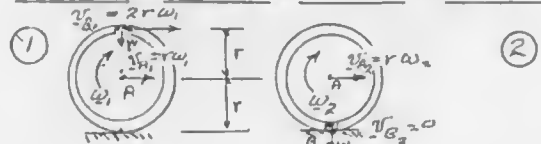
$$\vec{v}_2^2 = \frac{2gS}{1 + 1} = gS$$

$$\vec{v}_2 = \sqrt{gS} \downarrow$$

17.28



GIVEN: HOOP OF MASS  $m$   
ROLLS TO RIGHT, WITH  
COLLAR B OF MASS  $m$  AT  
TOP  $\omega = \omega_1$  AND AT  
BOTTOM  $\omega = 3\omega_1$ .  
FIND:  $\omega_1$  IN TERMS  
OF  $g$  AND  $r$ .



$$U_{1 \rightarrow 2} = W(2r) = mg(2r) = 2mgr$$

$$T_1 = \frac{1}{2} m \vec{v}_1^2 + \frac{1}{2} \bar{I} \omega_1^2 + \frac{1}{2} m \vec{v}_B^2$$

$$= \frac{1}{2} m (r\omega_1)^2 + \frac{1}{2} (mr^2) \omega_1^2 + \frac{1}{2} m (2r\omega_1)^2 = 3mr^2 \omega_1^2$$

$$T_2 = \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 + \frac{1}{2} m \vec{v}_B^2$$

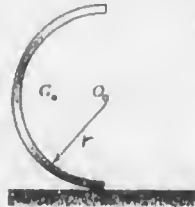
$$= \frac{1}{2} m (r\omega_2)^2 + \frac{1}{2} mr^2 \omega_2^2 + 0 = mr^2 \omega_2^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2: \quad 3mr^2 \omega_1^2 + 2mgr = mr^2 \omega_2^2$$

GIVEN:  $\omega_2 = 3\omega_1$   $3mr^2 \omega_1^2 + 2mgr = mr^2 (3\omega_1)^2$

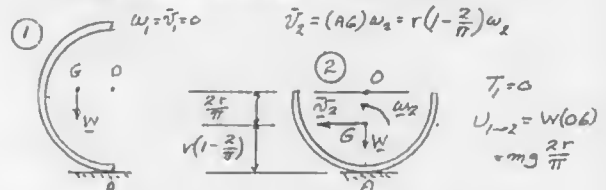
$$2mgr = 6mr^2 \omega_1^2; \quad \omega_1^2 = \frac{g}{3r}; \quad \omega_1 = \sqrt{\frac{g}{3r}} \leftarrow$$

17.29



GIVEN: HALF SECTION  
OF PIPE OF MASS  $m$ ,  
RELEASED FROM REST  
AFTER ROLLING THROUGH  
 $90^\circ$

FIND: (a)  $\omega$   
(b) REACTION



$$\bar{I} = mr^2 - m(0.6)^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2 \left( 1 - \frac{4}{\pi^2} \right)$$

$$T_2 = \frac{1}{2} m \vec{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} m \left( 1 - \frac{4}{\pi^2} \right) r^2 \omega_2^2 + \frac{1}{2} m r^2 \left( 1 - \frac{4}{\pi^2} \right) \omega_2^2$$

$$(a) = \frac{1}{2} m r^2 \left[ \left( 1 - \frac{4}{\pi^2} \right) + \left( 1 - \frac{4}{\pi^2} \right) \right] = \frac{1}{2} m r^2 \left( 2 - \frac{4}{\pi^2} \right)$$

$$T_1 + U_{1 \rightarrow 2} = T_2: \quad 0 + mg \frac{2r}{\pi} = \frac{1}{2} m r^2 \left( 2 - \frac{4}{\pi^2} \right) \omega_2^2$$

$$\omega_2^2 = \frac{2}{\pi \left( 1 - \frac{2}{\pi^2} \right)} \cdot \frac{g}{r} = 1.7519 \frac{g}{r} \quad \omega_2 = 1.324 \sqrt{\frac{g}{r}} \leftarrow$$

(b) KINEMATICS: SINCE O MOVES HORIZONTALLY,  $(a_O)_y = 0$

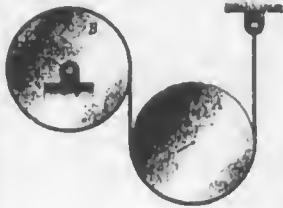
$$\vec{a}_O = (0.6) \omega_2^2 = \frac{2r}{\pi} \left( 1.7519 \frac{g}{r} \right) = 1.1153 g \uparrow$$

KINETICS:

$$\vec{W} = mg \quad \vec{a}_O = 1.1153 mg$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_e: \quad A - mg = 1.1153 mg; \quad A = 2.12 mg \leftarrow$$

# 17.30 and 17.31



GIVEN: 14-16 CYLINDERS OF 5-IN. RADIUS.

PROBLEM 17.30:

$(\omega_B)_1 = 30 \text{ rad/s}$

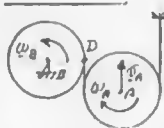
FIND: (a) DISTANCE A

WILL MOVE BEFORE  $(\omega_B)_2 = 5 \text{ rad/s}$

(b) TENSION IN CORD A-B

PROBLEM 17.31: SYSTEM IS RELEASED FROM REST

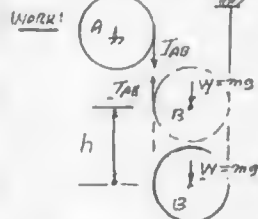
FIND: (a)  $\bar{v}_A$  AFTER 3 FT OF MOTION, (b) T IN CORD A-B



$$\begin{aligned} \bar{v}_D &= r\omega_B & \omega_A &= \frac{\bar{v}_D}{2r} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B \\ \bar{v}_A &= r\omega_A = \frac{1}{2}r\omega_B \end{aligned} \quad (1)$$

KINETIC ENERGY:  $T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}\bar{I}_B\omega_B^2$

$$T = \frac{1}{2}m\left(\frac{1}{2}r\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2 = \frac{7}{16}mr^2\omega_B^2$$



SINCE CORD IS INEXTENSIBLE, WORK IS DONE ONLY BY THE WEIGHT OF CYLINDER B

$$U_{1-2} = -Wh = -mgh$$

$$r = \frac{5}{12} \text{ ft}$$

PROBLEM 17.30:  $(\omega_B)_1 = 30 \text{ rad/s}$ ;  $(\omega_B)_2 = 5 \text{ rad/s}$

$$T_1 + U_{1-2} = T_2: \quad \frac{7}{16}mr^2(\omega_B)_1^2 - mgh = \frac{7}{16}mr^2(\omega_B)_2^2$$

$$h = \frac{7}{16} \frac{r}{g} [(\omega_B)_1^2 - (\omega_B)_2^2] \quad (2)$$

$$h = \frac{7}{16} \left( \frac{5}{12} \text{ ft} \right) \frac{(30)^2 - (5)^2}{32.2 \text{ ft/s}^2} = 2.064 \text{ ft} \quad h = 2.06 \text{ ft} \quad \blacktriangleleft$$

TENSION  $T_{AB}$ : WE NOTE THAT POINT D MOVE TWICE THE DISTANCE THAT A MOVES

$$U_{1-2} = -T_{AB}(2h)$$

FOR ONLY CYLINDER B,  $T = \frac{1}{2}\bar{I}_B\omega_B^2$

$$T_1 + U_{1-2} = T_2: \quad \frac{1}{2}\bar{I}_B(\omega_B)_1^2 - 2hT_{AB} = \frac{1}{2}\bar{I}_B(\omega_B)_2^2$$

$$T_{AB} = \frac{1}{4}\bar{I}_B \left[ \frac{(\omega_B)_1^2 - (\omega_B)_2^2}{h} \right] = \frac{1}{4} \left( \frac{1}{2}mr^2 \right) \left( \frac{7}{16} \frac{r}{g} [(\omega_B)_1^2 - (\omega_B)_2^2] \right)$$

$$T_{AB} = \frac{1}{8} \frac{16}{7} mg = \frac{2}{7} W = \frac{2}{7} (14 \text{ lb}) \quad T_{AB} = 4 \text{ lb} \quad \blacktriangleleft$$

NOTE:  $T_{AB}$  IS INDEPENDENT OF  $(\omega_B)_1$  AND  $(\omega_B)_2$

PROBLEM 17.31  $(\omega_B)_1 = 0$ ,  $h = 3 \text{ ft}$ ,  $r = \frac{5}{12} \text{ ft}$

SINCE  $h$  AND  $\bar{v}_A$  ARE NOW DOWNWARD,

$U_{1-2} = +Wh = +mgh$  AND EQ. 2 IS:

$$h = -\frac{7}{16} \frac{r^2}{g} [(\omega_B)_1^2 - (\omega_B)_2^2]$$

$$3 \text{ ft} = -\frac{7}{16} \frac{\left( \frac{5}{12} \text{ ft} \right)^2}{32.2 \text{ ft/s}^2} [0 - (\omega_B)_2^2]$$

$$(\omega_B)_2 = 127.18 \quad (\omega_B)_2 = 35.66 \text{ rad/s}$$

$$\text{EQ. (1)} \quad \bar{v}_A = \frac{1}{2}r(\omega_B)_2 = \frac{1}{2} \left( \frac{5}{12} \text{ ft} \right) (35.66 \text{ rad/s}) = 7.430 \text{ ft/s}$$

$$\bar{v}_A = 7.43 \text{ ft/s} \quad \blacktriangleleft$$

TENSION  $T_{AB}$ : SINCE  $T_{AB}$  IS INDEPENDENT OF VELOCITY, WE AGAIN HAVE

$$T_{AB} = 4 \text{ lb} \quad \blacktriangleleft$$

# 17.32

GIVEN:  $m_B = 5 \text{ kg}$

$m_A = 6 \text{ kg}$

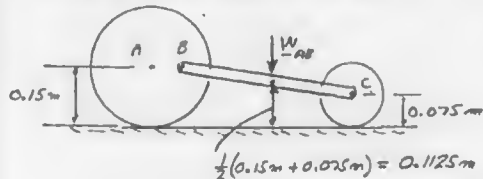
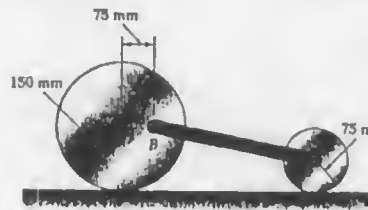
$m_C = 1.5 \text{ kg}$

SYSTEM IS RELEASED FROM REST

FIND:  $\bar{v}_B$  AFTER

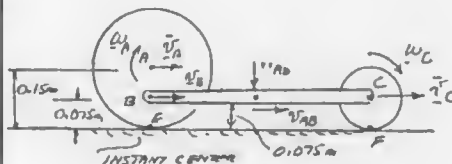
DISK A HAS

ROTATED  $90^\circ$



(1)

$$\frac{1}{2}(0.15 \text{ m} + 0.075 \text{ m}) = 0.1125 \text{ m}$$



(2)

$$\bar{v}_B = \bar{v}_{AB} \quad \omega_A = \frac{\bar{v}_B}{BE} = \frac{\bar{v}_{AB}}{0.075 \text{ m}} \quad \bar{v}_A = 2\bar{v}_B = 2\bar{v}_{AB}$$

$$\bar{v}_C = \bar{v}_{BC} \quad \omega_C = \frac{\bar{v}_C}{CF} = \frac{\bar{v}_{AB}}{0.075 \text{ m}}$$

$$U_{1-2} = W(0.1125 \text{ m} - 0.075 \text{ m}) = (5 \text{ kg})(9.8)(0.0375 \text{ m})$$

$$U_{1-2} = 1.8394 \text{ J}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_B\bar{v}_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2 + \frac{1}{2}\bar{I}_C\omega_C^2$$

$$= \frac{1}{2}[(6 \text{ kg})(2\bar{v}_{AB})^2 + \frac{1}{2}(6 \text{ kg})\left(\frac{\bar{v}_{AB}}{0.075 \text{ m}}\right)^2 + (5 \text{ kg})\bar{v}_{AB}^2 + (1.5 \text{ kg})(\bar{v}_{AB})^2 + \frac{1}{2}(1.5 \text{ kg})\left(\frac{\bar{v}_{AB}}{0.075 \text{ m}}\right)^2]$$

$$= \frac{1}{2}[24 + 12 + 5 + 1.5 + 0.75]\bar{v}_{AB}^2$$

$$T_2 = 21.625 \bar{v}_{AB}^2$$

WORK ENERGY

$$T_1 + U_{1-2} = T_2$$

$$0 + 1.8394 \text{ J} = 21.625 \bar{v}_{AB}^2$$

$$\bar{v}_{AB}^2 = 0.08506$$

$$\bar{v}_{AB} = 0.2916 \text{ m/s}$$

$$\bar{v}_{AB} = 292 \text{ mm/s} \rightarrow \quad \blacktriangleleft$$



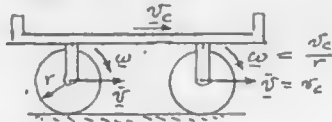
17.33



GIVEN: 9-kg CRADLE WITH  
6-kg WHEELS OF  $r = 80 \text{ mm}$   
INITIALLY AT REST

FIND:  $\vec{v}_C$  OF CRADLE  
AFTER 250 mm MOVEMENT

KINEMATICS:



$$T_1 = 0; T_2 = \frac{1}{2} m_C v_C^2 + 2 \left[ \frac{1}{2} m_W \bar{v}^2 + \frac{1}{2} I \omega^2 \right]$$

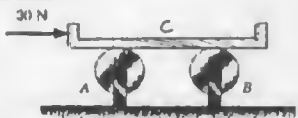
$$= \frac{1}{2} (9 \text{ kg}) v_C^2 + 2 \left[ \frac{1}{2} (6 \text{ kg}) v_C^2 + \frac{1}{2} \left( \frac{1}{2} (6 \text{ kg}) r^2 \right) \left( \frac{v_C}{r} \right)^2 \right] = 13.5 v_C^2$$

$$U_{1 \rightarrow 2} = (30 \text{ N})(0.25 \text{ m}) = 7.5 \text{ J}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; 0 + 7.5 \text{ J} = 13.5 v_C^2$$

$$v_C^2 = 0.5556 \quad v_C = 0.745 \text{ m/s} \rightarrow$$

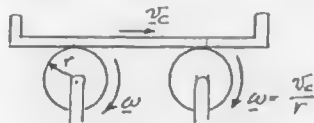
17.34



GIVEN: 9-kg CRADLE WITH  
6-kg WHEELS OF  $r = 80 \text{ mm}$   
INITIALLY AT REST

FIND:  $\vec{v}_C$  OF CRADLE  
AFTER 250 mm OF MOVEMENT

KINEMATICS



$$T_1 = 0; T_2 = \frac{1}{2} m_C v_C^2 + 2 \left[ \frac{1}{2} I \omega^2 \right]$$

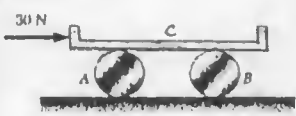
$$= \frac{1}{2} (9 \text{ kg}) v_C^2 + 2 \left[ \frac{1}{2} \left( \frac{1}{2} (6 \text{ kg}) r^2 \right) \left( \frac{v_C}{r} \right)^2 \right] = 7.5 v_C^2$$

$$U_{1 \rightarrow 2} = (30 \text{ N})(0.25 \text{ m}) = 7.5 \text{ J}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; 0 + 7.5 \text{ J} = 7.5 v_C^2$$

$$v_C^2 = 1.000 \quad v_C = 1.000 \text{ m/s} \rightarrow$$

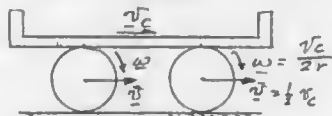
17.35



GIVEN: 9-kg CRADLE WITH  
6-kg WHEELS OF  $r = 80 \text{ mm}$   
INITIALLY AT REST

FIND:  $\vec{v}_C$  OF CRADLE  
AFTER 250 mm OF MOVEMENT

KINEMATICS:



$$T_1 = 0; T_2 = \frac{1}{2} m_C v_C^2 + 2 \left[ \frac{1}{2} m_W \bar{v}^2 + \frac{1}{2} I \omega^2 \right]$$

$$= \frac{1}{2} (9 \text{ kg}) v_C^2 + 2 \left[ \frac{1}{2} (6 \text{ kg}) \left( \frac{1}{2} v_C \right)^2 + \frac{1}{2} \left( \frac{1}{2} (6 \text{ kg}) r^2 \right) \left( \frac{v_C}{2r} \right)^2 \right]$$

$$T_2 = 6.75 v_C^2$$

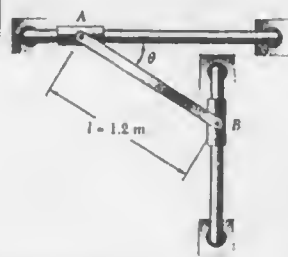
$$U_{1 \rightarrow 2} = (30 \text{ N})(0.25 \text{ m}) = 7.5 \text{ J}$$

$$T_1 + U_{1 \rightarrow 2} = T_2; 0 + 7.5 \text{ J} = 6.75 v_C^2$$

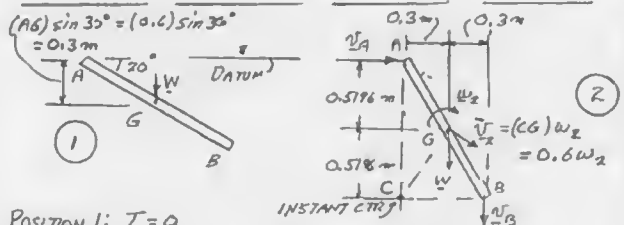
$$v_C^2 = 1.111$$

$$v_C = 1.054 \text{ m/s} \rightarrow$$

17.36



GIVEN:  $m = 10 \text{ kg}$   
ROD RELEASED  
FROM REST  
WHEN  $\theta = 30^\circ$   
FIND:  $\vec{v}_A$  AND  $\vec{v}_B$   
WHEN  $\theta = 60^\circ$

POSITION 1:  $T_1 = 0$ 

$$V_1 = -W(0.3 \text{ m}) = -(10 \text{ kg})(9.81)(0.3) = -29.43 \text{ J}$$

POSITION 2:  $V_2 = -W(0.5196 \text{ m}) = -(10 \text{ kg})(9.81)(0.5196) = -50.9774 \text{ J}$ 

$$T_2 = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (10 \text{ kg}) (0.6 \omega_2)^2 + \frac{1}{2} \left( \frac{1}{12} (10 \text{ kg}) (1.2 \text{ m})^2 \right) \omega_2^2 = 2.4 \omega_2^2$$

$$T_1 + V_1 = T_2 + V_2; 0 - 29.43 \text{ J} = 2.4 \omega_2^2 - 50.9774$$

$$\omega_2^2 = 8.9768 \quad \omega_2 = 2.996 \text{ rad/s}$$

VELOCITY OF COLLARS WHEN  $\theta = 60^\circ$ 

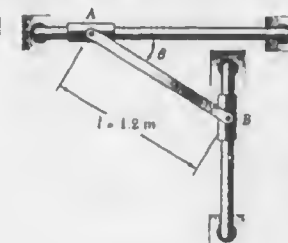
$$\vec{v}_A = (AC) \omega_2 = (2.105196 \text{ m})(2.996 \text{ rad/s})$$

$$\vec{v}_A = 3.11 \text{ m/s} \rightarrow$$

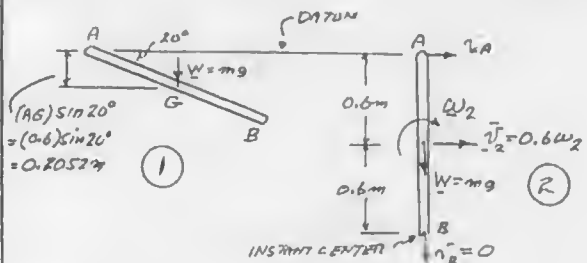
$$\vec{v}_B = (BC) \omega_2 = (2 \times 0.3 \text{ m})(2.996 \text{ rad/s})$$

$$\vec{v}_B = 1.798 \text{ m/s} \downarrow$$

17.37



GIVEN:  $m = 10 \text{ kg}$   
ROD RELEASED  
FROM REST  
WHEN  $\theta = 20^\circ$   
FIND:  $\vec{v}_A$  AND  $\vec{v}_B$   
WHEN  $\theta = 90^\circ$

POSITION 1:  $T_1 = 0$ 

$$V_1 = -W(0.2052 \text{ m}) = -m g (0.2052)$$

POSITION 2:  $V_2 = -W(0.6 \text{ m}) = -m g (0.6)$ 

$$T_2 = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m (0.6 \omega_2)^2 + \frac{1}{2} \left( \frac{1}{12} m (1.2)^2 \right) \omega_2^2$$

$$T_2 = 0.24 m \omega_2^2$$

$$T_1 + V_1 = T_2 + V_2; 0 - 0.2052 m g = 0.24 m \omega_2^2 - 0.6 m g$$

$$\omega_2^2 = 1.645 g = 1.645 (9.81) = 16.137$$

$$\omega_2 = 4.017 \text{ rad/s}$$

VELOCITY OF COLLARS WHEN  $\theta = 90^\circ$ 

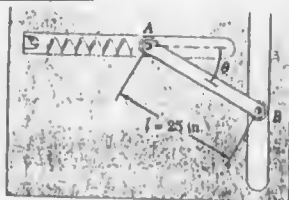
$$\vec{v}_A = (AB) \omega_2 = (1.2 \text{ m})(4.017 \text{ rad/s})$$

$$\vec{v}_A = 4.82 \text{ m/s} \rightarrow$$

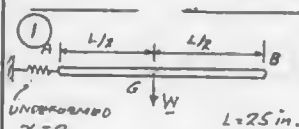
$$\vec{v}_B = 0$$

$$\vec{v}_B = 0$$

17.38



GIVEN:  $W_{AB} = 9/16$ ,  $k = 3 \text{ lb/in}$   
 SPRING TENSION IS ZERO  
 WHEN  $\theta = 0$ .  
 ROD IS RELEASED FROM  
 REST WHEN  $\theta = 0$ .  
 FIND:  $\omega$  AND  $v_B$  WHEN  
 $\theta = 30^\circ$



POSITION 1:  $T_1 = 0$ ,  $V_1 = 0$   
 POSITION 2:  $V_2 = -W \frac{L}{4} + \frac{1}{2} \omega x_2$

$x_2 = L - L \cos 30^\circ = (25 \text{ in})(1 - \cos 30^\circ)$   
 $x_2 = 3.349 \text{ in}$

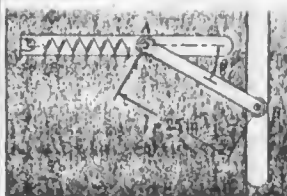
$T_2 = \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m \left( \frac{L}{4} \omega \right)^2 + \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega^2$   
 $= \frac{1}{8} m L^2 \omega^2 = \frac{1}{8} \left( \frac{9/16}{32.2} \right) \left( \frac{25}{12} \right)^2 \omega^2 = 0.2022 \omega^2$

$T_1 + V_1 = T_2 + V_2$ :  $0 + 0 = 0.2022 \omega^2 - 3.285 \text{ ft} \cdot \text{lb}$

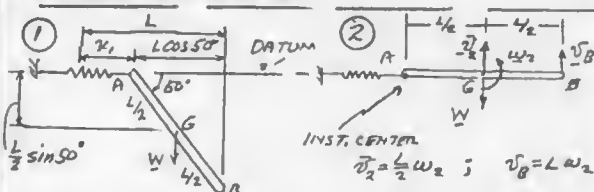
$\omega^2 = 16.25$   $\omega = 4.03$   $\omega = 4.03 \text{ rad/s}$

VELOCITY OF B:  $v_B = (L \cos 30^\circ) \omega = \left( \frac{25}{12} \right) \cos 30^\circ (4.03)$   
 $v_B = 7.27 \text{ ft/s}$

17.39



GIVEN:  $W_{AB} = 9/16$ ,  $k = 3 \text{ lb/in}$ .  
 SPRING TENSION IS  
 ZERO WHEN  $\theta = 0$ .  
 ROD IS RELEASED FROM  
 REST WHEN  $\theta = 50^\circ$ .  
 FIND:  $\omega$  AND  $v_B$  WHEN  
 $\theta = 0$



$x_1 = L - L \cos 50^\circ = (25 \text{ in})(1 - \cos 50^\circ) = 8.9303 \text{ in}$

POSITION 1:  $V_1 = -W \frac{L}{2} \sin 50^\circ + \frac{1}{2} k x_1^2$

$V_1 = -(9/16) \left( \frac{25}{2} \right) \sin 50^\circ + \frac{1}{2} (3 \text{ lb/in}) (8.9303 \text{ in})^2$   
 $= -86.18 + 119.63 = 33.45 \text{ in} \cdot \text{lb} = 2.787 \text{ ft} \cdot \text{lb}$

$T_1 = 0$

POSITION 2:  $V_2 = (V_G)_2 + (V_B)_2 = 0$

$T_2 = \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m \left( \frac{L}{2} \omega \right)^2 + \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega^2$   
 $= \frac{1}{8} m L^2 \omega^2 = \frac{1}{8} \left( \frac{9/16}{32.2} \right) \left( \frac{25}{12} \right)^2 \omega^2 = 0.2022 \omega^2$

$T_1 + V_1 = T_2 + V_2$ :  $0 + 2.787 \text{ ft} \cdot \text{lb} = 0.2022 \omega^2$

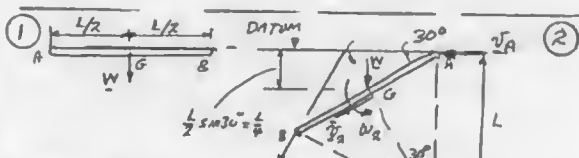
$\omega^2 = 13.7849$   $\omega = 3.713 \text{ rad/s}$   $\omega = 3.71 \text{ rad/s}$

VELOCITY OF B:  $v_B = L \omega = \left( \frac{25 \text{ in}}{12} \right) (3.713 \text{ rad/s}) = 7.735 \text{ ft/s}$   
 $v_B = 7.74 \text{ ft/s}$

17.40



GIVEN: ROD IS  
 RELEASED FROM  
 REST WHEN  $\theta = 0$   
 FIND:  $v_A$  AND  $v_B$   
 WHEN  $\theta = 30^\circ$



NOTE: FOR  $\theta = 30^\circ$ ,

$\triangle ABC$  IS EQUILATERAL.

IN  $\triangle AGC$ :  $CG = L \cos 30^\circ$ ,  $\vec{r}_2 = L \omega \cos 30^\circ$

POSITION 1:  $V_1 = 0$ ,  $T_1 = 0$

POSITION 2:  $V_2 = -W \frac{L}{4} = -\frac{1}{4} m g L$

$T_2 = \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m (L \omega \cos 30^\circ)^2 + \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega^2$

$T_2 = \left( \frac{1}{2} \cos^2 30^\circ + \frac{1}{12} \right) m L^2 \omega^2 = \left( \frac{3}{8} + \frac{1}{12} \right) m L^2 \omega^2 = \frac{5}{12} m L^2 \omega^2$

$T_1 + V_1 = T_2 + V_2$ :  $0 + 0 = -\frac{1}{4} m g L + \frac{5}{12} m L^2 \omega^2$

$\omega^2 = 0.6 \frac{g}{L}$   $\omega = \sqrt{0.6 \frac{g}{L}}$

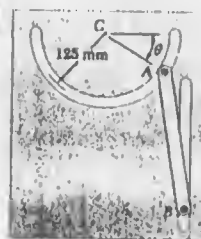
$v_A = (AC) \omega = L (\sqrt{0.6 \frac{g}{L}})$

$v_A = \sqrt{0.6 g L}$

$v_B = (BC) \omega = L (\sqrt{0.6 \frac{g}{L}})$

$v_B = \sqrt{0.6 g L} \angle 60^\circ$

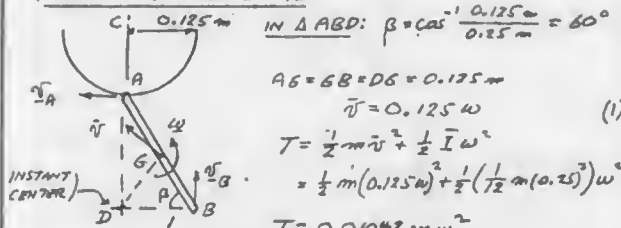
17.41



GIVEN: 250-mm ROD AB  
 IS RELEASED WHEN  
 $\theta = 0$

FIND:  $v_B$  WHEN  $\theta = 90^\circ$

KINEMATICS WHEN  $\theta = 90^\circ$



$AG = BG = DG = 0.125 \text{ m}$

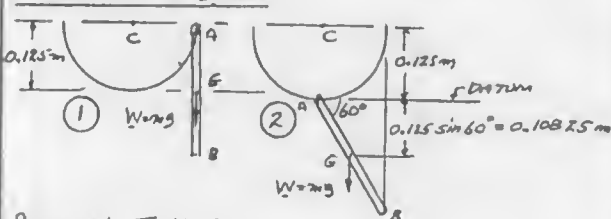
$\vec{r} = 0.125 \omega$

$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \omega^2$

$= \frac{1}{2} m (0.125 \omega)^2 + \frac{1}{2} \left( \frac{1}{12} m (0.25)^2 \right) \omega^2$

$T = 0.01042 m \omega^2$

CONSERVATION OF ENERGY



POSITION 1:  $T_1 = V_1 = 0$

POSITION 2:  $V_2 = -m g (0.10825)$

$T_2 = 0.01042 m \omega^2$

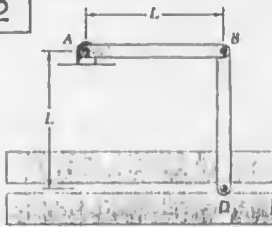
$T_1 + V_1 = T_2 + V_2$ :  $0 + 0 = -m g (0.10825) + 0.01042 m \omega^2$

$\omega^2 = 10.389 g = 10.389 (9.81) = 101.916$

VELOCITY OF B WHEN  $\theta = 90^\circ$   $\omega = 10.095 \text{ rad/s}$

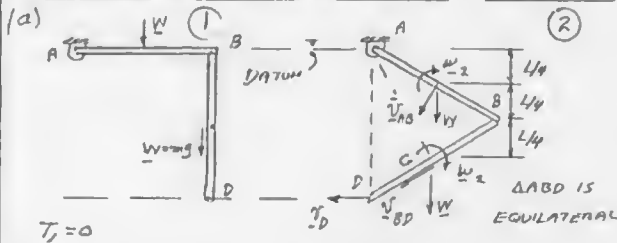
EO(1):  $v_B = 0.125 \omega = 0.125 (10.095)$   $v_B = 1.262 \text{ m/s}$

17.42



GIVEN: IDENTICAL RODS  
RELEASED FROM  
POSITION SHOWN WITH  
D MOVED SLIGHTLY TO  
THE LEFT.

FIND:  $\underline{v}_D$  WHEN  
(a) D IS BELOW A,  
(b) AB IS VERTICAL.



IN POSITION 2 POINT A IS THE INSTANTANEOUS CENTER  
OF BOTH AB AND BD. ANGULAR VELOCITY OF EACH ROD IS  $\omega_2$ .  
 $AG_{BD} = 0.866L$ ,  $\underline{v}_{BD} = (AG_{BD})\omega_2$ ,  $\underline{v}_{AB} = \frac{L}{2}\omega_2$

$$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m\bar{v}_{BD}^2 + \frac{1}{2}\bar{I}_{BD}\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 + \frac{1}{2}m(0.866L\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$T_2 = \frac{7}{12}mL^2\omega_2^2$$

$$V_1 = -\frac{1}{2}mgL$$

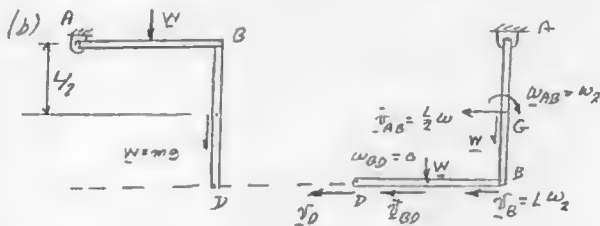
$$V_2 = -mg\frac{L}{4} - mg\frac{3L}{4} = -mgL$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2: 0 - \frac{1}{2}mgL = \frac{7}{12}mL^2\omega_2^2 - mgL$$

$$\omega_2^2 = \frac{6}{7}gL \quad \omega_2 = 0.9258\sqrt{gL}$$

$$\underline{v}_D = (AD)\omega_2 = L(0.9258\sqrt{gL}) \quad \underline{v}_D = 0.926\sqrt{gL} \quad \leftarrow$$



IN POSITION 2:  $\underline{v}_D = \underline{v}_A = \underline{v}_{BD} = L\omega_2 \leftarrow$ , AND  $\omega_2 = 0$

POSITION 1:  $T_1 = 0$ ,  $V_1 = -mg\frac{L}{2}$

POSITION 2:  $V_2 = -mg\frac{L}{2} - mgL = -\frac{3}{2}mgL$

$$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m\bar{v}_{BD}^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 + \frac{1}{2}m(L\omega_2)^2$$

$$T_2 = \frac{3}{2}mL^2\omega_2^2$$

CONSERVATION OF ENERGY

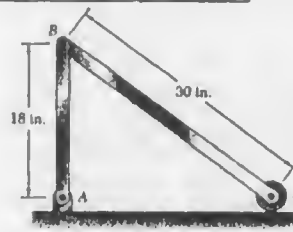
$$T_1 + V_1 = T_2 + V_2: 0 - mg\frac{L}{2} = \frac{3}{2}mL^2\omega_2^2 - \frac{3}{2}mgL$$

$$mgL = \frac{3}{2}mL^2\omega_2^2$$

$$\omega_2^2 = \frac{2}{3}gL \quad \omega_2 = 1.225\sqrt{gL}$$

$$\underline{v}_D = L\omega_2 = L(1.225\sqrt{gL}) \quad \underline{v}_D = 1.225\sqrt{gL} \quad \leftarrow$$

17.43 and 17.44



GIVEN:  $\underline{v}_{AB} = 2.4$  ft/s

$\underline{v}_{BC} = 4$  ft/s

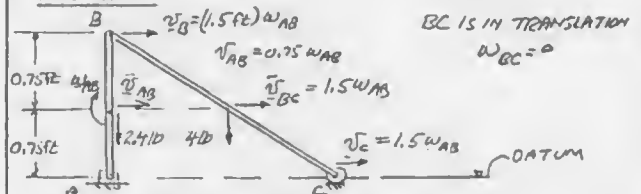
FIND:  $\underline{v}_B$  AFTER AB HAS  
ROTATED THROUGH  $90^\circ$

PROBLEM 17.43

IF WHEEL IS MOVED  
SLIGHTLY TO RIGHT AND  
RELEASED.

PROBLEM 17.44: IF IN  
POSITION SHOWN  $\underline{v}_C = 6$  ft/s  $\rightarrow$

POSITION 1:



$$V_1 = (2.4)(0.75\text{ ft}) + (4)(0.75\text{ ft}) = 4.8\text{ ft}\cdot\text{lb} \quad (1)$$

IF  $\underline{v}_C = 0$ , THEN  $\omega_{AB} = 0$  AND  $T_1$

IF  $\underline{v}_C = 6$  ft/s,  $6\text{ ft/s} = (1.5\text{ ft})\omega_{AB}$   $\omega_{AB} = 4\text{ rad/s}$

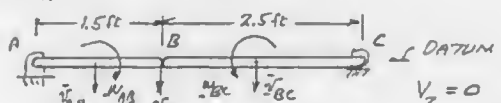
$$T_1 = \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}_{BC}^2$$

$$T_1 = \frac{1}{2}\left(\frac{2.4}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{2.4}{g}(1.5)^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{4}{g}\right)(1.5\omega_{AB})^2$$

$$T_1 = 0.675\frac{\omega_{AB}^2}{g} + 0.225\frac{\omega_{AB}^2}{g} + 4.5\frac{\omega_{AB}^2}{g} = 5.4\frac{\omega_{AB}^2}{g}$$

$$\text{For } \underline{v}_C = 6\text{ ft/s} \rightarrow T_1 = 5.4\frac{(4\text{ rad/s})^2}{32.2} = 2.683\text{ ft}\cdot\text{lb} \quad (2)$$

POSITION 2:



$$\underline{v}_{AB} = 0.75\omega_{AB} \downarrow \quad \underline{v}_B = 1.5\omega_{AB} \downarrow \quad (3)$$

$$\underline{v}_{BC} = \frac{1}{2}\underline{v}_C = 0.75\omega_{AB} \downarrow \quad \omega_{BC} = \frac{v_B}{2.5} = \frac{1.5\omega_{AB}}{2.5} = 0.6\omega_{AB}$$

$$T_2 = \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}_{BC}^2 + \frac{1}{2}\bar{I}_{BC}\omega_{BC}^2$$

$$= \frac{1}{2}\left(\frac{2.4}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{2.4}{g}(1.5)^2\right)\omega_{AB}^2$$

$$+ \frac{1}{2}\left(\frac{4}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{4}{g}(2.5)^2\right)(0.6\omega_{AB})^2$$

$$T_2 = (0.675 + 0.225 + 1.125 + 0.375)\frac{\omega_{AB}^2}{g} = \frac{2.4}{32.2}\omega_{AB}^2 = 0.07453\omega_{AB}^2$$

PROBLEM 17.43  $\underline{v}_C = 0$ ,  $T_1 = 0$ ,

$$\text{EQ(1)} \quad V_1 = 4.8\text{ ft}\cdot\text{lb}, \quad V_2 = 0 \quad T_2 = 0.07453\omega_{AB}^2$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + 4.8\text{ ft}\cdot\text{lb} = 0.07453\omega_{AB}^2 + 0$$

$$\omega_{AB}^2 = 64.4 \quad \omega_{AB} = 8.025$$

$$\text{EQ(2)}: \underline{v}_B = 1.5\omega_{AB} = 1.5(8.025) \quad \underline{v}_B = 12.04\text{ ft/s} \downarrow \quad \leftarrow$$

PROBLEM 17.44  $\underline{v}_C = 6$  ft/s  $\rightarrow$

$$\text{EQ(2)}: T_1 = 2.683\text{ ft}\cdot\text{lb} \quad T_2 = 0.07453\omega_{AB}^2$$

$$\text{EQ(1)}: V_1 = 4.8\text{ ft}\cdot\text{lb} \quad V_2 = 0$$

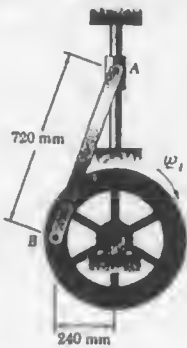
$$T_1 + V_1 = T_2 + V_2 \quad 2.683\text{ ft}\cdot\text{lb} + 4.8\text{ ft}\cdot\text{lb} = 0.07453\omega_{AB}^2 + 0$$

$$7.483 = 0.07453\omega_{AB}^2$$

$$\omega_{AB}^2 = 100.4 \quad \omega_{AB} = 10.02\text{ rad/s}$$

$$\text{EQ(3)}: \underline{v}_B = 1.5\omega_{AB} = 1.5(10.02) \quad \underline{v}_B = 15.03\text{ ft/s} \downarrow \quad \leftarrow$$

# 17.45 and 17.46



GIVEN:  $m_{AB} = 4 \text{ kg}$   
 $m_{\text{WHEEL}} = 16 \text{ kg}$   
 $R = 180 \text{ mm}$

PROBLEM 17.45:

IF  $\omega_1 = 60 \text{ rpm}$ ,

FIND:  $\omega_2$  WHEN B IS DIRECTLY BELOW C

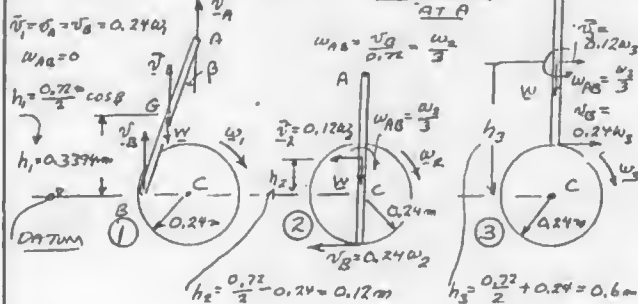
PROBLEM 17.46:

FIND:  $\omega_1$  SO ANGULAR VELOCITY IS THE SAME IN POSITION SHOWN AND WHEN B IS DIRECTLY ABOVE C.

KINEMATICS

$$\beta = \sin^{-1} \frac{.24}{.72} = 19.47^\circ$$

INST. CENTER AT A



POTENTIAL ENERGY:

$$V_1 = Wh_1 = mg h_1 = (4 \text{ kg})(9.81)(0.3374 \text{ m}) = 13.318 \text{ J}$$

$$V_2 = Wh_2 = mg h_2 = (4 \text{ kg})(9.81)(0.12 \text{ m}) = 4.709 \text{ J}$$

$$V_3 = Wh_3 = mg h_3 = (4 \text{ kg})(9.81)(0.6 \text{ m}) = 23.544 \text{ J}$$

KINETIC ENERGY

$$\bar{I}_C = I_{\text{WHEEL}} = (16 \text{ kg})(0.18 \text{ m})^2 = 0.5184 \text{ kg} \cdot \text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{2} m_{AB} L^2 = \frac{1}{2} (4 \text{ kg})(0.72 \text{ m})^2 = 0.1728 \text{ kg} \cdot \text{m}^2$$

$$T_1 = \frac{1}{2} \bar{I}_C \omega_1^2 + \frac{1}{2} m_{AB} \bar{v}_1^2 = \frac{1}{2} (0.5184) \omega_1^2 + \frac{1}{2} (4) (0.24 \omega_1)^2$$

$$= 0.2592 \omega_1^2 + 0.1152 \omega_1^2 \quad T_1 = 0.3744 \omega_1^2$$

$$T_2 = \frac{1}{2} \bar{I}_C \omega_2^2 + \frac{1}{2} m_{AB} \bar{v}_2^2 + \frac{1}{2} \bar{I}_{AB} \omega_2^2$$

$$= \frac{1}{2} (0.5184) \omega_2^2 + \frac{1}{2} (4) (0.12 \omega_2)^2 + \frac{1}{2} (0.1728) \left( \frac{\omega_2}{3} \right)^2$$

$$= 0.2592 \omega_2^2 + 0.0288 \omega_2^2 + 0.0036 \omega_2^2 \quad T_2 = 0.2916 \omega_2^2$$

$$T_3 = \text{SAME COEFFICIENT AS } T_2: \quad T_3 = 0.2916 \omega_3^2$$

PROBLEM 17.45: POSITION 1 TO POSITION 2

$$\omega_1 = 60 \text{ rpm} \left( \frac{2\pi}{60} \right) = 2\pi \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0.3744 \omega_1^2 + 13.318 \text{ J} = 0.2916 \omega_2^2 + 4.709 \text{ J}$$

$$0.3744 (2\pi)^2 + 13.318 = 0.2916 \omega_2^2 + 4.709$$

$$0.2916 \omega_2^2 = 23.39 \quad \omega_2^2 = 79.80 \text{ rad}^2/\text{s}^2$$

$$\omega_2 = 8.934 \text{ rad/s} \left( \frac{60}{2\pi} \right) \quad \omega_2 = 84.7 \text{ rpm}$$

PROBLEM 17.46: POSITION 1 TO POSITION 3 WITH  $\omega_1 = \omega_3$

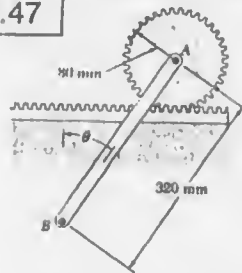
$$T_1 + V_1 = T_3 + V_3: \quad 0.3744 \omega_1^2 + 13.318 \text{ J} = 0.2916 \omega_3^2 + 23.544 \text{ J}$$

$$0.3744 \omega_1^2 + 13.318 = 0.2916 \omega_3^2 + 23.544$$

$$0.0768 \omega_1^2 = 10.226 \quad \omega_1^2 = 133.2$$

$$\omega_1 = 11.54 \text{ rad/s} \left( \frac{60}{2\pi} \right) \quad \omega_1 = 110.2 \text{ rpm}$$

# 17.47



GIVEN:

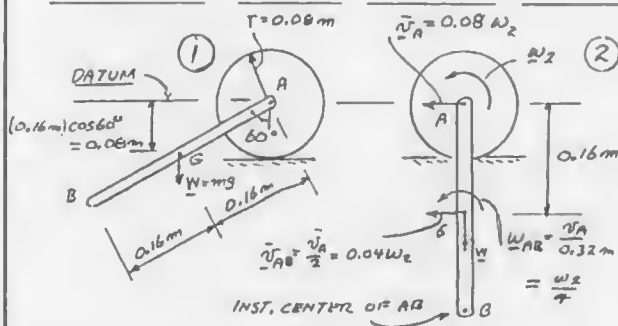
5-kg GEAR,  $\bar{R} = 60 \text{ mm}$

4-kg ROD AB

SYSTEM IS RELEASED FROM REST WHEN  $\theta = 60^\circ$

FIND:

$\bar{v}_A$  WHEN  $\theta = 0$



POSITION 1:  $T_1 = 0$

$$V_1 = -W(0.08 \text{ m}) = -(4 \text{ kg})(9.81)(0.08) = -3.139 \text{ J}$$

POSITION 2:  $T_2 = \frac{1}{2} m_A \bar{v}_A^2 + \frac{1}{2} \bar{I}_A \omega_2^2 + \frac{1}{2} m_B \bar{v}_B^2 + \frac{1}{2} \bar{I}_B \omega_2^2$

$$T_2 = \frac{1}{2} (5 \text{ kg})(0.08 \omega_2)^2 + \frac{1}{2} ((5 \text{ kg})(0.08 \text{ m})^2) \omega_2^2 + \frac{1}{2} (4 \text{ kg})(0.04 \omega_2)^2 + \frac{1}{2} ((4 \text{ kg})(0.22 \text{ m})^2) \omega_2^2$$

$$T_2 = 0.006 \omega_2^2 + 0.009 \omega_2^2 + 0.0032 \omega_2^2 + 0.00107 \omega_2^2 = 0.02927 \omega_2^2$$

$$V_2 = -W(0.16 \text{ m}) = -(4 \text{ kg})(9.81)(0.16) = -6.278 \text{ J}$$

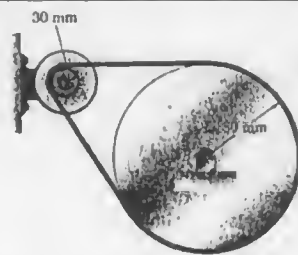
$$T_1 + V_1 = T_2 + V_2: \quad 0 - 3.139 \text{ J} = 0.02927 \omega_2^2 - 6.278 \text{ J}$$

$$\omega_2^2 = 107.26 \quad \omega_2 = 10.357 \text{ rad/s}$$

$$\text{VELOCITY OF A: } \bar{v}_A = 0.08 \omega_2 = 0.08 (10.357) = 0.829 \text{ m/s}$$

$$\bar{v}_A = 829 \text{ mm/s}$$

# 17.48



GIVEN:

$\omega_A = 22.5 \text{ Hz}$

MOTOR

DEVELOPS 3 kW

FIND:

(a)  $M_A$

(b)  $M_B$

$$\omega_A = 22.5 \text{ Hz} \left( \frac{2\pi \text{ rad}}{\text{cycle}} \right) = 45\pi \text{ rad/s}$$

$$r_A \omega_A = r_B \omega_B: \quad (0.03 \text{ m})(45\pi \text{ rad/s}) = (0.180 \text{ m}) \omega_B$$

$$\omega_B = 7.5\pi \text{ rad/s}$$

(a) PULLEY A: Power =  $M_A \omega_A$

$$3000 \text{ W} = M_A (45\pi \text{ rad/s})$$

$$M_A = 21.2 \text{ N} \cdot \text{m}$$

(b) PULLEY B: Power =  $M_B \omega_B$

$$3000 \text{ W} = M_B (7.5\pi \text{ rad/s})$$

$$M_B = 127.3 \text{ N} \cdot \text{m}$$

17.49

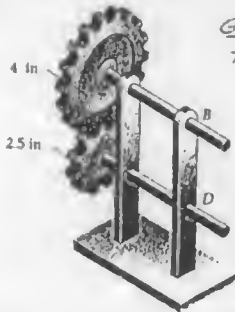
GIVEN: MAXIMUM COUPLE THAT CAN BE APPLIED TO A SHAFT IS  $15.5 \text{ kip}\cdot\text{in.}$   
 FIND: MAXIMUM HORSEPOWER THAT CAN BE TRANSMITTED AT (a)  $180 \text{ rpm}$ , (b)  $480 \text{ rpm}$ .

$$M = 15.5 \text{ kip}\cdot\text{in.} = 1.2917 \text{ kip}\cdot\text{ft} = 1291.7 \text{ lb}\cdot\text{ft}$$

(a)  $\omega = 180 \text{ rpm} \left( \frac{2\pi}{60} \right) = 6\pi \text{ rad/s}$   
 $\text{Power} = M\omega = (1291.7 \text{ lb}\cdot\text{ft})(6\pi \text{ rad/s}) = 24,348 \frac{\text{ft}\cdot\text{lb}}{\text{s}}$   
 $\text{Horsepower} = \frac{24,348}{550} = 44.3 \text{ hp}$

(b)  $\omega = 480 \text{ rpm} \left( \frac{2\pi}{60} \right) = 16\pi \text{ rad/s}$   
 $\text{Power} = M\omega = (1291.7 \text{ lb}\cdot\text{ft})(16\pi \text{ rad/s}) = 64,930 \frac{\text{ft}\cdot\text{lb}}{\text{s}}$   
 $\text{Horsepower} = \frac{64,930}{550} = 118.1 \text{ hp}$

17.50



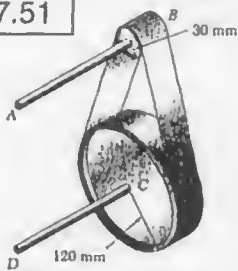
GIVEN: MOTOR ATTACHED TO SHAFT AB DEVELOPES  $4.5 \text{ hp}$  WHEN  $\omega_{AB} = 720 \text{ rpm}$

FIND: MAGNITUDE OF COUPLE EXERTED ON  
 (a) SHAFT AB  
 (b) SHAFT CD

(a) SHAFT AB:  $\omega_{AB} = 720 \text{ rpm} \left( \frac{2\pi}{60} \right) = 75.398 \text{ rad/s}$   
 $\text{Power} = 4.5 \text{ hp} \left( \frac{550 \text{ ft}\cdot\text{lb/s}}{\text{hp}} \right) = 2475 \text{ ft}\cdot\text{lb/s}$   
 $\text{Power} = M_{AB} \omega_{AB}$ ;  $2475 \text{ ft}\cdot\text{lb/s} = M_{AB}(75.398 \text{ rad/s})$   
 $M_{AB} = 32.826 \text{ lb}\cdot\text{ft}$   $M_{AB} = 32.8 \text{ lb}\cdot\text{ft}$

(b) SHAFT CD:  $\omega_{CD} = \frac{r_A}{r_C} \omega_{AB} = \frac{4 \text{ in.}}{2.5 \text{ in.}} (75.398) = 120.64 \text{ rad/s}$   
 $\text{Power} = M_{CD} \omega_{CD}$ ;  $2475 \text{ ft}\cdot\text{lb/s} = M_{CD}(120.64 \text{ rad/s})$   
 $M_{CD} = 20.5 \text{ lb}\cdot\text{ft}$

17.51



GIVEN:  $2.4 \text{ kW}$  TO BE TRANSMITTED FROM A TO D  
 ALLOWABLE COUPLES ARE

$$M_{AB} = 25 \text{ N}\cdot\text{m}$$

$$M_{CD} = 50 \text{ N}\cdot\text{m}$$

FIND: REQUIRED MINIMUM SPEED OF SHAFT AB

SHAFT AB:  $\text{Power} = M_{AB} \omega_{AB}$   
 $2400 \text{ W} = (25 \text{ N}\cdot\text{m}) \omega_{AB}$   $\omega_{AB} = 96 \text{ rad/s}$

SHAFT CD:  $\text{Power} = M_{CD} \omega_{CD}$   
 $2400 \text{ W} = (50 \text{ N}\cdot\text{m}) \omega_{CD}$   $\omega_{CD} = 30 \text{ rad/s}$   
 For  $\omega_{CD} = 30 \text{ rad/s}$ ,  $\omega_{AB} = \frac{r_C}{r_B} \omega_{CD} = \frac{120 \text{ mm}}{30 \text{ mm}} (30 \text{ rad/s})$   
 $\omega_{AB} = 120 \text{ rad/s}$

WE CHOOSE THE LARGER  $\omega_{AB}$ :  $\omega_{AB} = (120 \text{ rad/s}) \left( \frac{60}{2\pi} \right)$   
 $\omega_{AB} = 1146 \text{ rpm}$

17.52

GIVEN:  $30\text{-kg}$  ROTOR WITH  $\bar{r}_G = 200 \text{ mm}$   
 COASTS TO REST IN  $5.3 \text{ mm}$  FROM INITIAL ANGULAR VELOCITY OF  $3600 \text{ rpm}$ .  
 FIND: MAGNITUDE OF COUPLE DUE TO FRICTION

$$\bar{I} = m \bar{r}_G^2 = (30 \text{ kg})(0.2 \text{ m})^2 = 1.2 \text{ kg}\cdot\text{m}^2, \quad \omega_i = 3600 \text{ rpm} \left( \frac{2\pi}{60} \right) = 377 \text{ rad/s}$$



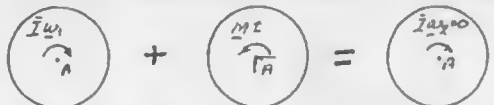
SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>  
 +) MOMENTS ABOUT A:  $\bar{I} \omega_i - M_L = 0$   
 $(1.2 \text{ kg}\cdot\text{m}^2)(377 \text{ rad/s}) - M(5.3 \text{ mm} \times \frac{60 \text{ s}}{\text{min}}) = 0$   
 $M = 1.423 \text{ N}\cdot\text{m}$

17.53

GIVEN:  $4000\text{-lb}$  FLYWHEEL WITH  $\bar{r}_G = 27 \text{ in.}$   
 COASTS TO REST FROM ANGULAR VELOCITY OF  $450 \text{ rpm}$ . FRICTIONAL COUPLE IS OF MAGNITUDE  $125 \text{ lb}\cdot\text{in.}$   
 FIND: TIME REQUIRED TO COAST TO REST

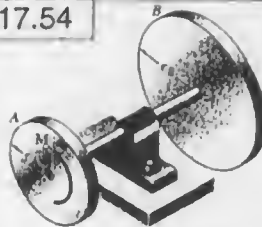
$$\bar{I} = m \bar{r}_G^2 = \left( \frac{4000 \text{ lb}}{32.2} \right) (27 \text{ in.})^2 = 628.88 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\omega_i = 450 \text{ rpm} \left( \frac{2\pi}{60} \right) = 47.125 \text{ rad/s}, \quad M = 125 \text{ lb}\cdot\text{in.} = 10.417 \text{ lb}\cdot\text{ft}$$



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>  
 +) MOMENTS ABOUT A:  $\bar{I} \omega_i - M_L = 0$   
 $t = \frac{\bar{I} \omega_i}{M} = \frac{(628.88 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(47.125 \text{ rad/s})}{10.417 \text{ lb}\cdot\text{ft}} = 2845 \text{ s}$   
 $t = 2845 \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \quad t = 47.4 \text{ min.}$

17.54



GIVEN:  $W_A = 8 \text{ lb}$ ,  $r_A = 3 \text{ in.}$ ,  $r_B = 4.5 \text{ in.}$   
 DISKS OF SAME MATERIAL AND THICKNESS.  
 $M = 20 \text{ lb}\cdot\text{in.}$ ,  $\omega_i = 0$   
 FIND: TIME UNTIL  $\omega_2 = 960 \text{ rpm}$

$$W_B = \left( \frac{r_A}{r_B} \right)^2 W_A = \left( \frac{3 \text{ in.}}{4.5 \text{ in.}} \right)^2 (8 \text{ lb}) = 18 \text{ lb}$$

$$\bar{I} = \bar{I}_A + \bar{I}_B = \frac{1}{2} \frac{W_A}{g} \left( \frac{r_A}{12} \right)^2 + \frac{1}{2} \frac{W_B}{g} \left( \frac{r_B}{12} \right)^2 = 0.04707 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\omega_2 = 960 \text{ rpm} \left( \frac{2\pi}{60} \right) = 100.53 \text{ rad/s}, \quad M = 20 \text{ lb}\cdot\text{in.} = 1.667 \text{ lb}\cdot\text{ft}$$



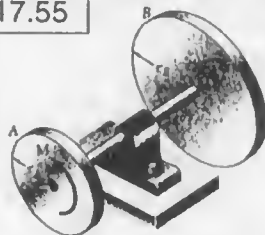
SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+) MOMENTS ABOUT C:  $0 + M_L = \bar{I} \omega_2$   
 $t = \frac{\bar{I} \omega_2}{M} = \frac{(0.04707 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(100.53 \text{ rad/s})}{1.667 \text{ lb}\cdot\text{ft}}$

$$t = 2.839 \text{ s}$$

$$t = 2.84 \text{ s}$$

17.55



GIVEN:  $m_A = 3R_0$ ,  $r_A = 100 \text{ mm}$ ,  
 $r_B = 125 \text{ mm}$ . DISKS OF SAME  
 MATERIAL AND THICKNESS.  
 $\omega_1 = 200 \text{ rpm}$ ,  $\omega_2 = 800 \text{ rpm}$   
 $\ell_{1-2} = 35$ .  
 FIND: MAGNITUDE OF  
 COUPLER  $M$

$$m_B = \left(\frac{r_A}{r_B}\right)^2 m_A = \left(\frac{100 \text{ mm}}{125 \text{ mm}}\right)^2 3R_0 = 4.6875 R_0$$

$$\bar{I} = \bar{I}_A + \bar{I}_B = \frac{1}{2}(3R_0)(0.1 \text{ m})^2 + \frac{1}{2}(4.6875 R_0)(0.125 \text{ m})^2 = 0.05162 R_0 \cdot \text{m}^2$$

$$\omega_1 = 200 \text{ rpm} \left(\frac{2\pi}{60}\right) = 20.944 \text{ rad/s}; \quad \omega_2 = 800 \text{ rpm} \left(\frac{2\pi}{60}\right) = 83.776 \text{ rad/s}$$



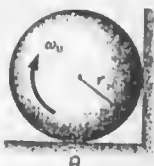
$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+\circlearrowleft \text{ MOMENTS ABOUT G: } \bar{I}\omega_1 + M\ell = \bar{I}\omega_2$$

$$M = \frac{\bar{I}}{\ell}(\omega_2 - \omega_1) = \frac{0.05162 R_0 \cdot \text{m}^2}{35} (83.776 \text{ rad/s} - 20.944 \text{ rad/s})$$

$$M = 1.081 \text{ N} \cdot \text{m}$$

17.56



GIVEN: SPHERE OF WEIGHT  $W$   
 $\mu_k = \text{COEF. OF KINETIC FRICTION}$   
 FIND: EXPRESSION FOR TIME  
 REQUIRED FOR SPHERE TO  
 COME TO REST.

$$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}\frac{W}{g}r^2$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} + \text{SYST. MOMENTA}_2$$

$$+\circlearrowleft \text{ COMPONENTS: } 0 + N_B \ell + \mu_k N_B \ell - W \ell = 0 \quad (1)$$

$$+\rightarrow \text{ COMPONENTS: } 0 + N_B \ell - \mu_k N_B \ell = 0 \quad (2)$$

$$\text{FROM EQ. (2): } N_A = \mu_k N_B \quad (3)$$

$$\text{SUBSTITUTE INTO EQ. (1): } N_B \ell + \mu_k (\mu_k N_B) \ell - W \ell = 0$$

$$N_B = \frac{1}{1 + \mu_k^2} W$$

$$\text{EQ. (2): } N_A = \frac{\mu_k}{1 + \mu_k^2} W$$

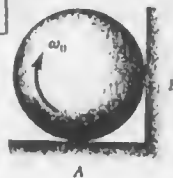
$$+\circlearrowleft \text{ MOMENTS ABOUT G: } \bar{I}\omega_0 - (\mu_k N_A \ell)r - (\mu_k N_B \ell)r = 0$$

$$\frac{2}{5}\frac{W}{g}r^2\omega_0 - \frac{\mu_k^2}{1 + \mu_k^2}rW\ell - \frac{\mu_k}{1 + \mu_k^2}rW\ell = 0$$

$$\frac{2}{5}\frac{r}{g}\omega_0 - \frac{\mu_k + \mu_k^2}{1 + \mu_k^2}\ell = 0$$

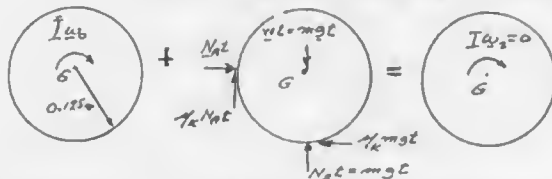
$$\ell = \frac{1 + \mu_k^2}{\mu_k + \mu_k^2} \cdot \frac{2}{5}\frac{r}{g}\omega_0$$

17.57



GIVEN:  $m = 3R_0$ ,  $r = 125 \text{ mm}$   
 $\omega_0 = 90 \text{ rad/s}$   
 $\mu_k = 0.10$   
 FIND: TIME REQUIRED FOR  
 SPHERE TO COME TO REST

$$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}(3R_0)(0.125 \text{ m})^2 = 18.75 \times 10^{-3} R_0 \cdot \text{m}^2$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+\circlearrowleft \text{ COMPONENTS: } 0 + N_B \ell - \mu_k N_B \ell - mg \ell = 0 \quad (1)$$

$$+\rightarrow \text{ COMPONENTS: } 0 + N_B \ell - \mu_k N_B \ell = 0 \quad (2)$$

$$\text{EQ. (2): } N_A = \mu_k N_B$$

$$\text{EQ. (1): } N_B \ell - \mu_k (\mu_k N_B) \ell - mg \ell = 0$$

$$N_B = \frac{mg}{1 + \mu_k^2} = \frac{(3R_0)9.81 \text{ m/s}^2}{1 + (0.10)^2} = 29.139 \text{ N}$$

$$N_A = \mu_k N_B = 0.1(29.139 \text{ N}) = 2.9139 \text{ N}$$

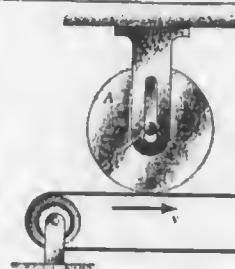
$$+\circlearrowleft \text{ MOMENTS ABOUT G: } \bar{I}\omega_0 - \mu_k N_A \ell r - (\mu_k N_B \ell)r = 0$$

$$\ell = \frac{\bar{I}\omega_0}{\mu_k r(N_A + N_B)} = \frac{(18.75 \times 10^{-3} R_0 \cdot \text{m}^2)(90 \text{ rad/s})}{(0.10)(0.125 \text{ m})(2.9139 \text{ N} + 29.139 \text{ N})}$$

$$\ell = 4.212 \text{ s}$$

$$\ell = 4.21 \text{ s}$$

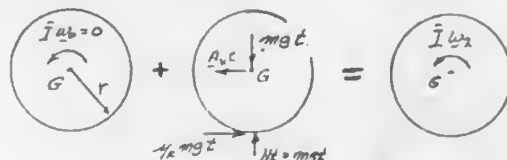
17.58 and 17.59



GIVEN: DISK AT REST PLACED  
 IN CONTACT WITH BELT.  
 COEF. OF KINETIC FRICTION =  $\mu_k$ .

FIND: TIME REQUIRED FOR  
 DISK TO REACH CONSTANT  $\omega$ .  
 PROBLEM 17.58: IN TERMS  
 OF  $\tau$ ,  $g$ , AND  $\mu_k$ .  
 PROBLEM 17.59: FOR  $r = 3 \text{ in.}$ ,  
 $W = 6 \text{ lb}$ ,  $\tau = 50 \text{ ft/s}$ ,  $\mu_k = 0.20$ .

$$W = mg \quad \bar{I} = \frac{1}{2}mr^2$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+\circlearrowleft \text{ MOMENTS ABOUT G: } 0 + (\mu_k mg \ell)r = \bar{I}\omega \quad (1)$$

$$\text{FINAL ANGULAR VELOCITY: } \tau = r\omega; \quad \omega_2 = \tau/r$$

$$\text{EQ. (1): } (\mu_k mg \ell)r = \frac{1}{2}mr^2\left(\frac{\tau}{r}\right)$$

$$\text{PROBLEM 17.58:}$$

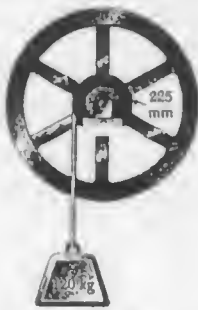
$$\ell = \frac{\tau}{2g\mu_k}$$

NOTE: RESULT IS INDEPENDENT OF  $W$  AND  $r$ .

$$\text{PROBLEM 17.59: DATA: } \tau = 50 \text{ ft/s}, \mu_k = 0.20$$

$$\ell = \frac{\tau}{2g\mu_k} = \frac{50 \text{ ft/s}}{2(32.2 \text{ ft/s}^2)(0.20)}; \quad \ell = 3.88 \text{ s}$$

# 17.60 and 17.61



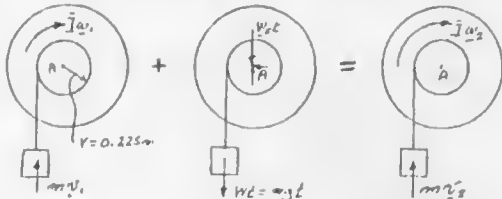
GIVEN: 350-kg FLYWHEEL OF  $\bar{r} = 600 \text{ mm}$ .

$\omega_1 = 100 \text{ rpm}$  WHEN POWER IS CUT OFF AND SYSTEM COMES TO REST.

PROBLEM 17.60: FIND TIME REQUIRED FOR SYSTEM TO COME TO REST.

PROBLEM 17.61: FIND TIME WHEN  $\omega_2 = 40 \text{ rpm}$ .

$$\bar{I} = m\bar{r}^2 = (350 \text{ kg})(0.6 \text{ m})^2 = 126 \text{ kg}\cdot\text{m}^2$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP}_{1 \rightarrow 2} = \text{SYST. MOMENTA}_2$$

$$+2 \text{ MOMENTS ABOUT A: } m\omega_1 r + \bar{I}\omega_1 - m\omega_2 r - \bar{I}\omega_2 = m\omega_2 r + \bar{I}\omega_2$$

$$\text{SUBSTITUTE: } T_1 = r\omega_1, \text{ AND } T_2 = r\omega_2$$

$$(mr^2 + \bar{I})\omega_1 - mgr = (mr^2 + \bar{I})\omega_2$$

$$t = \frac{mr^2 + \bar{I}}{mg} (\omega_1 - \omega_2) = \frac{(120 \text{ kg})(0.225 \text{ m})^2 + 126 \text{ kg}\cdot\text{m}^2}{(10 \text{ kg})(9.81 \text{ m/s}^2)} (\omega_1 - \omega_2)$$

$$t = \frac{6.075 + 126}{98.1} (\omega_1 - \omega_2) \quad t = 0.47864 (\omega_1 - \omega_2) \quad (1)$$

PROBLEM 17.60:

$$\omega_1 = 100 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10.472 \text{ rad/s} \quad \omega_2 = 0$$

$$\text{EQ (1): } t = 0.47864 (10.472 \text{ rad/s}) \quad t = 5.02 \text{ s}$$

PROBLEM 17.61:

$$\omega_1 = 100 \text{ rpm} \left( \frac{2\pi}{60} \right) = 10.472 \text{ rad/s}$$

$$\omega_2 = 40 \text{ rpm} \left( \frac{2\pi}{60} \right) = 4.189 \text{ rad/s}$$

$$\text{EQ (1): } t = 0.47864 (10.472 - 4.189) \quad t = 3.13 \text{ s}$$

## 17.62

GIVEN:  $m_A = 600 \text{ g}$ ,  $\bar{r}_A = 32 \text{ mm}$ ,  $m_B = 260 \text{ g}$ ,  $\bar{r}_B = 20 \text{ mm}$ . SPEED OF TAPE INCREASES  $4 \text{ m/s}$  IN  $0.3 \text{ s}$ .

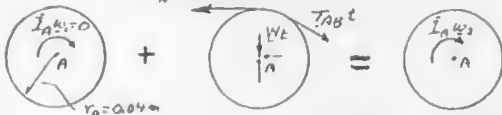
FIND: (a)  $T_B$ , (b)  $T_{AB}$ .

$$\bar{I}_A = m_A \bar{r}_A^2 = (0.6 \text{ kg})(0.032 \text{ m})^2 = 6.144 \times 10^{-6} \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_B = m_B \bar{r}_B^2 = (0.26 \text{ kg})(0.020 \text{ m})^2 = 10.4 \times 10^{-6} \text{ kg}\cdot\text{m}^2$$

$$\text{DRUM A: ASSUME } \omega_1 = 0 \text{ INCH } \omega_2 = \frac{\Delta v}{\bar{r}_A} = \frac{4 \text{ m/s}}{0.04 \text{ m}} = 100 \text{ rad/s}$$

$$T_A t = (3.5 \text{ N})t$$



$$+2 \text{ MOMENTS ABOUT A: } 0 - 3.5t(0.04 \text{ m}) + T_{AB}t(0.04 \text{ m}) = \bar{I}_A \omega_2$$

$$t = 0.35 \text{ s} \quad -3.5(0.35)(0.04) + T_{AB}(0.35)(0.04) = (6.144 \times 10^{-6})(100 \text{ rad/s})$$

$$-0.492 + 0.014 T_{AB} = 0.06144$$

$$T_{AB} = 8.62 \text{ N}$$

(CONTINUED)

## 17.62 continued

DRUM B WE RECALL:  $T_B = 8.62 \text{ N}$

$$\omega_1 = 0 \quad \omega_2 = \frac{\Delta v}{\bar{r}_B} = \frac{4 \text{ m/s}}{0.02 \text{ m}} = 166.67 \text{ rad/s}$$

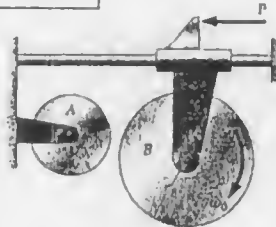
$$\bar{I}_B \omega_1 + \text{SYST. EXT. IMP}_{1 \rightarrow 2} = \text{SYST. MOMENTA}_2$$

$$+2 \text{ MOMENTS ABOUT B: } 0 + T_A t r - T_B t r = \bar{I}_B \omega_2$$

$$t = 0.35 \text{ s: } T_B(0.35)(0.02 \text{ m}) - (8.62 \text{ N})(0.35)(0.02 \text{ m}) = (10.4 \times 10^{-6} \text{ kg}\cdot\text{m}^2)(166.67 \text{ rad/s})$$

$$T_B = 11.03 \text{ N}$$

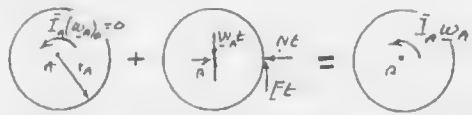
## 17.63



GIVEN: DISK A IS AT REST WHEN DISKS A AND B ARE BROUGHT INTO CONTACT.

SHOW THAT FINAL  $\omega_B$  DEPENDS ON ONLY  $\omega_A$  AND  $\frac{m_A}{m_B}$ .

DISK A:  $(\omega_A)_0 = 0$

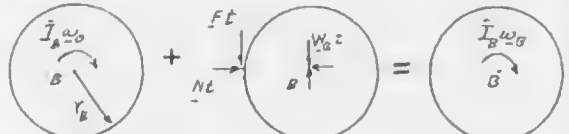


$$\text{SYST. MOMENTA}_1 + \text{SYST. MOMENTA}_{1 \rightarrow 2} = \text{SYST. MOMENTA}_2$$

$$+2 \text{ MOMENTS ABOUT A: } 0 + (Ft)r_A = \bar{I}_A \omega_A$$

$$Ft = \frac{\bar{I}_A \omega_A}{r_A} \quad (1)$$

DISK B:  $(\omega_B)_0 = \omega_0$



$$+2 \text{ MOMENTS ABOUT B: } \bar{I}_B \omega_0 + (Ft)r_B = \bar{I}_B \omega_B$$

SUBSTITUTE FOR  $Ft$  FROM EQ (1)

$$\bar{I}_B \omega_0 - \bar{I}_A \omega_A \frac{r_B}{r_A} = \bar{I}_B \omega_B \quad (2)$$

$$\text{FOR FINAL ANGULAR VELOCITIES: } r_A \omega_A = r_B \omega_B; \quad \omega_A = \frac{r_B}{r_A} \omega_B$$

$$\text{EQ (2): } \bar{I}_B \omega_0 - \bar{I}_A \omega_B \left( \frac{r_B}{r_A} \right)^2 = \bar{I}_B \omega_B$$

$$\omega_B = \frac{\omega_0}{1 + \frac{\bar{I}_A}{\bar{I}_B} \left( \frac{r_B}{r_A} \right)^2} \quad (3)$$

$$\text{BUT FOR UNIFORM DISCS: } \frac{\bar{I}_A}{\bar{I}_B} = \frac{\frac{1}{2} m_A r_A^2}{\frac{1}{2} m_B r_B^2} = \frac{m_A}{m_B} \left( \frac{r_A}{r_B} \right)^2$$

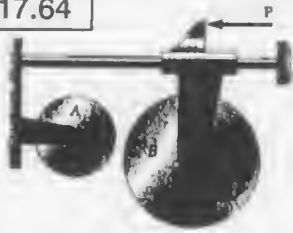
SUBSTITUTE INTO EQ (3):

$$\omega_B = \frac{\omega_0}{1 + \frac{m_A}{m_B}}$$

THUS,  $\omega_B$  DEPENDS ON ONLY  $\omega_0$  AND  $\frac{m_A}{m_B}$ .



17.64



GIVEN:  $W_B = 7.5 \text{ lb}$ ,  $r_A = 6 \text{ in}$   
 $W_D = 10 \text{ lb}$ ,  $r_B = 8 \text{ in}$   
 $\omega_0 = 900 \text{ rpm}$

FIND: (a) FINAL  $\omega_A$  AND  $\omega_B$   
 (b) IMPULSE OF FRICTION FORCE EXERTED ON DISK A.

$$\bar{I}_A = \frac{1}{2} \frac{W_A}{g} r_A^2 = \frac{1}{2} \frac{7.5(32)}{g} \left(\frac{6}{12}\right)^2 = \frac{7.5}{8g}$$

$$\bar{I}_B = \frac{1}{2} \frac{W_B}{g} r_B^2 = \frac{1}{2} \frac{10(32)}{g} \left(\frac{8}{12}\right)^2 = \frac{20}{3g}$$

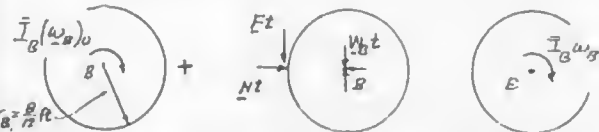
DISK A:



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA.

+ MOMENTS ABOUT A:  $0 + (Ft)r_A = \bar{I}_A \omega_A$   
 $(Ft)(0.5 \text{ ft}) = \frac{7.5}{8g} \omega_A$   $Ft = \frac{7.5}{4g} \omega_A$  (1)

DISK B:  $(\omega_B)_0 = \omega_0 = 900 \text{ rpm} \left(\frac{2\pi}{60}\right) = 30\pi \text{ rad/s}$



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA.

+ MOMENTS ABOUT B:  $\bar{I}_B (\omega_B)_0 - (Ft)r_B = \bar{I}_B \omega_B$   
 $\frac{20}{3g} \cdot 30\pi - (Ft)\left(\frac{8}{12}\right) = \frac{20}{3g} \omega_B$

SUBSTITUTE FROM EQ (1):

$$\frac{20}{3g} \cdot 30\pi - \left(\frac{7.5}{4g} \omega_A\right)\left(\frac{8}{12}\right) = \frac{20}{3g} \omega_B$$

$$30\pi - 0.5625 \omega_A = \omega_B$$
 (2)

FINAL VELOCITIES OCCUR WHEN:

$$v_A r_A = r_B \omega_B \quad \omega_B = \frac{r_A}{r_B} \omega_A = \frac{6 \text{ in}}{8 \text{ in}} \omega_A = 0.75 \omega_A$$
 (3)

SUBSTITUTE FOR  $\omega_B$  FROM (2) INTO (3)

$$30\pi - 0.5625 \omega_A = 0.75 \omega_A$$

$$30\pi = 1.3125 \omega_A \quad \omega_A = 71.807 \text{ rad/s}$$

$$\omega_A = 71.807 \text{ rad/s} \left(\frac{60}{2\pi}\right) \quad \omega_A = 685.7 \text{ rpm}$$

EQ (3):  $\omega_B = 0.75 \omega_A = 0.75(685.7 \text{ rpm}) \quad \omega_B = 514.3 \text{ rpm}$

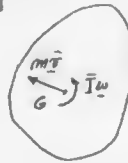
IMPULSE OF  $F$  EXERTED ON DISK A:

EQ (1):  $Ft = \frac{7.5}{4g} \omega_A = \frac{7.5}{4(32.2)} (71.807 \text{ rad/s})$   
 $Ft = 4.18 \text{ lb}\cdot\text{s}$

$$\omega_A = 686 \text{ rpm}$$

$$\omega_B = 574 \text{ rpm}$$

17.65



SHOW THAT SYSTEM OF MOMENTA IS EQUIVALENT TO A SINGLE VECTOR AND EXPRESS THE DISTANCE FROM G TO THE LINE OF ACTION OF THE VECTOR IN TERMS OF  $\bar{r}_G$ ,  $\bar{v}$ , AND  $\omega$ .

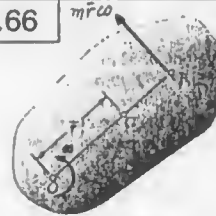
+ MOMENTS ABOUT G

$$\bar{I} \omega = (m \bar{v}) d$$

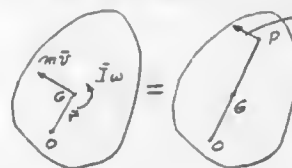
$$d = \frac{\bar{I} \omega}{m \bar{v}} = \frac{m \bar{r}_G^2 \omega}{m \bar{v}}$$

$$d = \frac{\bar{r}_G^2 \omega}{\bar{v}}$$

17.66



SHOW THAT SYSTEM OF MOMENTA IS EQUIVALENT TO  $m \bar{v} \omega$  LOCATED AT P WHERE  $GP = \frac{\bar{r}_G^2 \omega}{\bar{v}}$



+ MOMENTS ABOUT G

$$\bar{I} \omega = (m \bar{v} \omega) GP$$

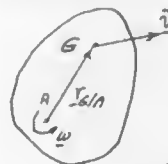
$$GP = \frac{\bar{I} \omega}{m \bar{v} \omega} = \frac{m \bar{r}_G^2 \omega}{m \bar{v} \omega}$$

$$GP = \frac{\bar{r}_G^2 \omega}{\bar{v}}$$

17.67

FOR A RIGID SLAB IN PLANE MOTION,

SHOW THAT  $\bar{H}_A$  IS EQUAL TO  $\bar{I}_A \omega$ , IF AND ONLY IF (a) A IS THE MASS CENTER, (b) A IS THE INSTANTANEOUS CENTER OF ROTATION, (c)  $\bar{v}_A$  IS DIRECTED ALONG LINE AG.



FOR GENERAL PLANE MOTION

$$\bar{v} = \bar{v}_A + \bar{v} = \bar{v}_A + \omega \times \bar{r}_{G/A}$$

SYSTEM OF MOMENTA:

MOMENTS ABOUT A

$$\bar{H}_A = \bar{I}_A \omega + \bar{r}_{G/A} \times m \bar{v}$$

$$\bar{H}_A = \bar{I}_A \omega + m \bar{r}_{G/A} \times (\bar{v}_A + \omega \times \bar{r}_{G/A})$$

$$\bar{H}_A = \bar{I}_A \omega + m \bar{r}_{G/A} \times \bar{v}_A + m \bar{r}_{G/A} \times (\omega \times \bar{r}_{G/A})$$

SINCE  $\omega \perp \bar{r}_{G/A}$  THE TRIPLE VECTOR PRODUCT

CAN BE WRITTEN:  $\bar{r}_{G/A} \times (\omega \times \bar{r}_{G/A}) = \bar{r}_{G/A}^2 \omega$

THUS  $\bar{H}_A = \bar{I}_A \omega + m \bar{r}_{G/A} \times \bar{v}_A + m \bar{r}_{G/A}^2 \omega$

BY PARALLEL-AXIS THEOREM:  $\bar{I}_A = \bar{I} + m \bar{r}_{G/A}^2$

WE NOW HAVE  $\bar{H}_A = \bar{I}_A \omega + m \bar{r}_{G/A} \times \bar{v}_A$

$\therefore \bar{H}_A = \bar{I}_A \omega$ , ONLY WHEN  $\bar{r}_{G/A} \times \bar{v}_A = 0$

(a)  $\bar{r}_{G/A} = 0$ : A COINCIDES WITH G

(b)  $\bar{v}_A = 0$ : A IS INSTANT. CENTER

(c)  $\bar{r}_{G/A}$  AND  $\bar{v}_A$  ARE COLLINEAR:  $\bar{v}_A$  IS DIRECTED ALONG AG.



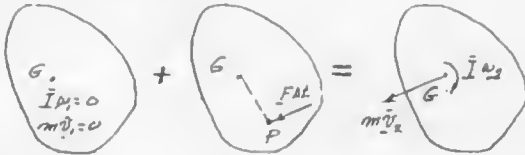
17.68



GIVEN: IMPULSIVE FORCE  $F$  IS APPLIED TO SLAB.  
SHOW THAT: (a) INST. CENTER IS AT C AND  $GC = \frac{R^2}{GP}$ .  
(b) IF  $F$  WERE APPLIED AT C THEN P IS THE INST. CENTR

$\Delta t$  = TIME OF APPLICATION OF  $F$  AT THE CENTER OF PERCUSSION P.

(a)

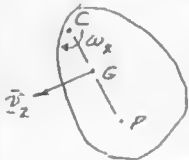


SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+ COMPONENTS:  $F \Delta t = m \bar{v}_2$ ;  $\bar{v}_2 = \frac{F \Delta t}{m}$  (1)

+ MOMENTS ABOUT G:  $(F \Delta t)(GP) = I \omega_2$

$\omega_2 = \frac{F \Delta t}{I} (GP) = \frac{F \Delta t}{m R^2} (GP)$  (2)



KINEMATICS: THE INSTANTANEOUS CENTER MUST BE LOCATED ON A LINE  $\perp$  TO  $\bar{v}_2$ , THAT IS, ON GP.

ALSO,  $\bar{v}_2 = (GP) \omega_2$   $GC = \frac{\bar{v}_2}{\omega_2}$

SUBSTITUTE FROM (1) AND (2)

$$GC = \frac{F \Delta t}{m \bar{v}_2} (GP) \quad GC = \frac{R^2}{GP}$$

(b)



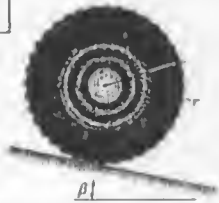
WE NOW ASSUME THAT  $F$  IS APPLIED SO THAT THE NEW CENTER OF PERCUSSION  $P'$  IS LOCATED AT C.

FROM PART (a), WE NOTE THAT NEW INST. CENTER WILL BE LOCATED AT  $C'$  WHERE

$$GC' = \frac{R^2}{GP'} = \frac{R^2}{R/GP} = GP$$

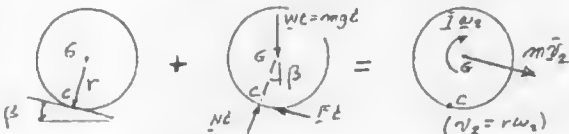
THUS, NEW INSTANTANEOUS CENTER IS LOCATED AT P

17.69



GIVEN:  $R$  = RADIUS OF CYCLATION  $\bar{v}_1 = 0$

FIND: (a)  $\bar{v}_2$  AT TIME  $t$   
(b)  $\Delta t$  REQUIRED TO PREVENT SLIPPING



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT C:  $(W \sin \beta) r = I \omega_2 + m \bar{v}_2 r$   
 $m g \sin \beta = m \bar{v}_2 + m r \omega_2$

$$\omega_2 = \frac{r g \sin \beta}{r^2 + R^2}$$
 (1)

(CONTINUED)

17.69 continued

$$(a) \bar{v}_2 = r \omega_2: \quad \bar{v}_2 = \frac{r^2}{r^2 + R^2} g \sin \beta \quad \triangleleft$$

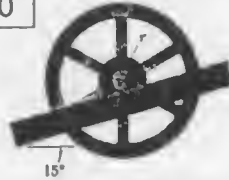
(b) + COMPONENTS:  $N \sin \beta = m g \cos \beta$   
 $N = m g \cos \beta$

+ MOMENTS ABOUT G:  $(F \Delta t) r = I \omega_2$

$$F = \frac{I}{r \Delta t} \omega_2 = \frac{m R^2}{r \Delta t} \cdot \frac{r^2 g \sin \beta}{r^2 + R^2} \sin \beta = \frac{R^2}{r^2 + R^2} m g \sin \beta$$

$$\Delta t = \frac{F}{N} = \frac{R^2}{r^2 + R^2} \cdot \frac{m g \sin \beta}{m g \cos \beta}; \quad \Delta t = \frac{R^2}{r^2 + R^2} \tan \beta \quad \triangleleft$$

17.70

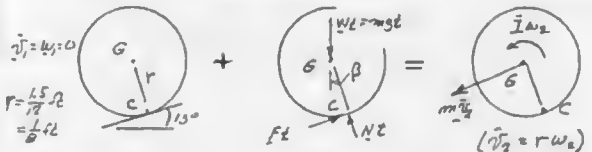


GIVEN:  $r = 1.5 \sin \beta$

WHEEL STARTS FROM REST AND ROLLS WITHOUT SLIDING.

$\bar{v}_2 = 6 \text{ in/s}$  AT  $t = 30 \text{ s}$ .

FIND:  $\bar{v}_2$



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT C:  $m g \sin \beta (r \sin \beta) = I \omega_2 + m \bar{v}_2 r$   
 $m g \sin \beta (r \sin \beta) = m R^2 \omega_2 + m r^2 \omega_2$   
 $m g \sin \beta (r \sin \beta) = (R^2 + r^2) \omega_2$  (1)

DATA:  $r = \frac{1}{8} R$ ,  $\bar{v}_2 = 6 \text{ in/s} = 0.5 \text{ ft/s}$ ,  $t = 30 \text{ s}$

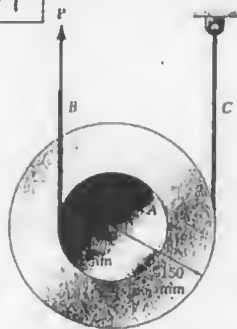
$$\omega_2 = \frac{\bar{v}_2}{r} = \frac{0.5 \text{ ft/s}}{1/8 R} = 4 \text{ rad/s}$$

$$\triangleleft (1) \quad (32.2 \text{ ft/s}^2)(30 \text{ s}) \left( \frac{1}{8} R \right) \sin 15^\circ = [R^2 + (0.15 \text{ ft})^2] (4 \text{ rad/s})$$

$$R^2 + 0.015625 = 7.8131 \quad R^2 = 7.7975$$

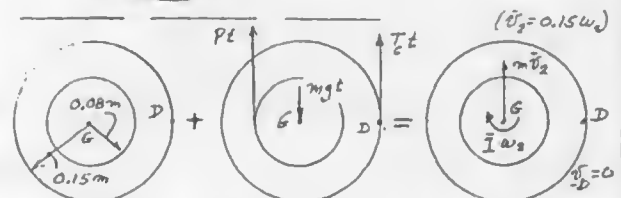
$$R = 2.7924 \text{ ft} \quad R = 2.79 \text{ ft} \quad \triangleleft$$

17.71



GIVEN:  $m = 3 \text{ kg}$ ,  $R = 100 \text{ mm}$   
PULLEY IS AT REST WHEN  $P = 24 \text{ N}$  IS APPLIED TO B

FIND: (a)  $\bar{v}$  AFTER 1.5 S  
(b) TENSION IN CORD C



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT D:  $P t (0.08 + 0.15) - m g t (0.15) = I \omega_2 + m \bar{v}_2 (0.15)$   
 $(24 \text{ N})(1.5 \text{ s})(0.23) - (3 \text{ kg})(9.81 \text{ m/s}^2)(1.5 \text{ s})(0.15) = (3 \text{ kg})(0.1 \text{ m})^2 \omega_2 + (3 \text{ kg})(0.15 \text{ m}) \bar{v}_2$   
 $1.6583 = (0.03 + 0.0675) \omega_2$ ;  $\omega_2 = 17.008 \text{ rad/s}$   
 $\bar{v}_2 = (0.15) \omega_2 = (0.15)(17.008) = 2.551 \text{ m/s}$   $\bar{v}_2 = 2.55 \text{ m/s} \uparrow$

(CONTINUED)

# 17.71 continued

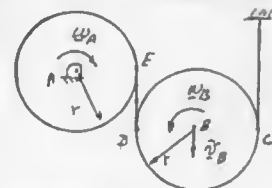
WE HAVE FOUND  $\vec{v}_x = 2.55 \text{ m/s} \uparrow$

COMPONENTS:  $Pt + T_c t - mgt = m\vec{v}_x$   
 $(24)(1.5) + T_c(1.5) - (32)(9.8)(1.5) = (32)(2.55 \text{ m/s})$   
 $36 + 1.5T_c - 44.195 = 7.653$   
 $1.5T_c = 15.798$   
 $T_c = 10.53 \text{ N}$

# 17.72

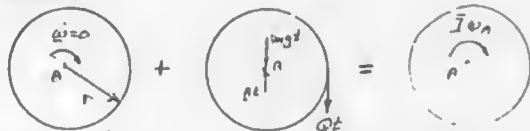


GIVEN: Two 14-lb cylinders of radius  $r = 5 \text{ in.}$   
 SYSTEM IS RELEASED FROM REST WHEN  $t = 0$ .  
 FIND: (a)  $\vec{v}_B$  AT  $t = 3 \text{ s.}$   
 (b) TENSION IN BELT CONNECTING CYLINDERS



KINEMATICS CYLINDER B  
 INSTANT. CENTER OF B IS AT C.  
 $\vec{v}_B = r\omega_B$   
 $\vec{v}_D = \vec{v}_E = 2r\omega_B$   
 CYLINDER A:  $\omega_A = \frac{\vec{v}_E}{r} = 2\omega_B$

CYLINDER A:



SYST. MOMENTA, + SYST. EXT. IMP.,  $\Sigma =$  SYST. MOMENTA,

+) MOMENTS ABOUT A:  $(Qt)r = \vec{I}\omega_A$   
 $(Qt)r = \frac{1}{2}mr^2(2\omega_B)$   
 $Qt = mr\omega_B$  (1)

CYLINDER B:



+) MOMENTS ABOUT C:  $(mgl)r - (Qt)2r = \vec{I}\omega_B + m\vec{v}_B r$   
 $mglr - (Qt)2r = \frac{1}{2}mr^2\omega_B + m(r\omega_B)r$   
 $mgl - 2(Qt)r = \frac{3}{2}mr\omega_B$  (2)

SUBSTITUTE FOR  $(Qt)$  FROM (1):  $mgl - 2(mr\omega_B)r = \frac{3}{2}mr\omega_B$

$$\omega_B = \frac{2}{7} \frac{gl}{r}$$

$$\vec{v}_B = r\omega_B; \quad \vec{v}_B = \frac{2}{7} gl$$

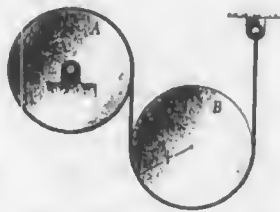
EG(1):  $Qt = mr\omega_B; \quad Qt = mr\left(\frac{2}{7} \frac{gl}{r}\right)$   
 $Q = \frac{2}{7} mg = \frac{2}{7} W$

DATA:  $W = 14 \text{ lb}, \quad t = 3 \text{ s}$

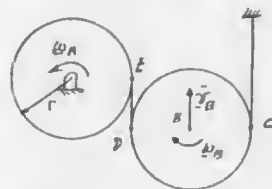
(a)  $\vec{v}_B = \frac{2}{7} gl = \frac{2}{7}(32.2)(14.5)(3 \text{ s}) \quad \vec{v}_B = 27.4 \text{ ft/s} \downarrow$

(b)  $Q = \frac{2}{7} W = \frac{2}{7}(14 \text{ lb}) = 4 \text{ lb}$   
 TENSION IN CONNECTING BELT = 4 lb

# 17.73



GIVEN: Two 14-lb cylinders of radius  $5 \text{ in.}$ , initially  $\omega_A = 30 \text{ rad/s}$   
 FIND: (a) TIME REQUIRED FOR  $\omega_A$  TO BE REDUCED TO  $\omega_A = 5 \text{ rad/s}$   
 (b) TENSION IN BELT CONNECTING CYLINDERS



KINEMATICS: CYLINDER B  
 INSTANT. CENTER OF B IS AT C.  
 $\vec{v}_B = r\omega_B$   
 $\vec{v}_D = \vec{v}_E = 2r\omega_B$   
 CYLINDER A:  $\omega_A = \frac{\vec{v}_E}{r} = 2\omega_B$

CYLINDER A:



SYST. MOMENTA, + SYST. EXT. IMP.,  $\Sigma =$  SYST. MOMENTA,

+) MOMENTS ABOUT A:  $\vec{I}(\omega_A)_1 - (Qt)r = \vec{I}(\omega_A)_2$   
 $(Qt)r = \frac{1}{2}mr^2[(\omega_A)_1 - (\omega_A)_2]$   
 $(Qt)r = \frac{1}{2}mr^2[2(\omega_B)_1 - 2(\omega_B)_2]$   
 $Qt = mr[(\omega_B)_1 - (\omega_B)_2]$  (1)

CYLINDER B:



SYST. MOMENTA, + SYST. EXT. IMP.,  $\Sigma =$  SYST. MOMENTA,

+) MOMENTS ABOUT C:  $\vec{I}(\omega_B)_1 + m(\vec{v}_B)_1 r + Qt(2r) - (mgl)r = \vec{I}(\omega_B)_2 + m(\vec{v}_B)_2 r$

SUBSTITUTE  $\vec{I} = \frac{1}{2}mr^2; \quad (\vec{v}_B)_1 = r(\omega_B)_1; \quad \text{AND} \quad (\vec{v}_B)_2 = r(\omega_B)_2$

$$Qt(2r) - (mgl)r = mr^2[(\omega_B)_1 - (\omega_B)_2] + \frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2]$$

$$2Qt - mgl = \frac{3}{2}mr[(\omega_B)_1 - (\omega_B)_2]$$
 (2)

SUBSTITUTE FOR  $Qt$  FROM (1):

$$2mr[(\omega_B)_1 - (\omega_B)_2] - mgl = \frac{3}{2}mr[(\omega_B)_1 - (\omega_B)_2]$$

$$t = \frac{7}{9} \frac{r}{g} [(\omega_B)_1 - (\omega_B)_2]$$
 (3)

SUBSTITUTE FOR  $t$  FROM (3) INTO (1)

$$Q \left[ \frac{7}{9} \frac{r}{g} [(\omega_B)_1 - (\omega_B)_2] \right] = mr[(\omega_B)_1 - (\omega_B)_2]$$

$$Q = \frac{2}{7} mg = \frac{2}{7} W$$
 (4)

DATA:  $(\omega_A)_1 = 30 \text{ rad/s} \rightarrow (\omega_B)_1 = \frac{1}{2}(\omega_A)_1 = 15 \text{ rad/s}$

$(\omega_B)_2 = 5 \text{ rad/s} \rightarrow (\omega_A)_2 = \frac{1}{2}(\omega_A)_2 = 2.5 \text{ rad/s}$

$W = 14 \text{ lb}, \quad r = 5 \text{ in.} = \frac{5}{12} \text{ ft}$

(a) EG(3):  $t = \frac{7}{9} \frac{(5/12 \text{ ft})}{32.2 \text{ ft/s}^2} [15 \text{ rad/s} - 2.5 \text{ rad/s}]$

$$t = 0.561 \text{ s} \quad t = 0.566 \text{ s}$$

(b) EG(4):  $Q = \frac{2}{7} W = \frac{2}{7}(14 \text{ lb}) = 4 \text{ lb}$

TENSION IN CONNECTING BELT = 4 lb

17.74



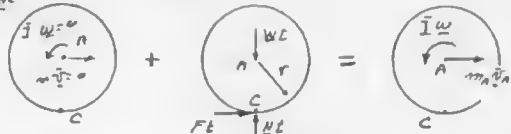
GIVEN:

CYLINDER:  $m_A = 8 \text{ kg}$ ,  $r = 240 \text{ mm}$ CARRIAGE:  $m_B = 3 \text{ kg}$ 

SYSTEM AT REST WHEN

 $P = 10 \text{ N}$  APPLIED FOR 1.25FIND: (a)  $\vec{v}_B$ , (b)  $\vec{v}_A$ 

CYLINDER

SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>+ MOMENTS ABOUT C:  $0 = I\omega - m_A \vec{v}_A r$ 

$$\omega = \frac{m_A \vec{v}_A r}{I} = \frac{m_A \vec{v}_A r}{\frac{1}{2} m_A r^2}; \quad \omega = \frac{2 \vec{v}_A}{r} \quad (1)$$

+ COMPONENTS:  $F = m_A \vec{v}_A$ 

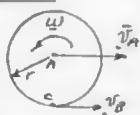
CARRIAGE:

SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>+ COMPONENTS:  $P - F = m_B \vec{v}_B$ 

$$P - m_A \vec{v}_A = m_B \vec{v}_B$$

$$P = m_A \vec{v}_A + m_B \vec{v}_B \quad (2)$$

KINEMATICS: ASSUME ROLLING



$$\vec{v}_B = \vec{v}_A + r\omega$$

SUBSTITUTE FROM EQ (1)

$$\vec{v}_B = \vec{v}_A + r \left( \frac{2 \vec{v}_A}{r} \right); \quad \vec{v}_B = 3 \vec{v}_A \quad (4)$$

$$\text{EQ (2)} \quad P = m_A \vec{v}_A + m_B (3 \vec{v}_A)$$

$$\vec{v}_A = \frac{P}{m_A + 3 m_B} \quad (5)$$

DATA:  $m_A = 8 \text{ kg}$ ,  $m_B = 3 \text{ kg}$  $P = 10 \text{ N}$ ,  $t = 1.25$ 

EQ (5)

$$\vec{v}_A = \frac{(10 \text{ N})(1.25)}{8 \text{ kg} + 3(3 \text{ kg})} = \frac{12.5}{17} \text{ m/s}$$

$$\vec{v}_A = 0.735 \text{ m/s} \rightarrow$$

EQ (4)

$$\vec{v}_B = 3 \vec{v}_A = 3 \left( \frac{12.5}{17} \text{ m/s} \right) = \frac{37.5}{17} \text{ m/s}$$

$$\vec{v}_B = 2.12 \text{ m/s} \rightarrow$$

17.75



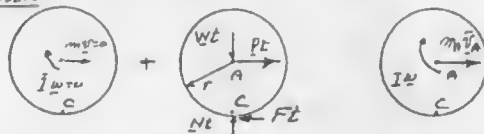
GIVEN:

CYLINDER:  $m_A = 8 \text{ kg}$ ,  $r = 240 \text{ mm}$ CARRIAGE:  $m_B = 3 \text{ kg}$ 

SYSTEM AT REST WHEN

 $P = 10 \text{ N}$  APPLIED FOR 1.25FIND: (a)  $\vec{v}_B$ , (b)  $\vec{v}_A$ 

CYLINDER

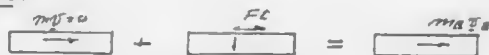
SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>+ MOMENTS ABOUT A:  $(F)r = I\omega$ 

$$(F)r = \frac{1}{2} m_A r^2 \omega$$

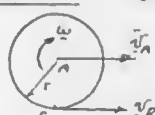
$$F = \frac{1}{2} m_A \omega \quad (1)$$

+ COMPONENTS:  $P - F = m_A \vec{v}_A$  (2)+ MOMENTS ABOUT C:  $P = I\omega + m_A \vec{v}_A r$  (3)

CARRIAGE:

SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>+ COMPONENTS:  $F = m_B \vec{v}_B$ 

KINEMATICS: ASSUME ROLLING



$$\vec{v}_B = \vec{v}_A - r\omega \quad (4)$$

$$\text{EQ (1)} \quad F = m_B (\vec{v}_A - r\omega) \quad (5)$$

SUBSTITUTE EQ (1) -> EQ (5):  $P - \frac{1}{2} m_A r\omega = m_B \vec{v}_A$ 

$$\vec{v}_A = \frac{P}{m_A} - \frac{1}{2} r\omega \quad (6)$$

SUBSTITUTE EQ (1) -&gt; EQ (5):

$$\frac{1}{2} m_A r\omega = m_B (\vec{v}_A - r\omega) \quad (7)$$

SUBSTITUTE EQ (6) -&gt; EQ (7):

$$\frac{1}{2} m_A r\omega = m_B \left( \frac{P}{m_A} - \frac{1}{2} r\omega - r\omega \right)$$

$$\left( \frac{1}{2} m_A r + \frac{3}{2} m_B r \right) \omega = \frac{m_B P}{m_A}$$

$$\omega = \frac{2 P t}{r \left( \frac{m_A}{m_B} \right)} \frac{1}{m_A + 3 m_B} \quad (8)$$

DATA:  $m_A = 8 \text{ kg}$ ,  $m_B = 3 \text{ kg}$  $P = 10 \text{ N}$ ,  $t = 1.25$ ,  $r = 0.24 \text{ m}$ 

$$\text{EQ (8)}: \quad \omega = \frac{2(10 \text{ N})(1.25)}{0.24 \text{ m}} \cdot \frac{3 \text{ kg}}{8 \text{ kg} + 3(3 \text{ kg})}$$

$$\omega = \frac{37.5}{17} \text{ rad/s}$$

$$\text{EQ (6)}: \quad \vec{v}_A = \frac{P}{m_A} - \frac{1}{2} r\omega = \frac{10 \text{ N}}{8 \text{ kg}} - \frac{1}{2} (0.24 \text{ m}) \left( \frac{37.5}{17} \text{ rad/s} \right)$$

$$\vec{v}_A = 1.5 - 0.2647 = 1.235 \text{ m/s}$$

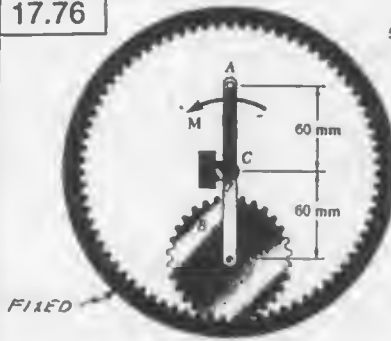
$$\vec{v}_A = 1.235 \text{ m/s} \rightarrow$$

$$\text{EQ (4)}: \quad \vec{v}_B = \vec{v}_A - r\omega = 1.235 \text{ m/s} - (0.24 \text{ m}) \left( \frac{37.5}{17} \text{ rad/s} \right)$$

$$= 1.235 - 0.529 = 0.706 \text{ m/s}$$

$$\vec{v}_B = 0.706 \text{ m/s} \rightarrow$$

17.76



GIVEN:

GEAR:  $m_B = 1.8 \text{ kg}$ ,  $\bar{I}_B = 32 \times 10^{-3}$ ROD:  $m_{AD} = 2.5 \text{ kg}$ SYSTEM IS AT REST AT  $t = 0$  $M = 1.25 \text{ N-m}$  IS APPLIED FOR 1.5 sFIND: (a)  $\omega_{AB}$   
(b)  $v_D$ 

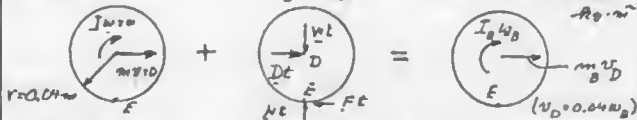
KINEMATICS:

ROD ACD:  $\bar{\omega}_D = (0.06 \text{ m}) \omega_{AD}$  (1)

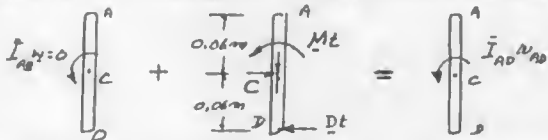
GEAR B: INST. CENTER AT E

 $\bar{v}_D = (0.04 \text{ m}) \omega_B$  $v_D = v_E$ :  $0.06 \omega_{AD} = 0.04 \omega_B$  $\omega_B = 1.5 \omega_{AD}$ 

GEAR B:

 $\bar{I}_B = m_B \bar{k}^2 = (1.8 \text{ kg})(0.032 \text{ m})^2 = 1.893 \times 10^{-3} \text{ kg-m}^2$ 

SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA\_2

+) MOMENTS ABOUT E:  $(DL)r = \bar{I}_B \omega_B + m_B v_D r$  $DL(0.04 \text{ m}) = (1.893 \times 10^{-3} \text{ kg-m}^2) \omega_B + (1.8 \text{ kg})(0.04 \text{ m}) \omega_B$  $DL = 0.11808 \omega_B$  $DL = 0.11808(1.5 \omega_{AD}) = 0.1771 \omega_{AD}$  (2)ROD ACD:  $\bar{I}_{AD} = \frac{1}{12} m_{AD} (AD)^2 = \frac{1}{12} (2.5 \text{ kg})(0.12 \text{ m})^2 = 3 \times 10^{-3} \text{ kg-m}^2$ 

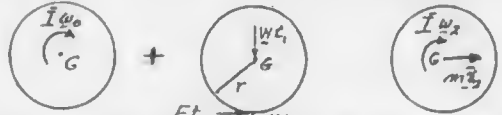
SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA\_2

+) MOMENTS ABOUT C:  $ML - (DL)(0.06 \text{ m}) = \bar{I}_{AD} \omega_{AD}$  $(1.25 \text{ N-m})(1.5 \text{ s}) - (DL)(0.06 \text{ m}) = (3 \times 10^{-3} \text{ kg-m}^2) \omega_{AD}$  $1.875 - 0.06(DL) = 3 \times 10^{-3} \omega_{AD}$ 

SUBSTITUTE FOR DL FROM EQ(2)

 $1.875 - 0.06(0.1771 \omega_{AD}) = 3 \times 10^{-3} \omega_{AD}$  $\omega_{AD} = 137.6 \text{ rad/s}$ EQ(1):  $\bar{v}_D = (0.06 \text{ m}) \omega_{AD} = (0.06 \text{ m})(137.6 \text{ rad/s})$  $\bar{v}_D = 8.26 \text{ m/s}$ 

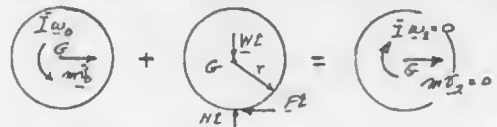
17.77

GIVEN: SPHERE OF RADIUS  $r$  PLACED ON FLOOR (AT  $t = 0$ ) WITH  $\bar{v} = 0$  AND  $\omega = \omega_0$ .COEF. OF KINETIC FRICTION =  $\mu_k$  FIND: (a) TIME  $t$ , WHEN ROLLING WITHOUT SLIDING STARTS(b)  $\bar{v}$  AND  $\omega$  AT  $t = t_1$ 

SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA\_2

+ y COMPONENTS:  $N - W = 0$   $N = W = mg$  (1)+ x COMPONENTS:  $F_k = m \bar{a}$  (2)+) MOMENTS ABOUT G:  $\bar{I} \omega_0 - F_k r t_1 = \bar{I} \omega_2$  (3)SINCE  $F = \mu_k N = \mu_k mg$ , EQ(2) YIELDS $\mu_k mg t_1 = m \bar{a}$   $\bar{a} = \mu_k g t_1$  (4)EQ(3): SINCE  $\bar{I} = \frac{2}{5} m r^2$  $\frac{2}{5} m r^2 \omega_0 - (\mu_k mg) r t_1 = \frac{2}{5} m r^2 \omega_2$  $\omega_2 = \omega_0 - \frac{5}{2} \frac{\mu_k g t_1}{r}$  (5)SLIDING STOPS WHEN  $\bar{v}_2 = r \omega_2$  $\mu_k g t_1 = r \omega_0 - \frac{5}{2} \mu_k g t_1$  $\frac{7}{2} \mu_k g t_1 = r \omega_0$   $t_1 = \frac{2}{7} \frac{r \omega_0}{\mu_k g}$ EQ(4):  $\bar{v}_2 = \mu_k g t_1 = \mu_k g \left( \frac{2}{7} \frac{r \omega_0}{\mu_k g} \right)$   $\bar{v}_2 = \frac{2}{7} r \omega_0$ EQ(5):  $\omega_2 = \omega_0 - \frac{5}{2} \frac{\mu_k g}{r} \left( \frac{2}{7} \frac{r \omega_0}{\mu_k g} \right)$   $\omega_2 = \frac{2}{7} \omega_0$ 

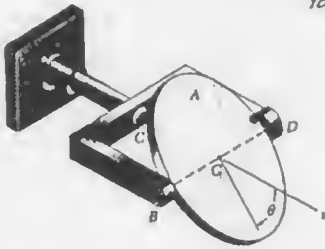
17.78

GIVEN: SPHERE OF RADIUS  $r$  PLACED ON FLOOR WITH VELOCITIES SHOWN IF FINAL VELOCITY IS TO BE ZEROFIND: (a)  $\omega_0$  IN TERMS OF  $\bar{v}_0$  AND  $r$  (b) TIME REQUIRED TO COME TO REST

SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA\_2

+ y COMPONENTS:  $N - W = 0$   $N = W = mg$  (1)+ x COMPONENTS:  $m \bar{v}_0 - F_k t = 0$   $F_k = m \bar{v}_0$  (2)+) MOMENTS ABOUT G:  $\bar{I} \omega_0 - (F_k t) r = 0$  (3)SUBSTITUTE FOR  $F_k$  (FROM EQ 2) AND  $\bar{I} = \frac{2}{5} m r^2$ EQ(3):  $\frac{2}{5} m r^2 \omega_0 - (m \bar{v}_0) r = 0$  $\omega_0 = \frac{5}{2} \frac{\bar{v}_0}{r}$ EQ(2):  $t = \frac{m \bar{v}_0}{F_k} = \frac{m \bar{v}_0}{\mu_k mg}$   $t = \frac{\bar{v}_0}{\mu_k g}$

17.79



GIVEN: DISK:  $W_0 = 2.5 \text{ lb}$ ,  $r = 4 \text{ in.}$   
 YOKE:  $W_y = 1.5 \text{ lb}$ ,  $\bar{r}_y = 3 \text{ in.}$   
 WHEN  $\theta = 0$ ,  $\omega_x = 120 \text{ rpm}$

FIND:  $\omega_x$  WHEN  $\theta = 90^\circ$

ROTATION ABOUT X AXIS:



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP.<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT THE X AXIS.  $\bar{I}_1 \omega_1 = \bar{I}_2 \omega_2$  (1)

$$\begin{aligned} \bar{I}_1 &= \bar{I}_{\text{disk}} + \bar{I}_{\text{yoke, } \theta=0} = m_y \bar{r}_y^2 + \frac{1}{2} m_D r^2 \\ &= \frac{1.5 \text{ lb}}{g} \left( \frac{3}{12} \text{ ft} \right)^2 + \frac{1}{2} \frac{2.5 \text{ lb}}{g} \left( \frac{4}{12} \text{ in} \right)^2 \\ &= \frac{0.09375}{g} + \frac{0.06944}{g} = 0.16319 \frac{1}{g} \end{aligned}$$

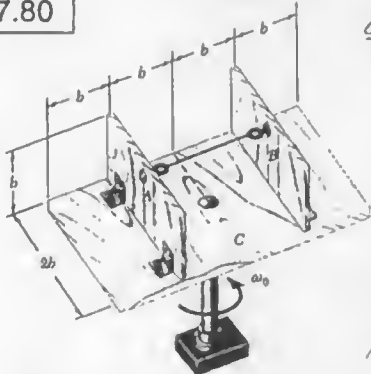
$$\begin{aligned} \bar{I}_2 &= \bar{I}_{\text{yoke}} + \bar{I}_{\text{disk, } \theta=90^\circ} = m_y \bar{r}_y^2 + \frac{1}{2} m_D r^2 \\ &= \frac{1.5 \text{ lb}}{g} \left( \frac{3}{12} \text{ ft} \right)^2 + \frac{1}{2} \frac{2.5 \text{ lb}}{g} \left( \frac{4}{12} \text{ in} \right)^2 = 0.23264 \frac{1}{g} \end{aligned}$$

$$\omega_1 = 120 \text{ rpm}$$

$$\text{EQ(1): } 0.16319 \frac{1}{g} (120 \text{ rpm}) = 0.23264 \frac{1}{g} \omega_2$$

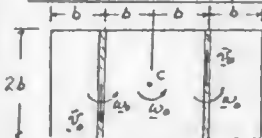
$$\omega_2 = 84.17 \text{ rpm} \quad \omega_2 = 84.2 \text{ rpm}$$

17.80

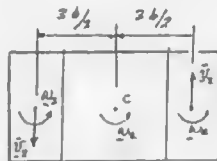


GIVEN: PANELS AND PLATE ARE MADE OF SAME MATERIAL AND ARE OF SAME THICKNESS. IN THE POSITION SHOWN ANGULAR VELOCITY  $= \omega_0$   
 FIND: AFTER WIDE BREAKS ANGULAR VELOCITY WHEN PANELS HAVE COME TO REST AGAINST PLATE

GEOMETRY AND KINEMATICS:



PANELS IN UP POSITION  
 $\bar{r}_0 = b \omega_0$



PANELS IN DOWN POSITION  
 $\bar{r}_0 = \frac{3}{2} b \omega_0$

LET  $\rho$  = MASS DENSITY,  $t$  = THICKNESS

$$\text{PLATE: } m_{\text{plate}} = \rho t (2b \times b) = 2 \rho t b^2$$

$$\bar{I}_{\text{plate}} = \frac{1}{12} (2 \rho t b^2) [(2b)^2 + (b)^2] = \frac{160}{12} \rho t b^4 = \frac{40}{3} \rho t b^4$$

(CONTINUED)

17.80 continued

$$\text{EACH PANEL: } m_{\text{panel}} = \rho t (b \times 2b) = 2 \rho t b^2$$

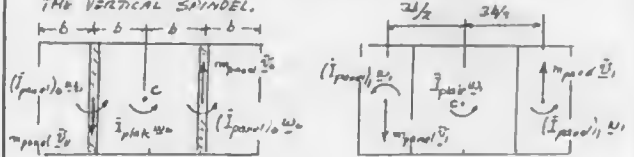
PANEL IN UP POSITION

$$(\bar{I}_{\text{panel}})_0 = \frac{1}{12} (2 \rho t b^2) (2b)^2 = \frac{8}{12} \rho t b^4 = \frac{2}{3} \rho t b^4$$

PANEL IN DOWN POSITION

$$(\bar{I}_{\text{panel}})_1 = \frac{1}{12} (2 \rho t b^2) [b^2 + (2b)^2] = \frac{10}{12} \rho t b^4 = \frac{5}{6} \rho t b^4$$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT THE VERTICAL SPINDEL.



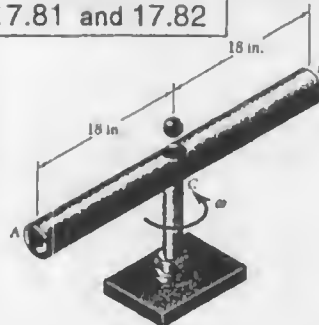
INITIAL MOMENTA

FINAL MOMENTA

1) MOMENTS ABOUT C:

$$\begin{aligned} \bar{I}_{\text{plate}} \omega_0 + 2 \left[ (\bar{I}_{\text{panel}})_0 \omega_0 + m_{\text{panel}} v_0 (b) \right] &= \bar{I}_{\text{plate}} \omega_1 + 2 \left[ (\bar{I}_{\text{panel}})_1 \omega_1 + m_{\text{panel}} v_1 \left( \frac{3}{2} b \right) \right] \\ \frac{40}{3} \rho t b^4 \omega_0 + 2 \left[ \frac{2}{3} \rho t b^4 \omega_0 + 2 \rho t b^2 (b \omega_0) (b) \right] &= \frac{40}{3} \rho t b^4 \omega_1 + 2 \left[ \frac{5}{6} \rho t b^4 \omega_1 + 2 \rho t b^2 \left( \frac{3}{2} b \omega_1 \right) \left( \frac{3}{2} b \right) \right] \\ \left[ \frac{40}{3} + \frac{4}{3} + 4 \right] \rho t b^4 \omega_0 &= \left[ \frac{40}{3} + \frac{10}{3} + 9 \right] \rho t b^4 \omega_1 \\ \frac{54}{3} \omega_0 = 24 \omega_1; \quad \omega_1 &= \frac{54}{3(24)} \omega_0 \quad \omega_1 = \frac{7}{9} \omega_0 \end{aligned}$$

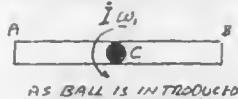
17.81 and 17.82



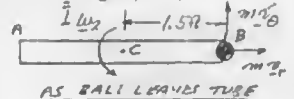
GIVEN: 4-16 TUBE AB INITIALLY  $\omega = 8 \text{ rad/s}$   
 BALLS INTRODUCED TO TUBE  
 PROBLEM 17.81:  
 FIND: (a)  $\omega$  AS A 0.8-16 BALL LEAVES TUBE  
 (b)  $\omega$  AS A SECOND 0.8-16 BALL LEAVES TUBE.  
 PROBLEM 17.82:

FIND:  $\omega$  AS A SINGLE 1.6-16 BALL LEAVES TUBE.

CONSERVATION OF MOMENTUM ABOUT C.



AS BALL IS INTRODUCED



AS BALL LEAVES TUBE

$$\begin{aligned} \text{1) MOMENTS ABOUT C: } \bar{I}_1 \omega_1 &= \bar{I}_2 \omega_2 + m_{\text{ball}} (1.5 \text{ ft}) v_2 \quad (1) \\ v_2 &= (1.5 \text{ ft}) \omega_2 \quad \bar{I} = \frac{1}{12} \frac{4}{9} (3 \text{ ft})^2 = \frac{3}{9} \end{aligned}$$

PROBLEM 17.82: (a) FIRST 0.8-16 BALL,  $\omega_1 = 8 \text{ rad/s}$

$$\text{EQ(1): } \frac{3}{9} (8 \text{ rad/s}) = \frac{3}{9} \omega_2 + \frac{0.8}{9} (1.5 \omega_2) (1.5 \text{ ft})$$

$$24 = (3 + 1.8) \omega_2 \quad \omega_2 = 5 \text{ rad/s}$$

AS FIRST BALL LEAVES TUBE:  $\omega = 5 \text{ rad/s}$

(b) SECOND 0.8-16 BALL,  $\omega_1 = 5 \text{ rad/s}$

$$\text{EQ(1): } \frac{3}{9} (5 \text{ rad/s}) = \frac{3}{9} \omega_2 + \frac{0.8}{9} (1.5 \omega_2) (1.5 \text{ ft})$$

$$15 = (3 + 1.8) \omega_2 \quad \omega_2 = 3.125 \text{ rad/s}$$

AS SECOND BALL LEAVES TUBE:  $\omega = 3.125 \text{ rad/s}$

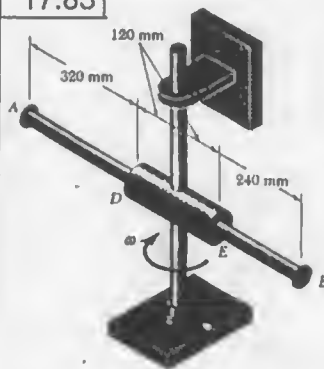
PROBLEM 17.83: A 1.6-16 BALL IS INTRODUCED,  $\omega_1 = 8 \text{ rad/s}$

$$\text{EQ(1): } \frac{3}{9} (8 \text{ rad/s}) = \frac{3}{9} \omega_2 + \frac{1.6}{9} (1.5 \omega_2) (1.5 \text{ ft})$$

$$24 = (3 + 3.6) \omega_2 \quad \omega_2 = 3.636 \text{ rad/s}$$

AS 1.6-16 BALL LEAVES THE TUBE:  $\omega = 3.64 \text{ rad/s}$

17.83



GIVEN: 3-RG ROD AB  
FOR CYLINDER DE:  $\bar{I} = 0.025 R_g \cdot m^2$   
IN POSITION SHOWN:  
 $\omega = 40 \text{ rad/s}$  A/D  
END B OF ROD IS MOVING  
TOWARD E AT  $76 \text{ mm/s}$ .

FIND: ANGULAR VELOCITY  
OF ASSEMBLY AS END B  
STRIKES CYLINDER AT E.

## KINEMATICS AND GEOMETRY

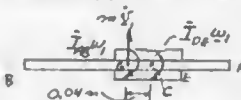


$$\bar{\omega}_1 = (0.04 \text{ m}) \omega_1 = (0.04 \text{ m}) (40 \text{ rad/s})$$

$$\bar{\omega}_1 = 1.6 \text{ rad/s}$$

## INITIAL POSITION

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT C.



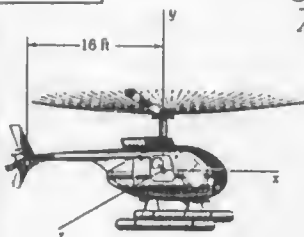
$$+ \text{MOMENTS ABOUT C: } \bar{I}_{AB} \omega_1 + m \bar{r}_{CD}^2 \omega_1 = \bar{I}_{AB} \omega_2 + m \bar{r}_{CD}^2 \omega_2 + \bar{I}_{CE} \omega_2$$

$$(0.16 \text{ kg} \cdot m^2) (40 \text{ rad/s}) + (3 \text{ kg}) (0.04 \text{ m})^2 (40 \text{ rad/s}) = (0.16 \text{ kg} \cdot m^2) \omega_2 + (3 \text{ kg}) (0.28 \text{ m})^2 \omega_2 + (0.025 \text{ kg} \cdot m^2) \omega_2$$

$$(6.4 + 0.48 + 1.00) = (0.16 + 0.2352 + 0.025) \omega_2$$

$$7.88 = 0.4202 \omega_2; \omega_2 = 18.75 \text{ rad/s}; \omega_2 = 18.75 \text{ rad/s}$$

17.84



GIVEN:  $\bar{I}_{CAB} = \bar{I}_C = 650 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
EACH BLADE WEIGHS  $55 \text{ lb}$   
INITIAL ANGULAR VELOCITY  
OF CAB = ZERO

FIND:  $\omega_C$  AS  $\omega_{\text{BLADES}}$  IS  
INCREASED FROM  $180 \text{ rpm}$   
TO  $240 \text{ rpm}$

$$(\omega_1)_1 = 180 \text{ rpm}$$

$$(\omega_1)_2 = 240 \text{ rpm}$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_2 = \text{SYST. MOMENTA}_2$$

$$\bar{I}_C = 4 \left[ \frac{55 \text{ lb}}{32.2} (14 \text{ ft})^2 \right] = 446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

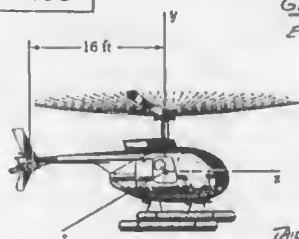
$$(\omega_2)_1 = (\omega_2)_2 = 240 \text{ rpm}; (\omega_2)_1 = (\omega_2)_2 = 240 \text{ rpm}$$

$$+ \text{MOMENTS ABOUT G: } \bar{I}_B (\omega_1)_1 + 0 = \bar{I}_B (\omega_1)_2 + \bar{I}_C (\omega_2)_2$$

$$(446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (180 \text{ rpm}) = (446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (\omega_2)_2 + (650 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (\omega_2)_2$$

$$(\omega_2)_2 = \frac{26784}{1096.4} = 24.4 \text{ rpm}$$

17.85



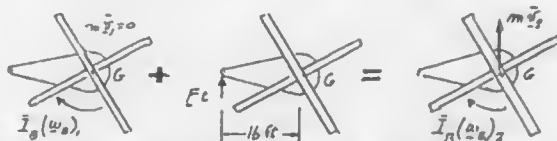
GIVEN:  $\bar{I}_{CAB} = \bar{I}_A = 650 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
EACH BLADE WEIGHS  $55 \text{ lb}$

$$W_{CAB} = 1250 \text{ lb}$$

TAIL PROPELLER PREVENTS  
ROTATION OF CAB AS  $\omega$  OF  
BLADES IS INCREASED FROM

$180 \text{ rpm}$  TO  $240 \text{ rpm}$  IN  $12 \text{ s}$ .

FIND: FORCE EXERTED BY  
TAIL PROPELLOR AND FINAL  $\bar{\omega}_C$ .



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_2 = \text{SYST. MOMENTA}_2$$

$$+ \text{MOMENTS ABOUT G: } \bar{I}_B (\omega_1)_1 + F \bar{r}_{CG} = \bar{I}_B (\omega_1)_2$$

$$+ \text{COMPLEMENTS: } 0 + F \bar{r}_{CG} = m \bar{r}_{CG}^2 \omega_2$$

$$\bar{I}_B = 4 \left[ \frac{55 \text{ lb}}{32.2} (14 \text{ ft})^2 \right] = 446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$m = m_C + m_B = \frac{1}{32.2} [1250 \text{ lb} + 4(55 \text{ lb})] = 45.65 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$(\omega_1)_1 = 180 \text{ rpm} = 18.85 \text{ rad/s}; (\omega_1)_2 = 240 \text{ rpm} = 25.13 \text{ rad/s}$$

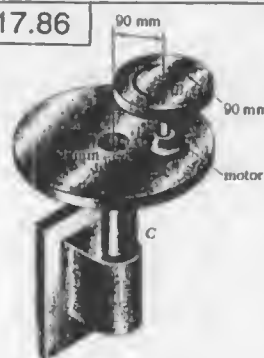
$$\text{EG(1): } (446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (18.85 \text{ rad/s}) + F(16 \text{ ft}) = (446.4 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (25.13 \text{ rad/s})$$

$$F \bar{r}_{CG} = 175.3 \text{ lb} \cdot \text{ft}$$

$$\text{EG(2): } 175.3 \text{ lb} \cdot \text{ft} = (45.65 \text{ lb} \cdot \text{s}^2/\text{ft}) \bar{\omega}_2; \bar{\omega}_2 = 3.84 \text{ rad/s}$$

$$\text{FOR } t = 12 \text{ s: } F \bar{r}_{CG} = F(12 \text{ s}) = 175.3 \text{ lb} \cdot \text{ft}; F = 14.61 \text{ lb}$$

17.86



GIVEN:  $m_B = 4 \text{ kg}$

$$\bar{I}_A = 0.20 \text{ kg} \cdot \text{m}^2$$

SYSTEM IS INITIALLY AT REST

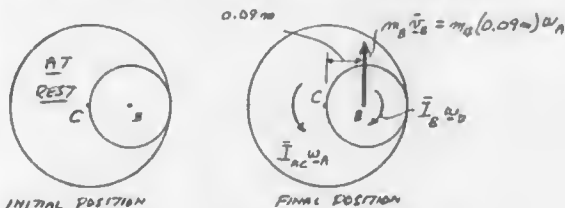
FIND:  $\omega_B$  AND  $\omega_A$  WHEN  
SPIND OF MOTOR REACHES  
 $360 \text{ rpm}$

$$\text{FOR DISK B}$$

$$\bar{I}_B = \frac{1}{2} (4 \text{ kg}) (0.09 \text{ m})^2$$

$$\bar{I}_B = 16.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT C



$$+ \text{MOMENTS ABOUT C: } \bar{I}_{AC} \omega_A + m_B \bar{r}_{CB}^2 \omega_B = \bar{I}_B \omega_B$$

$$(0.20 \text{ kg} \cdot \text{m}^2) \omega_A + (4 \text{ kg}) (0.09 \text{ m})^2 \omega_B = (16.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \omega_B$$

$$0.2324 \omega_A - 0.0362 \omega_B = 0$$

$$\omega_B = 14.346 \omega_A$$

$$\omega_{\text{MOTOR}} = \omega_A + \omega_B$$

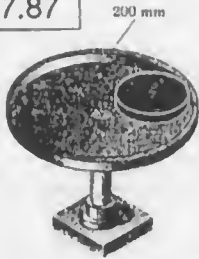
$$360 \text{ rpm} = \omega_A + 14.346 \omega_A$$

$$\omega_A = 23.5 \text{ rpm}$$

$$\omega_B = 337 \text{ rpm}$$

$$\text{EG(1): } \omega_B = 14.346 (23.5) = 336.55 \text{ rpm}$$

17.87



GIVEN: FOR 200-mm RADIUS PLATFORM-RIM UNIT:

$$m_p = 5 \text{ kg}, \quad \bar{R} = 175 \text{ mm}$$

INITIAL ANGULAR VELOCITY:  $\omega_1 = 50 \text{ rpm}$

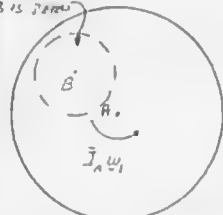
$$\text{DISK: } m_d = 2 \text{ kg}, \quad r_d = 50 \text{ mm}$$

DISK PLACED, WITH NO VELOCITY, ON PLATFORM

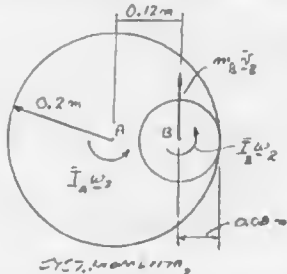
FIND: FINAL ANGULAR VELOCITY

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT

VELOCITY OF B IS ZERO



SYST. MOMENTUM,



SYST. MOMENTUM,

$$+ \text{ MOMENTS ABOUT A: } \bar{I}_A \omega_1 = \bar{I}_A \omega_2 + \bar{I}_B \omega_2 + m_d \bar{r}_B^2 (0.12 \text{ m}) \quad (1)$$

$$\bar{I}_A = m_p \bar{R}^2 = (5 \text{ kg})(0.175 \text{ m})^2 = 0.153125 \text{ kg} \cdot \text{m}^2$$

$$\bar{I}_B = \frac{1}{2} m_d r_d^2 = \frac{1}{2} (2 \text{ kg})(0.05 \text{ m})^2 = 9.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\bar{r}_B = 0.12 \text{ m}$$

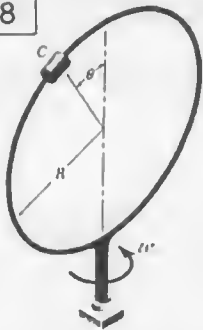
$$\text{EQU: } (0.153125 \text{ kg} \cdot \text{m}^2) \omega_1 = (0.153125 \text{ kg} \cdot \text{m}^2) \omega_2 + (9.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2) \omega_2 + (3 \text{ kg})(0.12 \text{ m})^2 \omega_2$$

$$0.153125 \omega_1 = 0.20593 \omega_2$$

$$\omega_2 = 0.7436 \omega_1 = 0.7436 (50 \text{ rpm})$$

$$\omega_2 = 37.2 \text{ rpm}$$

17.88



GIVEN: R-Ring COLLAR C

RING:  $m_R = 3 \text{ kg}$

$$R = 250 \text{ mm}$$

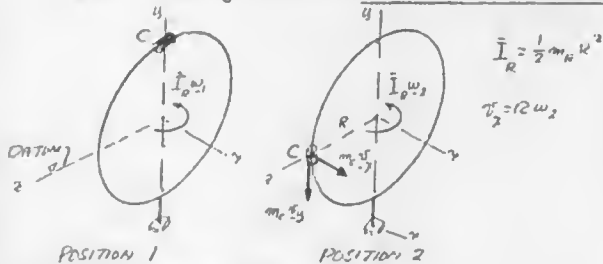
WHEN  $\theta = 0$ ,  $\omega_1 = 35 \text{ rad/s}$

AND  $v_C = 0$

FIND: (a)  $\omega$  WHEN  $\theta = 90^\circ$

(b) VELOCITY OF COLLAR RELATIVE TO RING WHEN  $\theta = 90^\circ$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT VERTICAL y AXIS AND CONSERVATION OF ENERGY



POSITION 1

POSITION 2

CONSERVATION OF ANGULAR MOMENTUM

MOMENTS ABOUT y AXIS:  $\bar{I}_R \omega_1 = \bar{I}_R \omega_2 + m_C \bar{r}_C^2 \omega_2$

$$\frac{1}{2} m_R R^2 \omega_1 = \frac{1}{2} m_R R^2 \omega_2 + m_C R^2 \omega_2$$

$$m_R R^2 \omega_1 = (m_R + 2 m_C) R^2 \omega_2$$

$$\omega_2 = \frac{m_R}{m_R + 2 m_C} \omega_1 \quad (1)$$

(CONTINUED)

17.88 continued

$$T_1 = \frac{1}{2} \bar{I}_R \omega_1^2 = \frac{1}{2} \left( \frac{1}{2} m_R R^2 \right) \omega_1^2 = \frac{1}{4} m_R R^2 \omega_1^2$$

$$V_1 = V_C R = m_C g R$$

$$T_2 = \frac{1}{2} \bar{I}_R \omega_2^2 + \frac{1}{2} m_C (\bar{r}_C^2 + R^2) \omega_2^2 = \frac{1}{4} m_R R^2 \omega_2^2 + \frac{1}{2} m_C R^2 \omega_2^2 + \frac{1}{2} m_C \bar{r}_C^2 \omega_2^2$$

$$V_2 = 0$$

CONSERVATION OF ENERGY:  $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{4} m_R R^2 \omega_1^2 + m_C g R = \left( \frac{1}{4} m_R + \frac{1}{2} m_C \right) R^2 \omega_2^2 + \frac{1}{2} m_C \bar{r}_C^2 \omega_2^2 \quad (2)$$

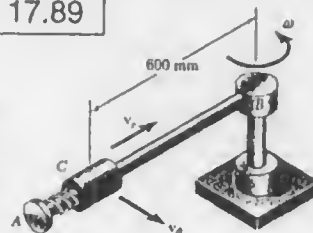
DATA:  $m_C = 2 \text{ kg}$ ,  $m_R = 3 \text{ kg}$ ,  $R = 0.25 \text{ m}$ ,  $\omega_1 = 35 \text{ rad/s}$

$$\text{EQ (1): } \omega_2 = \frac{3 \text{ kg}}{3 \text{ kg} + 2(2 \text{ kg})} (35 \text{ rad/s}) \quad \omega_2 = 15 \text{ rad/s}$$

$$\text{EQ (2): } \frac{1}{4} (3 \text{ kg}) (0.25 \text{ m})^2 (35 \text{ rad/s})^2 + (2 \text{ kg}) (2.8 \text{ m}) (0.25 \text{ m}) = \left[ \left( \frac{1}{4} (3 \text{ kg}) + \frac{1}{2} (2 \text{ kg}) \right) (0.25 \text{ m})^2 (15 \text{ rad/s})^2 + \frac{1}{2} (2 \text{ kg}) (0.15 \text{ m})^2 \omega_2^2 \right]$$

$$57.422 + 4.905 = 24.609 + \omega_2^2 \quad \omega_2^2 = 37.716 \quad \omega_2 = 6.14 \text{ rad/s}$$

17.89



GIVEN: IN POSITION, SHOWN

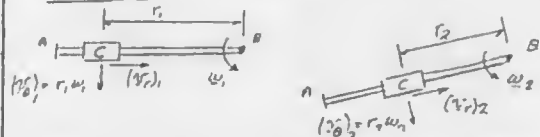
$$\omega_1 = 1.5 \text{ rad/s}, \quad (v_C)_1 = 1.5 \text{ m/s}$$

$m_C = 8 \text{ kg}$ , FOR ROD AND

$$\text{SPRING } \bar{I}_B = 1.2 \text{ kg} \cdot \text{m}^2$$

FIND: (a) MINIMUM DISTANCE BETWEEN C AND B, (b) CORRESPONDING ANGULAR VELOCITY

KINEMATICS



KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT B ARE ZERO,

WE HAVE:  $(H_B)_1 = (H_B)_2$ :  $\bar{I}_B \omega_1 + m_C (r_C)_1 r_1 = \bar{I}_B \omega_2 + m_C (r_C)_2 r_2$

$$(\bar{I}_B + m_C r_1^2) \omega_1 = (\bar{I}_B + m_C r_2^2) \omega_2 \quad (1)$$

$$[1.2 \text{ kg} \cdot \text{m}^2 + (8 \text{ kg})(0.6 \text{ m})^2] (1.5 \text{ rad/s}) = [1.2 \text{ kg} \cdot \text{m}^2 + (8 \text{ kg}) r_2^2] \omega_2 \quad (2)$$

CONSERVATION OF ENERGY SINCE  $v_1 = v_2$ , WE HAVE  $T_1 = T_2$

$$T_1 = \frac{1}{2} \bar{I}_B \omega_1^2 + \frac{1}{2} m_C (r_C)_1^2 \omega_1^2 + \frac{1}{2} m_C (r_1)^2 \omega_1^2 = \frac{1}{2} (1.2 \text{ kg} \cdot \text{m}^2) (1.5 \text{ rad/s})^2 + \frac{1}{2} (8 \text{ kg})(0.6 \text{ m})^2 (1.5 \text{ rad/s})^2 + \frac{1}{2} (8 \text{ kg})(1.5 \text{ m})^2 \omega_1^2$$

$$T_1 = 13.59 \text{ J}$$

$$T_2 = \frac{1}{2} \bar{I}_B \omega_2^2 + \frac{1}{2} m_C (r_C)_2^2 \omega_2^2 + \frac{1}{2} m_C (r_2)^2 \omega_2^2 = \frac{1}{2} (1.2 \text{ kg} \cdot \text{m}^2) \omega_2^2 + \frac{1}{2} (8 \text{ kg}) r_2^2 \omega_2^2 + \frac{1}{2} (8 \text{ kg}) (r_2)^2 \omega_2^2$$

$$T_2 = 0.6 \omega_2^2 + 4 r_2^2 \omega_2^2 + 4 (r_2)^2 \omega_2^2$$

$$T_1 = T_2: \quad 13.59 = (0.6 + 4 r_2^2) \omega_2^2 + 4 (r_2)^2 \omega_2^2 \quad (3)$$

$$\text{EQ (1): } 6.12 = (1.2 + 8 r_2^2) \omega_2 \quad \omega_2 = \frac{6.12}{1.2 + 8 r_2^2} = \frac{3.06}{0.6 + 4 r_2^2} \quad (4)$$

$$\text{EQ (2): } 13.59 = (0.6 + 4 r_2^2) \left[ \frac{3.06}{0.6 + 4 r_2^2} \right]^2 + 4 (r_2)^2 \left[ \frac{3.06}{0.6 + 4 r_2^2} \right]^2$$

FOR MINIMUM WE HAVE  $(r_C)_2 = 0$

$$13.59 = \frac{(3.06)^2}{0.6 + 4 r_2^2} \quad 0.154 + 5436 r_2^2 = 9.364$$

$$r_2^2 = 22.25 \times 10^{-3} \text{ m}^2$$

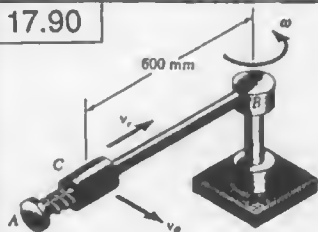
$$r_2 = 0.1492 \text{ m}$$

$$r_2 = 149.2 \text{ mm}$$

$$\text{EQ (4): } \omega_2 = \frac{3.06}{0.6 + 4 r_2^2} = \frac{3.06}{0.6 + 4 (22.25 \times 10^{-3})} = 4.441 \text{ rad/s}$$

$$\omega_2 = 4.44 \text{ rad/s}$$

17.90



GIVEN: IN POSITION SHOWN

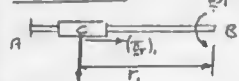
$$\omega_1 = 1.5 \text{ rad/s}$$

$$m_C = 8 \text{ kg}$$

$$\text{FOR ROD + SPRING } I_B = 1.2 \text{ kg}\cdot\text{m}^2$$

FIND:  $\omega_2$  IF MINIMUM DISTANCE FROM COLLAR TO B IS: TO BE 300 mm

KINEMATICS



$$(v_1)_\theta = r_1 \omega_1$$

KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT B ARE ZERO,  $(H_B)_1 = (H_B)_2$ 

$$I_B \omega_1 + m_C (v_1)_\theta r_1 = I_B \omega_2 + m_C (v_2)_\theta r_2$$

$$(I_B + m_C r_1^2) \omega_1 = (I_B + m_C r_2^2) \omega_2$$

$$\text{DATA: } I_B = 1.2 \text{ kg}\cdot\text{m}^2, m_C = 8 \text{ kg}, r_1 = 0.6 \text{ m}, r_2 = r_{\text{min}} = 0.3 \text{ m}$$

$$[1.2 \text{ kg}\cdot\text{m}^2 + (8 \text{ kg})(0.6 \text{ m})^2](1.5 \text{ rad/s}) = [1.2 \text{ kg}\cdot\text{m}^2 + (8 \text{ kg})(0.3 \text{ m})^2] \omega_2$$

$$6.12 = 1.92 \omega_2$$

$$\omega_2 = 3.1875 \text{ rad/s}$$

CONSERVATION OF ENERGY

SINCE  $v_1 = v_2$ , WE HAVE  $T_1 = T_2$ 

$$T_1 = \frac{1}{2} I_B \omega_1^2 + \frac{1}{2} m_C (v_1)_\theta^2 + \frac{1}{2} m (v_1)_r^2$$

$$= \frac{1}{2} (1.2 \text{ kg}\cdot\text{m}^2)(1.5 \text{ rad/s})^2 + \frac{1}{2} (8 \text{ kg})(0.6 \text{ m})^2 (1.5 \text{ rad/s})^2 + \frac{1}{2} (8 \text{ kg})(v_1)_r^2$$

$$T_1 = 4.59 \text{ J} + 4(v_1)_r^2$$

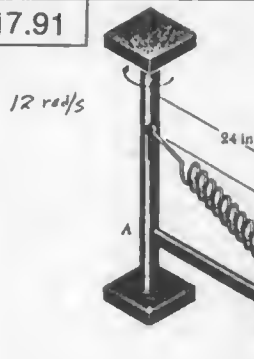
$$T_2 = \frac{1}{2} I_B \omega_2^2 + \frac{1}{2} m_C (v_2)_\theta^2 + \frac{1}{2} m_C (v_2)_r^2$$

$$= \frac{1}{2} (1.2 \text{ kg}\cdot\text{m}^2)(3.1875 \text{ rad/s})^2 + \frac{1}{2} (8 \text{ kg})(0.3 \text{ m})^2 (3.1875 \text{ rad/s})^2 + 0$$

$$T_2 = 9.754 \text{ J}$$

$$T_1 = T_2: 4.59 \text{ J} + 4(v_1)_r^2 = 9.754 \text{ J}; (v_1)_r = 1.136 \text{ m/s}$$

17.91

GIVEN:  $\omega_1 = 6 \text{ rad/s}$ 

MASS MOMENT OF INERTIA OF ROD

AB AND SPRING ABOUT A,

$$I_A = 0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

SPRING:  $k = 15 \text{ lb/in.}$  AND

UNDEFORMED LENGTH = 10 in.

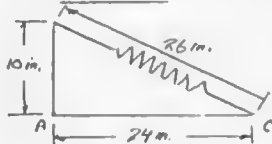
$$\omega_1 = 12 \text{ rad/s}$$

 $(v_1)_\theta = 0$  OF COLLAR.WHEN  $AC = 7.5 \text{ in.}$ FIND: (a)  $\omega_2$ (b)  $(v_2)_\theta$  OF

COLLAR.

POTENTIAL ENERGY OF SPRING UNDEFORMED LENGTH = 10 in.

POSITION 1:



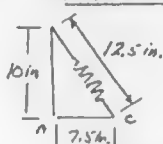
$$\Delta = 26 \text{ in.} - 10 \text{ in.} = 16 \text{ in.}$$

$$V_1 = \frac{1}{2} k \Delta^2 = \frac{1}{2} (15 \text{ lb/in.})(16 \text{ in.})^2$$

$$= 1920 \text{ in}\cdot\text{lb}$$

$$V_1 = 160 \text{ ft}\cdot\text{lb}$$

POSITION 2:



$$\Delta = 12.5 \text{ in.} - 10 \text{ in.} = 2.5 \text{ in.}$$

$$V_2 = \frac{1}{2} k \Delta^2 = \frac{1}{2} (15 \text{ lb/in.})(2.5 \text{ in.})^2$$

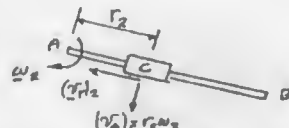
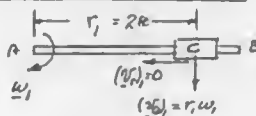
$$= 46.875 \text{ in}\cdot\text{lb}$$

$$V_2 = 3.91 \text{ ft}\cdot\text{lb}$$

(CONTINUED)

17.91 continued

KINEMATICS:

KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT SHFT AT A ARE ZERO,  $(H_A)_1 = (H_A)_2$ 

$$I_A \omega_1 + m_C (v_1)_\theta r_1 = I_A \omega_2 + m_C (v_2)_\theta r_2$$

$$(I_A + m_C r_1^2) \omega_1 = (I_A + m_C r_2^2) \omega_2$$

$$\text{DATA: } I_A = 0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2, m_C = \frac{6 \text{ lb}}{32.2}, r_1 = 2 \text{ ft}, r_2 = \frac{7.5}{12} \text{ ft}, \omega_1 = 12 \text{ rad/s}$$

$$[0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6 \text{ lb}}{32.2} (2 \text{ ft})^2](12 \text{ rad/s}) = [0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6 \text{ lb}}{32.2} (\frac{7.5}{12} \text{ ft})^2] \omega_2$$

$$13.1441 = 0.4227 \omega_2; \omega_2 = 31.087 \text{ rad/s}; \omega_2 = 31.1 \text{ rad/s}$$

CONSERVATION OF ENERGY

$$T_1 = \frac{1}{2} I_A \omega_1^2 + \frac{1}{2} m_C (v_1)_\theta^2 + \frac{1}{2} m_C (v_1)_r^2$$

$$= \frac{1}{2} (0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(12 \text{ rad/s})^2 + \frac{1}{2} (\frac{6 \text{ lb}}{32.2})(2 \text{ ft})^2 (12 \text{ rad/s})^2 + 0$$

$$T_1 = 78.965 \text{ ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2} I_A \omega_2^2 + \frac{1}{2} m_C (v_2)_\theta^2 + \frac{1}{2} m_C (v_2)_r^2$$

$$= \frac{1}{2} (0.35 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(31.087 \text{ rad/s})^2 + \frac{1}{2} (\frac{6 \text{ lb}}{32.2})(\frac{7.5}{12} \text{ ft})^2 (31.087 \text{ rad/s})^2 + \frac{1}{2} (\frac{6 \text{ lb}}{32.2})(v_2)_r^2$$

$$T_2 = 704.32 + 0.09317 (v_2)_r^2$$

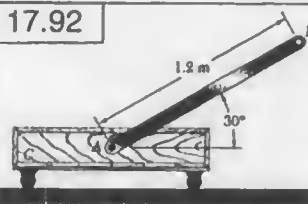
RECALL:  $V_1 = 160 \text{ ft}\cdot\text{lb}$  AND  $V_2 = 3.91 \text{ ft}\cdot\text{lb}$ 

$$T_1 + V_1 = T_2 + V_2: 78.965 + 160 = 704.32 + 0.09317 (v_2)_r^2 + 3.91$$

$$30.638 = 0.09317 (v_2)_r^2$$

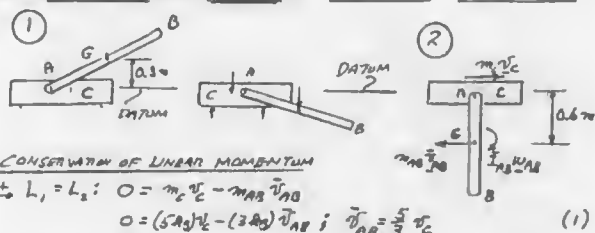
$$(v_2)_r = 18.13 \text{ ft/s}$$

17.92

GIVEN:  $m_{AB} = 3.4 \text{ kg}$ 

$$m_C = 5 \text{ kg}$$

SYSTEM RELEASED FROM REST

FIND: (a)  $\vec{v}_B$  AS AB IS VERTICAL  
(b)  $\vec{v}_C$ 

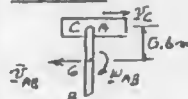
CONSERVATION OF LINEAR MOMENTUM

$$\pm L_1 = L_2: 0 = m_C v_C - m_{AB} \bar{v}_{AB}$$

$$0 = (5 \text{ kg}) v_C - (3.4 \text{ kg}) \bar{v}_{AB}; \bar{v}_{AB} = \frac{5}{3.4} v_C$$

(1)

KINEMATICS:



$$\bar{v}_{AB} = -v_C + 0.6 \omega_{AB}$$

$$\frac{5}{3.4} v_C = -v_C + 0.6 \omega_{AB}$$

$$\frac{8.5}{3.4} v_C = 0.6 \omega_{AB}; v_C = 0.215 \omega_{AB}$$

$$\text{EG. (1): } \bar{v}_{AB} = \frac{5}{3.4} v_C = \frac{5}{3.4} (0.215 \omega_{AB}); \bar{v}_{AB} = 0.375 \omega_{AB}$$

CONSERVATION OF ENERGY

$$V_1 = m_{AB} g (0.3 \text{ m}) = (3.4 \text{ kg})(9.81 \text{ m/s}^2)(0.3 \text{ m}) = 8.229 \text{ J}$$

$$V_2 = m_{AB} g (0.6 \text{ m}) = (3.4 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) = 17.658 \text{ J}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m_C v_C^2 + \frac{1}{2} m_{AB} \bar{v}_{AB}^2 + \frac{1}{2} I_{AB} \omega_{AB}^2$$

$$= \frac{1}{2} (5 \text{ kg})(0.215 \omega_{AB})^2 + \frac{1}{2} (3.4 \text{ kg})(0.375 \omega_{AB})^2 + \frac{1}{2} (\frac{1}{12} (3.4 \text{ kg})(1.2 \text{ m})^2) \omega_{AB}^2$$

$$T_2 = (0.1266 + 0.2109 + 0.1800) \omega_{AB}^2 = 0.5175 \omega_{AB}^2$$

$$T_1 + V_1 = T_2 + V_2: 0 + 8.229 = 0.5175 \omega_{AB}^2 + 17.658$$

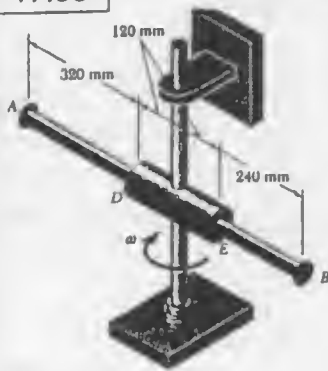
$$-9.429 = 0.5175 \omega_{AB}^2; \omega_{AB} = 7.154 \text{ rad/s}$$

$$v_C = 0.215 \omega_{AB} = 0.215 (7.154) \quad v_C = 1.610 \text{ m/s}$$

$$\bar{v}_B = \bar{v}_C + (\bar{v}_B)_\theta = -1.610 + (1.2 \text{ m})(7.154 \text{ rad/s}) \quad \bar{v}_B = 6.975 \text{ m/s}$$

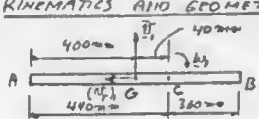


17.93



GIVEN: 3-kg ROD AB  
FOR CYLINDER DE:  $I = 0.025 \text{ kg} \cdot \text{m}^2$   
IN POSITION SHOWN  
 $\omega = 40 \text{ rad/s}$  AND  
END B OF ROD IS MOVING  
TOWARD E AT  $75 \text{ mm/s}$   
FIND: VELOCITY OF AB  
RELATIVE TO DE AS  
END B STRIKES END E  
OF THE CYLINDER

## KINEMATICS AND GEOMETRY

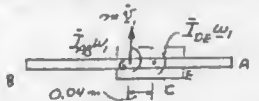


$$\vec{v}_E = (0.04 \text{ m}) \omega_1 = (0.04 \text{ m})(40 \text{ rad/s})$$

$$\vec{v}_E = 1.6 \text{ m/s}$$

## INITIAL POSITION

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT C.



+) MOMENTS ABOUT C:  $\vec{I}_{AB} \omega_1 + m \vec{v}_E (0.04 \text{ m}) = \vec{I}_{AB} \omega_2 + m \vec{v}_E (0.28 \text{ m}) + \vec{I}_{DE} \omega_2$

$$(0.16 \text{ kg} \cdot \text{m}^2)(40 \text{ rad/s}) + (3 \text{ kg})(1.6 \text{ m/s})(0.04 \text{ m}) = (0.16 \text{ kg} \cdot \text{m}^2) \omega_2 + (3 \text{ kg})(1.6 \text{ m/s})(0.28 \text{ m}) + (0.025 \text{ kg} \cdot \text{m}^2) \omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.84 + 0.025) \omega_2$$

$$7.592 = 0.995 \omega_2; \quad \omega_2 = 7.63 \text{ rad/s}; \quad \omega_2 = 7.63 \text{ rad/s}$$

CONSERVATION OF ENERGY ( $v_E = 0.075 \text{ m/s}$ )

$$V_1 = V_2 = 0$$

$$T_1 = \frac{1}{2} \vec{I}_{AB} \omega_1^2 + \frac{1}{2} \vec{I}_{AB} \omega_1^2 + \frac{1}{2} m_{AB} v_E^2 + \frac{1}{2} m_{AB} (v_E)^2$$

$$= \frac{1}{2} (0.025 \text{ kg} \cdot \text{m}^2)(40 \text{ rad/s})^2 + \frac{1}{2} (0.16 \text{ kg} \cdot \text{m}^2)(40 \text{ rad/s})^2$$

$$+ \frac{1}{2} (3 \text{ kg})(1.6 \text{ m/s})^2 + \frac{1}{2} (3 \text{ kg})(0.075 \text{ m/s})^2$$

$$T_1 = 20 \text{ J} + 128 \text{ J} + 3.84 \text{ J} + 0.009 \text{ J} = 151.85 \text{ J}$$

$$\vec{v}_E = (0.28 \text{ m}) \omega_2 + (0.04 \text{ m})(7.63 \text{ rad/s}) = 5.059 \text{ m/s}$$

$$T_2 = \frac{1}{2} \vec{I}_{AB} \omega_2^2 + \frac{1}{2} \vec{I}_{AB} \omega_2^2 + \frac{1}{2} m_{AB} v_E^2 + \frac{1}{2} m_{AB} (v_E)^2$$

$$= \frac{1}{2} (0.025 \text{ kg} \cdot \text{m}^2)(7.63 \text{ rad/s})^2 + \frac{1}{2} (0.16 \text{ kg} \cdot \text{m}^2)(7.63 \text{ rad/s})^2$$

$$+ \frac{1}{2} (3 \text{ kg})(5.059 \text{ m/s})^2 + \frac{1}{2} (3 \text{ kg})(v_E)^2$$

$$T_2 = 4.081 \text{ J} + 26.116 \text{ J} + 38.371 \text{ J} + 1.5 (v_E)^2$$

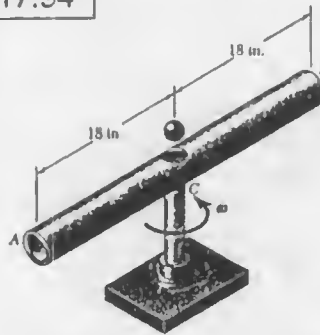
$$T_2 = 68.567 \text{ J} + 1.5 (v_E)^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 151.85 \text{ J} + 0 = 68.567 \text{ J} + 1.5 (v_E)^2$$

$$83.283 = 1.5 (v_E)^2$$

$$(v_E)_2 = 7.45 \text{ m/s}$$

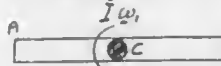
17.94



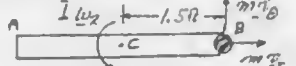
GIVEN: 4-1b TUBE AB  
INITIALLY  $\omega_1 = 8 \text{ rad/s}$

AN 0.8-1b BALL IS  
INTRODUCED INTO TUBE  
AND LEAVE TUBE AT B.  
A SECOND 0.8-1b BALL  
IS THEN PUT INTO TUBE  
FIND: VELOCITY OF  
EACH BALL RELATIVE TO  
TUBE AS IT LEAVES THE  
TUBE

## CONSERVATION OF MOMENTUM ABOUT C.



AS BALL IS INTRODUCED



AS BALL LEAVES TUBE

MOMENTS ABOUT C:  $\vec{I}_{AB} \omega_1 = \vec{I}_{AB} \omega_2 + m_B v_B (1.5 \text{ ft})$  (1)

$$v_B = (1.5 \text{ ft}) \omega_2 \quad \vec{I} = \frac{1}{12} (4 \text{ lb})(3 \text{ ft})^2 = \frac{3}{2}$$

FIRST 0.8-1b BALL,  $\omega_1 = 8 \text{ rad/s}$

EQ (1):  $\frac{3}{2} (8 \text{ rad/s}) = \frac{3}{2} \omega_2 + \frac{0.8 \text{ lb}}{9} (1.5 \omega_2)(1.5 \text{ ft})$

$$24 = (3 + 1.6) \omega_2 \quad \omega_2 = 5 \text{ rad/s}$$

AS FIRST BALL LEAVES TUBE:  $\omega = 5 \text{ rad/s}$

SECOND 0.8-1b BALL,  $\omega_1 = 5 \text{ rad/s}$

EQ (1):  $\frac{3}{2} (5 \text{ rad/s}) = \frac{3}{2} \omega_2 + \frac{0.8 \text{ lb}}{9} (1.5 \omega_2)(1.5 \text{ ft})$

$$15 = (3 + 1.6) \omega_2 \quad \omega_2 = 3.125 \text{ rad/s}$$

AS SECOND BALL LEAVES TUBE:  $\omega = 3.125 \text{ rad/s}$

## CONSERVATION OF ENERGY

FIRST BALL:  $\omega_1 = 8 \text{ rad/s}$ ,  $\omega_2 = 5 \text{ rad/s}$

$$V_1 = 0, \quad T_1 = \frac{1}{2} \vec{I} \omega_1^2 = \frac{1}{2} \left( \frac{3}{2} \right) (8 \text{ rad/s})^2 = \frac{96}{2}$$

$$V_2 = 0, \quad T_2 = \frac{1}{2} \vec{I} \omega_2^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_r^2$$

$$= \frac{1}{2} \left( \frac{3}{2} \right) (5 \text{ rad/s})^2 + \frac{1}{2} \left( \frac{0.8 \text{ lb}}{9} \right) (1.5)^2 (5 \text{ rad/s})^2 + \frac{1}{2} \left( \frac{0.8 \text{ lb}}{9} \right) v_r^2$$

$$T_2 = \frac{37.5}{2} + \frac{22.5}{2} + \frac{0.4}{9} v_r^2$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{96}{2} + 0 = \frac{37.5}{2} + \frac{22.5}{2} + \frac{0.4}{9} v_r^2 + 0$$

$$v_r^2 = 90$$

$$v_r = 9.49 \text{ ft/s}$$

SECOND BALL  $\omega_1 = 5 \text{ rad/s}$ ,  $\omega_2 = 3.125 \text{ rad/s}$

$$V_1 = 0, \quad T_1 = \frac{1}{2} \vec{I} \omega_1^2 = \frac{1}{2} \left( \frac{3}{2} \right) (5 \text{ rad/s})^2 = \frac{37.5}{2}$$

$$V_2 = 0, \quad T_2 = \frac{1}{2} \vec{I} \omega_2^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_r^2$$

$$= \frac{1}{2} \left( \frac{3}{2} \right) (3.125 \text{ rad/s})^2 + \frac{1}{2} \left( \frac{0.8 \text{ lb}}{9} \right) (1.5)^2 (3.125 \text{ rad/s})^2 + \frac{1}{2} \left( \frac{0.8 \text{ lb}}{9} \right) v_r^2$$

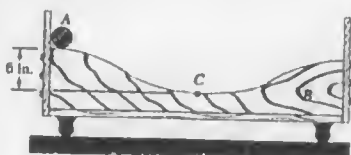
$$= \frac{14.6875}{2} + \frac{6.789}{2} + \frac{0.4}{9} v_r^2$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{37.5}{2} = \frac{14.6875}{2} + \frac{6.789}{2} + \frac{0.4}{9} v_r^2$$

$$v_r^2 = 35.156$$

$$v_r = 5.93 \text{ ft/s}$$

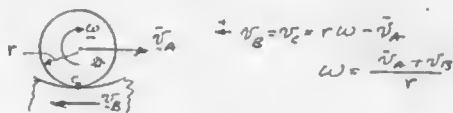
17.95

GIVEN:  $W_0 = 6 lb$  $W_0 = 10 lb$ 

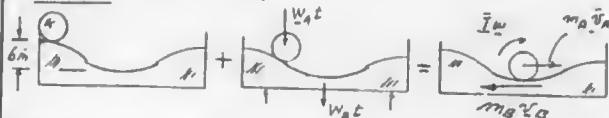
CYLINDER RELEASED FROM REST.

FIND:  $\vec{v}_B$  AS CYLINDER PASSES POINT C

KINEMATICS (WHEN CYLINDER IS PASSING C)



KINETICS:



SYST. OF MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA

 $\pm \sum \text{COMPONENTS: } m_A \vec{v}_A - m_B \vec{v}_B = 0$ 

$$\frac{6 lb}{g} \vec{v}_A = \frac{10 lb}{g} \vec{v}_B; \quad \vec{v}_B = 0.6 \vec{v}_A$$

PRINCIPLE OF WORK-ENERGY

$$U_{1 \rightarrow 2} = V_A (6 in.) = (6 lb) \left( \frac{6}{12} ft \right) = 3 ft \cdot lb; \quad T_1 = 0$$

$$T_2 = \frac{1}{2} m_A \vec{v}_A^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} m_B \vec{v}_B^2$$

$$\vec{v}_B = 0.6 \vec{v}_A; \quad \omega = \frac{\vec{v}_A + \vec{v}_B}{r} = \frac{\vec{v}_A + 0.6 \vec{v}_A}{r} = \frac{1.6 \vec{v}_A}{r}$$

$$T_2 = \frac{1}{2} \left( \frac{6 lb}{g} \right) \vec{v}_A^2 + \frac{1}{2} \left[ \frac{1}{2} \left( \frac{6 lb}{g} \right) r^2 \right] \left( \frac{1.6 \vec{v}_A}{r} \right)^2 + \frac{1}{2} \left( \frac{10 lb}{g} \right) (0.6 \vec{v}_A)^2$$

$$= \frac{3}{g} \vec{v}_A^2 + \frac{3.84}{g} \vec{v}_A^2 + \frac{1.8}{g} \vec{v}_A^2 = \frac{12.48}{g} \vec{v}_A^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + 3 ft \cdot lb = \frac{12.48}{g} \vec{v}_A^2$$

$$\vec{v}_A^2 = 11.181$$

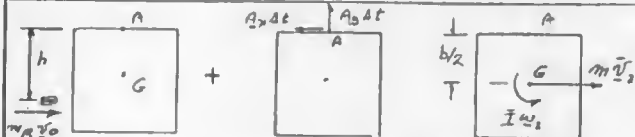
$$\vec{v}_A = 3.344 ft/s \rightarrow$$

$$\vec{v}_B = 0.6 \vec{v}_A = 2.01 ft/s \leftarrow$$

17.96 and 17.97

GIVEN: BULLET,  $m_B = 45 g$  $v_0 = 400 m/s$ PLATE:  $m = 9 kg$ ,  $b = 200 mm$ PROB. 17.96: FOR  $h = 200 mm$ FIND: (a)  $\vec{v}$  JUST AFTER IMPACT(b)  $A_x$  IF  $\Delta t = 2 ms$ 

PROB. 17.97: FIND

FIND: (a)  $h$  FOR  $A_x = 0$ (b)  $\vec{v}$  JUST AFTER IMPACT

SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA

$$\pm \sum \text{MOMENTS ABOUT A: } m_B v_0 h = \bar{I} \omega + m \vec{v} \frac{b}{2} \quad (1)$$

$$\pm \sum \text{COMPONENTS: } m_B v_0 - A_x \Delta t = m \vec{v} \quad (2)$$

$$\text{ROTATION ABOUT A: } \vec{v} = \frac{b}{2} \omega \quad \bar{I} = \frac{1}{8} m b^2 \quad (3)$$

(CONTINUED)

17.96 and 17.97 continued

SUBSTITUTE FROM (3) INTO (1)

$$m_B v_0 h = \frac{1}{8} m b^2 \omega + m \left( \frac{b}{2} \omega \right) \frac{b}{2}$$

$$m_B v_0 h = \frac{5}{12} m b^2 \omega \quad (4)$$

$$\text{EQ (2): } A_x \Delta t = m_B v_0 - m \vec{v} \quad (5)$$

DATA:  $m_B = 0.045 kg$ ,  $v_0 = 400 m/s$ ,  $b = 0.2 m$ ,  $m = 9 kg$ ,  $\Delta t = 0.002 s$ PROBLEM 17.96 FOR  $h = 0.2 m$ 

$$\text{EQ (4): } (0.045 kg)(400 m/s)(0.2 m) = \frac{5}{12} (9 kg)(0.2 m)^2 \omega$$

$$\omega = 24 rad/s$$

$$(a) \vec{v} = \frac{b}{2} \omega = \frac{0.2 m}{2} (24 rad/s) \quad \vec{v} = 2.4 m/s \rightarrow$$

$$(b) \text{EQ (5): } A_x (0.002 s) = (0.045 kg)(400 m/s) - (9 kg)(2.4 m/s)$$

$$0.002 A_x = 18 - 21.6 \quad A_x = -1.8 kN \quad A_x = 1.8 kN \rightarrow$$

PROBLEM 17.97 FOR  $A_x = 0$ 

$$\text{EQ (5): } m_B v_0 = m \vec{v}; \quad m_B v_0 = m \left( \frac{b}{2} \omega \right); \quad \omega = 2 \frac{m_B v_0}{m b}$$

$$\text{SUBSTITUTE INTO (4): } m_B v_0 h = \frac{5}{12} m b^2 \left( 2 \frac{m_B v_0}{m b} \right)$$

$$(a) \quad h = \frac{5}{6} b = \frac{5}{6} (200 mm) \quad h = 166.7 mm$$

$$(b) \quad m_B v_0 = m \vec{v}; \quad (0.045 kg)(400 m/s) = (9 kg) \vec{v}; \quad \vec{v} = 2 m/s \rightarrow$$

17.98



GIVEN:

BULLET:  $W_B = 0.08 lb$ ,  $v_0 = 1800 ft/s$ PLATE:  $W = 15 lb$ ,  $L = 30 in.$  $h = 12 in.$ FIND: (a)  $\omega$  JUST AFTER IMPACT(b)  $C_x$  FOR  $\Delta t = 0.001 s$ 

$$\bar{I} = \frac{1}{12} m L^2 = \frac{1}{12} \left( \frac{15 lb}{32.2} \right) (30 in.)^2$$

$$I = 0.24262 lb \cdot ft \cdot s^2$$

$$\vec{v}_2 = (0.75 ft) \omega$$

SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA

$$\pm \sum \text{MOMENTS ABOUT C: } m_B v_0 (2 ft) = \bar{I} \omega + m \vec{v}_2 (0.25 ft)$$

$$\left( \frac{0.08 lb}{32.2} \right) (1800 ft/s) (2 ft) = (0.24262 lb \cdot ft \cdot s^2) \omega + \left( \frac{15 lb}{32.2} \right) (0.25 ft) \omega$$

$$6.708 = (0.24262 + 0.02911) \omega$$

$$(a) \quad \omega = 24.68 rad/s \quad \omega = 24.7 rad/s \rightarrow$$

(b)  $\pm \sum \text{COMPONENTS:}$ 

$$C_x \Delta t - m_B v_0 = -m \vec{v}_2$$

$$C_x \Delta t = m_B v_0 - m (0.25 ft) \omega$$

$$= \left( \frac{0.08 lb}{32.2} \right) (1800 ft/s) - \left( \frac{15 lb}{32.2} \right) (0.25 ft) (24.68 rad/s)$$

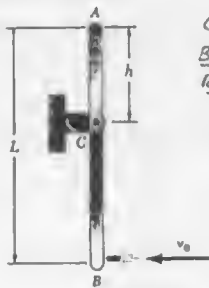
$$C_x \Delta t = 1.597 lb \cdot s$$

$$\Delta t = 0.001 s$$

$$C_x (0.001 s) = 1.597 lb \cdot s$$

$$C_x = 1.597 lb \rightarrow$$

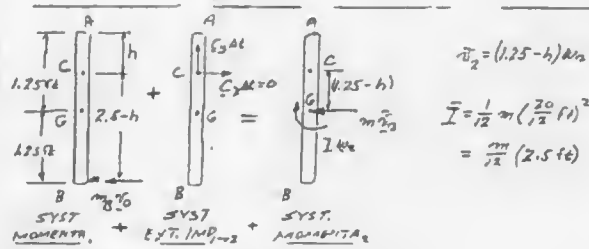
17.99



GIVEN:

BULLET:  $W_b = 0.08 \text{ lb}$ ,  $v_0 = 1800 \text{ ft/s}$   
 ROD:  $W = 15 \text{ lb}$ ,  $L = 30 \text{ in.}$

FIND: (a)  $h$  FOR  $C_2 = 0$   
 (b) CORRESPONDING  
 $\omega$  JUST AFTER  
 IMPACT



$$\bar{v}_2 = (1.25 - h)\omega_2$$

$$\bar{I} = \frac{1}{12} m \left( \frac{30}{12} \text{ ft} \right)^2 = \frac{m}{12} (2.5 \text{ ft})^2$$

SYST. MOMENTA. + SYST. EXT. IMP.  $\rightarrow$  = SYST. MOMENTA.  
 $\rightarrow$  X COMPONENTS:  $m_b v_0 = m \bar{v}_2$   
 $m_b v_0 = m(1.25 - h)\omega_2$  (1)

+ MOMENTS ABOUT G:  $m_b v_0 (1.25 \text{ ft}) = \bar{I} \omega_2$  (2)

SUBSTITUTE FOR  $m_b v_0$  FROM (1) INTO (2):

(a)  $\left[ m_b (1.25 - h)\omega_2 \right] (1.25 \text{ ft}) = \frac{m}{12} (2.5 \text{ ft})^2 \omega_2$   
 $(1.25 - h) = \frac{2.5^2}{12(1.25)} = 0.4167$ ;  $h = 0.8333 \text{ ft} = 10 \text{ in.}$

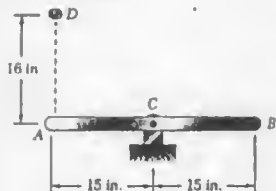
(b) FGI:  $\frac{0.08 \text{ lb}}{32} (1800 \text{ ft/s}) = \frac{15 \text{ lb}}{32} (1.25 - 0.8333)\omega_2$   
 $144 = 6.25 \omega_2$   $\omega_2 = 23.0 \text{ rad/s}$

17.100

GIVEN: 0.6-lb MAGNET D

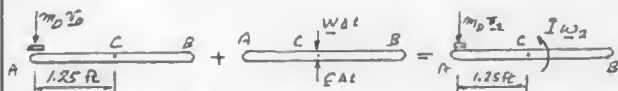
B-lb ROD AB

MAGNET RELEASED FROM  
 POSITION SHOWN

FIND:  $\omega$  AFTER IMPACT ( $C=0$ ) $v_D$  AFTER IMPACT ( $C=0$ )

MAGNET STRIKES BAR WITH VELOCITY  $v_0$

$$v_0 = \sqrt{2gh} = \sqrt{2(32.2 \text{ ft/s}^2)(16/12 \text{ ft})} = 9.266 \text{ ft/s} \downarrow$$



SYST. MOMENTA. + SYST. EXT. IMP.  $\rightarrow$  = SYST. MOMENTA.  
 $v_2 = (1.25 \text{ ft}) \omega_2$

(a) + MOMENTS ABOUT C:  $m_D v_0 (1.25 \text{ ft}) = \bar{I} \omega_2 + m_D v_2 (1.25 \text{ ft})$   
 $\frac{0.6 \text{ lb}}{32} (9.266 \text{ ft/s})(1.25 \text{ ft}) = \frac{1}{12} \frac{B \text{ lb}}{32} (2.5 \text{ ft})^2 \omega_2 + \frac{0.6 \text{ lb}}{32} (1.25 \text{ ft})^2 \omega_2$   
 $6.9498 = (4.1667 + 0.9375) \omega_2$   
 $6.9498 = 5.1042 \omega_2$   
 $\omega_2 = 1.3616 \text{ rad/s}$   $\omega_2 = 1.362 \text{ rad/s}$

(b)  $v_A = v_B = (1.25 \text{ ft}) \omega_2 = (1.25 \text{ ft})(1.3616 \text{ rad/s}) = 1.7020 \text{ ft/s}$   
 $v_A = 1.702 \text{ ft/s} \downarrow$

17.101 and 17.102

GIVEN:

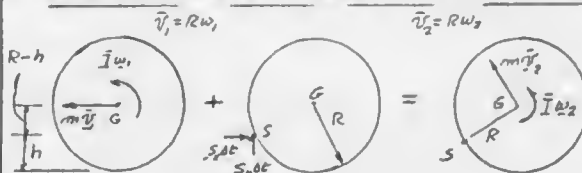
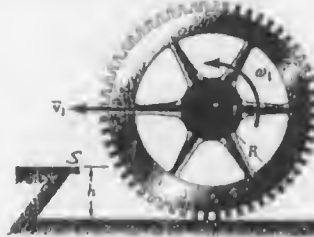
GEAR:  $R = 150 \text{ mm}$ ,  $\bar{R} = 125 \text{ mm}$   
 $\bar{v}_1 = 3 \text{ m/s}$

GEAR ROLL AND HITS  
 STEP, NO SLIPPING OCCURS  
 BETWEEN STEP & GEAR

PROBLEM 17.101

FIND:  $\omega_2$  FOR  $h = 75 \text{ mm}$ 

PROBLEM 17.102

FIND:  $\omega_2$  FOR  $h = 150 \text{ mm}$ 

+ MOMENTS ABOUT S:  $m \bar{v}_1 (R - h) + \bar{I} \omega_1 = m \bar{v}_2 R + \bar{I} \omega_2$

$$\frac{1}{2} (m R^2) (R - h) + \frac{1}{2} (m R^2) \omega_1 = \frac{1}{2} (m R^2) \omega_2 + \frac{1}{2} (m R^2) \omega_2$$

$$[R(R - h) + \bar{R}^2] \omega_1 = (R^2 + \bar{R}^2) \omega_2$$

$$\omega_2 = \frac{R^2 + \bar{R}^2 - R h}{R^2 + \bar{R}^2} \omega_1 \quad \omega_2 = \left[ 1 - \frac{R h}{R^2 + \bar{R}^2} \right] \omega_1 \quad (1)$$

DATA:  $R = 150 \text{ mm}$ ,  $\bar{R} = 125 \text{ mm}$ ,  $v_1 = 3 \text{ m/s}$ 

$$\omega_1 = \frac{v_1}{R} = \frac{3 \text{ m/s}}{0.150 \text{ m}} = 20 \text{ rad/s}$$

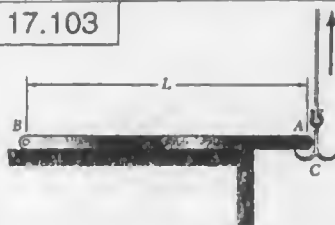
PROBLEM 17.101 FOR  $h = 75 \text{ mm}$ 

EQ(1):  $\omega_2 = \left[ 1 - \frac{(150)(75)}{(150^2 + 125^2)} \right] (20 \text{ rad/s}) = 0.7049(20)$ ;  $\omega_2 = 14.10 \text{ rad/s}$

PROBLEM 17.102 FOR  $h = 150 \text{ mm}$ 

EQ(1):  $\omega_2 = \left[ 1 - \frac{(150)(150)}{(150^2 + 125^2)} \right] (20 \text{ rad/s}) = 0.4082(20)$ ;  $\omega_2 = 8.20 \text{ rad/s}$

17.103

GIVEN: ROD OF MASS  $m$ 

PLASTIC IMPACT

BETWEEN HOOK C

AND ROD AT A.

FIND: IMPULSE

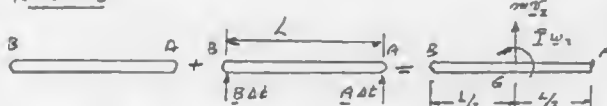
EXERCISED ON ROD

(a) AT A, (b) AT B

KINEMATICS: JUST AFTER IMPACT, ROD ROTATES ABOUT B

$v_B = 0$   $\omega$   $\bar{v} = \bar{v}_0$   $\omega = \frac{\bar{v}_0}{L}$ ,  $\bar{v} = \frac{1}{2} \bar{v}_0$

KINETICS



SYST. MOMENTA. + SYST. EXT. IMP.  $\rightarrow$  = SYST. MOMENTA.

+ MOMENTS ABOUT B:  $(A \Delta t) L = m \bar{v}_0 \frac{L}{2} + \bar{I} \omega_2$

$$(A \Delta t) L = m \left( \frac{\bar{v}_0}{2} \right) \frac{L}{2} + \frac{1}{12} m L^2 \left( \frac{\bar{v}_0}{L} \right)$$

$$A \Delta t = \left( \frac{1}{4} + \frac{1}{12} \right) m \bar{v}_0 L \quad A \Delta t = \frac{1}{3} m \bar{v}_0 L$$

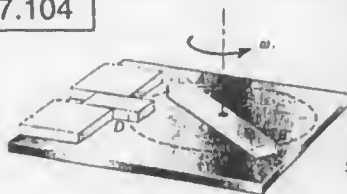
+ Y COMPONENTS:  $(A \Delta t) + (B \Delta t) = m \bar{v}_0$

$$\frac{1}{3} m \bar{v}_0 + (B \Delta t) = m \bar{v}_0$$

$$B \Delta t = \frac{2}{3} m \bar{v}_0$$

$$B \Delta t = \frac{2}{3} m \bar{v}_0$$

17.104



GIVEN: BAR AB OF MASS  $m$  AND LENGTH  $L$ .  
IMPACT IS PERFECTLY PLASTIC  
FIND:  $\omega$  AND  $\bar{v}$  JUST AFTER IMPACT

KINEMATICS: AFTER IMPACT  $\bar{v}_A = 0$  ( $e = 0$ )

$$\bar{v}_2 = \frac{1}{2} \omega_2 \uparrow \quad (1)$$

KINETICS

$$A \xrightarrow{\bar{v}_1} G \xrightarrow{\bar{v}_2} B + A \xrightarrow{\bar{v}_2} B = A \xrightarrow{\bar{v}_2} B$$

SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA  $2$

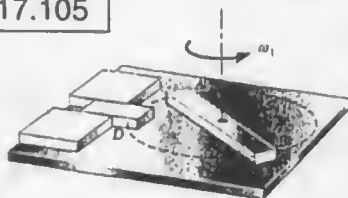
+) MOMENTS ABOUT A:  $\bar{I} \omega_1 = m \bar{v}_2 \frac{L}{2} + \bar{I} \omega_2$

$$\frac{1}{12} m L^2 \omega_1 = m \left( \frac{L}{2} \right) \omega_2 + \frac{1}{12} m L^2 \omega_2$$

$$\frac{\omega_1}{12} = \frac{\omega_2}{3} \quad \omega_2 = \frac{1}{4} \omega_1$$

EQ(1):  $\bar{v}_2 = \frac{1}{2} \omega_2 = \frac{1}{2} \left( \frac{1}{4} \omega_1 \right) \quad \bar{v}_2 = \frac{1}{8} \omega_1 \uparrow$

17.105



GIVEN: ROD AB OF MASS  $m$  AND LENGTH  $L$ .  
IMPACT IS PERFECTLY ELASTIC.  
FIND:  $\omega$  AND  $\bar{v}$  JUST AFTER IMPACT

KINEMATICS  
BEFORE IMPACT

$$A \xrightarrow{\bar{v}_1} G \xrightarrow{\bar{v}_2} B$$

$$\bar{v}_1, (\bar{v}_1)_1 = \frac{1}{2} \omega_1 \downarrow$$

AFTER IMPACT ( $e = 1$ )

$$A \xrightarrow{\bar{v}_2} G \xrightarrow{\bar{v}_2} B$$

$$(e = 1) (\bar{v}_2)_2 = -(\bar{v}_1)_1 = \frac{1}{2} \omega_1 \uparrow$$

$$\bar{v}_2 = (\bar{v}_2)_2 + \frac{1}{2} \omega_2 = \frac{1}{2} \omega_1 + \frac{1}{2} \omega_2$$

$$\bar{v}_2 = \frac{1}{2} (\omega_1 + \omega_2) \uparrow \quad (1)$$

KINETICS

$$A \xrightarrow{\bar{v}_1} G \xrightarrow{\bar{v}_2} B + A \xrightarrow{\bar{v}_2} B = A \xrightarrow{\bar{v}_2} B$$

SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA  $2$

+) MOMENTS ABOUT A:  $\bar{I} \omega_1 = m \bar{v}_2 \frac{L}{2} + \bar{I} \omega_2$

$$\frac{1}{12} m L^2 \omega_1 = m \left( \frac{L}{2} \right) (\omega_1 + \omega_2) \frac{1}{2} + \frac{1}{12} m L^2 \omega_2$$

$$\left( \frac{1}{12} - \frac{1}{6} \right) \omega_1 = \left( \frac{1}{6} + \frac{1}{12} \right) \omega_2$$

$$-\frac{1}{6} \omega_1 = \frac{1}{3} \omega_2 \quad \omega_2 = -\frac{1}{2} \omega_1$$

$$\omega_2 = \frac{1}{2} \omega_1$$

EQ(1):  $\bar{v}_2 = \frac{1}{2} (\omega_1 + \omega_2) = \frac{1}{2} (\omega_1 - \frac{1}{2} \omega_1)$

$$\bar{v}_2 = \frac{1}{4} \omega_1 \uparrow$$

17.106



GIVEN: ROD STRIKES WITH  $\bar{v}_1$  AND  $\omega_1 = 0$   
PERFECTLY ELASTIC IMPACT ( $e = 1$ )  
FIND:  $\bar{v}$  AND  $\omega$  AFTER ROD STRIKES (a) A, (b) B, (c) AGAIN A.

(a) ROD STRIKES A:  $(\bar{v}_1)_1 = \bar{v}_1 \downarrow$ ; SINCE  $e = 1$ ,  $(\bar{v}_1)_2 = \bar{v}_1 \uparrow$

$$\bar{v}_2 = -(\bar{v}_1)_1 + \frac{1}{2} \omega_2 \quad \bar{v}_2 = \left( \frac{1}{2} \omega_2 - \bar{v}_1 \right) \downarrow$$

KINETICS

$$A \xrightarrow{\bar{v}_1} G \xrightarrow{\bar{v}_2} B + A \xrightarrow{\bar{v}_2} B = A \xrightarrow{\bar{v}_2} B$$

SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA  $2$

+) MOMENTS ABOUT A:  $m \bar{v}_1 \frac{L}{2} = m \bar{v}_2 \frac{L}{2} + \bar{I} \omega_2$

$$m \bar{v}_1 \frac{L}{2} = m \left( \frac{1}{2} \omega_2 - \bar{v}_1 \right) \frac{L}{2} + \frac{1}{12} m L^2 \omega_2$$

$$\bar{v}_1 L = \frac{1}{3} L^2 \omega_2 \quad \omega_2 = 3 \frac{\bar{v}_1}{L} \downarrow$$

$$\bar{v}_2 = \frac{1}{2} \omega_2 - \bar{v}_1 = \frac{1}{2} \left( 3 \frac{\bar{v}_1}{L} \right) - \bar{v}_1$$

$$\bar{v}_2 = \frac{1}{2} \bar{v}_1 \downarrow$$

$$\bar{v}_2 = -(\bar{v}_1)_1 + \frac{1}{2} \omega_2 = -\bar{v}_1 + \left( 3 \frac{\bar{v}_1}{L} \right) \frac{L}{2} \quad (\bar{v}_2)_2 = 2 \bar{v}_1 \downarrow$$

(b) ROD STRIKES B: SINCE  $e = 1$ ,  $(\bar{v}_2)_2 = -(\bar{v}_1)_2 = 2 \bar{v}_1 \uparrow$

$$\bar{v}_3 = (\bar{v}_2)_2 + \frac{1}{2} \omega_3 \quad \bar{v}_3 = 2 \bar{v}_1 + \frac{1}{2} \omega_3 \uparrow$$

KINETICS

$$A \xrightarrow{\bar{v}_2} G \xrightarrow{\bar{v}_3} B + A \xrightarrow{\bar{v}_3} B = A \xrightarrow{\bar{v}_3} B$$

+) MOMENTS ABOUT B:  $\bar{I} \omega_2 - m \bar{v}_2 \frac{L}{2} = \bar{I} \omega_3 + m \bar{v}_3 \frac{L}{2}$

$$\frac{1}{12} m L^2 \left( 3 \frac{\bar{v}_1}{L} \right) - m \left( \frac{\bar{v}_1}{2} \right) \frac{L}{2} = \frac{1}{12} m L^2 \omega_3 + m \left( 2 \bar{v}_1 + \frac{1}{2} \omega_3 \right) \frac{L}{2}$$

$$0 = \frac{1}{6} m L^2 \omega_3 + m \bar{v}_1 L$$

$$\omega_3 = -3 \frac{\bar{v}_1}{L} \quad \omega_3 = \frac{3}{2} \frac{\bar{v}_1}{L} \downarrow$$

$$\bar{v}_3 = 2 \bar{v}_1 + \frac{1}{2} \omega_3 = 2 \bar{v}_1 + \frac{1}{2} \left( -3 \frac{\bar{v}_1}{L} \right) \frac{L}{2} = \frac{1}{2} \bar{v}_1$$

$$\bar{v}_3 = \frac{1}{2} \bar{v}_1 \uparrow$$

$$\bar{v}_3 = -(\bar{v}_2)_2 + \frac{1}{2} \omega_3 = -2 \bar{v}_1 + \left( 3 \frac{\bar{v}_1}{L} \right) \frac{L}{2} = \bar{v}_1 \downarrow$$

(c) ROD AGAIN STRIKES A: SINCE  $e = 1$ ,  $(\bar{v}_3)_3 = -(\bar{v}_2)_3 = \bar{v}_1 \uparrow$

$$\bar{v}_4 = (\bar{v}_3)_3 + \frac{1}{2} \omega_4 \quad \bar{v}_4 = \bar{v}_1 - \frac{1}{2} \omega_4 \uparrow$$

KINETICS

$$A \xrightarrow{\bar{v}_3} G \xrightarrow{\bar{v}_4} B + A \xrightarrow{\bar{v}_4} B = A \xrightarrow{\bar{v}_4} B$$

SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA  $2$

+) MOMENTS ABOUT A:  $\bar{I} \omega_3 + m \bar{v}_3 \frac{L}{2} = m \bar{v}_4 \frac{L}{2} + \bar{I} \omega_4$

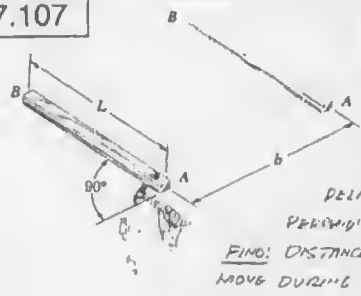
$$\frac{1}{12} m L^2 \left( 3 \frac{\bar{v}_1}{L} \right) + m \left( \frac{1}{2} \bar{v}_1 \right) \frac{L}{2} = m \left( \bar{v}_1 - \frac{1}{2} \omega_4 \right) \frac{L}{2} + \frac{1}{12} m L^2 \omega_4$$

$$\left( \frac{1}{4} + \frac{1}{6} - \frac{1}{2} \right) L \bar{v}_1 = \left( -\frac{1}{4} - \frac{1}{6} \right) L^2 \omega_4$$

$$0 = -\frac{1}{3} L^2 \omega_4 \quad \omega_4 = 0$$

$$\bar{v}_4 = \bar{v}_1 - \frac{1}{2} \omega_4 = \bar{v}_1 - 0 \quad \bar{v}_4 = \bar{v}_1 \uparrow$$

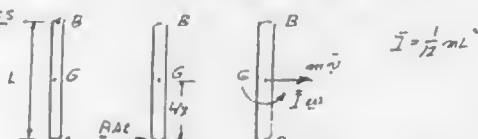
17.107



GIVEN: ROD AB IS AT REST ON A FRICTIONLESS HORIZONTAL TABLE. IT IS STRUCK BY A HAMMER THAT DELIVERS AN IMPULSE AT A PERPENDICULAR TO THE ROD. FIND: DISTANCE b THAT ROD WILL MOVE DURING A COMPLETE REVOLUTION

DERIVE BY ANG. IMPULS DELIVERED BY HAMMER

KINETICS



SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

± X COMPONENTS:  $APC = m\bar{v}$

(1)

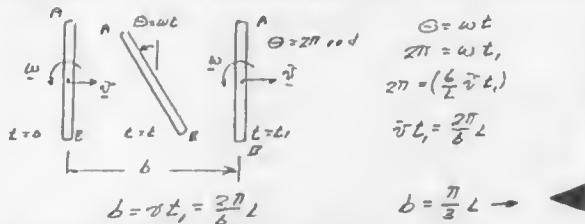
+ MOMENTS ABOUT G:  $(APC)\frac{L}{2} = \bar{I}\omega$

$$(APC)\frac{L}{2} = \frac{1}{12}mL^2\omega \quad \omega = \frac{6}{mL}(APC) \quad (2)$$

SUBSTITUTE (1) INTO (2)

$$\omega = \frac{6}{mL} \cdot m\bar{v} \quad \omega = \frac{6}{L}\bar{v} \quad (3)$$

KINEMATICS: LET  $t_1$  BE TIME REQUIRED FOR ONE REVOLUTION



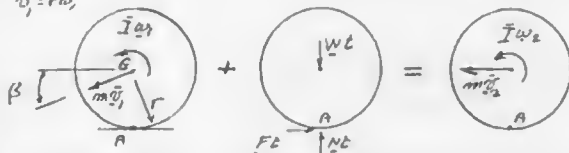
17.108



GIVEN: SPHERE ROLLS AND HITS HORIZONTAL SURFACE. AFTER SLIPPING IT STARTS ROLLING AGAIN. FIND:  $\bar{v}_2$  AND  $\omega_2$  AS IT ROLLS TO THE LEFT

POSITION 2, SPHERE HAS RESUMED ROLLING,  $\bar{v}_2 = r\omega_2$

$\bar{v}_1 = r\omega_1$



SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT A:  $\bar{I}\omega_1 + (m\bar{v}_1 \cos \beta)r = \bar{I}\omega_2 + m\bar{v}_2 r$

$$\frac{2}{5}mr^2\omega_1 + (mrv_1 \cos \beta)r = \frac{2}{5}mr^2\omega_2 + m(rv_2)r$$

$$(\frac{2}{5} + \cos \beta)\omega_1 = \frac{2}{5}\omega_2$$

$$\omega_2 = \frac{1}{2}(2 + 5\cos \beta)\omega_1$$

$$v_2 = r\omega_2 = \frac{1}{2}(2 + 5\cos \beta)rv_1$$

$$\bar{v}_2 = \frac{1}{2}(2 + 5\cos \beta)\bar{v}_1$$

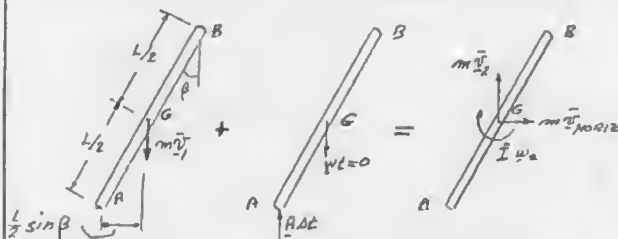
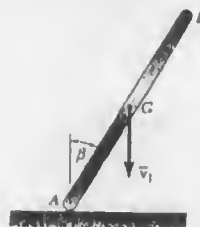
17.109 and 17.110

GIVEN: ROD AB STRIKES FRICTIONLESS SURFACE WITH THE VELOCITY SHOWN.

DERIVE AN EXPRESSION FOR  $\omega$  IMMEDIATELY AFTER IMPACT.

PROBLEM 17.109. ASSUME PERFECTLY ELASTIC IMPACT,  $(e=1)$

PROBLEM 17.110. ASSUME PERFECTLY PLASTIC IMPACT  $(e=0)$



SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA<sub>2</sub>

± X COMPONENTS:  $0 = m\bar{v}_{G2}$

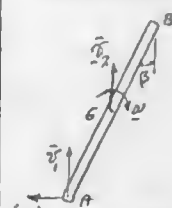
+ MOMENTS ABOUT A:

$$m\bar{v}_1(\frac{L}{2} \sin \beta) = \bar{I}\omega_2 - m\bar{v}_2(\frac{L}{2} \sin \beta) \quad (1)$$

PROBLEM 17.109: ELASTIC IMPACT AT A  $(e=1)$

$$(v_A)_1 = \bar{v}_1 \downarrow \quad \therefore [(v_A)_2]_y = \bar{v}_1 \uparrow$$

KINEMATICS:



$$(\bar{v}_A)_2 = \bar{v}_G + \bar{v}_{AG}$$

$$[\bar{v}_1 \uparrow] + [(v_A)_2 \leftarrow] = [\bar{v}_2 \uparrow] + [\frac{1}{2}\omega_2 \triangleright \beta]$$

+ Y COMPONENTS:

$$\bar{v}_1 = \bar{v}_2 + \frac{1}{2}\omega_2 \sin \beta$$

$$\bar{v}_2 = \bar{v}_1 - \frac{1}{2}\omega_2 \sin \beta$$

SUBSTITUTE INTO (1):

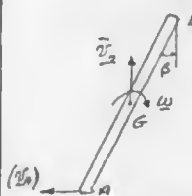
$$m\bar{v}_1 \frac{L}{2} \sin \beta = \frac{1}{12}mL^2\omega_2 - m(\bar{v}_1 - \frac{1}{2}\omega_2 \sin \beta)(\frac{L}{2} \sin \beta)$$

$$m\bar{v}_1 L \sin \beta = mL^2(\frac{1}{12} + \frac{1}{4} \sin^2 \beta)\omega_2$$

$$\omega_2 = \frac{\bar{v}_1}{L} \cdot \frac{12 \sin \beta}{3 \sin^2 \beta + 1}$$

PROBLEM 17.110: PLASTIC IMPACT  $(e=0)$

$$(v_A)_1 = \bar{v}_1 \downarrow \quad \therefore [(v_A)_2]_y = 0$$



$$(\bar{v}_A)_2 = \bar{v}_G + \bar{v}_{AG}$$

$$[(v_A)_2 \leftarrow] = [\bar{v}_2 \uparrow] + [\frac{1}{2}\omega_2 \triangleright \beta]$$

+ Y COMPONENTS:

$$0 = \bar{v}_2 + \frac{1}{2}\omega_2 \sin \beta$$

$$\bar{v}_2 = -\frac{1}{2}\omega_2 \sin \beta$$

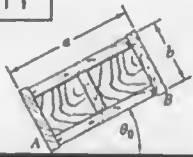
SUBSTITUTE INTO (1)

$$m\bar{v}_1 \frac{L}{2} \sin \beta = \frac{1}{12}mL^2\omega_2 - m(-\frac{1}{2}\omega_2 \sin \beta)(\frac{L}{2} \sin \beta)$$

$$m\bar{v}_1 \frac{L}{2} \sin \beta = mL^2(\frac{1}{12} + \frac{1}{4} \sin^2 \beta)\omega_2$$

$$\omega_2 = \frac{\bar{v}_1}{L} \cdot \frac{6 \sin \beta}{3 \sin^2 \beta + 1}$$

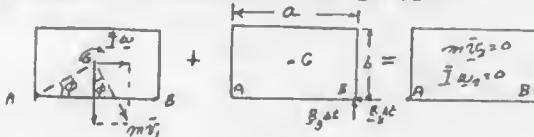
17.111



GIVEN: UNIFORM CRATE IS RELEASED FROM REST. IMPACT AT B IS PERFECTLY PLASTIC.

FIND: SMALLEST VALUE OF  $\frac{a}{b}$  FOR WHICH CORNER A REMAINS IN CONTACT WITH FLOOR.

WE CONSIDER THE LIMITING CASE WHEN THE CRATE IS JUST READY TO ROTATE ABOUT B. AT THAT INSTANT THE VELOCITIES MUST BE ZERO AND THE REACTION AT CORNER A MUST BE ZERO.



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

+2 MOMENTS ABOUT B

$$\bar{I} \omega + (m\bar{v}_y) \frac{b}{2} - (m\bar{v}_x) \frac{a}{2} + 0 = 0 \quad (1)$$

NOTE:  $\sin \phi = \frac{b}{\sqrt{a^2 + b^2}}, \cos \phi = \frac{a}{\sqrt{a^2 + b^2}}$

$$\bar{v}_y = (a\omega) \sin \phi = \frac{1}{2} \sqrt{a^2 + b^2} \omega$$

THUS:  $(m\bar{v}_x)_x = (m\bar{v}_y) \sin \phi = \frac{m}{2} \sqrt{a^2 + b^2} \omega \frac{b}{\sqrt{a^2 + b^2}} = \frac{1}{2} mb\omega$

ALSO,  $(m\bar{v}_x)_y = (m\bar{v}_y) \cos \phi = \frac{1}{2} ma\omega$

$$\bar{I} = \frac{1}{12} m(a^2 + b^2)$$

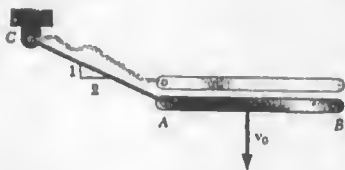
EQ(1):  $\frac{1}{12} m(a^2 + b^2) \omega + \frac{1}{2} (mb\omega) \frac{b}{2} - \frac{1}{2} (ma\omega) \frac{a}{2} = 0$

$$\frac{1}{3} mb^2 \omega - \frac{1}{6} ma^2 \omega = 0$$

$$\frac{b^2}{a^2} = 2$$

$$\frac{b}{a} = \sqrt{2}$$

17.112

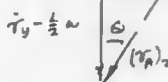
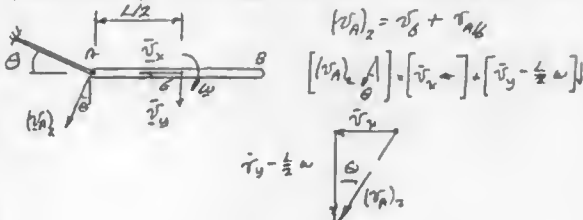


GIVEN: ROD OF LENGTH L.

ASSUMING PERFECTLY PLASTIC IMPACT.

FIND:  $\omega$  AND  $\bar{v}$  JUST AFTER COLLISION BECOMES TADT.

KINEMATICS (JUST AFTER IMPACT) LET  $\theta = \tan^{-1} \frac{1}{2}$

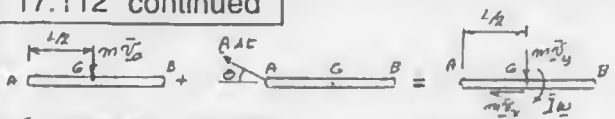


$$\tan \theta = \frac{\bar{v}_x}{\bar{v}_y - \frac{1}{2} \omega}$$

$$\bar{v}_x = (\bar{v}_y - \frac{1}{2} \omega) \tan \theta \quad (1)$$

(CONTINUED)

17.112 continued



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

+2 MOMENTS ABOUT A

$$m\bar{v}_y \frac{L}{2} = \bar{I} \omega + m\bar{v}_y \frac{L}{2}$$

$$m\bar{v}_y \frac{L}{2} = \frac{1}{12} mL^2 \omega + m\bar{v}_y \frac{L}{2}$$

$$\bar{v}_y = \frac{1}{6} L \omega + \bar{v}_y \quad (2)$$

+2 COMPONENTS:

$$m\bar{v}_y \cos \theta = m\bar{v}_x \sin \theta + m\bar{v}_y \cos \theta$$

$$\bar{v}_y = \bar{v}_x \tan \theta + \bar{v}_y \quad (3)$$

$$(1) \rightarrow (3) \quad \bar{v}_y = (\bar{v}_y - \frac{1}{2} \omega) \tan^2 \theta + \bar{v}_y$$

$$\bar{v}_y = \bar{v}_y (1 + \tan^2 \theta) - \frac{1}{2} \omega \tan^2 \theta$$

$$\omega = \frac{2}{L} \left( \bar{v}_y \frac{1 + \tan^2 \theta}{\tan^2 \theta} - \frac{\bar{v}_y}{\tan^2 \theta} \right) \quad (4)$$

$$(4) \rightarrow (2) \quad \bar{v}_y = \frac{L}{6} \cdot \frac{2}{L} \left( \bar{v}_y \frac{1 + \tan^2 \theta}{\tan^2 \theta} - \frac{\bar{v}_y}{\tan^2 \theta} \right) + \bar{v}_y$$

$$\bar{v}_y = \bar{v}_y \left( 1 + \frac{1}{3} \frac{1 + \tan^2 \theta}{\tan^2 \theta} \right) - \frac{1}{3} \frac{\bar{v}_y}{\tan^2 \theta}$$

$$3 \bar{v}_y \tan^2 \theta = \bar{v}_y (1 + 4 \tan^2 \theta) - \bar{v}_y$$

$$\bar{v}_y = \frac{1 + 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y \quad (5)$$

$$(5) \rightarrow (2) \quad \bar{v}_y = \frac{1}{6} \omega L + \frac{1 + 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y$$

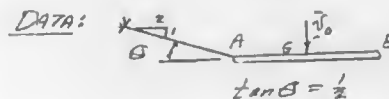
$$\omega = \frac{6}{L} \left[ 1 - \frac{1 + 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \right] \bar{v}_y = \frac{6}{L} \left[ \frac{1 + 4 \tan^2 \theta - 1 - 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \right] \bar{v}_y$$

$$\omega = \frac{6}{L} \frac{\tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y \quad (6)$$

$$(6) \text{ AND } (5) \rightarrow (1) \quad \bar{v}_x = (\bar{v}_y - \frac{1}{2} \omega) \tan \theta$$

$$\bar{v}_x = \left[ \frac{1 + 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y - \frac{1}{2} \cdot \frac{6}{L} \cdot \frac{\tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y \right] \tan \theta$$

$$\bar{v}_x = \frac{\tan \theta}{1 + 4 \tan^2 \theta} \bar{v}_y \quad (7)$$



DATA:  $\tan \theta = \frac{1}{2}$

EQ(6):  $\omega = \frac{6}{L} \frac{\tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y = \frac{6}{L} \frac{0.25}{1 + 4(0.25)} \bar{v}_y = \frac{4.5}{2} \frac{\bar{v}_y}{L}$

EQ(7):  $\bar{v}_x = \frac{\tan \theta}{1 + 4 \tan^2 \theta} \bar{v}_y = \frac{0.5}{1 + 4(0.25)} \bar{v}_y = \frac{0.5}{2} \bar{v}_y$

EQ(5):  $\bar{v}_y = \frac{1 + 3 \tan^2 \theta}{1 + 4 \tan^2 \theta} \bar{v}_y = \frac{1 + 3(0.25)}{1 + 4(0.25)} \bar{v}_y = \frac{1.25}{2} \bar{v}_y$

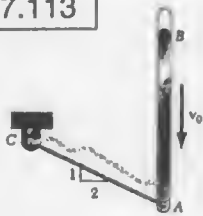
CHECK: EQ(1)  $\bar{v}_x = (\bar{v}_y - \frac{1}{2} \omega) \tan \theta$

$$= \left( \frac{1.25}{2} \bar{v}_y - \frac{1}{2} \cdot \frac{4.5}{2} \frac{\bar{v}_y}{L} \right) (0.5)$$

$$= \left( \frac{1.25}{2} \bar{v}_y \right) (0.5)$$

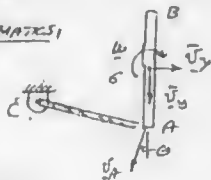
$$\bar{v}_x = \frac{1}{4} \bar{v}_y \quad \checkmark$$

17.113



GIVEN: ROD AB OF LENGTH  $L$ .  
 ASSUMING PERFECTLY  
 PLASTIC IMPACT  
 FIND:  $\omega$  AND  $\bar{v}$  IMMEDIATELY  
 AFTER CORD BECOMES TIGHT.

KINEMATICS:

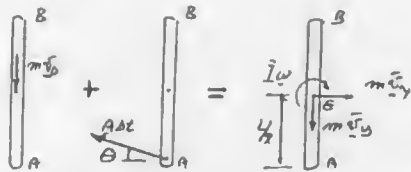


$$\bar{v} = v_A / \theta + \frac{1}{2} \omega \rightarrow$$

$$\bar{v}_x = (\frac{1}{2} \omega - v_A \sin \theta) \rightarrow$$

$$\bar{v}_y = v_A \cos \theta \downarrow$$

KINETICS:



SYST. MOMENTA, + SYST. EXT. IMP. = SYST. MOMENTA

+2 MOMENTS ABOUT A:  $0 = \bar{I} \omega + m \bar{v}_x \frac{L}{2}$

$$0 = \frac{1}{12} m L^2 \omega + m (\frac{1}{2} \omega - v_A \sin \theta) \frac{L}{2}$$

$$0 = \frac{1}{3} \omega L^2 - \omega L v_A \sin \theta$$

$$\omega = \frac{3}{2} \frac{v_A}{L} \sin \theta \quad (1)$$

+1 COMPONENTS:  $\omega v_A \cos \theta = \omega \bar{v}_y \cos \theta - \omega \bar{v}_x \sin \theta$

$$v_A \cos \theta = v_A \cos \theta - (\frac{1}{2} \omega - v_A \sin \theta) \sin \theta$$

$$v_A \cos \theta = v_A (\cos \theta + \sin^2 \theta) - \frac{1}{2} \omega \sin \theta$$

$$v_A = v_A \cos \theta + \frac{1}{2} \omega \sin \theta \quad (2)$$

(2) - (1)  $\omega = \frac{3}{2L} (v_A \cos \theta + \frac{1}{2} \omega \sin \theta) \sin \theta$

$$\omega = \frac{3 v_A \cos \theta \sin \theta}{2L} + \frac{3}{4} \omega \sin^2 \theta$$

$$\omega = \frac{3 v_A}{2L} \frac{\cos \theta \sin \theta}{1 - \frac{3}{4} \sin^2 \theta}$$

For  $\theta = \tan^{-1} 0.5$ ,  $\cos \theta = \frac{2}{\sqrt{5}}$  AND  $\sin \theta = \frac{1}{\sqrt{5}}$

$$\omega = \frac{3}{2} \frac{v_0}{L} \cdot \frac{(\frac{2}{\sqrt{5}})(\frac{1}{\sqrt{5}})}{1 - \frac{3}{4}(\frac{1}{\sqrt{5}})^2} = \frac{3}{5} \cdot \frac{1}{0.85} \frac{v_0}{L}$$

$$\omega = 0.7059 \frac{v_0}{L} \quad \omega = 0.706 \frac{v_0}{L}$$

EQ(2)  $v_A = v_0 \cos \theta + \frac{1}{2} \omega \sin \theta$

$$= v_0 \frac{2}{\sqrt{5}} + \frac{1}{2} (0.7059 \frac{v_0}{L}) \frac{1}{\sqrt{5}}$$

$$= (0.8944 + 0.1528) v_0$$

$$v_A = 1.0522 v_0$$

$$\bar{v}_x = \frac{1}{2} \omega - v_A \sin \theta = \frac{1}{2} (0.7059 \frac{v_0}{L}) - (1.0522 v_0) \frac{1}{\sqrt{5}}$$

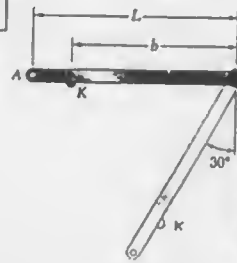
$$= (0.35295 - 0.47059) v_0 = -0.1176 v_0$$

$$\bar{v}_x = 0.1176 v_0 \leftarrow$$

$$\bar{v}_y = v_A \cos \theta = (1.0522 v_0) \frac{2}{\sqrt{5}} = 0.9411 v_0$$

$$\bar{v}_y = 0.941 v_0 \downarrow$$

17.114

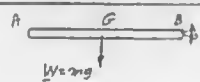


GIVEN: ROD IS  
 RELEASED FROM  
 POSITION SHOWN  
 AND REBOUNDS TO  
 $30^\circ$  WITH THE VERTIC.  
 FIND: (a) COEF. OF  
 RESTITUTION, (b)  
 SHOW THAT REBOUND  
 IS INDEPENDENT OF  
 POSITION OF KNOB K

CONSERVATION OF ENERGY:

BEFORE IMPACT

(1)



$$V_1 = 0$$

$$V_2 = -W \frac{L}{2} = -mg \frac{L}{2}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \bar{I} \omega^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2} (\frac{1}{12} m L^2) \omega^2 + \frac{1}{2} m (\frac{1}{2} \omega L)^2 = \frac{1}{8} m L^2 \omega^2$$

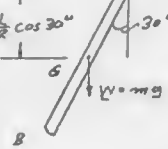
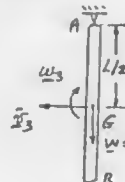
$$T_1 + V_1 = T_2 + V_2: 0 = \frac{1}{8} m L^2 \omega^2 - mg \frac{L}{2}; \omega^2 = 3 \frac{g}{L}$$

AFTER IMPACT:

DATUM

$$\bar{v}_3 = \frac{1}{2} \omega_3$$

(3)



(4)

$$V_3 = -W L/2 = -mg \frac{L}{2}$$

$$V_4 = -W \frac{L}{2} \cos 30^\circ$$

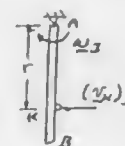
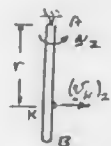
$$T_3 = \frac{1}{2} \bar{I} \omega_3^2 + \frac{1}{2} m \bar{v}_3^2 = \frac{1}{2} (\frac{1}{12} m L^2) \omega_3^2 + \frac{1}{2} m (\frac{1}{2} \omega_3 L)^2 = \frac{1}{8} m L^2 \omega_3^2$$

$$T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4: \frac{1}{8} m L^2 \omega_3^2 - mg \frac{L}{2} = 0 - mg \frac{L}{2} \cos 30^\circ$$

$$\omega_3^2 = 3 \frac{g}{L} (1 - \cos 30^\circ)$$

IMPACT



$$(v_K)_1 = r \omega_1 = r \sqrt{3 \frac{g}{L}}$$

$$(v_K)_2 = r \omega_2 = r \sqrt{3 \frac{g}{L} (1 - \cos 30^\circ)}$$

COEFFICIENT OF RESTITUTION

$$C = \frac{(v_K)_2}{(v_K)_1} = \frac{r \sqrt{3 \frac{g}{L} (1 - \cos 30^\circ)}}{r \sqrt{3 \frac{g}{L}}}$$

$$E = \sqrt{1 - \cos 30^\circ}$$

$$C = \sqrt{1 - \cos 30^\circ} = \sqrt{1 - 0.86603}$$

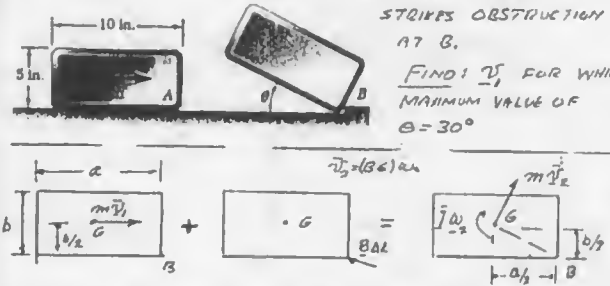
$$C = 0.366$$

WE NOTE THAT RESULT IS INDEPENDENT OF THE  
 POSITION OF THE KNOB.

17.115

GIVEN: UNIFORM BLOCK  
STRIKES OBSTACLE  
AT B.

FIND:  $\vec{v}_1$  FOR WHICH  
MAXIMUM VALUE OF  
 $\theta = 30^\circ$



SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA<sub>2</sub>

+2) MOMENTS ABOUT B:  $m \vec{v}_1 \cdot \frac{b}{2} = \bar{I} \omega_1 + m \vec{v}_2 \cdot (B\delta)$

$m \vec{v}_1 \cdot \frac{b}{2} = \bar{I} \omega_2 + m (B\delta)^2 \omega_2$

$$2\delta^2 = (a/2)^2 + (b/2)^2 = \frac{1}{4}(a^2 + b^2)$$

$$\bar{I} = \frac{1}{12} m (a^2 + b^2)$$

$$\lambda \vec{v}_1 \cdot \frac{b}{2} = \frac{1}{12} \lambda (a^2 + b^2) \omega_2 + \lambda \frac{1}{2} (a^2 + b^2) \omega_2$$

$$\vec{v}_1 \cdot \frac{b}{2} = \frac{1}{3} (a^2 + b^2) \omega_2 \quad \omega_2 = \frac{3}{2} \frac{b}{a^2 + b^2} \vec{v}_1$$

DATA:  $a = \frac{10}{12}$  ft  $b = \frac{5}{12}$  ft

$$\omega_2 = \frac{3}{2} \frac{5/12}{[(10/12)^2 + (5/12)^2]} \vec{v}_1 \quad \omega_2 = 0.720 \vec{v}_1 \quad (1)$$

CONSERVATION OF ENERGY:



$$T_1 = \frac{1}{2} \bar{I} \omega_1^2 + \frac{1}{2} m \vec{v}_1^2 = \frac{1}{2} \frac{1}{12} m (a^2 + b^2) \omega_1^2 + \frac{1}{2} m \frac{1}{4} (a^2 + b^2) \omega_1^2$$

$$T_1 = \frac{1}{6} m (a^2 + b^2) \omega_1^2$$

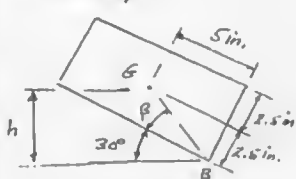
$$V_1 = W \frac{b}{2} = mg \frac{b}{2}$$

$$V_2 = W h = mg \theta h \quad T_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: \frac{1}{6} m (a^2 + b^2) \omega_1^2 + \lambda g \frac{b}{2} = \lambda g \theta h$$

$$\omega_1^2 = \frac{6(h - b/2)}{(a^2 + b^2)} g \quad (2)$$

For  $\theta_{\max} = 30^\circ$



$$\tan \theta = \frac{2.5 \text{ m}}{5 \text{ in.}}$$

$$\theta = 26.565^\circ$$

$$BG = \sqrt{2.5^2 + 5^2} = 5.5902 \text{ m}$$

$$BG = 0.46525 \text{ ft}$$

$$h = (BG) \tan(30^\circ + \mu) = (0.46525 \text{ ft}) \sin(30^\circ + 26.565^\circ)$$

$$h = 0.38876 \text{ ft}$$

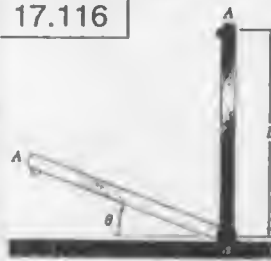
$$EQ(2): \omega_1^2 = \frac{6(h - b/2)}{g(a^2 + b^2)} = \frac{6(0.38876 - \frac{2.5}{12})}{[(\frac{10}{12})^2 + (\frac{5}{12})^2]} = 22.7 = 40.156$$

$$\omega_1 = 6.337 \text{ rad/s}$$

$$EQ(1): \omega_2 = 0.720 \vec{v}_1; \quad 6.337 = 0.720 \vec{v}_1$$

$$\vec{v}_1 = 8.80 \text{ ft/s}$$

17.116

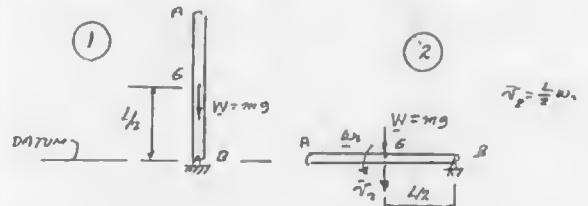


GIVEN: ROD AB IS GIVEN  
SLIGHT NUDGE AND ROTATES  
COUNTERCLOCKWISE HITS  
SURFACE AND REBOUNDS

$e = 0.40$

FIND: MAXIMUM  $\theta$  OF  
REBOUND.

CONSERVATION OF ENERGY:



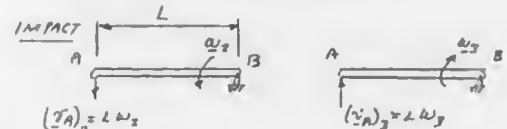
$$T_1 = 0 \quad V_1 = mg \frac{L}{2}$$

$$T_2 = \frac{1}{2} \bar{I} \omega_2^2 + \frac{1}{2} m \vec{v}_2^2 = \frac{1}{2} \cdot \frac{1}{12} mL^2 \omega_2^2 + \frac{1}{2} m \left( \frac{L}{2} \omega_2 \right)^2 = \frac{1}{6} mL^2 \omega_2^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: 0 + mg \frac{L}{2} = \frac{1}{6} mL^2 \omega_2^2 + 0$$

$$\omega_2^2 = 3 \frac{g}{L}$$



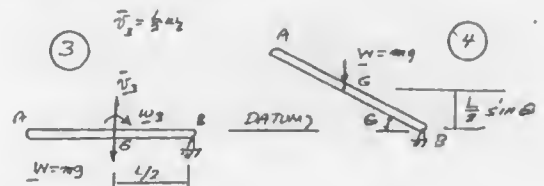
$$(\vec{v}_A)_2 = L \omega_2$$

$$(\vec{v}_A)_3 = L \omega_3$$

$$e = \frac{(\vec{v}_A)_3}{(\vec{v}_A)_2} = \frac{L \omega_3}{L \omega_2}: \quad \omega_3 = e \omega_2$$

$$\text{OR: } \omega_3^2 = e^2 \omega_2^2$$

CONSERVATION OF ENERGY



$$V_3 = 0, \quad T_3 = \frac{1}{2} \bar{I} \omega_3^2 + \frac{1}{2} m \vec{v}_3^2 = \frac{1}{2} \cdot \frac{1}{12} mL^2 \omega_3^2 + \frac{1}{2} m \left( \frac{L}{2} \omega_3 \right)^2 = \frac{1}{6} mL^2 \omega_3^2$$

$$V_4 = mg \frac{L}{2} \sin \theta, \quad T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4: \frac{1}{6} mL^2 \omega_3^2 + 0 = mg \frac{L}{2} \sin \theta$$

$$\frac{1}{6} mL^2 (e^2 \omega_2^2) = mg \frac{L}{2} \sin \theta$$

$$\frac{1}{6} \lambda L^2 e^2 (3 \frac{g}{L}) = \lambda g \frac{L}{2} \sin \theta$$

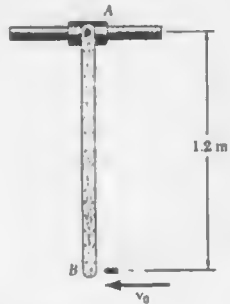
$$\sin \theta = e^2$$

$$\text{For } e = 0.40 \quad \sin \theta = (0.40)^2 = 0.16$$

$$\theta = 9.21^\circ$$



# 17.117 and 17.118

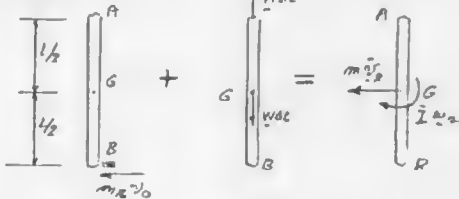


**GIVEN:** 30-g BULLET  
FIRED INTO THE 0.8-m BEAM.  
COLLAR A SLIDES FREELY.  
**PROBLEM 17.117**  
FIND: MAXIMUM ANGLE  
OF ROTATION OF BEAM FOR  
 $v_0 = 350 \text{ m/s}$

**PROBLEM 17.118:**

FIND:  $v_0$  FOR WHICH  
MAXIMUM ANGLE OF  
ROTATION OF BEAM IS  $70^\circ$

**IMPACT:**

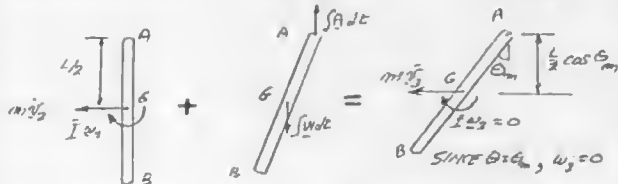


$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.} = \text{SYST. MOMENTA}_2$$

$$\pm \text{COMPONENTS: } m_B v_0 = m \bar{v}_2 \quad (1)$$

$$+ \text{MOMENTS ABOUT G: } m_B v_0 \frac{L}{2} = \bar{I} \omega_2; \omega_2 = \frac{m_B v_0 L}{2 \bar{I}} \quad (2)$$

$$\text{SUBSTITUTE FOR } m_B v_0: m \bar{v}_2 \frac{L}{2} = \frac{1}{12} m L^2 \omega_2 \quad \bar{v}_2 = \frac{1}{6} \omega_2$$



$$\text{SYST. MOMENTA}_2 + \text{SYST. EXT. IMP.} = \text{SYST. MOMENTA}_3$$

$$\pm \text{COMPONENTS: } m \bar{v}_2 = m \bar{v}_3 \quad \bar{v}_2 = \bar{v}_3$$

**CONSERVATION OF ENERGY: CHOOSE DATUM AT A.**

$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} m \left( \frac{1}{6} \omega_2 \right)^2 + \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega_2^2 = \frac{1}{18} m L^2 \omega_2^2$$

$$V_2 = -W \frac{L}{2} = -m g \frac{L}{2}$$

$$T_3 = \frac{1}{2} m \bar{v}_3^2 + \frac{1}{2} \bar{I} \omega_3^2 = \frac{1}{2} m \bar{v}_3^2 + 0 = \frac{1}{2} m \left( \frac{1}{6} \omega_3 \right)^2 = \frac{1}{12} m L^2 \omega_3^2$$

$$V_3 = -W \frac{L}{2} \cos \theta_m = -m g \frac{L}{2} \cos \theta_m$$

$$T_2 + V_2 = T_3 + V_3: \frac{1}{18} m L^2 \omega_2^2 - m g \frac{L}{2} = \frac{1}{12} m L^2 \omega_3^2 - m g \frac{L}{2} \cos \theta_m$$

$$\left( \frac{1}{18} - \frac{1}{12} \right) L^2 \omega_2^2 = g \frac{L}{2} (1 - \cos \theta_m)$$

**SUBSTITUTE FOR  $\omega_2$  FROM EQ (2):**

$$\frac{1}{24} L^2 \left( \frac{m_B v_0 L}{2 \bar{I}} \right)^2 = g \frac{L}{2} (1 - \cos \theta_m)$$

$$\frac{L m_B^2 v_0^2 L^2}{48 g (\frac{1}{12} m L^2)} = (1 - \cos \theta_m); \cos \theta_m = 1 - \frac{3 m_B^2 v_0^2}{g L m^2} \quad (3)$$

**DATA:**  $L = 1.2 \text{ m}$ ,  $m_B = 0.03 \text{ kg}$ ,  $m = 8 \text{ kg}$

**PROBLEM 17.117:** FOR  $v_0 = 350 \text{ m/s}$

$$\text{EQ (2): } \cos \theta_m = 1 - \frac{3(0.03 \text{ kg})^2 (350 \text{ m/s})^2}{(7.68 \text{ kg m}^2) (1.2 \text{ m}) (8 \text{ kg})} = 1 - 0.4391$$

$$\cos \theta_m = 0.5609 \quad \theta_m = 55.9^\circ$$

**PROBLEM 17.118:** FOR  $\theta_m = 90^\circ$ ,  $\cos \theta_m = 0$

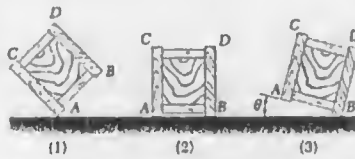
$$\text{EQ (2): } 0 = 1 - \frac{3(0.03 \text{ kg})^2 v_0^2}{(7.68 \text{ kg m}^2) (1.2 \text{ m}) (8 \text{ kg})}; 1 - 3.5837 \times 10^{-6} v_0^2 = 0$$

$$v_0^2 = 279.04 \times 10^3 \quad v_0 = 528 \text{ m/s}$$

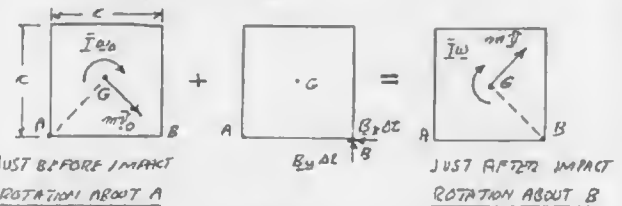
# 17.119

**GIVEN:** UNIFORM CRATE

RELEASED FROM  
POSITION 1. NO SLIPPING  
FIND: (a)  $\omega$  JUST AFTER  
IMPACT, (b) ENERGY LOST  
IN IMPACT, (c) MAXIMUM  
ANGLE  $\theta$ .



DENOTE BY  $\omega_0$  ANGULAR VELOCITY ABOUT A JUST BEFORE  
COLLAR R STOPS FLOOR.



JUST BEFORE IMPACT  
ROTATION ABOUT A

JUST AFTER IMPACT  
ROTATION ABOUT B

$$\text{SYST. MOMENTA}_0 + \text{SYST. EXT. IMP.} = \text{SYST. MOMENTA}_1$$

+ MOMENTS ABOUT B:

$$\bar{I} \omega_0 + 0 = \bar{I} \omega + m \bar{v} (BG) \quad (1)$$

$$AG = BG = \sqrt{2} \frac{c}{2} = \frac{c}{\sqrt{2}} \quad \bar{I} = \frac{1}{6} m c^2$$

$$\bar{v}_0 = (AG) \omega_0 = \frac{c}{\sqrt{2}} \omega_0 \quad \bar{v} = (BG) \omega = \frac{c}{\sqrt{2}} \omega$$

$$\text{EQ (1): } \frac{1}{6} m c^2 \omega_0 = \frac{1}{6} m c^2 \omega + m \left( \frac{c}{\sqrt{2}} \omega \right) \frac{c}{\sqrt{2}}$$

$$(a) \frac{1}{6} m c^2 \omega_0 = \frac{2}{3} m c^2 \omega \quad \omega = \frac{1}{4} \omega_0$$

(b) **KINETIC ENERGY LOST:**

SEE EQ. 17.10 PAGE 1049, FOR ROTATION ABOUT A AND B

$$T_0 = \frac{1}{2} \bar{I}_A \omega_0^2 \quad \text{NOTE: } \bar{I}_A = \bar{I}_B$$

$$T_1 = \frac{1}{2} \bar{I}_B \omega^2$$

$$\frac{\Delta T}{T_0} = \frac{T_0 - T_1}{T_0} = \frac{\frac{1}{2} \bar{I}_A \omega_0^2 - \frac{1}{2} \bar{I}_B \omega^2}{\frac{1}{2} \bar{I}_A \omega_0^2} = \frac{\omega_0^2 - \omega^2}{\omega_0^2} = 1 - \left( \frac{\omega}{\omega_0} \right)^2$$

$$\text{ENERGY LOST} = 1 - \left( \frac{\omega}{\omega_0} \right)^2 = 1 - \left( \frac{1}{4} \right)^2 = \frac{15}{16}$$

(c)



**INITIAL POSITION**

$$h_0 = \frac{c}{\sqrt{2}} - \frac{c}{2}$$

$$T_0 = 0$$

$$V_0 = W h_0$$

**FINAL POSITION**

$$h_f = (BG) \sin(45^\circ + \theta) - \frac{c}{2}$$

$$h_f = \frac{c}{\sqrt{2}} \sin(45^\circ + \theta) - \frac{c}{2}$$

$$T_f = 0 \quad V_f = W h_f$$

BUT, FROM PART (b) WE KNOW THAT  $\frac{15}{16}$  OF THE  
ENERGY IS LOST, THUS

$$V_f = \frac{1}{16} V_h; \quad W h_f = \frac{1}{16} W h_0$$

$$h_f = \frac{1}{16} h_0$$

$$\frac{c}{\sqrt{2}} \sin(45^\circ + \theta) - \frac{c}{2} = \frac{1}{16} \left( \frac{c}{\sqrt{2}} - \frac{c}{2} \right)$$

$$\frac{1}{\sqrt{2}} \sin(45^\circ + \theta) = \frac{1}{16} \left( \frac{1}{\sqrt{2}} - \frac{1}{2} \right) + \frac{1}{2}$$

$$\sin(45^\circ + \theta) = \frac{2 + 15\sqrt{2}}{32} = 0.72541$$

$$45^\circ + \theta = 46.503^\circ$$

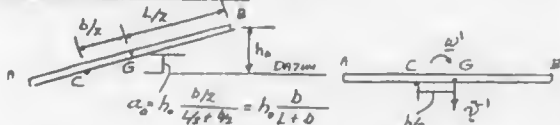
$$\theta = 1.50^\circ$$

17.120



GIVEN: ROD OF LENGTH  $L = 30$  in.  
 $b = 5$  in. ROD IS RELEASED WHEN  $h_0 = 4$  in.  
 FIND: (a)  $h_1$  AFTER FIRST IMPACT, (b)  $h_2$  AFTER SECOND IMPACT.

CONSERVATION OF ENERGY



POSITION "O"

$$V_0 = m g a_0; T_0 = 0$$

POSITION "PRIME"

$$V' = 0; T' = \frac{1}{2} I \omega'^2 + \frac{1}{2} m v'^2$$

$$T' = \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega'^2 + \frac{1}{2} m \left( \frac{b}{2} \omega' \right)^2$$

$$T' = \frac{1}{24} m \omega'^2 (L^2 + 3b^2)$$

$$T_0 + V_0 = T' + V': 0 + m g a_0 = \frac{1}{24} m \omega'^2 (L^2 + 3b^2)$$

$$\frac{1}{24} g h_0 \frac{b}{L+b} = \omega'^2 (L^2 + 3b^2)$$

$$(\omega')^2 = \frac{24 g b h_0}{(L+b)(L^2 + 3b^2)} \quad (1)$$

NOTE THIS EXPRESSION ALSO RELATES THE HEIGHT THE END OF THE ROD RISES WHEN ANGULAR VELOCITY  $\omega'$  OCCURS WHEN ROD IS HORIZONTAL

IMPACT



SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA  
 POSITION "PRIME" POSITION "DOUBLE PRIME"

$$v' = \frac{b}{2} \omega'$$

$$v'' = \frac{b}{2} \omega''$$

+ MOMENTS ABOUT D:

$$\bar{I} \omega' - m \bar{v}' \frac{b}{2} = \bar{I} \omega'' + m \bar{v}'' \frac{b}{2}$$

$$\frac{1}{12} m L^2 \omega' - m \left( \frac{b}{2} \right)^2 \omega' = \frac{1}{12} m L^2 \omega'' + m \left( \frac{b}{2} \right)^2 \omega''$$

$$\omega'' = \frac{\frac{1/12 - b^2/4}{L^2 + 3b^2} \omega'}{\frac{L^2 - 3b^2}{L^2 + 3b^2}} \quad (2)$$

FIRST IMPACT:



$$EQ(1): (\omega')^2 = \frac{24 g b}{(L+b)} \cdot \frac{h_0}{(L^2 + 3b^2)} \quad (\omega'')^2 = \frac{24 g b}{(L+b)} \cdot \frac{h_1}{(L^2 + 3b^2)}$$

SQUARE EQ(2):

$$(\omega'')^2 = \frac{(L^2 - 3b^2)^2}{(L^2 + 3b^2)^2} (\omega')^2$$

$$\left[ \frac{24 g b}{(L+b)} \cdot \frac{h_1}{(L^2 + 3b^2)} \right] = \frac{(L^2 - 3b^2)^2}{(L^2 + 3b^2)^2} \cdot \left[ \frac{24 g b}{(L+b)} \cdot \frac{h_0}{(L^2 + 3b^2)} \right]$$

$$h_1 = \left[ \frac{L^2 - 3b^2}{L^2 + 3b^2} \right]^2 h_0 \quad (3)$$

SECOND IMPACT:  $h_0 \rightarrow h_1, h_1 \rightarrow h_2$ 

$$h_2 = \left[ \frac{L^2 - 3b^2}{L^2 + 3b^2} \right]^2 h_1$$

$$DATA: h_0 = 4 \text{ in.}, L = 30 \text{ in.}, b = 5 \text{ in.}, \frac{L^2 - 3b^2}{L^2 + 3b^2} = \frac{30^2 - 3(5)^2}{30^2 + 3(5)^2} = 0.8445$$

$$EQ(3): h_1 = (0.8445)^2 (4 \text{ in.}) = 2.8637 \text{ in.} \quad h_1 = 2.86 \text{ in.}$$

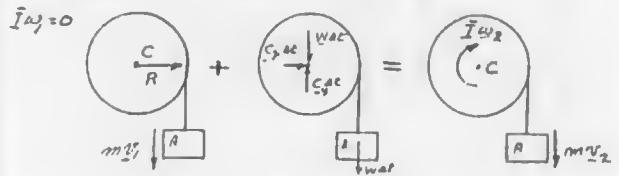
$$EQ(4): h_2 = (0.8445)^4 (4 \text{ in.}) = 2.0505 \text{ in.} \quad h_2 = 2.05 \text{ in.}$$

17.121 and 17.122



GIVEN: 3-lb COLLAR A DROPS  $h = 15$  in.  
 8-lb DISK OF RADIUS  $R = 9$  in.  
 IMMEDIATELY AFTER IMPACT  
 FIND: (a)  $\bar{I}_A$ , (b)  $\omega$   
 PROBLEM 17.121 ASSUME PERFECTLY PLASTIC IMPACT.  
 PROBLEM 17.122 ASSUME PERFECTLY ELASTIC IMPACT.

COLLAR A FALLS A DISTANCE  $h$ :  $v_1 = \sqrt{2gh}$   
 PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA + SYST. EXT. IMP. = SYST. MOMENTA  
 + 2 MOMENTS ABOUT C:

$$m v_1 R = \bar{I} \omega_2 + m v_2 R \quad (1)$$

PROBLEM 17.121

PLASTIC IMPACT  $e = 0$   $v_2 = R \omega_2$ ;  $\omega_2 = \frac{v_2}{R}$ M = MASS OF DISK;  $\bar{I} = \frac{1}{2} M R^2$ 

$$EQ(1): m v_1 R = \frac{1}{2} M R^2 \left( \frac{v_2}{R} \right) + m v_2 R$$

$$m v_1 = \frac{1}{2} M v_2 + m v_2$$

$$v_2 = \frac{2m}{2m + M} v_1 \quad (3)$$

DATA:  $m = \frac{3}{4}$ ;  $M = \frac{8}{3}$ ;  $h = 15$  in.

$$v_1 = \sqrt{2gh} = \sqrt{2(32.2 \text{ ft/s}^2)(\frac{15}{12} \text{ ft})} = 8.972 \text{ ft/s}$$

$$EQ(3): v_2 = \frac{2(\frac{3}{4})}{2(\frac{3}{4}) + \frac{8}{3}} (8.972 \text{ ft/s}) = 3.845 \text{ ft/s}$$

$$v_2 = 3.85 \text{ ft/s} \downarrow$$

$$\omega_2 = \frac{v_2}{R} = \frac{3.845 \text{ ft/s}}{(\frac{9}{12} \text{ ft})} = 5.127 \text{ rad/s} \quad \omega_2 = 5.13 \text{ rad/s} \downarrow$$

PROBLEM 17.122:

ELASTIC IMPACT  $e = 1$   $(v_2)_2 - (v_1)_2 = (v_1)_1 - (v_2)_1$ 

$$(v_2)_1 = 0; (v_2)_2 = R \omega_2$$

$$R \omega_2 - v_2 = v_1; \quad \omega_2 = (v_1 + v_2)/R \quad (2)$$

$$EQ(1): m v_1 R = \frac{1}{2} M R^2 \left( \frac{v_1 + v_2}{R} \right) + m v_2 R$$

$$m v_1 = \frac{1}{2} M (v_1 + v_2) + m v_2$$

$$v_2 = \frac{2m - M}{2m + M} v_1 \quad (4)$$

DATA:  $m = \frac{3}{4}$ ;  $M = \frac{8}{3}$ ;  $h = 15$  in.  $v_1 = 8.972 \text{ ft/s}$ 

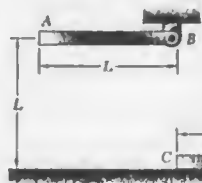
$$EQ(4): v_2 = \frac{2(\frac{3}{4}) - \frac{8}{3}}{2(\frac{3}{4}) + \frac{8}{3}} (8.972 \text{ ft/s}) = -1.2817 \text{ ft/s}$$

$$v_2 = 1.282 \text{ ft/s} \uparrow$$

$$EQ(2): \omega_2 = \frac{v_1 + v_2}{R} = \frac{8.972 \text{ ft/s} - 1.282 \text{ ft/s}}{(\frac{9}{12} \text{ ft})} = 10.254 \text{ rad/s}$$

$$\omega_2 = 10.25 \text{ rad/s} \downarrow$$

# 17.123 and 17.124



GIVEN: IDENTICAL RODS AB & CD  
ROD AB IS RELEASED FROM  
REST IN POSITION SHOWN.

FIND: VELOCITY OF  
CD JUST AFTER IMPACT  
PROBLEM 17.123: FOR  
COEFF. OF RESTITUTION  
EQUAL TO  $e = 0.50$

PROBLEM 17.125: FOR  $e = 1$

ROD AB SWINGS TO VERTICAL POSITION

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = V_1 = 0$$

$$T_2 = \frac{1}{2} \bar{I} \omega_2^2 + \frac{1}{2} m \bar{v}_B^2$$

$$= \frac{1}{2} \left( \frac{1}{12} m L^2 \right) \omega_2^2 + \frac{1}{2} m \left( \frac{L}{2} \omega_2 \right)^2$$

$$T_2 = \frac{1}{6} m L^2 \omega_2^2$$

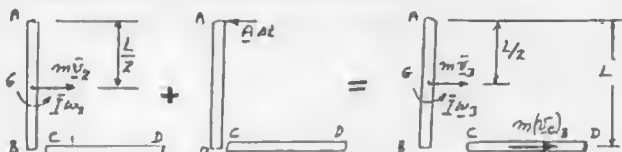
$$V_2 = -m g \frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: 0 = \frac{1}{6} m L^2 \omega_2^2 - m g \frac{L}{2}$$

$$\omega_2^2 = \frac{3g}{L} \quad \omega_2 = \sqrt{\frac{3g}{L}} \quad (1)$$

$$(\bar{v}_B)_2 = L \omega_2 = L \sqrt{\frac{3g}{L}} = \sqrt{3g} L \quad (2)$$

PRINCIPLE OF IMPULSE-MOMENTUM AT IMPACT



SYST. MOMENTA<sub>2</sub> + SYST. EXT. IMP.<sub>2-3</sub> = SYST. MOMENTA<sub>3</sub>

+ MOMENTS ABOUT A:

$$\bar{I} \omega_2 + m \bar{v}_G \frac{L}{2} = \bar{I} \omega_3 + m \bar{v}_G' \frac{L}{2} + m (\bar{v}_B)_3 L$$

$$\frac{1}{12} m L^2 \omega_2 + m \left( \frac{L}{2} \right) \omega_2 = \frac{1}{12} m L^2 \omega_3 + m \left( \frac{L}{2} \right) \omega_3 + m (\bar{v}_B)_3 L$$

$$\frac{1}{3} L \omega_2 - \frac{1}{3} L \omega_3 = + (\bar{v}_B)_3; \quad \omega_2 - \omega_3 = \frac{2(\bar{v}_B)_3}{L} \quad (3)$$

IMPACT:  $(\bar{v}_B)_2 = (\bar{v}_B)_3 - L \omega_3; \quad (\bar{v}_B)_3 = (\bar{v}_B)_2 + L \omega_3 \quad (4)$

$$EQ(3): \omega_2 - \omega_3 = \frac{2}{L} ((\bar{v}_B)_2 + L \omega_3); \quad \omega_2 - \omega_3 = \frac{2}{L} (\bar{v}_B)_2 + 2 \omega_3$$

$$\omega_2 - 4 \omega_3 = \frac{2}{L} (\bar{v}_B)_2$$

SUBSTITUTE FROM (1) AND (2)

$$\sqrt{\frac{3g}{L}} - 4 \omega_3 = \frac{2}{L} \sqrt{3g} L$$

$$4 \omega_3 = \sqrt{\frac{3g}{L}} - \frac{2}{L} \sqrt{3g} L; \quad 4 \omega_3 = \frac{1}{L} \sqrt{3g} L - \frac{2}{L} \sqrt{3g} L$$

$$\omega_3 = \frac{1}{4} (1 - 2) \sqrt{\frac{3g}{L}}$$

$$EQ(4): (\bar{v}_B)_3 = (\bar{v}_B)_2 + L \omega_3 = (\sqrt{3g} L) + L \left[ \frac{1}{4} (1 - 2) \sqrt{\frac{3g}{L}} \right]$$

$$(\bar{v}_B)_3 = \sqrt{3g} L \left( 1 + \frac{1}{4} - \frac{2}{4} \right) = \sqrt{3g} L \left( \frac{1}{2} + \frac{1}{4} \right)$$

$$(\bar{v}_B)_3 = \frac{3}{4} (1 + e) \sqrt{3g} L \quad (5)$$

PROBLEM 17.123: FOR  $e = 0.5$

$$EQ(5): (\bar{v}_B)_3 = \frac{3}{4} (1 + 0.5) \sqrt{3g} L = \frac{9}{8} \sqrt{3g} L$$

$$\bar{v}_{CD} = \frac{3}{8} \sqrt{3g} L \rightarrow$$

PROBLEM 17.124: FOR  $e = 1$

$$EQ(5): (\bar{v}_B)_3 = \frac{3}{4} (1 + 1) \sqrt{3g} L$$

$$\bar{v}_D = \frac{1}{2} \sqrt{3g} L \rightarrow$$

# 17.125 and 17.126



GIVEN: GYMNAST A IS  
AT REST. GYMNAST B  
JUMPS ON TO PLANK AT E.

$h = 2.5 \text{ m}$

MASS OF PLANK:  $m_p = 15 \text{ kg}$

ASSUMING PERFECTLY

PLASTIC IMPACT,

FIND: HEIGHT THAT

GYMNAST RISES

PROBLEM 17.125: USE  $m_A = 55 \text{ kg}$  AND  $m_B = 70 \text{ kg}$

PROBLEM 17.126: USE  $m_B = 70 \text{ kg}$  AND  $m_G = 55 \text{ kg}$

VELOCITY OF B AS IT STRIKES E:  $\bar{v}_B = \sqrt{2gh}$

PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP.<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT D:  $\bar{v}_2 = L \omega_2$

$$m_A \bar{v}_B L = \bar{I}_D \omega_2 + m_A \bar{v}_B L + m_B \bar{v}_B L$$

$$m_B \bar{v}_B L = \frac{1}{12} m_p (2L)^2 \omega_2 + (m_A + m_B) (L \omega_2) L$$

$$m_B \bar{v}_B L = \frac{1}{3} m_p L^2 \omega_2 + (m_A + m_B) L^2 \omega_2$$

$$\omega_2 = \frac{m_B}{\frac{1}{3} m_p + m_A + m_B} \frac{\bar{v}_B}{L}$$

$$\bar{v}_2 = L \omega_2 = \frac{3 m_B}{m_p + 3 m_A + 3 m_B} \bar{v}_B \quad (1)$$

$$\text{For } h = 2.5 \text{ m} \quad \bar{v}_B = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(2.5 \text{ m})} = 7.0036 \text{ m/s}$$

PROBLEM 17.125

$$m_p = 15 \text{ kg} \quad m_A = 55 \text{ kg} \quad m_B = 70 \text{ kg}$$

$$EQ(1): \bar{v}_2 = \frac{3(70)}{15 + 3(55) + 3(70)} (7.0036 \text{ m/s}) = \frac{210}{370} (7.0036)$$

$$\bar{v}_2 = 3.771 \text{ m/s} \uparrow$$

$$\bar{v}_2 = \sqrt{2gh_2} \quad h_2 = \frac{\bar{v}_2^2}{2g} = \frac{(3.771 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.725 \text{ m}$$

GYMNAST A RISES 725 mm

PROBLEM 17.126

$$m_p = 15 \text{ kg} \quad m_A = 70 \text{ kg} \quad m_B = 55 \text{ kg}$$

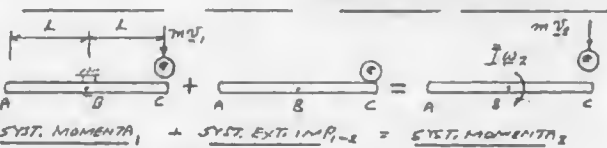
$$EQ(1): \bar{v}_2 = \frac{3(55)}{15 + 3(70) + 3(55)} (7.0036 \text{ m/s}) = \frac{165}{390} (7.0036)$$

$$\bar{v}_2 = 2.983 \text{ m/s} \uparrow$$


$$\bar{v}_2 = \sqrt{2gh_2} \quad h_2 = \frac{\bar{v}_2^2}{2g} = \frac{(2.983 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.447 \text{ m}$$

GYMNAST A RISES 447 mm

17.127



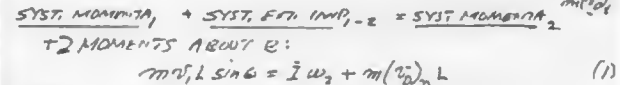
+2 MOMENTS ABOUT B:


  
 $V_1$ 
  
 $V_2$ 
  
 $\omega_2$ 
  
 $V_2 = L\omega_2$ 
  
 $V_1 = V_2 - V_2$ ;  $V_1 = L\omega_2 - V_2$ ;  $V_2 = L\omega_2 - V_1$  (2)

DATA:  $m = 0.8 \text{ kg}$ ,  $M = 2.4 \text{ kg}$ ,  $v_i = 3 \text{ m/s}$ ,  $e = 0.5$ ,  $L = 0.75 \text{ m}$

$$4.8 = 1.6 \omega_2 \quad \omega_2 = 3 \text{ rad/s} \quad \omega_2 = 3 \text{ rad/s}$$

17.128

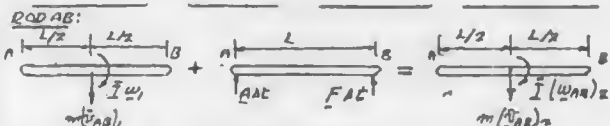


$$\begin{aligned} F_G(y): \quad m v_1 \sin \theta &= \frac{1}{12} M (2L) \omega_1 + m [L \omega_2 - (v_1 \sin \theta) c] L \\ m v_1 \sin \theta &= \frac{1}{3} M L \omega_1 + m [L \omega_2 - m (v_1 \sin \theta) c] \\ m (1+c) \frac{v_1}{L} \sin \theta &= \left( \frac{1}{3} M + m \right) \omega_2 \\ (0.28 \text{ kg}) (1+0.5) \frac{3 \text{ m/s}}{0.2 \text{ m}} \sin 60^\circ &= \left[ \frac{1}{3} (2.4 \text{ kg}) + 0.8 \text{ kg} \right] \omega_2 \end{aligned}$$

解(2):  $(v_D)_1 = (0.75 - 12.598 \text{ rad/s}) - (3 \text{ m/s}) \sin 60^\circ (0.5)$ ;  $(v_D)_1 = 0.6495 \text{ m/s}$   
 $(v_D)_2 = 0.6495 \text{ m/s} \angle 30^\circ$   $(v_D)_3 = (3 \text{ m/s}) \cos 60^\circ = 1.5 \text{ m/s}$

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17.129



+2 MOMENTS ABOUT A:

$$\frac{1}{12} m L^2 \omega, + m \left( \omega, \frac{L}{2} \right) \frac{L}{2} - (F \Delta L) L = \frac{1}{12} m L^2 (\omega_{AB})_2 + m \frac{L}{2} (\omega_{AB})_2 \frac{L}{2}$$

Red CE:

$$\underline{\text{SYST MOMENTA}_1} + \underline{\text{SYST EXT IMP}_{1 \rightarrow 2}} = \underline{\text{SYST MOMENTA}_2}$$

$$(F\Delta t) \frac{L}{\hbar} = \bar{I}(\omega_{cf}),$$

SUBSTITUTE FOR (Fol) FROM (1)

$$\omega_1 - (\omega_{AQ})_2 = \frac{1}{2} (\omega_{CE})_2 \quad (2)$$

$$\omega_1 - (\omega_{AQ})_2 = \frac{1}{2} (\omega_{CE})_2 \quad (2)$$

$$\begin{aligned} (v_B)_c &= (v_C)_2 - (v_B)_2 \\ L w_c &= \frac{1}{2} (w_{ct})_2 - L (w_{Ar})_2 \\ (w_{Ar})_2 &= \frac{1}{2} (w_{Ar})_2 - w_c \end{aligned} \quad (3)$$

$$w_i(1+\epsilon) = (w_{ct})_i$$

$$(\omega_{ce})_2 = \omega_1(1+e)$$

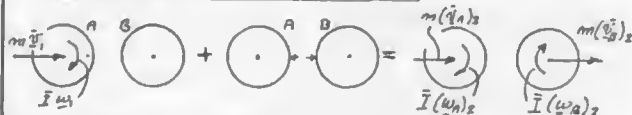
$$(\omega_{ce})_2 = \omega_1 (1 + e)$$

$$(\underline{w}_n)_2 = \frac{1}{2} \omega_1 (1 - c)$$



# 17.131 continued

CONSIDER BOTH SPHERES AS A SYSTEM



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP.<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

± COMPONENTS:  $m\vec{v}_1 = m(\vec{v}_A)_2 + m(\vec{v}_B)_2$   
 $\vec{v}_1 = (\vec{v}_A)_2 + (\vec{v}_B)_2$  (1)

RELATIVE VELOCITIES ( $e=1$ )

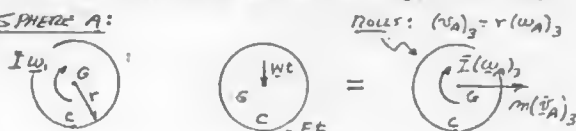
$(\vec{v}_B)_2 - (\vec{v}_A)_2 = e\vec{v}_1 = \vec{v}_1$   $\vec{v}_1 = (\vec{v}_B)_2 - (\vec{v}_A)_2$  (2)

ADD EGS. (1) AND (2):  $2\vec{v}_1 = 2(\vec{v}_A)_2$  ;  $(\vec{v}_B)_2 = \vec{v}_1$  →  
 SUBTRACT EG. (1) FROM EQ. (2):  $0 = 2(\vec{v}_A)_2$  ;  $(\vec{v}_A)_2 = 0$

(b) MOTION AFTER SPHERES START ROLLING UNIFORMLY

NOTE: TIME INTERVAL IS NOT SMALL AND IMPULSES OF FRICTION FORCES MUST BE INCLUDED

SPHERE A:

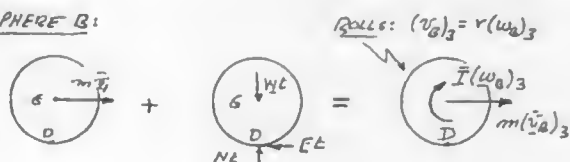


SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP.<sub>2-3</sub> = SYST. MOMENTA<sub>3</sub>

+ MOMENTS ABOUT C:  $I\omega_A = I(\omega_A)_3 + m(\vec{v}_A)_3 r$   
 $\frac{2}{5}mr^2\omega_A = \frac{2}{5}mr^2(\omega_A)_3 + mr^2(\omega_A)_3$   
 $(\omega_A)_3 = \frac{2}{7}\omega_A$

$(\vec{v}_A)_3 = r(\omega_A)_3 = \frac{2}{7}r\omega_A = \frac{2}{7}\vec{v}_1$   $(\vec{v}_A)_3 = \frac{2}{7}\vec{v}_1$

SPHERE B:



SYST. MOMENTA<sub>2</sub> + SYST. EXT. IMP.<sub>2-3</sub> = SYST. MOMENTA<sub>3</sub>

+ MOMENTS ABOUT D:

$m\vec{v}_1 r = I(\omega_B)_3 + m(\vec{v}_B)_3 r$

$m\vec{v}_1 r = \frac{2}{5}mr^2(\omega_B)_3 + mr^2(\omega_B)_3$

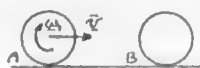
$(\omega_B)_3 = \frac{5}{7}\frac{\vec{v}_1}{r}$  BUT,  $\omega_B = \frac{\vec{v}_1}{r}$

$(\omega_B)_3 = \frac{5}{7}\omega_B$

$(\vec{v}_B)_3 = r(\omega_B)_3 = r(\frac{5}{7}\frac{\vec{v}_1}{r})$   $(\vec{v}_B)_3 = \frac{5}{7}\vec{v}_1$

SUMMARY

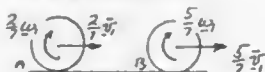
INITIAL MOTION



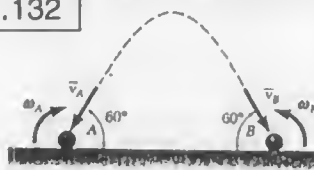
JUST AFTER IMPACT



FINAL (UNIFORM) MOTION



# 17.132



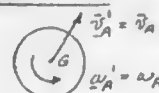
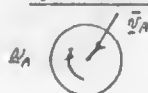
GIVEN: BALL BOUNCES AS SHOWN.  $\omega_0 = \omega_0$ ,  $\vec{v}_A = \vec{v}_0$   
 FIND:  $\omega_0$  IN TERMS OF  $\vec{v}_0$  AND  $r$

SINCE THE LINEAR AND ANGULAR VELOCITIES ARE CHANGED DURING A SHORT INTERVAL  $\Delta t$ , BOTH THE NORMAL AND FRICTION FORCES ARE IMPULSIVE. WE ASSUME THAT NO SLIPPING OCCURS.

FOR THE VELOCITY OF THE BALL TO BE REVERSED AT EACH IMPACT, WE MUST HAVE AT POINT A.

BEFORE IMPACT

AFTER IMPACT



IMPACT AT A:



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP.<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

+ MOMENTS ABOUT C:

$I\omega_A - (m\vec{v}_A \cos 60^\circ)r = -I\omega_A' + (m\vec{v}_A' \cos 60^\circ)r$

SUBSTITUTE:  $\omega_A' = \omega_A = \omega_0$   
 $\vec{v}_A' = \vec{v}_A = \vec{v}_0$

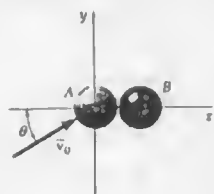
$2I\omega_0 = 2(m\vec{v}_0 \cos 60^\circ)r$

$\omega_0 = \frac{(m\vec{v}_0 \cos 60^\circ)r}{I}$

$\omega_0 = \frac{m\vec{v}_0(\frac{1}{2})r}{\frac{2}{5}mr^2}$

$\omega_0 = \frac{5}{4}\frac{\vec{v}_0}{r}$

# 17.133 and 17.134



GIVEN: BALL A IS ROLLING WITHOUT SLIPPING WHEN IT HITS BALL B. COEF. OF KINETIC FRICTION IS  $\mu_k$ . ASSUMING PERFECTLY ELASTIC IMPACT,

PROBLEM 17.133

FIND: (a)  $\vec{v}$  AND  $\omega$  OF EACH BALL, (b)  $\vec{v}_B$  AFTER IT STARTS ROLLING

PROBLEM 17.134: FIND EQUATION OF PATH OF BALL A

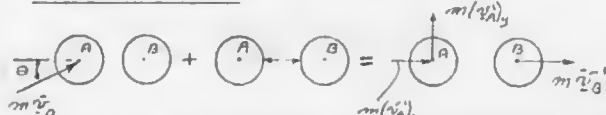
PROBLEM 17.133

(a) MOTION IMMEDIATELY AFTER IMPACT

FRICTION FORCES ARE NON IMPULSIVE, THUS ANGULAR MOMENTUM (AND THUS  $\omega$ ) OF EACH BALL IS UNCHANGED. WE HAVE  $\omega_B' = 0$  AND SINCE BALL A WAS ROLLING:

$$\omega_A' = \frac{\vec{v}_0}{r} \quad \omega_A' = \frac{\vec{v}_0}{r} (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

LOOKING DOWNWARD



SYST. MOMENTA<sub>0</sub> + SYST. EXT. IMP<sub>0-1</sub> = SYST. MOMENTA<sub>1</sub>

$$\pm \text{COMPONENTS: } m\vec{v}_0 \cos \theta = m(\vec{v}_A')_x + m\vec{v}_B' \quad (1)$$

$$+ \uparrow \text{COMPONENTS OF BALL A: } m\vec{v}_0 \sin \theta = m(\vec{v}_A')_y$$

$$(\vec{v}_A')_y = \vec{v}_0 \sin \theta \quad (2)$$

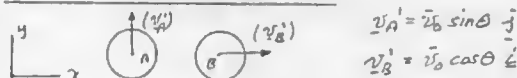
FOR ELASTIC IMPACT  $e = 1$

$$\vec{v}_B' - (\vec{v}_A')_x = \vec{v}_0 \cos \theta \quad (3)$$

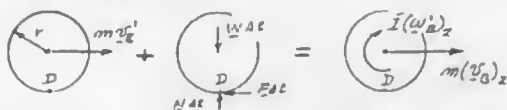
SOVING SIMULTANEOUSLY EQS. (1) AND (2)

$$(\vec{v}_A')_x = 0 \quad (\vec{v}_B') = \vec{v}_0 \cos \theta \quad (4)$$

MOTION IMMEDIATELY AFTER IMPACT:



(b) FINAL VELOCITY OF BALL B:



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

$$+ \uparrow \text{MOMENTS ABOUT D: } m\vec{v}_B' r = \bar{I}(\omega_B')_2 + m(\vec{v}_B')_2 r \quad (5)$$

WE RECALL:  $\vec{v}_B' = \vec{v}_0 \cos \theta$  AND  $\bar{I} = \frac{2}{5} m r^2$

ROLL ROLLS:  $(\vec{v}_B')_2 = r(\omega_B')_2$

$$\text{EQ. (5): } m r \vec{v}_0 \cos \theta = \frac{2}{5} m r^2 (\omega_B')_2 + m r (\omega_B')_2 r$$

$$(\omega_B')_2 = \frac{5}{7} \frac{\vec{v}_0 \cos \theta}{r} \quad (\vec{v}_B')_2 = r(\omega_B')_2 \rightarrow$$

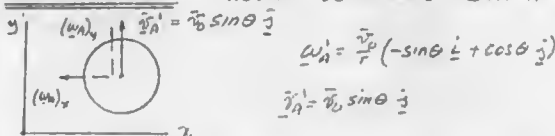
$$(\vec{v}_B')_2 = \frac{5}{7} \vec{v}_0 \cos \theta \hat{i}$$

(CONTINUED)

# 17.133 and 17.134 continued

PROBLEM 17.134

LOOKING DOWNWARD ON BALL A



WE ASSUME THAT BALL A ROLLS WITHOUT SLIPPING IN Y DIRECTION

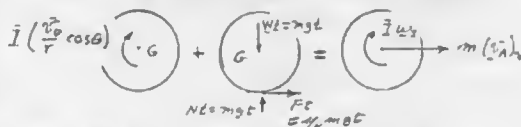
$$(\vec{v}_A')_y = (\omega_A')_x \times r = -\left(\frac{\vec{v}_0}{r} \sin \theta \hat{i}\right) \times r \hat{i} = + \vec{v}_0 \sin \theta \hat{j}$$

ASSUMPTION IS CORRECT

THE Y COMPONENT OF VELOCITY IS CONSTANT AND THUS THE Y COORDINATE AT ANY TIME  $t$  IS

$$y = (\vec{v}_A')_y t = (\vec{v}_0 \sin \theta) t \quad (1)$$

$\mu_k$  = COEF. OF KINETIC FRICTION BETWEEN BALLS AND TABLE  
BALL A ROLLS AND SLIDES IN THE X DIRECTION



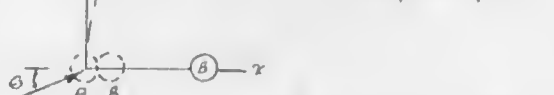
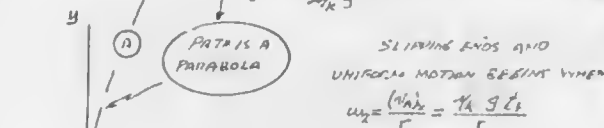
$$\pm X \text{ COMPONENTS: } 0 + \mu_k m g t = m(\vec{v}_A')_x \quad m(\vec{v}_A')_x = \mu_k g t$$

$$(\vec{v}_A')_x = \mu_k g = \text{CONSTANT} \quad x = \frac{1}{2} a t^2 \quad x = \frac{1}{2} \mu_k g t^2 \quad (2)$$

ELIMINATE  $t$  BETWEEN EQS. (1) AND (2)

$$t = y / (\vec{v}_0 \sin \theta); \quad x = \frac{1}{2} \mu_k g \left( \frac{y^2}{\vec{v}_0^2 \sin^2 \theta} \right);$$

$$x = \frac{y^2}{2 \vec{v}_0^2 \sin^2 \theta} \frac{\mu_k g}{\sin^2 \theta}$$



$$+ \uparrow \text{MOMENTS ABOUT G: } \bar{I} \left( \frac{\vec{v}_0}{r} \cos \theta \right) - (\mu_k m g t) = \bar{I} \omega_2$$

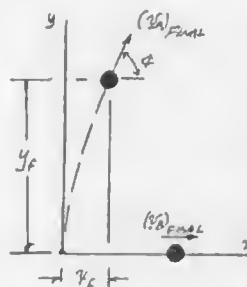
$$\frac{2}{5} m r \left( \frac{\vec{v}_0}{r} \cos \theta \right) - \mu_k m g t = \frac{2}{5} m r^2 \left( \frac{\omega_2}{r} \right)$$

$$\frac{2}{5} \vec{v}_0 \cos \theta = \frac{2}{5} \mu_k g t$$

ROLLING WITHOUT SLIDING BEGINS WHEN  $t_f = \frac{2}{7} \frac{\vec{v}_0 \cos \theta}{\mu_k g}$

$$\text{EQ. (2): } x_f = \frac{1}{2} \mu_k g t_f^2 = \frac{2}{7} \frac{\vec{v}_0^2 \cos^2 \theta}{\mu_k g}$$

$$\text{EQ. (1): } y_f = \vec{v}_0 \sin \theta t_f = \frac{2}{7} \frac{\vec{v}_0^2 \sin \theta \cos \theta}{\mu_k g}$$



FINAL VELOCITIES (UNIFORM MOTION)

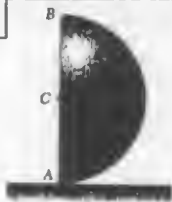
$$(\vec{v}_A')_x = \mu_k g t_f = \frac{2}{7} \vec{v}_0 \cos \theta$$

$$(\vec{v}_A')_y = \vec{v}_0 \sin \theta$$

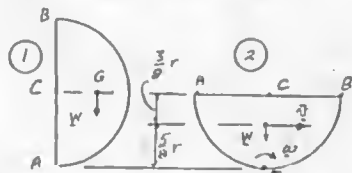
$$(\vec{v}_A')_{\text{FINAL}} = \left( \frac{2}{7} \vec{v}_0 \cos \theta \right) \hat{i} + (\vec{v}_0 \sin \theta) \hat{j}$$

$$(\vec{v}_B')_{\text{FINAL}} = (\vec{v}_B')_2 = \frac{5}{7} \vec{v}_0 \cos \theta \hat{i}$$

17.135



GIVEN: UNIFORM HEMISPHERE IS RELEASED FROM REST AND ROLLS WITHOUT SLIDING. AFTER HEMISPHERE ROLLS THROUGH  $90^\circ$ , FIND: (a)  $\omega$ , (b) NORMAL REACTION.



$$\bar{v} = \frac{5}{8} r \omega$$

$$\bar{I} = \frac{2}{5} m r^2 - m \left( \frac{3}{8} r \right)^2$$

$$\bar{I} = \left( \frac{2}{5} - \frac{9}{64} \right) m r^2$$

(a) WORK-ENERGY:  $U_{1 \rightarrow 2} = W \left( \frac{3}{8} r \right) = \frac{3}{8} m g r$

$$T_1 = 0, \quad T_2 = \frac{1}{2} \bar{I} \omega^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2} \left( \frac{2}{5} - \frac{9}{64} \right) m r^2 \omega^2 + \frac{1}{2} m \left( \frac{5}{8} r \right)^2 \omega^2$$

$$T_2 = \frac{13}{40} m r^2 \omega^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + \frac{3}{8} m g r = \frac{13}{40} m r^2 \omega^2$$

$$\omega^2 = \frac{15}{13} \frac{g}{r} \quad \omega = 1.074 \sqrt{\frac{g}{r}}$$

(b) REACTION AT D:  $a = 0 \quad a_c = 0$



$$\bar{a} = a_c + a_{c/c} = \left( \frac{7}{8} r \right) \omega^2 \uparrow$$

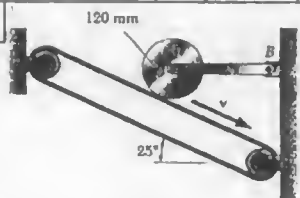
$$m \bar{a} = m \left( \frac{7}{8} r \right) \omega^2 \uparrow$$

$$\sum F = \sum F_R: N - mg = m \left( \frac{7}{8} r \right) \omega^2$$

$$N = mg + m \left( \frac{7}{8} r \right) \left( \frac{15}{13} \frac{g}{r} \right) = \frac{149}{104} mg$$

$$N = 1.433 mg \uparrow$$

17.136



GIVEN:  $\mu_k = 0.15$   
 $v = 25 \text{ m/s}$

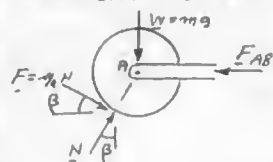
CYLINDER IS AT REST WHEN PLACED ON BELT.

FIND: (a) NUMBER OF REVOLUTIONS

BEFORE CYLINDER REACHES CONSTANT VELOCITY.

(b) TIME REQUIRED TO REACH CONSTANT VELOCITY.

WHILE SLIPPING OCCURS:



$$\sum F_y = 0$$

$$N \cos \beta - \mu_k N \sin \beta - mg = 0$$

$$N = \frac{mg}{\cos \beta - \mu_k \sin \beta} \quad (1)$$

SLIPPING OCCURS UNTIL:

$$\omega = \frac{v}{r}$$

WORK-ENERGY:  $M_R = Fr =$  MOMENT OF  $F$  ABOUT A.

$$U_{1 \rightarrow 2} = M_R \theta = Fr \theta = \mu_k N r \theta$$

$$T_1 = 0; \quad T_2 = \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \left( \frac{v}{r} \right)^2 = \frac{1}{4} m v^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + \mu_k N r \theta = \frac{1}{4} m v^2$$

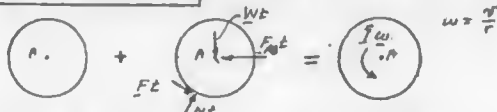
$$\theta = \frac{1}{4} \frac{m v^2}{\mu_k N} = \frac{1}{4} \frac{m v^2}{\mu_k N} \cdot \frac{\cos \beta - \mu_k \sin \beta}{mg}$$

$$\theta = \frac{1}{4} \frac{v^2}{\mu_k g} \cdot (\cos \beta - \mu_k \sin \beta) \quad (2)$$

(CONTINUED)

17.136 continued

PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA, + SYST. EXT. IMP.  $\rightarrow 2 =$  SYST. MOMENTA<sub>2</sub>

+1) MOMENTS ABOUT A:  $FLr = \bar{I} \omega$

$$\mu_k N r = \frac{1}{2} m r^2 \left( \frac{v}{r} \right)$$

SUBSTITUTE FOR N:

$$\mu_k \left( \frac{mg}{\cos \beta - \mu_k \sin \beta} \right) r = \frac{1}{2} m r v$$

$$t = \frac{1}{2} \frac{v}{\mu_k g} (\cos \beta - \mu_k \sin \beta) \quad (3)$$

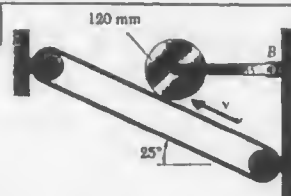
DATA:  $\mu_k = 0.15, \beta = 25^\circ, v = 25 \text{ m/s}, r = 0.12 \text{ m}$

$$\text{EQ(2): } \theta = \frac{(25 \text{ m/s})^2}{4(0.15)(0.12 \text{ m})(9.81 \text{ m/s}^2)} [\cos 25^\circ - (0.15) \sin 25^\circ]$$

$$\theta = 745.86 \text{ rad} \left( \frac{9.81 \text{ m/s}^2}{2\pi \text{ rad}} \right); \quad \theta = 118.7 \text{ revolutions}$$

$$\text{EQ(3): } t = \frac{25 \text{ m/s}}{2(0.15)(9.81 \text{ m/s}^2)} [\cos 25^\circ - (0.15) \sin 25^\circ]; \quad t = 7.16 \text{ s}$$

17.137

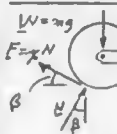


GIVEN:  $\mu_k = 0.15$   
 $v = 25 \text{ m/s}$

CYLINDER AT REST PLACED ON BELT. UNTIL MOTION BECOMES UNIFORM

FIND: (a) NUMBER

OF REVOLUTIONS REQUIRED. (b) TIME INTERVAL REQUIRED



WHILE SLIPPING OCCURS:

$$\sum F_y = 0; \quad N \cos \beta - \mu_k N \sin \beta - mg = 0$$

$$N = \frac{mg}{\cos \beta - \mu_k \sin \beta} \quad (1)$$

FOR CYLINDER SLIPPING OCCURS UNTIL  $\omega = \frac{v}{r}$

WORK-ENERGY:  $M_R = Fr =$  MOMENT OF  $F$  ABOUT A.

$$U_{1 \rightarrow 2} = M_R \theta = Fr \theta = \mu_k N r \theta$$

$$T_1 = 0; \quad T_2 = \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \left( \frac{v}{r} \right)^2 = \frac{1}{4} m v^2$$

$$T_1 + U_{1 \rightarrow 2} = T_2; \quad 0 + \mu_k N r \theta = \frac{1}{4} m v^2$$

$$\theta = \frac{1}{4} \frac{m v^2}{\mu_k N} = \frac{1}{4} \frac{m v^2}{\mu_k N} \cdot \frac{\cos \beta - \mu_k \sin \beta}{mg}$$

$$\theta = \frac{1}{4} \frac{v^2}{\mu_k g} (\cos \beta + \mu_k \sin \beta) \quad (2)$$

PRINCIPLE OF IMPULSE-MOMENTUM



+2) MOMENTS ABOUT A:  $FLr = \bar{I} \omega$

$$\mu_k N r = \frac{1}{2} m r^2 \left( \frac{v}{r} \right)$$

SUBSTITUTE FOR N:

$$\mu_k \left( \frac{mg}{\cos \beta + \mu_k \sin \beta} \right) r = \frac{1}{2} m r v$$

$$t = \frac{1}{2} \frac{v}{\mu_k g} (\cos \beta + \mu_k \sin \beta) \quad (3)$$

DATA:  $\mu_k = 0.15, \beta = 25^\circ, v = 25 \text{ m/s}, r = 0.12 \text{ m}$

$$\text{EQ(2): } \theta = \frac{(25 \text{ m/s})^2}{4(0.15)(0.12 \text{ m})(9.81 \text{ m/s}^2)} [\cos 25^\circ + (0.15) \sin 25^\circ]$$

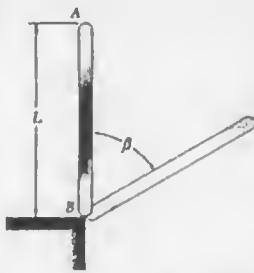
$$\theta = 858.05 \text{ rad} \left( \frac{9.81 \text{ m/s}^2}{2\pi \text{ rad}} \right) \quad \theta = 136.6 \text{ revolutions}$$

$$\text{EQ(3): } t = \frac{25 \text{ m/s}}{2(0.15)(9.81 \text{ m/s}^2)} [\cos 25^\circ + (0.15) \sin 25^\circ]$$

$$t = 8.24 \text{ s}$$

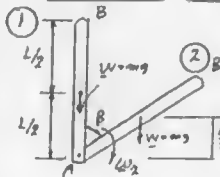


17.138



GIVEN: ROD AB IS  
GIVEN A SLIGHT MOTION  
CLOCK WISE

FIND: (a) ANGLE  $\beta$   
WHEN ROD LOSES  
CONTACT WITH  
CORNER  
(b) CORRESPONDING  $\omega_A$



WORK-ENERGY:

$$U_{1-2} = W = \frac{1}{2} (1 - \cos \beta) = mg \frac{L}{2} (1 - \cos \beta)$$

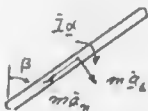
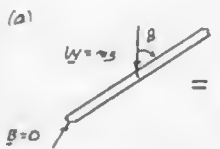
$$T_1 = 0$$

$$T_2 = \frac{1}{2} I_A \omega_2^2 = \frac{1}{2} \left( \frac{1}{3} mL^2 \right) \omega_2^2 = \frac{1}{6} mL^2 \omega_2^2$$

$$T_1 + U_{1-2} = T_2$$

$$0 + mg \frac{L}{2} (1 - \cos \beta) = \frac{1}{6} mL^2 \omega_2^2$$

$$\omega_2^2 = \frac{3g}{L} (1 - \cos \beta) \quad (1)$$



$$a_n = \frac{1}{2} \omega_2^2$$

$$a_n = \frac{3}{2} g (1 - \cos \beta)$$

$$\pm \sum F = \sum F_c: mg \cos \beta = ma_n = m \frac{3}{2} g (1 - \cos \beta)$$

$$\cos \beta = \frac{3}{5} = \frac{3}{5} \cos \beta$$

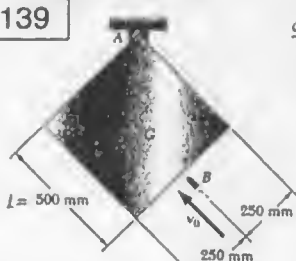
$$2.5 \cos \beta = 1.5; \cos \beta = 0.6; \beta = 53.1^\circ$$

(b) WHEN  $\cos \beta = 0.6$

$$EQ(1) \quad \omega_2^2 = \frac{3g}{L} (1 - 0.6) = 1.2 \frac{g}{L}; \quad \omega_2 = \sqrt{1.2 \frac{g}{L}}$$

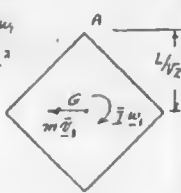
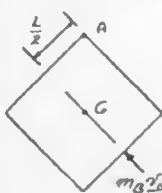
$$\tau_A = L \omega_2 = L \sqrt{1.2 \frac{g}{L}} \quad \tau_A = \sqrt{1.2 g L} \approx 53.1^\circ$$

17.139



GIVEN: 35-g BULLET FIRED  
WITH  $v_0 = 400 \text{ m/s}$  BECOMES  
EMBEDDED IN  $\Delta t = 1.5 \text{ ms}$ .  
MASS OF PLATE = 3 kg

FIND: (a)  $\omega$  JUST AFTER  
BULLET BECOMES EMBEDDED.  
(b) IMPULSIVE REACTION AT A.



$$\text{SYST. MOMENTA}_0 + \text{SYST. EXT. IMP}_{0-1} = \text{SYST. MOMENTA}_1$$

$$+2 \text{ MOMENTS ABOUT A: } m_B v_0 \frac{L}{2} = I \omega + m \bar{v}_1 \frac{L}{2}$$

$$m_B v_0 \frac{L}{2} = \frac{1}{6} mL^2 \omega + m \left( \frac{L}{2} \right)^2 \omega$$

$$\omega_1 = \frac{3 \cdot m_B v_0}{4 mL} = \frac{3 \cdot (0.035 \text{ kg}) (400 \text{ m/s})}{4 (3 \text{ kg}) (0.5 \text{ m})}; \quad \omega_1 = 7 \text{ rad/s}$$

$$\bar{v}_1 = \frac{1}{2} \omega_1 = \frac{0.5}{2} (7 \text{ rad/s}) = 2.75 \text{ m/s}$$

$$\pm \text{COMPONENTS: } A_x \Delta t - m_B v_0 \frac{1}{\sqrt{2}} = -m \bar{v}_1$$

$$A_x (0.0015) - (0.035 \text{ kg}) (400 \text{ m/s}) \frac{1}{\sqrt{2}} = -(3 \text{ kg}) (2.75 \text{ m/s})$$

$$A_x (0.0015) - 9.897 = -7.475; \quad A_x = 1650 \text{ N}$$

$$\pm \text{COMPONENTS: } A_y \Delta t - m_B v_0 = 0$$

$$A_y (0.0015) - (0.035 \text{ kg}) (400 \text{ m/s}) \frac{1}{\sqrt{2}}; \quad A_y = 6600 \text{ N}$$

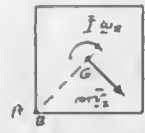
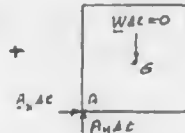
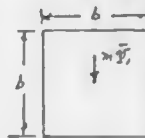
$$\theta = 6.80^\circ \text{ N } 76.0^\circ$$

17.140



GIVEN: UNIFORM BLOCK

JUST AFTER A PERFECTLY  
PLASTIC IMPACT AT B  
FIND: (a)  $\omega$ , (b)  $\bar{v}$ .



BLOCK ROTATES ABOUT B

$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+2 \text{ MOMENTS ABOUT A: } m \bar{v}_1 \frac{b}{2} = I \omega_2 + m \bar{v}_2 (b)$$

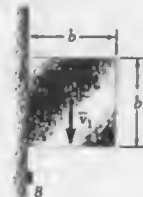
$$(b) = \frac{b}{\sqrt{2}} \quad \bar{v}_2 = (b) \omega_2 = \frac{b}{\sqrt{2}} \omega_2 \quad I = \frac{1}{8} m b^2$$

$$m \bar{v}_1 \frac{b}{2} = \frac{1}{8} m b^2 \omega_2 + m \left( \frac{b}{\sqrt{2}} \right)^2 \omega_2$$

$$\frac{1}{2} \bar{v}_1 = \frac{3}{8} b \omega_2; \quad \omega_2 = \frac{4}{3} \frac{\bar{v}_1}{b}$$

$$\bar{v}_2 = (b) \omega_2 = \frac{b}{\sqrt{2}} \cdot \frac{4}{3} \frac{\bar{v}_1}{b}; \quad \bar{v}_2 = \frac{2\sqrt{2}}{3} \bar{v}_1 \angle 45^\circ$$

17.141



GIVEN: UNIFORM BLOCK

JUST AFTER A PERFECTLY  
ELASTIC IMPACT AT B  
FIND: (a)  $\omega$ , (b)  $\bar{v}$ .

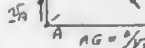
KINEMATICS:

AFTER ELASTIC IMPACT ( $e=1$ )

$$\bar{v}_A = \bar{v}_1 \uparrow$$

$$\bar{v}_2 = \bar{v}_A + \bar{v}_{G/A}$$

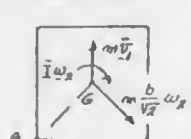
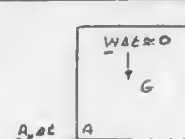
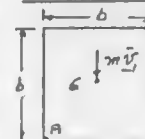
$$\bar{v}_2 = [\bar{v}_1 \uparrow] + \left[ \frac{b}{\sqrt{2}} \omega_2 \angle 45^\circ \right] \quad (1)$$



$$AG = \frac{b}{\sqrt{2}}$$

PRINCIPLE OF IMPULSE-MOMENTUM

$$AG = \frac{b}{\sqrt{2}}$$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+2 \text{ MOMENTS ABOUT A:}$$

$$m \bar{v}_1 \frac{b}{2} = I \omega_2 - m \bar{v}_2 \frac{b}{2} + m \left( \frac{b}{\sqrt{2}} \right)^2 \omega_2$$

$$I = \frac{1}{8} m b^2 \quad (AG) = \frac{b}{\sqrt{2}}$$

$$m \bar{v}_1 \frac{b}{2} = \frac{1}{8} m b^2 \omega_2 - m \bar{v}_2 \frac{b}{2} + m \left( \frac{b}{\sqrt{2}} \right)^2 \omega_2$$

$$m \bar{v}_1 \frac{b}{2} = \frac{3}{8} b^2 \omega_2 \quad \omega_2 = \frac{3}{2} \frac{\bar{v}_1}{b}$$

$$EQ(1) \quad \bar{v}_2 = [\bar{v}_1 \uparrow] + \left[ \frac{b}{\sqrt{2}} \cdot \frac{3}{2} \frac{\bar{v}_1}{b} \angle 45^\circ \right]$$

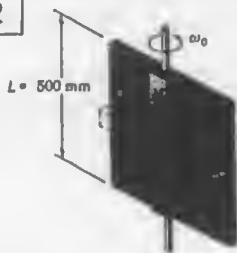
$$= [\bar{v}_1 \uparrow] + \left[ \frac{3}{2\sqrt{2}} \bar{v}_1 \sin 45^\circ \right] + \left[ \frac{3}{2\sqrt{2}} \bar{v}_1 \cos 45^\circ \right]$$

$$= [\bar{v}_1 \uparrow] + \left[ \frac{3}{4} \bar{v}_1 \uparrow \right] + \left[ \frac{3}{4} \bar{v}_1 \rightarrow \right]$$



$$\bar{v}_2 = 0.791 \bar{v}_1 \angle 18.4^\circ$$

17.142



GIVEN: 3-in. BAR AB  
4-in. PLATE  
 $\omega_0 = 120 \text{ rpm}$

AFTER BAR SWUNG TO HORIZONTAL,  
FIND: (a)  $\omega$ , (b) ENERGY LOST DURING PLASTIC IMPACT AT C

(a) LOOKING DOWNWARD



CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT

$$I_0 \omega_0 = I_1 \omega_1$$

(1)

$$I_0 = I_{\text{PLATE}} = \frac{1}{12} (4 \text{ in})^2 L^2$$

$$I_1 = I_{\text{PLATE}} + I_{\text{BAR}} = \frac{1}{12} (4 \text{ in})^2 L^2 + \frac{1}{12} (3 \text{ in})^2 L^2 = \frac{1}{12} (7 \text{ in})^2 L^2$$

$$\text{EQ (1): } \frac{1}{12} (4 \text{ in})^2 L^2 (120 \text{ rpm}) = \frac{1}{12} (7 \text{ in})^2 L^2 \omega_1$$

$$\omega_1 = \frac{4}{7} (120 \text{ rpm})$$

$$\omega_1 = 68.6 \text{ rpm}$$

(b) ENERGY: (WE MUST USE rad/s)

$$\omega_0 = 120 \text{ rpm} \left( \frac{2\pi}{60} \right) = 4\pi \text{ rad/s} = 12.566 \text{ rad/s}$$

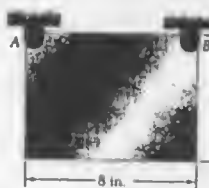
$$\omega_1 = \frac{4}{7} \omega_0 = \frac{4}{7} (4\pi \text{ rad/s}) = 2.181 \text{ rad/s}$$

$$T_0 = \frac{1}{2} I_0 \omega_0^2 = \frac{1}{2} \left[ \frac{1}{12} (4 \text{ in})^2 (0.5 \text{ m})^2 \right] (12.566 \text{ rad/s})^2 = 6.580 \text{ J}$$

$$T_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} \left[ \frac{1}{12} (7 \text{ in})^2 (0.5 \text{ m})^2 \right] (2.181 \text{ rad/s})^2 = 3.760 \text{ J}$$

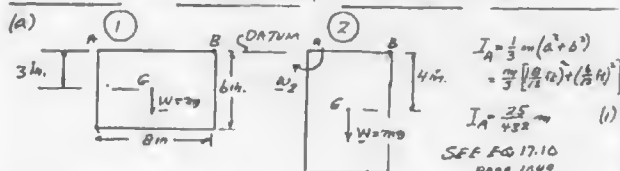
$$\text{ENERGY LOST} = 6.580 \text{ J} - 3.760 \text{ J} = 2.82 \text{ J}$$

17.143



GIVEN: PIN B IS REMOVED AND PLATE SWINGS ABOUT A

FIND: (a)  $\omega$  AFTER 90° ROTATION, (b) MAXIMUM  $\omega$



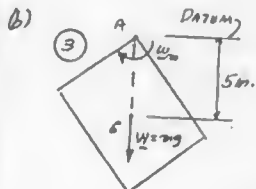
$$U_{1-2} = W(4 \text{ in.} - 3 \text{ in.}) = mg \left( \frac{1}{12} \text{ ft} \right)$$

$$T_1 = 0; T_2 = \frac{1}{2} I_A \omega_2^2 = \frac{1}{2} \left( \frac{25}{432} \right) m \omega_2^2$$

$$T_1 + U_{1-2} = T_2; mg \left( \frac{1}{12} \right) = \frac{1}{2} \left( \frac{25}{432} \right) m \omega_2^2$$

$$\omega_2^2 = \frac{18}{25} g = \frac{18}{25} (32.2) = 23.184$$

$$\omega_2 = 4.81 \text{ rad/s}$$



$$AG = \sqrt{(3 \text{ in.})^2 + (4 \text{ in.})^2} = 5 \text{ in.}$$

$$\text{EQ (1): } I_A = \frac{25}{432} m$$

$$U_{1-3} = W(5 \text{ in.} - 3 \text{ in.}) = mg \left( \frac{2}{12} \text{ ft} \right)$$

$$T_1 = 0$$

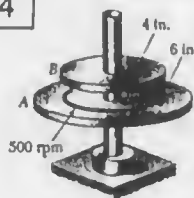
$$T_3 = \frac{1}{2} I_A \omega_3^2 = \frac{1}{2} \left( \frac{25}{432} \right) m \omega_3^2$$

$$T_1 + U_{1-3} = T_3; mg \left( \frac{2}{12} \right) = \frac{1}{2} \left( \frac{25}{432} \right) m \omega_3^2$$

$$\omega_3^2 = \frac{36}{25} g = \frac{36}{25} (32.2) = 46.372$$

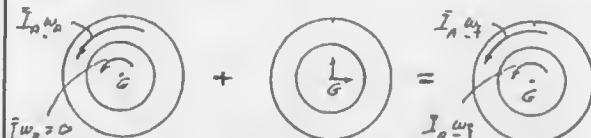
$$\omega_3 = 6.81 \text{ rad/s}$$

17.144



GIVEN: DISK OF SAME THICKNESS AND SAME MATERIAL  
DISC B IS AT REST WHEN IT IS DROPPED ON DISC A  
KNOWING  $\omega_A = 18 \text{ rad/s}$ ,  
FIND: (a) FINAL  $\omega$  OF DISKS  
(b) CHANGE IN KINETIC ENERGY

(a) FOR A DISK:  $m = \rho V = \rho \pi r^2 L$ ;  $I = \frac{1}{2} m r^2 = \frac{1}{2} (\rho \pi r^2 L) r^2 = \frac{1}{2} \rho \pi L r^4$



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+ \text{MOMENTS ABOUT G: } I_A \omega_A = I_A \omega_f + I_B \omega_f$$

$$\frac{1}{2} \rho \pi L r_A^4 \omega_A = \left( \frac{1}{2} \rho \pi L r_A^4 + \frac{1}{2} \rho \pi L r_B^4 \right) \omega_f$$

$$\omega_f = \frac{r_A^4}{r_A^4 + r_B^4} \omega_A$$

$$\omega_f = \frac{(6 \text{ in.})^4}{(6 \text{ in.})^4 + (4 \text{ in.})^4} (500 \text{ rpm}) = 417.57 \text{ rpm}$$

$$\omega_f = 418 \text{ rpm}$$

$$(b) \text{ ENERGY: } W_A = \pi r_A^2 g = \rho g \pi L r_A^2 \quad \left\{ \begin{array}{l} \frac{W_A}{W_B} = \frac{r_A^2}{r_B^2} \\ W_B = \pi r_B^2 g = \rho g \pi L r_B^2 \end{array} \right.$$

$$W_A = 18 \text{ lb}; W_B = \left( \frac{r_B^2}{r_A^2} \right) W_A = \left( \frac{4 \text{ in.}}{6 \text{ in.}} \right)^2 (18 \text{ lb}) = 8 \text{ lb}$$

$$\text{INITIAL KINETIC ENERGY } \omega_A = 500 \text{ rpm} \left( \frac{2\pi}{60} \right) = 52.36 \text{ rad/s}$$

$$T_1 = \frac{1}{2} I_A \omega_A^2 = \frac{1}{2} \left[ \frac{1}{2} \frac{18 \text{ lb}}{32.2} \left( \frac{6}{12} \text{ ft} \right)^2 \right] (52.36 \text{ rad/s})^2 = 95.784 \text{ ft} \cdot \text{lb}$$

$$\omega_f = 417.56 \text{ rpm} \left( \frac{2\pi}{60} \right) = 43.723 \text{ rad/s}$$

$$T_2 = \frac{1}{2} (I_A + I_B) \omega_f^2 = \frac{1}{2} \left[ \frac{1}{2} \frac{18 \text{ lb}}{32.2} \left( \frac{6}{12} \text{ ft} \right)^2 + \frac{1}{2} \frac{8 \text{ lb}}{32.2} \left( \frac{4}{12} \text{ ft} \right)^2 \right] (43.723 \text{ rad/s})^2$$

$$T_2 = 79.985 \text{ ft} \cdot \text{lb}$$

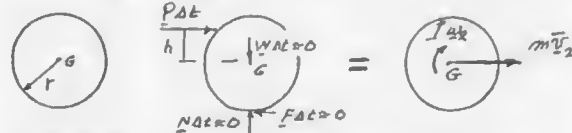
$$\text{ENERGY LOSS: } \Delta T = T_1 - T_2 = 95.784 \text{ ft} \cdot \text{lb} - 79.985 \text{ ft} \cdot \text{lb}$$

$$\Delta T = -15.80 \text{ ft} \cdot \text{lb}$$

17.145



FIND: DISTANCE h IF BALL IS TO START ROLLING WITHOUT SLIDING



$$\text{SYST. MOMENTA}_1 + \text{SYST. EXT. IMP.}_{1-2} = \text{SYST. MOMENTA}_2$$

$$+ \text{MOMENTS ABOUT G: } (P \Delta t) h = I \omega_2$$

$$+ \text{COMPONENTS: } P \Delta t = m \bar{v}_2$$

DIVIDE EQ (1) BY EQ (2) MEMBER BY MEMBER

$$h = \frac{I}{m} \cdot \frac{\omega_2}{\bar{v}_2}$$

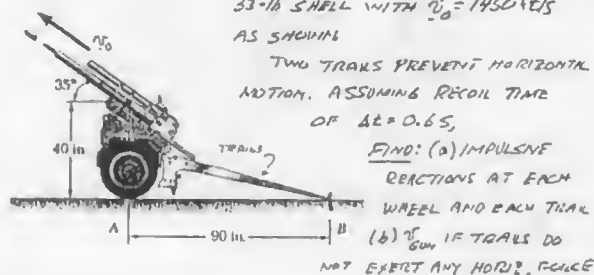
FOR ROLLING  $\bar{v} = r \omega_2$ 

$$h = \frac{I}{m} \cdot \frac{\omega_2}{r \omega_2} = \frac{I}{m r}$$

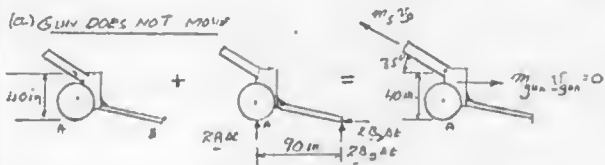
$$h = \frac{2}{5} r$$

17.146

GIVEN: 4980-lb GUN FIRES  
33-lb SHELL WITH  $\vec{v}_0 = 1450 \text{ ft/s}$   
AS SHOWN



(a) GUN DOES NOT MOVE



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

+ COMPONENTS:  $2B_y \Delta t = m_s \vec{v}_0 \cos 35^\circ$

$$2B_y(0.65) = \left(\frac{33 \text{ lb}}{32.2}\right)(1450 \text{ ft/s}) \cos 35^\circ$$

$$B_y = 1014.4 \text{ lb} \quad B_y = 1014 \text{ lb} \leftarrow$$

+ MOMENTS ABOUT A:  $2B_y \Delta t (90 \text{ in}) = (m_s \vec{v}_0 \cos 35^\circ)(40 \text{ in})$

$$2B_y(0.65)(90 \text{ in}) = \left(\frac{33 \text{ lb}}{32.2}\right)(1450 \text{ ft/s}) \cos 35^\circ (40 \text{ in})$$

$$B_y = 450.8 \text{ lb} \quad B_y = 451 \text{ lb} \uparrow$$

+ COMPONENTS:  $2A \Delta t + 2B_y \Delta t = m_s \vec{v}_0 \sin 35^\circ$

$$2(A + B_y)(0.65) = \left(\frac{33 \text{ lb}}{32.2}\right)(1450 \text{ ft/s}) \sin 35^\circ$$

$$A + B_y = 710.3 \text{ lb}$$

$$A + 450.8 \text{ lb} = 710.3 \text{ lb} \quad A = 259 \text{ lb} \uparrow$$

(b) TRAILS NOT EMBEDDED  $B_y = 0$ ,  $\vec{v}_{\text{GUN}} \neq 0$

+ COMPONENTS:  $0 + 0 = -m_s \vec{v}_0 \sin 35^\circ + m_{\text{GUN}} \vec{v}_{\text{GUN}}$

$$0 = -\left(\frac{33 \text{ lb}}{g}\right)(1450 \text{ ft/s}) \sin 35^\circ + \left(\frac{4980 \text{ lb}}{g}\right) \vec{v}_{\text{GUN}}$$

$$\vec{v}_{\text{GUN}} = 7.87 \text{ ft/s} \quad \vec{v}_{\text{GUN}} = 7.87 \text{ ft/s} \leftarrow$$

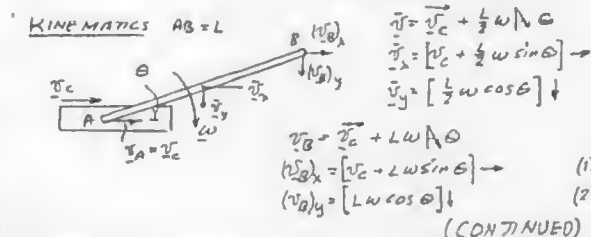
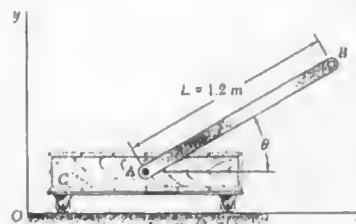
17.C1

GIVEN: 3-lb ROD AB  
5-lb CART C

ROD RELEASED FROM  
REST WHEN  $\theta = 30^\circ$

FIND:  $\vec{v}_C$  AND  $\vec{v}_B$   
FOR  $\theta = 30^\circ$  TO  
 $-90^\circ$  USING  $10^\circ$   
DECREMENTS.

ALSO, FIND  $\theta$  FOR  
MAXIMUM  $\vec{v}_C$  TO LEFT AND  
CORRESPONDING  $\vec{v}_C$



$$\vec{v}_B = \vec{v}_C + L\omega \hat{n}$$

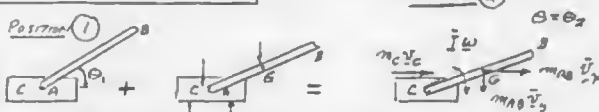
$$(v_B)_x = [v_C + L\omega \sin \theta] \rightarrow \quad (1)$$

$$(v_B)_y = [L\omega \cos \theta] \downarrow \quad (2)$$

(CONTINUED)

17.C1 continued

POSITION (2)



SYST. MOMENTA<sub>1</sub> + SYST. EXT. IMP<sub>1-2</sub> = SYST. MOMENTA<sub>2</sub>

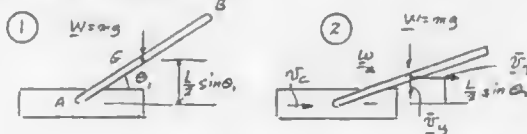
$$\rightarrow 0 + 0 = m_C \vec{v}_C + m_{AB} \vec{v}_x$$

$$0 = m_C \vec{v}_C + m_{AB} \left( \vec{v}_C + \frac{1}{2} L \omega \sin \theta \right)$$

$$(m_C + m_{AB}) \vec{v}_C = -m_{AB} \frac{1}{2} L \omega \sin \theta$$

$$\vec{v}_C = -\left[ \left( \frac{m_{AB}}{m_C + m_{AB}} \right) \frac{1}{2} L \omega \sin \theta \right] \omega = [\text{COEFF (1)}] \omega \quad (3)$$

CONSERVATION OF ENERGY



$$V_1 = mg \frac{1}{2} L \sin \theta_1 \quad V_2 = m_C \frac{1}{2} L \sin \theta_2 \quad I = \frac{1}{12} m_{AB} L^2$$

$$T_1 = 0 \quad T_2 = \frac{1}{2} m_C \vec{v}_C^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} m_{AB} \vec{v}^2$$

$$\vec{v}^2 = \vec{v}_C^2 + \vec{v}_B^2 = (\vec{v}_C + \frac{1}{2} L \omega \sin \theta)^2 + (\frac{1}{2} L \omega \cos \theta)^2$$

$$\vec{v}^2 = \vec{v}_C^2 + L \omega \sin \theta + \frac{1}{4} L^2 \omega^2$$

$$= [\text{COEFF (1)}]^2 + [\text{COEFF (1)}] L \omega \sin \theta + \frac{1}{4} L^2 \omega^2$$

$$\vec{v}^2 = [\text{COEFF (2)}] \omega^2 \quad (4)$$

$$T_2 = \frac{1}{2} \left\{ m_C [\text{COEFF (1)}]^2 + I + m_{AB} [\text{COEFF (2)}] \right\} \omega^2$$

$$T_2 = \frac{1}{2} [\text{COEFF (3)}] \omega^2 \quad (5)$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + m_{AB} \frac{1}{2} L \sin \theta_1 = \frac{1}{2} [\text{COEFF (3)}] \omega^2 + m_{AB} \frac{1}{2} L \sin \theta_2$$

$$\omega^2 = \frac{m_{AB} L (\sin \theta_1 - \sin \theta_2)}{[\text{COEFF (3)}]} \quad (6)$$

OUTLINE OF PROGRAM:

ENTER DATA:  $L = 1.2 \text{ m}$ ,  $m_C = 5 \text{ lb}$ ,  $m_{AB} = 3 \text{ lb}$ ,  $\theta_1 = 30^\circ$

PROGRAM IN SEQUENCE EQS. (3), (4), AND (5) WHICH  
CONTAIN THE THREE COEFFICIENTS, THEN PROGRAM  
EQS. (1) AND (2) THAT INVOLVE  $(\vec{v}_B)_x$  AND  $(\vec{v}_B)_y$

EVALUATE AND PRINT

$$\theta, \omega, \vec{v}, \vec{v}_C, (\vec{v}_B)_x, (\vec{v}_B)_y$$

Linear velocities positive to the right and up  
Omega positive clockwise

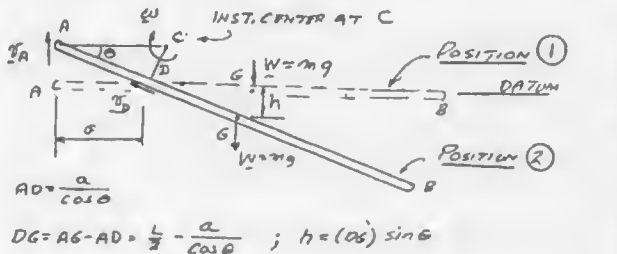
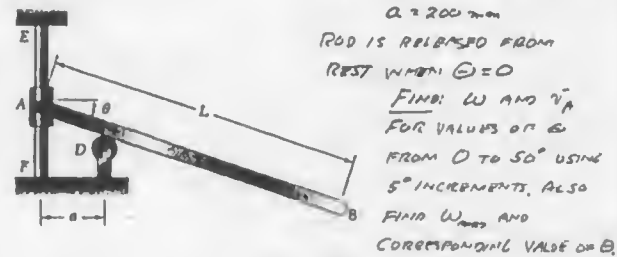
theta deg.	omega rad/s	vAB=0 m/s	vC m/s	vBx m/s	vBy m/s
30.00	0.000	0.000	0.000	0.000	0.000
20.00	2.002	1.157	-0.154	0.667	-2.257
10.00	2.841	1.689	-0.111	0.481	-3.358
0.00	3.502	2.101	0.000	0.000	-4.202
-10.00	4.082	2.427	0.159	-0.691	-4.824
-20.00	4.621	2.672	0.356	-1.541	-5.211
-30.00	5.136	2.837	0.578	-2.504	-5.338
-40.00	5.631	2.923	0.814	-3.529	-5.177
-50.00	6.098	2.933	1.051	-4.555	-4.704
-60.00	6.516	2.881	1.270	-5.502	-3.910
-70.00	6.854	2.795	1.449	-6.279	-2.813
-80.00	7.076	2.715	1.568	-6.795	-1.475
-90.00	7.154	2.683	1.610	-6.975	-0.000

Find maximum velocity of cart to the left.

19.70	2.0315	1.1760	-0.15408479
19.69	2.0325	1.1766	-0.15408483
19.68	2.0335	1.1772	-0.15408479

$$(\vec{v}_C)_{\text{max}} \text{ TO LEFT} = 0.1541 \text{ m/s WHEN } \theta = 19.7^\circ$$

## 17.C2



$CD = (AG) \tan \theta = \frac{a \tan \theta}{\cos \theta}$ ;  $AC = \frac{AD}{\cos \theta} = \frac{a}{\cos^2 \theta}$   
 MASS MOMENT OF INERTIA ABOUT INST. CENTER  
 $I_C = \bar{I} + m[(CD)^2 + (DG)^2]$   
 $\bar{I}_C = \frac{1}{12} mL^2 + m[a^2 \frac{\tan^2 \theta}{\cos^2 \theta} - (\frac{L}{2} \sin \theta - a \tan \theta)^2]$  (1)

## CONSERVATION OF ENERGY

$$T_1 = 0, V_1 = 0,$$

$$V_2 = -Wh = -mgh$$

$$T_2 = \frac{1}{2} I_C \omega^2 \quad (\text{SEE EQ 17.10 PAGE 1047})$$

$T_1 + V_1 = T_2 + V_2$   $0 + 0 = \frac{1}{2} I_C \omega^2 - mgh$   
 $\omega^2 = \frac{2mgh}{I_C}$   $\omega = \sqrt{\frac{2mgh}{I_C}}$  (2)

VELOCITY OF A:  $\bar{v}_A = (AC)\omega$

## OUTLINE OF PROGRAM

PROGRAM IN SEQUENCE, AD, DG, h, CD, AC,  $I_C$ ,  $\omega$ ,  $\bar{v}_A$ .

EVALUATE AND PRINT  $\theta$ , h,  $\omega$ , AND  $\bar{v}_A$  FOR VALUES OF  $\theta$   
 FROM  $0$  TO  $50^\circ$  AT  $5^\circ$  INTERVALS.

theta deg	h mm	omega rad / s	vA m/s
0.000	0.000	0.000	0.000
5.000	17.365	1.911	0.385
10.000	34.194	2.680	0.553
15.000	49.938	3.235	0.693
20.000	64.014	3.648	0.826
25.000	75.786	3.934	0.958
30.000	84.530	4.079	1.088
35.000	89.389	4.051	1.208
40.000	89.295	3.811	1.299
45.000	82.843	3.325	1.330
50.000	68.067	2.592	1.255

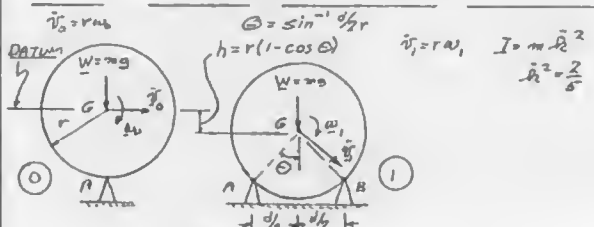
Find theta for max omega

theta deg	h mm	omega <sub>max</sub> rad / s
31.810	86.788	4.0907731056
31.820	86.799	4.0907735823
31.830	86.810	4.0907735823
31.840	86.821	4.0907731056

## 17.C3



GIVEN: 10-in. RADIUS SPHERE ROLLS ON THE BARS  
 WITHOUT SLIPPING. KNOWING THAT  $\omega_0 = 1.5 \text{ rad/s}$   
 AND ASSUMING PERFECTLY PLASTIC IMPACTS, FOR  $d = 1 \text{ in.}$   
 TO  $6 \text{ in.}$  USING 0.5-in. INCREMENTS,  
 FIND: (a)  $\omega_1$  AS G PASSES DIRECTLY ABOVE B  
 (b) NUMBER OF BARS THE SPHERE WILL  
 ROLL OVER AFTER LEAVING BAR A



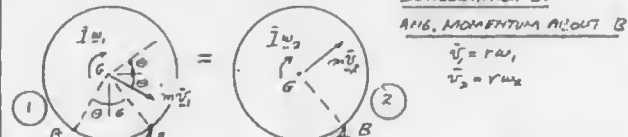
## CONSERVATION OF ENERGY

$V_0 = 0$ ;  $T_0 = \frac{1}{2} I \omega_0^2 = \frac{1}{2} m \bar{r}^2 \omega_0^2 = \frac{1}{2} m (\bar{r}^2 + r^2) \omega_0^2$   
 $V_1 = -mgh$ ;  $T_1 = \frac{1}{2} m (\bar{r}^2 + r^2) \omega_1^2$   
 $T_0 + V_0 = T_1 + V_1$ ;  $\frac{1}{2} m (\bar{r}^2 + r^2) \omega_0^2 = \frac{1}{2} m (\bar{r}^2 + r^2) \omega_1^2 - mgh$   
 $\omega_1^2 = \omega_0^2 + \frac{2gh}{\bar{r}^2 + r^2}$  (1)

AFTER IMPACT AT B: SPHERE ROTATES ABOUT B

## CONSERVATION OF

ANG. MOMENTUM ABOUT B

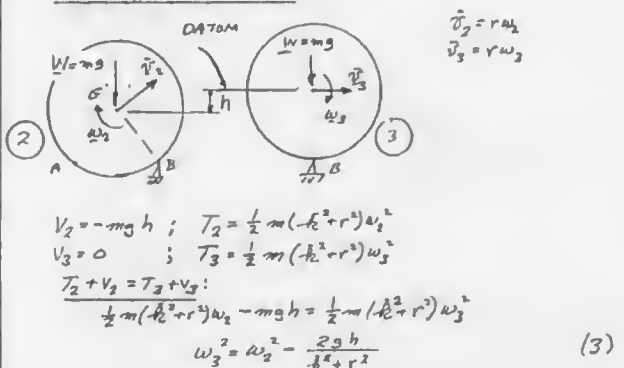


BEFORE IMPACT AT B AFTER IMPACT AT B

+2 MOMENTS ABOUT B:  $\bar{I}_B \omega_1 + (m \bar{r}^2 \cos 2\theta) r = \bar{I}_B \omega_2 + m \bar{r}^2 r$   
 $m \bar{r}^2 \omega_1 + m \bar{r}^2 \cos 2\theta \omega_1 = m \bar{r}^2 \omega_2 + m \bar{r}^2 \omega_2$   
 $\omega_2 = \frac{\bar{r}^2 + r^2 \cos 2\theta}{\bar{r}^2 + r^2} \omega_1$  (2)

SPHERE ROTATES ABOUT B UNTIL G IS ABOVE B

## CONSERVATION OF ENERGY



(CONTINUED)

# 17.C3 continued

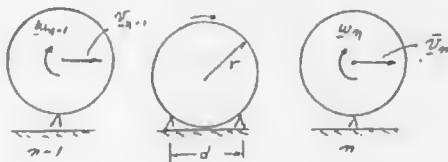
WE HAVE FOUND:  $\omega_1^2 = \omega_0^2 + \frac{2gh}{\bar{h}^2 + r^2}$  (1)

$\omega_2 = \frac{\bar{h}^2 + r^2 \cos 2\theta}{\bar{h}^2 + r^2} \omega_1$  (2)

$\omega_3^2 = \omega_2^2 - \frac{2gh}{\bar{h}^2 + r^2}$  (3)

$\omega_3$  IS ANGULAR VELOCITY OF SPHERE AS G PASSES OVER B. (THIS IS SHOWN AS  $\omega$ , IN PROBLEM FIGURE)

AS SPHERE ROLLS FROM THE  $(n-1)^{\text{th}}$  BAR TO THE  $n^{\text{th}}$  BAR.



## OUTLINE OF PROGRAM

ENTER DATA:  $r = \frac{70}{12}$  ft,  $\omega_0 = 1.5 \text{ rad/s}$ ,  $\bar{h} = 0.4$

FOR  $d = \frac{1}{12}$  ft TO  $\frac{6}{12}$  ft; INCREMENT  $\frac{0.5}{12}$  ft  
 $\omega_n = \omega_0$   
 FOR  $n = 1$  TO 100; INCREMENT = 1  
 $\Theta = \sin^{-1}(d/\bar{h})$   
 $h = \bar{h}(1 - \cos \Theta)$   
 $\omega_1 = \{\omega_n^2 + 2gh/(\bar{h}^2 + r^2)\}^{1/2}$   
 $\omega_2 = \{(\bar{h}^2 + r^2 \cos 2\theta)/(\bar{h}^2 + r^2)\} \omega_1$   
 $\omega_3 = \{\omega_2^2 - 2gh/(\bar{h}^2 + r^2)\}^{1/2}$   
 IF  $n = 1$  PRINT  $\omega_3$  (G IS ABOVE B)  
 IF  $\omega_3 < 0 \rightarrow \text{STOP}$ ,  
 ( $n$  IS NUMBER OF BARS ROLLED OVER)

NEXT

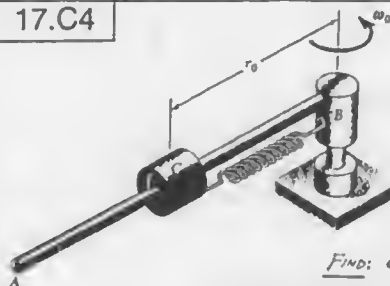
PRINT

$r = 10.000 \text{ in.}$        $\omega_0 = 1.500 \text{ rad/s}$

Distance between bars in.	omega when G is over B rad/s	Number of bars sphere rolls over
1.0	1.494	491
1.5	1.487	169
2.0	1.476	76
2.5	1.460	40
3.0	1.438	23
3.5	1.409	14
4.0	1.370	9
4.5	1.319	6
5.0	1.252	4
5.5	1.164	3
6.0	1.047	2

NOTE: FOR  $d = 7 \text{ in.}$ , SPHERE FAILS TO REACH A POSITION WITH G ABOVE B

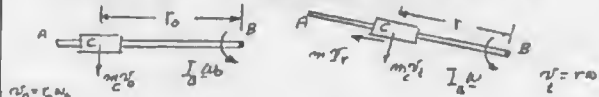
# 17.C4



GIVEN:  $m_c = 2.5 \text{ kg}$   
 SPRING:  $k = 750 \text{ N/m}$   
 UNSTRETCHED LENGTH:  $r_0 = 500 \text{ mm}$   
 ROD AND NUB:  $I_B = 0.3 \text{ kg} \cdot \text{m}^2$   
 INITIALLY:  $r_0 = 500 \text{ mm}$   
 $\omega_0 = 10 \text{ rad/s}$

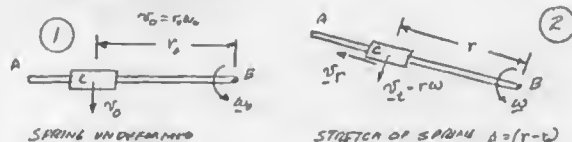
FIND:  $\omega$  AND  $v_c/r_B$  FOR VALUES OF  $r$  FROM 500 mm TO 700 mm AT 25-mm INCREMENTS. ALSO FIND  $r_{\text{MAX}}$ .

## CONSERVATION OF ANGULAR MOMENTUM ABOUT B



+J) MOMENTS ABOUT B:  $I_B \omega_0 + m_c v_0 r_0 = I_B \omega + m_c r^2 \omega$   
 $I_B \omega_0 + m_c r_0^2 \omega_0 = I_B \omega + m_c r^2 \omega$   
 $\omega = \frac{I_B + m_c r_0^2}{I_B + m_c r^2} \omega_0$  (1)

## CONSERVATION OF ENERGY



$T_1 = \frac{1}{2} I_B \omega_0^2 + \frac{1}{2} m_c v_0^2 = \frac{1}{2} I_B \omega_0^2 + \frac{1}{2} m_c r_0^2 \omega_0^2 = \frac{1}{2} (I_B + m_c r_0^2) \omega_0^2$        $V_1 = 0$   
 $T_2 = \frac{1}{2} I_B \omega^2 + \frac{1}{2} m_c v^2 = \frac{1}{2} I_B \omega^2 + \frac{1}{2} m_c r^2 \omega^2 = \frac{1}{2} (I_B + m_c r^2) \omega^2$   
 $V_2 = \frac{1}{2} k (r - r_0)^2$   
 $T_1 + V_1 = T_2 + V_2: \frac{1}{2} (I_B + m_c r_0^2) \omega_0^2 = \frac{1}{2} (I_B + m_c r^2) \omega^2 + \frac{1}{2} k (r - r_0)^2$   
 $v_r = \left\{ \frac{1}{m} [(I_B + m_c r_0^2) \omega_0^2 - (I_B + m_c r^2) \omega^2 - k (r - r_0)^2] \right\}^{1/2}$  (2)

## OUTLINE OF PROGRAM:

ENTER DATA:  $m = 2.5 \text{ kg}$ ,  $I_B = 0.3 \text{ kg} \cdot \text{m}^2$ ,  $r_0 = 0.5 \text{ m}$ ,  $k = 750 \text{ N/m}$  AND  $\omega_0 = 10 \text{ rad/s}$

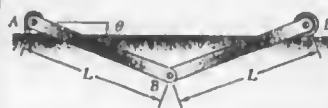
PROGRAM EQ(1) AND THEN EQ(2). EVALUATE AND PRINT  $\omega$  AND  $v_r$  FOR VALUES OF  $r$  FROM 0.5 m TO 0.7 m AT 0.025 m INCREMENTS. THEN SEEK  $r_{\text{MAX}}$  WHERE  $v_r = 0$

r mm	omega rad/s	v radial m/s
500.00	10.000	0.000
525.00	9.352	1.486
550.00	8.757	1.962
575.00	8.211	2.221
600.00	7.708	2.341
625.00	7.246	2.346
650.00	6.820	2.239
675.00	6.428	2.007
700.00	6.066	1.599

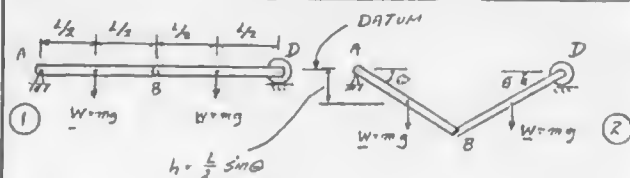
Find r maximum (where  $v_r = 0$ )

r mm	omega rad/s	[v radial]^-2
731.75	5.645	0.0014211
731.76	5.645	0.0004968
731.77	5.645	-0.0004275
731.78	5.645	-0.0013555

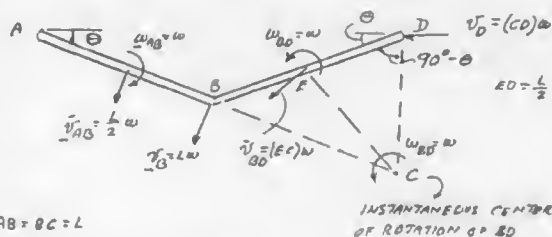
## 17.C5



GIVEN:  $L = 30$  in.  
BARS ARE RELEASED  
FROM REST WHEN  $\theta = 0$ .  
FIND:  $\omega_{AB}$  AND  $v_D$   
FOR VALUES OF  $\theta$  FROM  
0 TO 90° USING 10° INCREMENTS



KINEMATICS OF POSITION 2:



$$AB = BC = L$$

IN  $\triangle ACD$ :  $CD = 2L \sin \theta$

IN  $\triangle CED$  (LAW OF COSINES)

$$(EC)^2 = (CD)^2 + (L/2)^2 - 2(CD)(L/2) \cos(90^\circ - \theta)$$

$$EC = [(CD)^2 + (L/2)^2 - 2(CD)(L/2) \cos(90^\circ - \theta)]^{1/2} \quad (2)$$

CONSERVATION OF ENERGY

$$V_1 = 0 \quad T_1 = 0$$

$$V_2 = -2mg \left( \frac{L}{2} \sin \theta \right) = -mgL \sin \theta$$

$$T_2 = \frac{1}{2} m \bar{v}_{AB}^2 + \frac{1}{2} I_{AB} \omega_{AB}^2 + \frac{1}{2} m \bar{v}_{BD}^2 + \frac{1}{2} I_{BD} \omega_{BD}^2$$

$$= \frac{1}{2} m \left( \frac{L}{2} \omega \right)^2 + \frac{1}{2} \left( \frac{1}{12} mL^2 \right) \omega^2 + \frac{1}{2} m (EC \omega)^2 + \frac{1}{2} \left( \frac{1}{12} mL^2 \right) \omega^2$$

$$T_2 = \left[ \frac{1}{8} + \frac{1}{24} + \frac{1}{2} \left( \frac{EC}{L} \right)^2 + \frac{1}{24} \right] mL^2 \omega^2$$

$$T_2 = \frac{1}{24} \left[ 5 + 12 \left( \frac{EC}{L} \right)^2 \right] mL^2 \omega^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = -mgL \sin \theta + \frac{1}{24} \left[ 5 + 12 \left( \frac{EC}{L} \right)^2 \right] mL^2 \omega^2$$

$$\omega = \left[ \frac{24g}{L} \cdot \frac{\sin \theta}{5 + \left( \frac{EC}{L} \right)^2} \right]^{1/2} \quad (3)$$

VELOCITY OF D:  $v_D = (CD) \omega \quad (4)$

OUTLINE OF PROGRAM:

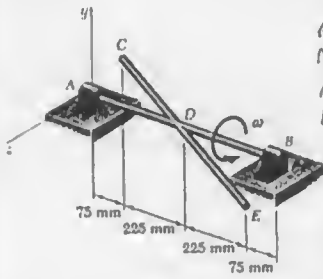
ENTER  $L = 30$  in.  $= 2.5$  ft,  $g = 32.2$  ft/s<sup>2</sup>

PROGRAM, IN SEQUENCE, EQS. (1), (2), (3), AND (4)

EVALUATE AND PRINT  $\omega$  AND  $v_D$  FOR VALUES  
OF  $\theta$  FROM 0 TO 90° USING 10° INCREMENTS.

theta deg.	omega rad/s	vD. ft/s
0	0.0000	0.0000
10	2.4806	2.1537
20	3.1277	5.3487
30	3.3226	8.3066
40	3.3302	10.7031
50	3.2746	12.5423
60	3.2088	13.8945
70	3.1544	14.8210
80	3.1198	15.3622
90	3.1081	15.5403

18.1



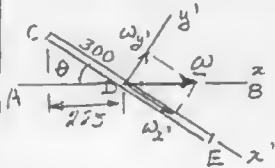
GIVEN:

TWO UNIFORM RODS AB AND CE ARE WELDED AT MIDPOINTS D.  
 MASS OF EACH ROD = 1.5 kg  
 LENGTH = 600 mm  
 ASSEMBLY HAS CONSTANT ANG. VEL  
 $\omega = 12 \text{ rad/s}$ .

FIND:

ANG. MOMENTUM  $H_D$ .

SINCE ROD AB HAS MOM. OF INERTIA  $\approx 0$  ABOUT AXIS OF ROTATION, ONLY ROD CE CONTRIBUTES TO ANGULAR MOMENTUM.



$$\omega_x = \omega \cos \theta$$

$$\omega_y = \omega \sin \theta$$

$$\omega_z = 0$$

EQU. (18.10):

$$H_x = \bar{I}_x \omega_x = 0$$

$$H_y = \bar{I}_y \omega_y = \frac{1}{12} m l^2 \omega \sin \theta$$

$$= \frac{1}{12} (1.5 \text{ kg})(0.6 \text{ m})^2 (12 \text{ rad/s}) \sin 41.41^\circ = 0.357$$

$$H_D = 0.357 \text{ kg} \cdot \text{m}^2/\text{s}; \theta_x = 48.6^\circ, \theta_y = 41.4^\circ, \theta_z = 90^\circ$$

$$\text{SINCE } CD = 300 \text{ mm},$$

$$\cos \theta = \frac{225}{300} \quad \theta = 41.41^\circ$$

USING THE PRINCIPAL CENTROIDAL AXES  $x', y', z'$ , WE HAVE

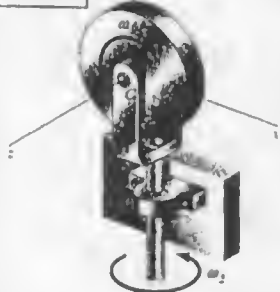
$$\bar{I}_x = 0$$

$$\bar{I}_y = \frac{1}{12} m l^2$$

$$\bar{I}_z = \frac{1}{12} m l^2$$

$$H_z = \bar{I}_z \omega_z = 0$$

18.2



GIVEN:

THIN, HOMOGENEOUS DISK OF MASS  $m$  AND RADIUS  $r$  SPINS AT CONSTANT RATE  $\omega_1$ .  
 FORK-ENDED ROD SPINS AT CONSTANT RATE  $\omega_2$ .

FIND:

ANGULAR MOMENTUM  $H_G$  OF DISK.

SINCE THE  $x, y, z$  AXES ARE PRINCIPAL CENTROIDAL AXES, WE CAN USE EQS. (18.10) WITH

$$\bar{I}_x = \bar{I}_y = \frac{1}{4} m r^2, \quad \bar{I}_z = \frac{1}{2} m r^2$$

$$\omega_x = 0, \quad \omega_y = \omega_2, \quad \omega_z = \omega_1$$

AND WRITE

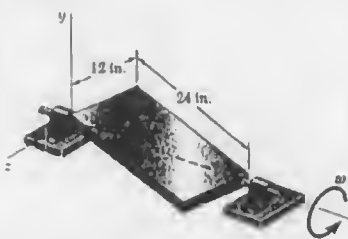
$$H_x = \bar{I}_x \omega_x = 0$$

$$H_y = \bar{I}_y \omega_y = \frac{1}{4} m r^2 \omega_2$$

$$H_z = \bar{I}_z \omega_z = \frac{1}{2} m r^2 \omega_1$$

$$H_G = \frac{1}{4} m r^2 (\omega_2 \underline{j} + 2 \omega_1 \underline{k})$$

18.3

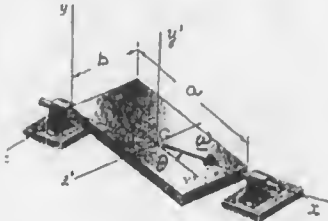


GIVEN:

RECTANGULAR PLATE SHOWN WEIGHS 18 lb AND ROTATES WITH CONSTANT  $\omega = 5 \text{ rad/s}$

FIND:

ANGULAR MOMENTUM  $H$  ABOUT MASS CENTER  $G$ .



WE USE THE PRINCIPAL CENTROIDAL AXES  $Gx'y'z'$

WE HAVE

$$\omega_x = \omega \cos \theta$$

$$\omega_y = 0$$

$$\omega_z = -\omega \sin \theta$$

$$\bar{I}_x = \frac{1}{12} m b^2, \quad \bar{I}_y = \frac{1}{12} m (a^2 + b^2), \quad \bar{I}_z = \frac{1}{12} m a^2$$

USING EQS. (18.10):

$$H_x = \bar{I}_x \omega_x = \frac{1}{12} m b^2 \omega \cos \theta$$

$$H_y = \bar{I}_y \omega_y = 0$$

$$H_z = \bar{I}_z \omega_z = -\frac{1}{12} m a^2 \omega \sin \theta$$

WE HAVE

$$H_G = H_x \underline{i}' + H_y \underline{j}' + H_z \underline{k}'$$

WHERE  $\underline{i}', \underline{j}', \underline{k}'$  ARE THE UNIT VECTORS ALONG THE  $x', y', z'$  AXES.

$$H_G = \frac{1}{12} m b^2 \omega \cos \theta \underline{i}' - \frac{1}{12} m a^2 \omega \sin \theta \underline{k}'$$

TO RETURN TO THE ORIGINAL  $x, y, z$  AXES, WE NOTE THAT

$$\underline{i}' = \underline{i} \cos \theta + \underline{k} \sin \theta \quad \underline{k}' = -\underline{i} \sin \theta + \underline{k} \cos \theta$$

THEREFORE

$$H_G = \frac{1}{12} m b^2 \omega (\cos^2 \theta \underline{i} + \cos \theta \sin \theta \underline{k}) + \frac{1}{12} m a^2 \omega (\sin^2 \theta \underline{i} - \sin \theta \cos \theta \underline{k})$$

$$H_G = \frac{1}{12} m \omega [(a^2 \sin^2 \theta + b^2 \cos^2 \theta) \underline{i} - (a^2 - b^2) \sin \theta \cos \theta \underline{k}]$$

GIVEN DATA:

$$m = (18 \text{ lb}) / (32.2 \text{ ft/s}^2) = 0.55901 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$a = 24 \text{ in.} = 2 \text{ ft} \quad b = 12 \text{ in.} = 1 \text{ ft}$$

$$\tan \theta = \frac{b}{a} = 0.5 \quad \theta = 26.565^\circ$$

THUS:

$$H_G = \frac{1}{12} (0.55901 \text{ lb} \cdot \text{s}^2/\text{ft}) \omega [(4 \sin^2 26.565^\circ + \cos^2 26.565^\circ) \underline{i} - (4 - 1) \sin 26.565^\circ \cos 26.565^\circ \underline{k}] (\text{ft})$$

$$H_G = (0.046584 \text{ lb} \cdot \text{s}^2/\text{ft}) \omega (1.600 \underline{i} - 1.200 \underline{k}) (\text{ft})$$

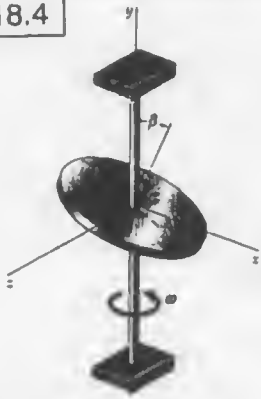
$$H_G = [(0.074534 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} - (0.055901 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}] \omega \quad (1)$$

LETTING  $\omega = 5 \text{ rad/s}$ ,

$$H_G = (0.3727 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} - (0.2795 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k} \quad (2)$$

$$H_G = (0.373 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} - (0.280 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}$$

18.4

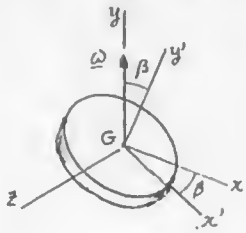


GIVEN:

HOMOGENEOUS DISK OF MASS  $m$  AND RADIUS  $r$  MOUNTED ON SHAFT AB WITH  $\beta = 25^\circ$ . SHAFT ROTATES WITH CONSTANT  $\omega$ .

FIND:

ANGLE  $\theta$  FORMED BY AB AND ANG. MOMENTUM  $\underline{H}_G$  OF DISK ABOUT G.



WE USE THE PRINCIPAL CENTROIDAL AXES  $Gx'y'z'$ . WE HAVE:

WE HAVE:

$$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} m r^2$$

$$\bar{I}_{y'} = \frac{1}{2} m r^2$$

$$\omega_{x'} = -\omega \sin \beta$$

$$\omega_{y'} = \omega \cos \beta$$

$$\omega_{z'} = 0$$

USING EQS. (18.10):

$$H_{x'} = \bar{I}_{x'} \omega_{x'} = -\frac{1}{4} m r^2 \omega \sin \beta$$

$$H_{y'} = \bar{I}_{y'} \omega_{y'} = \frac{1}{2} m r^2 \omega \cos \beta$$

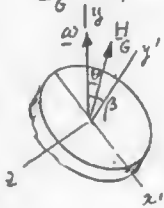
$$H_{z'} = \bar{I}_{z'} \omega_{z'} = 0$$

WE HAVE

$$\underline{H}_G = H_{x'} \underline{i}' + H_{y'} \underline{j}' + H_{z'} \underline{k}'$$

WHERE  $\underline{i}'$ ,  $\underline{j}'$ ,  $\underline{k}'$  ARE THE UNIT VECTORS ALONG THE  $x'y'z'$  AXES.

$$\underline{H}_G = -\frac{1}{4} m r^2 \omega \sin \beta \underline{i}' + \frac{1}{2} m r^2 \omega \cos \beta \underline{j}' \quad (1)$$



$$H_G = \frac{1}{4} m r^2 \omega (-\sin \beta \underline{i}' + 2 \cos \beta \underline{j}')$$

FROM EQ. (3.24) WE HAVE

$$\underline{H}_G \cdot \underline{\omega} = |\underline{H}_G| \omega \cos \theta$$

$$\cos \theta = \frac{\underline{H}_G \cdot \underline{\omega}}{|\underline{H}_G| \omega} \quad (2)$$

$$\text{BUT } \underline{H}_G \cdot \underline{\omega} = \frac{1}{4} m r^2 \omega (-\sin \beta \underline{i}' + 2 \cos \beta \underline{j}') \cdot \omega \underline{j}'$$

OR, OBSERVING THAT  $\underline{i}' \cdot \underline{j}' = -\sin \beta$  AND  $\underline{j}' \cdot \underline{j}' = \cos \beta$ ,

$$\underline{H}_G \cdot \underline{\omega} = \frac{1}{4} m r^2 \omega^2 (\sin^2 \beta + 2 \cos^2 \beta)$$

$$= \frac{1}{4} m r^2 \omega^2 (1 + \cos^2 \beta)$$

$$\text{ALSO } |\underline{H}_G| \omega = \frac{1}{4} m r^2 \omega^2 \sqrt{\sin^2 \beta + 4 \cos^2 \beta}$$

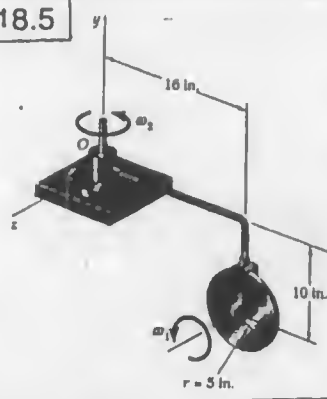
$$= \frac{1}{4} m r^2 \omega^2 \sqrt{1 + 3 \cos^2 \beta}$$

SUBSTITUTING FROM (3) AND (4) INTO (2),

$$\cos \theta = \frac{1 + \cos^2 \beta}{\sqrt{1 + 3 \cos^2 \beta}}$$

$$\text{FOR } \beta = 25^\circ, \cos \theta = 0.9786 \quad \theta = 11.88^\circ$$

18.5

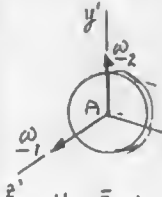


GIVEN:

HOMOGENEOUS DISK OF WEIGHT  $W = 8 \text{ lb}$  ROTATES AT CONSTANT RATE  $\omega_1 = 12 \text{ rad/s}$ . ARM OA ROTATES AT CONSTANT RATE  $\omega_2 = 4 \text{ rad/s}$ .

FIND:

ANGULAR MOMENTUM  $\underline{H}_A$  OF DISK ABOUT ITS CENTER A.



WE USE PRINCIPAL CENTROIDAL AXES  $Ax'y'z'$ . WE HAVE

$$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} m r^2, \quad \bar{I}_{y'} = \frac{1}{2} m r^2$$

$$\omega_{x'} = 0, \quad \omega_{y'} = \omega_2, \quad \omega_{z'} = \omega_1$$

FROM EQS. (18.10):

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i}' + \bar{I}_{y'} \omega_{y'} \underline{j}' + \bar{I}_{z'} \omega_{z'} \underline{k}' = \frac{1}{4} m r^2 (\omega_2 \underline{j}' + \omega_1 \underline{k}')$$

$$\text{GIVEN DATA: } m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}$$

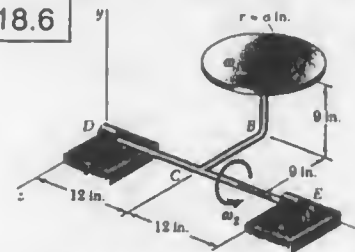
$$r = 5 \text{ in.} = \frac{5}{12} \text{ ft}, \quad \omega_1 = 12 \text{ rad/s}, \quad \omega_2 = 4 \text{ rad/s}$$

$$\underline{H}_A = \frac{1}{4} (0.24845) \left( \frac{5}{12} \right)^2 [4 \underline{j}' + 12 \underline{k}']$$

$$= (0.043133 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j}' + (0.25880 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}'$$

$$\underline{H}_A = (0.0431 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j}' + (0.259 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}'$$

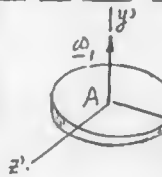
18.6



GIVEN:

HOMOGENEOUS DISK OF WEIGHT  $W = 6 \text{ lb}$  ROTATES AT CONSTANT RATE  $\omega_1 = 16 \text{ rad/s}$ . SHAFT DCE ROTATES AT CONSTANT RATE  $\omega_2 = 8 \text{ rad/s}$ .

FIND: ANG. MOMENTUM  $\underline{H}_A$  OF DISK ABOUT ITS CENTER A.



WE USE PRINCIPAL CENTROIDAL AXES  $Ax'y'z'$ . WE HAVE

$$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} m r^2, \quad \bar{I}_{y'} = \frac{1}{2} m r^2$$

$$\omega_{x'} = \omega_2, \quad \omega_{y'} = \omega_1, \quad \omega_{z'} = 0$$

FROM EQS. (18.10):

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i}' + \bar{I}_{y'} \omega_{y'} \underline{j}' + \bar{I}_{z'} \omega_{z'} \underline{k}' = \frac{1}{4} m r^2 (\omega_2 \underline{i}' + \omega_1 \underline{j}')$$

$$\text{GIVEN DATA: } m = \frac{W}{g} = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.186335 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$r = 6 \text{ in.} = \frac{6}{12} \text{ ft}, \quad \omega_1 = 16 \text{ rad/s}, \quad \omega_2 = 8 \text{ rad/s}$$

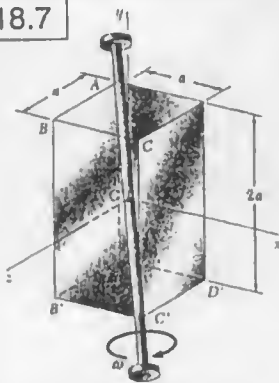
$$\underline{H}_A = \frac{1}{4} (0.186335) \left( \frac{6}{12} \right)^2 [8 \underline{i}' + 16 \underline{j}']$$

$$= (0.165631 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i}' + (0.66252 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j}'$$

$$\underline{H}_A = (0.1656 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i}' + (0.663 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j}'$$



18.7



GIVEN:

SOLID RECTANGULAR PARALLELEPIPED SHOWN, ROTATES ABOUT ITS DIAGONAL  $AC'$  AT CONSTANT  $\omega$ .

FIND:

(a) MAGNITUDE OF ANG. MOMENTUM  $H_G$ .

(b) ANGLE THAT  $H_G$  FORMS WITH  $AC'$ .

WE DENOTE BY  $\bar{I}_x, \bar{I}_y, \bar{I}_z$  THE PRINCIPAL CENTROIDAL MOMENTS OF INERTIA. WE HAVE

$$\underline{\omega} = \omega \frac{-a\hat{i} + 2a\hat{j} - a\hat{k}}{a\sqrt{6}} = \frac{\omega}{\sqrt{6}} (-\hat{i} + 2\hat{j} - \hat{k}) \quad (1)$$

$$\underline{H}_G = \bar{I}_x \omega_x \hat{i} + \bar{I}_y \omega_y \hat{j} + \bar{I}_z \omega_z \hat{k} = \frac{\omega}{\sqrt{6}} (-\bar{I}_x \hat{i} + 2\bar{I}_y \hat{j} - \bar{I}_z \hat{k}) \quad (2)$$

COMPUTATION OF THE MOMENTS OF INERTIA:

$$\bar{I}_x = \bar{I}_z = \frac{1}{12} m (a^2 + 4a^2) = \frac{5}{12} ma^2$$

$$\bar{I}_y = \frac{1}{12} m (a^2 + a^2) = \frac{1}{6} ma^2$$

SUBSTITUTE INTO (2):

$$\underline{H}_G = \frac{\omega}{\sqrt{6}} \left( -\frac{5}{12} ma^2 \hat{i} + \frac{2}{6} ma^2 \hat{j} - \frac{5}{12} ma^2 \hat{k} \right)$$

$$\underline{H}_G = \frac{ma^2 \omega}{12\sqrt{6}} (-5\hat{i} + 4\hat{j} - 5\hat{k})$$

$$(a) |\underline{H}_G| = \frac{ma^2 \omega}{12\sqrt{6}} \sqrt{25 + 16 + 25} = \frac{ma^2 \omega \sqrt{11}}{12}$$

$$|\underline{H}_G| = 0.276 ma^2 \omega$$

(b) FROM EQ. (3.24) WE HAVE

$$\underline{H}_G \cdot \underline{\omega} = |\underline{H}_G| \omega \cos \theta$$

$$\cos \theta = \frac{\underline{H}_G \cdot \underline{\omega}}{|\underline{H}_G| \omega}$$

RECALLING (1) AND (3):

$$\begin{aligned} \underline{H}_G \cdot \underline{\omega} &= \frac{ma^2 \omega}{12\sqrt{6}} (-5\hat{i} + 4\hat{j} - 5\hat{k}) \cdot \frac{\omega}{\sqrt{6}} (-\hat{i} + 2\hat{j} - \hat{k}) \\ &= \frac{ma^2 \omega^2}{72} (5 + 8 + 5) = \frac{1}{4} ma^2 \omega^2 \end{aligned} \quad (6)$$

$$\text{RECALLING (4): } |\underline{H}_G| \omega = \frac{\sqrt{11}}{12} ma^2 \omega^2 \quad (7)$$

SUBSTITUTING FROM (6) AND (7) INTO (5):

$$\cos \theta = \frac{1/4}{\sqrt{11}/12} = \frac{3}{\sqrt{11}} = 0.90453$$

$$\theta = 25.239^\circ$$

$$\theta = 25.2^\circ$$

18.8

GIVEN: SOLID PARALLELEPIPED OF PROB. 18.7 IS REPLACED BY HOLLOW ONE MADE OF 6 THIN METAL PLATES.

FIND: (a) MAGNITUDE OF ANG. MOMENTUM  $H_G$ .  
(b) ANGLE THAT  $H_G$  FORMS WITH  $AC'$ .

WE DENOTE BY  $\bar{I}_x, \bar{I}_y, \bar{I}_z$  THE PRINCIPAL CENTROIDAL MOMENTS OF INERTIA. WE HAVE

$$\underline{\omega} = \omega \frac{-a\hat{i} + 2a\hat{j} - a\hat{k}}{a\sqrt{6}} = \frac{\omega}{\sqrt{6}} (-\hat{i} + 2\hat{j} - \hat{k}) \quad (1)$$

$$\underline{H}_G = \bar{I}_x \omega_x \hat{i} + \bar{I}_y \omega_y \hat{j} + \bar{I}_z \omega_z \hat{k} = \frac{\omega}{\sqrt{6}} (-\bar{I}_x \hat{i} + 2\bar{I}_y \hat{j} - \bar{I}_z \hat{k}) \quad (2)$$

COMPUTATION OF MOMENTS OF INERTIA:

EACH OF THE TWO SQUARE PLATES HAS A MASS EQUAL TO  $m/10$  AND EACH OF THE RECTANGULAR PLATES HAS A MASS EQUAL TO  $m/5$ . USING THE PARALLEL-AXIS THEOREM WHEN NEEDED, WE OBTAIN:

	$\bar{I}_x$	$\bar{I}_y$	$\bar{I}_z$
SQUARE PLATES	$\frac{2m(a^2 + a^2)}{10 \cdot 12} = \frac{13}{60} ma^2$	$\frac{2m a^2}{10 \cdot 6} = \frac{ma^2}{30}$	$\frac{13}{60} ma^2$
RECTANG. PLATES // $yz$ PLANE	$\frac{2m a^2 + 4a^2}{5} = \frac{1}{6} ma^2$	$\frac{2m \left[ \frac{a^2}{12} + \left( \frac{a}{2} \right)^2 \right]}{5} = \frac{2}{15} ma^2$	$\frac{2m \left[ \frac{a^2}{3} + \left( \frac{a}{2} \right)^2 \right]}{5} = \frac{7}{30} ma^2$
RECT. PL. $xy$ PLANE	$\frac{7}{30} ma^2$	$\frac{2}{15} ma^2$	$\frac{1}{6} ma^2$
SUMS	$\frac{37}{60} ma^2$	$\frac{9}{30} ma^2$	$\frac{37}{60} ma^2$

SUBSTITUTE THE VALUES OBTAINED FOR  $\bar{I}_x, \bar{I}_y, \bar{I}_z$  INTO (2):

$$\underline{H}_G = \frac{ma^2 \omega}{60\sqrt{6}} (-37\hat{i} + 36\hat{j} - 37\hat{k}) \quad (3)$$

$$(a) |\underline{H}_G| = \frac{ma^2 \omega}{60\sqrt{6}} \sqrt{(37)^2 + (36)^2 + (37)^2} = \frac{ma^2 \omega \sqrt{4034}}{60\sqrt{6}} \quad (4)$$

$$|\underline{H}_G| = 0.432157 ma^2 \omega, \quad |\underline{H}_G| = 0.432 ma^2 \omega$$

(b) WE RECALL EQ. (5) IN SOLUTION OF PROB. 18.7:

$$\cos \theta = \frac{\underline{H}_G \cdot \underline{\omega}}{|\underline{H}_G| \omega} \quad (5)$$

RECALLING (1) AND (3) ABOVE:

$$\begin{aligned} \underline{H}_G \cdot \underline{\omega} &= \frac{ma^2 \omega}{60\sqrt{6}} (-37\hat{i} + 36\hat{j} - 37\hat{k}) \cdot \frac{\omega}{\sqrt{6}} (-\hat{i} + 2\hat{j} - \hat{k}) \\ &= \frac{ma^2 \omega^2}{360} (37 + 72 + 37) = \frac{146}{360} ma^2 \omega^2 \end{aligned} \quad (6)$$

RECALLING (4) ABOVE:

$$|\underline{H}_G| \omega = \frac{\sqrt{4034}}{60\sqrt{6}} ma^2 \omega^2 \quad (7)$$

SUBSTITUTING FROM (6) AND (7) INTO (5):

$$\cos \theta = \frac{146}{360} \frac{60\sqrt{6}}{\sqrt{4034}} = \frac{146}{\sqrt{6 \times 4034}} = 0.93845$$

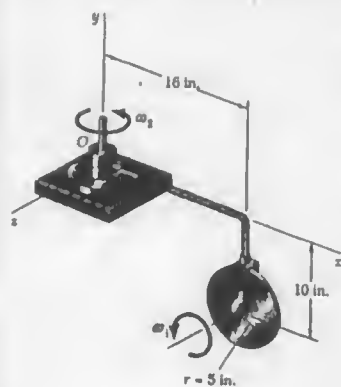
$$\theta = 20.208^\circ$$

$$\theta = 20.2^\circ$$

18.9

GIVEN: DISK OF PROB. 18.5 WITH  $W = 8 \text{ lb}$ ,  $\omega_1 = 12 \text{ rad/s}$ , AND  $\omega_2 = 4 \text{ rad/s}$ .

FIND: ANGULAR MOMENTUM  $H_D$  ABOUT POINT O.



WE USE EQ. (18.11):

$$H_D = \bar{r} \times m \bar{v} + H_G \quad (1)$$

WHERE

$$\bar{r} = \bar{r}_A = \left(\frac{16}{12} \text{ ft}\right) \underline{i} - \left(\frac{10}{12} \text{ ft}\right) \underline{j}$$

$$\bar{r} = \bar{r}_A = \left(\frac{4}{3} \text{ ft}\right) \underline{i} - \left(\frac{5}{6} \text{ ft}\right) \underline{j}$$

$$m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\bar{v} = \bar{v}_A = \omega_2 \times \bar{r}_A = (4 \text{ rad/s}) \underline{j} \times \left(\frac{4}{3} \underline{i} - \frac{5}{6} \underline{j}\right)$$

$$\bar{v} = -\left(\frac{16}{3} \text{ ft/s}\right) \underline{k}$$

FROM THE SOLUTION OF PROB. 18.5, WE RECALL THAT

$$H_G = \bar{H}_A = (0.0431 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j} + (0.259 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}$$

SUBSTITUTING INTO (1):

$$H_D = \left(\frac{4}{3} \underline{i} - \frac{5}{6} \underline{j}\right) \times 0.24845 \left(-\frac{16}{3} \underline{k}\right) + 0.0431 \underline{j} + 0.259 \underline{k}$$

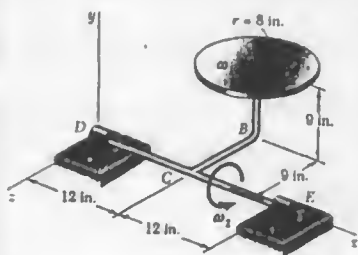
$$= 1.7668 \underline{j} + 1.1042 \underline{i} + 0.0431 \underline{j} + 0.259 \underline{k}$$

$$H_D = (1.104 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} + (1.810 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j} + (0.259 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}$$

18.10

GIVEN: DISK OF PROB. 18.6 WITH  $W = 6 \text{ lb}$ ,  $\omega_1 = 16 \text{ rad/s}$ , AND  $\omega_2 = 8 \text{ rad/s}$ .

FIND: ANGULAR MOMENTUM  $H_D$  ABOUT POINT D.



WE USE EQ. (18.11) WITH RESPECT TO D:

$$H_D = \bar{r} \times m \bar{v} + H_G \quad (1)$$

WHERE

$$\bar{r} = \bar{r}_A = (1 \text{ ft}) \underline{i} + (0.75 \text{ ft}) \underline{j} - (0.75 \text{ ft}) \underline{k}$$

$$m = \frac{W}{g} = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.186335 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\bar{v} = \bar{v}_A = \omega_2 \times \bar{r}_A = (8 \text{ rad/s}) \underline{k} \times (\underline{i} + 0.75 \underline{j} - 0.75 \underline{k})$$

$$\bar{v} = (6 \text{ ft/s}) \underline{j} + (6 \text{ ft/s}) \underline{k}$$

FROM THE SOLUTION OF PROB. 18.6, WE RECALL THAT

$$H_G = \bar{H}_A = (0.1656 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} + (0.663 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j}$$

SUBSTITUTING INTO (1):

$$H_D = (\underline{i} + 0.75 \underline{j} - 0.75 \underline{k}) \times 0.186335 (6 \underline{j} + 6 \underline{k}) + 0.1656 \underline{i} + 0.663 \underline{j}$$

$$= 1.1180 \underline{k} - 1.1180 \underline{j} + 0.8385 \underline{i} + 0.8385 \underline{j} + 0.1656 \underline{i} + 0.663 \underline{j}$$

$$H_D = (1.843 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{i} - (0.455 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{j} + (1.118 \text{ lb} \cdot \text{ft} \cdot \text{s}) \underline{k}$$

18.11

GIVEN:

PROJECTILE WITH  $m = 30 \text{ kg}$

$\bar{r}_x = 60 \text{ mm}$ ,  $\bar{r}_y = 250 \text{ mm}$

ANGLE  $\theta = 5^\circ$ ; ANG. MOM.

$H_G = (320 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{i} - (9 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{j}$

RESOLVE  $\omega$  INTO COMPONENTS

(a) ALONG GX (RATE OF SPIN)

(b) ALONG GD (RATE OF PRECESSION)

BECAUSE OF ASYMMETRY OF PROJECTILE, THE  $x$  AND  $y$  AXES ARE PRINCIPAL CENTROIDAL AXES.

$$\bar{I}_x = m \bar{r}_x^2 = (30 \text{ kg})(0.060 \text{ m})^2 = 0.108 \text{ kg} \cdot \text{m}^2$$

$$\bar{I}_y = m \bar{r}_y^2 = (30 \text{ kg})(0.250 \text{ m})^2 = 1.875 \text{ kg} \cdot \text{m}^2$$

$$\text{GIVEN: } H_x = 0.320 \text{ kg} \cdot \text{m}^2/\text{s}, H_y = -0.009 \text{ kg} \cdot \text{m}^2/\text{s}$$

FROM Eqs. (18.10):

$$\omega_x = \frac{H_x}{\bar{I}_x} = \frac{0.320 \text{ kg} \cdot \text{m}^2/\text{s}}{0.108 \text{ kg} \cdot \text{m}^2} = 2.9630 \text{ rad/s}$$

$$\omega_y = \frac{H_y}{\bar{I}_y} = \frac{-0.009 \text{ kg} \cdot \text{m}^2/\text{s}}{1.875 \text{ kg} \cdot \text{m}^2} = -0.00480 \text{ rad/s}$$

$$\text{THUS: } \omega = (2.9630 \text{ rad/s}) \underline{i} - (0.00480 \text{ rad/s}) \underline{j}$$

WE MUST NOW RESOLVE  $\omega$  INTO DESIRED COMPONENTS ALONG GX AND GD.

WE NOTE THAT

$$-\omega_y = \omega_p \sin \theta$$

$$\omega_p = \frac{-\omega_y}{\sin \theta} = \frac{+0.00480}{\sin 5^\circ}$$

$$\omega_p = 0.055074 \text{ rad/s}$$

$$\omega_s = \omega_x - \omega_p \cos \theta = 2.9630 - 0.055074 \cos 5^\circ = 2.908 \text{ rad/s}$$

$$\text{ANSWERS: (a) } \omega_s = 2.91 \text{ rad/s. (b) } \omega_p = 0.0551 \text{ rad/s}$$

18.12

GIVEN: PROJECTILE OF PROB. 18.11.

ADDITIONAL DATA:  $\bar{v} = 650 \text{ m/s}$ .

FIND: ANG. MOM.  $H_A$ . (RESOLVE INTO  $x, y, z$  COMP.)

RESOLVE  $\bar{v}$  INTO RECTANG. COMP. ALONG  $x$  AND  $y$  AXES.

$$\bar{v} = (650 \text{ m/s})(\cos 5^\circ \underline{i} - \sin 5^\circ \underline{j}) = (647.53 \text{ m/s}) \underline{i} - (56.65 \text{ m/s}) \underline{j}$$

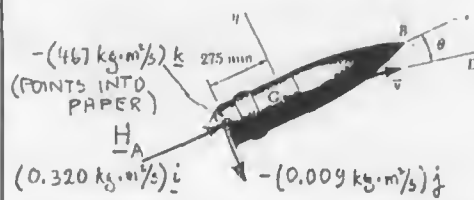
USING EQ. (18.11) AND RECALLING DATA FROM PROB. 18.11,

$$H_A = \bar{r} \times m \bar{v} + H_G$$

$$= (0.275 \text{ m}) \underline{i} \times (30 \text{ kg})[(647.53 \text{ m/s}) \underline{i} - (56.65 \text{ m/s}) \underline{j}] + (0.320 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{i} - (0.009 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{j}$$

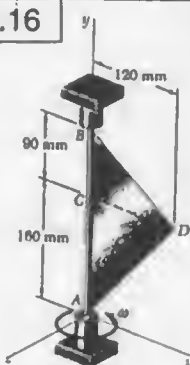
$$= -(467.57 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{k} + (0.320 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{i} - (0.009 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{j}$$

$$H_A = (0.320 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{i} - (0.009 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{j} - (467 \text{ kg} \cdot \text{m}^2/\text{s}) \underline{k}$$





18.16

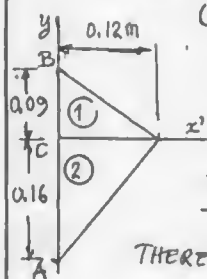


GIVEN:

TRIANGULAR PLATE SHOWN HAS MASS  $m = 7.5 \text{ kg}$  AND IS WELDED TO SHAFT AB. PLATE ROTATES AT CONSTANT RATE  $\omega = 12 \text{ rad/s}$ .

FIND:

- (a) ANG. MOMENTUM  $\underline{H}_C$   
 (b) ANG. MOMENTUM  $\underline{H}_A$   
 (FIND  $\bar{r}$  AND USE PROPERTY INDICATED IN PROB. 18.13a.)



(a) WE DIVIDE PLATE INTO TWO RIGHT TRIANGLES AND COMPUTE THEIR PRODUCTS OF INERTIA.

$$m_1 = \frac{1}{25} (7.5 \text{ kg}) = 2.7 \text{ kg}$$

$$m_2 = \frac{16}{25} (7.5 \text{ kg}) = 4.8 \text{ kg}$$

FROM SAMPLE PROB. 9.6, WE RECALL THAT  $I_{xy, \text{AREA}} = \frac{1}{24} b^2 h^2$

THEREFORE

$$I_{xy, \text{MASS}} = \frac{m}{2bh} \left( \frac{1}{24} b^2 h^2 \right) = \frac{mbh}{12}$$

$$\text{TRIANGLE 1: } (I_{xy})_1 = \frac{1}{12} (2.7 \text{ kg})(0.12 \text{ m})(0.09 \text{ m}) = 2.43 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{TRIANGLE 2: } (I_{xy})_2 = \frac{1}{12} (4.8 \text{ kg})(0.12 \text{ m})(0.16 \text{ m}) = -7.68 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{THUS, FOR THE PLATE, } I_{xy} = (2.43 - 7.68) \times 10^{-3} = -5.25 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

WE NOTE THAT  $I_{yz} = 0$ .

MOHENT OF INERTIA  $I_y$  OF ENTIRE PLATE:

$$I_{y, \text{AREA}} = \frac{1}{12} b h^3, \quad I_{y, \text{MASS}} = \frac{m}{12bh} \left( \frac{1}{12} b h^3 \right) = \frac{1}{6} m h^2$$

$$I_y = \frac{1}{6} (7.5 \text{ kg})(0.12 \text{ m})^2 = 0.018 \text{ kg} \cdot \text{m}^2 = 18 \text{ g} \cdot \text{m}^2$$

ANGULAR MOMENTUM  $\underline{H}_C$ 

WE USE EQS. (18.13) TO OBTAIN THE COMPONENTS  $H_x, H_y, H_z$  OF  $\underline{H}_C$

$$H_x = -I_{xy} \omega = -(-5.25 \text{ g} \cdot \text{m}^2)(12 \text{ rad/s}) = +63.0 \text{ g} \cdot \text{m}^2/\text{s}$$

$$H_y = I_y \omega = (18 \text{ g} \cdot \text{m}^2)(12 \text{ rad/s}) = 216 \text{ g} \cdot \text{m}^2/\text{s}$$

$$H_z = -I_{yz} \omega = 0$$

$$\text{THEREFORE } \underline{H}_C = (63.0 \text{ g} \cdot \text{m}^2/\text{s})\underline{i} + (216 \text{ g} \cdot \text{m}^2/\text{s})\underline{j}$$

(b) ANGULAR MOMENTUM  $\underline{H}_A$ 

WE APPLY THE EQUATION GIVEN IN PART a OF PROB. 18.13 TO POINTS A AND C.

$$\underline{H}_A = \underline{H}_C + \underline{r}_{C/A} \times m \bar{\underline{v}} \quad (1)$$

WHERE  $\underline{r}_{C/A} = (0.16 \text{ m})\underline{j}$ . NOTING THAT THE DISTANCE FROM THE AXIS OF ROTATION AB TO THE MASS CENTER G OF THE PLATE IS  $\bar{r} = \frac{1}{3} (0.12 \text{ m}) = 0.04 \text{ m}$ , WE HAVE

$$m \bar{\underline{v}} = m (\omega \times \bar{\underline{r}}) = (7.5 \text{ kg})(12 \text{ rad/s})\underline{j} \times (0.04 \text{ m})\underline{i}$$

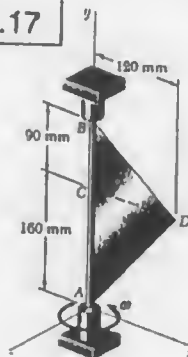
$$= -(3.60 \text{ kg} \cdot \text{m/s})\underline{k} = -(3600 \text{ g} \cdot \text{m/s})\underline{k}$$

$$\underline{r}_{C/A} \times m \bar{\underline{v}} = (0.16 \text{ m})\underline{j} \times (-3600 \text{ g} \cdot \text{m/s})\underline{k} = -(576 \text{ g} \cdot \text{m}^2/\text{s})\underline{i}$$

SUBSTITUTING FOR  $\underline{H}_C$  AND  $\underline{r}_{C/A} \times m \bar{\underline{v}}$  INTO (1):

$$\underline{H}_A = -(513 \text{ g} \cdot \text{m}^2/\text{s})\underline{i} + (216 \text{ g} \cdot \text{m}^2/\text{s})\underline{j}$$

18.17



GIVEN:

TRIANGULAR PLATE SHOWN HAS MASS  $m = 7.5 \text{ kg}$  AND IS WELDED TO SHAFT AB. PLATE ROTATES AT CONSTANT RATE  $\omega = 12 \text{ rad/s}$ .

FIND:

- (a) ANG. MOMENTUM  $\underline{H}_C$   
 (b) ANG. MOMENTUM  $\underline{H}_B$   
 (FIND  $\bar{r}$  AND USE PROPERTY INDICATED IN PROB. 18.13a.)

(a) SEE PART a OF SOLUTION OF PROB. 18.16. WE FIND

$$\underline{H}_C = (63.0 \text{ g} \cdot \text{m}^2/\text{s})\underline{i} + (216 \text{ g} \cdot \text{m}^2/\text{s})\underline{j}$$

(b) ANG. MOMENTUM  $\underline{H}_B$ 

WE APPLY THE EQUATION GIVEN IN PART a OF PROB. 18.13 TO POINTS B AND C.

$$\underline{H}_B = \underline{H}_C + \underline{r}_{C/B} \times m \bar{\underline{v}} \quad (1)$$

WHERE  $\underline{r}_{C/B} = -(0.09 \text{ m})\underline{j}$ .

NOTING THAT THE DISTANCE FROM THE AXIS OF ROTATION AB TO THE MASS CENTER G OF THE PLATE IS

$$\bar{r} = \frac{1}{3} (0.12 \text{ m}) = 0.04 \text{ m}$$

WE HAVE

$$m \bar{\underline{v}} = m (\omega \times \bar{\underline{r}}) = (7.5 \text{ kg})(12 \text{ rad/s})\underline{j} \times (0.04 \text{ m})\underline{i} = -(3.60 \text{ kg} \cdot \text{m/s})\underline{k} = -(3600 \text{ g} \cdot \text{m/s})\underline{k}$$

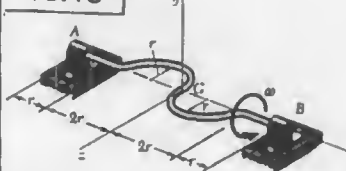
AND

$$\underline{r}_{C/B} \times m \bar{\underline{v}} = -(0.09 \text{ m})\underline{j} \times (-3600 \text{ g} \cdot \text{m/s})\underline{k} = + (324 \text{ g} \cdot \text{m}^2/\text{s})\underline{i}$$

SUBSTITUTING FOR  $\underline{H}_C$  AND  $\underline{r}_{C/B} \times m \bar{\underline{v}}$  INTO (1):

$$\underline{H}_B = (387 \text{ g} \cdot \text{m}^2/\text{s})\underline{i} + (216 \text{ g} \cdot \text{m}^2/\text{s})\underline{j}$$

18.18



GIVEN:

SHAFT OF PROB. 18.15

FIND:

ANG. MOM. OF SHAFT

(a) ABOUT A

(b) ABOUT B

WE FIRST DETERMINE  $\underline{H}_B$ . SEE SOLUTION OF PROB. 18.15. WE FOUND

$$\underline{H}_B = m \bar{r}^2 \omega (0.379 \underline{i} - 0.483 \underline{k})$$

FROM EQ. (18.11) WE HAVE

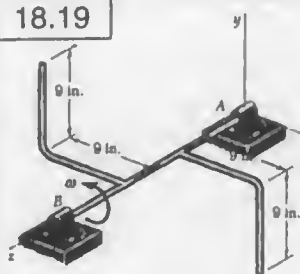
$$\underline{H}_A = \underline{r}_{B/A} \times m \bar{\underline{v}} + \underline{H}_B \quad \underline{H}_B = \underline{r}_{C/B} \times m \bar{\underline{v}} + \underline{H}_C$$

BUT  $\bar{\underline{v}} = 0$  SINCE G IS LOCATED ON AXIS AB. THUS

$$(a) \text{ AND } (b): \underline{H}_A = \underline{H}_B = \underline{H}_C = m \bar{r}^2 \omega (0.379 \underline{i} - 0.483 \underline{k})$$

NOTE. THE RESULT OBTAINED VERIFIES THE PROPERTY INDICATED IN PROB. 18.13b, NAMELY, THAT IF THE MASS CENTER G OF A BODY ROTATING ABOUT A FIXED AXIS IS LOCATED ON THE AXIS, THE ANGULAR MOMENTUM IS THE SAME ABOUT ANY TWO POINTS ON THE AXIS.

18.19



GIVEN:

TWO L-SHAPED ARMS,  
EACH WEIGHING 5 lb,  
ARE WELDED AT THE  
ONE-THIRD POINTS OF  
THE 27-in. SHAFT AB.  
THE ASSEMBLY ROTATES AT  
CONSTANT 360-rpm RATE.  
FIND: (a)  $H_A$   
(b) ANGLE FORMED BY  $H_A$   
AND AB.

MOMENTS AND PRODUCTS  
OF INERTIA

FOR EACH NUMBERED ELEMENT:

$$a = 9 \text{ in.} = 0.75 \text{ ft.}$$

$$m = \frac{1}{2} (5 \text{ lb}) / g = 2.5 / g$$

$$\text{FOR 1 AND 4: } I_x = \bar{I} + md^2 = \frac{1}{12} ma^2 + m\left(\frac{a}{4} + a\right)^2 = \frac{4}{3} ma^2$$

$$\text{FOR 2 AND 3: } I_x = \frac{1}{3} ma^2$$

$$\text{FOR ASSEMBLY: } I_x = 2\left(\frac{4}{3} m a^2 + \frac{1}{3} m a^2\right) = \frac{10}{3} m a^2$$

PRODUCTS OF INERTIA OF ASSEMBLY:

$$I_{xz} = (I_{xz})_1 + (I_{xz})_2 + (I_{xz})_3 + (I_{xz})_4$$

$$= m(-a)(2a) + m\left(-\frac{a}{2}\right)(2a) + m\left(\frac{a}{2}\right)a + m(a)a = -\frac{3}{2} m a^2$$

$$I_{yz} = m\left(\frac{a}{2}\right)(2a) + 0 + 0 + m\left(-\frac{a}{2}\right)a = \frac{1}{2} m a^2$$

(a) ANGULAR MOMENTUM ABOUT H

WE USE Eqs. (18.13) TO OBTAIN THE COMPONENTS OF  $H_A$ .

WE HAVE  $\omega_z = \omega = 360 \text{ rpm} = 6(2\pi) = 12\pi \text{ rad/s}$ ,  $\omega_x = \omega_y = 0$ .

$$H_x = -I_{xz} \omega_z = +\frac{3}{2} m a^2 (12\pi) = 18 m a^2 \pi$$

$$H_y = -I_{yz} \omega_z = -\frac{1}{2} m a^2 (12\pi) = -6 m a^2 \pi$$

$$H_z = I_z \omega_z = \frac{10}{3} m a^2 (12\pi) = 40 m a^2 \pi$$

$$\text{THUS: } H_A = H_x \hat{i} + H_y \hat{j} + H_z \hat{k} = 2 m a^2 \pi (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$= 2 \left( \frac{2.5}{32.2} \text{ lb} \cdot \text{s}^2 / \text{ft} \right) (\pi \text{ rad/s}) (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$= (0.2744 \text{ lb} \cdot \text{ft} \cdot \text{s}) (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$H_A = (2.47 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{i} - (0.823 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{j} + (5.49 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{k}$$

(b) ANGLE  $\theta$  FORMED BY  $H_A$  AND AB

WE NOTE THAT  $\theta = \theta_2$   
AND RECALL THAT

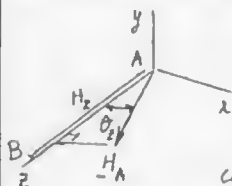
$$H_z = |H_A| \cos \theta_2$$

THUS:

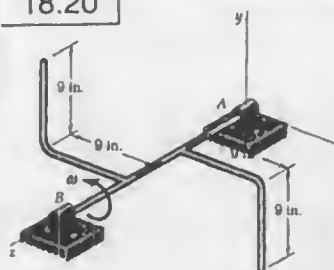
$$\cos \theta = \frac{H_z}{|H_A|} = \frac{2 m a^2 \pi (20)}{2 m a^2 \pi \sqrt{9^2 + 3^2 + 20^2}}$$

$$= \frac{20}{\sqrt{490}} = 0.90351$$

$$\theta = 25.4^\circ$$



18.20



GIVEN:

TWO L-SHAPED ARMS,  
EACH WEIGHING 5 lb,  
ARE WELDED AT THE  
ONE-THIRD POINTS OF  
THE 27-in. SHAFT AB.  
THE ASSEMBLY ROTATES AT  
CONSTANT 360-rpm RATE.  
FIND: (a)  $H_B$   
(b) ANGLE FORMED BY  $H_B$   
AND BA.

WE WILL USE AXES  $x', y', z'$  WITH  
ORIGIN AT B.

MOMENTS AND PRODUCTS OF INERTIA

FOR EACH NUMBERED ELEMENT:

$$a = 9 \text{ in.} = 0.75 \text{ ft.}$$

$$m = \frac{1}{2} (5 \text{ lb}) / g = 2.5 / g$$

$$\text{FOR 1 AND 4: } I_x = \bar{I} + md^2 = \frac{1}{12} ma^2 + m\left(\frac{a}{4} + a\right)^2 = \frac{4}{3} ma^2$$

$$\text{FOR 2 AND 3: } I_x = \frac{1}{3} ma^2$$

$$\text{FOR ASSEMBLY: } I_x = 2\left(\frac{4}{3} m a^2 + \frac{1}{3} m a^2\right) = \frac{10}{3} m a^2$$

PRODUCTS OF INERTIA OF ASSEMBLY:

$$I_{x'z'} = (I_{x'z'})_1 + (I_{x'z'})_2 + (I_{x'z'})_3 + (I_{x'z'})_4$$

$$= m(-a)(-a) + m\left(-\frac{a}{2}\right)(-a) + m\left(\frac{a}{2}\right)(-2a) + m(a)(-2a) = -\frac{3}{2} m a^2$$

$$I_{y'z'} = m\left(\frac{a}{2}\right)(-a) + 0 + 0 + m\left(-\frac{a}{2}\right)(-2a) = \frac{1}{2} m a^2$$

(a) ANGULAR MOMENTUM ABOUT B

WE USE Eqs. (18.13) TO OBTAIN THE COMPONENTS OF  $H_B$ .

WE HAVE  $\omega_z = \omega = 360 \text{ rpm} = 6(2\pi) = 12\pi \text{ rad/s}$ ,  $\omega_x = \omega_y = 0$ .

$$H_{x'} = -I_{x'z'} \omega_z = +\frac{3}{2} m a^2 (12\pi) = 18 m a^2 \pi$$

$$H_{y'} = -I_{y'z'} \omega_z = -\frac{1}{2} m a^2 (12\pi) = -6 m a^2 \pi$$

$$H_{z'} = I_{z'} \omega_z = \frac{10}{3} m a^2 (12\pi) = 40 m a^2 \pi$$

$$\text{THUS: } H_B = H_{x'} \hat{i} + H_{y'} \hat{j} + H_{z'} \hat{k} = 2 m a^2 \pi (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$= 2 \left( \frac{2.5}{32.2} \text{ lb} \cdot \text{s}^2 / \text{ft} \right) (\pi \text{ rad/s}) (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$= (0.2744 \text{ lb} \cdot \text{ft} \cdot \text{s}) (9 \hat{i} - 3 \hat{j} + 20 \hat{k})$$

$$H_B = (2.47 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{i} - (0.823 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{j} + (5.49 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{k}$$

NOTE. THIS IS THE SAME ANSWER THAT WAS OBTAINED FOR  $H_A$   
IN PROB. 18.19. THIS COULD HAVE BEEN ANTICIPATED, SINCE  
THE MASS CENTER G OF THE ASSEMBLY LIES ON THE FIXED  
AXIS AB (CF. PROB. 18.13 b).

(b) ANGLE  $\theta$  FORMED BY  $H_B$  AND BA

WE NOTE THAT  $\theta = \pi - \theta_2$   
AND RECALL THAT

$$H_z = |H_B| \cos \theta_2$$

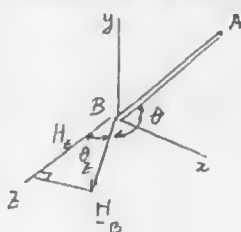
THUS:

$$\cos \theta = \cos(\pi - \theta_2) = -\cos \theta_2$$

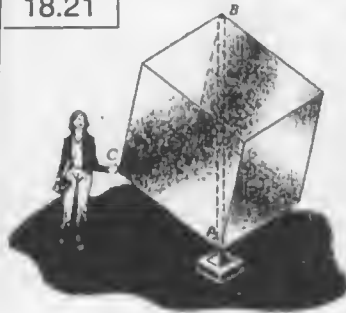
$$= -\frac{H_z}{|H_B|} = -\frac{2 m a^2 \pi (20)}{2 m a^2 \pi \sqrt{9^2 + 3^2 + 20^2}}$$

$$= -\frac{20}{\sqrt{490}} = -0.90351$$

$$\theta = 154.6^\circ$$



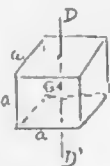
18.21



GIVEN:

HOLLOW CUBE CONSISTS OF SIX 5x5 ft ALUMINUM SHEETS AND CORD AT A IS A BODILY VERTICAL DIAGONAL AB. STUDENT PUSHES CORNER C FOR 1.2 s IN DIRECTION PERPENDICULAR TO PLANE ABC WITH FORCE OF 12.5 lb, CAUSING CUBE TO COMPLETE 1 REV IN 5 s. FIND: WEIGHT OF CUBE.

HINT: PERP. DISTANCE FROM C TO AB IS  $a\sqrt{2}/3$ , WHERE  $a$  IS SIDE OF CUBE.



FOR CUBE,  $I_{AB} = I_{DD'}$ , SINCE THE ELLIPSOID OF INERTIA AT G IS A SPHERE (SEC. 9.17).

FOR THE TWO HORIZONTAL FACES

$$(I_{DD'})_H = 2 \left( \frac{m}{6} \right) \left( \frac{a^2}{6} \right) = \frac{ma^2}{18}$$

WHERE  $m$  = MASS OF CUBE

FOR THE FOUR VERTICAL FACES

$$(I_{DD'})_V = 4 \left( \frac{m}{6} \right) \left[ \frac{a^2}{12} + \left( \frac{a}{2} \right)^2 \right] = \frac{2ma^2}{9}$$

FOR THE WHOLE CUBE:

$$I_{AB} = I_{DD'} = (I_{DD'})_H + (I_{DD'})_V = \frac{ma^2}{18} + \frac{2ma^2}{9} = \frac{5}{18} ma^2$$

IMPULSE-MOMENTUM PRINCIPLE

ANG. IMPULSE ABOUT AB = FINAL ANG. MOMENTUM ABOUT AB

$$(F \Delta t) a \sqrt{2}/3 = \frac{5}{18} ma^2 \omega \quad (1)$$

GIVEN DATA:  $F = 12.5$  lb,  $\Delta t = 1.2$  s,  $a = 5$  ft,  $\omega = \frac{2\pi \text{ rad}}{5 \text{ s}}$

SUBSTITUTE DATA AND  $m = W/g$  INTO (1):

$$(12.5 \text{ lb})(1.2 \text{ s})(5 \text{ ft}) \frac{\sqrt{2}}{3} = \frac{5}{18} \frac{W}{32.2 \text{ ft/s}^2} (5 \text{ ft})^2 \left( \frac{2\pi \text{ rad}}{5 \text{ s}} \right)$$

SOLVING FOR  $W$ :  $W = 225.96$  lb  $W = 226$  lb

18.22

GIVEN: ALUMINUM CUBE OF PROB. 18.21 IS

REPLACED BY CUBE CONSISTING OF SIX PLYWOOD SHEETS, WEIGHING 20 lb EACH. STUDENT PUSHES CORNER C AS IN PROB. 18.21 (FOR 1.2 s WITH 12.5-lb FORCE).

FIND: TIME REQUIRED FOR CUBE TO COMPLETE 1 REV.

SEE SOLUTION OF PROB. 18.21 FOR DERIVATION OF

$$(F \Delta t) a \sqrt{2}/3 = \frac{5}{18} m a^2 \omega \quad (1)$$

GIVEN DATA:  $F = 12.5$  lb,  $\Delta t = 1.2$  s,  $a = 5$  ft

$$m = \frac{W}{g} = \frac{6(20 \text{ lb})}{32.2 \text{ ft/s}^2} = 3.727 \text{ lb} \cdot \text{s}^2/\text{ft}$$

SUBSTITUTE DATA INTO (1):

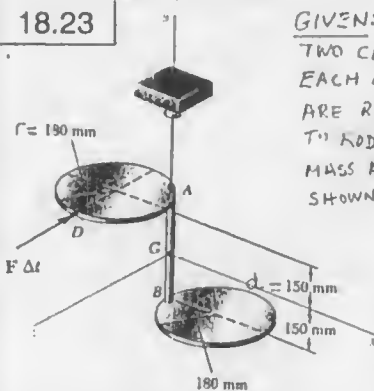
$$(12.5 \text{ lb})(1.2 \text{ s})(5 \text{ ft}) \frac{\sqrt{2}}{3} = \frac{5}{18} (3.727 \text{ lb} \cdot \text{s}^2/\text{ft})(5 \text{ ft})^2 \omega$$

SOLVING FOR  $\omega$ :  $\omega = 2.366 \text{ s}^{-1}$

$$\theta = \frac{2\pi}{\omega} = \frac{2\pi}{2.366 \text{ s}^{-1}} = 2.655 \text{ s}$$

$$\theta = 2.66 \text{ s}$$

18.23



GIVEN:

TWO CIRCULAR PLATES, EACH OF MASS  $m = 4$  kg, ARE RIGIDLY CONNECTED TO ROD AB OF NEGLIGIBLE MASS AND SUSPENDED AS SHOWN. AN IMPULSE  $F \Delta t = -(2.4 \text{ N} \cdot \text{s}) \mathbf{k}$  IS APPLIED AT D.

FIND:

- VELOCITY  $\bar{v}$  OF MASS CENTER G.
- ANGULAR VELOCITY  $\omega$  OF ASSEMBLY.

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

FOR UPPER PLATE:

$$I_x = \bar{I}_x + m d^2 = m \left( \frac{1}{4} r^2 + d^2 \right) = (4 \text{ kg}) \left[ \frac{1}{4} (0.18 \text{ m})^2 + (0.15 \text{ m})^2 \right] = 0.122 + \text{kg} \cdot \text{m}^2$$

$$I_y = \bar{I}_y + m r^2 = \frac{1}{2} m r^2 + m r^2 = \frac{3}{2} m r^2 = \frac{3}{2} (4 \text{ kg}) (0.18 \text{ m})^2 = 0.1944 \text{ kg} \cdot \text{m}^2$$

$$I_z = \bar{I}_z + m (r^2 + d^2) = \frac{1}{4} m r^2 + m (r^2 + d^2) = m \left( \frac{5}{4} r^2 + d^2 \right) = (4 \text{ kg}) \left[ \frac{5}{4} (0.18 \text{ m})^2 + (0.15 \text{ m})^2 \right] = 0.252 \text{ kg} \cdot \text{m}^2$$

$$I_{xy} = m (-r)(d) = -m r d = -(4 \text{ kg})(0.18 \text{ m})(0.15 \text{ m})$$

$$I_{xy} = -0.108 \text{ kg} \cdot \text{m}^2, \quad I_{yz} = 0, \quad I_{zx} = 0$$

FOR LOWER PLATE: WE OBTAIN THE SAME RESULTS

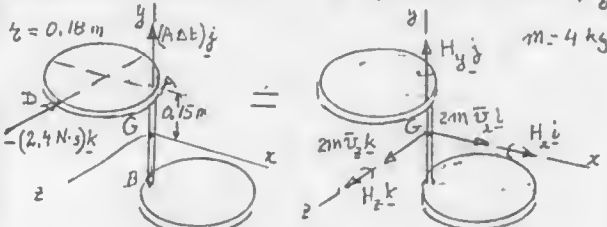
THUS, FOR ASSEMBLY, WE DOUBLE RESULTS FOR UPPER PLATE:

$$\begin{aligned} \bar{I}_x &= 0.2448 \text{ kg} \cdot \text{m}^2, \quad \bar{I}_y = 0.3888 \text{ kg} \cdot \text{m}^2, \quad \bar{I}_z = 0.504 \text{ kg} \cdot \text{m}^2 \\ \bar{I}_{xy} &= -0.216 \text{ kg} \cdot \text{m}^2, \quad \bar{I}_{yz} = 0, \quad \bar{I}_{zx} = 0 \end{aligned} \quad (1)$$

IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT THE IMPULSIVE FORCES ARE  $\bar{F}$  AND

POSSIBLY, THE FORCE AT A. ALSO, FROM CONSTRAINTS,  $\bar{v}_y = 0$



(a) VELOCITY OF MASS CENTER. EQUATE SUMS OF VECTORS:  $-(2.4 \text{ N} \cdot \text{s}) \mathbf{k} + (A \Delta t) \mathbf{j} = 2(4 \text{ kg})(v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k})$

THUS:  $A \Delta t = 0$ ,  $\bar{v}_x = 0$ ,  $\bar{v}_z = -0.3 \text{ m/s}$

$$\bar{v} = -(0.300 \text{ m/s}) \mathbf{k}$$

(b) ANGULAR VELOCITY. EQUATE SUMS OF MOMENTS ABOUT G:

$$\begin{aligned} [(-0.18 \text{ m}) \mathbf{i} + (0.15 \text{ m}) \mathbf{j}] \times (-2.4 \text{ N} \cdot \text{s}) \mathbf{k} &= H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k} \\ -(0.432 \text{ kg} \cdot \text{m}^2) \mathbf{j} - (0.360 \text{ kg} \cdot \text{m}^2) \mathbf{i} &= H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k} \\ H_x &= -0.360, \quad H_y = -0.432, \quad H_z = 0 \end{aligned} \quad (2)$$

SUBSTITUTE FROM (1) AND (2) INTO Eqs. (18.7):

$$H_x = \bar{I}_x \omega_x - \bar{I}_{yz} \omega_z = -0.360 = 0.2448 \omega_x + 0.216 \omega_z \quad (3)$$

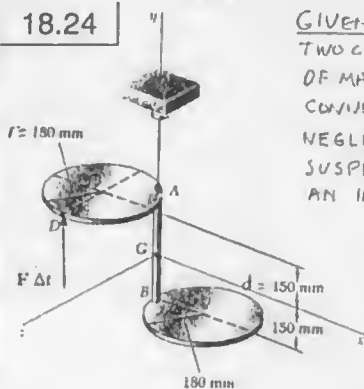
$$H_y = -\bar{I}_y \omega_y + \bar{I}_{xz} \omega_z = -0.432 = 0.3888 \omega_y + 0.108 \omega_z \quad (4)$$

$$H_z = -\bar{I}_z \omega_z + \bar{I}_{xy} \omega_x + \bar{I}_{yz} \omega_y = 0 = 0.504 \omega_z - 0.216 \omega_x - 0.216 \omega_y \quad (5)$$

SOLVE (3) AND (4):  $\omega_x = -0.96154$ ,  $\omega_y = -0.57692$

$$\omega = -(0.962 \text{ rad/s}) \mathbf{i} - (0.577 \text{ rad/s}) \mathbf{j}$$

18.24



GIVEN:

TWO CIRCULAR PLATES, EACH OF MASS  $m = 4 \text{ kg}$ , ARE RIGIDLY CONNECTED TO ROD AB OF NEGLECTIBLE MASS AND SUSPENDED AS SHOWN. AN IMPULSE

$$F\Delta t = (2.4 \text{ N}\cdot\text{s})\hat{j}$$

IS APPLIED AT D.

FIND:

- (a) VELOCITY  $\vec{v}$  OF MASS CENTER  $G$ ,  
(b) ANGULAR VELOCITY  $\vec{\omega}$  OF ASSEMBLY.

## COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

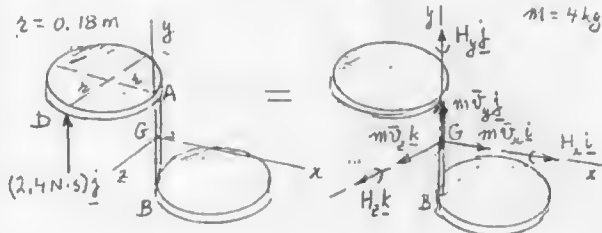
SEE SOLUTION OF PROB. 18.23 WHERE WE FOUND

$$\bar{I}_x = 0.2448 \text{ kg}\cdot\text{m}^2, \bar{I}_y = 0.3888 \text{ kg}\cdot\text{m}^2, \bar{I}_z = 0.504 \text{ kg}\cdot\text{m}^2 \quad (1)$$

$$\bar{I}_{xy} = -0.216 \text{ kg}\cdot\text{m}^2, \bar{I}_{yz} = 0, \bar{I}_{xz} = 0$$

## IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT THE CORD AT A WILL BECOME SLACK THUS, THE ONLY IMPULSIVE FORCE IS  $\vec{F}$ .



## (a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:

$$(2.4 \text{ N}\cdot\text{s})\hat{j} = 2(4 \text{ kg})(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$\text{THUS: } v_x = 0, v_y = 0.300 \text{ m/s}, v_z = 0$$

$$\vec{v} = (0.300 \text{ m/s})\hat{j}$$

## (b) ANGULAR VELOCITY

EQUATE SUMS OF MOMENTS ABOUT G:

$$[(-0.18 \text{ m})\hat{i} + (0.18 \text{ m})\hat{k}] \times (2.4 \text{ N}\cdot\text{s})\hat{j} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$(0.432 \text{ kg}\cdot\text{m}^2)(-\hat{k} - \hat{i}) = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$\text{THUS: } H_x = -0.432 \text{ kg}\cdot\text{m}^2, H_y = 0, H_z = -0.432 \text{ kg}\cdot\text{m}^2 \quad (2)$$

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: -0.432 = 0.2448\omega_x + 0.216\omega_y \quad (3)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: 0 = -0.216\omega_x + 0.3888\omega_y \quad (4)$$

$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: -0.432 = 0.504\omega_z \quad (5)$$

SOLVING (3) AND (4) SIMULTANEOUSLY,

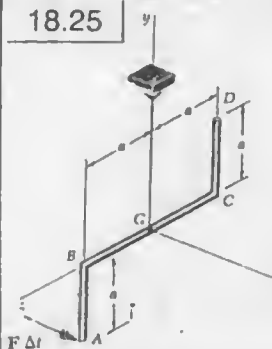
$$\omega_x = -3.4616 \text{ rad/s}, \omega_y = 1.9231 \text{ rad/s}$$

SOLVING (5) FOR  $\omega_z$ :  $\omega_z = -0.8571 \text{ rad/s}$ 

THUS:

$$\vec{\omega} = -(3.46 \text{ rad/s})\hat{i} + (1.923 \text{ rad/s})\hat{j} - (0.857 \text{ rad/s})\hat{k}$$

18.25



GIVEN:

UNIFORM BENT ROD OF MASS  $m$  IS SUSPENDED AS SHOWN. ROD IS HIT AT A WITH IMPULSE  $F\Delta t$  IN DIRECTION PERPENDICULAR TO PLANE CONTAINING ROD.

FIND:

- IMMEDIATELY AFTER IMPACT  
(a) VELOCITY  $\vec{v}$  OF MASS CENTER  
(b) ANGULAR VELOCITY  $\vec{\omega}$  OF ROD.

## COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

$$\text{PORTION BC: } (\bar{I}_x)_{BC} = (\bar{I}_y)_{BC} = \frac{1}{12}(m)(2a)^2 = \frac{1}{6}ma^2, (\bar{I}_z)_{BC} = 0$$

$$(\bar{I}_{yz})_{BC} = (\bar{I}_{zy})_{BC} = (\bar{I}_{xz})_{BC} = 0$$

PORTIONS AB AND CD:

$$(\bar{I}_x)_{AB} = (\bar{I}_x)_{CD} = \bar{I} + \left(\frac{m}{4}\right)a^2 = \frac{1}{12}\left(\frac{m}{4}\right)a^2 + \frac{m}{4}\left(a^2 + \frac{a^2}{4}\right) = \frac{1}{3}ma^2$$

$$(\bar{I}_y)_{AB} = (\bar{I}_y)_{CD} = \frac{m}{4}a^2, (\bar{I}_z)_{AB} = (\bar{I}_z)_{CD} = \frac{1}{3}\frac{m}{4}a^2 = \frac{1}{12}ma^2$$

$$(\bar{I}_{xy})_{AB} = (\bar{I}_{xy})_{CD} = 0, (\bar{I}_{xz})_{AB} = (\bar{I}_{xz})_{CD} = 0, (\bar{I}_{yz})_{AB} = (\bar{I}_{yz})_{CD} = -\frac{m}{4}\left(a\frac{a}{2}\right) = -\frac{ma^2}{8}$$

THE MOMENTS AND PRODUCTS OF INERTIA OF THE ROD ARE OBTAINED BY ADDING THE ABOVE VALUES:

$$\bar{I}_x = \frac{1}{6}ma^2 + \frac{1}{3}ma^2 + \frac{1}{3}ma^2 = \frac{5}{6}ma^2$$

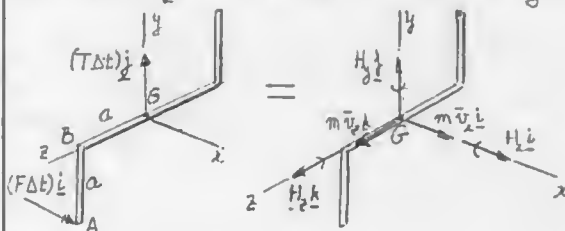
$$\bar{I}_y = \frac{1}{6}ma^2 + \frac{1}{4}ma^2 + \frac{1}{4}ma^2 = \frac{2}{3}ma^2$$

$$\bar{I}_z = 0 + \frac{1}{12}ma^2 + \frac{1}{12}ma^2 = \frac{1}{6}ma^2$$

$$\bar{I}_{xy} = 0, \bar{I}_{yz} = 0 - \frac{1}{8}ma^2 - \frac{1}{8}ma^2 = -\frac{1}{4}ma^2, \bar{I}_{xz} = 0$$

## IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF  $F\Delta t = (F\Delta t)\hat{i}$  AND, POSSIBLY, AN IMPULSE  $(T\Delta t)\hat{j}$  AT G. BECAUSE OF CONSTRAINTS,  $\vec{v}_y = 0$ .



## (a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:  $(F\Delta t)\hat{i} + (T\Delta t)\hat{j} = m\vec{v}_x\hat{i} + m\vec{v}_z\hat{k}$ 

$$\text{THUS: } \vec{v}_x = (F\Delta t)/m, \vec{v}_z = 0, T\Delta t = 0 \quad \vec{v} = (F\Delta t/m)\hat{i}$$

## (b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$(-a\hat{j} + a\hat{k}) \times (F\Delta t)\hat{i} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$aF\Delta t(\hat{k} + \hat{j}) = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$\text{THUS: } H_x = 0, H_y = aF\Delta t, H_z = aF\Delta t \quad (2)$$

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: 0 = \frac{5}{6}ma^2\omega_x \quad \omega_x = 0 \quad (3)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: aF\Delta t = \frac{2}{3}ma^2\omega_y + \frac{1}{4}ma^2\omega_z \quad (4)$$

$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: aF\Delta t = \frac{1}{4}ma^2\omega_y + \frac{1}{6}ma^2\omega_z \quad (5)$$

(CONTINUED)



### 18.25 continued

WE REPEAT THE FOLLOWING EQS.:

$$\omega_x = 0 \quad (3)$$

$$aF\Delta t = \frac{2}{3}ma^2\omega_y + \frac{1}{4}ma^2\omega_z \quad (4)$$

$$aF\Delta t = \frac{1}{2}ma^2\omega_y + \frac{1}{8}ma^2\omega_z \quad (5)$$

SOLVING (4) AND (5) SIMULTANEOUSLY, WE OBTAIN

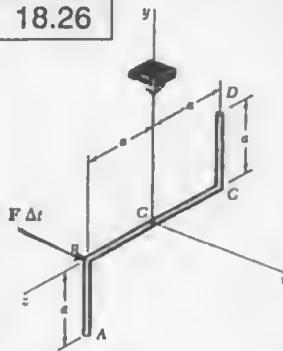
$$\omega_y = -\frac{12}{7} \frac{F\Delta t}{ma}$$

$$\omega_z = \frac{60}{7} \frac{F\Delta t}{ma}$$

THUS:

$$\underline{\omega} = (12F\Delta t/7ma)(-\hat{j} + 5\hat{k})$$

### 18.26



GIVEN:

UNIFORM BENT ROD OF MASS  $m$  IS SUSPENDED AS SHOWN. ROD IS HIT AT  $B$  WITH IMPULSE  $F\Delta t$  IN DIRECTION PERPENDICULAR TO PLANE CONTAINING ROD.

FIND:

IMMEDIATELY AFTER IMPACT  
(a) VELOCITY OF MASS CENTER  
(b) ANGULAR VELOCITY  $\underline{\omega}$  OF ROD.

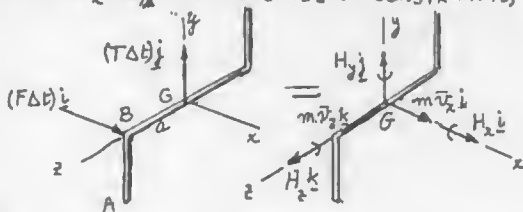
#### MOMENTS AND PRODUCTS OF INERTIA

SEE SOLUTION OF PROB. 18.25. WE OBTAINED

$$\bar{I}_x = \frac{2}{3}ma^2, \bar{I}_y = \frac{2}{3}ma^2, \bar{I}_z = \frac{1}{6}ma^2, \bar{I}_{xy} = -\frac{1}{4}ma^2, \bar{I}_{xz} = \bar{I}_{yz} = 0 \quad (1)$$

#### IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF  $F\Delta t = (F\Delta t)\hat{i}$  AND, POSSIBLY, AN IMPULSE  $(T\Delta t)\hat{j}$  AT  $G$ . BECAUSE OF CONSTRAINTS,  $\bar{v}_y = 0$ .



#### (a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:  $(F\Delta t)\hat{i} + (T\Delta t)\hat{j} = m\bar{v}_x\hat{i} + m\bar{v}_z\hat{k}$

THUS:  $\bar{v}_x = (F\Delta t)/m, \bar{v}_z = 0, T\Delta t = 0$

$$\underline{\bar{v}} = (F\Delta t/m)\hat{i}$$

#### (b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT  $G$ :

$$a\hat{k} \times (F\Delta t)\hat{i} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$(aF\Delta t)\hat{j} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

THUS:  $H_x = 0, H_y = aF\Delta t, H_z = 0$

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: 0 = \frac{2}{3}ma^2\omega_x \quad \omega_x = 0 \quad (3)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: aF\Delta t = \frac{2}{3}ma^2\omega_y - \frac{1}{4}ma^2\omega_z \quad (4)$$

$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: 0 = -\frac{1}{4}ma^2\omega_y + \frac{1}{6}ma^2\omega_z \quad (5)$$

SOLVING (4) AND (5) SIMULTANEOUSLY, WE OBTAIN

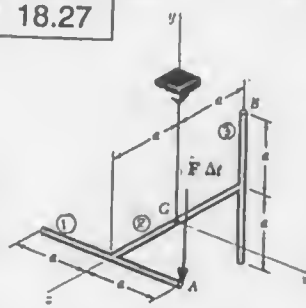
$$\omega_y = \frac{24}{7} \frac{F\Delta t}{ma}$$

$$\omega_z = -\frac{36}{7} \frac{F\Delta t}{ma}$$

THUS:

$$\underline{\omega} = (12F\Delta t/7ma)(2\hat{j} - 3\hat{k})$$

### 18.27



GIVEN:

THREE RODS, EACH OF MASS  $m$  AND LENGTH  $2a$  ARE WELDED TO FORM ASSEMBLY. ASSEMBLY IS HIT VERTICALLY AT  $A$  AS SHOWN.

FIND:

IMMEDIATELY AFTER IMPACT  
(a) VELOCITY OF MASS CENTER  
(b) ANGULAR VELOCITY  $\underline{\omega}$  OF ASSEMBLY.

#### COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

$$\bar{I}_x = (\bar{I}_x)_1 + (\bar{I}_x)_2 + (\bar{I}_x)_3 = ma^2 + \frac{1}{3}ma^2 + m(a^2 + \frac{a^2}{3}) = \frac{8}{3}ma^2$$

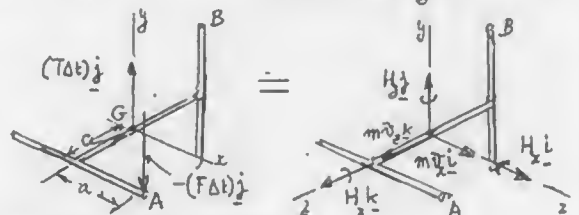
$$\bar{I}_y = (\bar{I}_y)_1 + (\bar{I}_y)_2 + (\bar{I}_y)_3 = m(a^2 + \frac{a^2}{3}) + \frac{1}{3}ma^2 + ma^2 = \frac{8}{3}ma^2 \quad (1)$$

$$\bar{I}_z = (\bar{I}_z)_1 + (\bar{I}_z)_2 + (\bar{I}_z)_3 = \frac{1}{3}ma^2 + 0 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$$

$$\bar{I}_{xy} = 0, \bar{I}_{yz} = 0, \bar{I}_{zx} = 0$$

#### IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF  $-(F\Delta t)\hat{j}$  APPLIED AT  $A$  AND  $(T\Delta t)\hat{j}$  APPLIED AT  $G$ . BECAUSE OF CONSTRAINTS,  $\bar{v}_y = 0$ .



#### (a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:  $(T\Delta t)\hat{j} - (F\Delta t)\hat{j} = m\bar{v}_x\hat{i} + m\bar{v}_z\hat{k}$

THUS:  $T\Delta t = F\Delta t, \bar{v}_x = 0, \bar{v}_z = 0$ . SINCE  $\bar{v}_y = 0$  FROM ABOVE,

$$\underline{\bar{v}} = 0$$

#### (b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT  $G$ :

$$(a\hat{i} + a\hat{k}) \times (F\Delta t)\hat{j} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$-(aF\Delta t)\hat{k} + (aF\Delta t)\hat{i} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

THUS:  $H_x = aF\Delta t, H_y = 0, H_z = -aF\Delta t$  (2)

SINCE THE THREE PRODUCTS OF INERTIA ARE ZERO, THE  $x, y,$  AND  $z$  AXES ARE PRINCIPAL CENTROIDAL AXES AND WE CAN USE EQS. (18.10). SUBSTITUTING FROM (1) AND (2) INTO THESE EQUATIONS, WE HAVE

$$H_x = \bar{I}_x\omega_x: aF\Delta t = \frac{8}{3}ma^2\omega_x \quad \omega_x = 3F\Delta t/8ma \quad (3)$$

$$H_y = \bar{I}_y\omega_y: 0 = \frac{8}{3}ma^2\omega_y \quad \omega_y = 0 \quad (4)$$

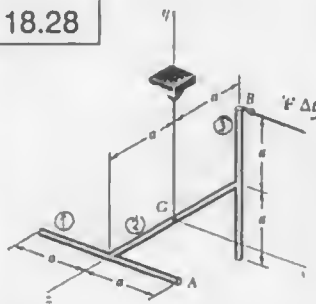
$$H_z = \bar{I}_z\omega_z: -aF\Delta t = \frac{2}{3}ma^2\omega_z \quad \omega_z = -3F\Delta t/2ma \quad (5)$$

THEREFORE:

$$\underline{\omega} = (3F\Delta t/8ma)(\hat{i} - 4\hat{k})$$



18.28



GIVEN:

THREE RODS, EACH OF MASS  $m$  AND LENGTH  $2a$  ARE WELDED TO FORM ASSEMBLY, WHICH IS HIT AT B IN DIRECTION OPPOSITE TO  $x$  AXIS.

FIND:

IMMEDIATELY AFTER IMPACT  
(a) VELOCITY OF MASS CENTER  
(b) ANGULAR VELOCITY  $\omega$ .

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

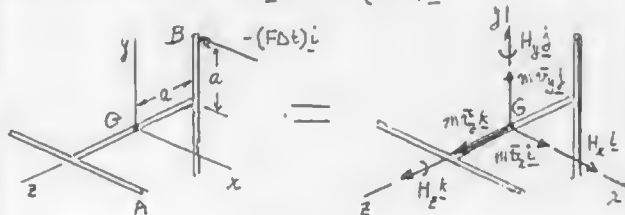
$$\bar{I}_x = (\bar{I}_x)_1 + (\bar{I}_x)_2 + (\bar{I}_x)_3 = ma^2 + \frac{1}{3}ma^2 + m(a^2 + \frac{a^2}{3}) = \frac{5}{3}ma^2$$

$$\bar{I}_y = (\bar{I}_y)_1 + (\bar{I}_y)_2 + (\bar{I}_y)_3 = m(a^2 + \frac{a^2}{3}) + \frac{1}{3}ma^2 + ma^2 = \frac{5}{3}ma^2 \quad (1)$$

$$\bar{I}_z = (\bar{I}_z)_1 + (\bar{I}_z)_2 + (\bar{I}_z)_3 = \frac{1}{3}ma^2 + 0 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$$

$$\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$$

IMPULSE-MOMENTUM PRINCIPLE

THE ONLY IMPULSE IS  $F\Delta t = -(F\Delta t)\hat{i}$ .

(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF MOMENTS:

$$-F\Delta t\hat{i} = m\bar{V}_G\hat{i} + m\bar{V}_G\hat{j} - m\bar{V}_G\hat{k}$$

$$\text{THUS: } \bar{V}_G = -F\Delta t/m, \quad \bar{V}_G = 0, \quad \bar{V}_G = 0$$

(b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$(a\hat{j} - a\hat{k}) \times (-F\Delta t)\hat{i} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$(aF\Delta t)\hat{k} + (aF\Delta t)\hat{j} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

$$\text{THUS: } H_x = 0, \quad H_y = aF\Delta t, \quad H_z = aF\Delta t \quad (2)$$

SINCE THE THIN RODS ARE PRINCIPAL CENTRAL AXES, THE  $x, y$ , AND  $z$  AXES ARE PRINCIPAL CENTRAL AXES AND WE CAN USE Eqs. (18.10). SUBSTITUTING FROM (1) AND (2) INTO THESE EQUATIONS, WE HAVE

$$H_x = \bar{I}_x\omega_x: \quad 0 = \frac{5}{3}ma^2\omega_x \quad \omega_x = 0 \quad (3)$$

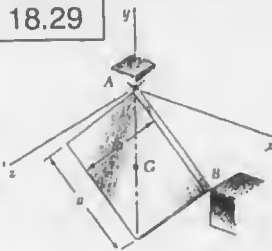
$$H_y = \bar{I}_y\omega_y: \quad aF\Delta t = \frac{5}{3}ma^2\omega_y \quad \omega_y = 3F\Delta t/5ma \quad (4)$$

$$H_z = \bar{I}_z\omega_z: \quad aF\Delta t = \frac{2}{3}ma^2\omega_z \quad \omega_z = 3F\Delta t/2ma \quad (5)$$

THEREFORE

$$\omega = (3F\Delta t/8ma)(\hat{j} + \hat{k})$$

18.29



GIVEN:

SQUARE PLATE OF MASS  $m$  SUPPORTED BY BALL AND SOCKET WITH ANGULAR VELOCITY  $\omega_0\hat{j}$  WHEN IT STRIKES OBSTACLE AT B IN  $xy$  PLANE ( $z=0$ ).

FIND:

IMMEDIATELY AFTER IMPACT  
(a) ANG. VELOCITY OF PLATE.  
(b) VELOCITY OF G.

ANGULAR MOMENTUM

BECAUSE OF SYMMETRY OF SQUARE PLATE,  $\bar{I}$  IS THE SAME ABOUT ANY AXIS THROUGH G WITHIN  $xy$  PLANE. (CF. SEC. 9.17),  $\bar{I} = \frac{1}{12}ma^2$ . IT FOLLOWS THAT  $H_G = \frac{1}{12}ma^2\omega_0\hat{j}$  FOR ANY  $\omega$ .

VELOCITIES AFTER IMPACT

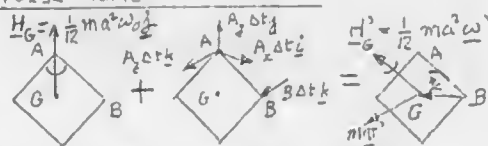
SINCE  $z=0$ , CORNER B REMAINS IN CONTACT WITH OBSTACLE AND PLATE ROTATES ABOUT AB.

$$\Delta_{BA} = -\cos 45^\circ\hat{i} + \sin 45^\circ\hat{j} = (-\hat{i} + \hat{j})/\sqrt{2}$$

$$\omega' = \omega'\Delta_{BA} = \omega'(-\hat{i} + \hat{j})/\sqrt{2} \quad (2)$$

$$\bar{v}' = \omega' \times \bar{r} = [\omega'(-\hat{i} + \hat{j})/\sqrt{2}] \times (a/\sqrt{2})(-\hat{i}) = \frac{1}{2}\omega'a\hat{k} \quad (3)$$

IMPULSE-MOMENTUM PRINCIPLE



EQUATING MOMENTS ABOUT LINE BA:

$$H_G \cos 45^\circ + 0 = H'_G + \Delta_{BA} \cdot (\bar{r} \times m\bar{v}')$$

RECALLING (1), (2), (3), AND VALUE OF  $\Delta_{BA}$ :

$$\frac{1}{12}ma^2\omega_0 \cos 45^\circ = \frac{1}{12}ma^2\omega' + [(-\hat{i} + \hat{j})/\sqrt{2}] \cdot [-\frac{a}{\sqrt{2}}\hat{i} \times \frac{1}{2}m\omega'a\hat{k}]$$

$$\frac{\omega_0}{12\sqrt{2}} = \omega'(\frac{1}{12} + \frac{1}{4}) \quad \omega' = \frac{1}{4\sqrt{2}}\omega_0 \quad (4)$$

(a) ANGULAR VELOCITY

$$\text{FROM (2) AND (4): } \omega' = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} \frac{\omega_0}{4\sqrt{2}} \quad \omega' = \frac{1}{8}\omega_0(-\hat{i} + \hat{j})$$

(b) VELOCITY OF G. FROM (3) AND (4):

$$\bar{v}' = \frac{1}{2} \frac{1}{4\sqrt{2}} \omega_0 a \hat{k} = 0.08839 \omega_0 a \hat{k} \quad \bar{v}' = 0.0884 \omega_0 a \hat{k}$$

18.30

GIVEN: IMPACT DESCRIBED IN PROB. 18.29.

FIND: IMPULSE ON PLATE AT (a) B, (b) A.

SEE SOLUTION OF PROB. 18.29 FOR IMPULSE-MOMENTUM DIAGRAM AND DETERMINATION OF  $\omega'$  AND  $\bar{v}'$ .

(a) EQUATING MOMENTS ABOUT A:

$$\frac{1}{12}ma^2\omega_0\hat{j} + \frac{a}{\sqrt{2}}(\hat{i} - \hat{j}) \times B\Delta t\hat{k} = \frac{1}{12}ma^2\omega'\hat{j} - \frac{a}{\sqrt{2}}\hat{j} \times m\bar{v}'$$

$$\text{SUBSTITUTING FOR } \omega' \text{ AND } \bar{v}' \text{ AND PERFORMING PRODUCTS}$$

$$\frac{1}{12}ma^2\omega_0\hat{j} - \frac{a}{\sqrt{2}}B\Delta t(\hat{j} + \hat{i}) = \frac{1}{12}ma^2\frac{\omega_0}{8}(-\hat{i} + \hat{j}) - \frac{a}{\sqrt{2}}\hat{j} \times m\frac{\omega_0 a}{8\sqrt{2}}\hat{k}$$

$$= ma^2\omega_0(-\frac{1}{96}\hat{i} + \frac{1}{96}\hat{j} - \frac{1}{16}\hat{i})$$

EQUATING THE COEFF. OF  $\hat{i}$ :

$$-\frac{a}{\sqrt{2}}B\Delta t = -\frac{7}{96}ma^2\omega_0$$

$$B\Delta t = 0.10312m\omega_0 a$$

$$B\Delta t = 0.1031m\omega_0 a \hat{k}$$

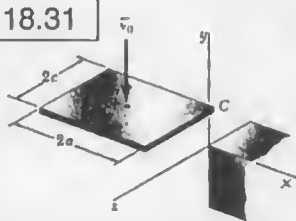
(b) EQUATING SUMS OF VECTORS:

$$A\Delta t + B\Delta t = m\bar{v}'$$

$$A\Delta t = m\bar{v}' - B\Delta t = m(0.08839\omega_0 a \hat{k}) - 0.10312m\omega_0 a \hat{k}$$

$$A\Delta t = -0.01473m\omega_0 a \hat{k}$$

18.31



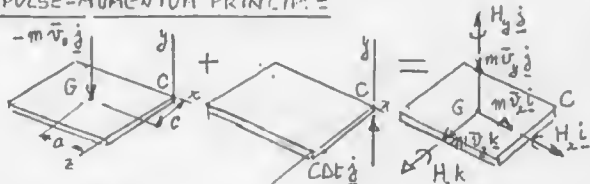
GIVEN:

RECTANGULAR PLATE OF MASS  $m$  FALLING WITH  $\vec{v}_0$  AND NO ANG. VELOCITY STRIKES OBSTRUCTION ( $e=0$ ).

FIND:

ANG. VELOCITY OF PLATE IMMEDIATELY AFTER IMPACT.

(IMPULSE-MOMENTUM PRINCIPLE)



$$x \text{ COMP.: } \vec{v}_x = 0 \quad \text{THUS: } \vec{v} = \vec{v}_y \hat{j} \quad (1)$$

$$y \text{ COMP.: } -m\vec{v}_0 + C\Delta t = m\vec{v}_y \quad C\Delta t = m(\vec{v}_0 + \vec{v}_y) \quad (2)$$

EQUATING MOMENTS ABOUT C:

$$(-a\hat{i} + c\hat{k}) \times (-m\vec{v}_0\hat{j}) = (-a\hat{i} + c\hat{k}) \times m\vec{v}_y\hat{j} + H_C\hat{i} + H_C\hat{j} + H_C\hat{k} \quad (3)$$

SINCE  $e=0$ , PLATE ROTATES ABOUT C IMMEDIATELY AFTER IMPACT

$$\vec{v} = \omega \times \vec{r}_{G/C}: \vec{v}_y\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ -a & 0 & c \end{vmatrix} = \omega_z c\hat{i} - (\omega_x c + \omega_z a)\hat{j} + \omega_y a\hat{k}$$

$$\text{EQUATE COEFF. OF UNIT VECTORS: } \omega_y = 0, \quad \vec{v}_y = -(\omega_z c + \omega_x a) \quad (4)$$

USE EQS. (1), (2):

$$H_x = I_x \omega_x = \frac{1}{12} m(2c)^2 \omega_x = \frac{1}{3} m c^2 \omega_x \quad (5)$$

$$H_y = I_y \omega_y = 0 \quad [\text{BECAUSE OF (4)}]$$

$$H_z = I_z \omega_z = \frac{1}{12} m(2a)^2 \omega_z = \frac{1}{3} m a^2 \omega_z$$

SUBSTITUTE FROM (4) AND (5) INTO (3):

$$m\vec{v}_0(a\hat{k} + c\hat{i}) = m(\omega_z c + \omega_x a)(a\hat{k} + c\hat{i}) + \frac{1}{3} m c^2 \omega_x \hat{i} + \frac{1}{3} m a^2 \omega_z \hat{k}$$

$$= \left(\frac{4}{3} m c^2 \omega_x + m a c \omega_z\right) \hat{i} + \left(\frac{4}{3} m a^2 \omega_z + m a c \omega_x\right) \hat{k}$$

DIVIDE BY  $m$  AND EQUATE COEFF. OF UNIT VECTORS:

$$\frac{4}{3} c^2 \omega_x + a c \omega_z = \vec{v}_0 c \quad (6)$$

$$\frac{4}{3} a c \omega_x + \frac{4}{3} a^2 \omega_z = \vec{v}_0 a \quad (7)$$

SOLVE (6) AND (7) SIMULTANEOUSLY:

$$\omega_x = 3\vec{v}_0/7c, \quad \omega_z = 3\vec{v}_0/7a \quad \omega = \frac{3\vec{v}_0}{7} \left(\frac{1}{c}\hat{i} + \frac{1}{a}\hat{k}\right)$$

18.32

GIVEN: IMPACT DESCRIBED IN PROB. 18.31

FIND:

(a) VELOCITY OF G IMMEDIATELY AFTER IMPACT,  
(b) IMPULSE ON PLATE DURING IMPACT.

(a) FROM SOLUTION OF PROB. 18.31:

$$\text{EQS. (1) AND (4): } \vec{v} = \vec{v}_y \hat{j} = -(\omega_x c + \omega_z a) \hat{j}$$

$$\text{FROM ANSWER TO PROB. 18.31: } \omega_x = \frac{3\vec{v}_0}{7c}, \quad \omega_z = \frac{3\vec{v}_0}{7a}$$

$$\text{THUS: } \vec{v} = -\left(\frac{3\vec{v}_0}{7} + \frac{3\vec{v}_0}{7}\right) \hat{j} = -\frac{6}{7} \vec{v}_0 \hat{j}$$

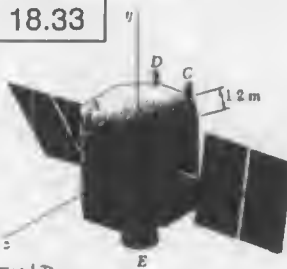
(b) FROM IMPULSE-MOMENTUM DIAGRAM OF PROB. 18.31;  
EQUATING SUMS OF VECTORS:

$$-m\vec{v}_0\hat{j} + C\Delta t\hat{j} = m\vec{v}$$

$$C\Delta t\hat{j} = m\vec{v} + m\vec{v}_0\hat{j} = -\frac{6}{7} m\vec{v}_0\hat{j} + m\vec{v}_0\hat{j} = \frac{1}{7} m\vec{v}_0\hat{j}$$

$$C\Delta t = \frac{1}{7} m\vec{v}_0\hat{j}$$

18.33



GIVEN: PROBE WITH

$m = 2500 \text{ kg}$ ,  $k_x = 0.98 \text{ m}^2$   
 $k_y = 1.06 \text{ m}^2$ ,  $k_z = 4.02 \text{ m}^2$ ;  
500-N MAIN THRUSTER E;  
20-N THRUSTERS A, B, C, D  
CAN EXPEL FUEL IN  $y$  DIR.  
PROBE HAS ANG. VELOCITY  
 $\omega = (0.04 \text{ rad/s})\hat{i}$   
 $+ (0.06 \text{ rad/s})\hat{k}$

FIND:

(a) WHICH TWO THRUSTERS SHOULD BE USED TO REDUCE  $\omega$  TO ZERO  
(b) OPERATING TIME OF THESE THRUSTERS,  
(c) HOW LONG SHOULD E BE ACTIVATED IF  $\vec{v}$  IS TO REMAIN UNCHANGED.

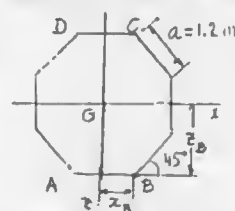
INITIAL ANGULAR MOMENTUM

$$H_G = \bar{I}_x \omega_x \hat{i} + \bar{I}_y \omega_y \hat{j} + \bar{I}_z \omega_z \hat{k} = m(k_x^2 \omega_x \hat{i} + k_y^2 \omega_y \hat{j} + k_z^2 \omega_z \hat{k})$$

$$= (2500 \text{ kg})[(0.98 \text{ m}^2)(0.04 \text{ rad/s})\hat{i} + 0 + (4.02 \text{ m}^2)(0.06 \text{ rad/s})\hat{k}]$$

$$= (96.04 \text{ kg}\cdot\text{m}^2/\text{s})\hat{i} + (156.06 \text{ kg}\cdot\text{m}^2/\text{s})\hat{k} \quad (1)$$

ANGULAR IMPULSE OF TWO 20-N THRUSTERS



LET US ASSUME THAT THRUSTERS A AND B WILL BE USED.

FROM GEOMETRY OF OCTAGON,

$$r_B = \frac{1}{2}a = \frac{1}{2}(1.2 \text{ m}) = 0.6 \text{ m}$$

$$r_C = \frac{1}{2}a + a \sin 45^\circ = 1.2071a$$

$$= 1.44853 \text{ m}$$

$$r_A = -r_B \quad r_C = r_D$$

$$\text{ANG. IMPULSE ABOUT G} = \sum \vec{r} \times (-F\Delta t\hat{j}) + \sum \vec{r} \times (-F\Delta t\hat{j})$$

$$= (-r_B\hat{i} + r_B\hat{k}) \times (-F\Delta t\hat{j}) + (r_C\hat{i} + r_C\hat{k}) \times (-F\Delta t\hat{j})$$

$$= r_B(F\Delta t\hat{i} - F\Delta t\hat{k}) + r_C(F\Delta t\hat{i} + F\Delta t\hat{k})$$

$$= (0.6 \text{ m})(F\Delta t\hat{i} - F\Delta t\hat{k}) + (1.44853 \text{ m})(F\Delta t\hat{i} + F\Delta t\hat{k}) \quad (2)$$

IMPULSE-MOMENTUM PRINCIPLE

SINCE THE FINAL ANGULAR VELOCITY AND, THUS, THE FINAL ANGULAR MOMENTUM MUST BE ZERO, THE SUM OF (1) AND (2) MUST BE ZERO. EQUATING THE COEFF. OF  $\hat{i}$  AND  $\hat{k}$  TO ZERO:

$$(1.44853 \text{ m})(F\Delta t_A + F\Delta t_B) + 96.04 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$

$$(0.6 \text{ m})(F\Delta t_A - F\Delta t_B) + 156.06 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$

$$\text{OR } F\Delta t_A + F\Delta t_B = -66.302 \text{ N}\cdot\text{s} \quad (3)$$

$$F\Delta t_A - F\Delta t_B = -260.1 \text{ N}\cdot\text{s} \quad (4)$$

SOLVING (3) AND (4) SIMULTANEOUSLY:

$$F\Delta t_A = -163.20 \text{ N}\cdot\text{s} \quad F\Delta t_B = 96.90 \text{ N}\cdot\text{s}$$

THE FACT THAT  $F\Delta t_A < 0$  INDICATES THAT THE DIAGONALLY OPPOSITE THRUSTER SHOULD BE USED INSTEAD OF A. THUS

(a) THRUSTERS B AND C

$$(b) F\Delta t_B = 96.90 \text{ N}\cdot\text{s}, \quad \Delta t_B = \frac{96.90 \text{ N}\cdot\text{s}}{20 \text{ N}} = 4.845 \text{ s}$$

$$F\Delta t_C = 163.20 \text{ N}\cdot\text{s}, \quad \Delta t_C = \frac{163.20 \text{ N}\cdot\text{s}}{20 \text{ N}} = 8.16 \text{ s}$$

(c) IF THE VELOCITY  $\vec{v}$  OF THE MASS CENTER IS TO BE UNCHANGED, THE RESULTANT OF THE LINEAR IMPULSES MUST BE ZERO.

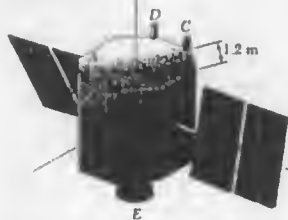
$$-(F\Delta t_B)\hat{j} - (F\Delta t_C)\hat{j} + (500 \text{ N})\Delta t_E\hat{j} = 0$$

$$-96.90 \text{ N}\cdot\text{s} - 163.20 \text{ N}\cdot\text{s} + (500 \text{ N})\Delta t_E = 0$$

$$\Delta t_E = \frac{260.1 \text{ N}\cdot\text{s}}{500 \text{ N}} = 0.5202 \text{ s}$$

$$\Delta t_E = 0.520 \text{ s}$$

18.34



**GIVEN:** PROBE WITH  
 $m = 2500 \text{ kg}$ ,  $k_x = 0.98 \text{ m}$ ,  
 $k_y = 1.06 \text{ m}$ ,  $k_z = 1.02 \text{ m}$ ;  
 500-N MAIN THRUSTER E;  
 20-N THRUSTERS A, B, C, D  
 CAN EXPEL FUEL IN  $\pm$  DIR.  
 PROBE HAS ANG. VELOCITY  
 $\underline{\omega} = (0.060 \text{ rad/s})\underline{i}$   
 $-(0.040 \text{ rad/s})\underline{k}$

**FIND:**

- (a) WHICH TWO THRUSTERS SHOULD BE USED TO REDUCE  $\underline{\omega}$  TO  $\underline{0}$ ?  
 (b) OPERATING TIME OF THESE THRUSTERS.  
 (c) HOW LONG SHOULD E BE ACTIVATED IF  $\underline{v}$  IS TO BE UNCHANGED?

**INITIAL ANGULAR MOMENTUM**

$$\begin{aligned} \underline{H}_G &= \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k} = m(k_x^2 \omega_x \underline{i} + k_y^2 \omega_y \underline{j} + k_z^2 \omega_z \underline{k}) \\ &= (2500 \text{ kg})[(0.98 \text{ m})^2(0.060 \text{ rad/s})\underline{i} + 0 + (1.02 \text{ m})^2(-0.040 \text{ rad/s})\underline{k}] \\ &= (144.06 \text{ kg}\cdot\text{m}^2/\text{s})\underline{i} - (104.04 \text{ kg}\cdot\text{m}^2/\text{s})\underline{k} \end{aligned} \quad (1)$$

**ANGULAR IMPULSE OF TWO 20-N THRUSTERS**

SEE SOLUTION OF PROB. 18.33. ASSUMING THAT THRUSTERS A AND B ARE USED, WE FOUND

**ANG. IMPULSE ABOUT G**

$$= (0.6 \text{ m})(F\Delta t_A - F\Delta t_B)\underline{k} + (1.44853 \text{ m})(F\Delta t_A + F\Delta t_B)\underline{i} \quad (2)$$

**IMPULSE-MOMENTUM PRINCIPLE**

SINCE THE FINAL ANG. VELOCITY AND, THUS, THE FINAL ANG. MOMENTUM MUST BE ZERO, THE SUM OF (1) AND (2) MUST BE ZERO. EQUATING THE COEFF. OF  $\underline{i}$  AND  $\underline{k}$  TO ZERO

$$(1.44853 \text{ m})(F\Delta t_A + F\Delta t_B) + 144.06 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$

$$(0.6 \text{ m})(F\Delta t_A - F\Delta t_B) - 104.04 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$

$$\text{OR } F\Delta t_A + F\Delta t_B = -99.453 \text{ N}\cdot\text{s} \quad (3)$$

$$F\Delta t_A - F\Delta t_B = 173.40 \text{ N}\cdot\text{s} \quad (4)$$

SOLVING (3) AND (4) SIMULTANEOUSLY

$$F\Delta t_A = 36.974 \text{ N}\cdot\text{s} \quad F\Delta t_B = -136.43 \text{ N}\cdot\text{s}$$

THE FACT THAT  $F\Delta t_B < 0$  INDICATES THAT THE THRUSTER D, WHICH IS DIAGONALLY OPPOSITE TO B SHOULD BE USED INSTEAD OF B. THUS:

**(c) THRUSTERS A AND D**

$$(b) F\Delta t_A = 36.974 \text{ N}\cdot\text{s}, \quad \Delta t_A = \frac{36.974 \text{ N}\cdot\text{s}}{20 \text{ N}} = 1.8487 \text{ s}$$

$$F\Delta t_D = 136.43 \text{ N}\cdot\text{s}, \quad \Delta t_D = \frac{136.43 \text{ N}\cdot\text{s}}{20 \text{ N}} = 6.8215 \text{ s}$$

$$\Delta t_A = 1.849 \text{ s}; \quad \Delta t_D = 6.82 \text{ s}$$

(c) IF THE VELOCITY  $\underline{v}$  OF THE PROBE IS TO BE UNCHANGED, THE RESULTANT OF THE LINEAR IMPULSES MUST BE ZERO.

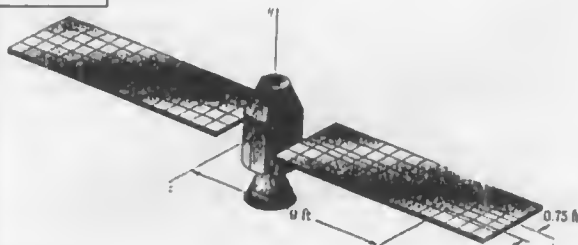
$$-(F\Delta t_A)\underline{j} - (F\Delta t_D)\underline{j} + (500 \text{ N})\Delta t_E \underline{j} = 0$$

$$-36.974 \text{ N}\cdot\text{s} - 136.43 \text{ N}\cdot\text{s} + (500 \text{ N})\Delta t_E = 0$$

$$\Delta t_E = \frac{173.40 \text{ N}\cdot\text{s}}{500 \text{ N}} = 0.3468 \text{ s}$$

$$\Delta t_E = 0.347 \text{ s}$$

18.35

**GIVEN:**

PROBE WITH PRINCIPAL CENTROIDAL AXES  $x, y, z$ , AND  
 $W = 3000 \text{ lb}$ ,  $k_x = 1.375 \text{ ft}$ ,  $k_y = 1.425 \text{ ft}$ ,  $k_z = 1.250 \text{ ft}$ .  
 PROBE HAS NO ANG. VELOCITY WHEN STRUCK AT A BY 5-oz  
 METEORITE WITH VELOCITY RELATIVE TO PROBE

$$\underline{v}_0 = (2400 \text{ ft/s})\underline{i} - (3000 \text{ ft/s})\underline{j} + (3200 \text{ ft/s})\underline{k}$$

METEORITE EMERGES ON OTHER SIDE OF PANEL MOVING  
 IN SAME DIRECTION WITH SPEED REDUCED BY 20%

**FIND: FINAL ANGULAR VELOCITY OF PROBE.****ANGULAR MOMENTUM OF METEORITE ABOUT G.**

$$\begin{aligned} (\underline{H}_G)_M &= \underline{r}_A \times m_M \underline{v}_0 \\ &= [(9 \text{ ft})\underline{i} + (0.75 \text{ ft})\underline{k}] \times \frac{(5/16) \text{ lb}}{32.2 \text{ ft/s}^2} [(2400 \text{ ft/s})\underline{i} - 3000 \underline{j} + 3200 \underline{k}] \\ &= (9.705 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}) (-27 \underline{k} - 28.8 \underline{j} + 1.8 \underline{j} + 2.25 \underline{i}) \times 10^3 \text{ ft/s} \\ &= (9.705 \text{ lb}\cdot\text{ft}\cdot\text{s}) (2.25 \underline{i} - 27 \underline{j} - 27 \underline{k}) \\ (\underline{H}_G)_M &= (21.836 \text{ lb}\cdot\text{ft}\cdot\text{s}) (\underline{i} - 12 \underline{j} - 12 \underline{k}) \end{aligned} \quad (1)$$

**FINAL ANGULAR MOMENTUM OF PROBE**

$$\begin{aligned} (\underline{H}_G)_P &= \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k} = m(k_x^2 \omega_x \underline{i} + k_y^2 \omega_y \underline{j} + k_z^2 \omega_z \underline{k}) \\ &= \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} [(1.375 \text{ ft})^2 \omega_x \underline{i} + (1.425 \text{ ft})^2 \omega_y \underline{j} + (1.250 \text{ ft})^2 \omega_z \underline{k}] \\ &= (176.15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2) \omega_x \underline{i} + (189.19 \text{ lb}\cdot\text{ft}\cdot\text{s}^2) \omega_y \underline{j} + (145.57 \text{ lb}\cdot\text{ft}\cdot\text{s}^2) \omega_z \underline{k} \end{aligned} \quad (2)$$

WE EXPRESS THAT  $(\underline{H}_G)_P = 0.20 (\underline{H}_G)_M$

RECALLING (1) AND (2):

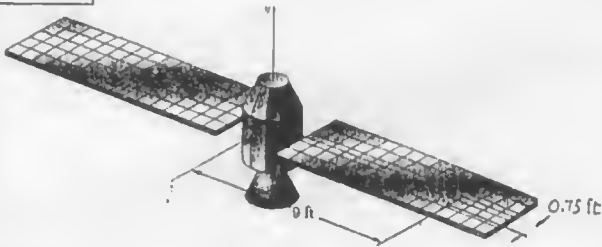
$$\begin{aligned} 176.15 \omega_x \underline{i} + 189.19 \omega_y \underline{j} + 145.57 \omega_z \underline{k} &= \\ &= 0.20 (21.836) (\underline{i} - 12 \underline{j} - 12 \underline{k}) \\ &= 4.3672 (\underline{i} - 12 \underline{j} - 12 \underline{k}) \end{aligned}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$\begin{aligned} 176.15 \omega_x &= 4.367 & \omega_x &= 0.02479 \text{ rad/s} \\ 189.19 \omega_y &= -52.406 & \omega_y &= -0.2770 \text{ rad/s} \\ 145.57 \omega_z &= -52.406 & \omega_z &= -0.3600 \text{ rad/s} \end{aligned}$$

$$\underline{\omega} = (0.0248 \text{ rad/s})\underline{i} - (0.277 \text{ rad/s})\underline{j} - (0.360 \text{ rad/s})\underline{k}$$

18.36



GIVEN:

PROBE WITH PRINCIPAL CENTROIDAL AXES  $x, y, z$ , AND  $W = 3000 \text{ lb}$ ,  $K_x = 1.375 \text{ ft}$ ,  $K_y = 1.425 \text{ ft}$ ,  $K_z = 1.250 \text{ ft}$ . PROBE HAS NO ANGULAR VELOCITY WHEN STRUCK AT A BY 5-oz METEORITE WHICH EMERGES ON OTHER SIDE OF PANEL MOVING IN SAME DIRECTION WITH SPEED REDUCED BY 25%.

FINAL ANGULAR VELOCITY OF PROBE IS  
 $\omega = (0.05 \text{ rad/s})\mathbf{i} - (0.12 \text{ rad/s})\mathbf{j} + \omega_z\mathbf{k}$   
 AND X COMPONENT OF CHANGE IN  $\mathbf{v}$  OF PROBE IS  $\Delta v_x = -0.675 \text{ in/s}$ .

FIND: (a)  $\omega_z$ .

(b) RELATIVE VELOCITY  $\mathbf{v}_0$  OF METEORITE WITH WHICH IT STRIKES PANEL.

CONSERVATION OF LINEAR MOMENTUM IN X DIRECTION  
 SINCE 25% OF LINEAR MOM. OF METEORITE IS TRANSFERRED TO PROBE:

$$0.25 \frac{(5/16) \text{ lb}}{g} (v_0)_x = \frac{3000 \text{ lb}}{g} \Delta v_x$$

$$(v_0)_x = 38.4 \times 10^3 \Delta v_x = 38.4 \times 10^3 (-0.675 \text{ in/s}) = 25.92 \times 10^3 \text{ in/s}$$

$$(v_0)_x = -2160 \text{ ft/s} \quad \triangleleft$$

CONSERVATION OF ANGULAR MOMENTUM ABOUT G  
 INITIAL ANG. MOM. OF METEORITE:

$$(H_G)_M = \mathbf{r}_A \times m_M \mathbf{v}_0 = [(9 \text{ ft})\mathbf{i} + (0.75 \text{ ft})\mathbf{k}] \times \frac{(5/16) \text{ lb}}{g} [(v_0)_x\mathbf{i} + (v_0)_y\mathbf{j} + (v_0)_z\mathbf{k}]$$

RECALLING THAT  $(v_0)_x = -2160 \text{ ft/s}$  AND USING DETERMINANT

$$(H_G)_M = \frac{(5/16) \text{ lb}}{32.2 \text{ ft/s}^2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & 0 & 0.75 \\ -2160 & (v_0)_y & (v_0)_z \end{vmatrix}$$

$$(H_G)_M = \frac{(5/16) \text{ lb}}{32.2 \text{ ft/s}^2} [-0.75(v_0)_y\mathbf{i} - (1620 + 9(v_0)_z)\mathbf{j} + 9(v_0)_z\mathbf{k}] \quad (1)$$

FINAL ANG. MOM. OF PROBE:

$$(H_G)_P = I_x \omega_x \mathbf{i} + I_y \omega_y \mathbf{j} + I_z \omega_z \mathbf{k} = m(K_x^2 \omega_x \mathbf{i} + K_y^2 \omega_y \mathbf{j} + K_z^2 \omega_z \mathbf{k})$$

$$= \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} [(1.375 \text{ ft})^2 (0.05 \text{ rad/s})\mathbf{i} - (1.425 \text{ ft})^2 (0.12 \text{ rad/s})\mathbf{j} + (1.250 \text{ ft})^2 \omega_z \mathbf{k}] \quad (2)$$

SINCE 25% OF ANGULAR MOM. OF METEORITE IS TRANSFERRED TO PROBE,  $(H_G)_P = 0.25(H_G)_M$  OR, RECALLING (1) AND (2):

$$3000 [(1.375)^2 (0.05)\mathbf{i} - (1.425)^2 (0.12)\mathbf{j} + (1.250)^2 \omega_z \mathbf{k}]$$

$$= 0.25 (5/16) [-0.75(v_0)_y\mathbf{i} - (1620 + 9(v_0)_z)\mathbf{j} + 9(v_0)_z\mathbf{k}]$$

EQUATE THE COEFF. OF UNIT VECTORS:

$$\textcircled{1} 203.59 = -0.058594(v_0)_y \quad (v_0)_y = -4840 \text{ ft/s} \quad \triangleleft$$

$$\textcircled{2} -731.03 = -126.56 - 0.70313(v_0)_z \quad (v_0)_z = 859.7 \text{ ft/s} \quad \triangleleft$$

$$\textcircled{3} 4687.5 \omega_z = 0.70313(-4840) \quad \omega_z = -0.726 \text{ rad/s} \quad \triangleleft$$

ANSWERS:

$$(a) \quad \omega_z = -0.726 \text{ rad/s} \quad \triangleleft$$

$$(b) \quad \mathbf{v}_0 = -(2160 \text{ ft/s})\mathbf{i} - (4840 \text{ ft/s})\mathbf{j} + (860 \text{ ft/s})\mathbf{k} \quad \triangleleft$$

18.37

GIVEN:

RIGID BODY WITH FIXED POINT O, ANG. VELOCITY  $\omega$ , ANGULAR MOMENTUM  $H_O$ , AND KINETIC ENERGY T.

SHOW THAT: (a)  $H_O \cdot \omega = 2T$ ,

(b)  $\theta < 90^\circ$ , WHERE  $\theta$  IS ANGLE BETWEEN  $\omega$  AND  $H_O$ .

(a) USING PRINCIPAL AXES AS COORDINATE AXES, WE WRITE

$$\begin{aligned} H_O \cdot \omega &= (H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k}) \cdot (\omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}) \\ &= H_x \omega_x + H_y \omega_y + H_z \omega_z \end{aligned} \quad (1)$$

SINCE  $x, y, z$  ARE PRINCIPAL AXES,

$$H_x = I_x \omega_x \quad H_y = I_y \omega_y \quad H_z = I_z \omega_z$$

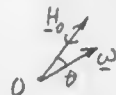
SUBSTITUTE INTO (1):

$$H_O \cdot \omega = I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 \quad (2)$$

BUT, FROM EQ. (18.20),  $T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$

WE CONCLUDE THAT

$$H_O \cdot \omega = 2T \quad (\text{Q.E.D.})$$



(b) WE CAN EXPRESS THE SCALAR PRODUCT AS

$$H_O \cdot \omega = H_O \omega \cos \theta$$

$$\text{THUS: } \cos \theta = \frac{H_O \cdot \omega}{H_O \omega} = \frac{2T}{H_O \omega} > 0, \text{ SINCE } T > 0$$

SINCE  $\cos \theta > 0$ , WE MUST HAVE  $\theta < 90^\circ$  (Q.E.D.)

18.38



GIVEN:

RIGID BODY WITH FIXED POINT O;  
 $\omega$  = INSTANTANEOUS ANG. VELOCITY;

$I_{OL}$  = MOMENT OF INERTIA OF BODY ABOUT LINE OF ACTION OL OF  $\omega$ .

SHOW THAT  $T = \frac{1}{2} I_{OL} \omega^2$

(a) USING EQS. (9.46) AND (18.19),

(b) CONSIDERING T AS THE SUM OF THE K.E. OF PARTICLES  $P_i$ .

(a) EQ. (18.19):

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 - 2I_{xy} \omega_x \omega_y - 2I_{yz} \omega_y \omega_z - 2I_{zx} \omega_z \omega_x)$$

$$\text{LET } \omega_x = \omega \cos \theta_x = \omega \lambda_x$$

$$\omega_y = \omega \cos \theta_y = \omega \lambda_y$$

$$\omega_z = \omega \cos \theta_z = \omega \lambda_z$$

SUBSTITUTE INTO EQ. (18.19):

$$T = \frac{1}{2} (I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x) \omega^2$$

BUT, BY EQ. (9.46) OF SEC. 9.16, EXPRESSION IN PARENTHESES IS  $I_{OL}$ . THUS:

$$T = \frac{1}{2} I_{OL} \omega^2 \quad (\text{Q.E.D.})$$

(b) EACH PARTICLE  $P_i$  DESCRIBES A CIRCLE OF RADIUS  $\rho_i$  CENTERED ON OL WITH A SPEED  $v_i = \rho_i \omega$  THEREFORE

$$T = \frac{1}{2} \sum (\Delta m_i) v_i^2 = \frac{1}{2} \sum (\Delta m_i) \rho_i^2 \omega^2$$

$$= \frac{1}{2} (\sum \rho_i^2 \Delta m_i) \omega^2$$

$$\text{BUT } \sum \rho_i^2 \Delta m_i = I_{OL}$$

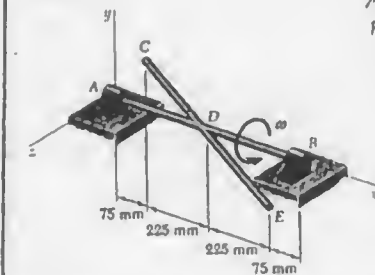
THEREFORE:

$$T = \frac{1}{2} I_{OL} \omega^2 \quad (\text{Q.E.D.})$$

18.39

GIVEN: ASSEMBLY OF PROB. 18.1. FOR EACH ROD:

$m = 1.5 \text{ kg}$   
 LENGTH = 600 mm  
 ASSEMBLY ROTATES WITH  $\omega = 12 \text{ rad/s}$ .  
 FIND: KINETIC ENERGY OF ASSEMBLY.



USING PRINCIPAL AXES  $x'y'z'$ :

$$\cos \theta = \frac{225}{300} \quad \theta = 41.41^\circ$$

$$\omega_x = \omega \cos \theta \quad \omega_y = \omega \sin \theta \quad \omega_z = 0$$

$$\bar{I}_{x'} = 0, \quad \bar{I}_{y'} = \frac{1}{12} m \ell^2, \quad \bar{I}_{z'} = \frac{1}{12} m \ell^2$$

EQ. (18.20):  $T = \frac{1}{2} (\bar{I}_{x'} \omega_x^2 + \bar{I}_{y'} \omega_y^2 + \bar{I}_{z'} \omega_z^2)$

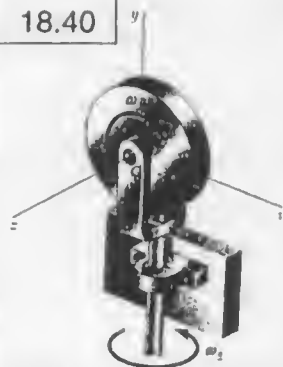
$$T = \frac{1}{2} (0 + \frac{1}{12} m \ell^2 \omega^2 \sin^2 \theta + 0)$$

$$= \frac{1}{24} (1.5 \text{ kg}) (0.6 \text{ m})^2 (12 \text{ rad/s})^2 \sin^2 41.41^\circ$$

$$T = 1.417 \text{ J}$$

18.40

GIVEN: DISK OF PROB. 18.2 OF MASS  $m$  AND RADIUS  $R$  ROTATING AS SHOWN.  
 FIND: KINETIC ENERGY OF DISK.



EQ. (18.20):

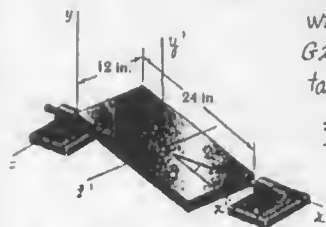
$$T = \frac{1}{2} (\bar{I}_x \omega_x^2 + \bar{I}_y \omega_y^2 + \bar{I}_z \omega_z^2)$$

$$= \frac{1}{2} (0 + \frac{1}{4} m R^2 \omega^2 \sin^2 \theta + \frac{1}{2} m R^2 \omega^2 \cos^2 \theta)$$

$$T = \frac{1}{8} m R^2 (\omega^2 \sin^2 \theta + 2 \omega^2 \cos^2 \theta)$$

18.41

GIVEN: 18.1b RECTANGULAR PLATE OF PROB. 18.3 ROTATING WITH  $\omega = 5 \text{ rad/s}$  ABOUT  $x$  AXIS.  
 FIND: KINETIC ENERGY OF PLATE



WE USE PRINCIPAL CENTROIDAL AXES  $x'y'z'$  WITH

$$\tan \theta = \frac{12 \text{ in.}}{24 \text{ in.}} = 0.5 \quad \theta = 26.565^\circ$$

$$\bar{I}_{x'} = \frac{1}{12} \frac{18 \text{ lb}}{g} (1 \text{ ft})^2 = \frac{1.5}{g}$$

$$\bar{I}_{z'} = \frac{1}{12} \frac{18 \text{ lb}}{g} (2 \text{ ft})^2 = \frac{6}{g}$$

EQ. (18.20):  $T = \frac{1}{2} (\bar{I}_{x'} \omega_x^2 + \bar{I}_{y'} \omega_y^2 + \bar{I}_{z'} \omega_z^2)$

$$T = \frac{1}{2} \left[ \frac{1.5}{g} (5 \text{ rad/s})^2 \cos^2 26.565^\circ + 0 + \frac{6}{g} (5 \text{ rad/s})^2 \sin^2 26.565^\circ \right]$$

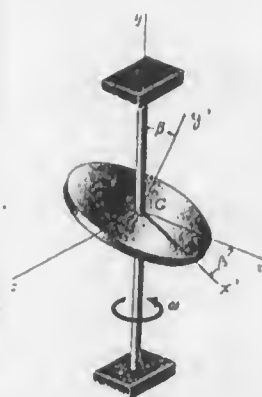
$$= \frac{1}{2} \left[ \frac{1.5}{32.2} (5)^2 (\cos^2 26.565^\circ + 4 \sin^2 26.565^\circ) \right]$$

$$= (0.58230 \text{ ft} \cdot \text{lb}) (0.8 + 4 \times 0.2) = 0.9317 \text{ ft} \cdot \text{lb}$$

$$T = 0.932 \text{ ft} \cdot \text{lb}$$

18.42

GIVEN: DISK OF PROB. 18.4. WITH  $\rho = 25^\circ$ .  
 FIND: KINETIC ENERGY OF DISK.



WE RESOLVE  $\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$  ALONG THE PRINCIPAL CENTROIDAL AXES  $Gx'y'z'$ :

$$\omega_x = -\omega \sin \rho, \quad \omega_y = \omega \cos \rho, \quad \omega_z = 0$$

EQ. (18.10):

$$T = \frac{1}{2} (\bar{I}_x \omega_x^2 + \bar{I}_y \omega_y^2 + \bar{I}_z \omega_z^2)$$

$$= \frac{1}{2} \left( \frac{1}{4} m R^2 \omega^2 \sin^2 \rho + \frac{1}{2} m R^2 \omega^2 \cos^2 \rho + 0 \right)$$

$$= \frac{1}{4} m R^2 \omega^2 (\sin^2 \rho + 2 \cos^2 \rho)$$

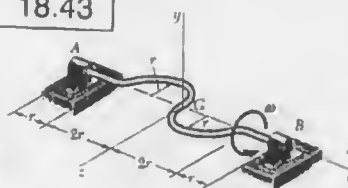
$$= \frac{1}{8} m R^2 \omega^2 (1 + \cos^2 \rho)$$

$$= \frac{1}{8} m R^2 \omega^2 (1 + \cos^2 25^\circ)$$

$$T = 0.228 m R^2 \omega^2$$

18.43

GIVEN: SHAFT OF PROB. 18.15 OF MASS  $m$ , ROTATING WITH ANG. VEL.  $\omega$ .  
 FIND: KINETIC ENERGY OF SHAFT.



$$\text{MASS PER UNIT LENGTH} = m' = \frac{m}{2R + 2\pi R} = \frac{m}{2R(\pi + 1)}$$

SINCE  $\omega_x = \omega_y = 0$ , EQ. (18.19) REDUCES TO  $T = \frac{1}{2} \bar{I}_z \omega_z^2$ . BUT  $\bar{I}_z$  OF BOTH SEMICIRCULAR PORTIONS OF SHAFT IS SAME AS OF FULL CIRCULAR SHAFT, THAT IS,

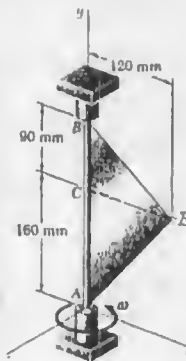
$$\bar{I}_z = \frac{1}{2} (2\pi R m') R^2 = \pi R^3 m' = \pi R^3 \frac{m}{2R(\pi + 1)} = \frac{\pi R^2 m}{2(\pi + 1)}$$

THEREFORE,  $T = \frac{1}{2} \frac{\pi R^2 m}{2(\pi + 1)} \omega^2 = \frac{\pi}{4(\pi + 1)} m R^2 \omega^2$

$$T = 0.1896 m R^2 \omega^2$$

18.44

GIVEN: TRIANGULAR PLATE OF PROB. 18.16 OF MASS  $m = 7.5 \text{ kg}$  WITH ANG. VEL.  $\omega = 12 \text{ rad/s}$ .  
 FIND: KINETIC ENERGY OF PLATE



SINCE  $\omega_x = \omega_y = 0$ , EQ. (18.19) REDUCES TO

$$T = \frac{1}{2} \bar{I}_y \omega^2 \quad (1)$$

BUT  $\bar{I}_y = \frac{1}{12} m h^3$

AND  $\bar{I}_y = \frac{m}{12} \left( \frac{1}{2} b h^3 \right) = \frac{1}{6} m h^3$

WHERE  $m = 7.5 \text{ kg}$ ,  $h = CB = (0.12 \text{ m})$

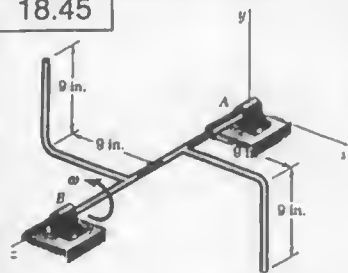
THUS  $\bar{I}_y = \frac{1}{6} (7.5 \text{ kg}) (0.12 \text{ m})^3 = 18.00 \times 10^{-3} \text{ kg} \cdot \text{m}^2$

SUBSTITUTING THIS VALUE FOR  $\bar{I}_y$  AND  $12 \text{ rad/s}$  FOR  $\omega$  INTO (1), WE HAVE

$$T = \frac{1}{2} (18.00 \times 10^{-3} \text{ kg} \cdot \text{m}^2) (12 \text{ rad/s})^2$$

$$T = 1.296 \text{ J}$$

18.45



GIVEN:

ASSEMBLY OF PROB. 18.19  
WHICH ROTATES AT 360 rpm.  
EACH L-SHAPED ARM  
WEIGHS 5 lb.

FIND:

KINETIC ENERGY OF  
ASSEMBLY.

SINCE  $\omega_x = \omega_y = 0$ , EQ. (18.19) REDUCES TO

$$T = \frac{1}{2} I_z \omega_z^2$$

FOR ONE ARM (OF MASS  $m$ ):

$$I_z = (\bar{I}_z) + \frac{m}{2} d^2 + (\bar{I}_z)$$

$$= \frac{1}{12} m a^2 + \frac{m}{2} (a^2 + \frac{a^2}{4}) + \frac{1}{3} m \frac{a^2}{2} = \frac{5}{6} m a^2$$

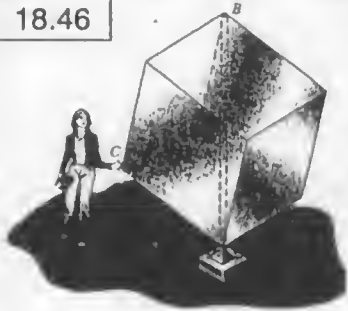
FOR BOTH ARMS:  $I_z = \frac{5}{3} m a^2 = \frac{5}{3} \frac{5 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{3}{4} \text{ ft})^2$

$$= 0.14557 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

AND  $\omega = 360 \frac{\text{rev}}{\text{min}} = 360 \frac{2\pi \text{ rad}}{60 \text{ s}} = 12\pi \text{ rad/s}$

THUS:  $T = \frac{1}{2} I_z \omega^2 = \frac{1}{2} (0.14557 \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (12\pi \text{ rad/s})^2$   
 $T = 103.5 \text{ ft} \cdot \text{lb}$

18.46



GIVE:

HOLLOW 5X5 FT ALUMINUM  
CUBE OF PROB. 18.21.  
STUFF IT WITH COTTON  
C FOR 1.25 IN DIRECTION  
PERPENDICULAR TO PLANE  
ABC WITH 12.5-LB FORCE,  
CAUSING CUBE TO COMPLETE  
1 REV IN 5 S.

FIND: KINETIC ENERGY  
IMPARTED TO CUBE.

DIRECT COMPUTATION OF K.E.

WE HAVE  $\omega = (2\pi \text{ rad})/5 \text{ s} = 1.2566 \text{ rad/s}$

WE RECALL FROM PROB. 18.21 THAT AB IS A PRINCIPAL  
AXIS AND THAT  $I_{AB} = \frac{5}{18} m a^2$  THUS, EQ. (18.14) YIELDS

$$T = \frac{1}{2} I_{AB} \omega^2 = \frac{1}{2} \frac{5}{18} m a^2 \omega^2 = \frac{5}{36} m (5 \text{ ft})^2 (1.2566 \text{ rad/s})^2$$

BUT WE FOUND IN PROB. 18.21 THAT  $W = 226 \text{ lb}$

THUS:  $T = \frac{5}{36} \frac{226 \text{ lb}}{32.2 \text{ ft/s}^2} (5 \text{ ft})^2 (1.2566 \text{ rad/s})^2 = 38.48 \text{ ft} \cdot \text{lb}$

$$T = 38.5 \text{ ft} \cdot \text{lb}$$

ALTERNATIVE SOLUTION

WE NOTE THAT THE K.E. IMPARTED TO THE CUBE IS  
EQUAL TO THE WORK  $U_{1 \rightarrow 2}$  DONE BY THE TANGENT

$$T = U_{1 \rightarrow 2} = F \Delta S$$

WHERE  $F = 12.5 \text{ lb}$  AND  $\Delta S = \frac{1}{2} v \Delta t = \frac{1}{2} \omega b \Delta t$   
 RECALLING THAT THE RADIUS  $b$  OF THE CIRCLE DESCRIBED  
 BY C IS (SEE HINT IN PROB. 18.21)

$$b = a \sqrt{2/3} = (5 \text{ ft}) \sqrt{2/3} = 4.0825 \text{ ft}$$

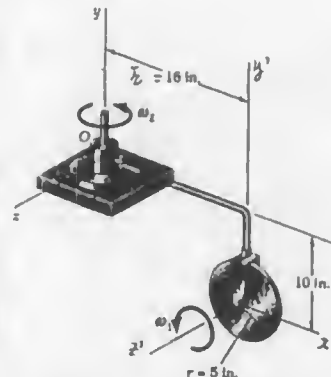
WE HAVE  $\Delta S = \frac{1}{2} (1.2566 \text{ rad/s}) (4.0825 \text{ ft}) (1.2 \text{ s}) = 3.078 \text{ ft}$

AND  $T = (12.5 \text{ lb}) (3.078 \text{ ft}) = 38.48 \text{ ft} \cdot \text{lb}$ ,  $T = 38.5 \text{ ft} \cdot \text{lb}$

18.47

GIVEN:

DISK OF PROB. 18.5 WITH WEIGHT  $W = 8 \text{ lb}$ ,  
AND ANGULAR VELOCITIES  $\omega_1 = 12 \text{ rad/s}$  AND  $\omega_2 = 4 \text{ rad/s}$ .  
FIND: KINETIC ENERGY OF DISK.



EQ. (18.17):

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} (\bar{I}_x \omega_1^2 + \bar{I}_y \omega_2^2 + \bar{I}_z \omega_3^2)$$

$$= \frac{1}{2} m \omega_1^2 \bar{r}^2 +$$

$$+ \frac{1}{2} (0 + \frac{1}{4} m \bar{r}^2 \omega_2^2 + \frac{1}{2} m \bar{r}^2 \omega_1^2)$$

$$= \frac{1}{2} \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} [(4 \text{ rad/s})^2 (\frac{16 \text{ ft}}{12})^2$$

$$+ \frac{1}{4} (\frac{5 \text{ ft}}{12})^2 (4 \text{ rad/s})^2 + \frac{1}{2} (\frac{5 \text{ ft}}{12})^2 (12)^2]$$

$$= (0.12422) [20.444 +$$

$$+ 0.6944 + 12.5]$$

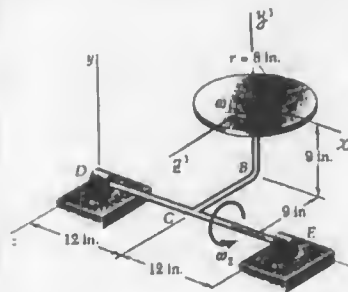
$$= 5.1724 \text{ ft} \cdot \text{lb}$$

$$T = 5.17 \text{ ft} \cdot \text{lb}$$

18.48

GIVEN:

DISK OF PROB. 18.6 WITH WEIGHT  $W = 6 \text{ lb}$   
AND ANGULAR VELOCITIES  $\omega_1 = 16 \text{ rad/s}$  AND  $\omega_2 = 8 \text{ rad/s}$ .  
FIND: KINETIC ENERGY OF DISK.



EQ. (18.17):

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} (\bar{I}_x \omega_1^2 + \bar{I}_y \omega_2^2 + \bar{I}_z \omega_3^2)$$

$$\text{WHERE } \bar{v}^2 = \omega_1^2 (AC)^2$$

$$\text{WITH } (AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = 2 (\frac{9 \text{ ft}}{12})^2 = 1.125$$

$$\bar{I}_x = \frac{1}{4} m \bar{r}^2 = \frac{1}{4} m (\frac{8 \text{ ft}}{12})^2$$

$$= 0.1111 m$$

$$\bar{I}_y = \frac{1}{2} m \bar{r}^2 = 0.2222 m$$

THUS:  $T = \frac{1}{2} (m \bar{v}^2 + \bar{I}_x \omega_1^2 + \bar{I}_y \omega_2^2 + 0)$

$$= \frac{1}{2} m [1.125 \omega_1^2 + 0.1111 \omega_2^2 + 0.2222 \omega_1^2]$$

$$= \frac{1}{2} \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} [1.236 (8 \text{ rad/s})^2 + 0.222 (16 \text{ rad/s})^2]$$

$$T = 12.67 \text{ ft} \cdot \text{lb}$$

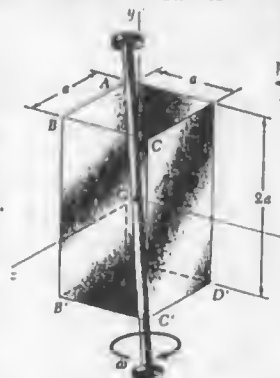
18.49 and 18.50

GIVEN: PARALLELEPIPED OF

18.49: PROB. 18.7 (SOLID)

18.50: PROB. 18.8 (HOLLOW)

FIND: KINETIC ENERGY



SINCE G IS FIXED AND  $x, y, z$   
ARE PRINCIPAL AXES, USE (18.20):

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$$

$$\text{WITH } \omega_x = -\frac{a}{\sqrt{a^2 + b^2 + c^2}} \omega$$

$$\omega_x = -\frac{a}{\sqrt{6}} \omega, \omega_y = \frac{2a}{\sqrt{6}} \omega, \omega_z = -\frac{a}{\sqrt{6}} \omega$$

$$\text{THUS: } T = \frac{1}{12} (I_x + 4I_y + I_z) \omega^2 \quad (1)$$

18.49 WE HAVE  $I_x = I_y = \frac{1}{12} m [a^2 + (2a)^2] = \frac{5}{12} m a^2$ ,  $I_z = \frac{1}{6} m a^2$   
 SUBSTITUTE IN (1):

$$T = \frac{1}{12} m a^2 (\frac{5}{12} + \frac{4}{6} + \frac{5}{12}) \omega^2 = \frac{1}{12} m a^2 (\frac{3}{2}) \omega^2 \quad T = \frac{1}{8} m a^2 \omega^2$$

(CONTINUED)

# 18.49 and 18.50 continued

WE RECALL FROM THE PREVIOUS PAGE

$$T = \frac{1}{12} (I_x + 4I_y + I_z) \omega^2 \quad (1)$$

18.50: SEE SOLUTION OF PROB. 18.8 FOR THE DETERMINATION OF THE PRINCIPAL MOMENTS OF INERTIA:

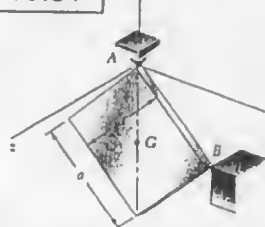
$$I_x = \frac{37}{60} ma^2 \quad I_y = \frac{9}{30} ma^2 \quad I_z = \frac{37}{60} ma^2$$

SUBSTITUTE IN EQ. (1):

$$T = \frac{1}{12} ma^2 \left( \frac{37}{60} + \frac{4 \times 9}{30} + \frac{37}{60} \right) \omega^2 = \frac{146}{720} ma^2 \omega^2$$

$$T = 0.203 ma^2 \omega^2$$

## 18.51



GIVEN:

SQUARE PLATE OF PROB. 18.29 OF MASS  $m$  WITH  $\vec{v}_0$  STRIKES  $B$  WITH  $e = 0$

FIND:

KINETIC ENERGY LOST IN IMPACT.

WE RECALL FROM PROB. 18.29 THAT  $\bar{I} = \frac{1}{12} ma^2$  ABOUT ANY AXIS THROUGH  $G$  IN THE PLANE OF THE PLATE. KINETIC ENERGY BEFORE IMPACT

$$T_0 = \frac{1}{2} \bar{I} \omega_0^2 = \frac{1}{2} \left( \frac{1}{12} ma^2 \right) \omega_0^2 = \frac{1}{24} ma^2 \omega_0^2$$

KINETIC ENERGY AFTER IMPACT

PLATE ROTATES ABOUT  $AB$ . WE FOUND IN PROB. 18.29 THAT  $\omega' = \frac{1}{\sqrt{2}} \omega_0$  AND  $\bar{v}' = \omega' (a/2)$

THEREFORE, FROM EQ. (18.17),

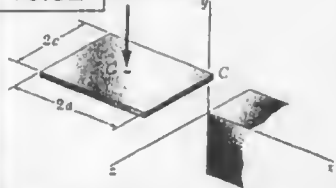
$$T = \frac{1}{2} m \bar{v}'^2 + \frac{1}{2} \bar{I} \omega'^2 = \frac{1}{2} m \omega'^2 \left( \frac{a}{2} \right)^2 + \frac{1}{2} \left( \frac{1}{12} ma^2 \right) \omega'^2$$

$$= \frac{1}{6} ma^2 \omega'^2 = \frac{1}{6} ma^2 \left( \frac{\omega_0}{\sqrt{2}} \right)^2 = \frac{1}{192} ma^2 \omega_0^2$$

KINETIC ENERGY LOST

$$= \frac{1}{24} ma^2 \omega_0^2 - \frac{1}{192} ma^2 \omega_0^2 = \frac{7}{192} ma^2 \omega_0^2$$

## 18.52



GIVEN:

RECTANGULAR PLATE OF PROB. 18.31 AND 18.32 OF MASS  $m$  FALLING WITH VELOCITY  $\vec{v}_0$  AND  $\omega_0 = 0$  HITS OBSTRUCTION ( $e = 0$ )

FIND: KINETIC ENERGY LOST IN IMPACT.

BEFORE IMPACT

$$T_0 = \frac{1}{2} m \bar{v}_0^2$$

AFTER IMPACT

FROM PROB. (18.31):  $\omega_x = 3\bar{v}_0/7c$ ,  $\omega_y = 0$ ,  $\omega_z = 3\bar{v}_0/7a$

FROM PROB. (18.32):  $\bar{v} = - (6\bar{v}_0/7) \hat{j}$

EQ. (18.17):

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} (\bar{I}_x \omega_x^2 + \bar{I}_y \omega_y^2 + \bar{I}_z \omega_z^2)$$

$$= \frac{1}{2} m \left( \frac{6}{7} \bar{v}_0 \right)^2 + \frac{1}{2} \left[ \frac{1}{3} m c^2 \left( \frac{3\bar{v}_0}{7c} \right)^2 + 0 + \frac{1}{3} m a^2 \left( \frac{3\bar{v}_0}{7a} \right)^2 \right]$$

$$= \frac{1}{2} m \bar{v}_0^2 \left( \frac{1}{7} \right) [36 + 3 + 3] = \frac{1}{2} m \bar{v}_0^2 \frac{42}{49} = \frac{1}{2} \frac{6}{7} m \bar{v}_0^2$$

KINETIC ENERGY LOST

$$T_0 - T = \frac{1}{2} m \bar{v}_0^2 \left( 1 - \frac{6}{7} \right)$$

$$T_0 - T = \frac{1}{14} m \bar{v}_0^2$$

## 18.53

GIVEN:

SPACE PROBE OF PROB. 18.35, WITH  $W = 3000 \text{ lb}$ ,  $k_x = 1.375 \text{ ft}$ ,  $k_y = 1.425 \text{ ft}$ ,  $k_z = 1.250 \text{ ft}$

FIND:

KINETIC ENERGY OF PROBE IN ITS MOTION ABOUT ITS MASS CENTER AFTER ITS COLLISION WITH METEORITE.

SEE SOLUTION OF PROB. 18.35 FOR DETERMINATION OF

$$\omega_x = 0.0248 \text{ rad/s}, \quad \omega_y = -0.277 \text{ rad/s}, \quad \omega_z = -0.360 \text{ rad/s}$$

IN MOTION ABOUT  $G$ ,  $G$  IS A FIXED POINT AND THE  $x, y, z$  AXES ARE PRINCIPAL AXES. WE USE EQ. (18.20):

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) = \frac{1}{2} m (k_x^2 \omega_x^2 + k_y^2 \omega_y^2 + k_z^2 \omega_z^2)$$

$$= \frac{1}{2} \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} [(1.375 \text{ ft} \times 0.0248 \text{ rad/s})^2 + (1.425 \text{ ft} \times 0.277 \text{ rad/s})^2 + (1.250 \text{ ft} \times 0.360 \text{ rad/s})^2]$$

$$= \frac{1}{2} \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} (0.3595 \text{ ft}^2/\text{s}^2) = 16.747 \text{ ft} \cdot \text{lb}$$

$$T = 16.75 \text{ ft} \cdot \text{lb}$$

## 18.54

GIVEN:

SPACE PROBE OF PROB. 18.36, WITH  $W = 3000 \text{ lb}$ ,  $k_x = 1.375 \text{ ft}$ ,  $k_y = 1.425 \text{ ft}$ ,  $k_z = 1.250 \text{ ft}$

FIND:

KINETIC ENERGY OF PROBE IN ITS MOTION ABOUT ITS MASS CENTER AFTER ITS COLLISION WITH METEORITE.

SEE STATEMENT AND SOLUTION OF PROB. 18.36 FOR THE VALUES OF  $\omega_x, \omega_y, \omega_z$  AFTER COLLISION:

$$\omega_x = 0.05 \text{ rad/s}, \quad \omega_y = -0.12 \text{ rad/s}, \quad \omega_z = -0.726 \text{ rad/s}$$

IN MOTION ABOUT  $G$ ,  $G$  IS A FIXED POINT AND THE  $x, y, z$  AXES ARE PRINCIPAL AXES. WE USE EQ. (18.20):

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) = \frac{1}{2} m (k_x^2 \omega_x^2 + k_y^2 \omega_y^2 + k_z^2 \omega_z^2)$$

$$= \frac{1}{2} \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} [(1.375 \text{ ft} \times 0.05 \text{ rad/s})^2 + (1.425 \text{ ft} \times 0.12 \text{ rad/s})^2 + (1.250 \text{ ft} \times 0.726 \text{ rad/s})^2]$$

$$= \frac{1}{2} \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2} (0.8575 \text{ ft}^2/\text{s}^2) = 39.946 \text{ ft} \cdot \text{lb}$$

$$T = 39.9 \text{ ft} \cdot \text{lb}$$

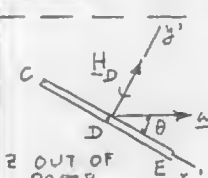
## 18.55

GIVEN: ASSEMBLY OF PROB. 18.1

FOR EACH ROD:  $m = 1.5 \text{ kg}$ ,  $\ell = 600 \text{ mm}$

ASSEMBLY ROTATES WITH  $\omega = 12 \text{ rad/s}$

FIND: RATE OF CHANGE  $\dot{H}_D$  OF ANG. MOMENTUM  $H_D$



FROM PROB. 18.1:  $\theta = 41.41^\circ$

USING PRINCIPAL AXES  $x', y', z'$ :

$$\omega = \omega (\cos \theta \hat{i}' + \sin \theta \hat{j}')$$

$$H_D = \frac{1}{2} m \ell^2 \omega \sin \theta \hat{j}'$$

EQ. (18.22) YIELDS

$$\dot{H}_D = (\dot{H}_D)_{Dx'y'z'} + \Omega \times H_D$$

BUT  $(\dot{H}_D)_{Dx'y'z'} = 0$  AND  $\Omega = \omega$ . THUS:

$$\dot{H}_D = \omega \times H_D = \omega (\cos \theta \hat{i}' + \sin \theta \hat{j}') \times \frac{1}{2} m \ell^2 \omega \sin \theta \hat{j}'$$

$$= \frac{1}{2} m \ell^2 \omega^2 \sin \theta \cos \theta \hat{k} = \frac{1}{24} m \ell^2 \omega^2 \sin 2\theta \hat{k}$$

WITH GIVEN DATA,

$$\dot{H}_D = \frac{1}{24} (1.5 \text{ kg}) (0.6 \text{ m})^2 (12 \text{ rad/s})^2 \sin 82.82^\circ \hat{k}$$

$$\dot{H}_D = (3.21 \text{ N} \cdot \text{m}) \hat{k}$$



18.56

GIVEN: DISK OF PROB. 18.2.

FIND: RATE OF CHANGE  $\dot{H}_G$  OF  $H_G$ .

FROM PROB. 18.2:

$$\omega = \omega_2 \hat{j} + \omega_1 \hat{k}$$

$$H_G = \frac{1}{4} m r^2 (\omega_2 \hat{j} + 2\omega_1 \hat{k})$$

WE NOTE THAT THE ANGULAR VELOCITY OF THE FRAME  $Gxyz$  IS

$$\Omega = \omega_2 \hat{j}$$

$$\text{EQ. (18.22): } \dot{H}_G = (\dot{H}_G)_{Gxyz} + \Omega \times H_G = 0 + \Omega \times H_G$$

$$\text{THUS: } \dot{H}_G = \omega_2 \hat{j} \times \frac{1}{4} m r^2 (\omega_2 \hat{j} + 2\omega_1 \hat{k})$$

$$\dot{H}_G = \frac{1}{2} m r^2 \omega_1 \omega_2 \hat{i}$$

18.57

GIVEN: PLATE OF PROB. 18.3 WEIGHING 18 lb, WHICH ROTATES WITH  $\omega = 5 \text{ rad/s}$ .FIND: RATE OF CHANGE  $\dot{H}_G$  OF  $H_G$ .

$$\text{WE HAVE } \omega = (5 \text{ rad/s}) \hat{i}$$

SEE SOLUTION OF PROB. 18.3 FOR THE DERIVATION OF EQ. (2):

$$H_G = (0.3727 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{i} - (0.2795 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{k}$$

EQ. (18.22):

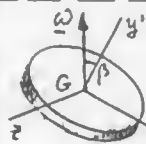
$$\dot{H}_G = (\dot{H}_G)_{xyz} + \Omega \times H_G = 0 + \omega \times H_G$$

$$\text{THUS: } \dot{H}_G = (5 \text{ rad/s}) \hat{i} \times [(0.3727 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{i} - (0.2795 \text{ lb} \cdot \text{ft} \cdot \text{s}) \hat{k}]$$

$$\dot{H}_G = (1.398 \text{ lb} \cdot \text{ft}) \hat{j}$$

18.58

GIVEN: DISK AND SHAFT OF PROB. 18.4.

FIND: RATE OF CHANGE  $\dot{H}_G$  OF  $H_G$ .USING THE PRINCIPAL AXES  $Gx'y'z'$ ,

WE FOUND IN PROB. 18.4 THAT

$$\omega = \omega (-\sin \beta \hat{i} + \cos \beta \hat{j})$$

$$H_G = \frac{1}{4} m r^2 \omega (-\sin \beta \hat{i} + 2 \cos \beta \hat{j})$$

$$\text{EQ. (18.22): } \dot{H}_G = (\dot{H}_G)_{Gx'y'z'} + \Omega \times H_G = 0 + \omega \times H_G$$

$$\dot{H}_G = \omega (-\sin \beta \hat{i} + \cos \beta \hat{j}) \times \frac{1}{4} m r^2 \omega (-\sin \beta \hat{i} + 2 \cos \beta \hat{j})$$

$$= \frac{1}{4} m r^2 \omega^2 (-2 \sin \beta \cos \beta \hat{k} + \cos^2 \beta \sin \beta \hat{i})$$

$$= -\frac{1}{8} m r^2 \omega^2 \sin 2\beta \hat{k} = -\frac{1}{8} m r^2 \omega^2 \sin 50^\circ$$

$$\dot{H}_G = -0.0958 m r^2 \omega^2 \hat{k}$$

18.59

GIVEN: DISK OF PROB. 18.5 WEIGHING 8 lb WITH  $\omega_1 = 12 \text{ rad/s}$  AND  $\omega_2 = 4 \text{ rad/s}$ .FIND: RATE OF CHANGE  $\dot{H}_A$  OF  $H_A$ .USING PRINCIPAL CENTROIDAL AXES  $Ax'y'z'$ :

$$\omega = \omega_2 \hat{j} + \omega_1 \hat{k} \quad \Omega = \omega_2 \hat{j}$$

$$H_A = \bar{I}_x \omega_1 \hat{i} + \bar{I}_y \omega_2 \hat{j} + \bar{I}_z \omega_1 \hat{k} = \frac{1}{4} m r^2 \omega_2 \hat{j} + \frac{1}{2} m r^2 \omega_1 \hat{k}$$

$$\text{EQ. (18.22): } \dot{H}_A = (\dot{H}_A)_{Ax'y'z'} + \Omega \times H_A = 0 + \omega_2 \hat{j} \times H_A$$

$$\dot{H}_A = \omega_2 \hat{j} \times (\frac{1}{4} m r^2 \omega_2 \hat{j} + \frac{1}{2} m r^2 \omega_1 \hat{k}) = \frac{1}{2} m r^2 \omega_1 \omega_2 \hat{i}$$

$$\text{WITH GIVEN DATA: } \dot{H}_A = \frac{1}{2} \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{5 \text{ ft}}{12})^2 (12 \text{ rad/s}) (4 \text{ rad/s}) \hat{i}$$

$$\dot{H}_A = (1.035 \text{ lb} \cdot \text{ft}) \hat{i}$$

18.60

GIVEN: DISK OF PROB. 18.6 WEIGHING 6 lb WITH  $\omega_1 = 16 \text{ rad/s}$  AND  $\omega_2 = 8 \text{ rad/s}$ .FIND: RATE OF CHANGE  $\dot{H}_A$  OF  $H_A$ .USING PRINCIPAL CENTROIDAL AXES  $Ax'y'z'$ :

$$\omega = \omega_2 \hat{i} + \omega_1 \hat{j} \quad \Omega = \omega_2 \hat{i}$$

$$H_A = \bar{I}_x \omega_2 \hat{i} + \bar{I}_y \omega_1 \hat{j} + \bar{I}_z \omega_2 \hat{i} = \frac{1}{4} m r^2 \omega_2 \hat{i} + \frac{1}{2} m r^2 \omega_1 \hat{j}$$

EQ. (18.22):

$$\dot{H}_A = (\dot{H}_A)_{Ax'y'z'} + \Omega \times H_A = 0 + \omega_2 \hat{i} \times H_A$$

$$= \omega_2 \hat{i} \times (\frac{1}{4} m r^2 \omega_2 \hat{i} + \frac{1}{2} m r^2 \omega_1 \hat{j}) = \frac{1}{2} m r^2 \omega_1 \omega_2 \hat{k}$$

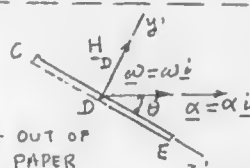
WITH GIVEN DATA:

$$\dot{H}_A = \frac{1}{2} \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{8 \text{ ft}}{12})^2 (16 \text{ rad/s}) (8 \text{ rad/s}) \hat{k}$$

$$\dot{H}_A = (5.30 \text{ lb} \cdot \text{ft}) \hat{k}$$

18.61

GIVEN: ASSEMBLY OF PROB. 18.1.

FOR EACH ROD:  $m = 1.5 \text{ kg}$ ,  $\ell = 600 \text{ mm}$ AT INSTANT CONSIDERED,  $\omega = (12 \text{ rad/s}) \hat{i}$ ,  $\alpha = (96 \text{ rad/s}^2) \hat{i}$ .FIND: RATE OF CHANGE  $\dot{H}_D$  OF  $H_D$ .FROM PROB. 18.1:  $\theta = 41.41^\circ$ USING PRINCIPAL AXES  $x'y'z'$ :

$$\omega = \omega (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$\alpha = \alpha (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$H_D = \frac{1}{12} m \ell^2 \omega \sin \theta \hat{j}$$

$$(\dot{H}_D)_{Dx'y'z'} = \frac{1}{12} m \ell^2 \alpha \sin \theta \hat{j} = \frac{1}{12} m \ell^2 \alpha \sin \theta \hat{j}$$

APPLY EQ. (18.22), OBSERVING THAT  $\Omega = \omega$ :

$$\dot{H}_D = (\dot{H}_D)_{Dx'y'z'} + \Omega \times H_D = (\dot{H}_D)_{Dx'y'z'} + \omega \times H_D$$

$$= \frac{1}{12} m \ell^2 \alpha \sin \theta \hat{j} + \omega (\cos \theta \hat{i} + \sin \theta \hat{j}) \times \frac{1}{12} m \ell^2 \omega \sin \theta \hat{j}$$

$$= \frac{1}{12} m \ell^2 \alpha \sin \theta \hat{j} + \frac{1}{12} m \ell^2 \omega^2 \cos \theta \sin \theta \hat{k}$$

$$\text{BUT } \hat{j} = \sin \theta \hat{i} + \cos \theta \hat{j}$$

THUS:

$$\dot{H}_D = \frac{1}{12} m \ell^2 \alpha \sin \theta (\sin \theta \hat{i} + \cos \theta \hat{j}) + \frac{1}{12} m \ell^2 \omega^2 \cos \theta \sin \theta \hat{k}$$

$$\dot{H}_D = \frac{1}{12} m \ell^2 \sin \theta (\alpha \sin \theta \hat{i} + \alpha \cos \theta \hat{j} + \omega^2 \cos \theta \hat{k}) \quad (1)$$

WITH GIVEN DATA:

$$m = 1.5 \text{ kg}, \ell = 0.6 \text{ m}, \omega = 12 \text{ rad/s}, \alpha = 96 \text{ rad/s}^2, \theta = 41.41^\circ$$

$$\dot{H}_D = \frac{1}{12} (1.5 \text{ kg}) (0.6 \text{ m})^2 \sin 41.41^\circ [(96 \text{ rad/s}^2) \sin 41.41^\circ \hat{i} + (96 \text{ rad/s}^2) \cos 41.41^\circ \hat{j} + (12 \text{ rad/s})^2 \cos 41.41^\circ \hat{k}]$$

$$\dot{H}_D = (1.890 \text{ N} \cdot \text{m}) \hat{i} + (2.14 \text{ N} \cdot \text{m}) \hat{j} + (3.21 \text{ N} \cdot \text{m}) \hat{k}$$

18.62

GIVEN: ASSEMBLY OF PROB. 18.1.

FOR EACH ROD:  $m = 1.5 \text{ kg}$ ,  $\ell = 600 \text{ mm}$ .AT INSTANT CONSIDERED,  $\omega = (12 \text{ rad/s})$ ,  $\alpha = -(96 \text{ rad/s}^2) \hat{i}$ .FIND: RATE OF CHANGE  $\dot{H}_D$  OF  $H_D$ .

SUBSTITUTE GIVEN DATA INTO EQ. (1) OF PROB. 18.61.

$$\dot{H}_D = \frac{1}{12} m \ell^2 \sin \theta (\alpha \sin \theta \hat{i} + \alpha \cos \theta \hat{j} + \omega^2 \cos \theta \hat{k}) \quad (1)$$

$$\dot{H}_D = \frac{1}{12} (1.5 \text{ kg}) (0.6 \text{ m})^2 \sin 41.41^\circ [-(96 \text{ rad/s}^2) \sin 41.41^\circ \hat{i} + (-96 \text{ rad/s}^2) \cos 41.41^\circ \hat{j} + (12 \text{ rad/s})^2 \cos 41.41^\circ \hat{k}]$$

$$\dot{H}_D = -(1.890 \text{ N} \cdot \text{m}) \hat{i} - (2.14 \text{ N} \cdot \text{m}) \hat{j} + (3.21 \text{ N} \cdot \text{m}) \hat{k}$$



18.63

GIVEN: AT INSTANT CONSIDERED, 18-1b PLATE OF PROB. 18.3 HAS  $\omega = (5 \text{ rad/s})\underline{i}$  AND  $\alpha = -(20 \text{ rad/s}^2)\underline{i}$ .

FIND: RATE OF CHANGE  $\dot{H}_G$  OF  $H_G$

SEE SOLUTION OF PROB. 18.3 FOR THE DERIVATION OF EQ. (1):

$$H_G = [(0.074534 \text{ lb} \cdot \text{ft} \cdot \text{s})\underline{i} - (0.055901 \text{ lb} \cdot \text{ft} \cdot \text{s})\underline{j}]\omega \quad (1)$$

SINCE  $\omega = \alpha$ , WE HAVE

$$(\dot{H}_G)_{xyz} = (0.074534 \underline{i} - 0.055901 \underline{j})\alpha$$

SINCE  $\Omega = \omega$ , EQ. (18.22) YIELDS

$$\begin{aligned} \dot{H}_G &= (\dot{H}_G)_{xyz} + \omega \times H_G \\ &= (0.074534 \underline{i} - 0.055901 \underline{j})\alpha \\ &\quad + \omega \underline{i} \times (0.074534 \underline{i} - 0.055901 \underline{j})\omega \\ &= 0.074534 \alpha \underline{i} - 0.055901 \alpha \underline{j} + 0.055901 \omega^2 \underline{j} \end{aligned}$$

LETTING  $\alpha = -20 \text{ rad/s}^2$  AND  $\omega = 5 \text{ rad/s}$ ,

$$\dot{H}_G = 0.074534(-20)\underline{i} + 0.055901(5)\underline{j} - 0.055901(-20)\underline{j}$$

$$\dot{H}_G = -(1.471 \text{ lb} \cdot \text{ft})\underline{i} + (1.398 \text{ lb} \cdot \text{ft})\underline{j} + (1.118 \text{ lb} \cdot \text{ft})\underline{j}$$

18.64

GIVEN: AT INSTANT CONSIDERED, SHAFT OF PROB. 18.4 HAS ANGLE OF PRECESSION

$\omega = \omega \underline{j}$  AND ANGLE OF PRECESSION  $\alpha = \alpha \underline{j}$

FIND: RATE OF CHANGE  $\dot{H}_G$  OF  $H_G$

SEE SOLUTION OF PROB. 18.4 FOR THE DETERMINATION OF  $H_G$ . USING THE PRINCIPAL CENTRIGUOUS AXES  $Gx'y'z$ , WE OBTAINED EQ. (1):

$$H_G = \frac{1}{4} m \omega^2 \omega (-\sin \beta \underline{i}' + 2 \cos \beta \underline{j}')$$

TO REVERT TO THE ORIGINAL AXES  $Gxyz$ , WE OBSERVE THAT

$$\begin{aligned} \underline{i}' &= \underline{i} \cos \beta - \underline{j} \sin \beta \\ \underline{j}' &= \underline{i} \sin \beta + \underline{j} \cos \beta \end{aligned}$$

SUBSTITUTING INTO (1):

$$\begin{aligned} H_G &= \frac{1}{4} m \omega^2 \omega [-\sin \beta (\underline{i} \cos \beta - \underline{j} \sin \beta) + 2 \cos \beta (\underline{i} \sin \beta + \underline{j} \cos \beta)] \\ &= \frac{1}{4} m \omega^2 \omega [\sin \beta \cos \beta \underline{i} + (1 + \cos^2 \beta) \underline{j}] \end{aligned}$$

SINCE  $\dot{\omega} = \alpha$

$$(\dot{H}_G)_{Gx'y'z} = \frac{1}{4} m \omega^2 \alpha [\sin \beta \cos \beta \underline{i} + (1 + \cos^2 \beta) \underline{j}]$$

WE USE EQ. (18.22) WITH  $\Omega = \omega = \omega \underline{j}$ :

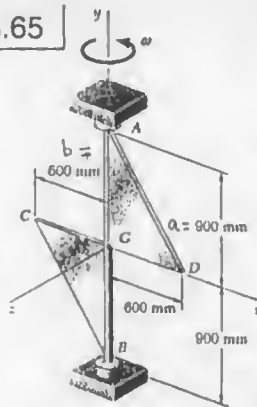
$$\begin{aligned} \dot{H}_G &= (\dot{H}_G)_{Gx'y'z} + \Omega \times H_G = \frac{1}{4} m \omega^2 \alpha [\sin \beta \cos \beta \underline{i} + (1 + \cos^2 \beta) \underline{j}] \\ &\quad + \omega \underline{j} \times \frac{1}{4} m \omega^2 \omega [\sin \beta \cos \beta \underline{i} + (1 + \cos^2 \beta) \underline{j}] \\ \dot{H}_G &= \frac{1}{4} m \omega^2 \alpha [\sin \beta \cos \beta \underline{i} + (1 + \cos^2 \beta) \underline{j}] - \frac{1}{4} m \omega^3 \sin \beta \cos \beta \underline{i} \end{aligned}$$

LETTING  $\beta = 25^\circ$ :

$$\dot{H}_G = \frac{1}{4} m \omega^2 \alpha (0.38302 \underline{i} + 1.8214 \underline{j}) - \frac{1}{4} m \omega^3 \sin 25^\circ (0.38302) \underline{i}$$

$$\dot{H}_G = m \omega^2 (0.0958 \alpha \underline{i} + 0.455 \alpha \underline{j} - 0.0958 \omega^2 \underline{i})$$

18.65



GIVEN:

ASSEMBLY CONSISTING OF TWO TRIANGULAR PLATES, EACH OF MASS  $m = 5 \text{ kg}$ , WELDED TO VERTICAL SHAFT. ASSEMBLY ROTATES WITH CONSTANT  $\omega = 8 \text{ rad/s}$ .

FIND:

DYNAMIC REACTIONS AT A AND B.

SINCE  $\omega = \omega \underline{j}$ ,

EQS. (18.7) YIELD

$$H_z = -I_{xz} \omega, H_y = I_{yz} \omega, H_x = -I_{xy} \omega \quad (1)$$

MOMENTS AND PRODUCTS OF INERTIA:

$$I_y = 2 \left( \frac{m}{A} I_{y, \text{AREA}} \right) = 2 \frac{m}{\frac{1}{2} ab} \left( \frac{1}{12} ab^3 \right) = \frac{1}{3} m b^2 \quad [\text{cf. front cover}]$$

$$I_{xz} = 2 \left( \frac{m}{A} I_{xz, \text{AREA}} \right) = 2 \frac{m}{\frac{1}{2} ab} \left( \frac{1}{24} a^2 b^2 \right) = \frac{1}{6} m ab \quad [\text{cf. Sample Prob. 9.6}]$$

$$I_{yz} = 0$$

$$\text{FROM EQ. (1): } H_G = -\frac{1}{6} m ab \omega \underline{i} + \frac{1}{3} m b^2 \omega \underline{j} \quad (2)$$

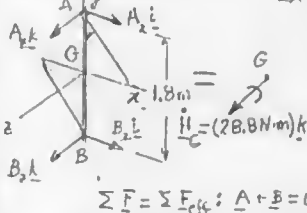
EQ. (18.22):

$$\dot{H}_G = (\dot{H}_G)_{xyz} + \Omega \times H_G = 0 + \omega \underline{j} \times H_G = \omega \underline{j} \times m \omega \left( -\frac{1}{6} ab \underline{i} + \frac{1}{3} b^2 \underline{j} \right)$$

$$\dot{H}_G = \frac{1}{6} m ab \omega^2 \underline{k} = \frac{1}{6} (5 \text{ kg})(0.9 \text{ m})(0.6 \text{ m})(8 \text{ rad/s})^2 = (28.8 \text{ N} \cdot \text{m}) \underline{k}$$

EQUATIONS OF MOTION:

WE EQUATE THE SYSTEMS OF EXTERNAL AND EFFECTIVE FORCES.



$$\Sigma \underline{M}_B = \Sigma (\underline{M}_B)_{\text{eff}}:$$

$$(1.8 \text{ m}) \underline{j} \times (A_z \underline{i} + A_x \underline{k}) = (28.8 \text{ N} \cdot \text{m}) \underline{k}$$

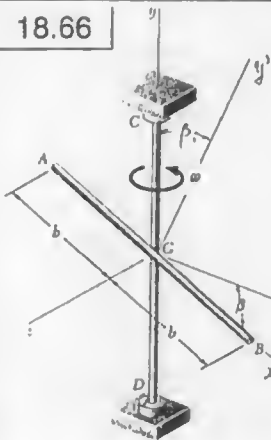
$$-1.8 A_x \underline{k} + 1.8 A_z \underline{i} = 28.8 \underline{k}$$

$$A_x = -16 \text{ N}, A_z = 0$$

$$\underline{A} = -(16.00 \text{ N}) \underline{i}$$

$$\underline{B} = (16.00 \text{ N}) \underline{i}$$

18.66



GIVEN:

ROD AB OF MASS  $m$  IS WELDED TO SHAFT CD, OF LENGTH  $2b$ , WHICH ROTATES AT CONSTANT RATE  $\omega$ .

FIND:

DYNAMIC REACTIONS AT C AND D.

USING THE PRINCIPAL AXES  $Gx'y'z$ :

$$I_x = 0, I_y = I_z = \frac{1}{3} m b^2$$

$$\omega_x = -\omega \sin \beta, \omega_y = \omega \cos \beta, \omega_z = 0$$

$$H_G = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

$$H_G = \frac{1}{3} m b^2 \omega \cos \beta \underline{j}$$

$$\text{OR, SINCE } \underline{j}' = \underline{i} \sin \beta + \underline{j} \cos \beta: H_G = \frac{1}{3} m b^2 \omega \cos \beta (\sin \beta \underline{i} + \cos \beta \underline{j}) \quad (1)$$

EQ. (18.22):

$$\dot{H}_G = (\dot{H}_G)_{xyz} + \Omega \times H_G = 0 + \omega \underline{j} \times H_G$$

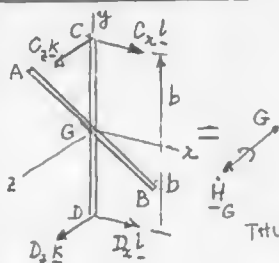
$$= \omega \underline{j} \times \frac{1}{3} m b^2 \omega \cos \beta (\sin \beta \underline{i} + \cos \beta \underline{j})$$

$$= -\frac{1}{3} m b^2 \omega^2 \sin \beta \cos \beta \underline{k}$$

(CONTINUED)

# 18.66 continued

## EQUATIONS OF MOTION



WE RECALL FROM PREVIOUS PAGE:

$$\dot{H}_G = -\frac{1}{3} m b^2 \omega^2 \sin \beta \cos \beta \hat{k}$$

WE EQUATE THE SYSTEMS OF EXTERNAL AND EFFECTIVE FORCES

$$\Sigma \vec{M}_D = \Sigma (\vec{M}_D)_{eff} :$$

$$2b \hat{j} \times (C_2 \hat{l} + C_2 \hat{k}) = \dot{H}_G$$

$$-2b C_2 \hat{k} + 2b C_2 \hat{i} =$$

$$-\frac{1}{3} m b^2 \omega^2 \sin \beta \cos \beta \hat{k}$$

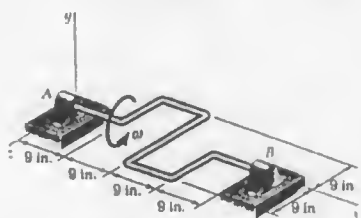
$$\text{THUS: } C_2 = \frac{1}{6} m b \omega^2 \sin \beta \cos \beta, \quad C_2 = 0$$

$$C_2 = \frac{1}{6} m b \omega^2 \sin \beta \cos \beta \hat{i}$$

$$D_2 = -\frac{1}{6} m b \omega^2 \sin \beta \cos \beta \hat{i}$$

$$\Sigma \vec{F} = \Sigma \vec{F}_{eff} : \quad C_2 + D_2 = 0$$

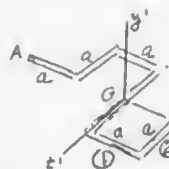
# 18.67



GIVEN: 16-lb SHAFT WITH UNIFORM CROSS-SECTION. ROTATES AT CONSTANT RATE  $\omega = 12 \text{ rad/s}$ .

FIND: DYNAMIC REACTIONS AT A AND B

MOMENTS AND PRODUCTS OF INERTIA



WE DENOTE BY  $a$  THE LENGTH OF AN ELEMENT. WE FIND  $\Sigma m$  ITS MASS, USING THE CENTROIDAL AXES  $Gx'y'$ :

$$\bar{I}_x = 2ma^2 + 4\left(\frac{1}{3}ma^2\right) = \frac{10}{3}ma^2$$

$$\bar{I}_{xy} = 0$$

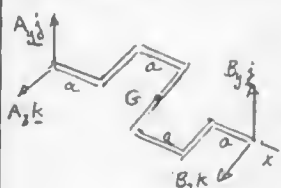
$$\bar{I}_{xz} = 2m\bar{x}_1\bar{z}_1 + 2m\bar{x}_2\bar{z}_2 = 2m\left(\frac{a}{2}\right)a + 2m\left(\frac{a}{2}\right)a = 2ma^2$$

$$\dot{H}_G = \bar{I}_x \omega \hat{i} - \bar{I}_{xy} \omega \hat{j} - \bar{I}_{xz} \omega \hat{k} = \frac{10}{3}ma^2 \omega \hat{i} - 2ma^2 \omega \hat{k} \quad (1)$$

$$\text{EQ. (18.22): } \dot{H}_G = (\dot{H}_G)_{Gx'y'z'} + \Omega \times H_G = 0 + \omega \hat{i} \times H_G$$

$$\dot{H}_G = \omega \hat{i} \times \left(\frac{10}{3}ma^2 \omega \hat{i} - 2ma^2 \omega \hat{k}\right) = 2ma^2 \omega^2 \hat{j}$$

## EQUATIONS OF MOTION



$$\Sigma \vec{M}_A = \Sigma (\vec{M}_A)_{eff} :$$

$$4a \hat{i} \times (B_2 \hat{j} + B_2 \hat{k}) = \dot{H}_G$$

$$4a B_2 \hat{k} - 4a B_2 \hat{j} =$$

$$2ma^2 \omega^2 \hat{j}$$

$$\text{THUS: } B_2 = 0, \quad B_2 = -\frac{1}{2} m a \omega^2$$

$$B_2 = -\frac{1}{2} m a \omega^2 \hat{k}$$

$$\Sigma \vec{F} = \Sigma \vec{F}_{eff} : \quad A + B = 0 \quad A = -B = \frac{1}{2} m a \omega^2 \hat{k}$$

$$\text{DATA: } m = \frac{1}{8} \frac{W}{g} = \frac{1}{8} \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.062112 \text{ lb} \cdot \text{s}^2/\text{ft}$$

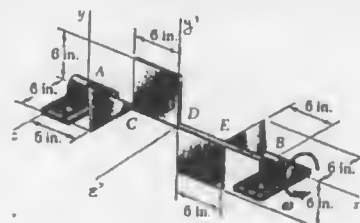
$$a = 9 \text{ in.} = 0.75 \text{ ft} \quad \omega = 12 \text{ rad/s}$$

THUS:

$$A = \frac{1}{2} (0.062112 \text{ lb} \cdot \text{s}^2/\text{ft}) (0.75 \text{ ft}) (12 \text{ rad/s})^2 = 3.354 \text{ lb}$$

$$A = (3.35 \text{ lb}) \hat{k} ; \quad B = -(3.35 \text{ lb}) \hat{k}$$

# 18.68



GIVEN:

ASSEMBLY WEIGHS 2.7 lb AND ROTATES AT CONSTANT RATE  $\omega = 240 \text{ rpm}$

FIND: DYNAMIC REACTIONS AT A AND B

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

WE USE THE CENTROIDAL AXES  $Dx'y'z'$ .

$$\text{FOR EACH SQUARE: } m_1 = \frac{1}{3} \frac{2.7 \text{ lb}}{g} = \frac{0.9}{g}$$

$$\bar{I}_x = \frac{1}{3} m a^2 = \frac{1}{3} \frac{0.9}{g} \left(\frac{1}{2} \text{ ft}\right)^2 = 0.075/g$$

$$\bar{I}_{xy} = m \left(\frac{a}{2}\right) \left(-\frac{a}{2}\right) = -\frac{1}{4} \frac{0.9}{g} \left(\frac{1}{2} \text{ ft}\right)^2 = -0.05625/g, \quad \bar{I}_{xz} = 0$$

$$\text{FOR EACH TRIANGLE: } m = \frac{1}{6} \frac{2.7 \text{ lb}}{g} = \frac{0.45}{g}$$

$$\bar{I}_{x, \text{mass}} = \bar{I}_{x, \text{area}} + \bar{I}_{x, \text{cm}} = \frac{1}{12} \frac{0.45}{g} \frac{m}{a^2} = \frac{1}{6} m a^2 = \frac{1}{6} \frac{0.45}{g} \left(\frac{1}{2}\right)^2 = \frac{0.01875}{g}$$

$$\bar{I}_{x, \text{area}} = A \bar{x}'^2 + \bar{I}_{x, \text{cm}} = \frac{1}{2} a^2 \left(\frac{1}{3} a - \frac{a}{3}\right) + \frac{1}{72} a^4 = -\frac{15}{72} a^4 \quad (\text{cf. SAMPLE PROB. 9.16})$$

$$\bar{I}_{x, \text{mass}} = \bar{I}_{x, \text{area}} + \bar{I}_{x, \text{cm}} = -\frac{15}{72} \frac{a^4}{g} = -\frac{5}{12} m a^2 = -\frac{5}{12} \frac{0.45}{g} \left(\frac{1}{2}\right)^2 = -0.046875/g$$

$$\bar{I}_{xy} = 0$$

FOR ASSEMBLY:

$$\bar{I}_x = (2 \times 0.075 + 2 \times 0.01875)/g = 0.1875/g$$

$$\bar{I}_{xy} = 2(-0.05625)/g = -0.1125/g$$

$$\bar{I}_{xz} = 2(-0.046875)/g = -0.09375/g$$

ANGULAR MOMENTUM  $H_D$

$$H_D = \bar{I}_x \omega \hat{i} - \bar{I}_{xy} \omega \hat{j} - \bar{I}_{xz} \omega \hat{k}$$

$$\dot{H}_D = (0.1875 \hat{i} + 0.1125 \hat{j} + 0.09375 \hat{k})(\omega/g) \quad (1)$$

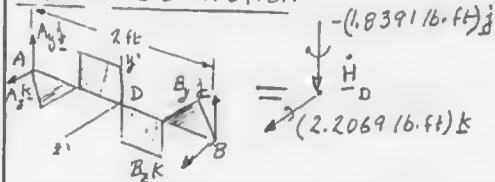
$$\text{EQ. (18.22): } \dot{H}_D = (\dot{H}_D)_{Dx'y'z'} + \Omega \times H_D = 0 + \omega \hat{i} \times H_D$$

$$\text{SINCE } \omega = 240 \text{ rpm} = 8\pi \text{ rad/s, AND } \hat{i} \times \hat{j} = \hat{k}, \quad \hat{i} \times \hat{k} = -\hat{j},$$

$$\dot{H}_D = (0.1125 \hat{k} - 0.09375 \hat{j})(8\pi)^2/g$$

$$\dot{H}_D = -(1.8391 \text{ lb} \cdot \text{ft}) \hat{j} + (2.2069 \text{ lb} \cdot \text{ft}) \hat{k}$$

EQUATIONS OF MOTION



$$\Sigma \vec{M}_A = \Sigma (\vec{M}_A)_{eff} : (2 \text{ ft}) \hat{i} \times (B_2 \hat{j} + B_2 \hat{k}) = -1.8391 \hat{j} + 2.2069 \hat{k}$$

$$2B_2 \hat{k} - 2B_2 \hat{j} = -1.8391 \hat{j} + 2.2069 \hat{k}$$

$$\text{THUS: } B_2 = \frac{1}{2} (2.2069) = 1.1034 \text{ lb}$$

$$B_2 = \frac{1}{2} (1.8391) = 0.9196 \text{ lb}$$

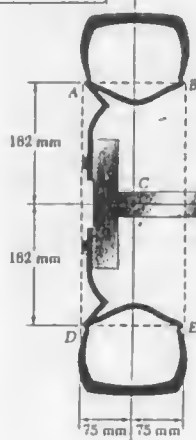
$$B_2 = (1.103 \text{ lb}) \hat{j} + (0.920 \text{ lb}) \hat{k}$$

$$\Sigma \vec{F} = \Sigma \vec{F}_{eff} : A + B = 0$$

$$A = -B$$

$$A = -(1.103 \text{ lb}) \hat{j} - (0.920 \text{ lb}) \hat{k}$$

18.69



GIVEN:

18-kg wheel is attached to balancing machine. When machine spins at the rate of 12.5 rev/s, wheel is found to exert on machine a force-couple  $\mathbf{F}$  consisting of

$$\mathbf{F} = (160 \text{ N})\mathbf{j} \text{ applied at } C$$

$$\text{and } \mathbf{M}_C = (14.7 \text{ N}\cdot\text{m})\mathbf{k}$$

FIND:

- (a) Distance  $\bar{z}$  from  $z$ -axis to  $G$ , and  $I_{xy}$  and  $I_{xz}$ .  
 (b) The two corrective masses required to balance the wheel and at which of points A, B, C, D they should be placed.

(a) THE FORCES EXERTED ON THE WHEEL MUST BE EQUIVALENT TO THE EFFECTIVE FORCES:



$$\begin{aligned} \sum \mathbf{F} &= \sum \mathbf{F}_{\text{eff}}: \\ - (160 \text{ N})\mathbf{j} &= - (18 \text{ kg})\bar{a}_y \mathbf{j} \\ \bar{a}_y &= 8.8889 \text{ m/s}^2 \\ \text{BUT } \bar{a}_y &= \bar{z}\omega^2 \\ \bar{z} &= \frac{\bar{a}_y}{\omega^2} = \frac{8.8889 \text{ m/s}^2}{(12.5 \times 2\pi \text{ rad/s})^2} = 1.4410 \times 10^{-3} \text{ m} \\ \bar{z} &= 1.441 \text{ mm} \end{aligned}$$

$$\sum \mathbf{M}_G = \sum (\mathbf{M}_G)_{\text{eff}}: - (14.7 \text{ N}\cdot\text{m})\mathbf{k} = \dot{\mathbf{H}}_G \quad (1)$$

$$\text{BUT } \dot{\mathbf{H}}_G = I_x \omega \dot{\mathbf{i}} - I_{xy} \omega \dot{\mathbf{j}} - I_{yz} \omega \dot{\mathbf{k}}$$

$$\text{AND } \dot{\mathbf{H}}_G = \omega \times \mathbf{H}_G = \omega \mathbf{i} \times (I_x \omega \mathbf{i} - I_{xy} \omega \mathbf{j} - I_{yz} \omega \mathbf{k})$$

$$\dot{\mathbf{H}}_G = -I_{xy} \omega^2 \mathbf{j} + I_{yz} \omega^2 \mathbf{k}$$

$$\text{SUBSTITUTE IN (1): } -14.7 \text{ k} = -I_{xy} \omega^2 \mathbf{j} + I_{yz} \omega^2 \mathbf{k}$$

$$\begin{aligned} \text{THUS: } I_{xy} &= \frac{14.7 \text{ N}\cdot\text{m}}{(12.5 \times 2\pi \text{ rad/s})^2} = 2.3831 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \text{ AND } I_{yz} = 0 \\ I_{xy} &= 2.38 \text{ g}\cdot\text{m}^2, \quad I_{yz} = 0 \end{aligned}$$

(b) WITH CORRECTIVE MASSES THE FORCES EXERTED ON THE WHEEL ARE EQUIVALENT TO ZERO. FOR THE EFFECTIVE FORCES TO ALSO BE EQUIVALENT TO ZERO, THE MASSES MUST BE PLACED AT A AND E:

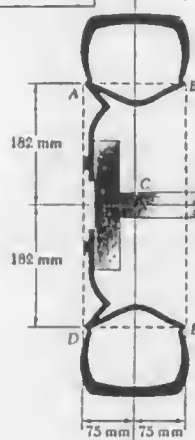
$$\begin{aligned} +\sum (F_y)_{\text{eff}} &= 0: m_E \bar{a}_E - m_A \bar{a}_A - m \bar{a}_y = 0 \\ (m_E - m_A)(0.182 \text{ m})\omega^2 - (18 \text{ kg})\bar{z}\omega^2 &= 0 \\ (m_E - m_A)(0.182 \text{ m}) &= (18 \text{ kg})(1.441 \times 10^{-3} \text{ m}) \\ m_E - m_A &= 0.14252 \text{ kg} \quad (2) \\ +\sum (M_C)_{\text{eff}} &= 0: \\ (m_E \bar{a}_E + m_A \bar{a}_A)(0.075 \text{ m}) - \dot{\mathbf{H}}_G &= 0 \\ (m_E + m_A)(0.182 \text{ m})\omega^2(0.075 \text{ m}) - 14.7 &= 0 \\ (m_E + m_A)(0.182 \times 12.5 \times 2\pi)^2(0.075) &= 14.7 \\ m_E + m_A &= 0.17458 \text{ kg} \quad (3) \end{aligned}$$

SOLVING (2) AND (3) SIMULTANEOUSLY:

$$m_A = 16.034 \times 10^{-3} \text{ kg}, \quad m_E = 158.55 \times 10^{-3} \text{ kg}$$

AT A AND E;  $m_A = 16.03 \text{ g}$ ,  $m_E = 158.6 \text{ g}$

18.70



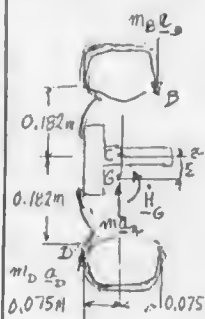
GIVEN:

18-kg wheel is attached to balancing machine and spins at the rate of 15 rev/s. Mechanic finds that a 170-g mass placed at B and a 56-g mass placed at D are needed to balance wheel.

FIND:

BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED:  
 (1) Distance  $\bar{z}$  from  $z$ -axis to  $G$ , and  $I_{xy}$  and  $I_{xz}$ .  
 (2) The force-couple system at  $G$  equivalent to the forces exerted by the wheel on the machine.

(a) AFTER THE CORRECTIVE MASSES HAVE BEEN ADDED, THE SYSTEM OF THE EXTERNAL FORCES IS ZERO THEREFORE, THE SYSTEM OF THE EFFECTIVE FORCES MUST ALSO BE EQUIVALENT TO ZERO SINCE THE LARGER OF THE TWO MASSES IS PLACED ABOVE THE  $z$  AXIS, THE MASS CENTER  $G$  OF THE UNBALANCED WHEEL MUST BE BELOW THAT AXIS



$$\begin{aligned} +\sum (F_y)_{\text{eff}} &= 0: m \bar{a}_y - m_B \bar{a}_B + m_D \bar{a}_D = 0 \\ (18 \text{ kg})\bar{z}\omega^2 - (0.170 \text{ kg})(0.182 \text{ m})\omega^2 + (0.056 \text{ kg})(0.075 \text{ m})\omega^2 &= 0 \\ 18\bar{z} &= (0.170)(0.182) - (0.056)(0.075) \\ \bar{z} &= 1.1527 \times 10^{-3} \text{ m} \quad \bar{z} = 1.153 \text{ mm} \end{aligned}$$

$$\begin{aligned} +\sum (M_G)_{\text{eff}} &= 0: \\ \dot{\mathbf{H}}_G - m_B \bar{a}_B(0.075 \text{ m}) - m_D \bar{a}_D(0.075 \text{ m}) &= 0 \\ \dot{\mathbf{H}}_G &= m_B \bar{a}_B \omega^2(0.075) + m_D \bar{a}_D \omega^2(0.075) \\ &= (0.170 + 0.056)(0.182)(0.075)\omega^2 \\ \dot{\mathbf{H}}_G &= 3.0849 \times 10^{-3} \omega^2 \text{ k} \quad (1) \end{aligned}$$

$$\text{SINCE } m \bar{a}_y \text{ PASSES THRU } G, \dot{\mathbf{H}}_G = \dot{\mathbf{H}}_G = 3.0849 \times 10^{-3} \omega^2 \text{ k} \quad (2)$$

$$\text{BUT } \dot{\mathbf{H}}_G = I_x \omega \dot{\mathbf{i}} - I_{xy} \omega \dot{\mathbf{j}} - I_{yz} \omega \dot{\mathbf{k}}$$

$$\text{AND } \dot{\mathbf{H}}_G = \omega \mathbf{i} \times (I_x \omega \mathbf{i} - I_{xy} \omega \mathbf{j} - I_{yz} \omega \mathbf{k}) = -I_{xy} \omega^2 \mathbf{j} + I_{yz} \omega^2 \mathbf{k} \quad (3)$$

$$\text{EQUATING (2) AND (3), WE HAVE } -I_{xy} = 3.0849 \times 10^{-3}, \quad I_{yz} = 0$$

$$I_{xy} = -3.08 \text{ g}\cdot\text{m}^2, \quad I_{yz} = 0$$

(b) THE FORCE-COUPLE SYSTEM EXERTED ON THE WHEEL BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED IS EQUAL TO THE EFFECTIVE FORCES:

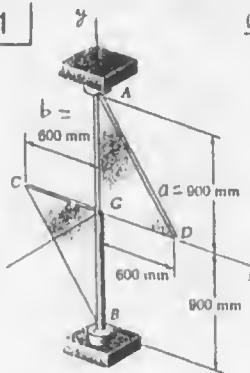
$$\begin{aligned} \mathbf{F} &= m \bar{a}_y = m \bar{z}\omega^2 \mathbf{j} = (18 \text{ kg})(1.1527 \times 10^{-3} \text{ m})(15 \times 2\pi \text{ rad/s})^2 \mathbf{j} = (184.3 \text{ N})\mathbf{j} \\ \mathbf{M}_C &= \dot{\mathbf{H}}_G = 3.0849 \times 10^{-3} (15 \times 2\pi \text{ rad/s})^2 \mathbf{k} = (27.4 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

THE FORCE-COUPLE SYSTEM EXERTED BY THE WHEEL ON THE MACHINE BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED

$$\mathbf{F}' = -\mathbf{F} = - (184.3 \text{ N})\mathbf{j}$$

$$\mathbf{M}'_C = -\mathbf{M}_C = - (27.4 \text{ N}\cdot\text{m})\mathbf{k}$$

18.71



GIVEN:

ASSEMBLY OF PROB. 18.66  
CONSISTING OF TWO TRIANGULAR  
PLATES, EACH OF MASS  
 $m = 5 \text{ kg}$ , IS AT REST  
WHEN A COUPLE OF  
MOMENT  $M_0 = (36 \text{ N}\cdot\text{m}) \hat{j}$   
IS APPLIED TO SHAFT AB.

FIND:

- (a) ANGULAR ACCELERATION  
OF ASSEMBLY,  
(b) INITIAL DYNAMIC  
REACTIONS AT A AND B.

SEE SOLUTION OF PROB. 18.65 FOR DERIVATION OF EQ. (2):

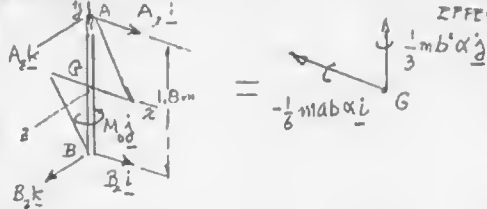
$$\underline{H}_G = -\frac{1}{6} m a b \omega \underline{i} + \frac{1}{3} m b^2 \dot{\omega} \underline{j} \quad (2)$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + 0$$

SINCE  $\underline{\Omega} = \dot{\omega} = 0$  WHEN COUPLE IS APPLIED, THUS

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} = -\frac{1}{6} m a b \alpha \underline{i} + \frac{1}{3} m b^2 \alpha \underline{j} \quad (3)$$

EQUATIONS OF MOTION: EQUIVALENCE OF APPLIED AND EFFECTIVE FORCES.



$$\Sigma \underline{M}_B = \Sigma (\underline{M}_B)_{\text{eff}};$$

$$(1.8 \text{ m}) \hat{j} \times (A_x \hat{i} + A_z \hat{k}) + M_0 \hat{j} = -\frac{1}{6} m a b \alpha \underline{i} + \frac{1}{3} m b^2 \alpha \underline{j}$$

$$-1.8 A_z \hat{k} + 1.8 A_x \hat{i} + M_0 \hat{j} = -\frac{1}{6} m a b \alpha \underline{i} + \frac{1}{3} m b^2 \alpha \underline{j}$$

EQUATING THE COEFF. OF  $\underline{i}, \underline{j}, \underline{k}$ :

$$\textcircled{1} (1.8 \text{ m}) A_z = -\frac{1}{6} m a b \alpha \quad (4)$$

$$\textcircled{2} M_0 = \frac{1}{3} m b^2 \alpha \quad (5)$$

$$\textcircled{3} A_x = 0 \quad (6)$$

(a) ANGULAR ACCELERATION

SUBSTITUTING GIVEN DATA IN (5):

$$36 \text{ N}\cdot\text{m} = \frac{1}{3} (5 \text{ kg}) (0.6 \text{ m})^2 \alpha$$

$$\alpha = 60.0 \text{ rad/s}^2$$

(b) INITIAL DYNAMIC REACTIONS

$$\text{EQ. (4): } (1.8 \text{ m}) A_z = -\frac{1}{6} (5 \text{ kg}) (0.9 \text{ m}) (60 \text{ rad/s}^2)$$

$$A_z = -15.00 \text{ N}$$

RECALLING EQ. (6),  $A_x = 0$ ,

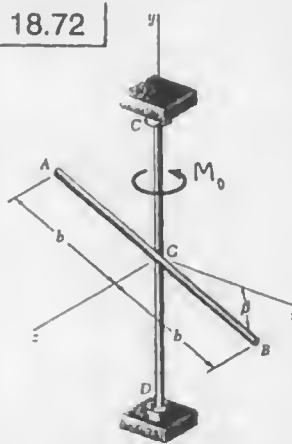
$$\underline{A} = -(15.00 \text{ N}) \underline{k}$$

$$\Sigma \underline{F} = \Sigma (\underline{F})_{\text{eff}};$$

$$\underline{A} + \underline{B} = 0, \quad \underline{B} = -\underline{A}$$

$$\underline{B} = (15.00 \text{ N}) \underline{k}$$

18.72



GIVEN:

ASSEMBLY OF PROB. 18.66,  
CONSISTING OF ROD OF MASS  $m$   
WELDED TO SHAFT CD OF  
LENGTH  $2b$ . ASSEMBLY IS AT  
REST WHEN COUPLE OF  
MOMENT  $M_0 = M_0 \hat{j}$  IS  
APPLIED TO SHAFT CD

FIND:

- (a) ANGULAR ACCELERATION  
OF ASSEMBLY,  
(b) INITIAL DYNAMIC  
REACTIONS AT C AND D.

SEE SOLUTION OF PROB. 18.66 FOR DERIVATION OF EQ. (1):

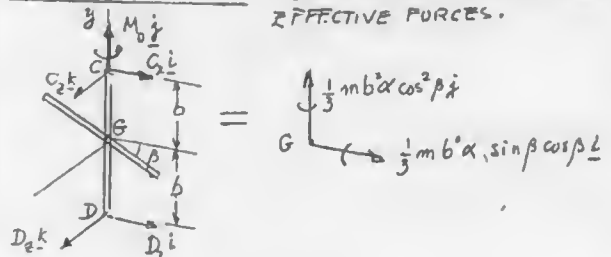
$$\underline{H}_G = \frac{1}{3} m b^2 \omega \cos \beta (\sin \beta \underline{i} + \cos \beta \underline{j}) \quad (1)$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + 0$$

SINCE  $\underline{\Omega} = \dot{\omega} = 0$  WHEN COUPLE IS APPLIED, THUS

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} = \frac{1}{3} m b^2 \alpha \cos \beta (\sin \beta \underline{i} + \cos \beta \underline{j}) \quad (2)$$

EQUATIONS OF MOTION: EQUIVALENCE OF APPLIED AND EFFECTIVE FORCES.



$$\Sigma \underline{M}_D = \Sigma (\underline{M}_D)_{\text{eff}};$$

$$2b \hat{j} \times (C_x \hat{i} + C_z \hat{k}) + M_0 \hat{j} = \frac{1}{3} m b^2 \alpha' \sin \beta \cos \beta \underline{i} + \frac{1}{3} m b^2 \alpha \cos^2 \beta \underline{j}$$

$$-2b C_z \hat{k} + 2b C_x \hat{i} + M_0 \hat{j} = \frac{1}{3} m b^2 \alpha \sin \beta \cos \beta \underline{i} + \frac{1}{3} m b^2 \alpha \cos^2 \beta \underline{j}$$

EQUATING THE COEFF. OF  $\underline{i}, \underline{j}, \underline{k}$ :

$$\textcircled{1} 2b C_z = \frac{1}{3} m b^2 \alpha \sin \beta \cos \beta \quad (3)$$

$$\textcircled{2} M_0 = \frac{1}{3} m b^2 \alpha \cos^2 \beta \quad (4)$$

$$\textcircled{3} C_x = 0 \quad (5)$$

(a) ANGULAR ACCELERATION

$$\text{FROM EQ. (4): } \alpha = 3 M_0 / m b^2 \cos^2 \beta$$

(b) INITIAL DYNAMIC REACTIONS

FROM EQ. (3):

$$C_z = \frac{1}{6} m b \alpha \sin \beta \cos \beta = \frac{1}{6} m b \sin \beta \cos \beta (3 M_0 / m b^2 \cos^2 \beta)$$

$$C_z = (M_0 / 2b) \tan \beta$$

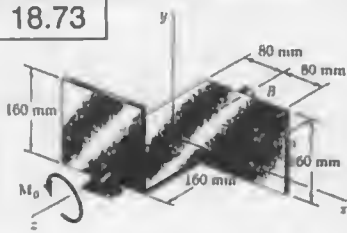
RECALLING EQ. (5),  $C_x = 0$ ,

$$\underline{C} = (M_0 / 2b) \tan \beta \underline{k}$$

$$\Sigma \underline{F} = \Sigma (\underline{F})_{\text{eff}};$$

$$\underline{C} + \underline{D} = 0, \quad \underline{D} = -\underline{C} \quad \underline{D} = -(M_0 / 2b) \tan \beta \underline{k}$$

18.73



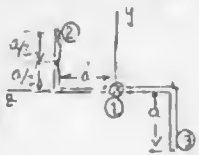
GIVEN:

2.4-KG COMPONENT SHOWN IS AT REST WHEN COUPLE  $M_0 = (0.8 \text{ N}\cdot\text{m})\mathbf{k}$  IS APPLIED TO IT.

FIND:

(a) ANG. ACCELERATION  
(b) DYNAMIC REACTIONS AT A AND B IMMEDIATELY AFTER COUPLE IS APPLIED

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

TOTAL MASS =  $m = 2.4 \text{ kg}$ ,  $a = 160 \text{ mm}$ 

PORTION 1:

$$I_1 = \frac{1}{12} \left( \frac{m}{2} \right) a^2 = \frac{1}{24} m a^2, \quad I_{yz} = I_{zx} = 0$$

PORTIONS 2 AND 3:

$$I_2 = 2 \left( \frac{m}{4} \right) \left[ \frac{a^2}{6} + \left( \frac{a}{2} \right)^2 \right] = \frac{5}{24} m a^2$$

$$I_{yz} = 2 \left( \frac{m}{4} \right) \left( \frac{a}{2} \right) a = \frac{1}{4} m a^2, \quad I_{zx} = 0$$

COMBINED:

$$I_z = \frac{1}{24} m a^2 + \frac{5}{24} m a^2 = \frac{1}{4} m a^2, \quad I_{yz} = \frac{1}{4} m a^2, \quad I_{zx} = 0$$

ANGULAR MOMENTUM:

$$\frac{H}{G} = -I_{xz} \omega \mathbf{i} - I_{yz} \omega \mathbf{j} + I_z \omega \mathbf{k} = 0 - \frac{1}{4} m a^2 \omega \mathbf{j} + \frac{1}{4} m a^2 \omega \mathbf{k}$$

$$\frac{H}{G} = \frac{1}{4} m a^2 \omega (-\mathbf{j} + \mathbf{k}) \quad (1)$$

RATE OF CHANGE:

EQ. (18.22) YIELDS, SINCE  $\Omega = \omega \mathbf{k}$ ,

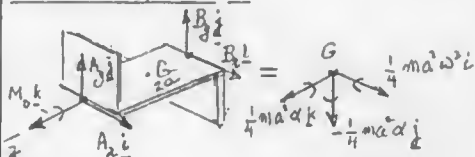
$$\dot{\frac{H}{G}} = \left( \frac{H}{G} \right)_{Gxyz} + \omega \mathbf{k} \times \frac{H}{G}$$

$$= \frac{1}{4} m a^2 \alpha (-\mathbf{j} + \mathbf{k}) + \omega \mathbf{k} \times \frac{1}{4} m a^2 \omega (-\mathbf{j} + \mathbf{k})$$

$$= \frac{1}{4} m a^2 \alpha (-\mathbf{j} + \mathbf{k}) + \frac{1}{4} m a^2 \omega^2 \mathbf{i}$$

$$\dot{\frac{H}{G}} = \frac{1}{4} m a^2 (\omega^2 \mathbf{i} - \alpha \mathbf{j} + \alpha \mathbf{k}) \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \mathbf{M}_G = \Sigma (\mathbf{M}_G)_{\text{eff}}: 2a \mathbf{k} \times (A_x \mathbf{i} + A_y \mathbf{j}) + M_0 \mathbf{k} = \frac{10}{3} m a^2 \alpha \mathbf{i} - 2 m a^2 \alpha \mathbf{k}$$

$$2a A_x \mathbf{j} - 2a A_y \mathbf{i} + M_0 \mathbf{k} = \frac{10}{3} m a^2 \alpha \mathbf{i} - \frac{1}{4} m a^2 \alpha \mathbf{j} + \frac{1}{4} m a^2 \alpha \mathbf{k} \quad (3)$$

(a) ANG. ACCELERATION

EQUATE COEFF. OF  $\mathbf{k}$  IN (3):

$$M_0 = \frac{1}{4} m a^2 \alpha \quad \alpha = \frac{4 M_0}{m a^2} = \frac{4(0.8 \text{ N}\cdot\text{m})}{(2.4 \text{ kg})(0.16 \text{ m})^2} = 52.083 \text{ rad/s}^2 \quad (4)$$

$$\alpha = 52.1 \text{ rad/s}^2$$

(b) DYNAMIC REACTIONS

EQUATE COEFF. OF  $\mathbf{j}$  IN (3):

$$2a A_x = -\frac{1}{4} m a^2 \alpha = -\frac{1}{4} m a^2 \frac{4 M_0}{m a^2} = -M_0$$

$$A_x = -\frac{M_0}{2a} = -\frac{0.8 \text{ N}\cdot\text{m}}{2(0.16 \text{ m})} = -2.50 \text{ N}$$

EQUATE COEFF. OF  $\mathbf{i}$  IN (3):

$$-2a A_y = \frac{1}{4} m a^2 \omega^2 \quad A_y = -\frac{1}{8} m a \omega^2$$

SINCE  $\omega = 0$ ,  $A_y = 0$ ; THUS:

$$\mathbf{A} = -(2.50 \text{ N})\mathbf{i}$$

$$\Sigma \mathbf{F} = \Sigma (\mathbf{F})_{\text{eff}}: \mathbf{A} + \mathbf{B} = 0, \quad \mathbf{B} = -\mathbf{A}$$

$$\mathbf{B} = (2.50 \text{ N})\mathbf{i}$$

18.74

GIVEN: COMPONENT OF PROB. 18.73.

FIND: DYNAMIC REACTIONS AT A AND B AFTER ONE FULL REVOLUTION

SEE SOLUTION OF PROB. 18.73 FOR DERIVATION OF Eqs. (4), (5), AND (6)

FROM EQ. (4),  $\alpha = 52.083 \text{ rad/s}^2$ FOR ONE FULL REVOLUTION,  $\theta = 2\pi \text{ rad}$ 

FROM Eqs. (15.16):

$$\omega^2 = 2\alpha\theta = 2(52.083 \text{ rad/s}^2)(2\pi \text{ rad}) = 654.49 \text{ rad/s}^2$$

$$\text{EQ. (5): } A_x = -2.50 \text{ N}$$

$$\text{EQ. (6): } A_y = -\frac{1}{8} (2.4 \text{ kg})(0.16 \text{ m})(654.49 \text{ rad/s}^2) = -31.4 \text{ N}$$

THEREFORE:

$$\mathbf{A} = -(2.50 \text{ N})\mathbf{i} - (31.4 \text{ N})\mathbf{j}; \quad \mathbf{B} = -\mathbf{A} = (2.50 \text{ N})\mathbf{i} + (31.4 \text{ N})\mathbf{j}$$

18.75

GIVEN:

16-16 SHAFT OF PROB. 18.67

IS AT REST WHEN A

COUPLE  $M_0$  IS APPLIED

TO IT, CAUSING ANGULAR

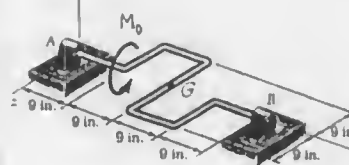
ACCEL.  $\alpha = (20 \text{ rad/s}^2)\mathbf{i}$ .

FIND:

(a) COUPLE  $M_0$ .

(b) DYNAMIC REACTIONS

AT A AND B IMMEDIATELY

AFTER  $M_0$  IS APPLIED.

SEE SOLUTION OF PROB. 18.67 FOR DERIVATION OF

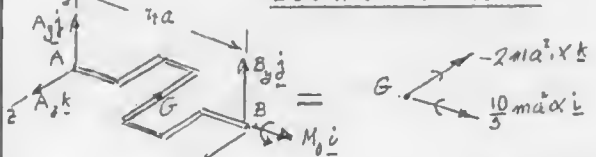
$$\frac{H}{G} = \frac{10}{3} m a^2 \omega \mathbf{i} - 2 m a^2 \omega \mathbf{k} \quad (1)$$

$$\text{EQ. (18.22): } \dot{\frac{H}{G}} = \left( \frac{H}{G} \right)_{Gxyz} + \Omega \times \frac{H}{G} = \left( \frac{H}{G} \right)_{Gxyz} + 0$$

$$\text{SINCE } \dot{\omega} = \alpha: \quad \dot{\frac{H}{G}} = \frac{10}{3} m a^2 \alpha \mathbf{i} - 2 m a^2 \alpha \mathbf{k} \quad (2)$$

WHERE  $a = 9 \text{ in.} = 0.75 \text{ ft}$  AND  $m = \frac{1}{8} (16 \text{ lb}) = 2 \text{ lb}$ 

EQUATIONS OF MOTION



$$\Sigma \mathbf{M}_A = \Sigma (\mathbf{M}_A)_{\text{eff}}: 4a \mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) + M_0 \mathbf{i} = \frac{10}{3} m a^2 \alpha \mathbf{i} - 2 m a^2 \alpha \mathbf{k}$$

$$4a B_y \mathbf{k} - 4a B_z \mathbf{j} + M_0 \mathbf{i} = \frac{10}{3} m a^2 \alpha \mathbf{i} - 2 m a^2 \alpha \mathbf{k} \quad (3)$$

(a) COUPLE  $M_0$ EQUATE COEFF. OF  $\mathbf{i}$  IN EQ. (3):

$$M_0 = \frac{10}{3} m a^2 \alpha = \frac{10}{3} \frac{2 \text{ lb}}{32.2 \text{ ft/s}^2} (0.75 \text{ ft})^2 (20 \text{ rad/s}^2) = 2.329 \text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_0 = (2.33 \text{ lb}\cdot\text{ft})\mathbf{i}$$

(b) DYNAMIC REACTIONS AT  $t = 0$ EQUATE COEFF. OF  $\mathbf{j}$  IN EQ. (3):  $B_z = 0$ EQUATE COEFF. OF  $\mathbf{k}$  IN EQ. (3):

$$4a B_y = -2 m a^2 \alpha$$

$$B_y = -\frac{1}{2} m a \alpha = -\frac{1}{2} \frac{2 \text{ lb}}{32.2 \text{ ft/s}^2} (0.75 \text{ ft}) (20 \text{ rad/s}^2) = -0.466 \text{ lb}$$

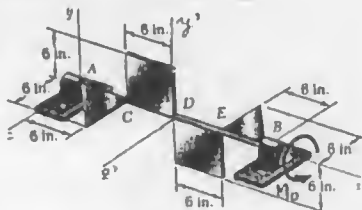
THEREFORE:  $\mathbf{B} = -(0.466 \text{ lb})\mathbf{j}$ 

$$\Sigma \mathbf{F} = \Sigma (\mathbf{F})_{\text{eff}}: \mathbf{A} + \mathbf{B} = 0 \quad \mathbf{A} = -\mathbf{B} = (0.466 \text{ lb})\mathbf{j}$$

THUS:

$$\mathbf{A} = (0.466 \text{ lb})\mathbf{j}; \quad \mathbf{B} = -(0.466 \text{ lb})\mathbf{j}$$

18.76



GIVEN:

THE 18.76 ASSEMBLY OF PROB. 18.68 IS AT REST WHEN A COUPLE  $M_0$  IS APPLIED TO AXLE AB, CAUSING AN ANGULAR ACCELERATION  $\alpha = (150 \text{ rad/s}^2) \mathbf{i}$ .

FIND: (a) THE COUPLE  $M_0$ ,  
(b) THE DYNAMIC REACTIONS AT A AND B IMMEDIATELY AFTER  $M_0$  IS APPLIED.

SEE SOLUTION OF PROB. 18.68 FOR DERIVATION OF EQ.(1):

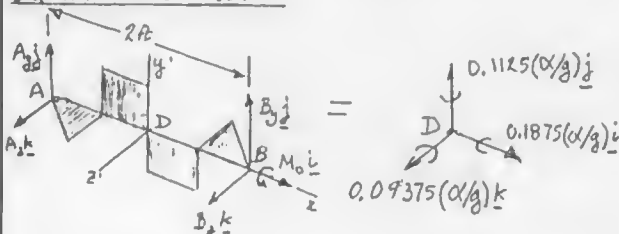
$$\mathbf{H}_D = (0.1875 \mathbf{i} + 0.1125 \mathbf{j} + 0.09375 \mathbf{k}) (\omega/g) \quad (1)$$

WHERE THE NUMERICAL VALUES ARE EXPRESSED IN  $\text{lb}\cdot\text{s}^2$

$$\text{EQ. (18.22): } \dot{\mathbf{H}}_D = (\dot{\mathbf{H}})_{Dxyz} + \boldsymbol{\Omega} \times \mathbf{H}_D = (\dot{\mathbf{H}})_{Dxyz} + 0$$

$$\text{SINCE } \dot{\omega} = \alpha: \dot{\mathbf{H}}_D = (0.1875 \mathbf{i} + 0.1125 \mathbf{j} + 0.09375 \mathbf{k}) (\alpha/g) \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \mathbf{M}_A = \Sigma (\mathbf{M}_A)_{\text{eff}}:$$

$$(2 \text{ ft}) \mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) + M_0 \mathbf{i} = 0.1875 (\alpha/g) \mathbf{i} + 0.1125 (\alpha/g) \mathbf{j} + 0.09375 (\alpha/g) \mathbf{k}$$

$$(2 \text{ ft}) B_y \mathbf{k} - (2 \text{ ft}) B_z \mathbf{j} + M_0 \mathbf{i} = 0.1875 (\alpha/g) \mathbf{i} + 0.1125 (\alpha/g) \mathbf{j} + 0.09375 (\alpha/g) \mathbf{k} \quad (3)$$

(a) COUPLE  $M_0$

EQUATE COEFF. OF  $\mathbf{i}$  IN EQ.(3):

$$M_0 = 0.1875 (\alpha/g) = (0.1875 \text{ lb}\cdot\text{ft}^2) \frac{150 \text{ rad/s}^2}{32.2 \text{ ft/s}^2} = 0.873 \text{ lb}\cdot\text{ft}$$

$$M_0 = (0.873 \text{ lb}\cdot\text{ft}) \mathbf{i}$$

(b) DYNAMIC REACTIONS AT  $t=0$ .

EQUATE COEFF. OF  $\mathbf{k}$  IN EQ.(3):

$$(2 \text{ ft}) B_y = 0.09375 (\alpha/g) = (0.09375 \text{ lb}\cdot\text{ft}^2) \frac{150 \text{ rad/s}^2}{32.2 \text{ ft/s}^2} = 0.43672 \text{ lb}\cdot\text{ft}$$

$$B_y = 0.218 \text{ lb}$$

EQUATE COEFF. OF  $\mathbf{j}$  IN EQ.(3):

$$-(2 \text{ ft}) B_z = 0.1125 (\alpha/g) = (0.1125 \text{ lb}\cdot\text{ft}^2) \frac{150 \text{ rad/s}^2}{32.2 \text{ ft/s}^2} = 0.52407 \text{ lb}\cdot\text{ft}$$

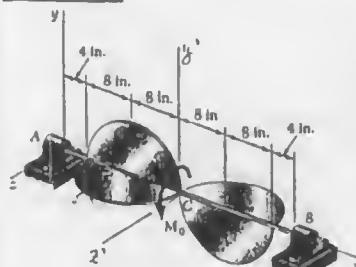
$$B_z = -0.262 \text{ lb}$$

$$\text{THUS: } \mathbf{B} = (0.218 \text{ lb}) \mathbf{j} - (0.262 \text{ lb}) \mathbf{k}$$

$$\Sigma \mathbf{F} = \Sigma (\mathbf{F})_{\text{eff}}: \mathbf{A} + \mathbf{B} = 0, \mathbf{A} = -\mathbf{B}$$

$$\mathbf{A} = -(0.218 \text{ lb}) \mathbf{j} + (0.262 \text{ lb}) \mathbf{k}$$

18.77



GIVEN:

ASSEMBLY WEIGHS 12 lb AND CONSISTS OF 4 SEMICIRCULAR PLATES. ASSEMBLY IS AT REST AT  $t=0$  WHEN COUPLE  $M_0$  IS APPLIED FOR ONE FULL REVOLUTION WHICH LASTS 2 s.

FIND: (a) THE COUPLE  $M_0$ ,  
(b) THE DYNAMIC REACTIONS AT A AND B AT  $t=0$

$$\text{MASS OF ASSEMBLY} = m = \frac{12 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.37267 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\text{RADIUS OF SEMICIRCULAR PLATES} = r = 8 \text{ in.} = \frac{2}{3} \text{ ft}$$

$$\text{MOMENTS AND PRODUCTS OF INERTIA FOR ASSEMBLY: } I_x = 2 \left( \frac{m}{2} \right) \frac{r^2}{4} = \frac{1}{4} m r^2$$

$$\text{FOR EACH VERTICAL PLATE: } I_{xz} = -\frac{m}{4} \bar{x} \bar{y} = -\frac{m}{4} r \left( \frac{4}{3} r \right)$$

$$I_{xy} = -\frac{m r^2}{3\pi} \quad I_{yz} = 0$$

$$\text{FOR EACH HORIZONTAL PLATE: } I_{xy} = 0 \quad I_{yz} = -\frac{m r^2}{3\pi}$$

$$\text{FOR ASSEMBLY: } I_{xy} = I_{yz} = 2 \left( -\frac{m r^2}{3\pi} \right) = -\frac{2 m r^2}{3\pi}$$

ANGULAR MOMENTUM

FROM EQS. (18.13) WITH  $\omega_x = \omega$ ,  $\omega_y = \omega_z = 0$ :

$$\mathbf{H}_C = I_x \omega \mathbf{i} - I_{yz} \omega \mathbf{j} - I_{zy} \omega \mathbf{k} = m r^2 \omega \left( \frac{1}{4} \mathbf{i} + \frac{2}{3\pi} \mathbf{j} + \frac{2}{3\pi} \mathbf{k} \right) \quad (1)$$

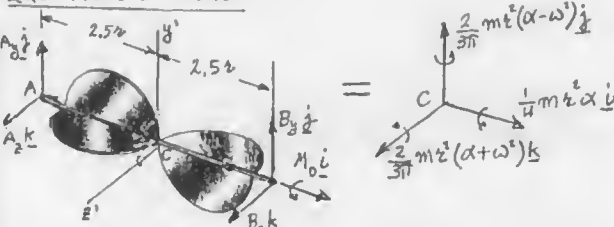
$$\text{EQ. (18.22): } \dot{\mathbf{H}}_C = (\dot{\mathbf{H}})_{Cxyz} + \boldsymbol{\Omega} \times \mathbf{H}_C$$

$$\dot{\mathbf{H}}_C = m r^2 \alpha \left( \frac{1}{4} \mathbf{i} + \frac{2}{3\pi} \mathbf{j} + \frac{2}{3\pi} \mathbf{k} \right) + \omega \mathbf{i} \times m r^2 \omega \left( \frac{1}{4} \mathbf{i} + \frac{2}{3\pi} \mathbf{j} + \frac{2}{3\pi} \mathbf{k} \right)$$

$$= m r^2 \alpha \left( \frac{1}{4} \mathbf{i} + \frac{2}{3\pi} \mathbf{j} + \frac{2}{3\pi} \mathbf{k} \right) + \frac{2}{3\pi} m r^2 \omega^2 (\mathbf{k} - \mathbf{j})$$

$$\dot{\mathbf{H}}_C = \frac{1}{4} m r^2 \alpha \mathbf{i} + \frac{2}{3\pi} m r^2 (\alpha - \omega^2) \mathbf{j} + \frac{2}{3\pi} m r^2 (\alpha + \omega^2) \mathbf{k} \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \mathbf{M}_A = \Sigma (\mathbf{M}_A)_{\text{eff}}: 5r \mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) + M_0 \mathbf{i} = \dot{\mathbf{H}}_C$$

$$5r B_y \mathbf{k} - 5r B_z \mathbf{j} + M_0 \mathbf{i} = \frac{1}{4} m r^2 \alpha \mathbf{i} + \frac{2}{3\pi} m r^2 (\alpha - \omega^2) \mathbf{j} + \frac{2}{3\pi} m r^2 (\alpha + \omega^2) \mathbf{k} \quad (3)$$

(a) COUPLE  $M_0$

$$\text{EQUATE COEFF. OF } \mathbf{i}: M_0 = \frac{1}{4} m r^2 \alpha$$

SINCE ASSEMBLY ROTATES THROUGH  $\theta = 2\pi \text{ rad}$  IN 2 s:

$$\theta = \frac{1}{2} \alpha t^2, \alpha = 2\theta/t^2 = 4\pi/4 = \pi \text{ rad/s}^2 \text{ .. THUS:}$$

$$M_0 = \frac{1}{4} (0.37267 \text{ lb}\cdot\text{s}^2/\text{ft}) \left( \frac{\pi}{2} \text{ ft} \right) (\pi \text{ rad/s}^2) = 0.1301 \text{ lb}\cdot\text{ft}$$

$$M_0 = (0.1301 \text{ lb}\cdot\text{ft}) \mathbf{i}$$

(b) DYNAMIC REACTIONS AT  $t=0$

EQUATING THE COEFF. OF  $\mathbf{j}$  AND  $\mathbf{k}$  IN (3) AND SETTING  $\omega=0$  AND  $\alpha=\pi \text{ rad/s}^2$ :

$$\text{A: } -5r B_z = \frac{2}{3\pi} m r^2 (\pi \text{ rad/s}^2), B_z = -\frac{2}{15} (0.37267) \left( \frac{\pi}{2} \right) = -0.0331 \text{ lb}$$

$$\text{B: } 5r B_y = \frac{2}{3\pi} m r^2 (\pi \text{ rad/s}^2), B_y = +0.0331 \text{ lb}$$

$$\text{THUS: } \mathbf{B} = (0.0331 \text{ lb}) \mathbf{j} - (0.0331 \text{ lb}) \mathbf{k}$$

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}}: \mathbf{A} + \mathbf{B} = 0, \mathbf{A} = -(0.0331 \text{ lb}) \mathbf{j} + (0.0331 \text{ lb}) \mathbf{k}$$

# 18.78 GIVEN: ASSEMBLY OF PROB. 18.77

FIND: DYNAMIC REACTIONS AT A AND B AT  $t = 2.5$ .

SEE SOLUTION OF PROB. 18.77 FOR DERIVATION OF EQ. (3):

$$5zB_y k - 5zB_z j + M_G i = \frac{1}{2} m l^2 \alpha i + \frac{2}{3} m l^2 (\alpha - \omega) j + \frac{2}{3} m l^2 (\alpha + \omega) k \quad (3)$$

WHERE  $m = 0.37267 \text{ lb} \cdot \text{s}^2/\text{ft}$  AND  $l = \frac{2}{3} \text{ ft}$

SINCE ASSEMBLY ROTATES THROUGH  $\theta = 2\pi \text{ rad}$  IN  $2 \text{ s}$ :

$$\theta = \frac{1}{2} \alpha t^2, \quad \alpha = 2\theta/t^2 = 4\pi/4 = \pi \text{ rad/s}^2$$

$$\text{AT } t = 2.5: \quad \omega = \alpha t = (\pi \text{ rad/s}^2)(2.5) = 2.5\pi \text{ rad/s}$$

EQUATING THE COEFF. OF  $i$  AND  $k$  IN EQ. (3) AND

SUBSTITUTING THE ABOVE VALUES:

$$\textcircled{1} -5zB_z = \frac{2}{3} m l^2 (\pi - 4\pi) \quad B_z = -\frac{2}{15} m l^2 (1 - 4\pi)$$

$$B_z = -\frac{2}{15} (0.37267)(\frac{2}{3})^2 (1 - 4\pi) = +0.383 \text{ lb}$$

$$\textcircled{2} 5zB_y = \frac{2}{3} m l^2 (\pi + 4\pi) \quad B_y = \frac{2}{15} m l^2 (1 + 4\pi)$$

$$B_y = \frac{2}{15} (0.37267)(\frac{2}{3})^2 (1 + 4\pi) = 0.449 \text{ lb}$$

THUS:

$$\underline{B} = (0.449 \text{ lb}) j + (0.383 \text{ lb}) k$$

$$\Sigma \underline{F} = \Sigma (\underline{F})_{eff}: \quad \underline{A} + \underline{B} = 0$$

$$\underline{A} = -\underline{B} = -(0.449 \text{ lb}) j - (0.383 \text{ lb}) k$$

# 18.79 GIVEN:

FLYWHEEL RIGIDLY ATTACHED TO CRANK-SHAFT OF AUTOMOBILE ENGINE IS EQUIVALENT TO 100-MM-DIAM, 15-MM-THICK STEEL FILN. (DENSITY =  $7860 \text{ kg/m}^3$ ). AUTOMOBILE TRAVELS ON UNEASED CURVE OF 200-M RADIUS AT 90 km/h WITH FLYWHEEL ROTATING AT 2700 rpm.

FIND:

MAGNITUDE OF COUPLE EXERTED BY FLYWHEEL ON CRANK-SHAFT, ASSUMING AUTOMOBILE TO HAVE

(a) REAR-WHEEL DRIVE WITH ENGINE MOUNTED LONGITUDINALLY

(b) FRONT-WHEEL DRIVE WITH ENGINE MOUNTED TRANSVERSELY

(a) REAR-WHEEL DRIVE (LONGITUDINAL MOUNTING)

ASSUME SENSES SHOWN FOR  $\omega_x, \omega_y, \bar{v}$ .

$$\underline{\Omega} = \underline{\omega}_y = \omega_y j$$

$$\bar{v} = 90 \text{ km/h} = 25 \text{ m/s}$$

$$\omega_x = 2700 \text{ rpm} \left( \frac{2\pi \text{ rad}}{60 \text{ s}} \right) = 282.74 \text{ rad/s}$$

$$\omega_y = \frac{\bar{v}}{R} = \frac{25 \text{ m/s}}{200 \text{ m}} = 0.125 \text{ rad/s}$$

$$\underline{I}_x = \frac{1}{2} m \bar{r}^2 = \frac{1}{2} (\rho \pi \bar{r}^2 l) \bar{r}^2$$

$$\underline{I}_x = \frac{1}{2} (\rho \pi \bar{r}^2 l) \bar{r}^2 = \frac{1}{2} (7860 \text{ kg/m}^3) \pi (0.02 \text{ m})^2 (0.015 \text{ m}) = 0.29632 \text{ kg} \cdot \text{m}^2$$

ANGULAR MOMENTUM ABOUT G:

$$\underline{H}_G = \underline{I}_x \omega_x i + \underline{I}_y \omega_y j$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y j \times (\underline{I}_x \omega_x i + \underline{I}_y \omega_y j)$$

$$\dot{\underline{H}}_G = -\underline{I}_x \omega_x \omega_y k = -(0.29632 \text{ kg} \cdot \text{m}^2)(282.74 \text{ rad/s})(0.125 \text{ rad/s}) k$$

$$\dot{\underline{H}}_G = -(10.47 \text{ N} \cdot \text{m}) k$$

THE COUPLE EXERTED ON THE FLYWHEEL, THEREFORE, MUST BE  $\underline{M} = \underline{H}_G = -(10.47 \text{ N} \cdot \text{m}) k$ , AND THE COUPLE EXERTED BY THE FLYWHEEL IS  $-\underline{M} = (10.47 \text{ N} \cdot \text{m}) k$

ANSWER:  $10.47 \text{ N} \cdot \text{m}$

(CONTINUED)

# 18.79 continued

(b) FRONT-WHEEL DRIVE (TRANSVERSE MOUNTING)

WE ASSUME THE SAME DIRECTION OF MOTION OF THE CAR AS IN PART (a) REFERRING TO THE NUMERICAL VALUES FOUND IN PART (a):

$$\omega_x = 282.74 \text{ rad/s}$$

$$\omega_y = 0.125 \text{ rad/s}$$

$$\underline{I}_x = 0.29632 \text{ kg} \cdot \text{m}^2$$

ANGULAR MOMENTUM ABOUT G:

$$\underline{H}_G = \underline{I}_x \omega_x i + \underline{I}_y \omega_y j$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y j \times (\underline{I}_x \omega_x i + \underline{I}_y \omega_y j)$$

$$\dot{\underline{H}}_G = -\underline{I}_x \omega_x \omega_y k = -(0.29632 \text{ kg} \cdot \text{m}^2)(282.74 \text{ rad/s})(0.125 \text{ rad/s}) k$$

$$\dot{\underline{H}}_G = -(10.47 \text{ N} \cdot \text{m}) k$$

THE COUPLE EXERTED ON THE FLYWHEEL, THEREFORE, MUST BE  $\underline{M} = \underline{H}_G = -(10.47 \text{ N} \cdot \text{m}) k$ , AND THE COUPLE EXERTED BY THE FLYWHEEL IS  $-\underline{M} = (10.47 \text{ N} \cdot \text{m}) k$

ANSWER:  $10.47 \text{ N} \cdot \text{m}$

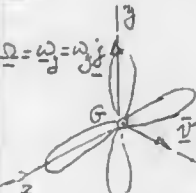
# 18.80 GIVEN:

FOUR-BLADED AIRPLANE PROPELLER

WITH  $m = 160 \text{ kg}$  AND  $\bar{r} = 800 \text{ mm}$  ROTATES AT 1600 rpm. AIRPLANE IS TRAVELING IN CIRCULAR PATH WITH  $R = 600 \text{ m}$  AT  $\bar{v} = 540 \text{ km/h}$ .

FIND:

MAGNITUDE OF COUPLE EXERTED BY PROPELLER ON ITS SHAFT.



WE ASSUME SENSES SHOWN

FOR  $\omega_x, \omega_y$ , AND  $\bar{v}$

$$\bar{v} = 540 \text{ km/h} = 150 \text{ m/s}$$

$$\omega_x = 1600 \text{ rpm} \left( \frac{2\pi \text{ rad}}{60 \text{ s}} \right)$$

$$= 167.55 \text{ rad/s}$$

$$\omega_y = \frac{\bar{v}}{R} = \frac{150 \text{ m/s}}{600 \text{ m}} = 0.25 \text{ rad/s}$$

$$\underline{I}_x = m \bar{r}^2 = (160 \text{ kg})(0.8 \text{ m})^2 = 102.4 \text{ kg} \cdot \text{m}^2$$

ANGULAR MOMENTUM ABOUT G:

$$\underline{H}_G = \underline{I}_x \omega_x i + \underline{I}_y \omega_y j$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y j \times (\underline{I}_x \omega_x i + \underline{I}_y \omega_y j)$$

$$\dot{\underline{H}}_G = -\underline{I}_x \omega_x \omega_y k = -(102.4 \text{ kg} \cdot \text{m}^2)(167.55 \text{ rad/s})(0.25 \text{ rad/s}) k$$

$$\dot{\underline{H}}_G = -(4289 \text{ N} \cdot \text{m}) k = -(4.29 \text{ kN} \cdot \text{m}) k$$

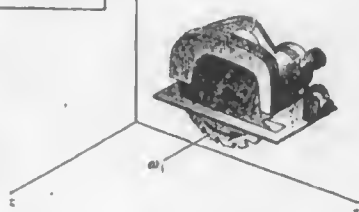
THE COUPLE EXERTED ON THE PROPELLER, THEREFORE, MUST BE  $\underline{M} = \underline{H}_G = -(4.29 \text{ kN} \cdot \text{m}) k$ , AND THE COUPLE EXERTED BY THE PROPELLER ON ITS SHAFT IS

$$-\underline{M} = (4.29 \text{ kN} \cdot \text{m}) k$$

ANSWER:  $4.29 \text{ kN} \cdot \text{m}$



18.81



GIVEN:

FOR BLADE AND ROTOR OF MOTOR OF PORTABLE SAW:

$W = 2.5 \text{ lb}$ ,  $R = 1.5 \text{ in}$ .  
BLADE ROTATES AT  
SHOWN AT RATE  
 $\omega_2 = 1500 \text{ rpm}$

FIND: COUPLE  $M$  THAT WORKER MUST EXERT ON HANDLE TO ROTATE SAW WITH CONSTANT  $\omega_2 = -(2.4 \text{ rad/s})\underline{i}$ .

USING AXES CENTERED AT MASS CENTER  $G$  OF BLADE AND ROTOR AND ROTATING WITH CASING:

$\omega_2 = \omega_1 = 1500 \text{ rpm} \left( \frac{2\pi \text{ rad}}{60 \text{ s}} \right) = 50\pi \text{ rad/s}$   
 $\Omega = \omega_1 = \omega_2 = -2.4 \text{ rad/s}$   
 $\bar{I}_x = \frac{W}{g} \bar{k}^2 = \frac{2.5 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{1.5 \text{ ft}}{12} \right)^2$   
 $= 1.2131 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$   
 ANGULAR MOMENTUM ABOUT  $G$ :  
 $\underline{H}_G = \bar{I}_x \omega_2 \underline{j} + \bar{I}_y \omega_2 \underline{k}$

EQ. (18.22):

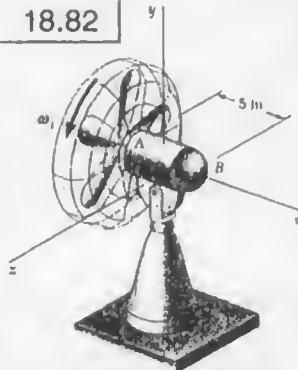
$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \Omega \times \underline{H}_G = 0 + \omega_2 \underline{j} \times (\bar{I}_x \omega_2 \underline{j} + \bar{I}_y \omega_2 \underline{k})$   
 $= \bar{I}_y \omega_2 \omega_2 \underline{i} = (1.2131 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2) (-2.4 \text{ rad/s}) (50\pi \text{ rad/s}) \underline{i}$   
 $\dot{\underline{H}}_G = -(0.457 \text{ lb} \cdot \text{ft}) \underline{i}$

THE COUPLE THAT THE WORKER MUST APPLY IS

$$\underline{M} = \dot{\underline{H}}_G$$

$$\underline{M} = -(0.457 \text{ lb} \cdot \text{ft}) \underline{i}$$

18.82



GIVEN:

FOR BLADE AND ROTOR OF MOTOR OF OSCILLATING FAN:  
 $W = 8 \text{ oz}$ ,  $\bar{k} = 3 \text{ in}$ .  
 BEARING SUPPORTS AT A AND B ARE 5 in. APART.  
 BLADE ROTATES AT RATE  
 $\omega_1 = 1800 \text{ rpm}$ .

FIND:

DYNAMIC REACTIONS AT A AND B WHEN MOTOR CASING HAS ANG. VEL.  $\omega_2 = (0.6 \text{ rad/s})\underline{j}$ .

ANGULAR MOMENTUM ABOUT MASS CENTER:

$$\underline{H}_G = \bar{I}_x \omega_2 \underline{i} + \bar{I}_y \omega_2 \underline{j} = \bar{I}_x \omega_1 \underline{i} + \bar{I}_y \omega_2 \underline{j}$$

EQ. (18.22):

$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \Omega \times \underline{H}_G = 0 + \omega_2 \underline{j} \times (\bar{I}_x \omega_1 \underline{i} + \bar{I}_y \omega_2 \underline{j})$   
 $\dot{\underline{H}}_G = -\bar{I}_x \omega_1 \omega_2 \underline{k} = -\left( \frac{8(16)}{32.2 \text{ ft/s}^2} \right) \left( \frac{3}{12} \text{ ft} \right) (1800 \text{ rpm}) \left( \frac{2\pi \text{ rad}}{60 \text{ s}} \right) (0.6 \text{ rad/s}) \underline{k}$   
 $= -(0.10976 \text{ lb} \cdot \text{ft}) \underline{k}$

THE REACTIONS AT A AND B FORM A COUPLE EQUIVALENT TO  $\dot{\underline{H}}_G$ :

$$A \left( \frac{5}{12} \text{ ft} \right) = 0.10976 \text{ lb} \cdot \text{ft}$$

$$A = 0.26343 \text{ lb} = 4.21 \text{ oz}$$

$$\underline{A} = (4.21 \text{ oz}) \underline{j}; \quad \underline{B} = -(4.21 \text{ oz}) \underline{j}$$

18.83

GIVEN:

AUTOMOBILE TRAVELS AROUND UNBANKED CURVE WITH  $R = 150 \text{ m}$  AT SPEED  $v = 95 \text{ km/h}$ .

FOR EACH WHEEL:  $m = 22 \text{ kg}$ ,  $\text{DIAM.} = 575 \text{ mm}$ ,  $\bar{r} = 225 \text{ mm}$ .  
 TRANSVERSE DISTANCE BETWEEN WHEELS = 1.5 m.

FIND: ADDITIONAL REACTION  $\Delta R$  EXERTED BY GROUND ON EACH OUTSIDE WHEEL DUE TO MOTION OF CAR.

FOR EACH WHEEL:  
 $v = 95 \text{ km/h} = 26.389 \text{ m/s}$   
 $\omega_1 = \frac{v}{R} = \frac{26.389 \text{ m/s}}{150 \text{ m}} = 0.17593 \text{ rad/s}$   
 $\omega_2 = -\frac{v}{\bar{r}} = -\frac{26.389 \text{ m/s}}{(0.575 \text{ m})/2} = -91.787 \text{ rad/s}$   
 $\bar{I}_2 = m \bar{k}^2 = (22 \text{ kg}) (0.225 \text{ m})^2 = 1.1138 \text{ kg} \cdot \text{m}^2$

ANGULAR MOMENTUM OF EACH WHEEL:

$$\underline{H}_G = \bar{I}_y \omega_1 \underline{j} + \bar{I}_z \omega_2 \underline{k}$$

EQ. (18.22):  $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \Omega \times \underline{H}_G = 0 + \omega_1 \underline{j} \times (\bar{I}_y \omega_1 \underline{j} + \bar{I}_z \omega_2 \underline{k})$

$$\dot{\underline{H}}_G = \bar{I}_z \omega_1 \omega_2 \underline{i} = (1.1138 \text{ kg} \cdot \text{m}^2) (0.17593 \text{ rad/s}) (-91.787 \text{ rad/s}) \underline{i} = -(17.986 \text{ N} \cdot \text{m}) \underline{i}$$

EQUATIONS OF MOTION FOR TWO WHEELS ON SAME AXLE:

$\Sigma \underline{M}_O = \Sigma (\underline{M}_O)_{\text{ext}} = -2(\Delta R)(0.75 \text{ m}) \underline{i} = -2(17.986 \text{ N} \cdot \text{m}) \underline{i}$   
 $\Delta R = 23.98 \text{ N} \quad \Delta R = 24.0 \text{ N} \uparrow$

18.84

GIVEN:

TYPE OF AIRCRAFT TURN INDICATOR.  
 UNIFORM DISK:  $m = 200 \text{ g}$ ,  $\bar{r} = 40 \text{ mm}$   
 SPINS AT RATE OF 10 000 rpm.  
 EACH SPRING HAS 500-N/m CONSTANT.  
 SPRINGS EXERT EQUAL FORCES ON YOKE AB IN STRAIGHT FLIGHT PATH.

FIND:

ANGLE OF ROTATION OF YOKE IN HORIZONTAL TURN OF 750-m RADIUS TO THE RIGHT WITH  $v = 800 \text{ km/h}$ . DOES A MOVE UP OR DOWN?

$\omega_2 = 10 000 \text{ rpm} = 1047.2 \text{ rad/s}$   
 $v = 800 \text{ km/h} = 222.2 \text{ m/s}$   
 $\omega_1 = -\frac{v}{R} = -\frac{222.2 \text{ m/s}}{750 \text{ m}} = -0.2963 \text{ rad/s}$   
 $\underline{H}_G = \bar{I}_x \omega_2 \underline{i} + \bar{I}_y \omega_1 \underline{j}$   
 EQ. (18.22):  $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \Omega \times \underline{H}_G = 0 + \omega_1 \underline{j} \times (\bar{I}_x \omega_2 \underline{i} + \bar{I}_y \omega_1 \underline{j})$   
 $\dot{\underline{H}}_G = -\bar{I}_x \omega_1 \omega_2 \underline{k} = -\frac{1}{2} (0.2 \text{ kg}) (0.04 \text{ m}) (1047.2 \text{ rad/s}) (-0.2963 \text{ rad/s}) \underline{k}$   
 $= + (0.049645 \text{ N} \cdot \text{m}) \underline{k}$

WE HAVE

$$(0.1 \text{ m}) F = \dot{\underline{H}}_G = 0.049645 \text{ N} \cdot \text{m}$$

$$F = 0.49645 \text{ N}$$

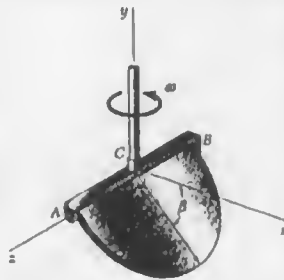
$$F = kx: x = \frac{0.49645 \text{ N}}{500 \text{ N/m}} = 0.9929 \text{ mm}$$

$$\theta = \frac{x}{GA} = \frac{0.9929 \text{ mm}}{50 \text{ mm}} = 0.01986 \text{ rad} = 1.14^\circ$$

SINCE SPRING AT A PULLS DOWN, A IS MOVING UP



18.85 and 18.86



GIVEN:

SEMICIRCULAR PLATE WITH  $r = 120 \text{ mm}$  IS HINGED TO CLEVIS WHICH ROTATES WITH CONSTANT  $\omega$ .

PROBLEM 18.85:

FIND:

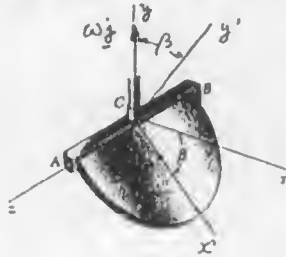
- (a)  $\beta$  WHEN  $\omega = 15 \text{ rad/s}$ ,  
(b) LARGEST  $\omega$  FOR WHICH PLATE REMAINS VERTICAL ( $\beta = 90^\circ$ )

PROBLEM 18.86:

FIND  $\omega$  FOR WHICH  $\beta = 50^\circ$ .

MOMENTS AND PRODUCTS OF INERTIA

WE USE THE AXES  $Cx'y'$  SHOWN.



WE NOTE THAT  $I_{x'}$  AND  $I_{y'}$  ARE HALF THOSE FOR A CIRCULAR PLATE, AND SO IS THE MASS  $m$ . THUS

$$I_{x'} = \frac{1}{4} m r^2$$

$$I_{y'} = \frac{1}{2} m r^2$$

BECAUSE OF SYMMETRY, ALL PRODUCTS OF INERTIA ARE EQUAL TO ZERO

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

ANGULAR MOMENTUM ABOUT C

$$\begin{aligned} \vec{H}_C &= I_{x'} \omega_x \hat{i}' + I_{y'} \omega_y \hat{j}' \\ &= \frac{1}{4} m r^2 (-\omega \sin \beta) \hat{i}' + \frac{1}{2} m r^2 (\omega \cos \beta) \hat{j}' \\ &= \frac{1}{4} m r^2 \omega (-\sin \beta \hat{i}' + 2 \cos \beta \hat{j}') \end{aligned}$$

SINCE C IS A FIXED POINT, WE CAN USE EQ. (18.28):

$$\Sigma \vec{M}_C = (\dot{\vec{H}}_C)_{Cx'y'z'} + \vec{\Omega} \times \vec{H}_C = 0 + \omega \hat{j} \times \vec{H}_C$$

OR, SINCE  $\hat{j} = -\hat{i}' \sin \beta + \hat{j}' \cos \beta$ :

$$\begin{aligned} \Sigma \vec{M}_C &= \omega (-\hat{i}' \sin \beta + \hat{j}' \cos \beta) \times \frac{1}{4} m r^2 \omega (-\sin \beta \hat{i}' + 2 \cos \beta \hat{j}') \\ &= \frac{1}{4} m r^2 \omega^2 (-2 \sin \beta \cos \beta \hat{k} + \cos \beta \sin \beta \hat{k}) \\ \Sigma \vec{M}_C &= -\frac{1}{4} m r^2 \omega^2 \sin \beta \cos \beta \hat{k} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{BUT } \Sigma \vec{M}_C &= -m g \bar{x}' \cos \beta \hat{k} \\ &= -m g \frac{4r}{3\pi} \cos \beta \hat{k} \quad (2) \end{aligned}$$

EQUATING (1) AND (2):

$$\begin{aligned} \frac{1}{4} m r^2 \omega^2 \sin \beta \cos \beta &= \frac{4 m g r}{3\pi} \cos \beta \\ \omega^2 \sin \beta &= \frac{16}{3\pi} \frac{g}{r} = \frac{16}{3\pi} \frac{9.81 \text{ m/s}^2}{0.12 \text{ m}} \quad \omega^2 \sin \beta = 138.78 \text{ s}^{-2} \quad (3) \end{aligned}$$

PROBLEM 18.85

(a) LET  $\omega = 15 \text{ rad/s}$  IN (3):  $\sin \beta = \frac{138.78}{(15)^2} = 0.61681$   
 $\beta = 38.1^\circ$

(b) LET  $\beta = 90^\circ$  IN (3):  $\omega^2 = 138.78 \text{ s}^{-2}$ ,  $\omega = 11.78 \text{ rad/s}$

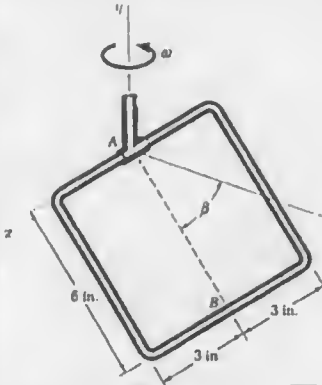
PROBLEM 18.86

LET  $\beta = 50^\circ$  IN EQ. (3):

$$\omega^2 = \frac{138.78 \text{ s}^{-2}}{\sin 50^\circ} = 181.17 \text{ s}^{-2}$$

$$\omega = 13.46 \text{ rad/s}$$

18.87 and 18.88



GIVEN:

ROD BENT TO FORM 6-in. SQUARE FRAME WHICH IS ATTACHED BY COLLAR A, & TO SHAFT ROTATING WITH CONSTANT  $\omega$ .

PROBLEM 18.87:

FIND:

- (a)  $\beta$  WHEN  $\omega = 9.8 \text{ rad/s}$ ,  
(b) LARGEST  $\omega$  FOR WHICH  $\beta = 90^\circ$ .

PROBLEM 18.88:

FIND  $\omega$  FOR WHICH  $\beta = 48^\circ$ .

MOMENTS AND PRODUCTS OF INERTIA. (MASS OF FRAME =  $m$ )

WE USE THE AXES  $Ax'y'z'$  SHOWN

FOR CD:

$$I_{x'} = I_{y'} = \frac{1}{12} m a^2 = \frac{1}{48} m a^2$$

$$\text{FOR EF: } I_{x'} = \frac{1}{48} m a^2$$

$$I_{y'} = \frac{1}{48} m a^2 + \frac{m}{4} a^2 = \frac{13}{48} m a^2$$

FOR CE OR DF:

$$I_{x'} = \frac{m}{4} \left(\frac{a}{2}\right)^2 = \frac{1}{16} m a^2$$

$$\begin{aligned} I_{y'} &= \frac{1}{48} m a^2 + \frac{m}{4} \left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right] \\ &= \left(\frac{1}{48} + \frac{1}{8}\right) m a^2 = \frac{7}{48} m a^2 \end{aligned}$$

FOR ENTIRE FRAME:

$$I_{x'} = \left[\frac{1}{48} + \frac{1}{48} + 2\left(\frac{1}{16}\right)\right] m a^2 = \frac{1}{6} m a^2; I_{y'} = \left[\frac{1}{48} + \frac{13}{48} + 2\left(\frac{7}{48}\right)\right] m a^2 = \frac{7}{12} m a^2$$

BECAUSE OF SYMMETRY:  $I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$

ANGULAR MOMENTUM ABOUT A

$$\vec{H}_A = I_{x'} \omega_x \hat{i}' + I_{y'} \omega_y \hat{j}' = \frac{1}{6} m a^2 (-\omega \sin \beta) \hat{i}' + \frac{7}{12} m a^2 (\omega \cos \beta) \hat{j}'$$

SINCE A IS FIXED, WE USE EQ. (18.28):

$$\Sigma \vec{M}_A = (\dot{\vec{H}}_A)_{Ax'y'z'} + \vec{\Omega} \times \vec{H}_A = 0 + \omega \hat{j} \times \vec{H}_A$$

OR, SINCE  $\hat{j} = -\hat{i}' \sin \beta + \hat{j}' \cos \beta$ :

$$\begin{aligned} \Sigma \vec{M}_A &= \omega (-\hat{i}' \sin \beta + \hat{j}' \cos \beta) \times \left[ \frac{1}{6} m a^2 (-\omega \sin \beta) \hat{i}' + \frac{7}{12} m a^2 (\omega \cos \beta) \hat{j}' \right] \\ &= \frac{1}{12} m a^2 \omega^2 (-7 \sin \beta \cos \beta \hat{k} + 2 \cos \beta \sin \beta \hat{k}) \end{aligned}$$

$$\Sigma \vec{M}_A = -\frac{5}{12} m a^2 \omega^2 \sin \beta \cos \beta \hat{k} \quad (1)$$

$$\text{BUT } \Sigma \vec{M}_A = -m g \left(\frac{a}{2}\right) \cos \beta \hat{k} \quad (2)$$

EQUATING (1) AND (2):

$$\frac{5}{12} m a^2 \omega^2 \sin \beta \cos \beta = -\frac{1}{2} m g a \cos \beta$$

$$\omega^2 \sin \beta = \frac{6}{5} \frac{g}{a} = \frac{6}{5} \frac{32.2 \text{ ft/s}^2}{(6/12) \text{ ft}} \quad \omega^2 \sin \beta = 77.28 \text{ s}^{-2} \quad (3)$$

PROBLEM 18.87

(a) LET  $\omega = 9.8 \text{ rad/s}$  IN (3):  $\sin \beta = \frac{77.28}{(9.8)^2} = 0.80466$   
 $\beta = 53.6^\circ$

(b) LET  $\beta = 90^\circ$  IN (3):

$$\omega^2 = 77.28 \text{ s}^{-2}$$

$$\omega = 8.79 \text{ rad/s}$$

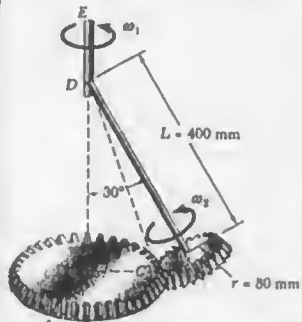
PROBLEM 18.88

LET  $\beta = 48^\circ$  IN EQ. (3):

$$\omega^2 = \frac{77.28 \text{ s}^{-2}}{\sin 48^\circ} = 103.99 \text{ s}^{-2}$$

$$\omega = 10.20 \text{ rad/s}$$

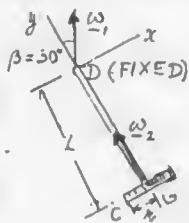
# 18.89 and 18.90



## PROBLEM 18.90:

FIND FORCE  $F$  EXERTED BY GEAR B ON GEAR A WHEN  $\omega_1 = 4 \text{ rad/s}$ . ( $F$  IS  $\perp$  CD.)

## ANGULAR VELOCITY OF GEAR A



WE USE THE AXES  $Gxy$  SHOWN AND EXPRESS THAT  $\vec{v}_C = 0$ :

$$\vec{v}_C = \omega \times \vec{DC} = 0$$

WHERE

$$\omega = (\omega_1 \cos \beta + \omega_2) \hat{j} + \omega_1 \sin \beta \hat{i} \quad (1)$$

$$\vec{DC} = -(L \hat{j} + \ell \hat{i})$$

THUS:

$$\vec{v}_C = -[(\omega_1 \cos \beta + \omega_2) \hat{j} + \omega_1 \sin \beta \hat{i}] \times (L \hat{j} + \ell \hat{i}) = 0$$

$$(\omega_1 \cos \beta + \omega_2) \ell \hat{k} - (\omega_1 \sin \beta) L \hat{k} = 0$$

$$\text{THUS: } \omega_1 \cos \beta + \omega_2 = (\omega_1 \sin \beta) (L/\ell)$$

$$\text{SUBSTITUTE INTO (1): } \omega = \omega_1 \sin \beta (\hat{i} + \frac{L}{\ell} \hat{j}) \quad (2)$$

## ANGULAR MOMENTUM ABOUT D:

$$\begin{aligned} H_D &= I_A \omega_2 \hat{i} + I_G \omega_1 \hat{j} = m(L^2 + \frac{\ell^2}{4}) \omega_1 \sin \beta \hat{i} + m \frac{\ell^2}{2} \omega_1 \frac{L}{\ell} \sin \beta \hat{j} \\ H_D &= m \omega_1 \sin \beta [(L^2 + \frac{\ell^2}{4}) \hat{i} + \frac{1}{2} \ell L \hat{j}] \quad (3) \end{aligned}$$

SINCE D IS A FIXED POINT, WE USE EQ. (18.2B):

$$\begin{aligned} \Sigma \vec{M}_D &= (\frac{d}{dt})_{D,xyz} H_D = 0 + (\omega_1 \sin \beta \hat{i} + \omega_1 \cos \beta \hat{j}) \times H_D \\ &= m \omega_1^2 \sin \beta [\frac{1}{2} \ell L \sin \beta - (L^2 + \frac{\ell^2}{4}) \cos \beta] \hat{k} \quad (4) \end{aligned}$$

PROB. 18.89: WHEN FORCE EXERTED BY GEAR B ON GEAR A BECOMES ZERO

$$\Sigma M_D = -mgL \sin \beta \hat{k} \quad (5)$$

EQUATING (4) AND (5):

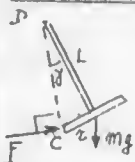
$$m \omega_1^2 \sin \beta [\frac{1}{2} \ell L \sin \beta - (L^2 + \frac{\ell^2}{4}) \cos \beta] = -mgL \sin \beta$$

$$\omega_1^2 [(L^2 + \frac{\ell^2}{4}) \cos \beta - \frac{1}{2} \ell L \sin \beta] = gL$$

$$\text{WITH } L = 0.4 \text{ m}, \ell = 0.08 \text{ m}, \beta = 30^\circ, g = 9.81 \text{ m/s}^2$$

$$0.1349 \omega_1^2 = 3.924 \quad \omega_1 = 5.45 \text{ rad/s}$$

## PROB. 18.90:



MOMENT OF FORCE AT D =  $F \sqrt{L^2 + \ell^2} \hat{k}$

THUS,  $\Sigma M_D$  (5) ABOVE MUST BE REPLACED BY

$$\Sigma M_D = (F \sqrt{L^2 + \ell^2} - mgL \sin \beta) \hat{k} \quad (6)$$

EQUATING (4) AND (6):

$$\begin{aligned} m \omega_1^2 \sin \beta [\frac{1}{2} \ell L \sin \beta - (L^2 + \frac{\ell^2}{4}) \cos \beta] &= F \sqrt{L^2 + \ell^2} - mgL \sin \beta \\ &= F \sqrt{L^2 + \ell^2} - mgL \sin \beta \end{aligned}$$

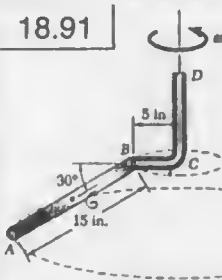
$$\text{WITH } L = 0.4 \text{ m}, \ell = 0.08 \text{ m}, \beta = 30^\circ, g = 9.81 \text{ m/s}^2, m = 0.95 \text{ kg}, \omega_1 = 4 \text{ rad/s}$$

$$0.95(4)^2 \sin 30^\circ - (0.1349) = F \sqrt{0.1664} - (0.95)(9.81)(0.4) \sin 30^\circ$$

$$0.40792 F = 1.8639 - 1.0028 = 0.8611 \quad F = 2.11 \text{ N}$$

$$\tan \delta = \frac{\ell}{L} = 0.2, \delta = 11.31^\circ, \beta - \delta = 18.7^\circ \quad F = 2.11 \text{ N} \angle 18.7^\circ$$

# 18.91



## GIVEN:

ROD AB IS ATTACHED BY A CLEVIS TO ARM BCD WHICH ROTATES WITH CONSTANT  $\omega$ .

## FIND:

MAGNITUDE OF  $\omega$

$$\text{LET } L = 15 \text{ in.} = 1.25 \text{ ft}$$

$$\text{THEN: } BC = 5 \text{ in.} = L/3$$

## ANGULAR MOMENTUM ABOUT G:

$$H_G = I_A \omega_2 \hat{i} + I_G \omega_1 \hat{j} = 0 + \frac{1}{12} mL^2 \omega \cos 30^\circ \hat{j}$$

$$\text{EQ. (18.22): } \dot{H}_G = (\frac{d}{dt})_{G,xyz} H_G + \omega \times H_G$$

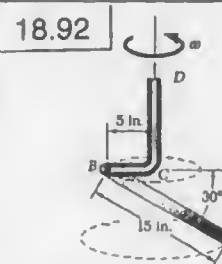
$$\dot{H}_G = 0 + \omega \times H_G = (\omega \sin 30^\circ \hat{i} + \omega \cos 30^\circ \hat{j}) \times \frac{1}{12} mL^2 \omega \cos 30^\circ \hat{j}$$

$$\dot{H}_G = \frac{1}{12} mL^2 \omega^2 \sin 30^\circ \cos 30^\circ \hat{k}$$

## EQUATIONS OF MOTION

$$\begin{aligned} \vec{M}_G &= \Sigma (\vec{M}_G)_{\text{ext}}: mg(\frac{L}{2} \cos 30^\circ) = \dot{H}_G + (m\vec{a}) (\frac{L}{2} \sin 30^\circ) \\ \frac{1}{2} mgL \cos 30^\circ &= \frac{1}{12} mL^2 \omega^2 \sin 30^\circ \cos 30^\circ + m(\frac{L}{2} \cos 30^\circ + \frac{L}{3}) \omega^2 (\frac{L}{2} \sin 30^\circ) \\ \frac{1}{2} \frac{g}{L} \cos 30^\circ &= (\frac{1}{3} \sin 30^\circ \cos 30^\circ + \frac{1}{6} \sin 30^\circ) \omega^2 \\ \frac{1}{2} \frac{32.2 \text{ ft/s}^2}{1.25 \text{ ft}} \cos 30^\circ &= 0.22767 \omega^2, \quad \omega^2 = 48.994 \\ \omega &= 7.00 \text{ rad/s} \end{aligned}$$

# 18.92



## GIVEN:

ROD AB IS ATTACHED BY A CLEVIS TO ARM BCD WHICH ROTATES WITH CONSTANT  $\omega$ .

## FIND:

MAGNITUDE OF  $\omega$

$$\text{LET } L = 15 \text{ in.} = 1.25 \text{ ft}$$

$$\text{THEN: } BC = 5 \text{ in.} = L/3$$

## ANGULAR MOMENTUM ABOUT G:

$$H_G = I_A \omega_2 \hat{i} + I_G \omega_1 \hat{j} = 0 + \frac{1}{12} mL^2 \omega \cos 30^\circ \hat{j}$$

$$\text{EQ. (18.22): } \dot{H}_G = (\frac{d}{dt})_{G,xyz} H_G + \omega \times H_G$$

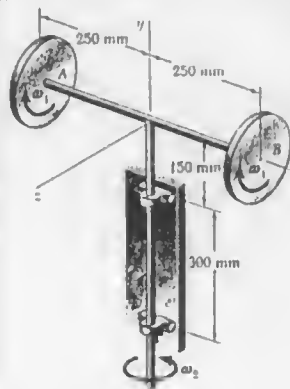
$$\dot{H}_G = (-\omega \sin 30^\circ \hat{i} + \omega \cos 30^\circ \hat{j}) \times \frac{1}{12} mL^2 \omega \cos 30^\circ \hat{j}$$

$$\dot{H}_G = -\frac{1}{12} mL^2 \omega^2 \sin 30^\circ \cos 30^\circ \hat{k}$$

## EQUATIONS OF MOTION

$$\begin{aligned} \vec{M}_G &= \Sigma (\vec{M}_G)_{\text{ext}}: mg(\frac{L}{2} \cos 30^\circ) = \dot{H}_G + (m\vec{a}) (\frac{L}{2} \sin 30^\circ) \\ \frac{1}{2} mgL \cos 30^\circ &= \frac{1}{12} mL^2 \omega^2 \sin 30^\circ \cos 30^\circ + m(\frac{L}{2} \cos 30^\circ - \frac{L}{3}) \omega^2 (\frac{L}{2} \sin 30^\circ) \\ \frac{1}{2} \frac{g}{L} \cos 30^\circ &= (\frac{1}{3} \sin 30^\circ \cos 30^\circ - \frac{1}{6} \sin 30^\circ) \omega^2 \\ \frac{1}{2} \frac{32.2 \text{ ft/s}^2}{1.25 \text{ ft}} \cos 30^\circ &= 0.061004 \omega^2, \quad \omega^2 = 182.85 \\ \omega &= 13.52 \text{ rad/s} \end{aligned}$$

# 18.93 and 18.94



## GIVEN:

FOR EACH DISK.

$$m = 5 \text{ kg}, z = 100 \text{ mm}$$

$$\omega_1 = 1500 \text{ rpm}$$

PROB 18.93:

FOR  $\omega_2 = 45 \text{ rpm}$  FIND DYNAMIC REACTIONS AT C AND D

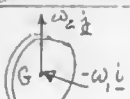
(a) BOTH DISKS ROTATE AS SHOWN

(b) DIRECTION OF SPIN OF B IS REVERSED

PROB. 18.94:

FIND MAX. ALLOWABLE  $\omega_2$  IF DYNAMIC REACTIONS AT C AND D ARE NOT TO EXCEED 250 N EACH.

ANGULAR MOMENTUM OF EACH DISK ABOUT ITS MASS CENTER



$$\vec{H}_A = \vec{I}_A \omega_1 \vec{k} = \frac{1}{2} m r^2 \omega_1 \vec{k}$$

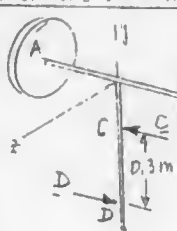
$$\vec{H}_B = \frac{1}{2} m r^2 (-\omega_2 \vec{k} + \omega_2 \vec{j}) \quad (1)$$

EQ. (18.22):

$$\dot{\vec{H}}_G = (\dot{\vec{H}}_G)_G + \vec{\Omega} \times \vec{H}_G = 0 + \omega_2 \vec{j} \times \frac{1}{2} m r^2 (-\omega_2 \vec{k} + \omega_2 \vec{j})$$

$$\dot{\vec{H}}_G = -\frac{1}{2} m r^2 \omega_2 \omega_2 \vec{i} \quad (2)$$

EQUATIONS OF MOTION



SINCE  $m\vec{a}_A$  AND  $m\vec{a}_B$  CANCEL OUT, EFFECTIVE FORCES REDUCE TO COUPLE  $2H = m r^2 \omega_1 \omega_2 \vec{k}$

IT FOLLOWS THAT THE REACTIONS FORM AN EQUIVALENT COUPLE WITH

$$-C = D = (m r^2 \omega_1 \omega_2 / 0.3 \text{ m}) \vec{i} \quad (3)$$

## PROBLEM 18.93

(a) WITH  $m = 5 \text{ kg}$ ,  $r = 0.1 \text{ m}$ ,  $\omega_1 = 1500 \text{ rpm} = 50\pi \text{ rad/s}$ , AND  $\omega_2 = 45 \text{ rpm} = 1.5\pi \text{ rad/s}$ , EQ. (3) YIELDS

$$C = D = (5 \text{ kg})(0.1 \text{ m})^2 (50\pi \text{ rad/s})(1.5\pi \text{ rad/s}) / 0.3 \text{ m} = 123.37 \text{ N}$$

$$C = -(123.4 \text{ N}) \vec{i}; D = (123.4 \text{ N}) \vec{i}$$

(b) WITH DIRECTION OF SPIN OF B REVERSED, ITS ANGULAR MOMENTUM WILL ALSO BE REVERSED AND THE EFFECTIVE FORCES (AND, THUS, THE APPLIED FORCES) REDUCE TO ZERO:

$$C = D = 0$$

## PROBLEM 18.94

MAKING  $C = D = 250 \text{ N}$  IN EQ. (3) YIELDS

$$\frac{m r^2 \omega_1 \omega_2}{0.3 \text{ m}} = 250 \text{ N}$$

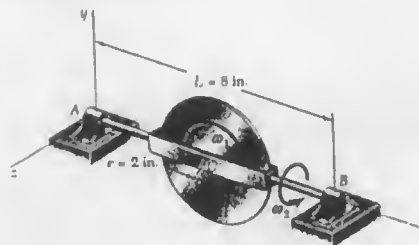
WITH  $m = 5 \text{ kg}$ ,  $r = 0.1 \text{ m}$ ,  $\omega_1 = 1500 \text{ rpm} = 50\pi \text{ rad/s}$

WE HAVE

$$\omega_2 = \frac{(250 \text{ N})(0.3 \text{ m})}{(5 \text{ kg})(0.1 \text{ m})^2 (50\pi \text{ rad/s})} = 9.5493 \text{ rad/s}$$

$$\omega_2 = 91.2 \text{ rpm}$$

# 18.95 and 18.96



GIVEN: 10-oz DISK SPINS AT RATE  $\omega_1 = 750 \text{ rpm}$

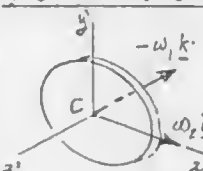
PROBLEM 18.95:

FOR  $\omega_2 = 6 \text{ rad/s}$  FIND THE DYNAMIC REACTIONS AT A AND B.

PROBLEM 18.96:

FIND MAX. ALLOWABLE  $\omega_2$  IF DYNAMIC REACTIONS AT A AND B ARE NOT TO EXCEED 0.25 lb EACH.

ANGULAR MOMENTUM ABOUT C



$$\vec{H}_C = \vec{I}_C \omega_1 \vec{k} + \vec{I}_C \omega_2 \vec{k}$$

$$= \frac{1}{2} m r^2 \omega_1 \vec{k} - \frac{1}{2} m r^2 \omega_2 \vec{k}$$

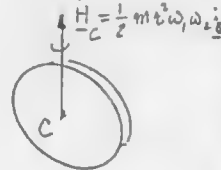
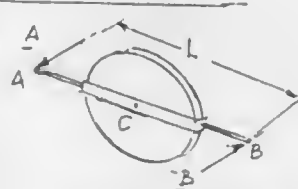
$$\vec{H}_C = \frac{1}{2} m r^2 (\omega_1 \vec{k} - \omega_2 \vec{k}) \quad (1)$$

EQ. (18.22):

$$\dot{\vec{H}}_C = (\dot{\vec{H}}_C)_C + \vec{\Omega} \times \vec{H}_C = 0 + \omega_2 \vec{j} \times \frac{1}{2} m r^2 (\omega_1 \vec{k} - \omega_2 \vec{k})$$

$$\dot{\vec{H}}_C = \frac{1}{2} m r^2 \omega_2 (\omega_1 \vec{i} + \omega_2 \vec{i}) \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \vec{M}_A = \Sigma (\vec{M}_A)_{\text{eff}}; BL = \frac{1}{2} m r^2 \omega_2 (\omega_1 + \omega_2) \quad A = B = \frac{m r^2 \omega_2 (\omega_1 + \omega_2)}{2L} \quad (3)$$

PROBLEM 18.95

$$\text{LETTING } m = \frac{W}{g} = \frac{(10/16) \text{ lb}}{32.2 \text{ ft/s}^2} = 0.01941 \text{ lb-s}^2/\text{ft}, r = \frac{1}{6} \text{ ft}, L = \frac{2}{3} \text{ ft}$$

$$\omega_1 = 750 \text{ rpm} = 25\pi \text{ rad/s}, \omega_2 = 6 \text{ rad/s IN EQ. (3):}$$

$$A = B = \frac{(0.01941 \text{ lb-s}^2/\text{ft})(\frac{1}{6} \text{ ft})^2 (25\pi \text{ rad/s})(6 \text{ rad/s})}{2(\frac{2}{3} \text{ ft})} = 0.1906 \text{ lb}$$

$$\vec{A} = (0.1906 \text{ lb}) \vec{k}; \vec{B} = -(0.1906 \text{ lb}) \vec{k}$$

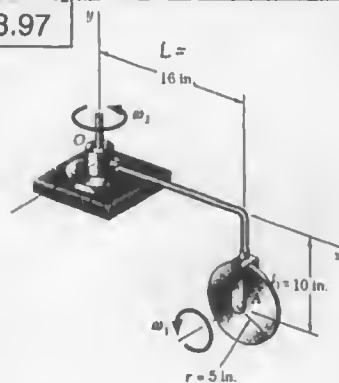
PROBLEM 18.96

LETTING  $A = B = 0.25 \text{ lb}$ ,  $m = 0.01941 \text{ lb-s}^2/\text{ft}$ ,  $r = \frac{1}{6} \text{ ft}$ ,  $L = \frac{2}{3} \text{ ft}$  AND  $\omega_1 = 750 \text{ rpm} = 25\pi \text{ rad/s}$  IN EQ. (3) AND SOLVING FOR  $\omega_2$ :

$$\omega_2 = \frac{2(\frac{2}{3} \text{ ft})(0.25 \text{ lb})}{(0.01941 \text{ lb-s}^2/\text{ft})(\frac{1}{6} \text{ ft})^2 (25\pi \text{ rad/s})} = 7.872 \text{ rad/s}$$

$$\omega_2 = 7.87 \text{ rad/s}$$

18.97



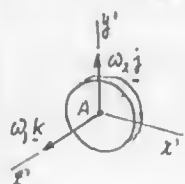
GIVEN:

DISK OF WEIGHT  
 $W = 8 \text{ lb}$  ROTATES AT  
 CONSTANT  $\omega_1 = 12 \text{ rad/s}$ .  
 ARM OF LENGTH  $L = 16 \text{ in.}$   
 ROTATES AT  
 CONSTANT  $\omega_2 = 4 \text{ rad/s}$ .

FIND:

FORCE-COUPLE  
 SYSTEM REPRESENTING  
 DYNAMIC REACTION  
 AT SUPPORT O.

ANGULAR MOMENTUM ABOUT A



$$\begin{aligned} \underline{H}_A &= \underline{I}_A \omega_1 \underline{j} + \underline{I}_A \omega_2 \underline{k} \\ &= \frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m L^2 \omega_2 \underline{k} \end{aligned} \quad (1)$$

$$\underline{H}_A = \frac{1}{4} m r^2 (\omega_1 \underline{j} + 2 \omega_2 \underline{k})$$

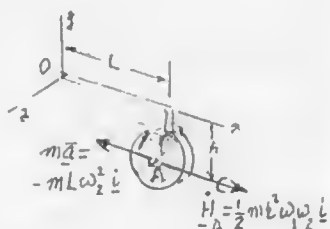
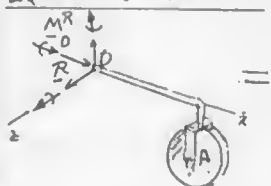
EQ. (18.22):

$$\dot{\underline{H}}_A = \left( \frac{d\underline{H}_A}{dt} \right)_{Axyz} + \underline{\Omega} \times \underline{H}_A = 0 + \omega_2 \underline{j} \times \underline{H}_A$$

$$\dot{\underline{H}}_A = \omega_2 \underline{j} \times \left( \frac{1}{4} m r^2 (\omega_1 \underline{j} + 2 \omega_2 \underline{k}) \right)$$

$$\dot{\underline{H}}_A = \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \underline{F} = \Sigma (\underline{F})_{\text{eff}}; \underline{R} = -m L \omega_2^2 \underline{i} \quad (3)$$

$$\begin{aligned} \Sigma \underline{M}_O &= \Sigma (\underline{M}_O)_{\text{eff}}; \\ \underline{M}_O^R &= \underline{H}_A + (\underline{L} \underline{i} - h \underline{j}) \times (-m L \omega_2^2 \underline{i}) \\ &= \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} - m h L \omega_2^2 \underline{k} \end{aligned} \quad (4)$$

WITH GIVEN DATA:

$$m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}, L = \frac{4}{3} \text{ ft}, h = \frac{5}{6} \text{ ft}$$

$$\omega_1 = 12 \text{ rad/s}, \omega_2 = 4 \text{ rad/s}, \underline{i} = \frac{5}{12} \text{ ft}$$

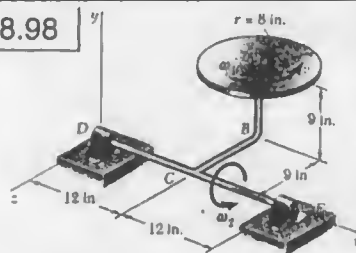
$$\text{EQ. (3): } \underline{R} = -(0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left( \frac{4}{3} \text{ ft} \right) (4 \text{ rad/s})^2 \underline{i} = -(5.300 \text{ lb}) \underline{i}$$

$$\begin{aligned} \text{EQ. (4): } \underline{M}_O^R &= \frac{1}{2} (0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left( \frac{5}{12} \text{ ft} \right) (12 \text{ rad/s}) (4 \text{ rad/s}) \underline{i} \\ &\quad - (0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left( \frac{5}{6} \text{ ft} \right) \left( \frac{4}{3} \text{ ft} \right) (4 \text{ rad/s})^2 \underline{k} \\ &= (1.0352 \text{ lb} \cdot \text{ft}) \underline{i} - (4.417 \text{ lb} \cdot \text{ft}) \underline{k} \end{aligned}$$

FORCE-COUPLE AT O:

$$\underline{R} = -(5.30 \text{ lb}) \underline{i}; \underline{M}_O^R = (1.035 \text{ lb} \cdot \text{ft}) \underline{i} - (4.42 \text{ lb} \cdot \text{ft}) \underline{k}$$

18.98

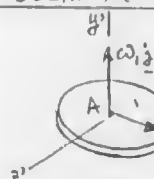


GIVEN:

DISK OF WEIGHT  
 $W = 6 \text{ lb}$  ROTATES AT  
 CONSTANT  $\omega_1 = 16 \text{ rad/s}$ .  
 ARM OF LENGTH  $L = 12 \text{ in.}$   
 ROTATES AT  
 CONSTANT  $\omega_2 = 8 \text{ rad/s}$ .

FIND: DYNAMIC REACTIONS AT D AND E.

ANGULAR MOMENTUM ABOUT A



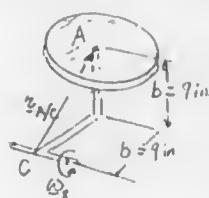
$$\begin{aligned} \underline{H}_A &= \underline{I}_A \omega_1 \underline{j} + \underline{I}_A \omega_2 \underline{k} \\ &= \frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m L^2 \omega_2 \underline{k} \end{aligned} \quad (1)$$

EQ. (18.22):

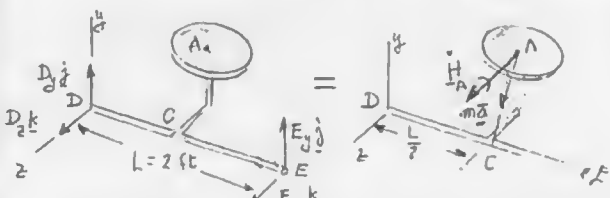
$$\dot{\underline{H}}_A = \left( \frac{d\underline{H}_A}{dt} \right)_{Axyz} + \underline{\Omega} \times \underline{H}_A = 0 + \omega_2 \underline{j} \times \underline{H}_A$$

$$\dot{\underline{H}}_A = \omega_2 \underline{j} \times \left( \frac{1}{4} m r^2 (\omega_1 \underline{j} + 2 \omega_2 \underline{k}) \right)$$

$$\dot{\underline{H}}_A = \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{k} \quad (2)$$

EFFECTIVE FORCE  $m \underline{\bar{a}}$ 

$$\begin{aligned} \underline{\bar{a}} &= -\underline{a}_C / \omega_2^2 \\ &= -(b \underline{j} - b \underline{k}) \omega_2^2 \\ m \underline{\bar{a}} &= m b \omega_2^2 (-\underline{j} + \underline{k}) \end{aligned} \quad (3)$$

EQUATIONS OF MOTION  
 APPLIED FORCES ARE  
 EQUIVALENT TO EFFECTIVE FORCES

$$\Sigma \underline{M}_D = \Sigma (\underline{M}_D)_{\text{eff}}; \underline{L} \underline{i} \times (\underline{E}_y \underline{j} + \underline{E}_z \underline{k}) = \dot{\underline{H}}_A + \left( \frac{\underline{L} \underline{i}}{L} \right) \times m \underline{\bar{a}}$$

$$\begin{aligned} \underline{L} \underline{i} \times (\underline{E}_y \underline{j} + \underline{E}_z \underline{k}) &= \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{k} + \frac{1}{2} \underline{L} \times m b \omega_2^2 (-\underline{j} + \underline{k}) \\ \underline{L} \underline{i} \times \underline{E}_y \underline{j} - \underline{L} \underline{i} \times \underline{E}_z \underline{k} &= \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{k} - \frac{1}{2} m b L \omega_2^2 \underline{j} - \frac{1}{2} m b L \omega_2^2 \underline{k} \end{aligned}$$

EQUATING COEFF. OF UNIT VECTORS:

$$\underline{E}_y = \frac{1}{2} m \left[ \left( \frac{L}{r} \right) \omega_1 \omega_2 - b \omega_2^2 \right] \quad \underline{E}_z = \frac{1}{2} m b \omega_2^2 \quad (4)$$

WITH GIVEN DATA:  $m = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.18634 \text{ lb} \cdot \text{s}^2/\text{ft}, \underline{i} = \frac{2}{3} \text{ ft}$   
 $L = 2 \text{ ft}, b = 0.75 \text{ ft}, \omega_1 = 16 \text{ rad/s}, \omega_2 = 8 \text{ rad/s}$ 

$$\underline{E}_y = -1.822 \text{ lb}, \underline{E}_z = 4.472 \text{ lb}$$

$$\underline{E} = -(1.822 \text{ lb}) \underline{j} + (4.472 \text{ lb}) \underline{k}$$

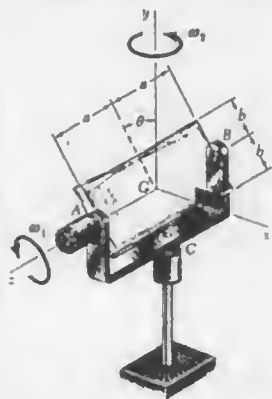
$$\Sigma \underline{F} = \Sigma \underline{F}_{\text{eff}}; \underline{D} + \underline{E} = m \underline{\bar{a}}$$

RECALLING (3) AND GIVEN DATA:

$$\begin{aligned} \underline{D} &= m \underline{\bar{a}} - \underline{E} = m b \omega_2^2 (-\underline{j} + \underline{k}) - \underline{E} \\ &= (0.18634)(0.75)(8)^2 (-\underline{j} + \underline{k}) + (1.822 \text{ lb}) \underline{j} - (4.472 \text{ lb}) \underline{k} \\ &= (1.822 - 8.944) \underline{j} + (0.944 - 4.472) \underline{k} \end{aligned}$$

$$\underline{D} = -(7.12 \text{ lb}) \underline{j} + (4.47 \text{ lb}) \underline{k}$$

# 18.99 and 18.100



## GIVEN:

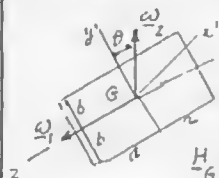
ADVERTISING PANEL  
 $m = 40 \text{ kg}$ ,  $2a = 2.4 \text{ m}$ ,  $2b = 1.6 \text{ m}$ .  
 MOTOR AT A KEEPS PANEL ROTATING ABOUT AB AT CONSTANT RATE  $\omega_1$ .  
 MOTOR AT C KEEPS FRAME ROTATING AT CONSTANT  $\omega_2$ .  
 PANEL COMPLETES FULL REVOLUTION IN 6 S.  
 FRAME COMPLETES FULL REVOLUTION IN 12 S.

## PROBLEM 18.99:

EXPRESS DYNAMIC REACTION AT D AS FUNCTION OF  $\theta$ .

## PROBLEM 18.100:

SHOW THAT (a) DYNAMIC REACTION AT D IS INDEPENDENT OF LENGTH  $2a$ ,  
 (b) AT ANY INSTANT  $M_1/M_2 = \omega_2/2\omega_1$ , WHERE  $M_1$  AND  $M_2$  ARE THE MAGNITUDES OF THE COUPLES EXERTED AT THE AXES AT A AND C, RESPECTIVELY.



USING AXES  $Gx'y'z'$  WITH  $x'$  PERPENDICULAR TO PANEL:  
 $\omega_{x'} = \omega_2 \sin \theta$ ,  $\omega_{y'} = \omega_2 \cos \theta$ ,  $\omega_{z'} = \omega_1$

$$\vec{H}_G = \vec{I}_{x'} \omega_{x'} \dot{\theta} \mathbf{i} + \vec{I}_{y'} \omega_{y'} \dot{\theta} \mathbf{j} + \vec{I}_{z'} \omega_{z'} \dot{\theta} \mathbf{k}$$

$$\vec{H}_G = \frac{1}{3} m (a^2 + b^2) \omega_2 \sin \theta \dot{\theta} \mathbf{i} + \frac{1}{3} m a^2 \omega_2 \cos \theta \dot{\theta} \mathbf{j} + \frac{1}{3} m b^2 \omega_1 \dot{\theta} \mathbf{k} \quad (1)$$

TO REVERT TO THE ORIGINAL FRAME  $Gxyz$ , WE NOTE THAT

$$\mathbf{i}' = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}$$

$$\mathbf{j}' = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j}$$

SUBSTITUTE IN (1):

$$\vec{H}_G = \frac{1}{3} m (a^2 + b^2) \omega_2 \sin \theta (\cos \theta \dot{\theta} \mathbf{i} + \sin \theta \dot{\theta} \mathbf{j}) + \frac{1}{3} m a^2 \omega_2 \cos \theta (-\sin \theta \dot{\theta} \mathbf{i} + \cos \theta \dot{\theta} \mathbf{j}) + \frac{1}{3} m b^2 \omega_1 \dot{\theta} \mathbf{k}$$

$$\vec{H}_G = \frac{1}{3} m [b^2 \omega_2 \sin \theta \cos \theta \dot{\theta} \mathbf{i} + (a^2 + b^2 \sin^2 \theta) \omega_2 \sin \theta \dot{\theta} \mathbf{j} + b^2 \omega_1 \dot{\theta} \mathbf{k}] \quad (2)$$

$$\text{EQ. (18.22): } \dot{\vec{H}}_G = (\dot{\vec{H}}_G)_{Gxyz} + \vec{\Omega} \times \vec{H}_G = (\dot{\vec{H}}_G)_{Gxyz} + \omega_2 \mathbf{j} \times \vec{H}_G \quad (3)$$

THE FIRST TERM IS OBTAINED BY DIFFERENTIATING (2) WITH RESPECT TO  $t$ , ASSUMING FRAME  $Gxyz$  TO BE FIXED:

$$(\dot{\vec{H}}_G)_{Gxyz} = \frac{1}{3} m [b^2 \omega_2 (\cos^2 \theta - \sin^2 \theta) \dot{\theta} \mathbf{i} + 2b^2 \omega_2 \sin \theta \cos \theta \dot{\theta} \mathbf{j} + b^2 \omega_1 \dot{\theta} \mathbf{k}]$$

OBSERVING THAT  $\dot{\theta} = \omega_2$ , AND SUBSTITUTING INTO (2):

$$\dot{\vec{H}}_G = \frac{1}{3} m b^2 \omega_2 \omega_2 [(\cos^2 \theta - \sin^2 \theta) \mathbf{i} + 2 \sin \theta \cos \theta \mathbf{j}] + \omega_2 \mathbf{j} \times \frac{1}{3} m b^2 \omega_2 \omega_2 [\cos \theta \mathbf{i} + \sin \theta \mathbf{j}] + \omega_1 \mathbf{k}$$

$$\dot{\vec{H}}_G = \frac{1}{3} m b^2 \omega_2^2 \omega_1 [(\cos^2 \theta - \sin^2 \theta) \mathbf{i} + 2 \sin \theta \cos \theta \mathbf{j}] + \frac{1}{3} m b^2 \omega_2 \omega_1 \omega_2 [-\sin \theta \mathbf{i} + \cos \theta \mathbf{j}]$$

$$\dot{\vec{H}}_G = \frac{1}{3} m b^2 \omega_2^2 \omega_1 \cos 2\theta \mathbf{i} + \frac{1}{3} m b^2 \omega_2^2 \omega_1 \sin 2\theta \mathbf{j} - \frac{1}{3} m b^2 \omega_2^2 \omega_1 \sin 2\theta \mathbf{i} + \frac{1}{3} m b^2 \omega_2^2 \omega_1 \cos 2\theta \mathbf{j}$$

$$\dot{\vec{H}}_G = \frac{1}{3} m b^2 \omega_2^2 \omega_1 \cos 2\theta \mathbf{i} + \frac{1}{3} m b^2 \omega_2^2 \omega_1 \sin 2\theta \mathbf{j} \quad (4)$$

THE REACTION AT D MUST BE EQUAL TO  $-\dot{\vec{H}}_G$

## PROBLEM 18.99, WITH GIVEN DATA:

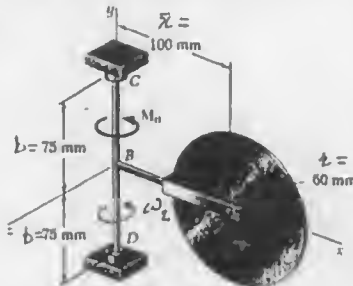
$$\vec{R}_D = \frac{1}{3} (40 \text{ kg}) (0.8 \text{ m})^2 [2 \left( \frac{2\pi}{6 \text{ s}} \right)^2 \cos 2\theta \mathbf{i} + 2 \left( \frac{2\pi}{12 \text{ s}} \right)^2 \sin 2\theta \mathbf{j} - \frac{1}{2} \left( \frac{2\pi}{6 \text{ s}} \right)^2 \sin 2\theta \mathbf{i}]$$

$$\vec{R}_D = (11.23 \text{ N/m}) \cos 2\theta \mathbf{i} + (5.61 \text{ N/m}) \sin 2\theta \mathbf{j} - (1.404 \text{ N/m}) \sin 2\theta \mathbf{i}$$

## PROBLEM 18.100

(a) EQ. (4) DOES NOT CONTAIN  $a$ .  
 (b) FROM (4):  $M_1 = \frac{1}{6} m b^2 \omega_2^2 \sin 2\theta$ ,  $M_2 = \frac{1}{3} m b^2 \omega_2 \omega_1 \sin 2\theta$   
 THUS:  $M_1/M_2 = \omega_2/2\omega_1$

# 18.101 and 18.102



## PROBLEM 18.101

GIVEN: 3-KG DISK SPINS AT CONSTANT  $\omega_1 = 60 \text{ rad/s}$ .  
 ARM AB AND SHAFT ARE AT REST WHEN  $M_1$  IS APPLIED FOR 3 S, WITH ANG. VELOCITY OF SHAFT REACHING  $18 \text{ rad/s}$ .  
 FIND: (a)  $M_0$ , (b) DYNAMIC REACTIONS AT C AND D AFTER  $M_0$  IS REMOVED.

## ANGULAR MOMENTUM ABOUT A

$$\vec{H}_A = \vec{I}_y \omega_1 \mathbf{j} + \vec{I}_z \omega_2 \mathbf{k} = \frac{1}{4} m r^2 \omega_1 \mathbf{j} + \frac{1}{2} m r^2 \omega_2 \mathbf{k}$$

$$\dot{\vec{H}}_A = \frac{1}{4} m r^2 (\dot{\omega}_1 \mathbf{j} + 2 \dot{\omega}_2 \mathbf{k}) \quad (1)$$

$$\text{EQ. (18.22): } \dot{\vec{H}}_A = (\dot{\vec{H}}_A)_{Axyz} + \vec{\Omega} \times \vec{H}_A$$

SINCE DISK HAS AN ANG. ACCEL.  $\alpha_2 \mathbf{j} = \dot{\omega}_2 \mathbf{j}$ , WE HAVE

$$(\dot{\vec{H}}_A)_{Axyz} = \frac{1}{4} m r^2 \alpha_2 \mathbf{j} = \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j}$$

ALSO,  $\vec{\Omega} = \omega_2 \mathbf{j}$

THUS:  $\dot{\vec{H}}_A = \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} + \omega_2 \mathbf{j} \times \frac{1}{2} m r^2 (\omega_1 \mathbf{j} + 2 \omega_2 \mathbf{k})$

$$\dot{\vec{H}}_A = \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} + \frac{1}{2} m r^2 \omega_2 \alpha_2 \mathbf{k} \quad (2)$$

## EQUATIONS OF MOTION

$$\vec{M}_0 \mathbf{j} = \dot{\vec{H}}_A$$

$$M_0 \mathbf{j} = \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} + \frac{1}{2} m r^2 \omega_2 \alpha_2 \mathbf{k}$$

$$M_0 = \frac{1}{4} m r^2 \dot{\omega}_2 + \frac{1}{2} m r^2 \omega_2 \alpha_2$$

$$M_0 = \frac{1}{4} m r^2 \dot{\omega}_2 + \frac{1}{2} m r^2 \omega_2 \alpha_2$$

$$M_0 = \frac{1}{4} m r^2 \dot{\omega}_2 + \frac{1}{2} m r^2 \omega_2 \alpha_2$$

FROM SYMMETRY AND INSPECTION OF EFFECTIVE FORCES, WE FIND THAT THE COMPONENTS OF THE REACTIONS AT C AND D ARE EQUAL IN MAGNITUDE AND DIRECTED AS SHOWN.

$$\sum M_y = \sum (M_y)_{eff}: M_0 = \frac{1}{4} m r^2 \alpha_2 + \frac{1}{2} m r^2 \omega_2 \alpha_2 = m \left( \frac{1}{4} \dot{\omega}_2 + \frac{1}{2} \omega_2 \alpha_2 \right)$$

$$M_0 = (3 \text{ kg}) \left[ \frac{1}{4} (0.06 \text{ m})^2 + (0.1 \text{ m})^2 \right] \alpha_2 \quad M_0 = 0.0327 \alpha_2 \quad (3)$$

$$\sum F_x = \sum (F_x)_{eff}: 2 R_x = m \ddot{x} = (3 \text{ kg}) (0.1 \text{ m}) \alpha_2^2, \quad R_x = 0.15 \omega_2^2 \quad (4)$$

$$\sum M_x = \sum (M_x)_{eff}: 2 b R_z = \frac{1}{2} m r^2 \omega_1 \omega_2$$

$$R_z = (m r^2 / 4 b) \omega_1 \omega_2 = (3 \text{ kg} \times 0.06 \text{ m}^2 / 4 \times 0.075 \text{ m}) (60 \text{ rad/s}) \omega_2, \quad R_z = 2.16 \omega_2 \quad (5)$$

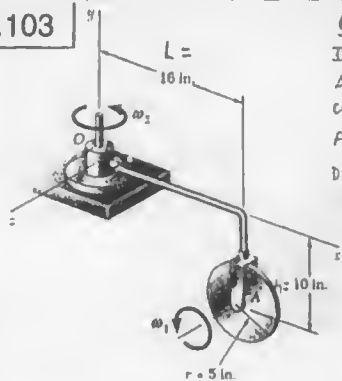
## PROBLEM 18.101

LET  $M_0 = 0.40 \text{ N}\cdot\text{m}$  IN (3):  $\alpha_2 = \frac{0.40}{0.0327} = 12.232 \text{ rad/s}^2$   
 FOR  $t = 2 \text{ s}$ :  $\omega_2 = \alpha_2 t = (12.232 \text{ rad/s}^2) (2 \text{ s}) = 24.464 \text{ rad/s}$   
 EQS. (4) AND (5):  $R_x = 0.15 \omega_2^2 = 89.8 \text{ N}$ ;  $R_z = 2.16 \omega_2 = 52.8 \text{ N}$   
 $\vec{C} = -(89.8 \text{ N}) \mathbf{i} + (52.8 \text{ N}) \mathbf{k}$ ;  $\vec{D} = -(89.8 \text{ N}) \mathbf{i} - (52.8 \text{ N}) \mathbf{k}$

## PROBLEM 18.102

$\omega_2 = \alpha_2 t$ :  $18 \text{ rad/s} = \alpha_2 (3 \text{ s})$ ,  $\alpha_2 = 6 \text{ rad/s}^2$   
 (a) EQ. (3):  $M_0 = 0.0327 (6) = 0.1962 \text{ N}\cdot\text{m}$   $M_0 = (0.1962 \text{ N}\cdot\text{m}) \mathbf{j}$   
 (b) EQ. (4):  $R_x = 0.15 (18 \text{ rad/s})^2 = 48.6 \text{ N}$   
 EQ. (5):  $R_z = 2.16 (18 \text{ rad/s}) = 38.88 \text{ N}$   
 $\vec{C} = -(48.6 \text{ N}) \mathbf{i} + (38.9 \text{ N}) \mathbf{k}$ ;  $\vec{D} = -(48.6 \text{ N}) \mathbf{i} - (38.9 \text{ N}) \mathbf{k}$

18.103



GIVEN:

DISK OF WEIGHT  $W = 8 \text{ lb}$ .  
AT INSTANT SHOWN  
 $\omega_1 = 12 \text{ rad/s}$  AND DECREASES  
AT RATE OF  $4 \text{ rad/s}^2$   
DUE TO BEARING FRICTION.  
ARM OR ROTATES AT  
CONSTANT  $\omega_2 = 4 \text{ rad/s}$

FIND:

FORCE-COUPLE SYSTEM  
REPRESENTING  
DYNAMIC REACTION  
AT SUPPORT O.

ANGULAR MOMENTUM ABOUT A

$$\begin{aligned} \mathbf{H}_A &= \bar{I}_y \omega_1 \mathbf{j} + \bar{I}_z \omega_2 \mathbf{k} = \frac{1}{4} m r^2 \omega_1 \mathbf{j} + \frac{1}{2} m r^2 \omega_2 \mathbf{k} \\ \mathbf{H}_A &= \frac{1}{4} m r^2 (\omega_1 \mathbf{j} + 2 \omega_2 \mathbf{k}) \quad (1) \\ \text{EQ. (18.22): } \dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Ax'y'z'} + \boldsymbol{\Omega} \times \mathbf{H}_A \end{aligned}$$

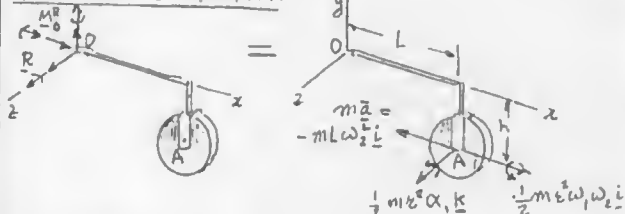
WHERE THE FIRST TERM IS OBTAINED BY DIFFERENTIATING  $\mathbf{H}_A$  ASSUMING THE FRAME  $Ax'y'z'$  TO BE FIXED:

$$(\dot{\mathbf{H}}_A)_{Ax'y'z'} = \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{k} = \frac{1}{2} m r^2 \alpha_1 \mathbf{k} \quad \text{WITH } \alpha_1 = -4 \text{ rad/s}^2$$

$$\text{THUS: } \dot{\mathbf{H}}_A = \frac{1}{2} m r^2 \alpha_1 \mathbf{k} + \omega_2 \mathbf{j} \times \frac{1}{4} m r^2 (\omega_1 \mathbf{j} + 2 \omega_2 \mathbf{k})$$

$$\dot{\mathbf{H}}_A = \frac{1}{2} m r^2 (\omega_1 \omega_2 \mathbf{i} + \alpha_1 \mathbf{k}) \quad (2)$$

EQUATION OF MOTION



$$\Sigma \mathbf{F} = \Sigma (\mathbf{F})_{\text{ext}}: \mathbf{R} = -mL\omega_1^2 \mathbf{i} \quad (3)$$

$$\Sigma \mathbf{M}_O = \Sigma (\mathbf{M}_O)_{\text{ext}}: \mathbf{M}_O^R = \dot{\mathbf{H}}_A + (L\mathbf{i} - h\mathbf{j}) \times (-mL\omega_1^2 \mathbf{i})$$

$$\mathbf{M}_O^R = \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{i} + \frac{1}{2} m r^2 \alpha_1 \mathbf{k} - m h L \omega_1^2 \mathbf{k}$$

$$\mathbf{M}_O^R = \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{i} + m \left( \frac{1}{2} r^2 \alpha_1 - h L \omega_1^2 \right) \mathbf{k} \quad (4)$$

WITH GIVEN DATA:

$$m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}, L = \frac{4}{3} \text{ ft}, h = \frac{5}{6} \text{ ft}, \mathbf{i} = \frac{5}{12} \text{ ft}$$

$$\omega_1 = 12 \text{ rad/s}, \alpha_1 = -4 \text{ rad/s}^2, \omega_2 = 4 \text{ rad/s}$$

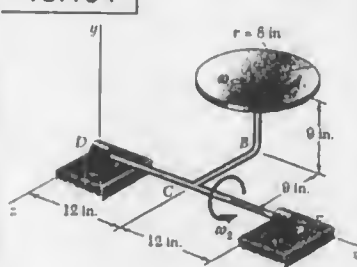
$$\text{EQ. (3): } \mathbf{R} = -(0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left( \frac{4}{3} \text{ ft} \right) (4 \text{ rad/s})^2 \mathbf{i} \\ = -(5.300 \text{ lb}) \mathbf{i}$$

$$\text{EQ. (4): } \mathbf{M}_O^R = \frac{1}{2} (0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left( \frac{5}{12} \text{ ft} \right) (12 \text{ rad/s}) (4 \text{ rad/s}) + \\ + (0.24845 \text{ lb} \cdot \text{s}^2/\text{ft}) \left[ \left( \frac{1}{2} \left( \frac{5}{12} \text{ ft} \right)^2 (-4 \text{ rad/s}^2) - \left( \frac{5}{6} \text{ ft} \right) \left( \frac{4}{3} \text{ ft} \right) (4 \text{ rad/s})^2 \right) \right] \mathbf{k} \\ = (1.0352 \text{ lb} \cdot \text{ft}) \mathbf{i} - 0.24845 (0.34722 + 17.778) \mathbf{k} \\ \mathbf{M}_O^R = (1.0352 \text{ lb} \cdot \text{ft}) \mathbf{i} - (4.503 \text{ lb} \cdot \text{ft}) \mathbf{k}$$

FORCE-COUPLE AT O:

$$\mathbf{R} = -(5.30 \text{ lb}) \mathbf{i}; \mathbf{M}_O^R = (1.035 \text{ lb} \cdot \text{ft}) \mathbf{i} - (4.50 \text{ lb} \cdot \text{ft}) \mathbf{k}$$

18.104



GIVEN:

DISK OF WEIGHT  $W = 6 \text{ lb}$   
ROTATES WITH CONSTANT  
 $\omega_1 = (16 \text{ rad/s}) \mathbf{i}$ .  
AT INSTANT SHOWN, SHAFT  
DCE HAS  $\omega_2 = (8 \text{ rad/s}) \mathbf{i}$   
AND  $\alpha_2 = (6 \text{ rad/s}^2) \mathbf{i}$ .

FIND:

(a) COUPLE APPLIED  
TO SHAFT

(b) DYNAMIC REACTIONS  
AT D AND E.

ANGULAR MOMENTUM ABOUT A

$$\begin{aligned} \mathbf{H}_A &= \bar{I}_x \omega_1 \mathbf{i} + \bar{I}_y \omega_2 \mathbf{j} = \frac{1}{4} m r^2 \omega_1 \mathbf{i} + \frac{1}{2} m r^2 \omega_2 \mathbf{j} \\ \mathbf{H}_A &= \frac{1}{4} m r^2 (\omega_1 \mathbf{i} + 2 \omega_2 \mathbf{j}) \quad (1) \\ \text{EQ. (18.22): } \dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Ax'y'z'} + \boldsymbol{\Omega} \times \mathbf{H}_A \end{aligned}$$

WHERE THE FIRST TERM IS OBTAINED BY DIFFERENTIATING  $\mathbf{H}_A$  ASSUMING THE FRAME  $Ax'y'z'$  TO BE FIXED:

$$(\dot{\mathbf{H}}_A)_{Ax'y'z'} = \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} = \frac{1}{4} m r^2 \alpha_2 \mathbf{j}$$

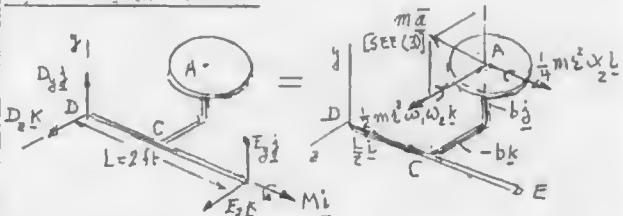
$$\text{THUS: } \dot{\mathbf{H}}_A = \frac{1}{4} m r^2 \alpha_2 \mathbf{j} + \omega_1 \mathbf{i} \times \frac{1}{4} m r^2 (\omega_1 \mathbf{i} + 2 \omega_2 \mathbf{j})$$

$$\dot{\mathbf{H}}_A = \frac{1}{4} m r^2 \alpha_2 \mathbf{j} + \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{k} \quad (2)$$

ACCELERATION OF MASS CENTER

$$\begin{aligned} \bar{\mathbf{a}} &= \alpha_2 \times \mathbf{r}_{A/C} - \omega_1^2 \mathbf{r}_{A/C} \\ &= \alpha_2 \mathbf{i} \times (b \mathbf{j} - b \mathbf{k}) - \omega_1^2 (b \mathbf{j} - b \mathbf{k}) \\ \bar{\mathbf{a}} &= b(\alpha_2 - \omega_1^2) \mathbf{j} + b(\alpha_2 + \omega_1^2) \mathbf{k} \quad (3) \end{aligned}$$

EQUATIONS OF MOTION



$$\Sigma \mathbf{M}_D = \Sigma (\mathbf{M}_D)_{\text{ext}}: \mathbf{L} \mathbf{i} \times (E_2 \mathbf{j} + E_2 \mathbf{k}) + \mathbf{M}_D^R = \left( \frac{1}{2} \mathbf{i} + b \mathbf{j} - b \mathbf{k} \right) \times m b [(\alpha_2 - \omega_1^2) \mathbf{j} + (\alpha_2 + \omega_1^2) \mathbf{k}] + \dot{\mathbf{H}}_A$$

$$L E_2 \mathbf{k} - L E_2 \mathbf{j} + \mathbf{M}_D^R = \frac{1}{2} m b L [(\alpha_2 - \omega_1^2) \mathbf{k} - (\alpha_2 + \omega_1^2) \mathbf{j}] + 2 m b^2 \alpha_2 \mathbf{i} + \frac{1}{4} m r^2 \alpha_2 \mathbf{j} + \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{k}$$

EQUATING COEFF. OF UNIT VECTORS:

$$\text{(i)} \quad M_D^R = m \left( 2b^2 + \frac{1}{4} r^2 \right) \alpha_2 \quad (4)$$

$$\text{(j)} \quad -L E_2 = -\frac{1}{2} m b L (\alpha_2 + \omega_1^2), \quad E_2 = \frac{1}{2} m b (\alpha_2 + \omega_1^2) \quad (5)$$

$$\text{(k)} \quad L E_2 = \frac{1}{2} m [b L (\alpha_2 - \omega_1^2) + \frac{1}{2} r^2 \omega_1 \omega_2] \\ E_2 = \frac{1}{2} m [b (\alpha_2 - \omega_1^2) + \frac{1}{2} r^2 \omega_1 \omega_2] \quad (6)$$

$$\Sigma F_y = \Sigma (F_y)_{\text{ext}}: D_y + E_y = m a_y, \quad D_y = m b (\alpha_2 - \omega_1^2) - E_y \\ D_y = \frac{1}{2} m [b (\alpha_2 - \omega_1^2) - \frac{1}{2} r^2 \omega_1 \omega_2] \quad (7)$$

$$\Sigma F_z = \Sigma (F_z)_{\text{ext}}: D_z + E_z = m a_z, \quad D_z = m b (\alpha_2 + \omega_1^2) - E_z \\ D_z = \frac{1}{2} m b (\alpha_2 + \omega_1^2)$$

WITH GIVEN DATA:

$$m = 6/32.2 = 0.18634, L = 2 \text{ ft}, b = \frac{3}{4} \text{ ft}, r = \frac{2}{3} \text{ ft}, \omega_1 = 16, \alpha_2 = 6$$

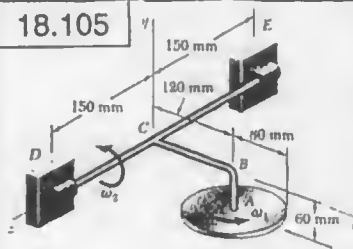
$$\text{(a)} \quad M_D^R = 1.382 \text{ lb} \cdot \text{ft}$$

$$M_D^R = (1.382 \text{ lb} \cdot \text{ft}) \mathbf{i}$$

$$\text{(b)} \quad D_y = -6.70 \text{ lb}, E_y = -1.403 \text{ lb}, D_z = E_z = 4.89 \text{ lb}$$

$$\mathbf{D} = -(6.70 \text{ lb}) \mathbf{j} + (4.89 \text{ lb}) \mathbf{k}; \mathbf{E} = -(1.403 \text{ lb}) \mathbf{j} + (4.89 \text{ lb}) \mathbf{k}$$

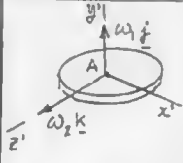
18.105



GIVEN:

2.5-KG DISK ROTATES WITH  
 $\omega_1 = \omega_1 \hat{j}$ ,  $\alpha_1 = -(15 \text{ rad/s}^2) \hat{j}$   
 SHAFT DCE ROTATES WITH  
 CONSTANT  $\omega_2 = (12 \text{ rad/s}) \hat{k}$ .  
 FIND:  
 DYNAMIC REACTIONS AT D  
 AND E WHEN  $\omega_1$  HAS  
 DECREASED TO 50 rad/s.

ANGULAR MOMENTUM ABOUT A



$$H_A = \bar{I}_A \omega_1 \hat{j} + \bar{I}_A \omega_2 \hat{k} = \frac{1}{2} m r^2 \omega_1 \hat{j} + \frac{1}{4} m r^2 \omega_2 \hat{k}$$

$$\text{Eq. (18.22): } \dot{H}_A = (\dot{H}_A)_{Ax'y'z'} + \underline{\Omega} \times H_A$$

THE FIRST TERM IS THE RATE OF CHANGE OF  $H_A$  WITH RESPECT TO THE ROTATING FRAME  $Ax'y'z'$ .

$$(\dot{H}_A)_{Ax'y'z'} = \frac{1}{2} m r^2 \dot{\omega}_1 \hat{j} = \frac{1}{2} m r^2 \alpha_1 \hat{j}; \quad \text{ALSO: } \underline{\Omega} = \omega_2 \hat{k}$$

$$\text{THUS: } \dot{H}_A = \frac{1}{2} m r^2 \alpha_1 \hat{j} + \omega_2 \hat{k} \times \left( \frac{1}{2} m r^2 \omega_1 \hat{j} + \frac{1}{4} m r^2 \omega_2 \hat{k} \right)$$

$$\dot{H}_A = \frac{1}{2} m r^2 \alpha_1 \hat{j} - \frac{1}{2} m r^2 \omega_1 \omega_2 \hat{i} = \frac{1}{2} m r^2 (-\omega_1 \omega_2 \hat{i} + \alpha_1 \hat{j})$$

$$= \frac{1}{2} (2.5 \text{ kg})(0.06 \text{ m})^2 [-(50 \text{ rad/s})(12 \text{ rad/s}) \hat{i} + (-15 \text{ rad/s}^2) \hat{j}]$$

$$\dot{H}_A = -(4.8 \text{ N}\cdot\text{m}) \hat{i} - (0.120 \text{ N}\cdot\text{m}) \hat{j} \quad (1)$$

ACCELERATION OF MASS CENTER

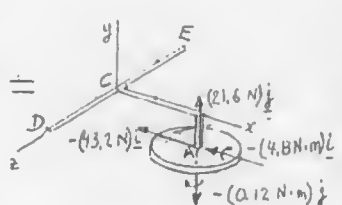
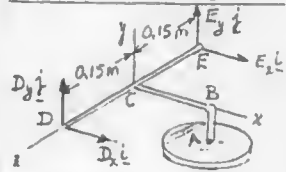
USING C AS THE FIXED ORIGIN, AND SINCE  $\alpha_2 = 0$ :

$$\underline{\underline{a}} = -\underline{r}_{AC} \omega_2^2 = -[(0.12 \text{ m}) \hat{i} - (0.06 \text{ m}) \hat{j}](12 \text{ rad/s})^2$$

$$\underline{\underline{a}} = -(17.28 \text{ m/s}^2) \hat{i} + (8.64 \text{ m/s}^2) \hat{j}$$

$$\text{THUS: } m \underline{\underline{a}} = (2.5 \text{ kg}) \underline{\underline{a}} \quad m \underline{\underline{a}} = -(43.2 \text{ N}) \hat{i} + (21.6 \text{ N}) \hat{j} \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \underline{M}_D = \Sigma (\underline{M}_D)_{\text{eff}}:$$

$$-(0.3 \text{ m}) \hat{k} \times (E_x \hat{i} + E_y \hat{j}) = -(4.8 \text{ N}\cdot\text{m}) \hat{i} - (0.12 \text{ N}\cdot\text{m}) \hat{j} + \underline{r}_{AD} \times m \underline{\underline{a}}$$

$$-0.3 E_x \hat{j} + 0.3 E_y \hat{i} = -4.8 \hat{i} - 0.12 \hat{j} + (-0.15 \hat{k} + 0.12 \hat{j} - 0.06 \hat{i}) \times (-43.2 \hat{i} + 21.6 \hat{j})$$

$$-0.3 E_x \hat{j} + 0.3 E_y \hat{i} = -4.8 \hat{i} - 0.12 \hat{j} + 6.48 \hat{i} + 3.24 \hat{j} + 12.54 \hat{k} - 2.52 \hat{k}$$

$$-0.3 E_x \hat{j} + 0.3 E_y \hat{i} = -1.56 \hat{i} + 16.36 \hat{j}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$-0.3 E_x = 6.36 \quad E_x = -21.2 \text{ N}$$

$$0.3 E_y = -1.56 \quad E_y = -5.20 \text{ N}$$

$$\Sigma F_x = \Sigma (F_x)_{\text{eff}}: D_x - 21.2 \text{ N} = -43.2 \text{ N} \quad D_x = -22.0 \text{ N}$$

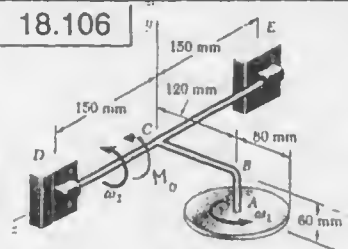
$$\Sigma F_y = \Sigma (F_y)_{\text{eff}}: D_y - 5.20 \text{ N} = 21.6 \text{ N} \quad D_y = 26.8 \text{ N}$$

ANSWER

$$\underline{D} = -(22.0 \text{ N}) \hat{i} + (26.8 \text{ N}) \hat{j}; \quad \underline{E} = -(21.2 \text{ N}) \hat{i} - (5.20 \text{ N}) \hat{j}$$

(ANSWER GIVEN WITH RESPECT TO ROTATING  $Cxyz$  AXES)

18.106

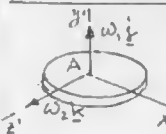


GIVEN:

2.5-KG DISK ROTATES WITH  
 CONSTANT  $\omega_1 = (50 \text{ rad/s}) \hat{j}$ .  
 AT INSTANT SHOWN, SHAFT  
 DCE ROTATES WITH  
 $\omega_2 = (12 \text{ rad/s}) \hat{k}$ ,  $\alpha_2 = (8 \text{ rad/s}^2) \hat{k}$ .  
 FIND:  
 (a) COUPLE  $M_0$  APPLIED  
 TO THE SHAFT,

(b) DYNAMIC REACTIONS AT D AND E

ANGULAR MOMENTUM ABOUT A



$$H_A = \bar{I}_A \omega_1 \hat{j} + \bar{I}_A \omega_2 \hat{k} = \frac{1}{2} m r^2 \omega_1 \hat{j} + \frac{1}{4} m r^2 \omega_2 \hat{k}$$

$$\text{Eq. (18.22): } \dot{H}_A = (\dot{H}_A)_{Ax'y'z'} + \underline{\Omega} \times H_A$$

THE FIRST TERM IS THE RATE OF CHANGE OF  $H_A$  WITH RESPECT TO THE FRAME  $Ax'y'z'$  WHICH ROTATES AT  $\underline{\Omega} = \omega_2 \hat{k}$ .

$$(\dot{H}_A)_{Ax'y'z'} = \frac{1}{4} m r^2 \dot{\omega}_2 \hat{k} = \frac{1}{4} m r^2 \alpha_2 \hat{k}$$

$$\text{THUS: } \dot{H}_A = \frac{1}{4} m r^2 \alpha_2 \hat{k} + \omega_2 \hat{k} \times \left( \frac{1}{2} m r^2 \omega_1 \hat{j} + \frac{1}{4} m r^2 \omega_2 \hat{k} \right)$$

$$\dot{H}_A = \frac{1}{4} m r^2 \alpha_2 \hat{k} - \frac{1}{2} m r^2 \omega_1 \omega_2 \hat{i} = \frac{1}{4} m r^2 (-2 \omega_1 \omega_2 \hat{i} + \alpha_2 \hat{k})$$

$$= \frac{1}{4} (2.5 \text{ kg})(0.06 \text{ m})^2 [-2(50 \text{ rad/s})(12 \text{ rad/s}) \hat{i} + (8 \text{ rad/s}^2) \hat{k}]$$

$$\dot{H}_A = -(4.8 \text{ N}\cdot\text{m}) \hat{i} + (0.032 \text{ N}\cdot\text{m}) \hat{k} \quad (1)$$

ACCELERATION OF MASS CENTER

USING C AS THE FIXED ORIGIN.

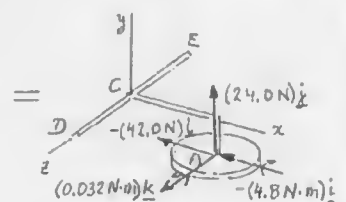
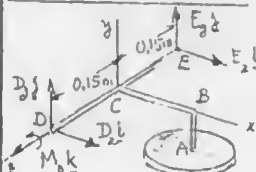
$$\underline{\underline{a}} = \alpha_2 \times \underline{r}_{AC} - \omega_2^2 \underline{r}_{AC} = (8 \text{ rad/s}^2) \hat{k} \times [(0.12 \text{ m}) \hat{i} - (0.06 \text{ m}) \hat{j}] - [(12 \text{ rad/s})^2] [(0.12 \text{ m}) \hat{i} - (0.06 \text{ m}) \hat{j}]$$

$$= (0.96 \text{ m/s}^2) \hat{j} + (0.48 \text{ m/s}^2) \hat{i} - (17.28 \text{ m/s}^2) \hat{i} + (8.64 \text{ m/s}^2) \hat{j}$$

$$\underline{\underline{a}} = -(16.8 \text{ m/s}^2) \hat{i} + (9.6 \text{ m/s}^2) \hat{j}$$

$$\text{THUS: } m \underline{\underline{a}} = (2.5 \text{ kg}) \underline{\underline{a}} \quad m \underline{\underline{a}} = -(42.0 \text{ N}) \hat{i} + (24.0 \text{ N}) \hat{j} \quad (2)$$

EQUATIONS OF MOTION



$$\Sigma \underline{M}_D = \Sigma (\underline{M}_D)_{\text{eff}}:$$

$$-(0.3 \text{ m}) \hat{k} \times (E_x \hat{i} + E_y \hat{j}) + M_0 \hat{k} = -(4.8 \text{ N}\cdot\text{m}) \hat{i} + (0.032 \text{ N}\cdot\text{m}) \hat{k} + \underline{r}_{AD} \times m \underline{\underline{a}}$$

$$-0.3 E_x \hat{j} + 0.3 E_y \hat{i} + M_0 \hat{k} = -4.8 \hat{i} + 0.032 \hat{k} + (-0.15 \hat{k} + 0.12 \hat{j} - 0.06 \hat{i}) \times (-42 \hat{i} + 24 \hat{j})$$

$$-0.3 E_x \hat{j} + 0.3 E_y \hat{i} + M_0 \hat{k} = -4.8 \hat{i} + 0.032 \hat{k} + 6.30 \hat{j} + 3.60 \hat{i} + 12.80 \hat{k} - 2.52 \hat{k}$$

$$\text{EQUATING THE COEFF. OF UNIT VECTORS:}$$

$$(a) \text{ (i) } M_0 = 0.032 + 2.88 - 2.52 = 0.392 \text{ N}\cdot\text{m}$$

$$\underline{M}_0 = (0.392 \text{ N}\cdot\text{m}) \hat{k}$$

$$(b) \text{ (i) } -0.3 E_x = 6.30 \quad E_x = -21.0 \text{ N}$$

$$\text{(ii) } 0.3 E_y = -4.8 + 3.6 = -1.20 \quad E_y = -4.00 \text{ N}$$

$$\Sigma F_x = \Sigma (F_x)_{\text{eff}}: D_x - 21.0 \text{ N} = -42.0 \text{ N} \quad D_x = -21.0 \text{ N}$$

$$\Sigma F_y = \Sigma (F_y)_{\text{eff}}: D_y - 4.00 \text{ N} = 24.0 \text{ N} \quad D_y = 28.0 \text{ N}$$

$$\underline{D} = -(21.0 \text{ N}) \hat{i} + (28.0 \text{ N}) \hat{j}; \quad \underline{E} = -(21.0 \text{ N}) \hat{i} - (4.00 \text{ N}) \hat{j}$$

(ANSWER GIVEN WITH RESPECT TO ROTATING  $Cxyz$  AXES)



# 18.107 and 18.108

## GIVEN:

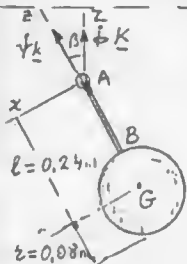
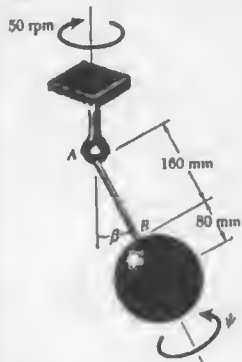
SOLID SPHERE WELDED TO END OF ROD AB OF NEGLIGIBLE MASS SUPPORTED BY BALL AND SOCKET AT A. SPHERE PRECEDES AT CONSTANT RATE OF 50 RPM AS SHOWN.

### PROBLEM 18.107:

FIND RATE OF SPIN  $\dot{\psi}$ , KNOWING THAT  $\beta = 25^\circ$ .

### PROBLEM 18.108:

FIND  $\beta$ , KNOWING THAT RATE OF SPIN IS  $\dot{\psi} = 800 \text{ rpm}$ .



### ANGULAR VELOCITIES:

SPHERE:  $\underline{\omega} = \dot{\phi} \underline{k} + \dot{\psi} \underline{k}$

$$\underline{\omega} = -\dot{\phi} \sin \beta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

FRAME  $AXYZ$ :  $\underline{\Omega} = \dot{\phi} \underline{k}$

$$\underline{\Omega} = -\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}$$

### ANGULAR MOMENTUM ABOUT A

$$\underline{H}_A = I_x \underline{\omega}_x \underline{i} + I_z \underline{\omega}_z \underline{k}$$

$$\underline{H}_A = -m \left( \frac{2}{5} z^2 + \ell^2 \right) \dot{\phi} \sin \beta \underline{i} + \frac{2}{5} m \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

SINCE A IS FIXED, WE USE EQ. (18.2B):

$$\begin{aligned} \Sigma \underline{M}_A &= (\underline{H}_A)_{\text{avg}} + \underline{\Omega} \times \underline{H}_A = 0 + (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times \underline{H}_A \\ &= (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times m \left[ -\left( \frac{2}{5} z^2 + \ell^2 \right) \dot{\phi} \sin \beta \underline{i} + \frac{2}{5} \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k} \right] \\ &= m \dot{\phi} \sin \beta \left[ \frac{2}{5} z^2 (\dot{\psi} + \dot{\phi} \cos \beta) - \left( \frac{2}{5} \ell^2 + \ell^2 \right) \dot{\phi} \cos \beta \right] \underline{j} \quad (1) \end{aligned}$$

$$\text{BUT } \Sigma \underline{M}_A = -\ell \underline{k} \times (-mg \underline{k}) = -mg \ell \sin \beta \underline{j} \quad (2)$$

EQUATING (1) AND (2):

$$\begin{aligned} m \dot{\phi} \sin \beta \left[ \frac{2}{5} z^2 (\dot{\psi} + \dot{\phi} \cos \beta) - \left( \frac{2}{5} \ell^2 + \ell^2 \right) \dot{\phi} \cos \beta \right] &= -mg \ell \sin \beta \\ \frac{2}{5} z^2 (\dot{\psi} + \dot{\phi} \cos \beta) &= \left( \frac{2}{5} \ell^2 + \ell^2 \right) \dot{\phi} \cos \beta - \frac{g \ell}{\dot{\phi}} \quad (3) \end{aligned}$$

GIVEN DATA: (NOTE THAT  $\dot{\phi}$  IS NEGATIVE)

$$z = 0.08 \text{ m}, \ell = 0.24 \text{ m}, g = 9.81 \text{ m/s}^2, \dot{\phi} = -50 \text{ rpm} = -5.236 \text{ rad/s}$$

$$2.56 \times 10^{-3} (\dot{\psi} - 5.236 \cos \beta) = 60.16 \times 10^{-3} (-5.236 \cos \beta) + 747.7 \times 10^{-3}$$

$$\dot{\psi} = -117.81 \cos \beta + 175.5 \quad (4)$$

### PROBLEM 18.107

LET  $\beta = 25^\circ$ , EQ. (4) READS

$$\begin{aligned} \dot{\psi} &= -117.81 \cos 25^\circ + 175.5 = +68.875 \text{ rad/s} \\ &= 657.7 \text{ rpm} \end{aligned}$$

$$\dot{\psi} = 658 \text{ rpm}$$

### PROBLEM 18.108

WITH  $\dot{\psi} = 800 \text{ rpm} = 83.776 \text{ rad/s}$ , EQ. (4) READS

$$83.776 = -117.81 \cos \beta + 175.5$$

$$\cos \beta = 0.77985 \quad \beta = 38.753^\circ$$

$$\beta = 38.8^\circ$$

# 18.109 and 18.110

CONE SUPPORTED BY BALL AND SOCKET AT A.

## PROBLEM 18.109:

### GIVEN:

PRECEDES AS SHOWN AT CONSTANT RATE OF 40 RPM WITH  $\beta = 40^\circ$

### FIND:

RATE OF SPIN  $\dot{\psi}$

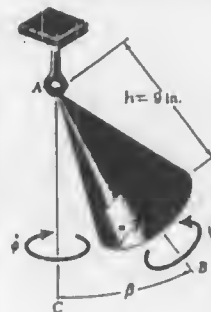
## PROBLEM 18.110

### GIVEN:

$\dot{\psi} = 3000 \text{ rpm}$ ,  $\beta = 60^\circ$

### FIND:

TWO POSSIBLE VALUES OF  $\dot{\phi}$



### ANGULAR VELOCITIES

CONE:  $\underline{\omega} = \dot{\phi} \underline{k} + \dot{\psi} \underline{k}$

$$\underline{\omega} = -\dot{\phi} \sin \beta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

FRAME  $AXYZ$ :  $\underline{\Omega} = -\dot{\phi} \underline{k}$

$$\underline{\Omega} = -\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}$$

### ANGULAR MOMENTUM ABOUT A

$$\underline{H}_A = I_x \underline{\omega}_x \underline{i} + I_z \underline{\omega}_z \underline{k}$$

$$\underline{H}_A = -\frac{3}{5} m \left( \frac{h^2}{4} + h^2 \right) \dot{\phi} \sin \beta \underline{i} + \frac{3}{5} m \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

SINCE A IS FIXED, WE USE EQ. (18.2B):

$$\begin{aligned} \Sigma \underline{M}_A &= (\underline{H}_A)_{\text{avg}} + \underline{\Omega} \times \underline{H}_A = 0 + (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times \underline{H}_A \\ &= (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times \left[ -\frac{3}{5} m \left( \frac{h^2}{4} + h^2 \right) \dot{\phi} \sin \beta \underline{i} + \frac{3}{5} m \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k} \right] \\ &= \frac{3}{5} m \dot{\phi} \sin \beta \left[ \frac{1}{2} \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) - \left( \frac{h^2}{4} + h^2 \right) \dot{\phi} \cos \beta \right] \underline{j} \quad (1) \end{aligned}$$

$$\text{BUT } \Sigma \underline{M}_A = -\frac{3}{4} h \underline{k} \times (-mg \underline{k}) = -\frac{3}{4} m g h \sin \beta \underline{j} \quad (2)$$

EQUATING (1) AND (2):

$$\frac{3}{5} m \dot{\phi} \sin \beta \left[ \frac{1}{2} \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) - \left( \frac{h^2}{4} + h^2 \right) \dot{\phi} \cos \beta \right] = -\frac{3}{4} m g h \sin \beta$$

$$\frac{1}{2} \ell^2 (\dot{\psi} + \dot{\phi} \cos \beta) - \left( \frac{h^2}{4} + h^2 \right) \dot{\phi} \cos \beta = -\frac{g h}{\dot{\phi}}$$

$$\frac{1}{2} \ell^2 \dot{\psi} - (h^2 - \frac{\ell^2}{2}) \dot{\phi} \cos \beta = -\frac{g h}{\dot{\phi}}$$

WITH  $\ell = \frac{1}{4} \text{ ft}$ ,  $h = \frac{3}{4} \text{ ft}$ ,  $g = 32.2 \text{ ft/s}^2$ , AND MULTIPLYING BY 32

$$\dot{\psi} - 17.5 \dot{\phi} \cos \beta = -966 / \dot{\phi} \quad (3)$$

### PROBLEM 18.109

LETTING  $\dot{\phi} = -40 \text{ rpm} = -4.1888 \text{ rad/s}$ ,  $\beta = 40^\circ$  IN (3),

$$\dot{\psi} - 17.5 (-4.1888) \cos 40^\circ = -966 / (-4.1888)$$

$$\dot{\psi} = -56.154 + 230.616 = 174.46 \text{ rad/s} = 1666.0 \text{ rpm}$$

$$\dot{\psi} = 1666 \text{ rpm}$$

### PROBLEM 18.110

LETTING  $\dot{\psi} = 3000 \text{ rpm} = 314.16 \text{ rad/s}$ ,  $\beta = 60^\circ$  IN (3),

$$314.16 - 17.5 \dot{\phi} \cos 60^\circ = -966 / \dot{\phi}$$

$$8.75 \dot{\phi}^2 - 314.16 \dot{\phi} - 966 = 0$$

$$\dot{\phi}^2 - 35.904 \dot{\phi} - 110.4 = 0$$

$$\dot{\phi} = \frac{1}{2} (35.904 \pm \sqrt{(35.904)^2 + 4(110.4)}) = \frac{1}{2} (35.904 \pm 41.602)$$

$$\dot{\phi} = +38.753 \text{ rad/s} = +370 \text{ rpm}$$

$$\dot{\phi} = (370 \text{ rpm}) \underline{k}$$

(SENSE OPPOSITE TO SENSE SHOWN)

$$\text{OR } \dot{\phi} = -2.849 \text{ rad/s} = -27.2 \text{ rpm}$$

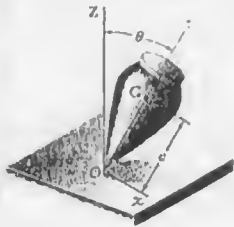
$$\dot{\phi} = -(27.2 \text{ rpm}) \underline{k}$$

(SAME SENSE AS SHOWN)



# 18.111 and 18.112

TOP SUPPORTED AT  
FIXED POINT O.



## PROBLEM 18.111:

GIVEN:

$m = 85 \text{ g}$ ,  $k_z = 21 \text{ mm}$ ,  $k_x = 45 \text{ mm}$   
 $c = 37.5 \text{ mm}$ ,  $\theta = 30^\circ$ ,  
RATE OF SPIN ABOUT Z AXIS =  
 $\dot{\psi} = 1800 \text{ rpm}$ .

FIND:

TWO POSSIBLE RATES  $\dot{\phi}$  OF  
STEADY PRECESSION.

## PROBLEM 18.112

GIVEN:  $I_z = I$ ,  $I_x = I'$ ,  $\omega_z = \text{RECTANGULAR}$   
COMPONENT OF  $\omega$  ALONG Z AXIS

(a) SHOW THAT  $(I\omega_z - I'\dot{\phi} \cos \theta)\dot{\phi} = Wc$

(b) SHOW THAT  $I\dot{\psi} \dot{\phi} \approx Wc$  IF  $\dot{\psi} \gg \dot{\phi}$

(c) FIND PERCENT ERROR WHEN EXPRESSION UNDER b  
IS USED TO APPROXIMATE THE SLOWER  $\dot{\phi}$  OF PROB. 18.111.

WE RECALL FROM PAGE 1150 THE FOLLOWING Eqs.

$$\omega = -\dot{\phi} \sin \theta \mathbf{i} + \omega_z \mathbf{k} \quad (18.40)$$

$$\mathbf{H}_O = -I'\dot{\phi} \sin \theta \mathbf{i} + I\omega_z \mathbf{k} \quad (18.41)$$

$$\dot{\mathbf{H}}_O = -\dot{\phi} \sin \theta \mathbf{i} + \dot{\phi} \cos \theta \mathbf{k} \quad (18.42)$$

SINCE O IS A FIXED POINT, WE USE EQ. (18.28):

$$\Sigma \mathbf{M}_O = (\dot{\mathbf{H}}_O)_{Oxyz} + \boldsymbol{\Omega} \times \mathbf{H}_O = \mathbf{0} + \boldsymbol{\Omega} \times \mathbf{H}_O$$

$$= (-\dot{\phi} \sin \theta \mathbf{i} + \dot{\phi} \cos \theta \mathbf{k}) \times (-I'\dot{\phi} \sin \theta \mathbf{i} + I\omega_z \mathbf{k})$$

$$= (I\omega_z \dot{\phi} \sin \theta - I'\dot{\phi}^2 \cos \theta \sin \theta) \mathbf{j}$$

$$= (I\omega_z - I'\dot{\phi} \cos \theta) \dot{\phi} \sin \theta \mathbf{j} \quad (1)$$

WHERE  $\mathbf{j}$  IS  $\perp$  PLANE OZE AND POINTS AWAY

$$\text{BUT } \Sigma \mathbf{M}_O = c \mathbf{k} \times (-W \mathbf{k}) = Wc \sin \theta \mathbf{j} \quad (2)$$

EQUATING EQS. (1) AND (2):

$$(I\omega_z - I'\dot{\phi} \cos \theta) \dot{\phi} = Wc \quad (3)$$

## PROBLEM 18.111

SINCE  $I = mk_z^2$ ,  $I' = mk_x^2$ ,  $W = mg$ , EQ. (3) YIELDS

$$(k_z^2 \omega_z - k_x^2 \dot{\phi} \cos \theta) \dot{\phi} = gc$$

WHERE  $\omega_z = \dot{\psi} + \dot{\phi} \cos \theta$

WITH GIVEN DATA AND  $\dot{\psi} = 1800 \text{ rpm} = 60\pi \text{ rad/s}$ :

$$[(0.021)^2 (60\pi + \dot{\phi} \cos 30^\circ) - (0.045)^2 \dot{\phi} \cos 30^\circ] \dot{\phi} = 9.81(0.0375)$$

$$[(0.045)^2 - (0.021)^2] \dot{\phi}^2 \cos 30^\circ - (0.021)^2 60\pi \dot{\phi} + 9.81(0.0375) = 0$$

$$\dot{\phi}^2 - 60.597 \dot{\phi} + 268.17 = 0$$

$$\text{SOLVING: } \dot{\phi} = 30.299 \pm 25.492$$

$$\dot{\phi} = 55.79 \text{ rad/s AND } \dot{\phi} = 4.807 \text{ rad/s}$$

$$\text{ANSWER: } 533 \text{ rpm AND } 45.7 \text{ rpm}$$

## PROBLEM 18.112

(a) SEE DERIVATION OF EQ. (3) ABOVE

(b) FOR  $\dot{\psi} \gg \dot{\phi}$ ,  $\omega_z \approx \dot{\psi}$ , AND EQ. (3) REDUCES TO

$$(I\dot{\psi} - I'\dot{\phi} \cos \theta) \dot{\phi} = Wc$$

AND, WITH  $\dot{\psi} \gg \dot{\phi}$ , TO

$$I\dot{\psi} \dot{\phi} = Wc \quad (\text{Q.E.D.})$$

(c) WITH DATA OF PROB. 18.111, ABOVE EQUATION YIELDS

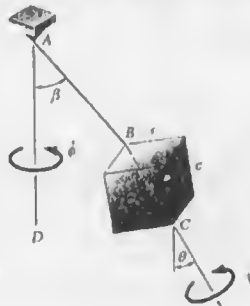
$$\dot{\phi} = \frac{Wc}{I\dot{\psi}} = \frac{mgc}{mk_z^2 \dot{\psi}} = \frac{9.81(0.0375)}{(0.021)^2 (60\pi \text{ rad/s})} = 4.455 \text{ rad/s}$$

$$= 42.26 \text{ rpm}$$

$$\% \text{ ERROR} = 100 \frac{42.26 - 45.90}{45.90} = -7.9\%$$

# 18.113 and 18.114

SOLID CUBE ATTACHED TO  
CORD AB



## PROBLEM 18.113:

GIVEN:

$c = 80 \text{ mm}$ ,  $\beta = 30^\circ$ ,  
 $\dot{\psi} = 40 \text{ rad/s}$ ,  $\dot{\phi} = 5 \text{ rad/s}$ .

FIND:  $\theta$

## PROBLEM 18.114:

GIVEN:

$c = 120 \text{ mm}$ ,  $AB = 240 \text{ mm}$ ,  
 $\theta = 25^\circ$ ,  $\beta = 40^\circ$

FIND:

(a)  $\dot{\psi}$ , (b)  $\dot{\phi}$

WE RECALL FROM SEC. 9.17 THAT, SINCE THE 3 PRINCIPAL  
MOMENTS OF INERTIA OF A CUBE ARE EQUAL, ITS MOMENT  
OF INERTIA ABOUT ANY LINE THROUGH G IS ALSO  $\bar{I} = \frac{1}{12} m c^2$   
USING Gxyz AXES WITH z ALONG CB, x IN ABD PLANE  
AND y  $\perp$  ABD AND POINTING AWAY, WE HAVE

$$\text{CUBE: } \omega = \dot{\phi} \sin \theta \mathbf{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{k}$$

$$\mathbf{H}_G = \frac{1}{6} m c^2 [\dot{\phi} \sin \theta \mathbf{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{k}]$$

FRAME Gxyz:

$$\boldsymbol{\Omega} = \dot{\phi} \sin \theta \mathbf{i} + \dot{\phi} \cos \theta \mathbf{k}$$

EQ. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$\dot{\mathbf{H}}_G = \mathbf{0} + (\dot{\phi} \sin \theta \mathbf{i} + \dot{\phi} \cos \theta \mathbf{k}) \times$$

$$\frac{1}{6} m c^2 [\dot{\phi} \sin \theta \mathbf{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{k}]$$

$$\dot{\mathbf{H}}_G = \frac{1}{6} m c^2 \dot{\phi} \sin \theta [-(\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{j} + \dot{\phi} \cos \theta \mathbf{j}]$$

$$\dot{\mathbf{H}}_G = -\frac{1}{6} m c^2 \dot{\phi} \dot{\psi} \sin \theta \mathbf{j} \quad (1)$$

$$\dot{\mathbf{H}}_G = -\frac{1}{6} m c^2 \dot{\phi} \dot{\psi} \sin \theta \mathbf{j}$$

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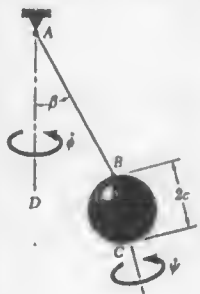
$$\dot{\mathbf{H}}_G = -\frac{1}{6} m c^2 \dot{\phi} \dot{\psi} \sin \theta \mathbf{j}$$

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$$\dot{\mathbf{H}}_G = -\frac{1}{6} m c^2 \dot{\phi} \dot{\psi} \sin \theta \mathbf{j}$$

# 18.115 and 18.116

SOLID SPHERE ATTACHED TO CORD AB.



## PROBLEM 18.115:

GIVEN:

$$c = 3 \text{ in.}, \beta = 40^\circ, \dot{\phi} = 6 \text{ rad/s}$$

FIND:

ANGLE  $\theta$ , KNOWING THAT

$$(a) \dot{\psi} = 0, (b) \dot{\psi} = 50 \text{ rad/s}, (c) \dot{\psi} = -50 \text{ rad/s}$$

## PROBLEM 18.116:

GIVEN:

$$c = 3 \text{ in.}, AB = 15 \text{ in.}, \theta = 20^\circ, \beta = 35^\circ$$

FIND:

$$(a) \dot{\psi}, (b) \dot{\phi}$$

USING  $Gxyz$  AXES WITH  $z$  ALONG CB,  $x$  IN ABD PLANE AND  $y$   $\perp$  ABD AND POINTING AWAY:

$$\text{SPHERE: } \underline{\omega} = \dot{\phi} \sin \theta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \underline{k}$$

$$\underline{H}_G = \frac{2}{5} m c^2 [\dot{\phi} \sin \theta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \underline{k}]$$

FRAME  $Gxyz$ :

$$\underline{\Omega} = \dot{\phi} \sin \theta \underline{i} + \dot{\phi} \cos \theta \underline{k}$$

EQ. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G$$

$$= 0 + (\dot{\phi} \sin \theta \underline{i} + \dot{\phi} \cos \theta \underline{k}) \times \underline{H}_G$$

$$\dot{\underline{H}}_G = (\dot{\phi} \sin \theta \underline{i} + \dot{\phi} \cos \theta \underline{k}) \times \frac{2}{5} m c^2 [\dot{\phi} \sin \theta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \theta) \underline{k}]$$

$$\dot{\underline{H}}_G = \frac{2}{5} m c^2 \dot{\phi} \sin \theta [-\dot{\psi} + \dot{\phi} \cos \theta] \underline{j} + \dot{\phi} \cos \theta \underline{j}$$

$$\dot{\underline{H}}_G = -\frac{2}{5} m c^2 \dot{\phi} \dot{\psi} \sin \theta \underline{j} \quad (1)$$

## EQUATIONS OF MOTION

$$\underline{T} = \frac{2}{5} m c^2 \dot{\phi} \dot{\psi} \sin \theta \quad \Sigma \underline{F} = \Sigma (\underline{F})_{eff}$$

$$\text{HORIZ. COMP.: } T \sin \beta = m \bar{a} \quad (2)$$

$$\text{VERTICAL COMP.: } T \cos \beta - W = 0 \quad (3)$$

$$\bar{a} = g \tan \beta \quad (4)$$

$$\text{DIVIDE (2) BY (3): } \tan \beta = \frac{\bar{a}}{g}$$

$$\text{+ } \Sigma M_G = \Sigma (M_G)_{eff}$$

$$-mg c \sin \theta = \frac{2}{5} m c^2 \dot{\phi} \dot{\psi} \sin \theta - (mg \tan \beta) c \cos \theta$$

$$\text{DIVIDE BY } mg c \cos \theta \text{ AND SOLVE FOR } \tan \theta:$$

$$\tan \theta = \frac{\tan \beta}{1 + (2c \dot{\phi} \dot{\psi} / 5g)}$$

## PROBLEM 18.115

$$\text{LETTING } \beta = 40^\circ, c = \frac{1}{4} \text{ ft}, \dot{\phi} = 6 \text{ rad/s}, g = 32.2 \text{ ft/s}^2 \text{ IN (3):}$$

$$\tan \theta = \tan 40^\circ / (1 + 0.018634 \dot{\psi})$$

$$(a) \text{ FOR } \dot{\psi} = 0: \tan \theta = \tan 40^\circ \quad \theta = 40.0^\circ$$

$$(b) \text{ FOR } \dot{\psi} = 50 \text{ rad/s: } \tan \theta = 0.43438 \quad \theta = 23.5^\circ$$

$$(c) \text{ FOR } \dot{\psi} = -50 \text{ rad/s: } \tan \theta = 12.285 \quad \theta = 85.3^\circ$$

## PROBLEM 18.116

$$\bar{a} = \frac{g \tan \beta}{1 + (2c \dot{\phi} \dot{\psi} / 5g)} \quad \text{RECALLING (4): } \dot{\phi} = \frac{\bar{a}}{c} = \frac{g \tan \beta}{(AB) \sin \beta + c \sin \theta}$$

$$\text{WITH } \beta = 35^\circ, \theta = 20^\circ, AB = 1.25 \text{ ft}, c = 0.25 \text{ ft}, g = 32.2 \text{ ft/s}^2:$$

$$\dot{\phi} = 5.3006 \text{ rad/s} \quad (b) \dot{\phi} = 5.30 \text{ rad/s}$$

$$\text{SOLVING (5) FOR } c \dot{\phi} \dot{\psi},$$

$$c \dot{\phi} \dot{\psi} = 2.5g \left( \frac{\tan \beta}{\tan \theta} - 1 \right) = 2.5(32.2) \left( \frac{\tan 35^\circ}{\tan 20^\circ} - 1 \right) = 74.366$$

$$\dot{\psi} = \frac{74.366}{(0.25)(5.3006)} = 56.12 \text{ rad/s} \quad (a) \dot{\psi} = 56.1 \text{ rad/s}$$

# 18.117

Axis of precession



(PRECESSION OF THE EQUINOXES)

GIVEN:

RATE OF PRECESSION OF EARTH ABOUT GA = 1 rev IN 25800 yr

FOR EARTH:  $P_{ave} = 5.51$

$$v_{ave} = 6370 \text{ km}, I = \frac{2}{5} m v_{ave}^2$$

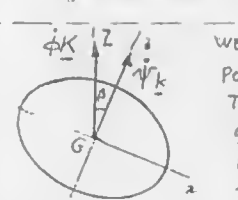
FIND:

AVERAGE VALUE OF CORIOLIS  $\underline{M}$

DUE TO GRAVITATIONAL

ATTRACTION OF SUN, MOON,

AND PLANETS.



WE USE  $Gxyz$  AXES (WITH  $y$  POINTING AWAY).

TOTAL ANG. VEL. OF EARTH

$$\underline{\omega} = -\dot{\phi} \sin \beta \underline{i} + (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

FRAME  $Gxyz$ :

$$\underline{\Omega} = -\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}$$

$$\underline{H}_G = -I \dot{\phi} \sin \beta \underline{i} + I (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

$$\text{EQ. (18.22): } \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times \underline{H}_G$$

$$\dot{\underline{H}}_G = (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times [-I \dot{\phi} \sin \beta \underline{i} + I (\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}]$$

$$= I \dot{\phi} \sin \beta (\dot{\psi} + \dot{\phi} \cos \beta - \dot{\phi} \cos \beta) \underline{j}$$

$$\dot{\underline{H}}_G = I \dot{\phi} \dot{\psi} \sin \beta \underline{j}$$

$$\underline{H}_G = I \dot{\phi} \dot{\psi} \sin \beta \underline{j}$$

## EQUATIONS OF MOTION

$$\Sigma \underline{M}_G = \Sigma (\underline{M}_G)_{eff}$$

$$\underline{M} = \dot{\underline{H}}_G = I \dot{\phi} \dot{\psi} \sin \beta \underline{j}$$

$$\text{(NOTE SENSE OF } \underline{j} \text{)}$$

$$\underline{M} = \dot{\underline{H}}_G = I \dot{\phi} \dot{\psi} \sin \beta \underline{j}$$

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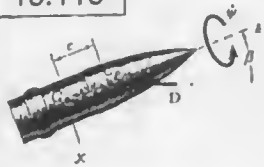
$$\underline{M} = \dot{\underline{H}}_G = I \dot{\phi} \dot{\psi} \sin \beta \underline{j}$$

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18.118



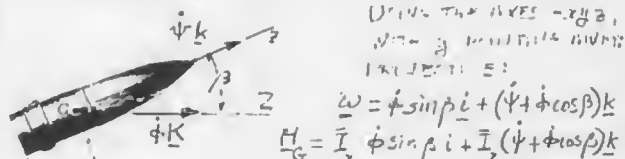
GIVEN:

PROJECTILE WITH  $m = 20 \text{ kg}$   
 $\bar{r}_x = 50 \text{ mm}$ ,  $\bar{r}_y = 200 \text{ mm}$   
 $\bar{v} = 600 \text{ m/s}$  (HORIZONTAL)  
 $\text{DRAG} = D = 120 \text{ N}$  (HORIZONTAL)  
 $\beta = 3^\circ$ ,  $c = 150 \text{ mm}$   
 $\dot{\psi} = 6000 \text{ rpm}$

FIND:

(a) APPROXIMATE VALUE OF RATE OF PRECESSION,  
 (b) EXACT VALUES OF TWO PRECESSION ANGLES

SINCE THE DRAG  $D$  IS A FORCE CONSTANT IN MAGNITUDE AND DIRECTION (LIKE THE WEIGHT OF A TOP), IT WILL PRECESS, LIKE A TOP, ABOUT AN AXIS  $GZ$  PARALLEL TO THAT FORCE.



USE THE AXES  $x, y, z$ ,  
 WITH  $z$  ALONG THE DRAG  
 DIRECTION:

$$\bar{\omega} = \dot{\phi} \sin \beta \bar{i} + (\dot{\psi} + \dot{\phi} \cos \beta) \bar{k}$$

$$\bar{H}_G = \bar{I}_x \dot{\phi} \sin \beta \bar{i} + \bar{I}_z (\dot{\psi} + \dot{\phi} \cos \beta) \bar{k}$$

$$\text{FRAME } Gxyz: \bar{\Omega} = \dot{\phi} \sin \beta \bar{i} + \dot{\phi} \cos \beta \bar{k}$$

$$\text{EQ. (18.22): } \dot{\bar{H}}_G = (\dot{\bar{H}}_G)_{Gxyz} + \bar{\Omega} \times \bar{H}_G = 0 + \bar{\Omega} \times \bar{H}_G$$

$$\dot{\bar{H}}_G = (\dot{\phi} \sin \beta \bar{i} + \dot{\phi} \cos \beta \bar{k}) \times [\bar{I}_x \dot{\phi} \sin \beta \bar{i} + \bar{I}_z (\dot{\psi} + \dot{\phi} \cos \beta) \bar{k}]$$

$$= \dot{\phi} \sin \beta [-\bar{I}_z (\dot{\psi} + \dot{\phi} \cos \beta) + \bar{I}_x \dot{\phi} \cos \beta] \bar{j}$$

$$\text{THUS: } \sum \bar{M}_G = \dot{\bar{H}}_G = \dot{\phi} \sin \beta [\bar{I}_x - \bar{I}_z] \dot{\phi} \cos \beta - \bar{I}_z \dot{\psi} \bar{j} \quad (1)$$

ON THE OTHER HAND,

$$\sum \bar{M}_G = c \bar{k} \times (-D \bar{k}) = -cD \sin \beta \bar{j} \quad (2)$$

$$\sum \bar{M}_G = \sum (\bar{M}_G)_{\text{eff}}: -cD = \dot{\phi} [(\bar{I}_x - \bar{I}_z) \dot{\phi} \cos \beta - \bar{I}_z \dot{\psi}] \quad (3)$$

(a) APPROXIMATE VALUE OF  $\dot{\phi}$ 

SINCE  $\dot{\psi} \gg \dot{\phi}$ , WE MUST NEGLECT THE FIRST TERM IN THE BRACKET IN (3). WE OBTAIN

$$\bar{I}_z \dot{\phi} \dot{\psi} = cD \quad (4)$$

WITH GIVEN DATA:  $\bar{I}_z = m \bar{r}_z^2 = (20 \text{ kg})(0.05 \text{ m})^2 = 0.05 \text{ kg} \cdot \text{m}^2$

$c = 0.15 \text{ m}$ ,  $D = 120 \text{ N}$ ,  $\dot{\psi} = 6000 \text{ rpm} = 200\pi \text{ rad/s}$

$$0.05 \dot{\phi} (200\pi) = (0.15)(120) \quad \dot{\phi} = 0.5730 \text{ rad/s}$$

$$\dot{\phi} = 5.47 \text{ rpm}$$

(b) EXACT VALUES OF  $\dot{\phi}$ 

USING EQ. (3) WITH THE ABOVE DATA AND WITH  $\beta = 3^\circ$  AND  $\bar{I}_x = m \bar{r}_x^2 = (20 \text{ kg})(0.2 \text{ m})^2 = 0.8 \text{ kg} \cdot \text{m}^2$ :

$$-(0.15 \text{ m})(120 \text{ N}) = \dot{\phi} [(0.8 - 0.05) \dot{\phi} \cos 3^\circ - 0.05 (200\pi)]$$

$$0.74897 \dot{\phi}^2 - 31.416 \dot{\phi} + 18 = 0$$

$$\dot{\phi}^2 - 41.945 \dot{\phi} + 24.033 = 0$$

$$\dot{\phi} = \frac{1}{2} (41.945 \pm \sqrt{41.945^2 - 4(24.033)})$$

$$= \frac{1}{2} (41.945 \pm 40.783) \text{ rad/s}$$

$$\dot{\phi} = 41.364 \text{ rad/s} \quad \text{or} \quad \dot{\phi} = 0.58101 \text{ rad/s}$$

$$\dot{\phi} = 395 \text{ rpm} \quad \text{AND} \quad \dot{\phi} = 5.55 \text{ rpm}$$

18.119

GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

$I$  = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY

$I'$  = — — — — — TRANSVERSE AXIS THRU  $G$ .

$\theta$  = ANG. MOM. ABOUT  $G$ .

SHOW THAT:

$$\dot{\phi} = \frac{H_G}{I'} \quad \text{AND} \quad \dot{\psi} = \frac{H_G \cos \theta (I' - I)}{I I'}$$

FROM EQ. (18.40), PAGE 1146:

$$\omega_z = -\dot{\phi} \sin \theta \quad (1)$$

FROM THE FIRST OF EQS. (18.41), PAGE 1147:

$$\omega_z = -\frac{H_G \sin \theta}{I'} \quad (2)$$

EQUATING THE R.H. MEMBERS OF (1) AND (2):

$$-\dot{\phi} \sin \theta = -\frac{H_G \sin \theta}{I'} \quad \dot{\phi} = \frac{H_G}{I'} \quad (\text{Q.E.D.}) \quad (3)$$

$$\text{FROM FIG. 18.21: } \dot{\psi} = \omega_z - \dot{\phi} \cos \theta \quad (4)$$

$$\text{FROM EQS. (18.42): } \omega_z = \frac{H_G \cos \theta}{I} \quad (5)$$

$$\text{FROM EQ. (3) ABOVE: } \dot{\phi} \cos \theta = \frac{H_G \cos \theta}{I'} \quad (6)$$

SUBSTITUTE FROM (5) AND (6) INTO (4):

$$\dot{\psi} = H_G \cos \theta \left( \frac{1}{I} - \frac{1}{I'} \right)$$

$$\dot{\psi} = \frac{H_G \cos \theta (I' - I)}{I I'} \quad (\text{Q.E.D.})$$

18.120

GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

$I$  = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY

$I'$  = — — — — — TRANSVERSE AXIS THRU  $G$

$\theta$  = ANG. BETWEEN AXES OF PRECESSION & SPIN

$\omega_z$  = COMPONENT OF  $\bar{\omega}$  ALONG AXIS OF SYMMETRY

SHOW THAT:

$$(a) \quad \dot{\phi} = \frac{I \omega_z}{I' \cos \theta}$$

(b) EQ. (18.44) IS SATISFIED

(a) SEE SOLUTION OF PROB. 18.119 FOR DERIVATION OF EQ. (3):

$$\dot{\phi} = \frac{H_G}{I'} \quad (3)$$

FROM EQS. (18.48):  $H_G = \frac{I \omega_z}{\cos \theta}$ SUBSTITUTE FOR  $H_G$  IN (3):

$$\dot{\phi} = \frac{I \omega_z}{I' \cos \theta} \quad (\text{Q.E.D.})$$

(b) FROM RELATION JUST OBTAINED, WE HAVE

$$I \omega_z - I' \dot{\phi} \cos \theta = 0$$

WHICH SHOWS THAT, FOR AN AXISYMMETRICAL BODY UNDER NO FORCE, THE R.H. MEMBER OF

$$\sum \bar{M}_G = (I \omega_z - I' \dot{\phi} \cos \theta) \dot{\phi} \sin \theta \bar{j} \quad (18.44)$$

IS EQUAL TO ZERO. BUT, SINCE THERE

IS NO FORCE, WE ALSO HAVE  $\sum \bar{M}_G = 0$  AND

EQ. (18.44) IS SATISFIED. (Q.E.D.)

18.121

GIVEN:

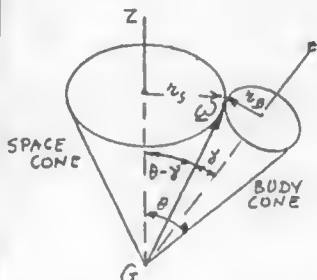
AXISYMMETRICAL BODY UNDER NO FORCE

 $I$  = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY. $I' = - - -$  TRANSVERSE AXIS THRU G $\omega_z$  = COMPONENT OF  $\omega$  ALONG AXIS OF SYMMETRY.

SHOW THAT:

ANGULAR VELOCITY  $\omega$  IS OBSERVED FROM THE BODY TO ROTATE ABOUT THE AXIS OF SYMMETRY AT THE RATE

$$\pi = \frac{I - I'}{I'} \omega_z$$



ASSUMING DIRECT PRECESSION ( $I' > I$ ), WE CONSIDER THE SPACE AND BODY CONES. THE PLANE ZGE ROTATES ABOUT THE Z AXIS AT THE RATE  $\dot{\phi}$ ; SO WILL THE VECTOR  $\omega$  CONTAINED IN THAT PLANE. THUS, THE TIP OF  $\omega$

WILL DESCRIBE AN ARC OF CIRCLE OF LENGTH  $\omega_z \dot{\phi} \Delta t$  IN THE TIME  $\Delta t$ . BUT, ACCORDING TO THE DEFINITION OF  $\pi$ , THE VECTOR  $\omega$  IS OBSERVED TO ROTATE AT THE RATE  $\pi$  WITH RESPECT TO THE BODY, THUS THE TIP OF  $\omega$  WILL DESCRIBE AN ARC OF CIRCLE OF LENGTH  $\omega \pi \Delta t$  IN THE TIME  $\Delta t$ . SINCE THE BODY CONE ROLLS ON THE SPACE CONE, WE HAVE

$$\omega_z \dot{\phi} \Delta t = \omega \pi \Delta t \quad (1)$$

BUT, FROM THE SKETCH ABOVE,

$$\omega_z = \omega \sin(\theta - \gamma) \quad \text{AND} \quad \omega = \omega \sin \gamma$$

SUBSTITUTING INTO (1):

$$\dot{\phi} \sin(\theta - \gamma) = \pi \sin \gamma$$

$$\pi = \dot{\phi} \frac{\sin(\theta - \gamma)}{\sin \gamma} \quad (2)$$

WE RECALL THE RELATION DERIVED IN PROB. 18.120:

$$\dot{\phi} = \frac{I \omega_z}{I' \cos \theta}$$

SUBSTITUTING INTO (2) AND EXPANDING  $\sin(\theta - \gamma)$ :

$$\pi = \frac{I \omega_z}{I' \cos \theta} \frac{\sin \theta \cos \gamma - \sin \gamma \cos \theta}{\sin \gamma}$$

$$= \frac{I \omega_z}{I'} \left( \frac{\tan \theta}{\tan \gamma} - 1 \right)$$

RECALLING FROM EQ. (18.49) THAT  $\frac{\tan \theta}{\tan \gamma} = \frac{I'}{I}$ , WE HAVE

$$\pi = \frac{I}{I'} \left( \frac{I'}{I} - 1 \right) \omega_z$$

$$\pi = \frac{I - I'}{I'} \omega_z \quad (\text{Q.E.D.})$$

NOTE. FOR  $I > I'$  (RETROGRADE PRECESSION), WE WOULD FIND

$$\pi = \frac{I - I'}{I'} \omega_z$$

18.122

GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

AND IN RETROGRADE PRECESSION ( $I > I'$ ).

SHOW THAT:

(a) RATE OF RETROGRADE PRECESSION CANNOT BE LESS THAN TWICE THE RATE OF SPIN:  $|\dot{\phi}| \geq 2|\dot{\psi}|$ ,

(b) THE AXIS OF SYMMETRY IN FIG. 18.24 CAN NEVER LIE WITHIN THE SPACE CONE.

(a) WE RECALL THE RELATION DERIVED IN PROB. 18.120:

$$\dot{\phi} = \frac{I \omega_z}{I' \cos \theta} \quad (1)$$

OR

$$I' \dot{\phi} \cos \theta = I \omega_z$$

SUBSTITUTING  $\omega_z = \dot{\psi} + \dot{\phi} \cos \theta$ , WE HAVE

$$I' \dot{\phi} \cos \theta = I (\dot{\psi} + \dot{\phi} \cos \theta)$$

SOLVING FOR  $\dot{\phi}$ ,

$$\dot{\phi} = -\frac{I}{I - I' \cos \theta} \dot{\psi} \quad \text{OR} \quad \dot{\phi} = -\frac{\sec \theta}{1 - (I'/I)} \dot{\psi} \quad (2)$$

FOR RETROGRADE PRECESSION,  $I'/I < 1$ ON THE OTHER HAND, THE SMALLEST POSSIBLE VALUE OF  $I'/I$  IS  $1/2$  (WHICH CORRESPONDS TO THE CASE OF A FLAT DISK OR ANNULUS).

THUS:

$$\frac{1}{2} \leq \frac{I'}{I} < 1 \quad \text{OR} \quad \frac{1}{2} \geq 1 - \frac{I'}{I} > 0$$

$$\text{OR} \quad \frac{1}{1 - (I'/I)} \geq 2$$

RECALLING THAT  $\sec \theta \geq 1$ , WE MUST HAVE FROM (2)

$$|\dot{\phi}| \geq 2|\dot{\psi}| \quad (\text{Q.E.D.})$$

(b) WE RECALL EQ. (18.49):

$$\tan \gamma = \frac{I}{I'} \tan \theta$$

SINCE  $\frac{I'}{I} \geq \frac{1}{2}$  AS SHOWN ABOVE,  $\frac{I}{I'} \leq 2$  AND

$$\tan \gamma \leq 2 \tan \theta \quad (3)$$

WE WRITE THE TRIGONOMETRIC IDENTITY

$$\tan(\gamma - \theta) = \frac{\tan \gamma - \tan \theta}{1 + \tan \gamma \tan \theta}$$

SINCE  $\gamma < \frac{\pi}{2}$  AND  $\theta < \frac{\pi}{2}$ , WE HAVE  $1 + \tan \gamma \tan \theta \geq 1$ 

AND, FROM (3):

$$\tan \gamma - \tan \theta \leq \tan \theta$$

THUS  $\tan(\gamma - \theta) \leq \frac{\tan \theta}{1}$ 

$$\tan(\gamma - \theta) \leq \tan \theta$$

$$\gamma - \theta \leq \theta$$

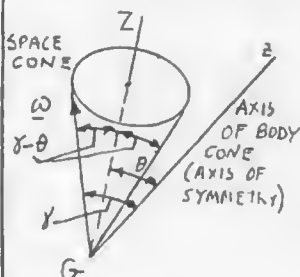
THE Z AXIS CANNOT

LIE WITHIN THE

SPACE CONE

(SEE SKETCH)

(Q.E.D.)



18.123

## (FREE PRECESSION OF THE EARTH)

GIVEN:

 $I$  = MOM. OF INERTIA OF EARTH ABOUT AXIS OF SYMMETRY. $I'$  = ———— TRANSVERSE AXIS

$$I' = 0.9967 I$$

RELATION DERIVED IN PROB. 18.121:

$$\Omega = \frac{I - I'}{I'} \omega_s \quad (\text{FOR } I > I')$$

WHERE  $\omega_s$  = COMPONENT OF  $\omega$  OF EARTH ALONG AXIS OF SYMMETRY, AND  $\Omega$  = RATE AT WHICH  $\omega$  IS OBSERVED FROM THE EARTH TO ROTATE ABOUT ITS AXIS OF SYMMETRY.

FIND:

PERIOD OF PRECESSION OF NORTH POLE.

PERIOD OF PRECESSION

$$= \frac{2\pi}{\Omega} = \frac{I'}{I - I'} \frac{2\pi}{\omega_s} = \frac{I'}{I - I'} (1 \text{ day})$$

$$\text{BUT } \frac{I'}{I - I'} = \frac{0.9967 I}{0.0033 I} = 302$$

THUS: PERIOD OF PRECESSION = 302 days

18.124

GIVEN:

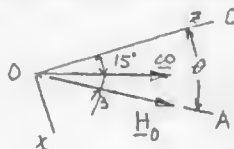
FOOTBALL KICKED WITH HORIZONTAL ANG. VEL.  $\omega$  OF MAGNITUDE 200 rpm. RATIO OF AXIAL AND

TRANSVERSE MOMENTS OF INERTIA IS  $I/I' = 1/3$ .

FIND:

(a) ANGLE  $\beta$  BETWEEN  $\omega$  AND PRECESSION AXIS OA.

(b) RATES OF PRECESSION AND SPIN.

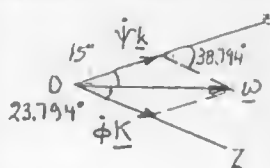
(a) USING REFERENCE FRAME  $Oxyz$  WITH  $y$  POINTING AWAY

$$\begin{aligned} \omega_x &= \omega \sin 15^\circ \\ \omega_y &= 0 \\ \omega_z &= \omega \cos 15^\circ \\ H_x &= I' \omega_x = I' \omega \sin 15^\circ \\ H_y &= I' \omega_y = 0 \\ H_z &= I' \omega_z = I' \omega \cos 15^\circ \end{aligned}$$

$$\tan \theta = \frac{H_x}{H_z} = \frac{I' \omega \sin 15^\circ}{I' \omega \cos 15^\circ} = \frac{1}{I/I'} \tan 15^\circ = 3 \tan 15^\circ$$

$$\tan \theta = 0.80385 \quad \theta = 38.794^\circ$$

$$\beta = \theta - 15^\circ = 38.794^\circ - 15^\circ = 23.794^\circ \quad \beta = 23.8^\circ$$

(b) USING THE OBLIQUE COMPONENTS OF  $\omega$  ALONG OA AND OC:

LAW OF SINES:

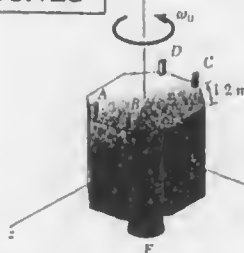
$$\frac{\omega}{\sin 38.794^\circ} = \frac{\dot{\phi}}{\sin 15^\circ} = \frac{\dot{\psi}}{\sin 23.794^\circ}$$

SETTING  $\omega = 200 \text{ rpm}$ , WE FIND

$$\text{RATE OF PRECESSION} = \dot{\phi} = 82.6 \text{ rpm}$$

$$\text{RATE OF SPIN} = \dot{\psi} = 128.8 \text{ rpm}$$

18.125



GIVEN:

2500-kg SPIN RATE, 2.4- $\pi$  HIGH WITH CENTER OF MASS

 $k_x = k_y = 0.30 \text{ m}$ ,  $k_z = 0.98 \text{ m}$ 

SYMMETRIC SPIN AXIS

KINETIC OF 3' (1.2 m) ...

BY WHEN 20-14 T- ...

AT 111.5 m = 11.15 m ...

FL. 25, EXPELLING ...

IN POSITIVE ...

FIND: (a) PRECESSION AXIS, (b)  $\dot{\phi}$ , (c)  $\dot{\psi}$ .

INITIAL ANG. VELOCITY:

$$\omega_0 = (36 \frac{\text{rev}}{\text{h}}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \hat{j} = (0.062832 \text{ rad/s}) \hat{j}$$

INITIAL ANG. VELOCITY:

$$\begin{aligned} (H_G)_0 &= m k_y^2 \omega_0 = (2500 \text{ kg}) (0.98 \text{ m})^2 (0.062832 \text{ rad/s}) \hat{j} \\ &= (150.86 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j} \end{aligned}$$

ANG. IMPULSE:

$$M_G \Delta t = (1.4485 \text{ m}) \hat{x}$$

$$\times 2(-20 \text{ N}) \hat{j} (25)$$

$$M_G \Delta t = (115.88 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{i}$$

PRINCIPLE OF IMPULSE AND MOMENTUM

FINAL MOMENTUM:

$$H_G = (H_G)_0 + M_G \Delta t = (150.86 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j} + (115.88 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{i}$$

$$H_G = (115.88 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{i} + (150.86 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j} \quad (1)$$

WE RECALL THAT

$$H_G = I_x \omega_x \hat{i} + I_y \omega_y \hat{j} + I_z \omega_z \hat{k}$$

$$H_G = (2500 \text{ kg}) (0.90 \text{ m})^2 \omega_x \hat{i} + (2500 \text{ kg}) (1.98 \text{ m})^2 \omega_y \hat{j} + I_z \omega_z \hat{k} \quad (2)$$

EQUATING THE COEFF OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  IN (1) AND (2):

$$2025 \omega_x = 115.88$$

$$2401 \omega_y = 150.86$$

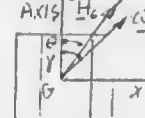
$$I_z \omega_z = 0$$

$$\omega_x = 57.225 \times 10^{-3} \text{ rad/s}$$

$$\omega_y = 62.832 \times 10^{-3} \text{ rad/s}$$

$$\omega_z = 0$$

SPIN AXIS PRECESSION AXIS



(c) FROM EQ. (1):

$$\tan \theta = \frac{H_x}{H_y} = \frac{115.88}{150.86} \quad \theta = 37.52^\circ$$

THUS:  $\theta_x = 52.5^\circ$ ,  $\theta_y = 37.5^\circ$ ,  $\theta_z = 90^\circ$ FROM EQ. (2):  $\tan \chi = \frac{\omega_x}{\omega_y} = \frac{57.225}{62.832} \quad \chi = 42.32^\circ$ 

$$\omega = \sqrt{\omega_x^2 + \omega_y^2} = 84.986 \times 10^{-3} \text{ rad/s}$$

LAW OF SINES:

$$\frac{\omega}{\sin \theta} = \frac{\dot{\phi}}{\sin \chi} = \frac{\dot{\psi}}{\sin (90^\circ - \theta)}$$

$$\frac{84.986 \times 10^{-3}}{\sin 37.52^\circ} = \frac{\dot{\phi}}{\sin 42.32^\circ} = \frac{\dot{\psi}}{\sin 47.48^\circ}$$

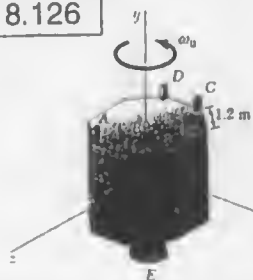
SOLVING FOR  $\dot{\phi}$  AND  $\dot{\psi}$ :

$$(b) \dot{\phi} = 93.94 \times 10^{-3} \text{ rad/s} \quad \dot{\phi} = 53.8 \text{ rev/h}$$

$$(c) \dot{\psi} = 11.667 \times 10^{-3} \text{ rad/s} \quad \dot{\psi} = 6.68 \text{ rev/h}$$

WE CHECK FROM DIAGRAM THAT PRECESSION IS RETROGRADE. (IT HAD TO BE, SINCE  $k_y > k_x$  AND, THUS,  $I > I'$ )

18.126



GIVEN:

2500-KG SATELLITE, 2.4-m HIGH WITH OCTAGONAL BASE.  
 $k_x = k_y = 0.10 \text{ m}$ ,  $k_z = 0.98 \text{ m}$ .  
 SATELLITE SPINNING AT RATE OF 36 rev/h ABOUT  $Gz$  WHEN 20-N THRUSTERS AT A AND D ARE ACTIVATED FOR 2 S, EXPULSION FUEL IN POSITIVE  $y$  DIRECTION.

FIND: (a) PRECESSION AXIS, (b)  $\dot{\phi}$ , (c)  $\dot{\psi}$ .

INITIAL ANG. VELOCITY:

$$\omega_0 = (36 \frac{\text{rev}}{\text{h}}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \hat{j} = (0.062832 \text{ rad/s}) \hat{j}$$

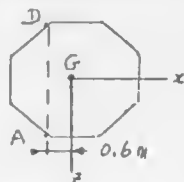
INITIAL ANG. MOMENTUM:

$$(H_G)_0 = m k_y^2 \omega_0 = (2500 \text{ kg})(0.98 \text{ m})^2 (0.062832 \text{ rad/s}) \hat{j} = (150.86 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j}$$

ANG. IMPULSE:

$$M_G \Delta t = -(0.6 \text{ m}) \hat{j} \times 2(-20 \text{ N}) \hat{j} (2 \text{ s})$$

$$M_G \Delta t = (48.0 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{k}$$



PRINCIPLE OF IMPULSE AND MOMENTUM

FINAL MOMENTUM:

$$H_G = (H_G)_0 + M_G \Delta t = (150.86 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j} + (48.0 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{k} \quad (1)$$

WE KNOW THAT

$$H_G = I_x \omega_x \hat{i} + I_y \omega_y \hat{j} + I_z \omega_z \hat{k}$$

$$H_G = I_x \omega_x \hat{i} + (2500 \text{ kg})(0.98 \text{ m})^2 \omega_y \hat{j} + (2500 \text{ kg})(0.90 \text{ m})^2 \omega_z \hat{k} \quad (2)$$

EQUATING THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ , &  $\hat{k}$  IN (1) AND (2):

$$I_x \omega_x = 0$$

$$\omega_x = 0$$

$$2401 \omega_y = 150.86$$

$$\omega_y = 62.832 \times 10^{-3} \text{ rad/s} \quad (3)$$

$$2025 \omega_z = 48.0$$

$$\omega_z = 23.704 \times 10^{-3} \text{ rad/s}$$

PREC. AXIS



(a) FROM EQ. (1):

$$\tan \theta = \frac{H_z}{H_y} = \frac{48.0}{150.86}$$

$$\theta = 17.650^\circ$$

$$\text{THUS: } \theta_x = 90^\circ, \theta_y = 17.65^\circ, \theta_z = 72.35^\circ$$

$$\text{FROM EQS. (3): } \tan \gamma = \frac{\omega_z}{\omega_y} = \frac{23.704}{62.832} \quad \gamma = 20.669^\circ$$

$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 67.155 \times 10^{-3} \text{ rad/s}$$

LAW OF SINES 1

$$\frac{\omega}{\sin \theta} = \frac{\dot{\phi}}{\sin \gamma} = \frac{\dot{\psi}}{\sin(\gamma - \theta)}$$

$$\frac{67.155 \times 10^{-3}}{\sin 17.650^\circ} = \frac{\dot{\phi}}{\sin 20.669^\circ} = \frac{\dot{\psi}}{\sin 3.019^\circ}$$

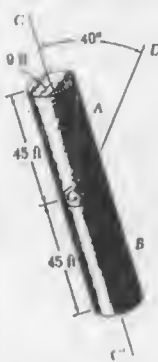
SOLVING FOR  $\dot{\phi}$  AND  $\dot{\psi}$ :

$$(b) \quad \dot{\phi} = 78.177 \times 10^{-3} \text{ rad/s} \quad \dot{\phi} = 44.8 \text{ rev/h}$$

$$(c) \quad \dot{\psi} = 11.665 \times 10^{-3} \text{ rad/s} \quad \dot{\psi} = 6.68 \text{ rev/h}$$

WE CHECK FROM DIAGRAM THAT PRECESSION IS RETROGRADE  
 (IT HAD TO BE, SINCE  $k_y > k_z$  AND, THUS,  $I > I'$ .)

18.127 and 18.128 GIVEN:



SPACE STATION CONSISTS OF TWO SECTIONS A AND B OF THE SAME WEIGHT WHICH ARE RIGIDLY CONNECTED. EACH SECTION IS DYNAMICALLY EQUIVALENT TO A HOMOGENEOUS CYLINDER. STATION IS PRECESSING ABOUT  $Gz$  AT THE CONSTANT RATE OF 2 rev/h.

PROBLEM 18.127:

FIND THE RATE OF SPIN OF THE STATION ABOUT  $CC'$ .

PROBLEM 18.128:

IF CONNECTION IS SEVERED BETWEEN A AND B, FIND FOR SECTION A:

(a) THE ANGLE BETWEEN  $CC'$  AND THE PRECESSION AXIS,  
 (b)  $\dot{\phi}$ , (c)  $\dot{\psi}$ .

FOR ENTIRE STATION:

$$\theta = 40^\circ$$

$$I = \frac{1}{2} m a^2 \quad I' = \frac{1}{12} m (3a^2 + L^2) \quad \frac{I}{I'} = \frac{6a^2}{3a^2 + L^2}$$

$$\text{EQ. (18.49): } \tan \delta = \frac{I}{I'} \tan \theta = \frac{6(9)^2}{3(9)^2 + (45)^2} \tan 40^\circ = 58.252 \times 10^{-3} \tan 40^\circ, \quad \delta = 2.7984^\circ$$

PROBLEM 18.127

LAW OF SINES:

$$\frac{\omega}{\sin \theta} = \frac{\dot{\phi}}{\sin \delta} = \frac{\dot{\psi}}{\sin(\theta - \delta)}$$

WITH  $\dot{\phi} = 2 \text{ rev/h}$ :

$$\frac{\omega}{\sin 40^\circ} = \frac{2 \text{ rev/h}}{\sin 2.7984^\circ} = \frac{\dot{\psi}}{\sin 37.202^\circ}$$

SOLVING FOR  $\omega$  AND  $\dot{\psi}$ :

$$\omega = 26.332 \text{ rev/h}$$

$$\dot{\psi} = 24.8 \text{ rev/h}$$

PROBLEM 18.128

FOR SECTION A:

(a) ANGLE BETWEEN SPIN AXIS AND  $\omega$  IS STILL  $\delta = 2.7984^\circ$ 

$$\text{NOW: } \frac{I}{I'} = \frac{6a^2}{3a^2 + L^2} = \frac{6(9)^2}{3(9)^2 + (45)^2} = 0.21429$$

$$\text{EQ. (18.49): } \tan \gamma = \frac{I}{I'} \tan \theta \quad \tan \gamma = 0.21429 \tan \theta$$

$$\tan \theta = \frac{\tan \gamma}{0.21429} = \frac{\tan 2.7984^\circ}{0.21429} = 0.22811$$

$$\theta = 12.850^\circ$$

$$\theta = 12.85^\circ$$

(b) AND (c) WE HAVE  $\omega = 26.332 \text{ rev/h}$ ,  $\delta = 2.7984^\circ$ AND  $\theta = 12.850^\circ$ 

LAW OF SINES:

$$\frac{\omega}{\sin \theta} = \frac{\dot{\phi}}{\sin \gamma} = \frac{\dot{\psi}}{\sin(\theta - \gamma)}$$

$$\frac{26.332 \text{ rev/h}}{\sin 12.850^\circ} = \frac{\dot{\phi}}{\sin 2.7984^\circ} = \frac{\dot{\psi}}{\sin 10.052^\circ}$$

SOLVING FOR  $\dot{\phi}$  AND  $\dot{\psi}$ :

$$(b) \quad \dot{\phi} = 5.781 \text{ rev/h}$$

$$\dot{\phi} = 5.78 \text{ rev/h}$$

$$(c) \quad \dot{\psi} = 20.665 \text{ rev/h}$$

$$\dot{\psi} = 20.7 \text{ rev/h}$$

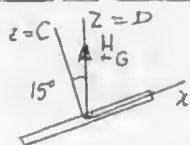
18.129

**GIVEN:**

COIN SPINS AT THE RATE OF 600 rpm ABOUT AXIS GC PERPENDICULAR TO COIN AND PRECESSES ABOUT VERTICAL DIRECTION GD.

**FIND:**

- (a) ANGLE BETWEEN  $\omega$  AND GD.  
(b) RATE OF PRECESSION ABOUT GD.



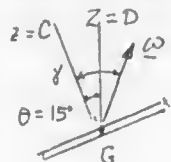
IT FOLLOWS FROM THE ABOVE STATEMENT THAT  $H_G$  IS DIRECTED AS SHOWN AND THAT THE ANGLE BETWEEN THE AXES OF SPIN AND PRECESSION IS  $\theta = 15^\circ$

FOR DISK:

$$I = I_z = \frac{1}{2} m a^2 \quad I' = I_x = \frac{1}{4} m a^2$$

EQ. (18.49):

$$\tan \gamma = \frac{I}{I'} \tan \theta = 2 \tan 15^\circ \quad \gamma = 28.187^\circ$$

(a) ANGLE BETWEEN  $\omega$  AND GD

THE ANGLE  $\gamma$  WE HAVE FOUND IS THE ANGLE BETWEEN  $\omega$  AND GC. THE ANGLE BETWEEN  $\omega$  AND GD IS

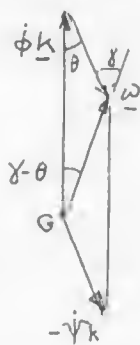
$$\gamma - \theta = 28.187^\circ - 15^\circ = 13.187^\circ$$

$$\gamma - \theta = 13.19^\circ$$

(b) RATE OF PRECESSION

THE RATE OF SPIN IS  $\dot{\psi} = 600 \text{ rpm}$ 

RESOLVING THE ANGULAR VELOCITY  $\omega$  INTO ITS SPIN COMPONENT  $\dot{\psi} \underline{k}$  AND ITS PRECESSION COMPONENT  $\dot{\phi} \underline{k}$ , WE DRAW THE FOLLOWING DIAGRAM:



LAW OF SINES:

$$\frac{\dot{\phi}}{\sin \gamma} = \frac{\dot{\psi}}{\sin(\gamma - \theta)}$$

$$\dot{\phi} = \dot{\psi} \frac{\sin \gamma}{\sin(\gamma - \theta)}$$

$$= (600 \text{ rpm}) \frac{\sin 28.187^\circ}{\sin 13.187^\circ}$$

$$\dot{\phi} = 1242 \text{ rpm}$$

WE NOTE FROM DIAGRAM THAT THE PRECESSION IS RETROGRADE

THIS COULD HAVE BEEN ANTICIPATED, SINCE  $I/I' = 2 > 1$ .

18.130

SOLVE SAMPLE PROB. 18.6, ASSUMING THAT THE METEORITE STRIKES THE SATELLITE AT C WITH  $\underline{v}_0 = (2000 \text{ m/s}) \underline{i}$ .

(a) ANGULAR VELOCITY AFTER IMPACT

FROM SAMPLE PROB. 18.6:

$$I = I_z = \frac{1}{2} m a^2 \quad I' = I_x = I_y = \frac{5}{4} m a^2$$

ANG. MOMENTUM AFTER IMPACT:

$$\underline{H}_G = \underline{r}_C \times m_0 \underline{v}_0 + I \omega_0 \underline{k}$$

$$= (-a \underline{j} - a \underline{k}) \times m_0 v_0 \underline{i} + I \omega_0 \underline{k}$$

$$\underline{H}_G = -m_0 v_0 a \underline{j} + (I \omega_0 + m_0 v_0 a) \underline{k} \quad (1)$$

$$\text{BUT } \underline{H}_G = I' \omega_x \underline{i} + I' \omega_y \underline{j} + I \omega_z \underline{k} \quad (2)$$

EQUATING THE COEFF. OF THE UNIT VECTORS IN (1) AND (2)

$$\omega_x = 0 \quad \omega_y = -\frac{m_0 v_0 a}{I'} = -\frac{4}{5} \frac{m_0 v_0}{m a}$$

$$\omega_z = \omega_0 + \frac{m_0 v_0 a}{I} = \omega_0 + 2 \frac{m_0 v_0}{m a}$$

GIVEN DATA:  $\omega_0 = 60 \text{ rpm} = 6.283 \text{ rad/s}$ 

$$\frac{m_0}{m} = 0.001 \quad a = 0.2 \text{ m} \quad v_0 = 2000 \text{ m/s}$$

WE FIND

$$\omega_x = 0 \quad \omega_y = -2 \text{ rad/s} \quad \omega_z = 11.283 \text{ rad/s}$$

$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 11.459 \text{ rad/s} \quad \omega = 109.4 \text{ rpm}$$

$$\cos \theta_x = 0 \quad \cos \theta_y = \frac{\omega_y}{\omega} = -0.17453 \quad \cos \theta_z = \frac{\omega_z}{\omega} = 0.98464$$

$$\theta_x = 90^\circ, \quad \theta_y = 100.05^\circ, \quad \theta_z = 10.05^\circ$$

(b) PRECESSION AXIS

SINCE IT IS DIRECTED ALONG  $\underline{H}_G$ , WE USE EQ (1) AND WRITE

$$\underline{H}_x = 0, \quad \underline{H}_y = -m_0 v_0 a = -\frac{m}{1000} (2000)(0.2) = -(1.6) \text{ m}$$

$$\underline{H}_z = I \omega_0 + m_0 v_0 a = \frac{1}{2} m a^2 \omega_0 + m_0 v_0 a$$

$$= \frac{1}{2} m (0.2)^2 (6.283) + (1.6) \text{ m} = (3.6100) \text{ m}$$

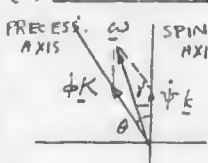
$$\underline{H}_G = \sqrt{\underline{H}_y^2 + \underline{H}_z^2} = (3.1442) \text{ m}$$

$$\cos \theta_x = 0, \quad \cos \theta_y = \frac{\underline{H}_y}{\underline{H}_G} = -0.40515, \quad \cos \theta_z = \frac{\underline{H}_z}{\underline{H}_G} = 0.91425$$

DIRECTION OF PRECESSION AXIS IS

$$\theta_x = 90^\circ, \quad \theta_y = 113.9^\circ, \quad \theta_z = 23.9^\circ$$

(c) RATES OF PRECESSION AND SPIN



WE HAVE

$$\theta = \theta_z = 23.9^\circ$$

$$\gamma = \theta_y = 100.05^\circ$$

$$\gamma - \theta = 13.85^\circ$$

LAW OF SINES:

$$\frac{\omega}{\sin \theta} = \frac{\dot{\phi}}{\sin \gamma} = \frac{\dot{\psi}}{\sin(\gamma - \theta)}$$

$$\frac{109.4 \text{ rpm}}{\sin 23.9^\circ} = \frac{\dot{\phi}}{\sin 100.05^\circ} = \frac{\dot{\psi}}{\sin 13.85^\circ}$$

SOLVING FOR  $\dot{\phi}$  AND  $\dot{\psi}$ 

$$\text{RATE OF PRECESSION} = \dot{\phi} = 47.1 \text{ rpm}$$

$$\text{RATE OF SPIN} = \dot{\psi} = 64.6 \text{ rpm}$$



# 18.131 and 18.132

## GIVEN:

DISK OF MASS  $m$  IS FREE TO ROTATE ABOUT AB. FORK-ENDED SHAFT OF NEGLIGIBLE MASS IS FREE TO ROTATE IN BEARING C.

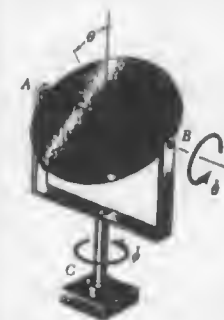
### PROBLEM 18.131:

INITIALLY,  $\theta_0 = 90^\circ$ ,  $\dot{\theta}_0 = 0$ ,  $\dot{\phi}_0 = 8 \text{ rad/s}$ .

IF DISK SLIGHTLY DISTURBED

FIND IN ENSUING MOTION

- (a) MINIMUM VALUE OF  $\dot{\phi}$ ,  
(b) MAXIMUM VALUE OF  $\dot{\phi}$ .

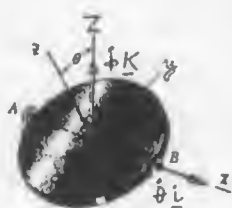


### PROBLEM 18.132:

INITIALLY  $\theta_0 = 30^\circ$ ,  $\dot{\theta}_0 = 0$ ,  $\dot{\phi}_0 = 8 \text{ rad/s}$ .

FIND IN ENSUING MOTION

- (a) RANGE OF VALUES OF  $\theta$ , (b) MINIMUM  $\dot{\phi}$ , (c) MAXIMUM  $\dot{\phi}$ .



USING THE AXES  $Gxyz$ :

$$\omega = \dot{\theta} \hat{i} + \dot{\phi} \sin \theta \hat{j} + \dot{\phi} \cos \theta \hat{k}$$

CONSERVATION OF ANGULAR MOMENTUM:

SINCE DISK IS FREE TO ROTATE ABOUT THE  $Z$  AXIS, WE HAVE

$$H_z = \text{constant} \quad (1)$$

$$\text{BUT } H_z = H_y \sin \theta + H_z \cos \theta$$

$$H_z = I_y \omega_y \sin \theta + I_z \omega_z \cos \theta = \frac{1}{4} m a^2 \dot{\phi} \sin^2 \theta + \frac{1}{2} m a^2 \dot{\phi} \cos^2 \theta$$

$$= \frac{1}{4} m a^2 \dot{\phi} (\sin^2 \theta + 2 \cos^2 \theta) = \frac{1}{4} m a^2 \dot{\phi} (1 + \cos^2 \theta)$$

USING THE INITIAL CONDITIONS, EQ. (1) YIELDS

$$\dot{\phi} (1 + \cos^2 \theta) = \dot{\phi}_0 (1 + \cos^2 \theta_0) \quad (2)$$

CONSERVATION OF ENERGY

SINCE NO WORK IS DONE, WE HAVE  $T = \text{constant}$  (3) WHERE

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$$

$$T = \frac{1}{2} \left( \frac{1}{4} m a^2 \dot{\theta}^2 + \frac{1}{4} m a^2 \dot{\phi}^2 \sin^2 \theta + \frac{1}{2} m a^2 \dot{\phi}^2 \cos^2 \theta \right)$$

$$= \frac{1}{8} m a^2 \left[ \dot{\theta}^2 + \dot{\phi}^2 (\sin^2 \theta + 2 \cos^2 \theta) \right] = \frac{1}{8} m a^2 \left[ \dot{\theta}^2 + \dot{\phi}^2 (1 + \cos^2 \theta) \right]$$

USING THE INITIAL CONDITIONS, INCLUDING  $\dot{\theta}_0 = 0$ , EQ. (3) YIELDS

$$\dot{\theta}^2 + \dot{\phi}^2 (1 + \cos^2 \theta) = \dot{\phi}_0^2 (1 + \cos^2 \theta_0) \quad (4)$$

### PROBLEM 18.131

(a) WITH  $\theta_0 = 90^\circ$  AND  $\dot{\phi}_0 = 8 \text{ rad/s}$ , EQ. (2) YIELDS  $\dot{\phi} = \frac{8}{1 + \cos^2 \theta}$

$\dot{\phi}$  IS MINIMUM FOR  $\theta = 0$ :  $\dot{\phi}_{\min} = 4.00 \text{ rad/s}$

(b) EQ. (4) YIELDS  $\dot{\theta}^2 = 64 - \dot{\phi}^2 (1 + \cos^2 \theta) = 64 \left( 1 - \frac{1}{1 + \cos^2 \theta} \right)$

$\dot{\theta}$  IS LARGEST FOR  $\theta = 0$ :  $\dot{\theta}_{\max} = 6.4 \text{ rad/s}$

### PROBLEM 18.132

(a) WITH  $\theta_0 = 30^\circ$ ,  $\dot{\phi}_0 = 8 \text{ rad/s}$  IN (2)

$$\dot{\phi} (1 + \cos^2 \theta) = 14 \quad \dot{\phi} = 14 / (1 + \cos^2 \theta) \quad (5)$$

SUBSTITUTE IN (4) AND SOLVE FOR  $\dot{\theta}^2$ :  $\dot{\theta}^2 = 112 - \frac{196}{1 + \cos^2 \theta}$  (6)

SINCE  $\dot{\theta}^2 \geq 0$ , WE MUST HAVE  $1 + \cos^2 \theta \geq \frac{196}{112}$ ,  $30^\circ \leq \theta \leq 150^\circ$

(b) FROM (5),  $\dot{\phi}$  IS MINIMUM FOR  $\theta = 0$ :  $\dot{\phi}_{\min} = 7.00 \text{ rad/s}$

(c) FROM (6),  $\dot{\theta}$  IS MAXIMUM FOR  $\theta = 0$ :  $\dot{\theta}_{\max} = 3.74 \text{ rad/s}$

# 18.133 and 18.134

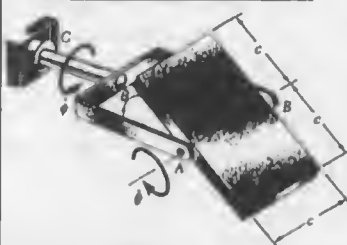
## GIVEN:

PLATE OF MASS  $m$  IS FREE TO ROTATE ABOUT AB.

FORK-ENDED SHAFT OF NEGLIGIBLE MASS IS FREE TO ROTATE IN BEARING C.

### PROBLEM 18.133:

INITIALLY  $\theta_0 = 30^\circ$ ,  $\dot{\theta}_0 = 0$ ,  $\dot{\phi}_0 = 6 \text{ rad/s}$ .



FIND IN ENSUING MOTION (a) RANGE OF VALUES OF  $\theta$ , (b) MINIMUM VALUE OF  $\dot{\phi}$ , (c) MAXIMUM VALUE OF  $\dot{\theta}$ .

### PROBLEM 18.134:

INITIALLY  $\theta_0 = 0$ ,  $\dot{\theta}_0 = 0$ ,  $\dot{\phi}_0 = 6 \text{ rad/s}$ . IF PLATE IS SLIGHTLY DISTURBED, FIND IN ENSUING MOTION

- (a) MINIMUM VALUE OF  $\dot{\phi}$ , (b) MAXIMUM VALUE OF  $\dot{\theta}$ .

USING THE AXES  $Gxyz$ :

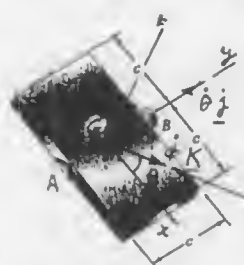
$$\omega = \dot{\phi} \cos \theta \hat{i} + \dot{\theta} \hat{j} + \dot{\phi} \sin \theta \hat{k}$$

CONSERVATION OF ANGULAR MOMENTUM

SINCE PLATE IS FREE TO ROTATE ABOUT  $Z$  AXIS,

$$H_z = \text{constant} \quad (1)$$

$$\text{BUT } H_z = H_x \cos \theta + H_z \sin \theta$$



$$H_z = I_x \omega_x \cos \theta + I_z \omega_z \sin \theta = \frac{1}{12} m c^2 \dot{\phi} \cos^2 \theta + \frac{5}{12} m c^2 \dot{\phi} \sin^2 \theta$$

$$= \frac{1}{12} m c^2 \dot{\phi} (\cos^2 \theta + 5 \sin^2 \theta) = \frac{1}{12} m c^2 \dot{\phi} (1 + 4 \sin^2 \theta)$$

USING THE INITIAL CONDITIONS, EQ. (1) YIELDS

$$\dot{\phi} (1 + 4 \sin^2 \theta) = \dot{\phi}_0 (1 + 4 \sin^2 \theta_0) \quad (2)$$

CONSERVATION OF ENERGY

SINCE NO WORK IS DONE, WE HAVE  $T = \text{constant}$  (3)

WHERE  $T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$

$$T = \frac{1}{2} \left( \frac{1}{12} m c^2 \dot{\phi}^2 \cos^2 \theta + \frac{1}{3} m c^2 \dot{\theta}^2 + \frac{5}{12} m c^2 \dot{\phi}^2 \sin^2 \theta \right)$$

$$= \frac{1}{24} m c^2 \left[ 4 \dot{\theta}^2 + \dot{\phi}^2 (\cos^2 \theta + 5 \sin^2 \theta) \right] = \frac{1}{24} m c^2 \left[ 4 \dot{\theta}^2 + \dot{\phi}^2 (1 + 4 \sin^2 \theta) \right]$$

USING THE INITIAL CONDITIONS, INCLUDING  $\dot{\theta}_0 = 0$ , EQ. (3) YIELDS

$$4 \dot{\theta}^2 + \dot{\phi}^2 (1 + 4 \sin^2 \theta) = \dot{\phi}_0^2 (1 + 4 \sin^2 \theta_0) \quad (4)$$

### PROBLEM 18.133

(a) WITH  $\theta_0 = 30^\circ$  AND  $\dot{\phi}_0 = 6 \text{ rad/s}$  IN (2) AND (4):

$$\dot{\phi} (1 + 4 \sin^2 \theta) = 12 \quad 4 \dot{\theta}^2 + \dot{\phi}^2 (1 + 4 \sin^2 \theta) = 72 \quad (2', 4')$$

ELIMINATING  $\dot{\phi}$  AND SOLVING FOR  $\dot{\theta}^2$ :  $\dot{\theta}^2 = 18 - \frac{36}{1 + 4 \sin^2 \theta}$  (5)

FOR  $\dot{\theta}^2 \geq 0$ :  $1 + 4 \sin^2 \theta \geq 2$ ,  $\sin^2 \theta \geq \frac{1}{4}$ ,  $30^\circ \leq \theta \leq 150^\circ$

(b) FROM (2'),  $\dot{\phi}$  IS MIN. FOR  $\theta = 90^\circ$ :  $\dot{\phi}_{\min} = 2.40 \text{ rad/s}$

(c) FROM (5),  $\dot{\theta}$  IS MAX. FOR  $\theta = 90^\circ$ :  $\dot{\theta}_{\max} = 3.29 \text{ rad/s}$

### PROBLEM 18.134

(a) WITH  $\theta_0 = 0$ ,  $\dot{\phi}_0 = 6 \text{ rad/s}$ , EQ. (2) YIELDS  $\dot{\phi} = \frac{6}{1 + 4 \sin^2 \theta}$

$\dot{\phi}$  IS MINIMUM FOR  $\theta = 90^\circ$ :  $\dot{\phi}_{\min} = 1.200 \text{ rad/s}$

(b) EQ. (4) YIELDS:  $4 \dot{\theta}^2 = 36 - \dot{\phi}^2 (1 + 4 \sin^2 \theta) = 36 \left( 1 - \frac{1}{1 + 4 \sin^2 \theta} \right)$

$\dot{\theta}$  IS LARGEST FOR  $\theta = 90^\circ$ :

$$4 \dot{\theta}_{\max}^2 = 36 \left( 1 - \frac{1}{5} \right) \quad \dot{\theta}_{\max}^2 = 7.20 \quad \dot{\theta}_{\max} = 2.68 \text{ rad/s}$$



18.135 and 18.136

GIVEN:

DISK WELDED TO ROD AC  
OF NEGLIGIBLE MASS CONNECTED  
BY CLEVIS TO SHAFT AB.

ROD AND DISK FREE TO ROTATE  
ABOUT AC; SHAFT FREE TO  
ROTATE ABOUT VERTICAL AXIS.

INITIALLY,  $\theta_0 = 90^\circ$ ,  $\dot{\theta}_0 = 0$ .

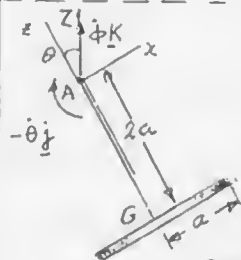
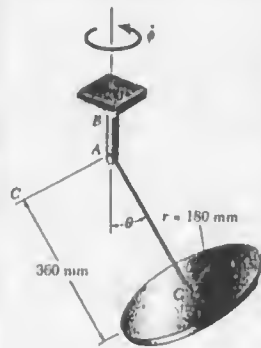
PROBLEM 18.135:

KNOWING THAT  $\dot{\phi}_{\max} = 2 \dot{\phi}_0$ 

FIND:

(a)  $\theta_{\min}$ , (b)  $\dot{\phi}_0$ 

PROBLEM 18.136:

KNOWING THAT  $\theta_{\min} = 30^\circ$ ,FIND: (a)  $\dot{\phi}_0$ , (b)  $\dot{\phi}_{\max}$ .

USING AXES  $Axyz$ , WITH  $y$   
POINTING INTO PAPER.

$$\omega = \dot{\phi} \sin \theta \mathbf{i} - \dot{\theta} \mathbf{j} + \dot{\phi} \cos \theta \mathbf{k}$$

$$I_x = I_y = \frac{17}{4} m a^2, \quad I_z = \frac{1}{2} m a^2$$

$$H_x = I_x \omega_x = \frac{17}{4} m a^2 \dot{\phi} \sin \theta$$

$$H_z = I_z \omega_z = \frac{1}{2} m a^2 \dot{\phi} \cos \theta$$

CONSERVATION OF ANG. MOM. ABOUT Z

SINCE ONLY FORCES ARE REACTION AT A AND  $\mathbf{W} = -mg \mathbf{k}$ ,  
WE HAVE  $\Sigma M_z = 0$  AND  $H_z = \text{constant}$ . THUS,

$$H_z = H_z \sin \theta + H_z \cos \theta = \frac{1}{2} m a^2 \dot{\phi} (17 \sin^2 \theta + 2 \cos^2 \theta)$$

$$H_z = \frac{1}{2} m a^2 \dot{\phi} (2 + 15 \sin^2 \theta) = \text{constant} \quad (1)$$

USING THE INITIAL CONDITIONS, EQ. (1) YIELDS  
 $\dot{\phi} (2 + 15 \sin^2 \theta) = 17 \dot{\phi}_0 \quad (2)$

CONSERVATION OF ENERGY

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) = \frac{1}{2} \frac{m a^2}{4} (17 \dot{\phi}^2 \sin^2 \theta + 17 \dot{\theta}^2 + 2 \dot{\phi}^2 \cos^2 \theta)$$

$$T = \frac{1}{8} m a^2 [(2 + 15 \sin^2 \theta) \dot{\phi}^2 + 17 \dot{\theta}^2] \quad V = -2 m g a \cos \theta$$

USING THE INITIAL CONDITIONS, WE WRITE  $T + V = \text{const.}$ 

$$(2 + 15 \sin^2 \theta) \dot{\phi}^2 + 17 \dot{\theta}^2 - 16 \frac{g}{a} \cos \theta = 17 \dot{\phi}_0^2 \quad (3)$$

PROBLEM 18.135

(a) LET  $\dot{\phi} = \dot{\phi}_{\max} = 2 \dot{\phi}_0$  IN (2):  $2 \dot{\phi}_0 (2 + 15 \sin^2 \theta) = 17 \dot{\phi}_0$ 

$$2 + 15 \sin^2 \theta = 8.5, \quad \sin \theta = \sqrt{0.4333}, \quad \theta = 41.169^\circ$$

(b) LET  $\theta = 41.169^\circ$ ,  $\dot{\theta} = 0$ ,  $\dot{\phi} = 2 \dot{\phi}_0$  IN (3):

$$(2 + 15 \sin^2 41.169^\circ) (4 \dot{\phi}_0^2) - 16 \frac{g}{a} \cos 41.169^\circ = 17 \dot{\phi}_0^2$$

$$17 \dot{\phi}_0^2 = 12.044 \left( \frac{9.81}{0.18} \right) \dot{\phi}_0^2 = 30.63 \quad \dot{\phi}_0 = 6.21 \text{ rad/s}$$

PROBLEM 18.136

(a) LET  $\theta = 30^\circ$  IN (2):  $\dot{\phi} (2 + 3.75) = 17 \dot{\phi}_0$ ,  $\dot{\phi} = \frac{17}{5.75} \dot{\phi}_0$  (4)LET  $\theta = 30^\circ$ ,  $\dot{\theta} = 0$  IN (3):

$$(2 + 3.75) \dot{\phi}^2 - 16 \left( \frac{9.81}{0.18} \right) \cos 30^\circ = 17 \dot{\phi}_0^2$$

SUBSTITUTE FOR  $\dot{\phi}$  FROM (4):

$$5.75 \left( \frac{17}{5.75} \right)^2 \dot{\phi}_0^2 - 872 \cos 30^\circ = 17 \dot{\phi}_0^2$$

$$17 \left( \frac{17}{5.75} - 1 \right) \dot{\phi}_0^2 = 872 \cos 30^\circ \quad \dot{\phi}_0 = 4.76 \text{ rad/s}$$

(b) FROM (4):  $\dot{\phi}_{\max} = \frac{17}{5.75} (4.7649)$ 

$$\dot{\phi}_{\max} = 14.07 \text{ rad/s}$$

\*18.137 and \*18.138

GIVEN:

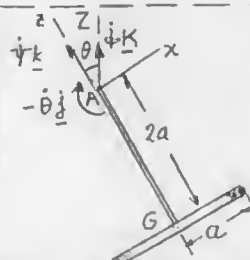
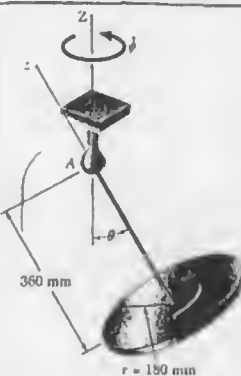
DISK WELDED TO ROD AC  
OF NEGLIGIBLE MASS SUPPORTED  
BY BALL AND SOCKET AT A.

INITIALLY,  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = \dot{\theta}_0 = 0$ .

PROBLEM 18.137:

KNOWING THAT  $\dot{\psi}_0 = 50 \text{ rad/s}$ ,FIND: (a)  $\theta_{\min}$ ,(b)  $\dot{\phi}$  AND  $\dot{\psi}$  FOR  $\theta = \theta_{\min}$ 

PROBLEM 18.138:

KNOWING THAT  $\theta_{\min} = 30^\circ$ ,FIND: (a)  $\dot{\psi}_0$ ,(b)  $\dot{\phi}$  AND  $\dot{\psi}$  FOR  $\theta = \theta_{\min}$ 

USING AXES  $Axyz$ , WITH  $y$  AXIS  
POINTING INTO PAPER.

$$\omega = \dot{\phi} \sin \theta \mathbf{i} - \dot{\theta} \mathbf{j} + (\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{k}$$

$$I_x = I_y = \frac{17}{4} m a^2, \quad I_z = \frac{1}{2} m a^2$$

$$H_x = \frac{m a^2}{4} [17 \dot{\phi} \sin \theta \mathbf{i} - 17 \dot{\theta} \mathbf{j} + 2(\dot{\psi} + \dot{\phi} \cos \theta) \mathbf{k}]$$

CONSERVATION OF ANG. MOMENTUM

SINCE THE ONLY EXTERNAL FORCES

ARE THE REACTION AT A AND THE WEIGHT  $\mathbf{W} = -mg \mathbf{k}$  AT G,  
WE HAVE  $\Sigma M_z = 0$ ,  $\Sigma M_x = 0$ . SINCE Z IS PART OF A NEWTONIAN  
FRAME, IT FOLLOWS THAT  $H_z = \text{const.}$ ; BECAUSE OF THE AXISYMMETRY  
OF THE DISK, IT ALSO FOLLOWS THAT  $H_x = \text{const.}$  (SEE PROB. 18.139).  
USING THE INITIAL CONDITIONS, WE WRITE

$$H_z = \text{const.} : \quad \dot{\psi} + \dot{\phi} \cos \theta = \dot{\psi}_0 \quad (1)$$

$$\text{NOTING THAT } H_x = \frac{m a^2}{4} [17 \dot{\phi} \sin^2 \theta + 2(\dot{\psi} + \dot{\phi} \cos \theta) \cos \theta]$$

AND SUBSTITUTING FROM (1) FOR THE INSIDE PARENTHESIS,

$$H_z = \text{const.} : \quad 17 \dot{\phi} \sin^2 \theta + 2 \dot{\psi}_0 \cos \theta = 0 \quad (2)$$

CONSERVATION OF ENERGY

$$T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) = \frac{1}{2} \frac{m a^2}{4} [17 \dot{\phi}^2 \sin^2 \theta + 17 \dot{\theta}^2 + 2(\dot{\psi} + \dot{\phi} \cos \theta)^2]$$

$$T = \frac{1}{8} m a^2 (17 \dot{\phi}^2 \sin^2 \theta + 17 \dot{\theta}^2 + 2 \dot{\psi}_0^2) \quad V = -2 m g a \cos \theta$$

$$T + V = \text{const.} : \quad 17 \dot{\phi}^2 \sin^2 \theta + 17 \dot{\theta}^2 + 2 \dot{\psi}_0^2 - 16 \frac{g}{a} \cos \theta = 2 \dot{\psi}_0^2$$

$$\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2 = \frac{16}{17} \frac{g}{a} \cos \theta \quad (3)$$

PROBLEM 18.137

$$(a) \text{ FROM (2): } \dot{\phi} = -\frac{2 \dot{\psi}_0 \cos \theta}{17 \sin^2 \theta} = -\frac{2}{17} (50 \text{ rad/s}) \frac{\cos \theta}{\sin^2 \theta}$$

CARRY INTO (3) AND LET  $\dot{\theta} = 0$  FOR  $\theta = \theta_{\min}$ :

$$\left( \frac{100}{17} \right) \frac{\cos^2 \theta}{\sin^4 \theta} = \frac{16}{17} \frac{9.81 \text{ m/s}^2}{0.18 \text{ m}} \quad \frac{10 \times 10^3}{16 \times 17} \frac{0.18}{9.81} \cos \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta + 0.67458 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2} (-0.67458 \pm 2.1107) = 0.71806 \quad \theta = 44.105^\circ$$

$$\text{OR } -1.3926 \text{ (IMPOSSIBLE)} \quad \theta = 44.1^\circ$$

(b) SUBSTITUTING  $\dot{\psi}_0 = 50 \text{ rad/s}$  AND  $\theta = 44.105^\circ$  IN (2) AND (1)

$$\text{EQ. (2): } \dot{\phi} = -\frac{2}{17} (50) \frac{\cos 44.105^\circ}{\sin^2 44.105^\circ} \quad \dot{\phi} = -8.72 \text{ rad/s}$$

$$\text{EQ. (1): } \dot{\psi} = 50 - (-8.72) \cos 44.105^\circ \quad \dot{\psi} = 56.3 \text{ rad/s}$$

PROBLEM 18.138

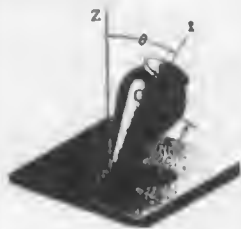
$$\text{LET } \theta = 30^\circ, \dot{\theta} = 0 \text{ IN (3): } \dot{\phi}^2 = \frac{16}{17} \frac{9.81 \text{ m/s}^2}{0.18 \text{ m}} \cos 30^\circ, \quad \dot{\phi} = \pm 13.33 \text{ rad/s}$$

FROM (2), WE NOTE THAT  $\dot{\phi} < 0$  FOR  $\dot{\psi}_0 > 0$ . THUS:  $\dot{\phi} = -13.33 \text{ rad/s}$ 

$$\text{EQ. (2): } \dot{\psi}_0 = -\frac{17}{2} (-13.33) \frac{\sin^2 30^\circ}{\cos 30^\circ} = 32.708, \quad \dot{\psi}_0 = 32.7 \text{ rad/s}$$

$$\text{EQ. (1): } \dot{\psi} = 32.708 - (-13.33) \cos 30^\circ \quad \dot{\psi} = 44.3 \text{ rad/s}$$

\*18.139



GIVEN:

TOP WITH FIXED POINT O

$\phi, \theta, \psi =$  EULERIAN ANGLES

$I =$  MOM. OF INERTIA ABOUT  $z$  AXIS

$I' =$  — — — TRANSVERSE

AXIS THROUGH O.

SHOW THAT:

$$(a) I' \dot{\phi} \sin^2 \theta + I(\dot{\psi} + \dot{\phi} \cos \theta) \cos \theta = \alpha \quad (1)$$

$$I(\dot{\psi} + \dot{\phi} \cos \theta) = \beta \quad (2)$$

$$(b) \omega_z = \text{const. AND } \dot{\phi} = \text{FUNCTION OF } \theta$$

WE USE FRAME  $Ox_1y_1z_1$  WITH  $y_1$  AXIS POINTING INTO PAPER

ANG. VELOCITY OF TOP:

$$\omega = -\dot{\phi} \sin \theta \hat{i} + \dot{\theta} \hat{j} + (\dot{\psi} + \dot{\phi} \cos \theta) \hat{k} \quad (A)$$

ANG. VELOCITY OF FRAME:

$$\Omega = -\dot{\phi} \sin \theta \hat{i} + \dot{\theta} \hat{j} + \dot{\phi} \cos \theta \hat{k} \quad (B)$$

ANG. MOMENTUM ABOUT O:

$$H_O = I_z \omega_z \hat{k} + I_y \omega_y \hat{j} + I_x \omega_x \hat{i}$$

$$H_O = -I' \dot{\phi} \sin \theta \hat{i} + I' \dot{\theta} \hat{j} + I(\dot{\psi} + \dot{\phi} \cos \theta) \hat{k} \quad (C)$$

$$(a) \text{ WE RECALL } \Sigma \underline{M}_O = \dot{H}_O \quad (18.27)$$

SINCE THE ONLY EXTERNAL FORCES ARE THE REACTION AT O AND THE WEIGHT  $\underline{W} = -mg \hat{k}$  AT G, WE HAVE

$\Sigma \underline{M}_O = 0$  AND FROM (18.27)  $\dot{H}_O = 0$ . SINCE THE  $z$  AXIS IS PART OF A NEWTONIAN FRAME OF REFERENCE, IT FOLLOWS THAT  $H_z = \text{constant}$ . BUT  $H_z = H \cdot \hat{k}$ .

SUBSTITUTING FOR  $H_O$  FROM (C) AND NOTING THAT  $\hat{i} \cdot \hat{k} = -\sin \theta$ ,  $\hat{j} \cdot \hat{k} = 0$ ,  $\hat{k} \cdot \hat{k} = \cos \theta$ , WE HAVE

$$H_z = H_O \cdot \hat{k} = -I' \dot{\phi} \sin \theta (-\sin \theta) + 0 + I(\dot{\psi} + \dot{\phi} \cos \theta) \cos \theta$$

$$\text{THUS: } I' \dot{\phi} \sin^2 \theta + I(\dot{\psi} + \dot{\phi} \cos \theta) \cos \theta = \alpha \quad (1)$$

WHERE  $\alpha$  IS A CONSTANT.

WE OBSERVE THAT WE ALSO HAVE  $\Sigma \underline{M}_x = 0$ , BUT WE CANNOT CONCLUDE THAT  $H_x = \text{const.}$ , SINCE THE  $z$  AXIS IS NOT PART OF A NEWTONIAN FRAME OF REFERENCE USING EQ. (18.28), WE WRITE

$$\Sigma \underline{M}_O = (\dot{H}_O)_{Ox_1y_1z_1} + \Omega \times H_O \quad (18.28)$$

SUBSTITUTING FROM (B) AND (C) INTO (18.28),

$$\Sigma \underline{M}_O = -I' \frac{d}{dt} (\dot{\phi} \sin \theta) \hat{i} + I' \dot{\theta} \hat{j} + I \frac{d}{dt} (\dot{\psi} + \dot{\phi} \cos \theta) \hat{k} + (-\dot{\phi} \sin \theta \hat{i} + \dot{\theta} \hat{j} + \dot{\phi} \cos \theta \hat{k}) \times [-I' \dot{\phi} \sin \theta \hat{i} + I' \dot{\theta} \hat{j} + I(\dot{\psi} + \dot{\phi} \cos \theta) \hat{k}]$$

CONSIDERING ONLY THE COEFFICIENTS OF  $\hat{k}$ , WE OBTAIN

$$\Sigma M_z = I \frac{d}{dt} (\dot{\psi} + \dot{\phi} \cos \theta) - I' \dot{\phi} \dot{\theta} \sin \theta + I \dot{\theta} \sin \theta = 0$$

BUT THE SECOND AND THIRD TERMS CANCEL OUT, DUE TO THE AXISYMMETRY OF THE TOP. THUS

$$\Sigma M_z = I \frac{d}{dt} (\dot{\psi} + \dot{\phi} \cos \theta) = 0$$

$$\text{AND } I(\dot{\psi} + \dot{\phi} \cos \theta) = \beta \quad (2)$$

WHERE  $\beta$  IS A CONSTANT

(b) FROM EQ. (A) WE HAVE  $\omega_z = \dot{\psi} + \dot{\phi} \cos \theta$  AND, IN VIEW OF (2):

$$\omega_z = \beta / I = \text{constant} \quad (3)$$

SUBSTITUTING FOR  $I(\dot{\psi} + \dot{\phi} \cos \theta)$  FROM (2) INTO (1):

$$I' \dot{\phi} \sin^2 \theta + \beta \cos \theta = \alpha \quad (4)$$

$$\dot{\phi} = \frac{\alpha - \beta \cos \theta}{I' \sin^2 \theta} \quad (\text{FUNCTION OF } \theta) \quad (5)$$

\*18.140

GIVEN:

TOP OF PROB. 18.139

SHOW THAT:

(a) A THIRD EQUATION OF MOTION CAN BE OBTAINED FROM THE PRINCIPLE OF CONSERVATION OF ENERGY.

(b) BY ELIMINATING  $\dot{\phi}$  AND  $\dot{\psi}$  FROM THAT EQUATION AND EQS. (1) AND (2) OF PROB. 18.140 AN EQUATION  $\dot{\theta}^2 = f(\theta)$  CAN BE OBTAINED, WHERE

$$f(\theta) = \frac{1}{I'} \left( 2E - \frac{\beta^2}{I} - 2mgc \cos \theta \right) - \left( \frac{\alpha - \beta \cos \theta}{I' \sin \theta} \right)^2 \quad (1)$$

(c) BY INTRODUCING THE VARIABLE  $x = \cos \theta$ , THE MAX. AND MIN. VALUES OF  $\theta$  CAN BE OBTAINED BY SOLVING THE CUBIC EQUATION

$$\left( 2E - \frac{\beta^2}{I} - 2mgcx \right) (1 - x^2) - \frac{1}{I'} (\alpha - \beta x)^2 = 0 \quad (2)$$

(a) CONSERVATION OF ENERGY.

$$T = \frac{1}{2} (I' \omega_x^2 + I' \omega_y^2 + I \omega_z^2)$$

REFERRING TO EQ. (A) OF PROB. 18.139:

$$T = \frac{1}{2} [I' \dot{\phi}^2 \sin^2 \theta + I' \dot{\theta}^2 + I(\dot{\psi} + \dot{\phi} \cos \theta)^2], \quad V = mgc \cos \theta$$

$$T + V = E;$$

$$\frac{1}{2} [I' \dot{\phi}^2 \sin^2 \theta + I' \dot{\theta}^2 + I(\dot{\psi} + \dot{\phi} \cos \theta)^2] + mgc \cos \theta = E \quad (6)$$

(b) SUBSTITUTING IN (6) FOR  $\dot{\phi}$  FROM EQ. (5) OF PROB. 18.139 AND FOR  $(\dot{\psi} + \dot{\phi} \cos \theta)$  FROM EQ. (2) OF PROB. 18.139, AND MULTIPLYING BY 2:

$$I' \left( \frac{\alpha - \beta \cos \theta}{I' \sin^2 \theta} \right)^2 \sin^2 \theta + I' \dot{\theta}^2 + I \left( \frac{\beta}{I} \right)^2 + 2mgc \cos \theta = 2E$$

$$\frac{(\alpha - \beta \cos \theta)^2}{I' \sin^2 \theta} + I' \dot{\theta}^2 + \frac{\beta^2}{I} + 2mgc \cos \theta = 2E$$

SOLVING FOR  $\dot{\theta}^2$ , WE OBTAIN

$$\dot{\theta}^2 = f(\theta)$$

WHERE

$$f(\theta) = \frac{1}{I'} \left( 2E - \frac{\beta^2}{I} - 2mgc \cos \theta \right) - \left( \frac{\alpha - \beta \cos \theta}{I' \sin \theta} \right)^2$$

(c) SETTING  $\cos \theta = x$ ,

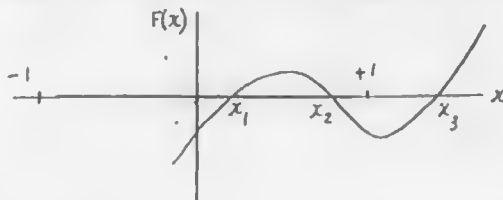
$$f(\theta) = \frac{1}{I'} \left( 2E - \frac{\beta^2}{I} - 2mgcx \right) - \frac{(\alpha - \beta x)^2}{I'^2 (1 - x^2)}$$

LETTING  $f(\theta) = 0$  AND MULTIPLYING BY  $I' (1 - x^2)$ ,

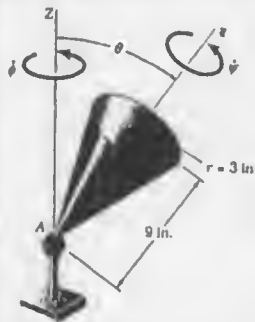
WE OBTAIN THE CUBIC EQUATION  $F(x) = 0$ :

$$\left( 2E - \frac{\beta^2}{I} - 2mgcx \right) (1 - x^2) - \frac{1}{I'} (\alpha - \beta x)^2 = 0$$

SOLVING THIS EQUATION WILL YIELD THREE VALUES OF  $x$ . THE TWO VALUES COMPRISED BETWEEN -1 AND +1 CORRESPOND TO THE MAX. AND MIN. VALUES OF  $\theta$ .



\* 18.141 and \* 18.142



**GIVEN:** SOLID CONE.  
INITIALLY,  $\theta_0 = 30^\circ$ ,  $\dot{\theta}_0 = 0$   
 $\dot{\psi}_0 = 300 \text{ rad/s}$ . USING EQ. (2)  
OF PROB. 18.140 AND  
PROBLEM 18.141:  
KNOWING THAT  $\dot{\phi}_0 = 20 \text{ rad/s}$   
FIND: (a)  $\theta_{\max}$ ,  
(b) CORRESPONDING  $\dot{\psi}$  AND  $\dot{\phi}$   
**PROBLEM 18.142:**  
KNOWING THAT  $\dot{\phi}_0 = -4 \text{ rad/s}$   
FIND: (a)  $\theta_{\max}$ ,  
(b) CORRESPONDING  $\dot{\psi}$  AND  $\dot{\phi}$   
(c) VALUE OF  $\theta$  FOR WHICH  
SENSE OF  $\dot{\phi}$  IS REVERSED

WE FIRST DETERMINE THE FOLLOWING CONSTANTS:

$$I = \frac{3}{10} m k^2 = \frac{3}{10} m (0.25 \text{ ft})^2 = (18.75 \times 10^{-3} \text{ ft}^2) m$$

$$I' = \frac{3}{5} m \left( \frac{1}{4} r^2 + h^2 \right) = \frac{3}{5} m \left( \frac{1}{4} (0.25 \text{ ft})^2 + (0.75 \text{ ft})^2 \right) m$$

$$= (346.875 \times 10^{-3} \text{ ft}^2) m$$

$$c = AG = \frac{3}{4} h = \frac{3}{4} (0.75 \text{ ft}) = 562.5 \times 10^{-3} \text{ ft}$$

NEXT, WE DETERMINE THE CONSTANTS  $\beta$ ,  $\alpha$ , AND  $\epsilon$  FROM EQS. (2) AND (4) OF THE SOLUTION OF PROB. 18.139 AND FROM EQ. (6) OF THE SOLUTION OF PROB. 18.140, USING THE APPROPRIATE INITIAL CONDITIONS.

**PROBLEM 18.141**

$$\beta = I (\dot{\psi}_0 + \dot{\phi}_0 \cos \theta_0) = (18.75 \times 10^{-3}) m (300 + 20 \cos 30^\circ)$$

$$= (5.94976) m$$

$$\alpha = I' \dot{\phi}_0 \sin^2 \theta_0 + \beta \cos \theta_0 = (346.875 \times 10^{-3}) m (20 \sin^2 30^\circ + (5.94976) m \cos 30^\circ)$$

$$= (6.84702) m$$

$$\epsilon = \frac{1}{2} [I' \dot{\phi}_0^2 \sin^2 \theta_0 + I' \dot{\theta}_0^2 + I (\dot{\psi}_0 + \dot{\phi}_0 \cos \theta_0)^2] + m g c \cos \theta_0$$

$$= \frac{1}{2} [(346.875 \times 10^{-3}) m (20)^2 \sin^2 30^\circ + 0 + (5.94976 m)^2] + m (32.2) (562.5 \times 10^{-3}) \cos 30^\circ = (977.020) m$$

SUBSTITUTE IN EQ. (2) OF PROB. 18.140:

$$(2\epsilon - \frac{\alpha^2}{\beta} - 2m g c x) (1 - x^2) - \frac{1}{2} (\alpha - \beta x)^2 = 0$$

$$(66.0593 - 36.225x)(1 - x^2) - 2.88288(6.84702 - 5.94976x)^2 = 0$$

(a) SOLVING:  $x = 0.743151$   $\theta_{\max} = 42.0^\circ$

(b) EQ. (5) OF PROB. 18.139:

$$\dot{\phi} = \frac{\alpha - \beta \cos \theta}{I' \sin^2 \theta} = \frac{6.84702 - 5.94976 \cos 42.0^\circ}{(346.875 \times 10^{-3}) m \sin^2 42.0^\circ} = 15.8748 \text{ rad/s}$$

FROM EQ. (2):  $\dot{\psi} = \frac{\beta}{I} - \dot{\phi} \cos \theta = \frac{5.94976}{18.75 \times 10^{-3}} - (15.8748) \cos 42.0^\circ$

$$\dot{\psi} = 306 \text{ rad/s}; \dot{\phi} = 15.87 \text{ rad/s}$$

**PROBLEM 18.142**

$$\beta = I (\dot{\psi}_0 + \dot{\phi}_0 \cos \theta_0) = (18.75 \times 10^{-3}) m (300 - 4 \cos 30^\circ) = (5.56005) m$$

$$\alpha = I' \dot{\phi}_0 \sin^2 \theta_0 + \beta \cos \theta_0 = (346.875 \times 10^{-3}) m (-4 \sin^2 30^\circ + (5.56005) m \cos 30^\circ) = (4.46827) m$$

$$\epsilon = \frac{1}{2} [I' \dot{\phi}_0^2 \sin^2 \theta_0 + I' \dot{\theta}_0^2 + I (\dot{\psi}_0 + \dot{\phi}_0 \cos \theta_0)^2] + m g c \cos \theta_0$$

$$= \frac{1}{2} [(346.875 \times 10^{-3}) m (-4)^2 \sin^2 30^\circ + 0 + (5.56005 m)^2] + m (32.2) (562.5 \times 10^{-3}) \cos 30^\circ = 840.76 m$$

SUBSTITUTE IN EQ. (2) OF PROB. 18.140:

$$(2\epsilon - \frac{\alpha^2}{\beta} - 2m g c x) (1 - x^2) - \frac{1}{2} (\alpha - \beta x)^2 = 0$$

$$(32.765 - 36.225x)(1 - x^2) - 2.88288(4.46827 - 5.56005x)^2 = 0$$

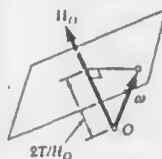
(a) SOLVING:  $x = 0.37166$ ,  $\theta_{\max} = 68.18^\circ$ ,  $\theta_{\min} = 68.2^\circ$

(b) EQ. (5):  $\dot{\phi} = \frac{\alpha - \beta \cos \theta}{I' \sin^2 \theta} = \frac{4.46827 - 5.56005 \cos 68.18^\circ}{(346.875 \times 10^{-3}) m \sin^2 68.18^\circ} = 8.0335 \text{ rad/s}$

EQ. (2):  $\dot{\psi} = (\beta/I) - \dot{\phi} \cos \theta = 296.536 - 8.0335 \cos 68.18^\circ = 293.55 \text{ rad/s}$   
 $\dot{\psi} = 294 \text{ rad/s}; \dot{\phi} = 8.03 \text{ rad/s}$

(c)  $\dot{\phi}$  REVERSES FOR  $\alpha - \beta \cos \theta = 0$ ,  $\cos \theta = \frac{4.46827}{5.56005}$ ,  $\theta = 36.5^\circ$

\* 18.143



**GIVEN:**

RIGID BODY OF ARBITRARY SHAPE  
SUPPORTED AT ITS MASS CENTER O  
AND SUBJECTED TO NO FORCE (EXCEPT  
AT SUPPORT O).

SHOW THAT:

(a)  $H_0 = \text{constant}$  (IN MAGNITUDE & DIR)  
 $T = \text{constant}$

PROJ. OF  $\omega$  ALONG  $H_0 = \text{constant}$

(b) TIP OF  $\omega$  DESCRIBES CIRCLE ON  
FIXED PLANE (THE "INVARIABLE PLANE")  
PERP. TO  $H_0$  AND AT DISTANCE  $2T/H_0$   
FROM O.

(c) WITH RESPECT TO PRINCIPAL AXES  
OxyZ ATTACHED TO BODY,  $\omega$

APPEARS TO DESCRIBE A CURVE ON ELLIPSOID OF EQUATION

$$I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 = 2T \quad (\text{POINCARÉ ELLIPSOID})$$

(a) FROM EQ. (18.27):  $\Sigma M_O = \dot{H}_0$   
SINCE  $\Sigma M_O = 0$ :  $\dot{H}_0 = \text{constant}$  (1)

$T + V = \text{const.}$ ; SINCE  $V = \text{const.}$ ,  $T = \text{constant}$  (2)

WE RECALL FROM PROB. 18.37 THAT  $H_0 \cdot \omega = 2T$

BUT  $H_0 \cdot \omega = H_0 \omega \cos \beta$

THUS  $H_0 \omega \cos \beta = 2T$

PROJ. OF  $\omega$  ON  $H_0 = \omega \cos \beta = \frac{2T}{H_0} = \text{const}$  (3)

(b) IT FOLLOWS FROM (3) THAT THE TIP OF  $\omega$  MUST REMAIN  
IN A PLANE  $\perp H_0$  AT A DISTANCE  $2T/H_0$  FROM O.

(c) FROM EQ. (18.20):

$$T = \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2)$$

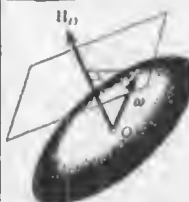
FROM (2) IT FOLLOWS THAT

$$I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 = 2T = \text{const.} \quad (4)$$

EQ. (4) IS THE EQUATION OF AN ELLIPSOID ON WHICH  
THE TIP OF  $\omega$  MUST LIE. THIS IS POINCARÉ ELLIPSOID.

COMPARING EQ. (4) WITH EQ. (9.49) OF SEC. 9.17, WE NOTE  
THAT POINCARÉ ELLIPSOID HAS THE SAME SHAPE AS THE  
ELLIPSOID OF INERTIA OF THE BODY, BUT A DIFFERENT SIZE.

\* 18.144



**GIVEN:**

POINCARÉ ELLIPSOID AND INVARIABLE  
PLANE DEFINED IN PROB. 18.143.

SHOW THAT:

(a) THE ELLIPSOID IS TANGENT TO THE  
PLANE.

(b) AS THE BODY MOVES THE POINCARÉ  
ELLIPSOID ROLLS ON THE INVARIABLE PLANE.

(a) AT THE TIP OF  $\omega$  THE DIRECTION OF THE NORMAL TO THE  
ELLIPSOID IS THAT OF  $\text{grad } F(\omega_1, \omega_2, \omega_3)$ , WHERE  $F$  DENOTES  
THE LEFT-HAND MEMBER OF EQ. (4) OF PROB. 18.143. FROM

SEC. 13.7:  $\text{grad } F = \frac{\partial F}{\partial \omega_1} \mathbf{i} + \frac{\partial F}{\partial \omega_2} \mathbf{j} + \frac{\partial F}{\partial \omega_3} \mathbf{k}$   
 $= 2 I_1 \omega_1 \mathbf{i} + 2 I_2 \omega_2 \mathbf{j} + 2 I_3 \omega_3 \mathbf{k}$   
 $= 2 (I_1 \omega_1 \mathbf{i} + I_2 \omega_2 \mathbf{j} + I_3 \omega_3 \mathbf{k}) = 2 H_0$

THUS, THE NORMAL TO POINCARÉ ELLIPSOID IS PARALLEL  
TO  $H_0$ . IT FOLLOWS THAT

POINCARÉ ELLIPSOID IS TANGENT TO THE INVARIABLE PLANE  
(CONTINUED)

\* 18.144 continued

(b) THE POINSET ELLIPSOID IS PART OF THE BODY WHOSE MOTION IS BEING ANALYZED, AND ITS POINT OF CONTACT WITH THE INVARIABLE PLANE IS THE TIP OF THE VECTOR  $\omega$ . SINCE  $\omega$  DEFINES THE INSTANTANEOUS AXIS OF ROTATION, THE POINT OF CONTACT HAS ZERO VELOCITY. THUS, THE POINSET ELLIPSOID ROLLS ON THE INVARIABLE PLANE (WITH ITS CENTER O REMAINING FIXED).

\* 18.145 GIVEN:

AXISYMMETRICAL RIGID BODY SUPPORTED AT ITS MASS CENTER O AND SUBJECTED TO NO FORCE (EXCEPT AT SUPPORT O).

USING THE RESULTS OBTAINED IN PROBS. 18.143-144, SHOW THAT THE POINSET ELLIPSOID IS AN ELLIPSOID OF REVOLUTION AND THE SPACE AND BODY CONES ARE BOTH CIRCULAR AND TANGENT TO EACH OTHER. FURTHER SHOW THAT

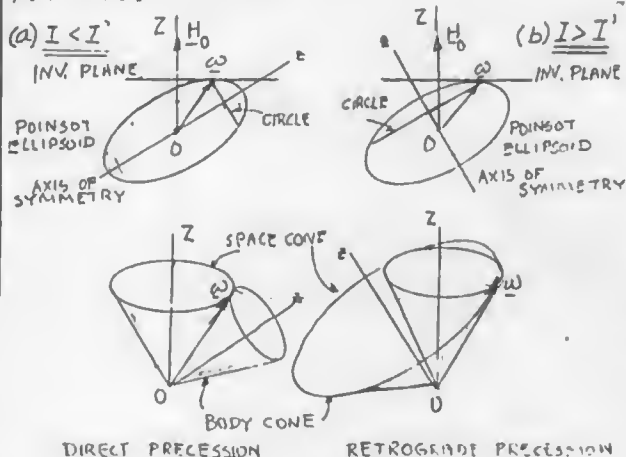
- (a) THE TWO CONES ARE TANGENT EXTERNALLY AND THE PRECESSION IS DIRECT WHEN  $I < I'$ , WHERE  $I$  = MOM. OF INERTIA ABOUT AXIS OF SYMMETRY  $I' = \text{--- -- -- -- -- TRANSVERSE AXIS}$ ,  
(b) THE SPACE CONE IS INSIDE THE BODY CONE AND THE PRECESSION IS RETROGRADE WHEN  $I > I'$ .

CHOOSING  $z$  ALONG THE AXIS OF SYMMETRY, WE HAVE  $I_x = I_y = I'$  AND  $I_z = I$ . SUBSTITUTE INTO (4) OF PROB. 18.143:

$$I'(\omega_x^2 + \omega_y^2) + I\omega_z^2 = \text{const.}$$

WHICH IS THE EQUATION OF AN ELLIPSOID OF REVOLUTION. IT FOLLOWS THAT THE TIP OF  $\omega$  DESCRIBES CIRCLES ON BOTH THE POINSET ELLIPSOID AND THE INVARIABLE PLANE, AND THAT THE VECTOR  $\omega$  ITSELF DESCRIBES CIRCULAR BODY AND SPACE CONES.

THE POINSET ELLIPSOID, THE INVARIABLE PLANE AND THE BODY AND SPACE CONES ARE SHOWN BELOW FOR CASES a AND b:



\* 18.146

GIVEN:

RIGID BODY OF ARBITRARY SHAPE AND ITS POINSET ELLIPSOID (CF. PROBS. 18.143 AND 18.144,

SHOW THAT:

(a) CURVE DESCRIBED BY TIP OF  $\omega$  ON POINSET ELLIPSOID IS DEFINED BY

$$I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 = 2T = \text{constant} \quad (1)$$

$$I_x^2 \omega_x^2 + I_y^2 \omega_y^2 + I_z^2 \omega_z^2 = H_0^2 = \text{constant} \quad (2)$$

AND CAN THUS BE OBTAINED BY INTERSECTING THE POINSET ELLIPSOID WITH THE ELLIPSOID DEFINED BY (2).

- (b) ASSUMING  $I_x > I_y > I_z$ , THE CURVES (CALLED POLHODES) OBTAINED FOR VARIOUS VALUES OF  $H_0$  HAVE THE SHAPES INDICATED IN FIGURE  
(c) THE BODY CAN ROTATE ABOUT A FIXED AXIS ONLY IF THAT AXIS COINCIDES WITH ONE OF THE PRINCIPAL AXES, THIS MOTION BEING STABLE IF THE AXIS IS THE MAJOR OR MINOR AXIS OF THE POINSET ELLIPSOID ( $x$  OR  $z$  AXIS) AND UNSTABLE IF IT IS THE INTERMEDIATE AXIS ( $y$  AXIS).

(a) EQ. (1) IN STATEMENT EXPRESSES CONSERVATION OF ENERGY; THIS IS EQ. (4) OF PROB. 18.143.

WE NOW EXPRESS THAT THE MAGNITUDE OF  $H$  IS CONSTANT:

$$H_0^2 = H_x^2 + H_y^2 + H_z^2 = I_x^2 \omega_x^2 + I_y^2 \omega_y^2 + I_z^2 \omega_z^2 = \text{const.}$$

WHICH IS EQ. (2) IN STATEMENT. SINCE THE COORDINATES  $\omega_x, \omega_y, \omega_z$  OF THE TIP OF  $\omega$  MUST SATISFY BOTH EQS. (1) AND (2), THE CURVE DESCRIBED BY THE TIP OF  $\omega$  IS THE INTERSECTION OF THE TWO ELLIPSOIDS.

(b) WE NOW WRITE THE EQUATIONS OF THE TWO ELLIPSOIDS IN THE STANDARD FORM

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

WHERE  $a, b, c$  ARE THE SEMI-AXES OF THE ELLIPSOID. WE HAVE

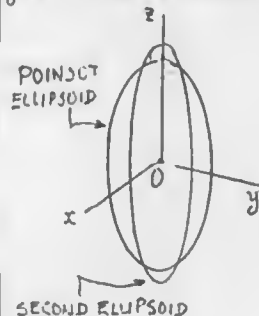
FOR POINSET ELLIPSOID:  $\frac{\omega_x^2}{2T/I_x} + \frac{\omega_y^2}{2T/I_y} + \frac{\omega_z^2}{2T/I_z} = 1 \quad (3)$

FOR SECOND ELLIPSOID:  $\frac{\omega_x^2}{H_0^2/I_x^2} + \frac{\omega_y^2}{H_0^2/I_y^2} + \frac{\omega_z^2}{H_0^2/I_z^2} = 1 \quad (4)$

SINCE WE ASSUMED THAT  $I_x > I_y > I_z$ , WE HAVE  $2T/I_x < 2T/I_y < 2T/I_z$  AND  $H_0^2/I_x^2 < H_0^2/I_y^2 < H_0^2/I_z^2$

THUS, FOR BOTH ELLIPSOIDS, THE MINOR AXIS IS DIRECTED ALONG THE  $x$  AXIS, THE INTERMEDIATE AXIS ALONG THE  $y$  AXIS, AND THE MAJOR AXIS ALONG THE  $z$  AXIS.

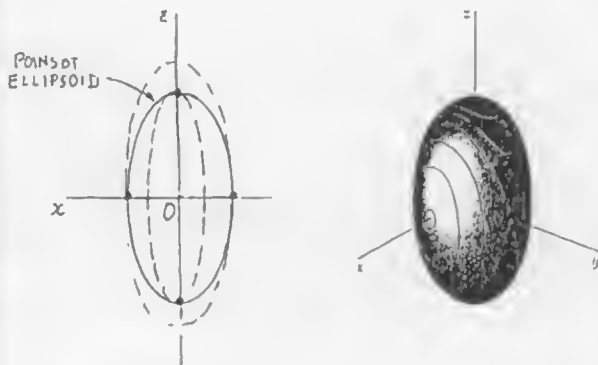
HOWEVER, BECAUSE THE RATIO OF THE MAJOR TO THE MINOR SEMIAXIS IS  $\sqrt{I_x/I_z}$  FOR THE POINSET ELLIPSOID AND  $I_z/I_x$  FOR THE SECOND ELLIPSOID, THE SHAPE OF THE LATTER WILL BE MORE "PRONOUNCED".



(CONTINUED)

# \* 18.146 continued

THE LARGEST ELLIPSOID OF THE SECOND TYPE TO BE IN CONTACT WITH THE POINSOT ELLIPSOID WILL BE OUTSIDE THAT ELLIPSOID AND TOUCH IT AT ITS POINTS OF INTERSECTION WITH THE  $x$  AXIS, AND THE SMALLEST WILL BE INSIDE THE POINSOT ELLIPSOID AND TOUCH IT AT ITS POINTS OF INTERSECTION WITH THE  $z$  AXIS (SEE LEFT-HAND SKETCH) ALL ELLIPSOIDS OF THE SECOND TYPE COMPRISED BETWEEN THESE TWO WILL INTERSECT THE POINSOT ELLIPSOID ALONG THE POLHODES AS SHOWN IN THE RIGHT-HAND FIGURE.



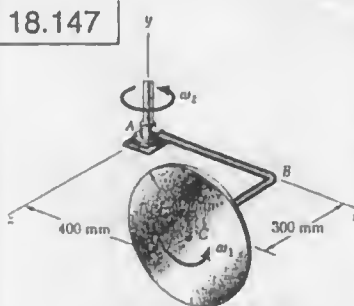
NOTE THAT THE ELLIPSOID OF THE SECOND TYPE WHICH HAS THE SAME INTERMEDIATE AXIS AS THE POINSOT ELLIPSOID INTERSECTS THAT ELLIPSOID ALONG TWO ELLIPSES WHOSE PLANES CONTAIN THE  $y$  AXIS. THESE CURVES ARE NOT POLHODES, SINCE THE TIP OF  $\omega$  WILL NOT DESCRIBE THEM, BUT THEY SEPARATE THE POLHODES INTO FOUR GROUPS: TWO GROUPS LOOP AROUND THE MINOR AXIS ( $x$  AXIS) AND THE OTHER TWO AROUND THE MAJOR AXIS ( $z$  AXIS).

(c) IF THE BODY IS SET TO SPIN ABOUT ONE OF THE PRINCIPAL AXES, THE POINSOT ELLIPSOID WILL REMAIN IN CONTACT WITH THE INVARIABLE PLANE AT THE SAME POINT (ON THE  $x$ ,  $y$ , OR  $z$  AXIS); THE ROTATION IS STEADY IN ANY OTHER CASE, THE POINT OF CONTACT WILL BE LOCATED ON ONE OF THE POLHODES AND THE TIP OF  $\omega$  WILL START DESCRIBING THAT POLHODE, WHILE THE POINSOT ELLIPSOID ROLLS ON THE INVARIABLE PLANE.

A ROTATION ABOUT THE MINOR OR THE MAJOR AXIS ( $x$  OR  $z$  AXIS) IS STABLE: IF THAT MOTION IS DISTURBED, THE TIP OF  $\omega$  WILL MOVE TO A VERY SMALL POLHODE SURROUNDING THAT AXIS AND STAY CLOSE TO ITS ORIGINAL POSITION.

ON THE OTHER HAND, A ROTATION ABOUT THE INTERMEDIATE AXIS ( $z$  AXIS) IS UNSTABLE: IF THAT MOTION IS DISTURBED, THE TIP OF  $\omega$  WILL MOVE TO ONE OF THE POLHODES LOCATED NEAR THAT AXIS AND START DESCRIBING IT, DEPARTING COMPLETELY FROM ITS ORIGINAL POSITION AND CAUSING THE BODY TO TUMBLE.

## 18.147

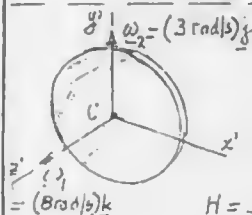


GIVEN:

DISK OF MASS  $m = 5 \text{ kg}$   
 $\omega_1 = 8 \text{ rad/s}$  (constant)  
 $\omega_2 = 3 \text{ rad/s}$  (constant)

FIND:

$$\frac{H_C}{C}$$



USING FRAME  $Cx'y'z'$ :

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2 \quad \bar{I}_{z'} = \frac{1}{2} m r^2$$

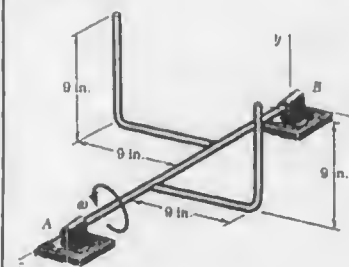
$$\frac{H_C}{C} = \bar{I}_{x'} \omega_2 \hat{j} + \bar{I}_{z'} \omega_1 \hat{k}$$

$$= \frac{1}{4} m r^2 (\omega_2 \hat{j} + 2 \omega_1 \hat{k})$$

$$\frac{H_C}{C} = \frac{1}{4} (5 \text{ kg}) (0.25 \text{ m})^2 [(3 \text{ rad/s}) \hat{j} + 2 (8 \text{ rad/s}) \hat{k}]$$

$$\frac{H_C}{C} = (0.234 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{j} + (1.250 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{k}$$

## 18.148



GIVEN:

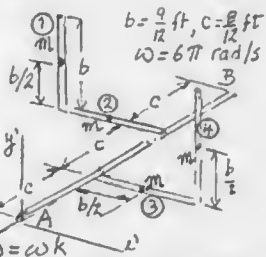
TWO L-SHAPED ARMS,  
 EACH WEIGHING 5 lb,  
 ARE WELDED TO ONE-  
 THIRD POINTS OF 24-in.  
 SHAFT AB.

$\omega = 180 \text{ rpm}$  (CONSTANT)

FIND:

(a)  $\frac{H_A}{A}$   
 (b) ANGLE THAT  $H_A$   
 FORMS WITH AB.

(a)



MOMENT OF INERTIA:

$$\begin{aligned} I_C &= 2 (I_C^D + I_C^P) \\ &= 2 \left[ \frac{1}{12} m b^2 + m \left( b^2 + \frac{b^2}{4} \right) \right] \\ &\quad + 2 \left[ \frac{1}{12} m b^2 + m \left( \frac{b^2}{4} \right) \right] \\ I_C &= \frac{10}{3} m b^2 \end{aligned}$$

PRODUCTS OF INERTIA  
 BECAUSE OF SYMMETRY OF  
 EACH ELEMENT ABOUT ITS  
 MASS CENTER:

$$\begin{aligned} I_{yz} &= m \bar{y}'_1 \bar{z}'_1 + m \bar{y}'_2 \bar{z}'_2 = m \left[ \left( \frac{b}{2} \right) (-c) + \left( \frac{b}{2} \right) (-c) \right] = -\frac{3}{2} m b c \\ I_{zx} &= m \bar{z}'_1 \bar{x}'_1 + m \bar{z}'_2 \bar{x}'_2 + m \bar{z}'_3 \bar{x}'_3 + m \bar{z}'_4 \bar{x}'_4 = m \left[ (-b)(-c) - \frac{b}{2}(-c) + b(-c) + \frac{b}{2}(-c) \right] = \frac{3}{2} m b c \end{aligned}$$

Eqs. (18.13):

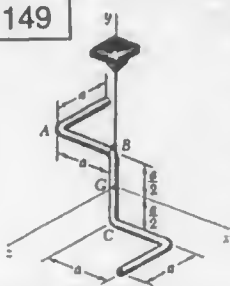
$$\begin{aligned} H_{x'} &= -I_{yz} \omega_2 = -\frac{3}{2} m b c \omega_2 = -\frac{3}{2} \frac{2.5}{32.2} \left( \frac{9}{12} \right) \left( \frac{9}{12} \right) (6 \pi \text{ rad/s}) = -1.0976 \text{ ft} \cdot \text{lb} \cdot \text{s} \\ H_{y'} &= -I_{yz} \omega_2 = +\frac{3}{2} m b c \omega_2 = +1.0976 \text{ ft} \cdot \text{lb} \cdot \text{s} \\ H_{z'} &= I_C \omega_2 = \frac{10}{3} m b^2 \omega_2 = \frac{10}{3} \frac{2.5}{32.2} \left( \frac{9}{12} \right)^2 (6 \pi \text{ rad/s}) = 2.744 \text{ ft} \cdot \text{lb} \cdot \text{s} \\ H_A &= (-1.098 \text{ ft} \cdot \text{lb} \cdot \text{s}) \hat{i} + (1.098 \text{ ft} \cdot \text{lb} \cdot \text{s}) \hat{j} + (2.74 \text{ ft} \cdot \text{lb} \cdot \text{s}) \hat{k} \end{aligned}$$

(b) SINCE THE UNIT VECTOR OF AB IS  $-\hat{k}$ , AND RECALLING EQ. (13.32), WE HAVE

$$\cos \theta = \frac{H_A \cdot (-\hat{k})}{|H_A|} = \frac{-2.744}{\sqrt{(1.0976)^2 + (2.744)^2}} = \frac{-2.744}{3.1526} = -0.87039$$

$$\theta = 150.5^\circ$$

18.149



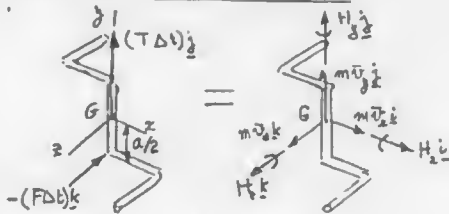
GIVEN:

ROD OF MASS  $m$  IS HIT AT C  
IN NEGATIVE  $z$  DIRECTION.  
IMPULSE  $= -(F\Delta t)\mathbf{k}$ .

FIND:

IMMEDIATELY AFTER IMPACT  
(a) ANG. VELOCITY OF ROD,  
(b) VELOCITY OF G.

IMPULSE-MOMENTUM PRINCIPLE



(WEIGHT IS OMITTED, SINCE NONIMPULSIVE)

(a) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$-\frac{a}{2}\mathbf{j} \times (-F\Delta t)\mathbf{k} = H_2\mathbf{i} + H_3\mathbf{j} + H_1\mathbf{k}$$

$$\frac{1}{2}aF\Delta t\mathbf{i} = H_2\mathbf{i} + H_3\mathbf{j} + H_1\mathbf{k}$$

$$\text{THUS: } H_2 = \frac{1}{2}aF\Delta t \quad H_3 = 0 \quad H_1 = 0$$

MOMENTS AND PRODUCTS OF INERTIA:

$$\bar{I}_x = \frac{1}{12}\frac{m}{2}a^2 + 2\frac{m}{2}\left(\frac{a}{2}\right)^2 + 2\frac{m}{2}\left(\frac{1}{2}a^2 + 2\frac{a^2}{4}\right) = 0.35ma^2$$

$$\bar{I}_y = 2\frac{m}{2}\frac{a^2}{2} + 2\frac{m}{2}\left(\frac{1}{2}a^2 + a^2 + \frac{a^2}{4}\right) = \frac{5}{2}ma^2$$

$$\bar{I}_z = \frac{1}{12}\frac{m}{2}a^2 + 2\frac{m}{2}\left(\frac{1}{2}a^2 + 2\frac{a^2}{4}\right) + 2\frac{m}{2}\left(a^2 + \frac{a^2}{4}\right) = 0.75ma^2$$

$$\bar{I}_{xy} = \frac{m}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{m}{2}\left(-a\right)\left(\frac{a}{2}\right) + \frac{m}{2}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + \frac{m}{2}\left(a\right)\left(-\frac{a}{2}\right) = -0.3ma^2$$

$$\bar{I}_{yz} = \frac{m}{2}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + \frac{m}{2}\left(-\frac{a}{2}\right)\left(\frac{a}{2}\right) = -0.1ma^2$$

$$\bar{I}_{zx} = \frac{m}{2}\left(-\frac{a}{2}\right)\left(-a\right) + \frac{m}{2}\left(\frac{a}{2}\right)\left(a\right) = 0.2ma^2$$

EQS. (18.17) AND DIVIDING BY  $ma^2$ :

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: \frac{F\Delta t}{2ma} = 0.35\omega_x + 0.3\omega_y - 0.2\omega_z \quad (1)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: 0 = 0.3\omega_x + \frac{5}{2}\omega_y + 0.1\omega_z \quad (2)$$

$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: 0 = -0.2\omega_x + 0.1\omega_y + 0.75\omega_z \quad (3)$$

SOLVING EQS. (1), (2), (3) SIMULTANEOUSLY:

$$\omega_x = \frac{30}{8}\frac{F\Delta t}{ma} \quad \omega_y = -\frac{15}{8}\frac{F\Delta t}{ma} \quad \omega_z = \frac{10}{8}\frac{F\Delta t}{ma}$$

$$\text{THUS: } \boldsymbol{\omega} = \frac{F\Delta t}{8ma}(30\mathbf{i} - 15\mathbf{j} + 10\mathbf{k})$$

(b) VELOCITY OF G

WE FIRST NOTE THAT THE GIVEN CONSTRAINTS REQUIRE  
THAT  $\dot{y}_G = 0$ . EQUATING THE COMPONENTS OF IMPULSE  
AND MOMENTUM:

$$x \text{ COMP: } 0 = m\dot{x}_G \quad \dot{x}_G = 0$$

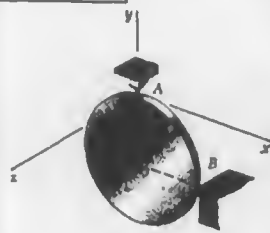
$$y \text{ COMP: } T\Delta t = m\dot{y}_G = 0 \quad T\Delta t = 0$$

$$z \text{ COMP: } -F\Delta t = m\dot{z}_G \quad \dot{z}_G = -\frac{F\Delta t}{m}$$

THEREFORE:

$$\bar{\mathbf{v}} = -\frac{F\Delta t}{m}\mathbf{k}$$

18.150



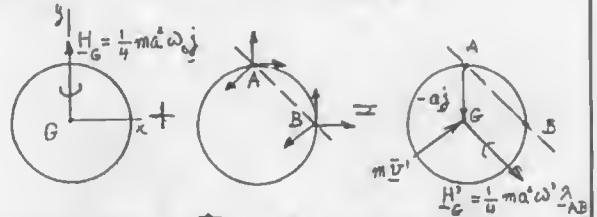
GIVEN:

DISK OF MASS  $m$  SUPPORTED BY  
BALL AND SOCKET AT A ROTATES  
WITH CONSTANT  $\boldsymbol{\omega} = \omega_0\mathbf{j}$  WHEN  
OBSTRUCTION IS INTRODUCED  
AT B. IMPACT PERFECTLY PLASTIC  
( $e=0$ ).

FIND:

IMMEDIATELY AFTER IMPACT  
(a) ANGULAR VELOCITY OF DISK,  
(b) VELOCITY OF G.

IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT  $\bar{I}_{\text{DIAM}} = \frac{1}{4}ma^2$  AND  $\mathbf{r}_{AB} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$ WE NOTE THAT  $\bar{\mathbf{v}}' = \boldsymbol{\omega}' \times \bar{\mathbf{A}}\bar{\mathbf{G}} = \omega'_0 \mathbf{j} \times (-a\mathbf{j}) = -\frac{1}{2}\omega'_0 a \mathbf{k}$ 

(a) EQUATE MOMENTS ABOUT AB OF ALL VECTORS AND COUPLES:

$$\mathbf{r}_{AB} \cdot \mathbf{H}_G + 0 = \mathbf{r}_{AB} \cdot (-a\mathbf{j} \times m\bar{\mathbf{v}}') + \mathbf{r}_{AB} \cdot \mathbf{H}'_G$$

$$\frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}) \cdot \frac{1}{4}ma^2\omega_0\mathbf{j} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}) \cdot [-a\mathbf{j} \times (-\frac{1}{2}m\omega'_0 a \mathbf{k})] + \mathbf{r}_{AB} \cdot \mathbf{H}'_G$$

$$-\frac{1}{4\sqrt{2}}ma^2\omega_0 = \frac{1}{2}ma^2\omega' + \frac{1}{4}ma^2\omega'$$

$$\omega' = -\frac{1}{3\sqrt{2}}\omega_0$$

$$\boldsymbol{\omega}' = \omega'_0 \mathbf{j} = -\frac{1}{3\sqrt{2}}\omega_0 \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}), \quad \boldsymbol{\omega}' = \frac{1}{6}\omega_0(-\mathbf{i} + \mathbf{j})$$

(b) RECALLING THAT  $\bar{\mathbf{v}}' = \boldsymbol{\omega}' \times \bar{\mathbf{A}}\bar{\mathbf{G}}$ ,

$$\bar{\mathbf{v}}' = \frac{1}{6}\omega_0(-\mathbf{i} + \mathbf{j}) \times (-a\mathbf{j}) \quad \bar{\mathbf{v}}' = \frac{1}{6}\omega_0 a \mathbf{k}$$

18.151

GIVEN:

DISK OF PROB. 18.150

FIND:

KINETIC ENERGY LOST WHEN DISK HITS OBSTRUCTION.

BEFORE IMPACT:

$$T_0 = \frac{1}{2}\bar{I}_{\text{DIAM}}\omega_0^2 = \frac{1}{2}\frac{1}{4}ma^2\omega_0^2 = \frac{1}{8}ma^2\omega_0^2$$

AFTER IMPACT:

$$T' = \frac{1}{2}m\bar{\mathbf{v}}'^2 + \frac{1}{2}\bar{I}_{\text{DIAM}}\omega'^2$$

BUT, FROM ANSWERS TO PROB. 18.150:

$$\bar{\mathbf{v}}'^2 = \left(\frac{1}{6}\omega_0 a\right)^2 = \frac{1}{36}\omega_0^2 a^2$$

$$\omega'^2 = \omega_x'^2 + \omega_y'^2 = \frac{\omega_0^2}{36}(1+1) = \frac{1}{18}\omega_0^2$$

THEREFORE:

$$T' = \frac{1}{2}m\left(\frac{1}{36}\omega_0^2 a^2\right) + \frac{1}{2}\frac{ma^2}{4}\left(\frac{1}{18}\omega_0^2\right) = \frac{1}{48}ma^2\omega_0^2$$

KINETIC ENERGY LOST

$$= T_0 - T' = \frac{1}{8}ma^2\omega_0^2 - \frac{1}{48}ma^2\omega_0^2$$

$$= \frac{5}{48}ma^2\omega_0^2$$

18.152



GIVEN:

TRIANGULAR PLATE OF MASS  $m$   
WELDED TO SHAFT SUPPORTED BY  
BEARINGS AT A AND B.  
PLATE ROTATES AT CONSTANT  
RATE  $\omega$ .

FIND:

DYNAMIC REACTIONS AT A  
AND B.

COMPUTATION OF MOMENT AND PRODUCTS OF INERTIA

FROM BACK COVER:

$$(I_y)_{\text{AREA}} = \frac{1}{12} b^3 h, \quad A = \frac{1}{2} b h, \quad (I_y)_{\text{MASS}} = (I_y)_{\text{AREA}} \frac{m}{A}$$

$$(I_y)_{\text{MASS}} = \frac{1}{12} b^3 h \left( \frac{m}{\frac{1}{2} b h} \right) = \frac{1}{6} m b^2$$

FROM SAMPLE PROB. 9.6 (PAGE 485 OF STATICS):

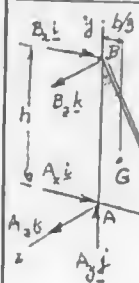
$$(I_y)_{\text{AREA}} = \frac{1}{24} b^3 h, \quad (I_y)_{\text{MASS}} = (I_y)_{\text{AREA}} \frac{m}{A} = \frac{1}{24} b^3 h \left( \frac{m}{\frac{1}{2} b h} \right) = \frac{1}{12} m b h$$

WE ALSO NOTE THAT  $I_{yz} = 0$ ANGULAR MOMENTUM  $H_A$ SINCE  $\omega_x = 0, \omega_y = \omega, \omega_z = 0$ , EQ. (18.13) YIELD

$$H_x = -I_{xy} \omega_y = -\frac{1}{12} m b h \omega, \quad H_y = I_y \omega_y = \frac{1}{6} m b^2 \omega, \quad H_z = 0$$

$$H_A = -\frac{1}{12} m b h \omega \mathbf{i} + \frac{1}{6} m b^2 \omega \mathbf{j} \quad (1)$$

EQUATIONS OF MOTION



(WEIGHT OMITTED FOR DYNAMIC REACTIONS)

FRAME OF REFERENCE  $xyz$  ROTATES  
WITH  $\Omega = \omega = \omega \mathbf{j}$ .

EQ. (18.28):

$$\Sigma \mathbf{M}_A = (\dot{\mathbf{H}}_A)_{xyz} + \Omega \times \mathbf{H}_A$$

$$= 0 + \omega \mathbf{j} \times \mathbf{H}_A$$

RECALLING (1) AND COMPUTING  $\Sigma \mathbf{M}_A$ :

$$h \mathbf{j} \times (B_x \mathbf{i} + B_z \mathbf{k}) = \omega \mathbf{j} \times \left( -\frac{1}{12} m b h \omega \mathbf{i} + \frac{1}{6} m b^2 \omega \mathbf{j} \right)$$

$$-h B_x \mathbf{k} + h B_z \mathbf{i} = \frac{1}{12} m b h \omega^2 \mathbf{k}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$B_x = -\frac{1}{12} m b \omega^2, \quad B_z = 0$$

$$B_y = -\frac{1}{12} m b \omega^2 \mathbf{i}$$

Σ Q. (18.1):

$$\Sigma \mathbf{F} = m \mathbf{a} \quad \text{WHERE } \mathbf{a} = -\omega^2 \mathbf{r} = -\frac{1}{3} b \omega^2 \mathbf{i}$$

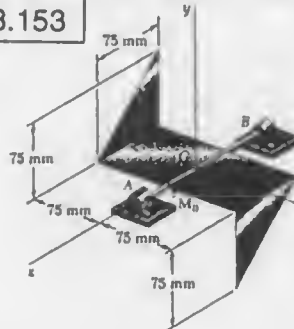
THUS:

$$A + B = -\frac{1}{3} b \omega^2 \mathbf{i}$$

$$A = -\frac{1}{3} b \omega^2 \mathbf{i} - \left( -\frac{1}{12} m b \omega^2 \mathbf{i} \right)$$

$$A = -\frac{1}{4} m b \omega^2 \mathbf{i}$$

18.153



GIVEN:

SHEET-METAL COMPONENT  
OF MASS  $m = 600 \text{ g}$ .  
LENGTH  $AB = 150 \text{ mm}$ .  
COMPONENT AT REST WHEN  
 $M_0 = (49.5 \text{ mN} \cdot \text{m}) \mathbf{k}$  IS  
APPLIED.

FIND:

DYNAMIC REACTIONS AT  
A AND B

(a) JUST AFTER COUPLE IS  
APPLIED

(b) 0.65 s LATER

MOMENT AND PRODUCTS OF INERTIA

RECTANGLE Z: MASS =  $\frac{2}{3} m$ 

$$I_z = \frac{1}{12} \left( \frac{2}{3} m \right) (2b)^2 = \frac{2}{9} m b^2$$

$$I_{xz} = I_{yz} = 0$$

TRIANGLE 1: MASS =  $\frac{1}{6} m$ 

FROM BACK COVER:

$$(\bar{I}_z)_{\text{AREA}} = \frac{1}{36} b^4, \quad A = \frac{1}{2} b^2, \quad (\bar{I}_z)_{\text{MASS}} = (\bar{I}_z)_{\text{AREA}} \frac{m}{A}$$

$$(\bar{I}_z)_{\text{MASS}} = \frac{1}{36} b^4 \left( \frac{\frac{1}{6} m}{\frac{1}{2} b^2} \right) = \frac{1}{108} m b^2$$

FROM SAMPLE PROB. 9.6 (PAGE 485 OF STATICS)

$$(\bar{I}_{yz})_{\text{AREA}} = -\frac{1}{72} b^4, \quad (\bar{I}_{yz})_{\text{MASS}} = -\frac{1}{72} b^4 \left( \frac{\frac{1}{6} m}{\frac{1}{2} b^2} \right) = -\frac{1}{216} m b^2$$

THEREFORE:

$$I_z = \bar{I}_z + \frac{m}{6} d^2 = \frac{1}{108} m b^2 + \frac{m}{6} \left[ b^2 + \left( \frac{b}{2} \right)^2 \right] = \left( \frac{1}{108} + \frac{10}{54} \right) m b^2 = \frac{7}{36} m b^2$$

$$I_{yz} = \bar{I}_{yz} + \frac{m}{6} \bar{y} \bar{z} = -\frac{1}{216} m b^2 + \frac{m}{6} \left( \frac{b}{3} \right) \left( -\frac{b}{6} \right) = -\frac{1}{72} m b^2$$

$$I_{xz} = \bar{I}_{xz} + \frac{m}{6} \bar{x} \bar{z} = 0 + \frac{m}{6} \left( -b \right) \left( -\frac{b}{6} \right) = \frac{1}{36} m b^2$$

TRIANGLE 3: BY SYMMETRY, SAME AS TRIANGLE 1.

FOR ENTIRE COMPONENT:

$$\bar{I}_z = \frac{2}{9} m b^2 + 2 \left( \frac{1}{36} m b^2 \right) = \frac{11}{18} m b^2$$

$$\bar{I}_{yz} = 2 \left( -\frac{1}{72} m b^2 \right) = -\frac{1}{36} m b^2$$

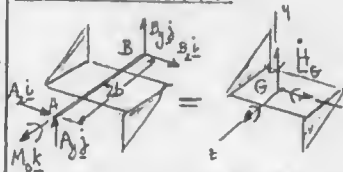
$$\bar{I}_{xz} = 2 \left( \frac{1}{36} m b^2 \right) = \frac{1}{18} m b^2$$

ANGULAR MOMENTUM  $H_G$ EQS. (18.7) WITH  $\omega_x = \omega_y = 0, \omega_z = \omega$ :

$$H_x = -\bar{I}_{xz} \omega_z = -\frac{1}{18} m b^2 \omega, \quad H_y = -\bar{I}_{yz} \omega_z = \frac{1}{36} m b^2 \omega, \quad H_z = \bar{I}_z \omega_z = \frac{11}{18} m b^2 \omega$$

$$\text{THUS: } H_G = \frac{1}{36} m b^2 \omega (-2\mathbf{i} + \mathbf{j} + 22\mathbf{k}) \quad (1)$$

EQUATIONS OF MOTION



EQ. (18.22), AND USING (1):

$$H_G = (\dot{\mathbf{H}}_G)_{xyz} + \Omega \times \mathbf{H}_G$$

$$= \frac{1}{36} m b^2 \alpha (-2\mathbf{i} + \mathbf{j} + 22\mathbf{k}) + \omega \mathbf{k} \times \frac{1}{36} m b^2 \omega (-2\mathbf{i} + \mathbf{j} + 22\mathbf{k})$$

EQUATING MOMENTS ABOUT B:

$$2b \mathbf{k} \times (A_x \mathbf{i} + A_y \mathbf{j}) + M_0 \mathbf{k} = \frac{1}{36} m b^2 [(-2\mathbf{i} + \mathbf{j} + 22\mathbf{k}) \alpha - (2\mathbf{j} + \mathbf{i}) \omega^2]$$

$$2b A_x \mathbf{j} - 2b A_y \mathbf{i} + M_0 \mathbf{k} = \frac{1}{36} m b^2 [(-2\alpha + \omega^2) \mathbf{i} + (\alpha - 2\omega^2) \mathbf{j} + 22\alpha \mathbf{k}]$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$\textcircled{1} -2b A_y = -\frac{1}{36} m b^2 (2\alpha + \omega^2) \quad A_y = \frac{1}{72} m b (2\alpha + \omega^2) \quad (2)$$

$$\textcircled{2} 2b A_x = \frac{1}{36} m b^2 (\alpha - 2\omega^2) \quad A_x = \frac{1}{72} m b (\alpha - 2\omega^2) \quad (3)$$

$$\textcircled{3} M_0 = \frac{11}{18} m b^2 \alpha \quad \alpha = 18 M_0 / 11 m b^2 \quad (4)$$

(CONTINUED)



### 18.153 continued

WE RECALL THE RESULTS OBTAINED:

$$A_y = \frac{1}{72} mb (2\alpha + \omega^2) \quad (2)$$

$$A_z = \frac{1}{72} mb (\alpha - 2\omega^2) \quad (3)$$

$$\alpha = 18 M_0 / 11 m b^2 \quad (4)$$

WITH GIVEN DATA:  $M_0 = 0.0495 \text{ N}\cdot\text{m}$ ,  $m = 0.6 \text{ kg}$ ,  $b = 0.075 \text{ m}$ :

$$\text{EQ. (4): } \alpha = 18(0.0495) / 11(0.6)(0.075)^2 = 24 \text{ rad/s}^2$$

$$\text{EQ. (3): } A_z = \frac{1}{72} (0.6)(0.075)(24 - 2(24)^2) = (15 - 1.25\omega^2) 10^{-3} \text{ N} \quad (3')$$

$$\text{EQ. (2): } A_y = \frac{1}{72} (0.6)(0.075)(2 \times 24 + \omega^2) = (30 + 0.625\omega^2) 10^{-3} \text{ N} \quad (2')$$

(a) JUST AFTER COUPLER IS ATTACHED:

$$\text{LETTING } \omega = 0 \text{ IN (3')} \text{ AND (2')}: A_z = 15 \times 10^{-3} \text{ N}, A_y = 30 \times 10^{-3} \text{ N}$$

$$\text{THUS: } \mathbf{A} = (15.00 \text{ mN})\mathbf{i} + (30.0 \text{ mN})\mathbf{j}$$

$$\Sigma \mathbf{F} = m\mathbf{\ddot{a}}: \mathbf{A} + \mathbf{B} = 0, \quad \mathbf{B} = -(15.00 \text{ mN})\mathbf{i} - (30.0 \text{ mN})\mathbf{j}$$

(b) AFTER 0.6 s:

$$\text{LETTING } \omega = \alpha t = (24 \text{ rad/s}^2)(0.6) = 14.40 \text{ rad/s IN (3')} \text{ AND (2')}: A_z = [15 - 1.25(14.4)^2] 10^{-3} \text{ N} = -244.2 \text{ mN}, A_y = [30 + 0.625(14.4)^2] 10^{-3} \text{ N} = 159.6 \text{ mN}$$

$$\text{THUS: } \mathbf{A} = -(244 \text{ mN})\mathbf{i} + (159.6 \text{ mN})\mathbf{j}$$

$$\Sigma \mathbf{F} = m\mathbf{\ddot{a}}: \mathbf{A} + \mathbf{B} = 0, \quad \mathbf{B} = (244 \text{ mN})\mathbf{i} - (159.6 \text{ mN})\mathbf{j}$$

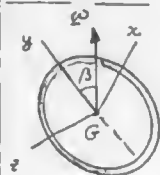
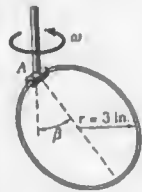
### 18.154

GIVEN:

RING ATTACHED BY COLLAR AT A TO VERTICAL SHAFT ROTATING AT CONSTANT RATE  $\omega$ .

FIND:

(a) CONSTANT ANGLE  $\beta$  THAT PLANE OF RING FORMS WITH VERTICAL WHEN  $\omega = 12 \text{ rad/s}$ ,  
(b) MAX. VALUE OF  $\omega$  FOR WHICH  $\beta = 0$ .



ANGULAR MOMENTUM  $\mathbf{H}_G$

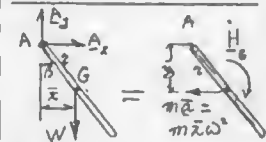
USING THE PRINCIPAL AXES  $Gx, Gy, Gz$  WITH  $x$  PERPENDICULAR TO PLANE OF RING:

$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$

$$= m r^2 \omega \sin \beta \mathbf{i} + \frac{1}{2} m r^2 \omega \cos \beta \mathbf{j}$$

$$\mathbf{H}_G = m r^2 \omega (\sin \beta \mathbf{i} + \frac{1}{2} \cos \beta \mathbf{j}) \quad (1)$$

EQUATIONS OF MOTION



$$x = r \sin \beta$$

$$y = r \cos \beta$$

EQUATING MOMENTS ABOUT A:

$$\sum \mathbf{M}_A = (m \bar{x} \omega^2) \mathbf{j} + \mathbf{H}_G$$

$$m g z \sin \beta = m (r \sin \beta) \omega^2 (r \cos \beta) + \frac{1}{2} m r^2 \omega^2 \sin \beta \cos \beta$$

$$g = \omega^2 r \cos \beta + \frac{1}{2} \omega^2 r \cos \beta$$

$$\cos \beta = \frac{2g}{3\omega^2} \quad (2)$$

EQ. (18.22) AND USING (1):

$$\mathbf{H}_G = (\mathbf{H}_G)_{Gx, Gy, Gz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$\dot{\mathbf{H}}_G = 0 + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= \omega (\sin \beta \mathbf{i} + \frac{1}{2} \cos \beta \mathbf{j}) \times m r^2 \omega (\sin \beta \mathbf{i} + \frac{1}{2} \cos \beta \mathbf{j})$$

$$= m r^2 \omega^2 (\frac{1}{2} \sin \beta \cos \beta \mathbf{j} - \sin \beta \cos \beta \mathbf{j})$$

$$= -\frac{1}{2} m r^2 \omega^2 \sin \beta \cos \beta \mathbf{k}$$

(a) LETTING  $g = 32.2 \text{ ft/s}^2$ ,  $t = 0.25 \text{ ft}$ ,  $\omega = 12 \text{ rad/s}$ :

$$\cos \beta = \frac{2}{3} \frac{32.2}{(12)^2} = 0.59630 \quad \beta = 53.4^\circ$$

(b) SOLVING EQ. (2) FOR  $\omega^2$  AND LETTING  $g = 32.2 \text{ ft/s}^2$ ,  $r = 0.25 \text{ ft}$ ,  $\beta = 0$ :

$$\omega^2 = \frac{2g}{3r} = \frac{2(32.2)}{3(0.25)} = 85.87 \quad \omega = 9.27 \text{ rad/s}$$

### 18.155

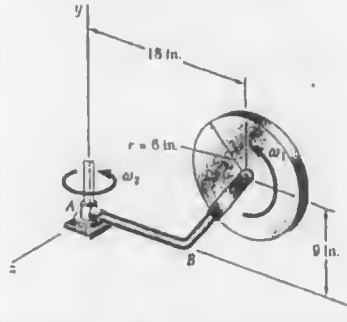
GIVEN:

10-lb DISK ROTATES AT CONSTANT RATE  $\omega_1 = 15 \text{ rad/s}$ .

ARM ABC ROTATES AT CONSTANT RATE  $\omega_2 = 5 \text{ rad/s}$ .

FIND:

FORCE-COUPLE SYSTEM REPRESENTING THE DYNAMIC REACTION AT SUPPORT A.



ANGULAR MOMENTUM OF DISK ABOUT C.

USING THE PRINCIPAL CENTROIDAL AXES  $Cx, Cy, Cz$ :

$$\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$$

ANG. VELOCITY OF FRAME  $Cx, Cy, Cz$ :

$$\boldsymbol{\Omega} = \omega_2 \mathbf{j}$$

$$\mathbf{H}_C = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$

$$= 0 + \frac{1}{4} m r^2 \omega_2 \mathbf{j} + \frac{1}{2} m r^2 \omega_1 \mathbf{k}$$

$$\mathbf{H}_C = \frac{1}{4} m r^2 (\omega_2 \mathbf{j} + 2\omega_1 \mathbf{k}) \quad (1)$$

RATE OF CHANGE OF  $\mathbf{H}_C$

EQ. (18.22) AND USING (1):

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Cx, Cy, Cz} + \boldsymbol{\Omega} \times \mathbf{H}_C = 0 + \omega_2 \mathbf{j} \times \frac{1}{4} m r^2 (\omega_2 \mathbf{j} + 2\omega_1 \mathbf{k})$$

$$\dot{\mathbf{H}}_C = \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{i}$$

WITH GIVEN DATA:

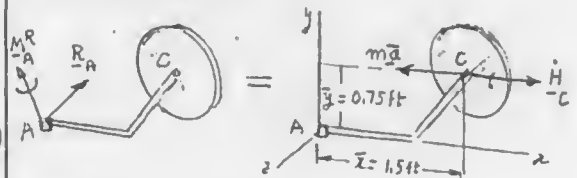
$$\dot{\mathbf{H}}_C = \frac{1}{2} \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{1}{2} \text{ ft})^2 (15 \text{ rad/s})(5 \text{ rad/s}) \mathbf{i} = (2.9115 \text{ lb}\cdot\text{ft}) \mathbf{i}$$

COMPUTATION OF  $m \bar{\mathbf{a}}$

$$\bar{\mathbf{a}} = -\bar{x} \omega_2^2 \mathbf{i} = -(1.5 \text{ ft})(5 \text{ rad/s})^2 \mathbf{i} = -(37.5 \text{ ft/s}^2) \mathbf{i}$$

$$m \bar{\mathbf{a}} = \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} (-37.5 \text{ ft/s}^2) \mathbf{i} = -(11.646 \text{ lb}) \mathbf{i}$$

EQUATIONS OF MOTION



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}}: \mathbf{R}_A = m \bar{\mathbf{a}} = -(11.646 \text{ lb}) \mathbf{i}$$

$$\Sigma \mathbf{M}_A = \Sigma (\mathbf{M}_A)_{\text{eff}}:$$

$$\mathbf{M}_A^R = \dot{\mathbf{H}}_C + (\bar{x} \mathbf{i} + \bar{y} \mathbf{j}) \times m \bar{\mathbf{a}}$$

$$= (2.9115 \text{ lb}\cdot\text{ft}) \mathbf{i} + [(1.5 \text{ ft}) \mathbf{i} + (0.75 \text{ ft}) \mathbf{j}] \times (-11.646 \text{ lb}) \mathbf{i}$$

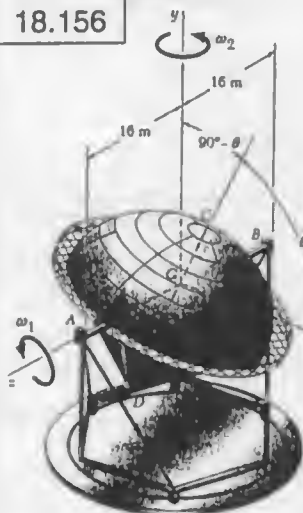
$$= (2.9115 \text{ lb}\cdot\text{ft}) \mathbf{i} + (8.734 \text{ lb}\cdot\text{ft}) \mathbf{k}$$

FORCE-COUPLE SYSTEM AT A:

$$\mathbf{R}_A = -(11.65 \text{ lb}) \mathbf{i}; \quad \mathbf{M}_A^R = (2.91 \text{ lb}\cdot\text{ft}) \mathbf{i} + (8.73 \text{ lb}\cdot\text{ft}) \mathbf{k}$$



18.156



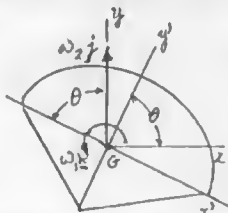
GIVEN:

SOLAR-ENERGY CONCENTRATOR:  $m = 30 \text{ kg}$   
 RADIUS OF GYRATION ABOUT CD:  $k = 12 \text{ m}$   
 ABOUT AB:  $k' = 10 \text{ m}$   
 $\omega_1 = 0.20 \text{ rad/s}$  (constant)  
 $\omega_2 = 0.25 \text{ rad/s}$  (constant)

FIND FOR  $\theta = 60^\circ$ :

(a) FORCES EXERTED ON CONCENTRATOR AT A AND B.  
 (b) COUPLE  $M_z$  APPLIED TO CONCENTRATOR AT THAT INSTANT.

ANGULAR MOMENTUM ABOUT G

USING THE PRINCIPAL AXES  $Gx'y'z$ .

$$\omega_2 = -\omega_2 \cos \theta, \omega_3 = \omega_2 \sin \theta, \omega_2 = \omega_2$$

$$\underline{H}_G = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

$$\underline{H}_G = -I_x \omega_2 \cos \theta \underline{i} + I_y \omega_2 \sin \theta \underline{j} + I_z \omega_2 \underline{k}$$

$$\text{WHERE } I = m \bar{k}^2 \text{ AND } I' = m \bar{k}'^2$$

WE NOW RETURN TO THE REFERENCE FRAME  $Gxyz$  ATTACHED TO THE STEEL FRAMEWORK ( $\underline{\Omega} = \omega_2 \underline{j}$ ).

$$\dot{\underline{H}}_G = -I_x \omega_2 \dot{\theta} (\underline{i} \sin \theta - \underline{j} \cos \theta) + I_y \omega_2 \sin \theta (\cos \theta \underline{i} + \sin \theta \underline{j}) + I_z \omega_2 \underline{k}$$

$$\underline{H}_G = (I - I') \omega_2 \sin \theta \cos \theta \underline{i} + (I' \cos \theta + I \sin \theta) \omega_2 \underline{j} + I_z \omega_2 \underline{k} \quad (1)$$

RATE OF CHANGE OF  $\underline{H}_G$ 

WE NOTE THAT  $\omega_1$  AND  $\omega_2$  ARE CONSTANT, BUT THAT  $\theta$  IS A FUNCTION OF  $t$  WITH DERIVATIVE  $\dot{\theta} = \omega_1$

EQ. (18.22):

$$\begin{aligned} \dot{\underline{H}}_G &= (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = (I - I') \omega_2 (\cos \theta \dot{\theta} - \sin \theta \dot{\theta}) \underline{i} + \\ &+ 2(I - I') \omega_2 \sin \theta \cos \theta \dot{\theta} \underline{j} + \omega_2 \underline{j} \times [(I - I') \omega_2 \sin \theta \cos \theta \underline{i} + \\ &+ (I' \cos \theta + I \sin \theta) \omega_2 \underline{j} + I_z \omega_2 \underline{k}] \\ &= (I - I') \omega_2 (\cos 2\theta \underline{i} + \sin 2\theta \underline{j}) - \frac{1}{2} (I - I') \omega_2^2 \sin 2\theta \underline{k} + I_z \omega_2 \underline{k} \\ \dot{\underline{H}}_G &= [I' + (I - I') \cos 2\theta] \omega_2 \dot{\theta} \underline{i} + (I - I') \omega_2 \sin 2\theta \dot{\theta} \underline{j} - \frac{1}{2} (I - I') \omega_2^2 \sin 2\theta \underline{k} \end{aligned}$$

WITH GIVEN DATA:  $I = m \bar{k}^2 = (30 \times 10^3 \text{ kg})(12 \text{ m})^2 = 4.32 \times 10^6 \text{ kg} \cdot \text{m}^2$ 

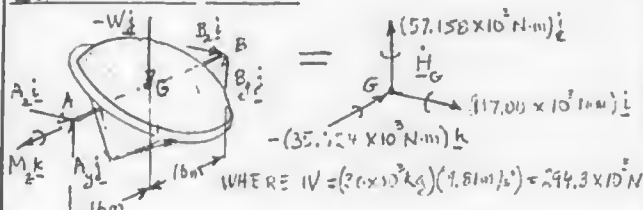
$$I' = m \bar{k}'^2 = (30 \times 10^3 \text{ kg})(10 \text{ m})^2 = 3.00 \times 10^6 \text{ kg} \cdot \text{m}^2$$

$$\omega_1 = 0.20 \text{ rad/s}, \omega_2 = 0.25 \text{ rad/s}, \theta = 60^\circ, 2\theta = 120^\circ$$

$$\dot{\underline{H}}_G = (3 + 1.32 \cos 120^\circ) 10^6 (0.20)(0.25) \underline{i} + (1.32 \sin 120^\circ) 10^6 (0.20)(0.25) \underline{j} - \frac{1}{2} (1.32 \sin 120^\circ) 10^6 (0.25)^2 \underline{k}$$

$$\dot{\underline{H}}_G = (117.00 \times 10^3 \text{ N} \cdot \text{m}) \underline{i} + (57.158 \times 10^3 \text{ N} \cdot \text{m}) \underline{j} - (35.724 \times 10^3 \text{ N} \cdot \text{m}) \underline{k} \quad (2)$$

EQUATIONS OF MOTION



$$\text{WHERE } IV = (2 \times 10^3 \text{ kg})(9.81 \text{ m/s}^2) = 19.62 \times 10^3 \text{ N}$$

(CONTINUED)

18.156 continued

$$\Sigma \underline{M}_B = \Sigma (\dot{\underline{H}}_B)_{\text{eff}}:$$

$$(57.158 \text{ m}) \underline{k} \times (A_x \underline{i} - A_y \underline{j}) + (16 \text{ m}) \underline{k} \times (-19.62 \times 10^3 \text{ N}) \underline{j} + M_z \underline{k} = \dot{\underline{H}}_B$$

$$(32 \text{ m}) A_x \underline{j} - (32 \text{ m}) A_y \underline{i} + (16 \text{ m})(19.62 \times 10^3 \text{ N}) \underline{i} + M_z \underline{k} = (117 \times 10^3 \text{ N} \cdot \text{m}) \underline{i} + (57.158 \times 10^3 \text{ N} \cdot \text{m}) \underline{j} - (35.724 \times 10^3 \text{ N} \cdot \text{m}) \underline{k}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$\textcircled{1} \quad -(32 \text{ m}) A_y + (16 \text{ m})(19.62 \times 10^3 \text{ N}) = 117 \times 10^3 \text{ N} \cdot \text{m}$$

$$A_y = 143.49 \times 10^3 \text{ N}$$

$$\textcircled{2} \quad (32 \text{ m}) A_x = 57.158 \times 10^3 \text{ N} \cdot \text{m}$$

$$A_x = 1.7862 \times 10^3 \text{ N}$$

$$\textcircled{3} \quad M_z = -35.724 \times 10^3 \text{ N} \cdot \text{m}$$

(a) FORCES AT A AND B

$$\underline{A} = A_x \underline{i} + A_y \underline{j} \quad \underline{A} = (1.786 \text{ kN}) \underline{i} + (143.5 \text{ kN}) \underline{j}$$

$$\Sigma \underline{F} = m \underline{\ddot{a}} = 0: \underline{A} + \underline{B} - W \underline{j} = 0$$

$$\underline{B} = (19.62 \text{ kN}) \underline{j} - (1.786 \text{ kN}) \underline{i} - (143.5 \text{ kN}) \underline{j}$$

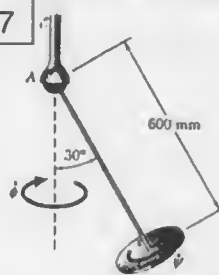
$$\underline{B} = -(1.786 \text{ kN}) \underline{i} + (15.08 \text{ kN}) \underline{j}$$

(b) COUPLE  $M_z$ 

$$M_z \underline{k} = -(35.724 \times 10^3 \text{ N} \cdot \text{m}) \underline{k}$$

$$M_z \underline{k} = -(35.7 \text{ kN} \cdot \text{m}) \underline{k}$$

18.157



GIVEN:

2-kg DISK OF 150-mm DIAMETER ATTACHED TO ROD SUPPORTED BY BALL AND SOCKET AT A.

$$\dot{\phi} = 36 \text{ rpm AS SHOWN}$$

FIND:

RATE OF SPIN  $\dot{\psi}$ 

USING THE FRAME  $Axyz$  (WITH THE  $y$  AXIS POINTING TOWARD US), AND NOTING THAT THE PRECESSION IS STEADY EQ. (18.43) YIELDS

$$\Sigma \underline{M}_A = \underline{\Omega} \times \underline{H}_A \quad (1)$$

WHERE

$$\underline{H}_A = I_x \omega_x \underline{i} + I_y \omega_y \underline{j}$$

$$\underline{H}_A = I'(-\dot{\phi} \sin \beta) \underline{i} + I(\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}$$

$$\text{AND } \underline{\Omega} = -\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}$$

$$\text{THUS: } \Sigma \underline{M}_A = (-\dot{\phi} \sin \beta \underline{i} + \dot{\phi} \cos \beta \underline{k}) \times [I'(-\dot{\phi} \sin \beta) \underline{i} + I(\dot{\psi} + \dot{\phi} \cos \beta) \underline{k}]$$

$$\Sigma \underline{M}_A = [I(\dot{\psi} + \dot{\phi} \cos \beta) - I' \dot{\phi} \cos \beta] \dot{\phi} \sin \beta \underline{j}$$

$$\text{BUT } \Sigma \underline{M}_A = \underline{AB} \times -mg \underline{k} = -l \underline{k} \times -mg(-\sin \beta \underline{i} + \cos \beta \underline{k})$$

$$= -mg l \sin \beta \underline{j} \quad (2)$$

EQUATING THE R.H. MEMBERS OF (2) AND (2):

$$[I(\dot{\psi} + \dot{\phi} \cos \beta) - I' \dot{\phi} \cos \beta] \dot{\phi} \sin \beta = -mg l \sin \beta$$

$$[I \dot{\psi} + (I - I') \dot{\phi} \cos \beta] \dot{\phi} = -mg l$$

$$\dot{\psi} = \frac{I' - I}{I} \dot{\phi} \cos \beta - \frac{mg l}{I \dot{\phi}} \quad (4)$$

FROM GIVEN DATA:  $I = \frac{1}{2} m r^2 = \frac{1}{2} (2 \text{ kg})(0.075 \text{ m})^2 = 5.625 \times 10^{-3}$ 

$$I' = m(l^2 + \frac{r^2}{4}) = 2[(0.6)^2 + \frac{1}{4}(0.075)^2] = 0.72281, \frac{I' - I}{I} = 127.5$$

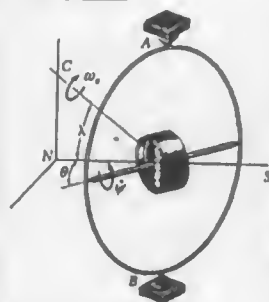
$$\dot{\phi} = -36 \text{ rpm} = -1.2 \pi \text{ rad/s}, \beta = 30^\circ$$

$$\text{EQ. (4): } \dot{\psi} = (127.5)(-1.2 \pi) \cos 30^\circ - \frac{2(9.81)(0.6)}{(5.625 \times 10^{-3})(-1.2 \pi)}$$

$$= -416.27 + 555.13 = 138.86 \text{ rad/s}$$

$$\dot{\psi} = 1326 \text{ rpm}$$

18.158

GIVEN:

GYROCOMPASS CONSISTS OF ROTOR SPINNING AT RATE  $\dot{\psi}$  ABOUT AXIS MOUNTED IN GIMBAL ROTATING FREELY ABOUT VERTICAL AB.  $\theta$  = ANGLE FORMED BY AXIS OF ROTOR AND MERIDIAN NS.

$\lambda$  = LATITUDE = ANGLE FORMED BY NS AND LINE OC PARALLEL TO EARTH AXIS

$\omega_e$  = ANG. VELOCITY OF EARTH ABOUT ITS AXIS.

SHOW THAT

(a) THE EQUATIONS OF MOTION OF THE GYROCOMPASS ARE

$$I'\ddot{\theta} + I\omega_e\omega_r \cos \lambda \sin \theta - I'\omega_e^2 \cos^2 \lambda \sin \theta \cos \theta = 0$$

$$I\dot{\omega}_e = 0$$

WHERE  $\omega_e$  = RECTANGULAR COMPONENT OF TOTAL ANG. VELOCITY ALONG AXIS OF ROTOR

(b) NEGLECTING TERMS IN  $\omega_e^2$  AND FOR SMALL VALUES OF  $\theta$ ,

$$\ddot{\theta} + \frac{I\omega_e\omega_r \cos \lambda}{I'} \theta = 0$$

AND THAT AXIS OF ROTOR OSCILLATES ABOUT THE LINE NS.

(a) ANGULAR MOMENTUM ABOUT O.

WE SELECT A FRAME OF REFERENCE  $Oxyz$  ATTACHED TO THE GIMBAL. THE ANG. VELOCITY OF  $Oxyz$  WITH RESPECT TO A NEWTONIAN FRAME IS  $\underline{\Omega} = \omega_e \underline{k} + \dot{\theta} \underline{j}$

WHERE

$$\underline{k} = -\cos \lambda \sin \theta \underline{i} + \sin \lambda \underline{j} + \cos \lambda \cos \theta \underline{k}$$

$$\text{THUS: } \underline{\Omega} = -\omega_e \cos \lambda \sin \theta \underline{i} + (\dot{\theta} + \omega_e \sin \lambda) \underline{j} + \omega_e \cos \lambda \cos \theta \underline{k} \quad (1)$$

THE ANG. VELOCITY  $\underline{\omega}$  OF THE ROTOR IS OBTAINED BY ADDING ITS SPIN  $\dot{\psi} \underline{k}$  TO  $\underline{\Omega}$ . SETTING  $\dot{\psi} + \omega_e \cos \lambda \cos \theta = \omega_e$ , WE HAVE

$$\underline{\omega} = -\omega_e \cos \lambda \sin \theta \underline{i} + (\dot{\theta} + \omega_e \sin \lambda) \underline{j} + \omega_e \underline{k} \quad (2)$$

THE ANG. MOMENTUM  $\underline{H}_O$  OF THE ROTOR IS

$$\underline{H}_O = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

WHERE  $I_x = I_y = I'$  AND  $I_z = I$ . RECALLING (2) WE WRITE

$$\underline{H}_O = -I'\omega_e \cos \lambda \sin \theta \underline{i} + I'(\dot{\theta} + \omega_e \sin \lambda) \underline{j} + I\omega_e \underline{k} \quad (3)$$

EQUATIONS OF MOTION

EQ. (10.2B):  $\sum \underline{M}_O = (\dot{\underline{H}})_O + \underline{\Omega} \times \underline{H}_O$  OR, FROM (1) & (3):

$$\sum \underline{M}_O = -I'\omega_e \cos \lambda \cos \theta \dot{\theta} \underline{i} + I'\ddot{\theta} \underline{j} + I\dot{\omega}_e \underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -\omega_e \cos \lambda \sin \theta & \dot{\theta} + \omega_e \sin \lambda & \omega_e \cos \lambda \cos \theta \\ -I'\omega_e \cos \lambda \sin \theta & I'(\dot{\theta} + \omega_e \sin \lambda) & I\omega_e \end{vmatrix} \quad (4)$$

WE OBSERVE THAT THE ROTOR IS FREE TO SPIN ABOUT THE  $z$  AXIS AND FREE TO ROTATE ABOUT THE  $y$  AXIS. THEREFORE THE  $y$  AND  $z$  COMPONENTS OF  $\sum \underline{M}_O$  MUST BE ZERO. IT FOLLOWS THAT THE COEFFICIENTS OF  $\underline{j}$  AND  $\underline{k}$  IN THE R.H. MEMBER OF EQ. (4) MUST ALSO BE ZERO.

(CONTINUED)

18.158 continued

SETTING THE COEFF. OF  $\underline{j}$  IN THE R.H. MEMBER OF EQ. (4) EQUAL TO ZERO:

$$I'\ddot{\theta} + (-I'\omega_e \cos \lambda \sin \theta)(\omega_e \cos \lambda \cos \theta) - (-\omega_e \cos \lambda \sin \lambda) I\omega_e = 0$$

$$I'\ddot{\theta} + I\omega_e^2 \cos \lambda \sin \theta - I'\omega_e^2 \cos \lambda \sin \theta \cos \theta = 0 \quad (5) \quad \text{(Q.E.D.)}$$

SETTING THE COEFF. OF  $\underline{k}$  EQUAL TO ZERO:

$$I\dot{\omega}_e + (-\omega_e \cos \lambda \sin \theta) I'(\dot{\theta} + \omega_e \sin \lambda) - (-I'\omega_e \cos \lambda \sin \theta)(\dot{\theta} + \omega_e \sin \lambda) = 0$$

OBSERVING THAT THE LAST TWO TERMS CANCEL OUT, WE HAVE

$$I\dot{\omega}_e = 0 \quad \text{(Q.E.D.)} \quad (6)$$

(b) IT FOLLOWS FROM EQ. (6) THAT

$$\omega_e = \text{CONSTANT} \quad (7)$$

REWRITE EQ. (5) AS FOLLOWS:

$$I'\ddot{\theta} + (I\omega_e - I'\omega_e \cos \lambda \cos \theta) \omega_e \cos \lambda \sin \theta = 0$$

IT IS EVIDENT THAT  $\omega_e \gg \omega_e \cos \lambda \cos \theta$ . WE CAN THEREFORE NEGLECT THE SECOND TERM IN THE PARENTHESES AND WRITE

$$I'\ddot{\theta} + I\omega_e^2 \cos \lambda \sin \theta = 0$$

OR

$$\ddot{\theta} + \frac{I\omega_e^2 \cos \lambda \sin \theta}{I'} = 0 \quad (8)$$

WHERE THE COEFFICIENT OF  $\sin \theta$  IS A CONSTANT. THE ROTOR, THEREFORE, OSCILLATES ABOUT THE LINE NS AS A SIMPLE PENDULUM.

FOR SMALL OSCILLATIONS,  $\sin \theta \approx \theta$ , AND EQ. (8) YIELDS

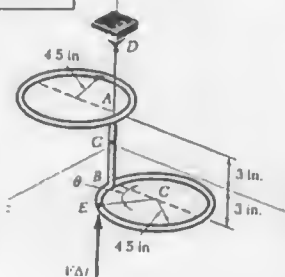
$$\ddot{\theta} + \frac{I\omega_e^2 \cos \lambda}{I'} \theta = 0 \quad \text{(Q.E.D.)} \quad (9)$$

EQ. (9) IS THE EQUATION OF SIMPLE HARMONIC MOTION WITH PERIOD

$$\tau = 2\pi \sqrt{\frac{I'}{I\omega_e^2 \cos \lambda}} \quad (10)$$

SINCE ITS ROTOR OSCILLATES ABOUT THE LINE NS, THE GYROCOMPASS CAN BE USED TO DETERMINE THE DIRECTION OF THAT LINE. WE SHOULD NOTE, HOWEVER THAT FOR VALUES OF  $\lambda$  CLOSE TO  $90^\circ$  OR  $-90^\circ$ , THE PERIOD OF OSCILLATION BECOMES VERY LARGE AND THE LINE ABOUT WHICH THE ROTOR OSCILLATES CANNOT BE DETERMINED. THE GYROCOMPASS, THEREFORE, CANNOT BE USED IN THE POLAR REGIONS.

## 18.C1



## GIVEN:

FIGURE SHOWN MADE OF WIRE WEIGHING  $\frac{5}{8}$  OZ/FT IS SUSPENDED FROM POINT AB. IMPULSE  $F\Delta t = (0.5 \text{ lb}\cdot\text{s})\hat{j}$  IS APPLIED AT E. FIND IMMEDIATELY AFTER IMPACT, FOR VALUES OF  $\theta$  FROM 0 TO  $180^\circ$  IN  $10^\circ$  INCREMENTS  
(a) VELOCITY OF G.  
(b) ANGULAR VELOCITY

## ANALYSIS

LET  $m' =$  MASS PER UNIT LENGTH  
 $2a =$  LENGTH OF ROD AB  
 $r =$  RADIUS OF EACH RING

## COMPUTATION OF MASSES:

$$AB: m_{AB} = 2a m' \quad (1)$$

$$\text{EACH RING: } m_R = 2\pi r m' \quad (2)$$

$$\text{ENTIRE FIGURE: } m = m_{AB} + 2m_R \quad (3)$$

## MOMENTS OF INERTIA:

$$AB: (I_x)_{AB} = (I_y)_{AB} = \frac{1}{12} m_{AB} a^2, (I_z)_{AB} = 0 \quad (4)$$

$$\text{EACH RING: } (I_x)_R = \frac{1}{2} m_R r^2 + m_R a^2 = m_R \left( \frac{1}{2} r^2 + a^2 \right) \quad (5)$$

$$(I_y)_R = m_R r^2 + m_R a^2 = 2m_R a^2 \quad (6)$$

$$(I_z)_R = \frac{1}{2} m_R r^2 + m_R (a^2 + r^2) = m_R \left( \frac{3}{2} r^2 + a^2 \right) \quad (7)$$

## ENTIRE FIGURE:

$$I_x = (I_x)_{AB} + 2(I_x)_R, I_y = 2(I_y)_R, I_z = (I_z)_{AB} + 2(I_z)_R \quad (8)$$

## PRODUCTS OF INERTIA:

THE ONLY NON-ZERO PRODUCTS OF INERTIA ARE  $(I_{xy})_R$

$$I_{xy} = 2(I_{xy})_R = -2m_R r a \quad (9)$$

## IMPULSE-MOMENTUM PRINCIPLE:

EQUATING IMPULSE AND MOMENTUM AFTER IMPACT

$$F\Delta t = m\bar{v}; \quad (F\Delta t)\hat{j} = m\bar{v} \quad (10)$$

$$\bar{v} = \frac{F\Delta t}{m}\hat{j} \quad (\text{FOR ALL VALUES OF } \theta)$$

EQUATING MOMENT OF IMPULSE ABOUT G AND ANGULAR MOMENTUM  $H_G$  AFTER IMPACT (NOTE THAT THERE IS NO IMPULSIVE FORCE EXCEPT  $F$ )

$$\underline{r}_E \times F\Delta t \hat{j} = H_G$$

$$H_G = [r(1-\cos\theta)\hat{i} - a\hat{j} + r\sin\theta\hat{k}] \times F\Delta t \hat{j}$$

$$= -rF\Delta t \sin\theta \hat{i} + rF\Delta t(1-\cos\theta)\hat{k}$$

$$\text{THUS: } H_x = -rF\Delta t \sin\theta, H_y = 0, H_z = rF\Delta t(1-\cos\theta) \quad (11)$$

USING EQS. (10.7) AND RECALLING THAT  $I_{yz} = I_{zy} = 0$

$$I_x \omega_x - I_{xy} \omega_y = H_x \quad (12)$$

$$-I_{xy} \omega_x + I_y \omega_y = 0 \quad (13)$$

$$I_z \omega_z = H_z \quad (14)$$

(CONTINUED)

## 18.C1 continued

SOLVING EQS. (12) AND (13) SIMULTANEOUSLY FOR  $\omega_x$  AND  $\omega_y$ , AND EQ. (14) FOR  $\omega_z$ , WE OBTAIN

$$\omega_x = \frac{I_y H_x}{I_x I_y - I_{xy}^2}, \quad \omega_y = \frac{I_{xy} H_x}{I_x I_y - I_{xy}^2}, \quad \omega_z = \frac{H_z}{I_z} \quad (15)$$

## OUTLINE OF PROGRAM

$$\text{ENTER } m' = \frac{[(5/8)/16] \text{ lb}}{32.2 \text{ ft/s}^2}, a = \frac{3}{12} \text{ ft}, r = \frac{4.5}{12} \text{ ft}, F\Delta t = 0.5 \text{ lb}\cdot\text{s}$$

COMPUTE  $m_{AB}$ ,  $m_R$ , AND  $m$  FROM EQS. (1), (2), AND (3)

COMPUTE  $(I_x)_{AB}$  AND  $(I_y)_{AB}$  FROM EQS. (4)

COMPUTE  $(I_x)_R$ ,  $(I_y)_R$ , AND  $(I_z)_R$  FROM EQS. (5), (6), AND (7)

COMPUTE  $I_x$ ,  $I_y$ , AND  $I_z$  FROM EQS. (8) AND  $I_{xy}$  FROM EQ. (9)

COMPUTE  $\bar{v} = F\Delta t/m$  AND PRINT

FOR  $\theta = 0$  TO  $180^\circ$  AND USING  $10^\circ$  INCREMENTS:

CALCULATE  $H_x$  AND  $H_z$  FROM EQS. (11)

CALCULATE  $\omega_x$ ,  $\omega_y$ , AND  $\omega_z$  FROM EQS. (15) AND TABULATE

## PROGRAM OUTPUT

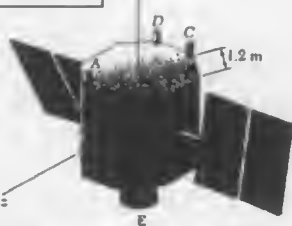
(a)

Velocity of mass center  
 $v_{bar} = 79.07 \text{ ft/s}$  (directed upward)

(b)

Theta degrees	Angular velocity (Omega) <sub>x</sub> rad/s	(Omega) <sub>y</sub> rad/s	(Omega) <sub>z</sub> rad/s
0.00	0.00	0.00	0.00
10.00	-54.88	18.29	1.81
20.00	-108.10	36.03	7.18
30.00	-158.03	52.68	15.94
40.00	-203.16	67.72	27.84
50.00	-242.12	80.71	42.50
60.00	-273.72	91.24	59.49
70.00	-297.00	99.00	78.29
80.00	-311.26	103.75	98.33
90.00	-316.06	105.35	118.99
100.00	-311.26	103.75	139.65
110.00	-297.00	99.00	159.68
120.00	-273.72	91.24	178.48
130.00	-242.12	80.71	195.47
140.00	-203.16	67.72	210.14
150.00	-158.03	52.68	222.03
160.00	-108.10	36.03	230.80
170.00	-54.88	18.29	236.17
180.00	0.00	-0.00	237.97

## 18.C2



## GIVEN:

PROBE WITH  $m = 2500 \text{ kg}$ ,  
 $k_x = 0.98 \text{ m}$ ,  $k_y = 1.06 \text{ m}$ ,  $k_z = 1.02 \text{ m}$ .  
 500-N MAIN THRUSTER E;  
 20-N THRUSTERS A, B, C, D  
 CAN EXPEL FUEL IN  $y$  DIRECTION.  
 PROBE HAS ANG. VELOCITY

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$$

FIND WHICH TWO OF THE 20-N THRUSTERS SHOULD BE USED TO REDUCE ANG. VELOCITY TO ZERO AND FOR HOW LONG EACH OF THEM SHOULD BE ACTIVATED, ASSUMING  
 (a)  $\underline{\omega} = (0.040 \text{ rad/s})\underline{i} + (0.060 \text{ rad/s})\underline{k}$ , AS IN PROB. 18.33,  
 (b)  $\underline{\omega} = (0.060 \text{ rad/s})\underline{i} - (0.040 \text{ rad/s})\underline{k}$ , AS IN PROB. 18.34,  
 (c)  $\underline{\omega} = (0.060 \text{ rad/s})\underline{i} + (0.020 \text{ rad/s})\underline{k}$ ,  
 (d)  $\underline{\omega} = -(0.060 \text{ rad/s})\underline{i} - (0.020 \text{ rad/s})\underline{k}$ .

## ANALYSIS

INITIAL ANG. MOMENTUM:

$$\underline{H}_G = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k} = m k_x^2 \omega_x \underline{i} + m k_y^2 \omega_y \underline{j} + m k_z^2 \omega_z \underline{k}$$

$$\text{THUS } H_x = m k_x^2 \omega_x \quad H_y = 0 \quad H_z = m k_z^2 \omega_z \quad (1)$$

ANGULAR IMPULSE OF TWO 20-N THRUSTERS:

LET US ASSUME THAT A AND B ARE ACTIVATED.  
 ANG. IMPULSE ABOUT G  
 $= \underline{z}_A \times (-F \Delta t_A) \underline{j} + \underline{z}_B \times (-F \Delta t_B) \underline{j}$   
 $= (-0.5 \underline{a} \underline{i} + 1.2071 \underline{a} \underline{k}) \times (-F \Delta t_A) \underline{j} +$   
 $+ (0.5 \underline{a} \underline{i} + 1.2071 \underline{a} \underline{k}) \times (-F \Delta t_B) \underline{j}$

$$\text{ANG. IMP.} = 1.2071 a F (\Delta t_A + \Delta t_B) \underline{i} + 0.5 a F (\Delta t_A - \Delta t_B) \underline{k} \quad (2)$$

IMPULSE-MOMENTUM PRINCIPLE

WE MUST HAVE  $\underline{H}_G + \text{ANG. IMP.} = 0$

OR, USING COMPONENTS:

$$H_x + 1.2071 a F (\Delta t_A + \Delta t_B) = 0 \quad \Delta t_A + \Delta t_B = -\frac{H_x}{1.2071 a F}$$

$$H_z + 0.5 a F (\Delta t_A - \Delta t_B) = 0 \quad \Delta t_A - \Delta t_B = -\frac{H_z}{0.5 a F}$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$$\Delta t_A = -\frac{H_x + 0.41421 H_z}{a F}, \quad \Delta t_B = -\frac{H_z - 0.41421 H_x}{a F} \quad (3)$$

IF  $\Delta t_A > 0$ , ASSUMPTION IS CORRECT, A SHOULD BE USED;

IF  $\Delta t_A < 0$ , ASSUMPTION IS WRONG; C SHOULD BE USED AND ACTIVATED FOR  $\Delta t_C = |\Delta t_A|$ .

SIMILARLY, IF  $\Delta t_B > 0$ , B SHOULD BE USED, AND IF  $\Delta t_B < 0$  D SHOULD BE USED WITH  $\Delta t_D = |\Delta t_B|$ .

## OUTLINE OF PROGRAM

ENTER PART: A, B, C, OR D

ENTER  $m = 2500 \text{ kg}$ ,  $k_x = 0.98 \text{ m}$ ,  $k_z = 1.02 \text{ m}$

ENTER  $a = 1.2 \text{ m}$ ,  $F = 20 \text{ N}$

ENTER VALUES OF  $\omega_x$  AND  $\omega_z$

COMPUTE  $H_x$  AND  $H_z$  FROM EQS. (1)

COMPUTE  $\Delta t_A$  AND  $\Delta t_B$  FROM EQS. (3)

IF  $\Delta t_A > 0$ , PRINT  $\Delta t_A$ ; IF NOT, PRINT  $\Delta t_C = |\Delta t_A|$

IF  $\Delta t_B > 0$ , PRINT  $\Delta t_B$ ; IF NOT, PRINT  $\Delta t_D = |\Delta t_B|$

## PROGRAM OUTPUT

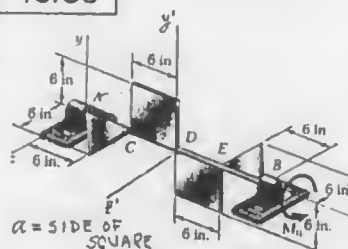
(a) C AND B;  $\Delta t_C = 8.160 \text{ s}$ ;  $\Delta t_B = 4.845 \text{ s}$

(b) A AND D;  $\Delta t_A = 1.849 \text{ s}$ ;  $\Delta t_D = 6.821 \text{ s}$

(c) C AND D;  $\Delta t_C = 4.654 \text{ s}$ ;  $\Delta t_D = 0.3188 \text{ s}$

(d) A AND B;  $\Delta t_A = 4.654 \text{ s}$ ;  $\Delta t_B = 0.3188 \text{ s}$

## 18.C3



## GIVEN:

A COUPLE  $M_0 = (0.03 \text{ lb} \cdot \text{ft})\underline{i}$   
 IS APPLIED AT  $t = 0$  TO  
 2.7-lb ASSEMBLY OF SHOT  
 ALUMINUM OF UNIFORM  
 THICKNESS

## FIND:

(a) COMPONENTS ALONG THE  
 ROTATING  $y$  AND  $z$  AXES  
 OF THE DYNAMIC REAC-

TIONS AT A AND B FROM  $t = 0$  TO  $t = 2 \text{ s}$  AT  $0.1 \text{ s}$  INTERVALS,  
 (b) THE TIME (WITH 3 SIGNIFICANT FIGURES) AT WHICH THE  
 $z$  COMPONENTS OF THESE REACTIONS ARE EQUAL TO ZERO.

## ANALYSIS

WE COMPUTE THE MOMENT AND PRODUCTS OF INERTIA OF  
 THE ASSEMBLY WITH RESPECT TO THE CENTROIDAL AXES  
 $Dx'y'z'$ . WE FIRST COMPUTE THE MOMENT AND PRODUCTS OF AREAS  
 FOR EACH SQUARE:  $(I_x)_{\text{AREA}} = \frac{1}{3} a^4$ ,  $(I_{xy})_{\text{AREA}} = -\frac{1}{4} a^4$ ,  $(I_{yz})_{\text{AREA}} = 0$

FOR EACH TRIANGLE:  $(I_x)_{\text{AREA}} = \frac{1}{12} a^4$ ,  $(I_{xy})_{\text{AREA}} = 0$

$(I_{xy})_{\text{AREA}} = \frac{1}{2} a^2 \bar{x}' \bar{y}' + I_{xy}' = -\frac{1}{2} a^2 (\frac{1}{3} a)(\frac{1}{3} a) + \frac{1}{72} a^4 = -\frac{15}{72} a^4$  [REF. SP. 9.6]

FOR ENTIRE ASSEMBLY:

$$(I_x)_{\text{AREA}} = 2(\frac{1}{3} a^4) + 2(\frac{1}{12} a^4) = \frac{5}{6} a^4$$

$$(I_{xy})_{\text{AREA}} = 2(-\frac{1}{4} a^4) = -\frac{1}{2} a^4 \quad (I_{yz})_{\text{AREA}} = 2(-\frac{15}{72} a^4) = -\frac{5}{12} a^4$$

THE MASS MOMENT AND PRODUCTS OF INERTIA ARE OBTAINED BY  
 MULTIPLYING THESE EXPRESSIONS BY THE MASS  $m$  OF THE  
 ASSEMBLY AND DIVIDING BY ITS AREA, WHICH IS EQUAL TO  $3a^2$ :

$$I_x = \frac{5}{18} m a^2, \quad I_{xy} = -\frac{1}{6} m a^2, \quad I_{yz} = -\frac{5}{36} m a^2 \quad (1)$$

WE DETERMINE  $\underline{H}_D$  AND ITS DERIVATIVE  $\dot{\underline{H}}_D$

SETTING  $\omega_x = \omega$ ,  $\omega_y = \omega_z = 0$  IN EQS. (18.7), WE HAVE

$$H_x = I_x \omega, \quad H_y = -I_{xy} \omega, \quad H_z = -I_{yz} \omega$$

$$\underline{H}_D = (I_x \underline{i} - I_{xy} \underline{j} - I_{yz} \underline{k}) \omega$$

$$\text{EB (18.12): } \dot{\underline{H}}_D = (\dot{\underline{H}}_D)_{Dx'y'z'} + \underline{\omega} \times \underline{H}_D$$

$$\dot{\underline{H}}_D = (I_x \underline{i} - I_{xy} \underline{j} - I_{yz} \underline{k}) \dot{\omega} + \omega \underline{i} \times (I_x \underline{i} - I_{xy} \underline{j} - I_{yz} \underline{k}) \omega$$

$$= (I_x \underline{i} - I_{xy} \underline{j} - I_{yz} \underline{k}) \dot{\omega} - I_{xy} \omega^2 \underline{k} + I_{yz} \omega^2 \underline{j}$$

$$\underline{H}_D = I_x \alpha \underline{i} + (I_{yz} \omega^2 - I_{xy} \alpha) \underline{j} - (I_{xy} \omega^2 + I_{yz} \alpha) \underline{k}$$

## EQUATIONS OF MOTION

$$\begin{aligned} \sum \underline{M}_B &= \sum (\underline{M}_B)_{\text{eff}}: \\ M_0 \underline{i} - 4a \underline{i} \times (A_y \underline{j} + A_z \underline{k}) &= \dot{\underline{H}}_D \\ M_0 \underline{i} - 4a A_y \underline{k} + 4a A_z \underline{j} &= I_x \alpha \underline{i} + (I_{yz} \omega^2 - I_{xy} \alpha) \underline{j} - (I_{xy} \omega^2 + I_{yz} \alpha) \underline{k} \end{aligned}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$$\textcircled{1} M_0 = I_x \alpha \quad \alpha = M_0 / I_x \quad (2)$$

$$\text{FROM WHICH WE OBTAIN } \omega = \alpha t \quad (3)$$

$$\textcircled{2} A_z = (I_{yz} \omega^2 - I_{xy} \alpha) / 4a \quad (4)$$

$$\textcircled{3} A_y = (I_{xy} \omega^2 + I_{yz} \alpha) / 4a \quad (5)$$

$$\sum F = \sum (F)_{\text{eff}} = 0: \underline{A} + \underline{B} = 0$$

$$\text{THUS: } B_y = -A_y \quad B_z = -A_z \quad (6)$$

(CONTINUED)

# 18.C3 continued

## OUTLINE OF PROGRAM

(a) ENTER  $M_0 = 0.03 \text{ lb}\cdot\text{ft}$ ,  $W = 2.7 \text{ lb}$ ,  $a = 0.5 \text{ ft}$   
 COMPUTE  $m = W/32.2$   
 COMPUTE  $I_x, I_{xy}, I_{yz}$  FROM EQS. (1)  
 COMPUTE  $\alpha$  FROM EQ. (2)  
 FOR  $t = 0$  TO  $t = 2.5$  AT 0.1 S INTERVALS:  
 COMPUTE  $\omega$  FROM EQ. (3)  
 COMPUTE  $A_x, A_y, B_x, B_z$  FROM EQS. (4), (5), AND (6)  
 AND TABULATE VS  $t$

(b) DETERMINE BY INSPECTION THE TIME INTERVAL IN WHICH  
 $A_z$  AND  $B_z$  CHANGE SIGN AND RUN THE PROGRAMS OVER THAT  
 INTERVAL, USING 0.01 S INCREMENTS. REPEAT THIS PROCEDURE,  
 USING 0.001 S INCREMENTS, THE DESIRED VALUE OF  $t$   
 IS THAT FOR WHICH  $|A_z|$  AND  $|B_z|$  ARE SMALLEST.

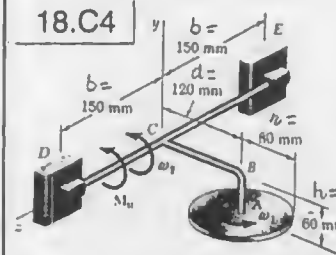
## PROGRAM OUTPUT

(a)	t s	Ay lb	Az lb	By lb	Bz lb
	0.00000	-0.00750	0.00900	0.00750	-0.00900
	0.10000	-0.00796	0.00861	0.00796	-0.00861
	0.20000	-0.00935	0.00745	0.00935	-0.00745
	0.30000	-0.01167	0.00552	0.01167	-0.00552
	0.40000	-0.01492	0.00282	0.01492	-0.00282
	0.50000	-0.01909	-0.00066	0.01909	0.00066
	0.60000	-0.02419	-0.00491	0.02419	0.00491
	0.70000	-0.03022	-0.00993	0.03022	0.00993
	0.80000	-0.03718	-0.01573	0.03718	0.01573
	0.90000	-0.04506	-0.02230	0.04506	0.02230
	1.00000	-0.05387	-0.02964	0.05387	0.02964
	1.10000	-0.06361	-0.03775	0.06361	0.03775
	1.20000	-0.07427	-0.04664	0.07427	0.04664
	1.30000	-0.08586	-0.05630	0.08586	0.05630
	1.40000	-0.09838	-0.06673	0.09838	0.06673
	1.50000	-0.11183	-0.07794	0.11183	0.07794
	1.60000	-0.12620	-0.08992	0.12620	0.08992
	1.70000	-0.14150	-0.10267	0.14150	0.10267
	1.80000	-0.15773	-0.11619	0.15773	0.11619
	1.90000	-0.17489	-0.13049	0.17489	0.13049
	2.00000	-0.19297	-0.14556	0.19297	0.14556

(b)	t s	Ay lb	Az lb	By lb	Bz lb
	0.40000	-0.01492	0.00282	0.01492	-0.00282
	0.41000	-0.01529	0.00250	0.01529	-0.00250
	0.42000	-0.01568	0.00218	0.01568	-0.00218
	0.43000	-0.01607	0.00186	0.01607	-0.00186
	0.44000	-0.01648	0.00152	0.01648	-0.00152
	0.45000	-0.01689	0.00118	0.01689	-0.00118
	0.46000	-0.01731	0.00082	0.01731	-0.00082
	0.47000	-0.01774	0.00046	0.01774	-0.00046
	0.48000	-0.01818	0.00010	0.01818	-0.00010
	0.49000	-0.01863	-0.00028	0.01863	0.00028
	0.50000	-0.01909	-0.00066	0.01909	0.00066

t s	Ay lb	Az lb	By lb	Bz lb
0.48000	-0.01818	0.00010	0.01818	-0.00010
0.48100	-0.01823	0.00006	0.01823	-0.00006
0.48200	-0.01827	0.00002	0.01827	-0.00002
0.48300	-0.01832	-0.00001	0.01832	0.00001
0.48400	-0.01836	-0.00005	0.01836	0.00005
0.48500	-0.01841	-0.00009	0.01841	0.00009
0.48600	-0.01845	-0.00013	0.01845	0.00013
0.48700	-0.01850	-0.00016	0.01850	0.00016
0.48800	-0.01854	-0.00020	0.01854	0.00020
0.48900	-0.01859	-0.00024	0.01859	0.00024
0.49000	-0.01863	-0.00028	0.01863	0.00028

# 18.C4



## GIVEN:

DISK:  $m = 2.5 \text{ kg}$ ,  $k = 80 \text{ mm}$   
 $\omega_1 = 60 \text{ rad/s}$  AT  $t = 0$  AND  
 DECREASES AT RATE OF  
 $15 \text{ rad/s}^2$ . AT  $t = 0$ ,  $\omega_2 = 0$   
 AND COUPLE  $M_0 = (0.5 \text{ N}\cdot\text{m})k$   
 IS APPLIED TO SHAFT DCE.

## FIND:

(a) COMPONENTS ALONG  
 THE ROTATING  $x$  AND  $y$  AXES OF THE DYNAMIC REACTIONS AT  
 D AND E FROM  $t = 0$  TO  $t = 4 \text{ s}$  AT 0.2-S INTERVALS,  
 (b) THE TIMES  $t_1$  AND  $t_2$  (WITH 3 SIGNIFICANT FIGURES)  
 AT WHICH  $E_x$  AND  $E_y$  ARE RESPECTIVELY EQUAL TO ZERO.

## ANALYSIS

$$H_A = I_y \omega_1 \hat{j} + I_z \omega_2 \hat{k} = \frac{1}{2} m k^2 \omega_1 \hat{j} + \frac{1}{4} m k^2 \omega_2 \hat{k}$$

$$\text{EQ. (18.22): } \dot{H}_A = (\dot{H}_A)_{xyz} + \Omega \times H_A$$

$$\dot{H}_A = \frac{1}{2} m k^2 \dot{\omega}_1 \hat{j} + \frac{1}{4} m k^2 \dot{\omega}_2 \hat{k} + \omega_2 \hat{k} \times \left( \frac{1}{2} m k^2 \omega_1 \hat{j} + \frac{1}{4} m k^2 \omega_2 \hat{k} \right)$$

$$= \frac{1}{2} m k^2 \alpha_1 \hat{j} + \frac{1}{4} m k^2 \alpha_2 \hat{k} - \frac{1}{2} m k^2 \omega_1 \omega_2 \hat{i}$$

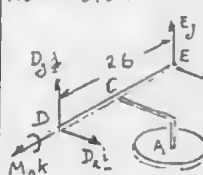
$$\dot{H}_A = \frac{1}{2} m k^2 (-\omega_1 \omega_2 \hat{i} + \alpha_1 \hat{j} + \alpha_2 \hat{k}) \quad (1)$$

$$m \bar{a} = m (\alpha_2 \times r_{AC} - \omega_2^2 r_{AC}) = m \alpha_2 k \times (d \hat{i} - h \hat{j}) - m \omega_2^2 (d \hat{i} - h \hat{j})$$

$$= m (d \alpha_2 \hat{j} + h \alpha_2 \hat{i} - d \omega_2^2 \hat{i} + h \omega_2^2 \hat{j})$$

$$m \bar{a} = m (h \alpha_2 - d \omega_2^2) \hat{i} + m (d \alpha_2 + h \omega_2^2) \hat{j} \quad (2)$$

## EDUNTION: OF MOTION



$$\sum M_D = \sum (M_D)_{rel}:$$

$$-2b k \times (E_x \hat{i} + E_y \hat{j}) + M_0 \hat{k} = -\frac{1}{2} m k^2 \omega_1 \omega_2 \hat{i} + \frac{1}{2} m k^2 \alpha_1 \hat{j} + \frac{1}{4} m k^2 \alpha_2 \hat{k}$$

$$+ (-b k + d \hat{i} - h \hat{j}) \times m [(h \alpha_2 - d \omega_2^2) \hat{i} + (d \alpha_2 + h \omega_2^2) \hat{j}]$$

$$-2b E_x \hat{j} + 2b E_y \hat{i} + M_0 \hat{k} = -\frac{1}{2} m k^2 \omega_1 \omega_2 \hat{i} + \frac{1}{2} m k^2 \alpha_1 \hat{j} + \frac{1}{4} m k^2 \alpha_2 \hat{k}$$

$$- m b (h \alpha_2 - d \omega_2^2) \hat{j} + m b (d \alpha_2 + h \omega_2^2) \hat{i} + m d (d \alpha_2 + h \omega_2^2) \hat{k} + m h (h \alpha_2 - d \omega_2^2) \hat{k}$$

## EQUATE THE COEFF. OF THE UNIT VECTORS:

$$\textcircled{K} M_0 = m \left( \frac{1}{4} k^2 + d^2 + h^2 \right) \alpha_2 \quad \alpha_2 = \frac{M_0}{m \left( \frac{1}{4} k^2 + d^2 + h^2 \right)} \quad (3)$$

$$\textcircled{J} E_x = \frac{m}{2b} \left( -\frac{1}{2} k^2 \alpha_1 + b h \alpha_2 - b d \omega_2^2 \right) \quad (4)$$

$$\textcircled{I} E_y = \frac{m}{2b} \left( -\frac{1}{2} k^2 \omega_1 \omega_2 + b d \alpha_2 + b h \omega_1^2 \right) \quad (5)$$

$$\sum F = \sum (F)_{eff}: D + E = m \bar{a}$$

$$D_x \hat{i} + D_y \hat{j} + E_x \hat{i} + E_y \hat{j} = m (h \alpha_2 - d \omega_2^2) \hat{i} + m (d \alpha_2 + h \omega_2^2) \hat{j}$$

## EQUATE THE COEFF. OF THE UNIT VECTORS:

$$\textcircled{I} D_x = m (h \alpha_2 - d \omega_2^2) - E_x \quad (6)$$

$$\textcircled{J} D_y = m (d \alpha_2 + h \omega_2^2) - E_y \quad (7)$$

## WE RECALL FROM THE GIVEN DATA THAT

$$m = 2.5 \text{ kg}, k = 0.08 \text{ m}, b = 0.15 \text{ m}, d = 0.12 \text{ m}, h = 0.06 \text{ m}$$

$$M_0 = 0.5 \text{ N}\cdot\text{m} \quad \omega_0 = 60 \text{ rad/s} \quad \alpha_1 = -15 \text{ rad/s}^2 \quad (8)$$

AND NOTE THAT AT TIME  $t$

$$\omega_1 = \omega_0 + \alpha_1 t \quad \omega_2 = \alpha_2 t \quad (9)$$

(CONTINUED)

## 18.C4 continued

### OUTLINE OF PROGRAM

- (a) ENTER DATA SHOWN IN (B) ON PREVIOUS PAGE  
 COMPUTE  $\alpha_2$  FROM EQ. (3)  
 FOR  $t = 0$  TO  $t = 4$  s AT 0.2-s INTERVALS  
 COMPUTE  $\omega_1$  AND  $\omega_2$  FROM EQS. (9)  
 COMPUTE  $E_x$  AND  $E_y$  FROM EQS. (4) AND (5)  
 COMPUTE  $D_x$  AND  $D_y$  FROM EQS. (6) AND (7)  
 AND TABULATE VS  $t$ .
- (b) TO FIND THE TIME  $t_1$  AT WHICH  $E_x = 0$ ,  
 DETERMINE BY INSPECTION THE TIME INTERVAL  
 IN WHICH  $E_x$  CHANGES SIGN AND RUN THE  
 PROGRAM OVER THAT INTERVAL, USING 0.01-s  
 INCREMENTS. REPEAT THIS PROCEDURE USING  
 0.001-s INCREMENTS. SELECT FOR  $t_1$  THE TIME  
 AT WHICH  $|E_x|$  IS SMALLEST.  
 A SIMILAR PROCEDURE IS USED TO DETERMINE  
 THE TIME  $t_2$  AT WHICH  $E_y = 0$ .

### PROGRAM OUTPUT

(a)	t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
	0.0000	0.3653	1.5306	1.1653	1.5306
	0.2000	-0.2594	4.9450	0.5406	-1.2591
	0.4000	-2.1337	8.6576	-1.3337	-3.0975
	0.6000	-5.2574	12.6685	-4.4574	-3.9846
	0.8000	-9.6305	16.9775	-8.8305	-3.9204
	1.0000	-15.2532	21.5848	-14.4532	-2.9050
	1.2000	-22.1253	26.4902	-21.3253	-0.9384
	1.4000	-30.2469	31.6939	-29.4469	1.9796
	1.6000	-39.6181	37.1958	-38.8180	5.8488
	1.8000	-50.2386	42.9958	-49.4386	10.6693
	2.0000	-62.1087	49.0941	-61.3087	16.4411
	2.2000	-75.2282	55.4906	-74.4282	23.1641
	2.4000	-89.5972	62.1854	-88.7973	30.8384
	2.6000	-105.2158	69.1783	-104.4158	39.4640
	2.8000	-122.0838	76.4694	-121.2837	49.0409
	3.0000	-140.2012	84.0588	-139.4012	59.5690
	3.2000	-159.5681	91.9463	-158.7681	71.0483
	3.4000	-180.1846	100.1321	-179.3845	83.4790
	3.6000	-202.0505	108.6160	-201.2504	96.8609
	3.8000	-225.1659	117.3982	-224.3658	111.1942
	4.0000	-249.5307	126.4786	-248.7306	126.4786

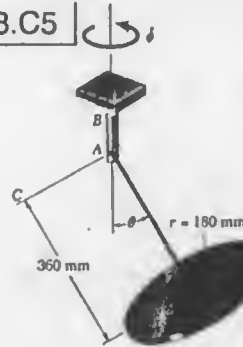
(b) LAST STEP IN DETERMINATION OF  $t_1$

t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
0.2700	-0.7733	6.2105	0.0267	-2.0107
0.2710	-0.7817	6.2289	0.0183	-2.0206
0.2720	-0.7902	6.2472	0.0098	-2.0305
0.2730	-0.7987	6.2656	0.0013	-2.0403
0.2740	-0.8073	6.2839	-0.0073	-2.0501
0.2750	-0.8158	6.3023	-0.0158	-2.0599
0.2760	-0.8244	6.3207	-0.0244	-2.0697
0.2770	-0.8331	6.3391	-0.0331	-2.0795
0.2780	-0.8418	6.3575	-0.0418	-2.0892
0.2790	-0.8505	6.3759	-0.0505	-2.0989
0.2800	-0.8592	6.3943	-0.0592	-2.1086

LAST STEP IN DETERMINATION OF  $t_2$

t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
1.2700	-24.8258	28.2776	-24.0258	-0.0253
1.2710	-24.8654	28.3034	-24.0655	-0.0114
1.2720	-24.9052	28.3292	-24.1052	0.0025
1.2730	-24.9449	28.3550	-24.1449	0.0165
1.2740	-24.9847	28.3808	-24.1847	0.0304
1.2750	-25.0245	28.4066	-24.2245	0.0444
1.2760	-25.0644	28.4325	-24.2644	0.0584
1.2770	-25.1042	28.4583	-24.3042	0.0724
1.2780	-25.1442	28.4842	-24.3441	0.0865
1.2790	-25.1841	28.5100	-24.3841	0.1006

## 18.C5



### GIVEN:

DISK WELDED TO ROD AB OF  
 NEGLIGIBLE MASS CONNECTED  
 BY CLEVIS TO SHAFT AB.

ROD AND DISK FREE TO ROTATE  
 ABOUT AC; SHAFT AB FREE  
 TO ROTATE ABOUT VERTICAL AXIS.

### INITIALLY:

(a) MINIMUM VALUE  $\theta_m$  OF  $\theta$   
 DURING ENSUING MOTION  
 AND TIME REQUIRED FOR  $\theta$  TO  
 RETURN TO  $\theta_0$  (PERIOD).

(b) ANG. VEL.  $\dot{\phi}$  FOR VALUES OF  $\theta$  FROM  $\theta_0$  TO  $\theta_m$  USING 2° INCREMENTS

CONSIDER SUCCESSIVELY THE INITIAL CONDITIONS

(i)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$ , (ii)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 10 \text{ rad/s}$ , (iii)  $\theta_0 = 60^\circ$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$ .

### ANALYSIS

USING THE ROTATING FRAME  $xyz$ :

$$\omega = \dot{\phi} \sin \theta \mathbf{i} - \dot{\theta} \mathbf{j} + \dot{\phi} \cos \theta \mathbf{k}$$

$$H_z = I_2 \omega_z = I \dot{\phi} \sin \theta$$

$$H_z = I_2 \omega_z = I \dot{\phi} \cos \theta$$

$$\text{WHERE } I = \frac{1}{2} m a^2 \quad (1)$$

$$I = \frac{1}{4} m a^2 + m (2a)^2 = \frac{17}{4} m a^2 \quad (2)$$

### CONSERVATION OF ANG. MOM ABOUT Z

SINCE THE FORCES CONSIST OF REACTION AT A AND WEIGHT

$\mathbf{W} = -mg \mathbf{K}$  AT G, WE HAVE  $\Sigma M_z = 0$  AND  $H_z = \text{CONSTANT}$

SINCE  $H_z = H_z \sin \theta + H_z \cos \theta = I \dot{\phi} \sin \theta + I \dot{\phi} \cos \theta$ ,

WE HAVE  $(I \dot{\phi} \sin \theta + I \dot{\phi} \cos \theta) \dot{\phi} = (I \dot{\phi}_0 \sin \theta_0 + I \dot{\phi}_0 \cos \theta_0) \dot{\phi}_0$

$$\text{SETTING } Q = I \dot{\phi} \sin \theta + I \dot{\phi} \cos \theta \quad (3)$$

$$\text{AND } Q_0 = I \dot{\phi}_0 \sin \theta_0 + I \dot{\phi}_0 \cos \theta_0 \quad (4)$$

$$\text{AND SOLVING FOR } \dot{\phi}; \quad \dot{\phi} = (Q_0 / Q) \dot{\phi}_0 \quad (5)$$

### CONSERVATION OF ENERGY

$$T + V = E = \text{CONSTANT}; \quad \frac{1}{2} (I_2 \omega_z^2 + I_2 \dot{\theta}^2 + I_2 \dot{\phi}^2 \cos^2 \theta) + W(-2a \cos \theta) = E$$

$$\frac{1}{2} (I \dot{\phi}^2 \sin^2 \theta + I \dot{\theta}^2 + I \dot{\phi}^2 \cos^2 \theta) - 2mg a \cos \theta = E$$

$$(I \dot{\phi}^2 \sin^2 \theta + I \dot{\theta}^2 + I \dot{\phi}^2 \cos^2 \theta) - 4mg a \cos \theta = 2E$$

RECALLING (3) AND SUBSTITUTING FOR  $\dot{\phi}$  FROM (5):

$$(Q_0^2 \dot{\phi}_0^2 / Q) + I \dot{\theta}^2 - 4mg a \cos \theta = 2E \quad (6)$$

SOLVING FOR  $\dot{\theta}^2$ :

$$\dot{\theta}^2 = \frac{1}{I} (2E + 4mg a \cos \theta - \frac{Q_0^2 \dot{\phi}_0^2}{Q})$$

$$\text{WHICH IS OF THE FORM } \dot{\theta}^2 = f(\theta) \quad (7)$$

$$\text{WHERE } f(\theta) = \frac{1}{I} (2E + 4mg a \cos \theta - \frac{Q_0^2 \dot{\phi}_0^2}{Q}) \quad (8)$$

AND  $Q$  IS THE FUNCTION OF  $\theta$  DEFINED IN (3). THE

CONSTANT  $2E$  IS OBTAINED BY MAXIMIZING  $\theta = \theta_0$ ,  $\dot{\theta} = 0$  AND

$Q = Q_0$  IN EQ. (6):  $E = \frac{1}{2} Q_0 \dot{\phi}_0^2 - 2mg a \cos \theta_0$  (9)

FROM (7) WE WRITE

$$\frac{d\theta}{dt} = \dot{\theta} = \sqrt{f(\theta)} \quad t = \int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{f(\theta)}} \quad (10)$$

(a) THE TIME  $\frac{1}{2} E$  NEEDED FOR  $\theta$  TO DECREASE TO  $\theta_m$

IS OBTAINED THROUGH NUMERICAL INTEGRATION,  $\theta_m$  BEING

DEFINED BY THE FACT THAT  $f(\theta_m) = 0$  ( $f$  CHANGES SIGN)

(b) FOR EACH DESIRED VALUE OF  $\theta$ , COMPUTE  $Q$  FROM EQ. (3)

AND  $\dot{\phi}$  FROM EQ. (5).

(CONTINUED)

## 18.C5 continued

### OUTLINE OF PROGRAM

ENTER  $a = 0.18 \text{ m}$ ,  $g = 9.81 \text{ m/s}^2$ . ASSUME  $m = 1$ .  
 ENTER INITIAL CONDITIONS:  $\theta_0$  AND  $\dot{\theta}_0$ .  
 ENTER DECREMENT  $\Delta\theta$  YOU WISH TO USE  
 COMPUTE  $I$  AND  $I'$  FROM (1) AND (2)  
 COMPUTE  $Q_0$  FROM (4) AND  $E$  FROM (9)  
 FOR  $\theta = \theta_0$  TO  $\theta = \theta_m$  (WHEN  $f(\theta)$  CHANGES SIGN), AND  
 USING DECREMENTS  $\Delta\theta$ :  
 COMPUTE  $Q$  FROM (3)  
 COMPUTE  $f(\theta)$  FROM (8)  
 CARRY OUT NUMERICALLY THE INTEGRATION  
 INDICATED IN (10)  
 AT  $2^\circ$  INTERVALS, COMPUTE  $\dot{\phi}$  FROM (5) AND PRINT  
 THE VALUES OF  $\theta$  AND  $\dot{\phi}$

THE PERIOD OF THE OSCILLATION IN  $\theta$  IS OBTAINED  
 BY DOUBLING THE VALUE OF  $t$  WHEN  $\theta$  REACHES  
 ITS MINIMUM VALUE  $\theta_m$ .

### PROGRAM OUTPUT

(i)

TH0= 90 PHID0= 5 DTH= .1	Theta (degrees)	Precession Rate (rad/s)
	90.000	5.000
	88.000	5.005
	86.000	5.022
	84.000	5.049
	82.000	5.087
	80.000	5.137
	78.000	5.198
	76.000	5.272
	74.000	5.359
	72.000	5.460
	70.000	5.575
	68.000	5.707
	66.000	5.855
	64.000	6.021
	62.000	6.207
	60.000	6.415
	58.000	6.647
	56.000	6.905
	54.000	7.193
	52.000	7.513
	50.000	7.869
	48.000	8.265
	46.000	8.708
	44.000	9.201
	42.000	9.752
	40.000	10.369
	38.000	11.060
	36.000	11.835
	34.000	12.705
	32.000	13.683
	31.852	13.704
Theta min = 32.0 degrees		
Period = 0.736 s		

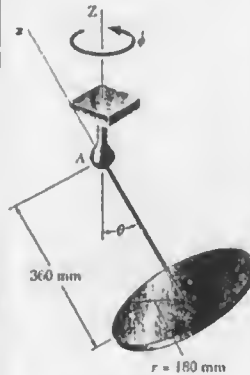
(ii)

TH0= 90 PHID0= 10 DTH= .1	Theta (degrees)	Precession Rate (rad/s)
90.000	10.000	
88.000	10.011	
86.000	10.043	
84.000	10.097	
82.000	10.174	
80.000	10.273	
78.000	10.397	
76.000	10.545	
74.000	10.719	
72.000	10.920	
70.000	11.151	
68.000	11.413	
66.000	11.709	
64.000	12.042	
62.052	12.404	
Theta min = 62.1 degrees		
Period = 0.577 s		

(iii)

TH0= 60 PHID0= 5 DTH= .1	Theta (degrees)	Precession Rate (rad/s)
	60.000	5.000
	58.000	5.181
	56.000	5.382
	54.000	5.606
	52.000	5.855
	50.000	6.133
	48.000	6.442
	46.000	6.787
	44.000	7.171
	42.000	7.601
	40.000	8.082
	38.000	8.620
	36.824	8.945
Theta min = 36.9 degrees		
Period = 0.725 s		

## 18.C6



### GIVEN:

DISK WELDED TO ROD AG OF  
 NEGLIGIBLE MASS SUPPORTED BY  
 BALL AND SOCKET AT A.

INITIALLY,  $\theta = \theta_0$ ,  $\dot{\theta} = 0$ ,  $\dot{\phi} = \dot{\phi}_0$   
 AND  $\dot{\phi} = \dot{\phi}_0$ .

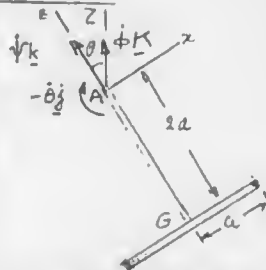
### FIND:

(a) MINIMUM VALUE  $\theta_m$  OF  $\theta$  IN  
 ENSUING MOTION AND PERIOD (TIME  
 REQUIRED FOR  $\theta$  TO RETURN TO  $\theta_0$ ).  
 (b)  $\dot{\phi}$  AND  $\dot{\phi}$  FOR VALUES OF  $\theta$   
 FROM  $\theta_0$  TO  $\theta_m$  USING  $2^\circ$  INCREMENTS.  
 CONSIDER SUCCESSIVELY THE INITIAL  
 CONDITIONS

(i)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 50 \text{ rad/s}$ ,  $\dot{\phi}_0 = 0$   
 (ii)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 0$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$

(iii)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 50 \text{ rad/s}$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$   
 (iv)  $\theta_0 = 90^\circ$ ,  $\dot{\phi}_0 = 10 \text{ rad/s}$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$   
 (v)  $\theta_0 = 60^\circ$ ,  $\dot{\phi}_0 = 0$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$   
 (vi)  $\theta_0 = 60^\circ$ ,  $\dot{\phi}_0 = 50 \text{ rad/s}$ ,  $\dot{\phi}_0 = 5 \text{ rad/s}$

### ANALYSIS



USING THE ROTATING FRAME:

AXES WITH Y AXIS POINTING  
 INTO THE PAPER:

$$\omega = \dot{\phi} \sin \theta \hat{i} - \dot{\theta} \hat{j} + (\dot{\phi} + \dot{\phi} \cos \theta) \hat{k}$$

$$H_A = I_x \omega_x \hat{i} + I_y \omega_y \hat{j} + I_z \omega_z \hat{k} \\ = I' \dot{\phi} \sin \theta \hat{i} - I' \dot{\theta} \hat{j} + I (\dot{\phi} + \dot{\phi} \cos \theta) \hat{k}$$

$$\text{WHERE } I = \frac{1}{2} m a^2 \quad (1)$$

$$I' = \frac{1}{4} m a^2 + m (2a)^2 = \frac{17}{4} m a^2 \quad (2)$$

### CONSERVATION OF ANGULAR MOMENTUM

SINCE THE ONLY EXTERNAL FORCES ARE THE REACTION  
 AT A AND THE WEIGHT  $\mathbf{W} = -mg \hat{k}$  AT G, WE HAVE  
 $\Sigma M_A = 0$  AND  $\Sigma M_z = 0$ . SINCE Z IS PART OF A  
 NEWTONIAN FRAME OF REFERENCE, IT FOLLOWS THAT  
 $H_z = \text{CONST.}$ . BECAUSE OF THE AXISYMMETRY OF  
 THE DISK, IT ALSO FOLLOWS THAT  $H_2 = \text{CONST.}$  (SEE  
 PROB. 15, 139). WE WRITE

$$H_2 = \text{CONST.} \quad I' \dot{\phi} \sin^2 \theta + I (\dot{\phi} + \dot{\phi} \cos \theta) \cos \theta = \beta \quad (3)$$

$$\text{WHERE FROM INIT. COND.} \quad \beta = I' \dot{\phi}_0 + \dot{\phi}_0 \cos \theta_0 \quad (4)$$

$$H_z = \text{CONST.} \quad H_z \sin \theta + H_z \cos \theta = \alpha \\ I' \dot{\phi} \sin^2 \theta + I (\dot{\phi} + \dot{\phi} \cos \theta) \cos \theta = \alpha \quad (5)$$

$$\text{RECALLING (3) WE HAVE } I' \dot{\phi} \sin^2 \theta + \beta \cos \theta = \alpha \quad (6)$$

$$\text{FROM INIT. CONDITIONS: } \alpha = I' \dot{\phi}_0 \sin^2 \theta_0 + \beta \cos \theta_0 \quad (7)$$

$$\text{CONSERVATION OF ENERGY} \quad T = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) \\ = \frac{1}{2} [I' \dot{\phi}^2 \sin^2 \theta + I' \dot{\theta}^2 + I (\dot{\phi} + \dot{\phi} \cos \theta)^2]$$

$$\text{SUBSTITUTE FOR ( ) FROM (3):} \\ T = \frac{1}{2} (I' \dot{\phi}^2 \sin^2 \theta + I' \dot{\theta}^2 + \frac{\beta^2}{I}) \quad V = -mg(2a) \cos \theta -$$

$$T + V = E; \quad \frac{1}{2} (I' \dot{\phi}^2 \sin^2 \theta + I' \dot{\theta}^2 + \frac{\beta^2}{I}) - 2mg(2a) \cos \theta = E \quad (8)$$

$$\text{FROM INIT. COND.: } E = \frac{1}{2} (I' \dot{\phi}_0^2 \sin^2 \theta_0 + \frac{\beta^2}{I}) - 2mg(2a) \cos \theta_0 \quad (9)$$

$$\text{SOLVING (8) FOR } \dot{\theta}^2: \quad \dot{\theta}^2 = f(\theta) \quad (10)$$

$$\text{WHERE} \quad f(\theta) = \frac{1}{I'} (2E - \frac{\beta^2}{I} + 4mg(2a) \cos \theta) - \dot{\phi}^2 \sin^2 \theta \quad (11)$$

(CONTINUED)



## 18.C6 continued

SUBSTITUTING FOR  $\phi$  FROM (7) INTO (11), WE HAVE

$$f(\theta) = \frac{1}{I} \left( 2E - \frac{I^2}{I} + 4mg\alpha \cos\theta \right) - \left( \frac{\alpha - \beta \cos\theta}{I' \sin\theta} \right)^2 \quad (12)$$

FROM EQ. (10) WE WRITE

$$\frac{d\theta}{dt} = \dot{\theta} = \sqrt{f(\theta)} \quad t = \int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{f(\theta)}} \quad (13)$$

(a) THE TIME  $\frac{1}{2}T$  NEEDED FOR  $\theta$  TO DECREASE TO  $\theta_m$  IS OBTAINED THROUGH NUMERICAL INTEGRATION,

$\theta_m$  BEING DEFINED BY THE FACT THAT  $f(\theta_m) = 0$ , THAT IS, THAT  $f(\theta)$  CHANGES SIGN FOR  $\theta = \theta_m$ .

(b) FOR EACH DESIRED VALUE OF  $\theta$ , COMPUTE  $\dot{\phi}$  FROM EQ. (7).

### OUTLINE OF PROGRAM

ENTER  $\alpha = 0.18m$ ,  $g = 9.81m/s^2$ . ASSUME  $m = 1$ .

ENTER INITIAL CONDITIONS:  $\theta_0$ ,  $\dot{\psi}_0$ , AND  $\dot{\phi}_0$ .

ENTER DECREMENT  $d\theta$  YOU WISH TO USE

COMPUTE  $I$  AND  $I'$  FROM (1) AND (2)

COMPUTE  $\beta$  FROM (4),  $\alpha$  FROM (6), AND  $E$  FROM (9)

FOR  $\theta = \theta_0$  TO  $\theta = \theta_m$  (WHEN  $f(\theta)$  CHANGES SIGN),

AND USING DECREMENTS  $d\theta$ :

COMPUTE  $\dot{\phi}$  FROM (7)

COMPUTE  $f(\theta)$  FROM (11)

CARRY OUT NUMERICALLY THE INTEGRATION DEFINED IN EQ. (13)

AT  $2^\circ$  INTERVALS, PRINT THE VALUES OF  $\theta$ ,  $\dot{\phi}$ ,

AND, FROM (3), OF  $\dot{\psi} = \frac{A}{I} - \dot{\phi} \cos\theta$

THE PERIOD OF THE OSCILLATION IN  $\theta$  IS OBTAINED BY DOUBLING THE VALUE OF  $t$  CORRESPONDING TO  $\theta = \theta_m$ .

### PROGRAM OUTPUT

(i)

TH0=90 PSID0=50 PHID0= 0  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	0.00	50.00
88.00	-0.21	50.01
86.00	-0.41	50.03
84.00	-0.62	50.06
82.00	-0.83	50.12
80.00	-1.05	50.18
78.00	-1.28	50.27
76.00	-1.51	50.37
74.00	-1.75	50.48
72.00	-2.01	50.62
70.00	-2.28	50.78
68.00	-2.56	50.96
66.00	-2.87	51.17
64.00	-3.19	51.40
62.00	-3.54	51.66
60.00	-3.92	51.96
58.00	-4.33	52.30
56.00	-4.79	52.68
54.00	-5.28	53.11
52.00	-5.83	53.59
50.00	-6.44	54.14
48.00	-7.13	54.77
46.00	-7.90	55.49
44.11	-8.72	56.26

Theta min = 44.1 degrees  
Period = 0.668 s

(ii)

TH0=90 PSID0= 0 PHID0= 5  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	0.00
88.00	5.01	-0.17
86.00	5.02	-0.35
84.00	5.06	-0.53
82.00	5.10	-0.71
80.00	5.16	-0.90
78.00	5.23	-1.09
76.00	5.31	-1.28
74.00	5.41	-1.49
72.00	5.53	-1.71
70.00	5.66	-1.94
68.00	5.82	-2.18
66.00	5.99	-2.44
64.00	6.19	-2.71
62.00	6.41	-3.01
60.00	6.67	-3.33
58.00	6.95	-3.68
56.00	7.27	-4.07
54.00	7.64	-4.49
52.00	8.05	-4.96
50.00	8.52	-5.48
48.00	9.05	-6.06
46.00	9.66	-6.71
44.00	10.36	-7.45
42.00	11.17	-8.30
40.00	12.10	-9.27
38.23	13.06	-10.26

Theta min = 38.2 degrees  
Period = 0.687 s

(iii)

TH0=90 PSID0=50 PHID0= 5  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	50.00
88.00	4.80	49.83
86.00	4.61	49.68
84.00	4.43	49.54
82.00	4.26	49.41
80.00	4.10	49.29
78.00	3.95	49.18
76.00	3.80	49.08
74.00	3.66	48.99
72.00	3.52	48.91
70.00	3.38	48.84
68.00	3.25	48.78
66.00	3.12	48.73
64.00	3.00	48.69
62.00	2.87	48.65
60.00	2.75	48.63
58.00	2.62	48.61
56.00	2.49	48.61
54.00	2.36	48.61
52.00	2.22	48.63
50.00	2.08	48.66
48.00	1.93	48.71
46.00	1.77	48.77
44.00	1.59	48.85
42.00	1.40	48.96
40.00	1.20	49.08
38.00	0.96	49.24
36.00	0.70	49.44
34.00	0.39	49.67
32.00	0.04	49.97
30.00	-0.38	50.33
28.00	-0.88	50.78
26.00	-1.49	51.34
24.00	-2.26	52.06
22.00	-3.24	53.00
20.00	-4.51	54.24
18.00	-6.23	55.92
16.00	-8.62	58.28
14.00	-12.09	61.73
12.00	-17.44	67.06
10.00	-26.30	75.90
8.00	-42.61	92.19
6.00	-77.82	127.40
5.62	-89.03	138.60

Theta min = 5.62 degrees  
Period = 0.542 s

(iv)

TH0=90 PSID0=10 PHID0= 5  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	10.00
88.00	4.96	9.83
86.00	4.94	9.66
84.00	4.93	9.48
82.00	4.93	9.31
80.00	4.94	9.14
78.00	4.97	8.97
76.00	5.01	8.79
74.00	5.06	8.61
72.00	5.13	8.42
70.00	5.21	8.22
68.00	5.30	8.01
66.00	5.42	7.80
64.00	5.55	7.57
62.00	5.71	7.32
60.00	5.88	7.06
58.00	6.09	6.78
56.00	6.32	6.47
54.00	6.58	6.13
52.00	6.89	5.76
50.00	7.23	5.35
48.00	7.63	4.90
46.00	8.08	4.38
44.00	8.61	3.81
42.00	9.21	3.15
40.00	9.92	2.40
38.00	10.75	1.53
36.00	11.72	0.52
34.00	12.87	-0.67
32.00	14.25	-2.09
30.00	15.93	-3.79
28.23	17.71	-5.60

Theta min = 28.2 degrees  
Period = 0.655 s

(v)

TH0=60 PSID0= 0 PHID0= 5  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
60.00	5.00	0.00
58.00	5.20	-0.26
56.00	5.43	-0.54
54.00	5.69	-0.84
52.00	5.98	-1.18
50.00	6.32	-1.56
48.00	6.70	-1.98
46.00	7.14	-2.46
44.00	7.64	-2.99
42.00	8.22	-3.61
40.33	8.77	-4.18

Theta min = 40.3 degrees  
Period = 0.661 s

(vi)

TH0=60 PSID0=50 PHID0= 5  
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
60.00	5.00	50.00
58.00	4.96	49.87
56.00	4.92	49.75
54.00	4.90	49.62
52.00	4.89	49.49
50.00	4.89	49.36
48.00	4.90	49.22
46.00	4.92	49.08
44.00	4.96	48.93
42.00	5.02	48.77
40.00	5.10	48.59
38.00	5.20	48.40
36.00	5.33	48.19
34.00	5.49	47.95
32.00	5.70	47.67
30.00	5.96	47.34
28.00	6.28	46.95
26.00	6.70	46.48
24.00	7.23	45.90
22.00	7.92	45.16
20.00	8.84	44.19
18.00	10.10	42.90
16.00	11.86	41.10
14.00	14.44	38.49
12.00	18.43	34.47
10.00	25.06	27.82
8.00	37.27	15.59
6.01	63.40	-10.55

Theta min = 6.01 degrees  
Period = 0.520 s



19.1

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION  
AMPLITUDE = 40 in., PERIOD = 1.4 s.

FIND:

MAXIMUM VELOCITY,  $v_m$ MAXIMUM ACCELERATION,  $a_m$ 

SIMPLE HARMONIC MOTION

$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = 2\pi/T_n = 2\pi/(1.4 \text{ s}) = 4.480 \text{ rad/s}$$

$$x_m = \text{AMPLITUDE} = 40 \text{ in} = 3.333 \text{ ft}$$

$$x = (3.333) \sin(4.480t + \phi)$$

$$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi) \quad \dot{x}_m = v_m = x_m \omega_n$$

$$v_m = (3.333 \text{ ft})(4.480 \text{ rad/s})$$

$$v_m = 14.96 \text{ ft/s}$$

$$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi) \quad \ddot{x}_m = a_m = -x_m \omega_n^2$$

$$a_m = (3.333 \text{ ft})(4.480 \text{ rad/s})^2$$

$$a_m = 67.1 \text{ ft/s}^2$$

19.2

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION  
MAXIMUM ACCELERATION  $72 \text{ m/s}^2$   
FREQUENCY  $f_n = 8 \text{ Hz}$ .

FIND:

AMPLITUDE,  $x_m$ MAXIMUM VELOCITY,  $v_m$ 

SIMPLE HARMONIC MOTION

$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = 2\pi f_n = (2\pi)(8 \text{ Hz}) = 16\pi \text{ rad/s}$$

$$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi) \quad v_m = x_m \omega_n$$

$$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi) \quad a_m = x_m \omega_n^2$$

$$a_m = 72 \text{ m/s}^2 = x_m (16\pi \text{ rad/s})^2$$

$$x_m = (72 \text{ m/s}^2) / (16\pi \text{ rad/s})^2$$

$$x_m = 2.849 \times 10^{-3} \text{ m}$$

$$x_m = 2.85 \text{ mm}$$

$$v_m = x_m \omega_n = (2.849 \text{ mm})(16\pi \text{ rad/s})$$

$$v_m = 143.2 \text{ mm/s}$$

19.3

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION  
AMPLITUDE = 300 mm  
MAXIMUM ACCELERATION =  $5 \text{ m/s}^2$

FIND:

MAXIMUM VELOCITY,  $v_m$ FREQUENCY,  $f$ 

SIMPLE HARMONIC MOTION

$$x = x_m \sin(\omega_n t + \phi) \quad x_m = 0.300 \text{ m}$$

$$x = (0.300) \sin(\omega_n t + \phi) \text{ (m)}$$

$$\dot{x} = (0.3)(\omega_n) \cos(\omega_n t + \phi) \text{ (m/s)}$$

$$\ddot{x} = -(0.3)(\omega_n^2) \sin(\omega_n t + \phi) \text{ (m/s}^2\text{)}$$

$$|a_m| = (0.3 \text{ m/s})(\omega_n^2) \quad a_m = 5 \text{ m/s}^2$$

$$\omega_n^2 = |a_m| / (0.3 \text{ m}) = (5 \text{ m/s}^2) / (0.3 \text{ m}) = 16.667 \text{ rad/s}^2$$

$$\omega_n = 4.082 \text{ rad/s} \quad f_n = \omega_n / 2\pi$$

$$f_n = (4.082 \text{ rad/s}) / (2\pi \text{ rad/cycle}) = 0.6497 \text{ Hz}$$

$$f_n = 0.650 \text{ Hz}$$

$$v_m = x_m \omega_n = (0.3 \text{ m})(4.082 \text{ rad/s})$$

$$v_m = 1.225 \text{ m/s}$$

19.4

GIVEN:

BLOCK  $W = 30 \text{ lb}$ SPRING  $k = 20 \text{ lb/in.}$ 

INITIAL DEFLECTION = 2.1 in.

RELEASED FROM REST

FIND:

(a) PERIOD  $T_n$  AND FREQUENCY,  $f_n$ (b) MAXIMUM VELOCITY,  $v_m$ AND ACCELERATION,  $a_m$ 

(a)



$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{k/m} \quad k = 20 \text{ lb/in} = 240 \text{ lb/ft}$$

$$\omega_n = \sqrt{(240 \text{ lb/ft}) / (30 \text{ lb})} = (32.2 \text{ ft/s}^2)$$

$$\omega_n = 16.050 \text{ rad/s} \quad T_n = \frac{2\pi}{\omega_n}$$

$$T_n = 2\pi / 16.050 = 0.3915 \text{ s}$$

$$T_n = 0.3915 \text{ s}$$

$$f_n = 1/T_n = 1/0.391 = 2.55 \text{ Hz}$$

(b)  $x_m = 2.1 \text{ in.} = 0.175 \text{ ft}$ 

$$x = 0.175 \sin(16.050t + \phi)$$

MAXIMUM VELOCITY

$$v_m = x_m \omega_n = (0.175 \text{ ft})(16.050 \text{ rad/s})$$

$$v_m = 2.81 \text{ ft/s}$$

$$a_m = x_m \omega_n^2 = (0.175 \text{ ft})(16.050 \text{ rad/s})^2$$

$$a_m = 45.1 \text{ ft/s}^2$$

19.5

GIVEN:

BLOCK  $m = 32 \text{ kg}$ SPRING  $k = 12 \text{ kN/m}$ 

INITIAL VELOCITY

 $v_0 = 250 \text{ mm/s}$ 

INITIAL DISPLACEMENT = 0

FIND:

(a) PERIOD  $T_n$  AND FREQ,  $f_n$ (b) AMPLITUDE  $x_m$ MAXIMUM ACCELERATION,  $a_m$ 

(a)

 $v = 0$  $x_m$  $v_0 = 250 \text{ mm/s}$ 

$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{k/m} = \sqrt{12 \times 10^3 \text{ N/m} / 32 \text{ kg}}$$

$$\omega_n = 19.365 \text{ rad/s}$$

$$T_n = 2\pi / \omega_n$$

$$T_n = 2\pi / 19.365$$

$$T_n = 0.324 \text{ s}$$

$$f_n = 1/T_n = 1/0.324 = 3.08 \text{ Hz}$$

$$(b) @ t = 0, x_0 = 0, \dot{x}_0 = v_0 = 250 \text{ mm/s}$$

THUS

$$x_0 = 0 = x_m \sin(\omega_n(0) + \phi)$$

$$\text{AND } \phi = 0$$

$$\dot{x}_0 = v_0 = x_m \omega_n \cos(\omega_n(0) + 0) = x_m \omega_n$$

$$v_0 = 0.250 \text{ m/s} = x_m (19.365 \text{ rad/s})$$

$$x_m = (0.250 \text{ m/s}) / (19.365 \text{ rad/s})$$

$$x_m = 12.91 \times 10^{-3} \text{ m}$$

$$x_m = 12.91 \text{ mm}$$

$$a_m = x_m \omega_n^2 = (12.91 \times 10^{-3} \text{ m})(19.365 \text{ rad/s})^2$$

$$a_m = 4.84 \text{ m/s}^2$$

19.6

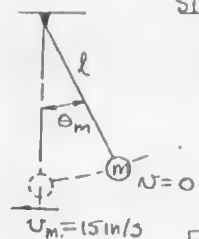
GIVEN:

PENDULUM IN SIMPLE HARMONIC MOTION  
PERIOD  $T_n = 1.35$   
MAXIMUM VELOCITY,  $v_m = 15 \text{ in./s}$

FIND:

- (a) AMPLITUDE OF THE MOTION,  $\theta_m$  IN DEGREES  
(b) THE MAXIMUM TANGENTIAL ACCELERATION  $(a_t)_m$

(a)



SIMPLE HARMONIC MOTION

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\omega_n = 2\pi/T_n = (2\pi)/(1.35)$$

$$\omega_n = 4.833 \text{ rad/s}$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_m = \theta_m \omega_n$$

$$v_m = l \dot{\theta}_m = l \theta_m \omega_n$$

$$\theta_m = v_m / l \omega_n$$

FOR A SIMPLE PENDULUM

$$\omega_n = \sqrt{g/l}$$

THUS

$$l = g / \omega_n^2 = \frac{32.2 \text{ ft/s}^2}{(4.833 \text{ rad/s})^2}$$

$$l = 1.378 \text{ ft}$$

$$\text{FROM (1)} \quad \theta_m = v_m / l \omega_n = (15/12 \text{ ft/s}) / (1.378 \text{ ft})(4.833 \text{ rad/s})$$

$$\theta_m = 0.18769 \text{ rad} = 10.75^\circ$$

$$(b) a_t = l \ddot{\theta}$$

MAX TANGENTIAL ACCELERATION OCCURS WHEN  $\ddot{\theta}$  IS MAXIMUM,  $\ddot{\theta} = -\theta_m \omega_n^2 \sin(\omega_n t + \phi)$

$$\ddot{\theta}_{\text{MAX}} = \theta_m \omega_n^2, (a_t)_{\text{MAX}} = l \theta_m \omega_n^2$$

$$(a_t)_{\text{MAX}} = (1.378 \text{ ft})(0.18769 \text{ rad})(4.833 \text{ rad/s})^2$$

$$(a_t)_m = 6.04 \text{ ft/s}^2$$

19.7

GIVEN:

SIMPLE PENDULUM  
 $l = 800 \text{ mm}$ ,  $\theta_{\text{MAX}} = 6^\circ$

FIND:

- (a) FREQUENCY OF OSCILLATION,  $f_n$   
(b) MAXIMUM VELOCITY  $v_m$  OF THE BOB



$$(a) \omega_n = \sqrt{g/l} = \sqrt{(9.81 \text{ m/s}^2)/(0.8 \text{ m})}$$

$$\omega_n = 3.502 \text{ rad/s}$$

$$f_n = \omega_n / 2\pi = (3.502 \text{ rad/s}) / 2\pi$$

$$f_n = 0.557 \text{ Hz}$$

$$(b) \theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_m = \theta_m \omega_n$$

$$v_m = l \dot{\theta}_m = l \theta_m \omega_n = (0.8 \text{ m})(6^\circ)(\pi \text{ rad}) / (180^\circ)(3.502 \text{ rad/s})$$

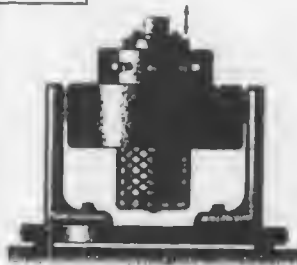
$$v_m = 293.4 \times 10^{-3} \text{ m/s}$$

$$v_m = 293 \text{ mm/s}$$

19.8

GIVEN:

PACKAGE A IN  
SIMPLE HARMONIC  
MOTION AT A  
FREQUENCY  
WHICH IS THE  
SAME AS THE  
MOTOR WHICH  
DRIVES IT.  
PEAK ACCELERATION  
=  $150 \text{ ft/s}^2$   
AMPLITUDE =  $2.310$



FIND:

REQUIRED SPEED OF THE MOTOR IN RPM

MAXIMUM VELOCITY OF THE TABLE (PACKAGE)  
IN SIMPLE HARMONIC MOTION

$$v_{\text{MAX}} = x_{\text{MAX}} \omega_n^2$$

$$150 \text{ ft/s}^2 = (2.3 \text{ ft}) \omega_n^2$$

$$\omega_n^2 = (782.6 \text{ rad/s})^2$$

$$\omega_n = 27.98 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{27.98}{2\pi} = 4.452 \text{ Hz (CYCLE/S)}$$

$$1 \text{ RPM} = 1 \text{ CYCLE} / (1 \text{ MIN.}) (60 \text{ S/MIN.}) = 1/60 \text{ (Hz.)}$$

$$(f \text{ Hz}) / (1/60 \text{ Hz}) = 4.452 = 267 \text{ RPM}$$

$$\text{MAXIMUM VELOCITY } v_{\text{MAX}} = x_{\text{MAX}} \omega_n = (2.3 \text{ ft})(27.98 \text{ rad/s})$$

$$v_{\text{MAX}} = 5.36 \text{ ft/s}$$

19.9

GIVEN:

PARTICLE MOTION

$$x = 5 \sin 2t + 4 \cos 2t \text{ (m, s)}$$

FIND:

- (a) PERIOD,  $T_n$   
(b) AMPLITUDE,  $x_m$   
(c) PHASE ANGLE,  $\phi$

FOR SIMPLE HARMONIC MOTION

$$x = x_m \sin(\omega_n t + \phi)$$

DOUBLE ANGLE FORMULA (TRIGONOMETRY)

$$\sin(A+B) = (\sin A)(\cos B) + (\sin B)(\cos A)$$

$$\text{LET } A = \omega_n t, B = \phi$$

$$\text{THEN } x = x_m \sin(\omega_n t + \phi)$$

$$x = x_m (\sin \omega_n t)(\cos \phi) + x_m (\sin \phi)(\cos \omega_n t)$$

$$x = (x_m \cos \phi)(\sin \omega_n t) + (x_m \sin \phi)(\cos \omega_n t)$$

$$\text{GIVEN } x = 5 \sin 2t + 4 \cos 2t$$

$$\text{COMPARING, } \omega_n = 2 \quad x_m \cos \phi = 5 \quad (1)$$

$$x_m \sin \phi = 4 \quad (2)$$

$$(a) T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{(2 \text{ rad/s})} = \pi \text{ s}$$

$$T = 3.14 \text{ s}$$

$$(b) \text{ SQUARING (1) AND (2) AND ADDING,}$$

$$x_m^2 \cos^2 \phi + x_m^2 \sin^2 \phi = 5^2 + 4^2$$

$$x_m^2 (\cos^2 \phi + \sin^2 \phi) = x_m^2 = 41 \text{ m}^2$$

$$x_m = 6.40 \text{ m}$$

$$(c) \text{ DIVIDE (2) BY (1)}$$

$$\tan \phi = \frac{4}{5} \quad \phi = 38.7^\circ$$

19.10

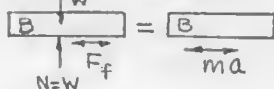


GIVEN:

TABLE C MOVES IN SIMPLE HARMONIC MOTION WITH AMPLITUDE 3 IN  
 $\mu_s = 0.65$  BETWEEN BLOCK B AND C

FIND:

LARGEST FREQUENCY ALLOWED FOR NO SLIDING



NEWTON'S LAW

$$F_f = ma$$

BLOCK B MOVES IN SIMPLE HARMONIC MOTION WITH THE SAME FREQUENCY AS C WHEN THERE IS NO SLIDING

$$x = x_m \sin(\omega_n t + \phi)$$

$$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

MAXIMUM ACCELERATION

$$a = x_m \omega_n^2$$

$$F_f = m x_m \omega_n^2$$

FOR NO SLIDING

$$F_f > \mu_s W$$

$$\text{OR } \mu_s W > \frac{W}{g} x_m \omega_n^2$$

$$\omega_n^2 > \mu_s g / x_m$$

$$\omega_n^2 > (0.65)(32.2 \text{ ft/s}^2) / (3/12 \text{ ft}) = 83.72 \text{ (rad/s)}^2$$

$$\omega_n > 9.150$$

$$f_n = \omega_n / 2\pi = (9.150) / 2\pi = 1.456 \text{ Hz}$$

19.11



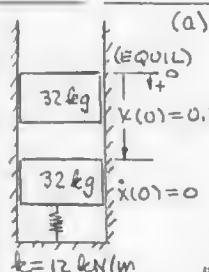
k = 12 kN/m

GIVEN:

INITIAL DISPLACEMENT OF THE BLOCK = 300 mm DOWNWARD

FIND:

1.5 S AFTER THE BLOCK IS RELEASED,  
 (a) TOTAL DISTANCE TRAVELED BY THE BLOCK  
 (b) ACCELERATION OF THE BLOCK



$$(a) \quad x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{k/m} = \sqrt{(12 \times 10^3 \text{ N/m}) / (32 \text{ kg})}$$

$$\omega_n = 19.365 \text{ rad/s}$$

$$T_n = 2\pi / \omega_n = (2\pi) / (19.365)$$

$$T_n = 0.3245 \text{ s}$$

INITIAL CONDITIONS

$$x(0) = 0.3 \text{ m}, \quad \dot{x}(0) = 0$$

$$0.3 = x_m \sin(0 + \phi)$$

$$\dot{x}(0) = 0 = x_m \omega_n \cos(0 + \phi)$$

$$\phi = \pi/2$$

$$x_m = 0.3$$

$$x(t) = (0.3) \sin(19.365t + \pi/2)$$

$$T_n = 0.3245 \text{ s}$$

$$x(1.5) = (0.3) \sin[(19.365)(1.5) + \pi/2] = -0.2147 \text{ m}$$

$$\dot{x}(1.5) = (0.3)(19.365) \cos[(19.365)(1.5) + \pi/2] = 4.057 \text{ m/s}$$

IN ONE CYCLE, BLOCK TRAVELS

$$(4)(0.3 \text{ m}) = 1.2 \text{ m}$$

TO TRAVEL 4 CYCLES IT TAKES

$$(4 \text{ cyc})(0.3245 \text{ s/cyc}) = 1.2980 \text{ s}$$

AT  $t = 1.5$  THUS, TOTAL DISTANCE TRAVELED

$$\text{IS } 4(1.2) + 0.6 + (0.3 - 0.2147) = 5.49 \text{ m}$$

(b)

$$\ddot{x}(1.5) = -(0.3)(19.365)^2 \sin[(19.365)(1.5) + \pi/2] = 80.5 \text{ m/s}^2$$

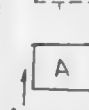
19.12

GIVEN:

$W_A = 3 \text{ lb}$ ,  $k = 2 \text{ lb/in}$   
 INITIAL VELOCITY OF A = 90 in./s

FIND:

(a) TIME REQUIRED FOR THE BLOCK TO MOVE 3 IN UPWARD  
 (b) CORRESPONDING VELOCITY AND ACCELERATION



$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{k}{m}}, \quad k = 2 \text{ lb/in} = 24 \frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{24 \text{ lb/ft}}{(3 \text{ lb}) / (32.2 \text{ ft/s}^2)}}$$

$$\omega_n = 16.05 \text{ rad/s}$$

$$x(0) = 0 = x_m \sin(0 + \phi)$$

$$\phi = 0$$

$$\dot{x}(0) = x_m \omega_n \cos(0 + 0) \quad \dot{x}(0) = 90 = 7.5 \text{ ft/s}$$

$$7.5 = x_m (16.05) \quad x_m = 0.4673 \text{ ft}$$

$$x = (0.4673) \sin(16.05t) \text{ (ft, s)} \quad (1)$$

(a) AT  $x = 3/12 = 0.25 \text{ ft}$ 

$$0.25 = 0.4673 \sin(16.05t)$$

$$t = \frac{\sin^{-1}(0.25 / (0.4673))}{16.05} = 0.0352 \text{ s}$$

$$(b) \quad \dot{x} = x_m \omega_n \cos(\omega_n t) \quad \ddot{x} = -x_m \omega_n^2 \sin \omega_n t$$

$$t = 0.0352, \quad \dot{x} = (0.4673)(16.05) \cos[(16.05)(0.0352)]$$

$$\dot{x} = 6.34 \text{ ft/s}$$

$$\ddot{x} = -(0.4673)(16.05)^2 \sin[(16.05)(0.0352)] = -64.4 \text{ ft/s}^2$$

19.13

REFER TO FIGURE IN PROBLEM 19.12 ABOVE

GIVEN:

$W_A = 3 \text{ lb}$ ,  $k = 2 \text{ lb/in}$ ,  $v_0 = 90 \text{ in./s}$  (SAME AS 19.12)

FIND:

AFTER 0.90 S, POSITION, VELOCITY AND ACCELERATION OF THE BLOCK

$$x = x_m \sin(\omega_n t + \phi)$$

$$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$$

$$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

SINCE THE GIVEN DATA IS THE SAME AS IN PROBLEM 19.12 ABOVE, THE EQUATION OF MOTION IS THE SAME AS EQUATION (1) IN 19.12  
 $\phi = 0$ ,  $x_m = 0.4673 \text{ ft}$ ,  $\omega_n = 16.05 \text{ rad/s}$  AND  $x, \dot{x}, \ddot{x}$  ARE +

$$x = (0.4673) \sin(16.05t) \text{ (ft, s)} \quad (1)$$

AT 0.90 S

$$x = (0.4673) \sin[(16.05)(0.90)] = 0.445 \text{ ft}$$

$$\dot{x} = (0.4673)(16.05) \cos[(16.05)(0.90)] = -2.27 \text{ ft/s}$$

$$\ddot{x} = -(0.4673)(16.05)^2 \sin[(16.05)(0.90)] = 114.7 \text{ ft/s}^2$$

19.14

GIVEN:

$l = 800 \text{ mm}$   
 AT  $t = 0$ ,  $\theta = +5^\circ$ ,  $\dot{\theta} = 0$   
 ASSUME SIMPLE HARMONIC MOTION

FIND:

1.6 s AFTER RELEASE  
 (a)  $\theta$   
 (b)  $v$  AND  $a$  OF THE BOB.



$$\theta = \theta_m \sin(\omega_n t + \phi) \quad \omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81 \text{ m/s}^2}{0.8 \text{ m}}}$$

INITIAL CONDITIONS

$$\theta(0) = 5^\circ = (5)(\pi/180) \text{ RAD}$$

$$\dot{\theta}(0) = 0$$

$$\theta(0) = \frac{5\pi}{180} = \theta_m \sin(0 + \phi)$$

$$\dot{\theta}(0) = 0 = \theta_m \omega_n \cos(0 + \phi) \quad \phi = \pi/2$$

$$\theta_m = \frac{5\pi}{180} \text{ RAD}$$

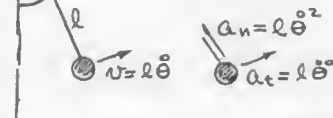
$$\theta = \frac{5\pi}{180} \sin(3.502t + \frac{\pi}{2})$$

(a)

AT  $t = 1.6 \text{ s}$   $\theta = \frac{5\pi}{180} \sin(3.502(1.6) + \frac{\pi}{2})$   
 $\theta = 0.06786 \text{ RAD} = 3.9^\circ$

(b)  $\theta$ ,  $\dot{\theta}$ ,  $\ddot{\theta}$ 

$$\theta(1.6) = 0.06786 \text{ RAD}$$



$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi) = \left(\frac{5\pi}{180}\right)(3.502) \cos(3.502(1.6) + \frac{\pi}{2})$$

$$\dot{\theta}(1.6 \text{ s}) = 0.19223 \text{ RAD/s}$$

$$v = l \dot{\theta} = (0.800 \text{ m})(0.19223 \text{ RAD/s}) = 0.1538 \text{ m/s}$$

$$\ddot{\theta} = -\theta_m \omega_n^2 \sin(\omega_n t + \phi) = -\left(\frac{5\pi}{180}\right)(3.502)^2 \sin(3.502(1.6) + \frac{\pi}{2})$$

$$\ddot{\theta} = -0.8319 \text{ RAD/s}^2$$

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$

$$a_t = l \ddot{\theta} = (0.8 \text{ m})(-0.8319 \text{ RAD/s}^2) = -0.6655 \frac{\text{m}}{\text{s}^2}$$

$$a_n = l \dot{\theta}^2 = (0.8 \text{ m})(0.19223 \text{ RAD/s})^2 = 0.02956 \frac{\text{m}}{\text{s}^2}$$

$$a = \sqrt{(0.6655)^2 + (0.02956)^2} = 0.6662 \text{ m/s}^2$$

$$a = 0.666 \text{ m/s}^2$$

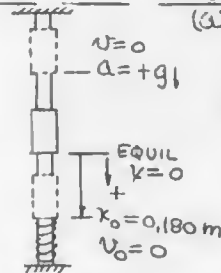
19.15

GIVEN:

$m = 5 \text{ kg}$ , UNATTACHED TO THE SPRING  
 WHEN COLLAR IS PUSHED DOWN  
 180 mm OR MORE AND RELEASED  
 IT LOSES CONTACT WITH THE  
 SPRING

FIND:

(a) THE SPRING CONSTANT  $k$   
 (b) POSITION, VELOCITY AND ACCELERATION  
 0.16 s AFTER IT IS PUSHED DOWN  
 180 mm AND RELEASED.



$$x = x_m \sin(\omega_n t + \phi)$$

$$x_0 = x_m \sin(0 + \phi) = 0.180 \text{ m}$$

$$x_0 = 0 = x_m \cos(0 + \phi)$$

$$\phi = \pi/2$$

$$x_m = 0.180 \text{ m}$$

$$x = 0.180 \sin(\omega_n t + \frac{\pi}{2})$$

WHEN THE COLLAR JUST  
 LEAVES THE SPRING, ITS  
 ACCELERATION IS  $g \downarrow$  AND  $v = 0$

$$\ddot{x} = (0.180) \omega_n^2 \cos(\omega_n t + \frac{\pi}{2})$$

$$v = 0 \quad 0 = (0.180) \omega_n \cos(\omega_n t + \frac{\pi}{2})$$

$$(\omega_n t + \frac{\pi}{2}) = \frac{\pi}{2}$$

$$a = -g = -(0.180) (\omega_n)^2 \sin(\omega_n t + \frac{\pi}{2})$$

$$-g = -(0.180) (\omega_n^2) \quad \omega_n = \sqrt{\frac{9.81 \text{ m/s}^2}{0.180 \text{ m}}}$$

$$\omega_n = 7.382 \text{ RAD/s}$$

$$\omega_n = \sqrt{k/m}$$

$$k = m \omega_n^2 = (5 \text{ kg})(7.382 \text{ RAD/s})^2 = 272.5 \text{ N/m}$$

$$k = 273 \text{ N/m}$$

(b)  $\omega_n = 7.382 \text{ RAD/s}$ 

$$x = 0.180 \sin[(7.382)t + \frac{\pi}{2}]$$

At  $t = 0.16 \text{ s}$ 

POSITION

$$x = 0.180 \sin[(7.382)(0.16) + \frac{\pi}{2}] = 0.06838 \text{ m}$$

$$x = 68.4 \text{ mm}$$

BELOW EQUILIBRIUM POSITION

VELOCITY

$$\dot{x} = (0.180)(7.382) \cos[(7.382)(0.16) + \frac{\pi}{2}] = -1.229 \text{ m/s}$$

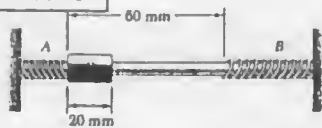
$$\dot{x} = 1.229 \text{ m/s} \uparrow$$

ACCELERATION

$$\ddot{x} = -(0.180)(7.382)^2 \sin[(7.382)(0.16) + \frac{\pi}{2}] = -3.716 \frac{\text{m}}{\text{s}^2}$$

$$\ddot{x} = 3.73 \text{ m/s}^2 \uparrow$$

19.16

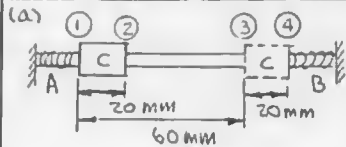


GIVEN:

$m_c = 8 \text{ kg}$   
 $k = 600 \text{ N/m}$   
 FOR EACH SPRING  
 INITIAL DEFLECTION  
 OF SPRING A  
 $= 20 \text{ mm}$ .  
 NO FRICTION

FIND:

- (a) PERIOD  
 (b) POSITION OF C AFTER 1.5 s



FOR EITHER SPRING

$$T_n = \frac{2\pi}{\sqrt{k/m_c}}$$

$$T_n = \frac{2\pi}{\sqrt{600 \text{ N/m} / 8 \text{ kg}}}$$

$$T_n = 0.7255 \text{ s}$$

COMPLETE CYCLE IS 1 2 3 4 4 3 2 1

TIME FROM 1 TO 2 IS  $T_n/4$  WHICH IS THE SAME  
 AS TIME FROM 3 TO 4, 4 TO 3 AND 2 TO 1

THUS THE TIME DURING WHICH THE SPRINGS ARE  
 COMPRESSED IS  $4(T_n/4) = T_n = 0.7255 \text{ s}$

VELOCITY AT 3 OR 2:

$$V_1 = 0 \quad T_1 = 0 \quad V_1 = \frac{1}{2} k x^2 = \frac{1}{2} (600 \text{ N/m}) (0.020 \text{ m})^2$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} (8 \text{ kg}) (v_2)^2 \quad T_2 = 4 v_2^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + 0.20 = 4 v_2^2 \quad v_2 = 0.1732 \text{ m/s}$$

$$\text{TIME FROM 2 TO 3 IS } t_{2-3} = \frac{(0.020 \text{ m})}{(0.1732 \text{ m/s})} = 0.11545 \text{ s}$$

AND IS THE SAME AS THE TIME FROM 3 TO 2  
 THUS

TOTAL TIME FOR A COMPLETE CYCLE IS

$$T_c = T_n + 2 t_{2-3} = 0.7255 + 2(0.11545) = 0.9564$$

$$T_c = 0.956 \text{ s}$$

(b) FROM (a), IN 0.9564 THE SPRING A IS AGAIN  
 FULLY COMPRESSED. SPRING B IS COMPRESSED  
 THE SECOND TIME IN 1.5 CYCLES OR  $(1.5)(0.9564) =$   
 $1.4346 \text{ s}$ . AT 1.5 s THE COLLAR IS STILL IN  
 CONTACT WITH SPRING B MOVING TO THE LEFT  
 AND IS AT A DISTANCE  $\Delta x$  FROM THE  
 MAXIMUM DEFLECTION OF B EQUAL TO

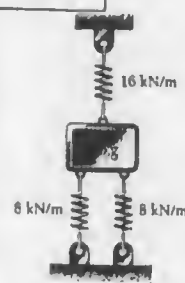
$$\Delta x = 20 - 20 \cos \left( \frac{2\pi}{0.7255} (1.5 - 1.4346) \right)$$

$$\Delta x = 20 - 16.877 = 3.123 \text{ mm}$$

THUS COLLAR C IS  $60 - 3.123 = 56.877 \text{ mm}$   
 FROM ITS INITIAL POSITION

56.9 mm  
 FROM INITIAL POSITION

19.17



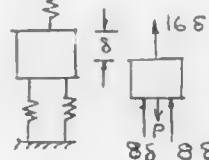
GIVEN:

MASS AND SPRINGS AS SHOWN  
 AFTER THE MASS IS PULLED  
 DOWN AND RELEASED FROM  
 REST THE AMPLITUDE OF THE  
 RESULTING MOTION IS 45 mm

FIND:

- (a) THE PERIOD AND FREQUENCY  
 OF THE MOTION  
 (b) THE MAXIMUM VELOCITY  
 AND ACCELERATION OF THE  
 BLOCK

(a) DETERMINE THE CONSTANT  $k$  OF A SINGLE  
 SPRING EQUIVALENT TO THE THREE SPRINGS



$$P = k \delta = 16 \delta + 8 \delta + 8 \delta$$

$$k = 32 \text{ kN/m}$$

$$\omega_n = \sqrt{k/m} = \sqrt{32 \times 10^3 \text{ N/m} / 35 \text{ kg}}$$

$$(1 \text{ N} = 1 \text{ kg} \cdot 1 \text{ m/s}^2)$$

$$\omega_n = 30.237 \text{ rad/s}$$

$$T_n = 2\pi / \omega_n = 2\pi / 30.23 = 0.2085$$

$$f_n = 1/T_n = 4.81 \text{ Hz}$$

$$(b) x = x_m \sin(\omega_n t + \phi) \quad x_0 = 0.045 \text{ m} = x_m$$

$$\omega_n = 30.24 \text{ rad/s}$$

$$x = 0.045 \sin(30.24 t + \phi)$$

$$\dot{x} = (0.045)(30.24) \cos(30.24 t + \phi) \quad v_{\text{MAX}} = 1.361 \text{ m/s}$$

$$\ddot{x} = -(0.045)(30.24)^2 \sin(30.24 t + \phi) \quad a_{\text{MAX}} = 41.1 \text{ m/s}^2$$

19.18

GIVEN:

MASS AND SPRINGS AS SHOWN  
 AMPLITUDE OF MOTION IS  
 45 mm AFTER MASS IS  
 PULLED DOWN AND RELEASED  
 FROM REST

FIND:

- (a) PERIOD AND FREQUENCY OF  
 MOTION  
 (b) MAXIMUM VELOCITY AND ACCELERATION

(a) DETERMINE THE CONSTANT  $k$  OF A SINGLE SPRING  
 EQUIVALENT TO THE TWO SPRINGS SHOWN

$$S = S_1 + S_2 = \frac{P}{16 \text{ kN/m}} + \frac{P}{16 \text{ kN/m}} = \frac{P}{k}$$

$$\frac{1}{k} = \frac{1}{16} + \frac{1}{16} \quad k = 8 \text{ kN/m}$$

$$T_n = \frac{2\pi}{\sqrt{k/m}} = \frac{2\pi}{\sqrt{8 \times 10^3 / 35}} = 0.416 \text{ s}$$

$$f_n = \frac{1}{T_n} = \frac{1}{0.416} = 2.41 \text{ Hz}$$

$$(b) \omega_n = 2\pi f_n = 2\pi(2.41) = 15.12 \text{ rad/s}$$

$$x = 0.045 \sin(15.12 t + \phi)$$

$$\dot{x} = (0.045)(15.12) \cos(15.12 t + \phi)$$

$$v_{\text{MAX}} = 0.680 \text{ m/s}$$

$$\ddot{x} = -(0.045)(15.12)^2 \sin(15.12 t + \phi)$$

$$a_{\text{MAX}} = 10.29 \text{ m/s}^2$$

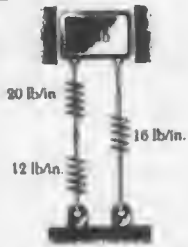
19.19

GIVEN:

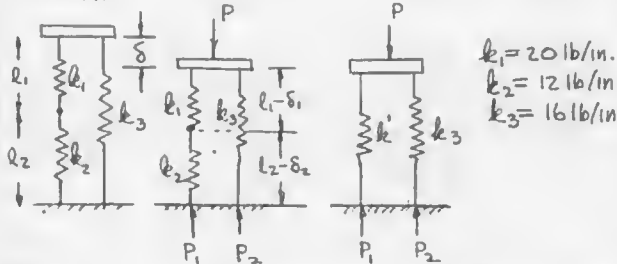
30 lb BLOCK  
AT  $t=0$ ,  $x=1.75$  in.  
DOWNWARD,  $v=0$

FIND:

- (a) PERIOD AND FREQUENCY OF MOTION  
(b) MAXIMUM VELOCITY AND ACCELERATION



DETERMINE THE CONSTANT  $k$  OF A SINGLE SPRING EQUIVALENT TO THE THREE SPRINGS SHOWN



SPRINGS 1 AND 2 (FORCE IN EACH SPRING IS THE SAME)

$$\delta = \delta_1 + \delta_2 \quad \frac{P}{k'} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$k' = \frac{k_1 k_2}{k_1 + k_2}$$

$k'$  IS THE SPRING CONSTANT OF A SINGLE SPRING EQUIVALENT TO SPRINGS 1 AND 2  
SPRINGS  $k'$  AND 3 (DEFLECTION IN EACH SPRING IS THE SAME)

$$P = P_1 + P_2 \quad P = k' \delta \quad P_1 = k_1 \delta \quad P_2 = k_3 \delta$$

$$k \delta = k' \delta + k_3 \delta$$

$$k = k' + k_3 = \frac{k_1 k_2}{k_1 + k_2} + k_3$$

$$k = \frac{(20)(12)}{(20+12)} + 16 = 23.5 \text{ lb/in.} = 282 \text{ lb/ft}$$

$$(a) \tau_n = \frac{2\pi}{\sqrt{k/m}} = \frac{2\pi}{\sqrt{(282 \text{ lb/ft})/(30 \text{ lb})}} = 0.361 \text{ s}$$

$$f_n = 1/\tau_n = 1/0.361 = 2.77 \text{ Hz}$$

$$(b) x = x_m \sin(\omega_n t + \phi)$$

$$x_m = 1.75 \text{ in} = 0.1458 \text{ ft}$$

$$\omega_n = 2\pi f_n = (2\pi)(2.77) = 34.9 \text{ rad/s}$$

$$x = (0.1458) \sin(34.9 t + \phi)$$

$$\dot{x} = (0.1458)(34.9) \cos(34.9 t + \phi)$$

$$v_{\text{MAX}} = (0.1458)(34.9) = 5.09 \text{ ft/s}$$

$$x = -(0.1458)(34.9) \cos(34.9 t + \phi)$$

$$a_{\text{MAX}} = 44.1 \text{ ft/s}^2$$

$$a_{\text{MAX}} = 44.1 \text{ ft/s}^2$$

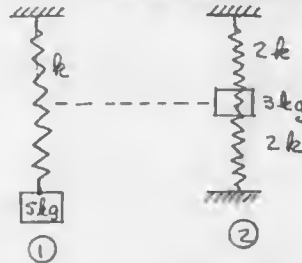
19.20

GIVEN:

5-kg BLOCK ATTACHED TO A SPRING  
FIXED AT THE OTHER END  
VIBRATES WITH A PERIOD  $\tau_n = 6.8$  s  
SPRING CONSTANT  $k$  IS  
INVERSELY PROPORTIONAL TO  
THE SPRING'S LENGTH.

FIND:

THE PERIOD  
FOR A 3 kg BLOCK ATTACHED TO THE CENTER  
OF THE SAME SPRING FIXED AT BOTH ENDS



EQUIVALENT

SPRING CONSTANT

$$k' = 2k + 2k = 4k$$

(DEFLECTION OF  
EACH SPRING IS  
THE SAME)

$$(\tau_n)_1 = 6.8 = 2\pi / \sqrt{k/(5 \text{ kg})} \quad (\tau_n)_2 = 2\pi / \sqrt{4k/(3 \text{ kg})}$$

$$k = \frac{(2\pi)^2}{(6.8)^2} (5 \text{ kg}) \quad (\tau_n)_2 = 2\pi / \sqrt{4(4.269 \text{ N/m})/(3 \text{ kg})}$$

$$k = 4.269 \text{ N/m}$$

$$(\tau_n)_2 = 2.63 \text{ s}$$

19.21

GIVEN:

SYSTEM AS SHOWN IS MOVED 0.8 in.  
DOWNWARD AND RELEASED FROM  
REST.  
PERIOD FOR RESULTING MOTION  
IS  $\tau_n = 1.5$  s

FIND:

- (a) CONSTANT  $k$   
(b) MAXIMUM VELOCITY AND  
ACCELERATION OF THE BLOCK



SINCE THE FORCE IN EACH SPRING IS THE SAME, THE  
CONSTANT  $k'$  OF A SINGLE EQUIVALENT SPRING IS

$$\frac{1}{k'} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{k} \quad k' = k/2.5 \quad (\text{SEE PROB 19.19})$$

$$(a) \tau_n = 1.5 \text{ s} = 2\pi / \sqrt{k'/m} \quad k = \frac{(2\pi)^2}{(1.5)^2} (m) (2.5)$$

$$k = \frac{(2\pi)^2}{(1.5)^2} \left( \frac{30 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (2.5) = 40.868 \text{ lb/ft}$$

$$k = 40.9 \text{ lb/ft}$$

$$(b) x = x_m \sin(\omega_n t + \phi)$$

$$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi) \quad v_{\text{MAX}} = x_m \omega_n$$

$$\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{1.5} = 4.189 \text{ rad/s}$$

$$x_m = 0.8 \text{ in.} = 0.06667 \text{ ft}$$

$$v_{\text{MAX}} = (0.06667 \text{ ft})(4.189 \text{ rad/s})$$

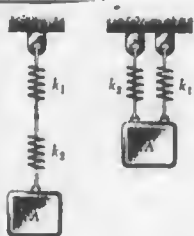
$$v_{\text{MAX}} = 0.279 \text{ ft/s}$$

$$\ddot{x} = -x_m \omega_n^2 \cos(\omega_n t + \phi)$$

$$|a_{\text{MAX}}| = x_m \omega_n^2 = (0.06667 \text{ ft})(4.189)^2$$

$$|a_{\text{MAX}}| = 1.170 \text{ ft/s}^2$$

## 19.22



GIVEN:

PERIOD FOR SPRINGS IN SERIES,  $T_s = 5$  s  
PERIOD FOR SPRINGS IN PARALLEL,  $T_p = 2$  s

FIND:

RATIO OF SPRING CONSTANTS  $k_1/k_2$

## EQUIVALENT SPRINGS

SERIES,  $k_s = \frac{k_1 k_2}{k_1 + k_2}$

PARALLEL,  $k_p = k_1 + k_2$

$$T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{\sqrt{k_s/m}}$$

$$T_p = \frac{2\pi}{\omega_p} = \frac{2\pi}{\sqrt{k_p/m}}$$

$$\left(\frac{T_s}{T_p}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{k_p}{k_s} = \frac{(k_1 + k_2)}{(k_1 k_2)/(k_1 + k_2)} = \frac{(k_1 + k_2)^2}{k_1 k_2}$$

$$(6.25)(k_1 k_2) = k_1^2 + 2k_1 k_2 + k_2^2$$

$$k_1^2 - 4.25 k_1 k_2 + k_2^2 = 0$$

$$k_1 = \frac{4.25 k_2 \pm \sqrt{(4.25)^2 k_2^2 - 4k_2^2}}{2}$$

$$\frac{k_1}{k_2} = 2.125 \pm \sqrt{3.516}$$

$$k_1/k_2 = 4$$

## 19.23



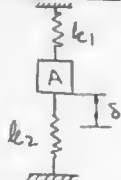
GIVEN:

PERIOD = 0.7 s =  $T$   
AFTER  $k_2$  IS REMOVED  
PERIOD = 0.9 s =  $T'$

FIND:

(a)  $k_1$   
(b) MASS OF  $m$

## EQUIVALENT SPRINGS



$$F_1 = k_1 \delta \quad F_2 = k_2 \delta$$

$$F_1 + F_2 = F = k_e \delta$$

$$k_1 \delta + k_2 \delta = k_e \delta$$

$$k_e = k_1 + k_2$$

(a) BOTH SPRINGS  $T = \frac{2\pi}{\omega} = 0.7$  s

$k_1$  ALONE,  $T' = \frac{2\pi}{\omega'} = 0.9$  s

$$\frac{T}{T'} = \frac{0.7}{0.9} = \sqrt{\frac{k_1}{k_e}} = \sqrt{\frac{k_1}{k_1 + k_2}}$$

$$\left(\frac{7}{9}\right)^2 = 0.6049 = \frac{k_1}{k_1 + 1.2}$$

## 19.23 CONTINUED

$$(0.6049)(k_1 + 1.2) = k_1$$

$$k_1 = 1.838 \text{ kN/m}$$

(b)  $T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{k_1/m}} = \frac{(2\pi)^2 m}{(1.838 \times 10^3 \text{ N/m})}$

$$m = \frac{(0.9 \text{ s})^2 (1.838 \times 10^3 \text{ N/m})}{(2\pi)^2}$$

$$m = 37.7 \text{ kg}$$

## 19.24



GIVEN:

PERIOD FOR SYSTEM SHOWN IS  $T = 1.6$  s

PERIOD AFTER A 7-kg COLLAR IS PLACED ON A, IS  $T' = 2.1$  s

FIND:

(a) MASS OF A  
(b)  $k$

INITIALLY  $T = \frac{2\pi}{\omega} = 1.6$  s

AFTER 7-kg MASS IS ADDED TO A,

$$T' = \frac{2\pi}{\omega'} = 2.1 \text{ s}$$

(a)

$$\frac{T'}{T} = \sqrt{\frac{(m_A + 7)}{m_A}}$$

$$\left(\frac{2.1}{1.6}\right)^2 = \frac{m_A + 7}{m_A}$$

$$(1.7227)(m_A) = m_A + 7$$

$$m_A = 9.69 \text{ kg}$$

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m_A}}$$

$$k = (2\pi)^2 (m_A) / (T)^2$$

$$k = (2\pi)^2 (9.69 \text{ kg}) / (1.6 \text{ s})^2$$

$$k = 149.4 \text{ kg/s}^2$$

$$k = 149.4 \text{ N/m}$$

19.25

GIVEN:

FOR SYSTEM SHOWN  
PERIOD  $T = 0.2$  s  
AFTER  $k_2$  IS REMOVED  
AND BLOCK A IS CONNECTED  
TO  $k_1$ , PERIOD  $T' = 0.12$  s

FIND:

- (a)  $k_1$   
(b) WEIGHT OF BLOCK A.



EQUIVALENT SPRING CONSTANT FOR  
SPRINGS IN SERIES.

$$k_e = \frac{k_1 k_2}{(k_1 + k_2)}$$

FOR  $k_1$  AND  $k_2$ 

$$T = \frac{2\pi}{\sqrt{k_e/m_A}} = \frac{2\pi}{\sqrt{(k_1 k_2)/(m_A)(k_1 + k_2)}}$$

FOR  $k_1$  ALONE

$$T' = \frac{2\pi}{\sqrt{k_1/m_A}}$$

(a)

$$\frac{T}{T'} = \sqrt{\frac{(k_1 + k_2)(k_1)}{k_1 k_2}} = \sqrt{\frac{k_1 + k_2}{k_2}}$$

$$k_2 \left(\frac{T}{T'}\right)^2 = k_1 + k_2$$

$$T/T' = 0.2/0.12 = 1.6667, \quad k_2 = 20 \text{ lb/in}$$

$$(20 \text{ lb/in.})(1.6667)^2 = k_1 + 20 \text{ lb/in}$$

$$k_1 = 35.6 \text{ lb/in.}$$

(b)

$$T' = \frac{2\pi}{\sqrt{k_1/m_A}}$$

$$m_A = W_A/g$$

$$m_A = \frac{(T')^2 k_1}{(2\pi)^2}, \quad k_1 = 35.6 \text{ lb/in.} = 426.7 \frac{\text{lb}}{\text{ft}}$$

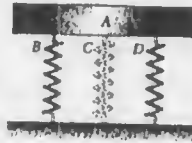
$$W_A = \frac{(32.2 \text{ ft/s}^2)(0.12 \text{ s})^2(426.7 \text{ lb/ft})}{(2\pi)^2}$$

$$W_A = 5.01 \text{ lb}$$

19.26

GIVEN:

$W_A = 100 \text{ lb}$   
 $k_B = k_D = k = 120 \text{ lb/ft}$   
FREQUENCY REMAINS  
THE SAME WHEN AN  
80 lb BLOCK IS ADDED  
TO A AND A SPRING OF  
CONSTANT  $k_c$  IS ADDED  
TO THE SYSTEM



FIND:

 $k_c$ 

FREQUENCY OF THE ORIGINAL SYSTEM

SPRINGS B AND D ARE IN PARALLEL

$$k_e = k_B + k_D = 2(120 \text{ lb/ft}) = 240 \text{ lb/ft}$$

$$\omega_n^2 = \frac{k_e}{m_A} = \frac{240 \text{ lb/ft}}{(100 \text{ lb})/(32.2 \text{ ft/s}^2)}$$

$$\omega_n^2 = 77.78 (\text{rad/s})^2$$

FREQUENCY OF NEW SYSTEM

SPRINGS A, B AND C ARE IN PARALLEL

$$k'_e = k_B + k_D + k_c = (2)(120) + k_c$$

$$(\omega'_n)^2 = \frac{k'_e}{m_A + m_B} = \frac{(240 + k_c)(32.2 \text{ ft/s}^2)}{(100 \text{ lb} + 80 \text{ lb})}$$

$$(\omega'_n)^2 = (0.1789)(240 + k_c)$$

$$\omega_n^2 = (\omega'_n)^2$$

$$77.78 = (0.1789)(240 + k_c)$$

$$k_c = 191.97 \text{ lb/ft}$$

$$k_c = 192.0 \text{ lb/ft}$$



19.27



GIVEN:

$$\begin{aligned}\delta &= PL/AE \\ L &= 450 \text{ mm} \\ E &= 200 \text{ GPa} \\ \text{ROD DIAMETER} &= 8 \text{ mm}, m = 8 \text{ kg}\end{aligned}$$

FIND:

- (a) EQUIVALENT SPRING CONSTANT OF THE ROD, ( $k_e$ )  
(b) FREQUENCY OF VERTICAL VIBRATIONS OF THE 8-kg MASS

$$\begin{aligned}(a) \quad P &= k_e \delta \quad \delta = PL/AE, P = \left(\frac{AE}{L}\right) \delta \\ k_e &= \frac{P}{\delta} = \frac{AE}{L} \\ A &= \pi d^2/4 = \pi (8 \times 10^{-3} \text{ m})^2/4 \\ A &= 5.027 \times 10^{-5} \text{ m}^2 \\ L &= 0.450 \text{ m} \\ E &= 200 \times 10^9 \text{ N/m}^2 \\ k_e &= \frac{(5.027 \times 10^{-5} \text{ m}^2)(200 \times 10^9 \text{ N/m}^2)}{(0.450 \text{ m})} \\ k_e &= 22.34 \times 10^6 \text{ N/m} \\ k_e &= 22.3 \text{ MN/m}\end{aligned}$$

$$\begin{aligned}(b) \quad f_n &= \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{22.3 \times 10^6 / 8}}{2\pi} = 265.96 \text{ Hz} \\ f_n &= 266 \text{ Hz}\end{aligned}$$

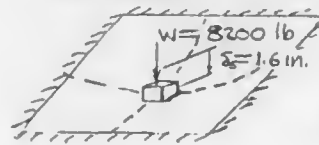
19.29

GIVEN:

STATIC DEFLECTION OF THE FLOOR OF A BUILDING UNDER AN 8200-lb PIECE OF MACHINERY EQUALS  $\delta_s = 1.6 \text{ in.}$

FIND:

- (a) EQUIVALENT SPRING CONSTANT  $k_e$   
(b) THE SPEED IN RPM OF THE MACHINERY THAT SHOULD BE AVOIDED SO AS NOT TO COINCIDE WITH THE NATURAL FREQUENCY OF THE SYSTEM.



$$(a) \quad W = k_e \delta_s$$

$$k_e = \frac{W}{\delta_s} = \frac{8200 \text{ lb}}{1.6 \text{ in.}}$$

$$k_e = 5130 \text{ lb/in.}$$

$$(b) \quad f_n = \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{(5130 \times 12 \text{ lb/ft}) / (8200 \text{ lb} / 32.2 \text{ ft/s}^2)}}{2\pi}$$

$$f_n = 2.473 \text{ Hz}$$

$$1 \text{ Hz} = 1 \text{ CYCLE/S} = 60 \text{ RPM}$$

$$\text{SPEED} = (2.473 \text{ Hz})(60 \text{ RPM}) = 148.4 \text{ RPM}$$

19.28

GIVEN:

$$\begin{aligned}\delta_B &= PL^3/3EI \\ L &= 10 \text{ ft} \\ E &= 29 \times 10^6 \text{ lb/in}^2 \\ I &= 12.4 \text{ in}^4\end{aligned}$$

FIND:

- (a) EQUIVALENT SPRING CONSTANT ( $k_e$ )  
(b) FREQUENCY OF A 120-lb BLOCK AT B

$$\begin{aligned}(a) \quad P &= k_e \delta_B \quad \delta_B = PL^3/3EI, P = \left(\frac{3EI}{L^3}\right) \delta_B \\ k_e &= \frac{3EI}{L^3} = \frac{(3)(29 \times 10^6 \text{ lb/in}^2)(12.4 \text{ in}^4)}{(10 \times 12 \text{ in})^3} \\ k_e &= 624.3 \text{ lb/in.} \\ k_e &= 624.3 \text{ lb/in.}\end{aligned}$$

$$\begin{aligned}(b) \quad f_n &= \frac{\sqrt{k_e/m}}{2\pi} \quad k_e = 624.3 \text{ lb/in.} \\ &= 7.492 \times 10^3 \text{ lb/ft} \\ f_n &= \frac{\sqrt{(7.492 \times 10^3 \text{ lb/ft}) / (120 \text{ lb} / 32.2 \text{ ft/s}^2)}}{2\pi} \\ f_n &= 3.428 \text{ Hz}\end{aligned}$$

$$f_n = 3.43 \text{ Hz}$$

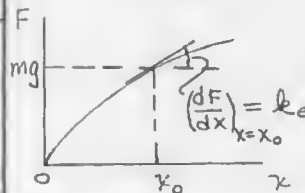
19.30

GIVEN:

FORCE-DEFLECTION EQUATION FOR A NON-LINEAR SPRING,  $F = 5x^{1/2} \text{ (N, m)}$

FIND:

- (a) STATIC DEFLECTION  $x_0$  UNDER A 120-g BLOCK  
(b) FREQUENCY OF VIBRATION OF THE BLOCK FOR SMALL OSCILLATIONS AT  $x_0$



$$(a) \quad mg = (0.120 \text{ kg})(9.81 \text{ m/s}^2)$$

$$mg = 1.177 \text{ N}$$

$$F = mg = 5x_0^{1/2}$$

$$x_0 = \left(\frac{1.177}{5}\right)^2 = 0.0554 \text{ m}$$

$$x_0 = 55.4 \text{ mm}$$

$$(b) \quad \text{AT } x_0, \left(\frac{dF}{dx}\right)_{x_0} = \frac{5}{2}(x_0)^{-1/2} = \frac{5}{2}(0.0554)^{-1/2}$$

$$\left(\frac{dF}{dx}\right)_{x_0} = 10.618 \text{ N/m}$$

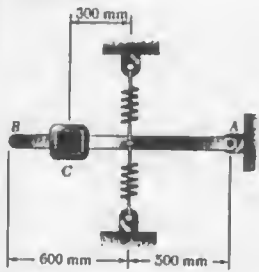
$$k_e = 10.618 \text{ N/m}$$

$$f_n = \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{(10.618 \text{ N/m}) / (0.120 \text{ kg})}}{2\pi}$$

$$f_n = 1.4971 \text{ Hz}$$

$$f_n = 1.497 \text{ Hz.}$$

19.31

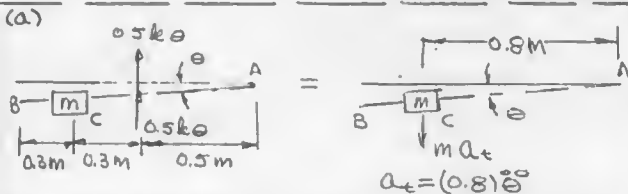


GIVEN:

$\tau = 3 \text{ s}$   
 $k = 900 \text{ N/m}$ , EACH  
 SPRING, TENSION OR  
 COMPRESSION  
 NEGLECT THE MASS  
 OF ROD AB

FIND:

- (a) MASS  $m$  AT C  
 (b)  $(v_c)_{\text{MAX}}$  IF B IS  
 DEPRESSED 40 mm AND RELEASED



NEWTON'S LAW  $\sum M_A = -(0.5)k\theta - (0.5)k\theta = (0.8)m\ddot{\theta}$

$$(0.5\bar{m})(900 \text{ N/m})\theta = (0.64 \text{ m}^2) m \ddot{\theta}$$

$$\ddot{\theta} + \frac{(450 \text{ N}\cdot\text{m})}{(0.64 \text{ m}^2) m} \theta = 0$$

$$\omega_n^2 = \frac{(703.1 \text{ N/m})}{m} \quad \tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m}{703.1 \text{ N/m}}}$$

$$m = \frac{\tau_n^2}{(2\pi)^2} (703.1 \text{ N/m}) = \frac{(3 \text{ s})^2 (703.1 \text{ N/m})}{(2\pi)^2}$$

(b)  $\theta = \theta_m \sin(\omega_n t + \phi)$

AT  $t=0$ 

$$\theta = \frac{(0.04 \text{ m})}{1.1} = 0.03636 \text{ RAD}$$

$$\dot{\theta} = 0$$

$$0.03636 = \theta_m \sin \phi$$

$$0 = \omega_n \theta_m \cos \phi \quad \phi = \pi/2, \theta_m = 0.03636 \text{ RAD}$$

$$\dot{\theta} = \omega_n \theta_m \cos(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{450}{(0.64)(160.3)}} = 2.094 \text{ RAD/s}$$

$$\dot{\theta} = (0.03636)(2.094 \text{ RAD/s}) \cos(2.094 t + \pi/2)$$

$$\dot{\theta}_{\text{MAX}} = 0.03636(2.094) = 0.07615 \text{ RAD/s}$$

$$(v_c)_{\text{MAX}} = (0.800)(\dot{\theta}_{\text{MAX}}) = (0.800)(0.07615) \text{ RAD/s}$$

$$(v_c)_{\text{MAX}} = 0.0609 \text{ m/s}$$

19.32

GIVEN:

$\delta_{\text{ST}}$ , STATIC DEFLECTION OF A BEAM UNDER LOAD  $W$

SHOW:

$$\text{THAT, } f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta_{\text{ST}}}}$$



$$k = W/\delta_{\text{ST}} \\ m = W/g$$

$$\omega_n^2 = k/m = \frac{W/\delta_{\text{ST}}}{W/g} = g/\delta_{\text{ST}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{g/\delta_{\text{ST}}}$$

19.33

GIVEN:

$$\tau_n = 4\sqrt{\frac{g}{\bar{g}}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{\sin^2(\theta_m/2) - \sin^2\phi}}$$

SHOW:

BY EXPANDING THE INTEGRAND OF THE  
 ABOVE EQUATION,  $\tau_n = 2\pi \sqrt{\frac{g}{\bar{g}}} \left(1 + \frac{1}{4} \sin^2 \frac{\theta_m}{2}\right)$

USING THE BINOMIAL THEOREM, WE WRITE

$$\frac{1}{\sqrt{1 - \sin^2(\theta_m/2) \sin^2\phi}} = [1 - \sin^2(\theta_m/2) \sin^2\phi]^{-1/2} \\ = 1 + \frac{1}{2} \sin^2 \frac{\theta_m}{2} \sin^2\phi + \dots$$

WHERE WE NEGLECT TERMS OF ORDER HIGHER THAN 2  
 SETTING  $\sin^2\phi = \frac{1}{2}(1 - \cos 2\phi)$ , WE HAVE

$$\tau_n = 4\sqrt{\frac{g}{\bar{g}}} \int_0^{2\pi} \left\{ 1 + \frac{1}{2} \sin^2 \frac{\theta_m}{2} \left[ \frac{1}{2}(1 - \cos 2\phi) \right] \right\} d\phi \\ = 4\sqrt{\frac{g}{\bar{g}}} \int_0^{2\pi} \left\{ 1 + \frac{1}{4} \sin^2 \frac{\theta_m}{2} - \frac{1}{4} \sin^2 \frac{\theta_m}{2} \cos 2\phi \right\} d\phi \\ = 4\sqrt{\frac{g}{\bar{g}}} \left[ \phi + \frac{1}{4} \left( \sin^2 \frac{\theta_m}{2} \right) \phi - \frac{1}{8} \sin^2 \frac{\theta_m}{2} \sin 2\phi \right]_0^{2\pi} \\ = 4\sqrt{\frac{g}{\bar{g}}} \left[ \frac{\pi}{2} + \frac{1}{4} \left( \sin^2 \frac{\theta_m}{2} \right) \frac{\pi}{2} + 0 \right]$$

$$\tau_n = 2\pi \sqrt{\frac{g}{\bar{g}}} \left( 1 + \frac{1}{4} \sin^2 \frac{\theta_m}{2} \right)$$

19.34

GIVEN:

$$\tau_n = 2\pi \sqrt{\frac{g}{\bar{g}}} \left( 1 + \frac{1}{4} \sin^2 \frac{\theta_m}{2} \right) \quad (\text{PROB. 19.33})$$

FIND:

AMPLITUDE  $\theta_m$  OF A PENDULUM FOR  
 WHICH THE PERIOD OF A SIMPLE  
 PENDULUM IS  $\frac{1}{2}$  PERCENT LONGER THAN  
 THE PERIOD OF THE SAME PENDULUM FOR  
 SMALL OSCILLATIONS

$$\text{FOR SMALL OSCILLATIONS } (\tau_n)_0 = 2\pi \sqrt{\frac{g}{\bar{g}}}$$

$$\text{WE WANT } \tau_n = 1.005(\tau_n)_0 = 1.005 \cdot 2\pi \sqrt{\frac{g}{\bar{g}}}$$

USING THE FORMULA OF PROB 19.33, WE WRITE

$$\tau_n = (\tau_n)_0 \left( 1 + \frac{1}{4} \sin^2 \frac{\theta_m}{2} \right) = 1.005(\tau_n)_0$$

$$\sin^2 \frac{\theta_m}{2} = 4[1.005 - 1] = 0.02$$

$$\sin \frac{\theta_m}{2} = \sqrt{0.02}$$

$$\frac{\theta_m}{2} = 8.130^\circ$$

$$\theta_m = 16.3^\circ$$

19.35

GIVEN:

DATA OF TABLE 19.1

PENDULUM LENGTH,  $L = 750 \text{ mm}$

FIND:

(a) PERIOD FOR SMALL OSCILLATIONS

(b) PERIOD FOR AMPLITUDE  $\theta_m = 60^\circ$

(c) PERIOD FOR AMPLITUDE  $\theta_m = 90^\circ$

(a)  $T_n = 2\pi \sqrt{\frac{L}{g}}$  (EQ. 19.18 FOR SMALL OSCILLATIONS)

$$T_n = 2\pi \sqrt{\frac{0.750 \text{ m}}{9.81 \text{ m/s}^2}} = 1.7375 \text{ s}$$

$$T_n = 1.7375$$

(b) FOR LARGE OSCILLATIONS (EQ. 19.20)

$$T_n = \left(\frac{2k}{\pi}\right) \left(2\pi \sqrt{\frac{L}{g}}\right) = \frac{2k}{\pi} (1.7375)$$

FOR  $\theta_m = 60^\circ$ ,  $k = 1.686$  (TABLE 19.1)

$$T_n(60^\circ) = \frac{2(1.686)(1.7375)}{\pi} = 1.8645$$

$$T_n(60^\circ) = 1.8645$$

(c) FOR  $\theta_m = 90^\circ$ ,  $k = 1.854$

$$T_n = \frac{2(1.854)(1.7375)}{\pi} = 2.05 \text{ s}$$

19.36

GIVEN:

DATA OF TABLE 19.1

PERIOD = 2 s, AMPLITUDE =  $90^\circ$

FIND:

LENGTH  $L$  OF A SIMPLE PENDULUM (in.)

FOR LARGE OSCILLATIONS (EQ. 19.20)

$$T_n = \left(\frac{2k}{\pi}\right) \left(2\pi \sqrt{\frac{L}{g}}\right) \text{ FOR } \theta_m = 90^\circ$$

$k = 1.854$  (TABLE 19.1)

$$(2 \text{ s}) = \frac{2(1.854)(2)}{\pi} \sqrt{\frac{L}{32.2 \text{ ft/s}^2}}$$

$$L = \frac{(2.5)^2 (32.2 \text{ ft/s}^2)}{[(4)(1.854)]^2} = 2.342 \text{ ft}$$

$$L = 28.1 \text{ in.}$$

19.37

GIVEN:

5-kg ROD AC

SPRING B,  $k = 500 \text{ N/m}$

SPRING C,  $k = 620 \text{ N/m}$

(TENSION OR COMP.)

FIND:

WHEN END C IS

DEPRESSED SLIGHTLY

(a) FREQUENCY OF

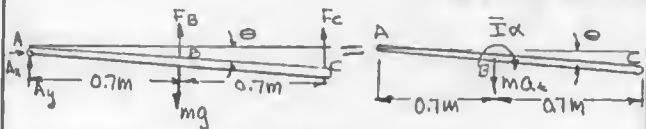
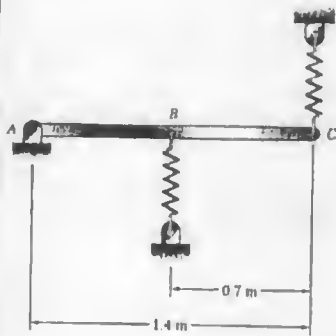
VIBRATION

(b) AMPLITUDE OF

POINT C KNOWING

THAT ITS MAXIMUM

VELOCITY IS  $0.9 \text{ m/s}$



$$F_B = k_B (\chi_B + (\delta_{st})_B) = k_B (0.7\theta + (\delta_{st})_B)$$

$$F_C = k_C (\chi_C + (\delta_{st})_C) = k_C (1.4\theta + (\delta_{st})_C)$$

19.37 CONTINUED

IN MOTION  $\sum M_A = (\sum M_A)_{eff}$

$$(0.7) [k_B (0.7\theta + (\delta_{st})_B) - mg] + 1.4 [k_C (1.4\theta + (\delta_{st})_C) - \bar{I} \alpha - (0.7)(m a_C)] = 0 \quad (1)$$

BUT IN EQUILIBRIUM ( $\theta = 0$ )

$$\sum M_A = 0 = 0.7 [k_B (\delta_{st})_B - mg] + 1.4 [k_C (\delta_{st})_C] \quad (2)$$

SUBSTITUTING (2) INTO (1)

$$\bar{I} \alpha + 0.7 m a_C + (0.7)^2 k_B \theta + (1.4)^2 k_C \theta = 0$$

KINEMATICS ( $\alpha = \ddot{\theta}$ )

$$a_C = 0.7 \alpha = 0.7 \ddot{\theta}$$

$$[\bar{I} + m(0.7)^2] \ddot{\theta} + [(0.7)^2 k_B + (1.4)^2 k_C] \theta = 0$$

$$\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12} (5 \text{ kg})(1.4 \text{ m})^2 = 0.8167 \text{ kg-m}^2$$

$$(0.7)^2 m = (0.49 \text{ m}^2)(5 \text{ kg}) = 2.45 \text{ kg-m}^2$$

$$(0.7)^2 k_B + (1.4)^2 k_C = (0.49 \text{ m}^2)(500 \text{ N/m}) + (1.96 \text{ m}^2)(620 \text{ N/m}) = 245 + 1215.2 = 1460.2 \text{ N-m}$$

$$[0.8167 + 2.45] \ddot{\theta} + 1460.2 \theta = 0$$

$$\ddot{\theta} + \frac{(1460.2 \text{ N-m})}{(3.267 \text{ kg-m}^2)} \theta = 0$$

$$\ddot{\theta} + 447 \theta = 0 \quad \left(\frac{\text{N}}{\text{kg-m}} = \text{s}^{-2}\right)$$

$$\omega_n = \sqrt{447 \text{ s}^{-2}} = 21.14 \text{ RAD/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{21.14}{2\pi} = 3.36 \text{ Hz}$$

$$(b) \theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = (\theta_m)(\omega_n) \cos(\omega_n t + \phi)$$

$$\text{MAXIMUM ANGULAR VELOCITY, } \dot{\theta}_m = \theta_m \omega_n$$

MAXIMUM VELOCITY AT C

$$(\dot{\chi}_C)_m = 1.4 \dot{\theta}_m = (1.4 \text{ m})(\theta_m)(\omega_n)$$

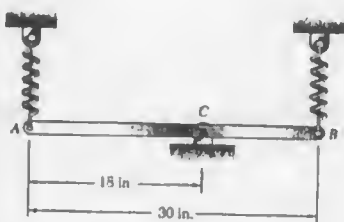
$$\theta_m = \frac{(0.9 \text{ m/s})}{(1.4 \text{ m})(21.14 \text{ RAD/s})} = 0.03041 \text{ RAD} \quad \omega_n = 21.14 \text{ RAD/s}$$

MAXIMUM AMPLITUDE AT C

$$(\chi_C)_m = (1.4 \text{ m})(\theta_m) = (1.4 \text{ m})(0.03041)$$

$$(\chi_C)_m = 0.0426 \text{ m}$$

19.38



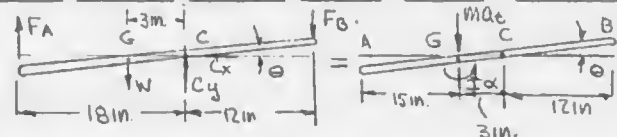
GIVEN:

18-lb ROD  
 $k = 6 \text{ lb/in}$  FOR EACH SPRING  
 END A IS DEPRESSED SLIGHTLY AND RELEASED

FIND:

(a) FREQUENCY

(b) AMPLITUDE OF ANGULAR MOTION KNOWING THAT THE MAXIMUM VELOCITY OF A IS  $22 \text{ in./s}$



$$F_A = k \left[ \left( \frac{18}{12} \text{ ft} \right) \theta + (\delta_{ST})_A \right] = k \left[ (1.5 \text{ ft}) \theta + (\delta_{ST})_A \right]$$

$$F_B = k \left[ \left( \frac{12}{12} \text{ ft} \right) \theta + (\delta_{ST})_B \right] = k \left[ (1 \text{ ft}) \theta + (\delta_{ST})_B \right]$$

(a)  $\sum M_C = (\sum M)_{\text{eff}}$

$$-\frac{18}{12} (k) \left[ (1.5 \text{ ft}) \theta + (\delta_{ST})_A \right] + \frac{3}{12} W - \frac{12}{12} (k) \left[ (1 \text{ ft}) \theta + (\delta_{ST})_B \right] = \bar{I} \ddot{\alpha} + \frac{3}{12} m a_c \quad (1)$$

BUT IN EQUILIBRIUM ( $\theta = 0$ )

$$\sum M_C = 0 = -\frac{18}{12} k (\delta_{ST})_A + \frac{3}{12} W - \frac{12}{12} k (\delta_{ST})_B \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\bar{I} \ddot{\alpha} + (0.25) m a_c + (3.25) k \theta = 0$$

KINEMATICS ( $\alpha = \ddot{\theta}$ )

$$a_c = \frac{3}{12} \ddot{\theta} = 0.25 \ddot{\theta} \quad \bar{I} = \frac{1}{12} W l^2$$

$$\bar{I} + (0.25)^2 m = \frac{1}{12} \frac{18 \text{ lb}}{(32.2 \text{ ft/s}^2)} \left( \frac{30 \text{ ft}}{12} \right)^2 + (0.25 \text{ ft})^2 \left( \frac{18 \text{ lb}}{(32.2 \text{ ft/s}^2)} \right)$$

$$= 0.2912 + 0.03494 = 0.3261 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$(0.3261 \text{ lb} \cdot \text{s}^2/\text{ft}) \ddot{\theta} + (3.25 \text{ ft}^2) \left( 6 \cdot \frac{12 \text{ lb}}{\text{ft}} \right) \theta = 0$$

$$\ddot{\theta} + (717.6 \text{ s}^{-2}) \theta = 0$$

$$\omega_n = \sqrt{717.6} = 26.78 \text{ RAD/S} \quad f_n = \omega_n / 2\pi$$

$$f_n = \frac{26.78}{2\pi} = 4.26 \text{ Hz}$$

(b)  $\theta = \theta_m \sin(\omega_n t + \phi)$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{\text{MAX}} = \theta_m \omega_n \quad (\dot{x}_A)_{\text{MAX}} = \left( \frac{18}{12} \text{ ft} \right) (\dot{\theta})_{\text{MAX}}$$

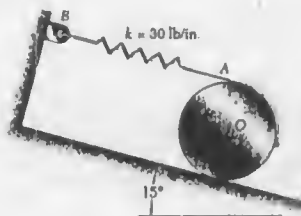
$$(\dot{x}_A)_{\text{MAX}} = (22 \text{ in/s}) / (12 \text{ in/ft}) = 1.833 \text{ ft/s}$$

$$1.833 \text{ ft/s} = (1.5 \text{ ft}) (\theta_m) (26.78 \text{ RAD/S})$$

$$\theta_m = 0.04564 \text{ RAD}$$

$$\theta_m = 2.61^\circ$$

19.39

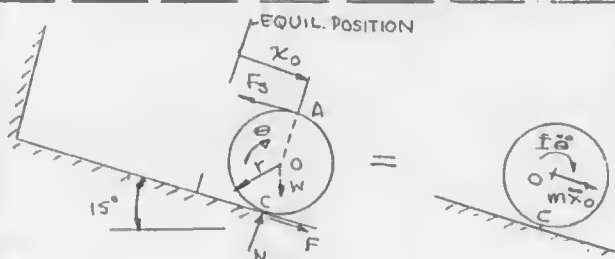


GIVEN:

30-lb CYLINDER  
 ROLLS WITHOUT SLIDING.  
 DATA AS SHOWN.  
 INITIAL DISPLACEMENT = 2 in DOWN

(a) FIND:

(a) PERIOD  
 (b) MAXIMUM ACCELERATION OF C

SPRING DEFLECTION,  $x_A = x_0 + x_{A/O}$ 

$$x_{A/O} = r \theta \quad \theta = x_0 / r$$

$$x_A = 2x_0$$

$$F_s = k (x_A + \delta_s) = k (2x_0 + \delta_{ST})$$

(a)  $\sum M_C = (\sum M)_{\text{eff}}$

$$-2rk(x_0 + \delta_{ST}) + rW \sin 15^\circ = r m \ddot{x}_0 + \bar{I} \ddot{\theta} \quad (1)$$

BUT IN EQUILIBRIUM,  $x_0 = 0$ 

$$\sum M_C = 0 = -2rk \delta_{ST} + rW \sin 15^\circ \quad (2)$$

SUBSTITUTE (2) INTO (1) AND NOTING THAT

$$\theta = x_0 / r, \quad \ddot{\theta} = \ddot{x}_0 / r$$

$$r m \ddot{x}_0 + \bar{I} \frac{\ddot{x}_0}{r} + 4rk x_0 = 0 \quad \bar{I} = \frac{1}{2} m r^2$$

$$\frac{3}{2} m r \ddot{x}_0 + 4rk x_0 = 0$$

$$\ddot{x}_0 + \left( \frac{8}{3} \frac{k}{m} \right) x_0 = 0$$

$$\omega_n = \sqrt{\frac{8}{3} \frac{k}{m}} = \sqrt{\frac{(8)(30 \times 12 \text{ lb/ft})}{(3)(30 \text{ lb})(32.2 \text{ ft/s}^2)}} = 32.1 \text{ s}^{-1}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{32.1} = 0.1957 \text{ s}$$

$$T_n = 0.1957 \text{ s}$$

(b)  $x_0 = (x_0)_m \sin(\omega_n t + \phi)$

$$\text{at } t=0 \quad x_0 = \frac{2}{12} \text{ ft} \quad \dot{x}_0 = 0$$

$$\dot{x}_0 = (x_0)_m \omega_n \cos(\omega_n t + \phi), \quad t=0, \quad 0 = (x_0)_m \omega_n \cos \phi$$

THUS  $\phi = \pi/2$ 

$$t=0 \quad x_0(0) = \frac{1}{6} \text{ ft} = (x_0)_m \sin \phi = (x_0)_m (1)$$

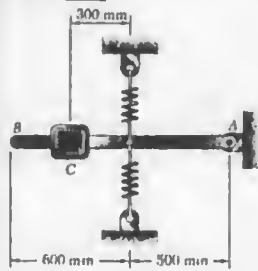
$$(x_0)_m = \frac{1}{6} \text{ ft}$$

$$\ddot{x}_0 = -(x_0)_m \omega_n^2 \sin(\omega_n t + \phi)$$

$$(\ddot{x}_0)_{\text{MAX}} = (\ddot{x}_0)_{\text{MAX}} = -(x_0)_m \omega_n^2 = -\left( \frac{1}{6} \text{ ft} \right) (32.1 \text{ s}^{-1})^2 = -171.7 \text{ ft/s}^2$$

$$(\ddot{x}_0)_{\text{MAX}} = 171.7 \text{ ft/s}^2$$

19.40

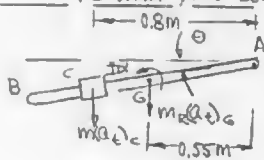
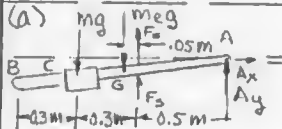


GIVEN:

750-g ROD AB  
 $k = 300 \text{ N/m}$  FOR EACH SPRING.  
 PERIOD  $\tau = 0.4 \text{ s}$

FIND:

- (a) MASS  $m$  OF BLOCK C  
 (b) MAXIMUM VELOCITY OF BLOCK C IF END B IS DEPRESSED 40 mm AND RELEASED



$$\sum M_A = (\sum M_A)_{\text{eff}}$$

$$G^+ 0.8 mg + 0.55 m_R g - (0.5)(2F_s) = \bar{I} \alpha + (0.55) m_R (a_c)_G + (0.8) m (a_c)_C \quad (1)$$

$$F_s = k(0.5\theta + \delta_{st})$$

BUT AT EQUILIBRIUM ( $\theta = 0$ )

$$F_s = k(\delta_{st}) \text{ AND } \sum M_A = 0$$

$$\sum M_A = 0.8 mg + 0.55 m_R g - (0.5)(2) k \delta_{st} = 0 \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\bar{I} \alpha + (0.55) m_R (a_c)_G + (0.8) m (a_c)_C + (0.5)^2 k \theta = 0$$

$$\alpha = \ddot{\theta} \quad (a_c)_G = (0.55)(\ddot{\theta}) \quad (a_c)_C = (0.8)(\ddot{\theta})$$

$$\bar{I} = \frac{1}{12} m_R L^2 = \frac{1}{12} (0.750 \text{ kg})(1.1 \text{ m})^2$$

$$\bar{I} = 0.07563 \text{ kg} \cdot \text{m}^2$$

$$(0.07563 + (0.55)^2(0.750) + (0.8)^2 m) \ddot{\theta} + (0.5)^2 (2)(300) \theta = 0$$

$$\ddot{\theta} + \frac{(150 \text{ N} \cdot \text{m})}{(0.3025 \text{ kg} \cdot \text{m}^2 + (0.64 \text{ m}^2) m)} \theta = 0$$

$$\omega_n^2 = (2\pi f_n)^2 = \frac{(2\pi)^2}{\tau^2} = \frac{(2\pi)^2}{(0.4 \text{ s})^2} = 246.7 \text{ (s}^{-2}\text{)}$$

$$\omega_n^2 = \frac{150 \text{ N} \cdot \text{m}}{(0.3025 \text{ kg} \cdot \text{m}^2 + (0.64 \text{ m}^2) m)} = 246.7 \text{ (s}^{-2}\text{)}$$

$$150 \text{ N} = 246.7 \text{ (s}^{-2}\text{)} [0.3025 \text{ kg} \cdot \text{m}^2 + (0.64 \text{ m}^2) m]$$

$$m = \frac{(150 - 74.64) \text{ (N)}}{(246.7)(0.64) \text{ (m/s}^2\text{)}} = 0.477 \text{ kg}$$

$$(b) (\omega_c)_{\text{MAX}} = (0.8)(\ddot{\theta})_{\text{MAX}} \quad \theta_m = \frac{y_B}{1.1} = \frac{0.04}{1.1}$$

$$\theta = \theta_m \sin(\omega_n t + \phi) \quad \theta_m = 0.03636 \text{ rad}$$

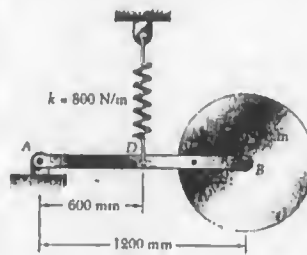
$$\ddot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\ddot{\theta}_{\text{MAX}} = \theta_m \omega_n = (0.03636)(246.7) = 0.5712 \text{ RAD/s}$$

$$(\omega_c)_{\text{MAX}} = (0.8 \text{ m})(0.5712 \text{ s}^{-1}) = 0.4569 \text{ m/s}$$

$$(\omega_c)_{\text{MAX}} = 457 \frac{\text{mm}}{\text{s}}$$

19.41

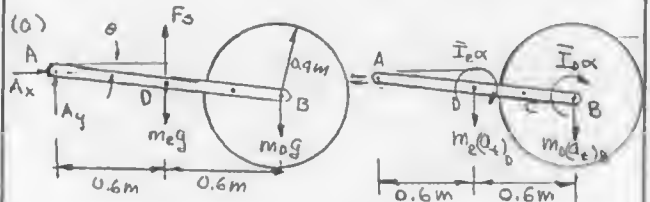


GIVEN:

8 kg ROD AB  
 12 kg DISK OF RADIUS 0.4 m  
 DISK IS RIGIDLY ATTACHED TO THE ROD AT B AND C.  
 POINT B MOVED DOWN 0.025 m AND RELEASED

FIND:

- (a) THE PERIOD  
 (b) MAXIMUM VELOCITY OF B



$$\sum M_A = (\sum M_A)_{\text{eff}} \quad F_s = k(0.6\theta + \delta_{st})$$

$$G^+ 0.6(m_R g - F_s) + 1.2 m_D g = (\bar{I}_A + \bar{I}_B) \alpha + 0.6(m_R)(a_c)_D + 1.2(m_D)(a_c)_B \quad (1)$$

AT EQUILIBRIUM ( $\theta = 0$ )  $F_s = k \delta_{st}$ 

$$\sum M_A = 0 = 0.6(m_R g - k(\delta_{st})) + 1.2 m_D g \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$(\bar{I}_A + \bar{I}_B) \alpha + 0.6 m_R (a_c)_D + 1.2 m_D (a_c)_B + (0.6)^2 k \theta = 0$$

$$= \ddot{\theta} \quad (a_c)_D = 0.6 \ddot{\theta} \quad (a_c)_B = 1.2 \ddot{\theta}$$

$$\bar{I}_R = \frac{1}{12} m_R L^2 = \frac{1}{12} (8)(1.2)^2 = 0.960 \text{ kg} \cdot \text{m}$$

$$\bar{I}_D = \frac{1}{2} m_D R^2 = \frac{1}{2} (12)(0.4)^2 = 0.960 \text{ kg} \cdot \text{m}$$

$$[(0.960 + 0.960 + (0.6)^2(8) + (1.2)^2(12)] \ddot{\theta} + (0.6)^2(800) \theta = 0$$

$$\ddot{\theta} + \frac{288 \text{ N} \cdot \text{m}}{(22.08 \text{ kg} \cdot \text{m}^2)} \theta = 0 \quad \omega_n = \sqrt{\frac{288}{22.08}} = 3.612 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 1.740 \text{ s}$$

$$(b) (\omega_B)_{\text{MAX}} = (1.2)(\ddot{\theta})_{\text{MAX}} \quad \theta_m = \frac{y_B}{1.2} = \frac{0.025}{1.2}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\ddot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

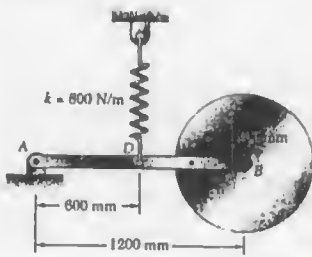
$$\ddot{\theta}_{\text{MAX}} = \theta_m \omega_n = (0.02083)(3.612) = 0.07524 \text{ rad/s}$$

$$(\omega_B)_{\text{MAX}} = (1.2)(\ddot{\theta}_{\text{MAX}}) = (1.2 \text{ m})(0.07524 \text{ rad/s})$$

$$(\omega_B)_{\text{MAX}} = 90.29 \text{ m/s}$$

$$(\omega_B)_{\text{MAX}} = 90.3 \text{ m/s}$$

19.42

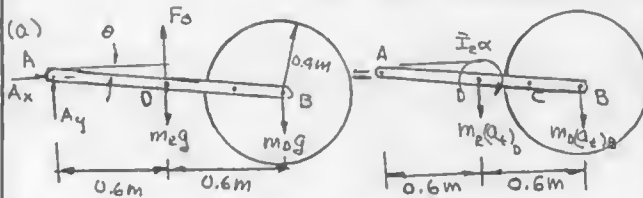


GIVEN:

8 kg ROD AB  
12 kg DISK OF  
RADIUS 94 mm  
PIN C REMOVED  
AND DISK CAN  
ROTATE FREELY  
ABOUT PIN B.  
POINT B MOVED  
DOWN 0.025 m  
AND RELEASED

FIND:

- (a) PERIOD  
(b) MAX VELOCITY OF B



NOTE: THIS PROBLEM IS THE SAME AS PROB 19.41 EXCEPT THAT THE DISK DOES NOT ROTATE, SO THAT THE EFFECTIVE MOMENT  $I_D \alpha = 0$ .  
 $\sum M_A = (\sum M)_{\text{eff}} \quad F_s = k(0.60 + \delta_{ST})$

$$\uparrow + (0.6)(m_2 g - F_s) + 1.2 m_D g = \bar{I}_D \alpha + (0.6)(m_2)(a_t)_D + 1.2(m_D)(a_t)_B \quad (1)$$

AT EQUILIBRIUM ( $\theta = 0$ )  $F_s = k \delta_{ST}$

$$\sum M_A = 0 = 0.6(m_2 g - \delta_{ST}) + 1.2 m_D g \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$-\bar{I}_D \alpha + 0.6 m_2 (a_t)_D + 1.2 m_D (a_t)_B + (0.6)k\theta = 0$$

$$\alpha = \ddot{\theta} \quad (a_t)_D = 0.6 \ddot{\theta} \quad (a_t)_B = 1.2 \ddot{\theta}$$

$$I_D = \frac{1}{12} m_2 L^2 = \frac{1}{12} (8)(1.2)^2 = 0.960 \text{ kg} \cdot \text{m}^2$$

$$[0.960 + (0.6)^2(8) + (1.2)^2(12)]\ddot{\theta} + (0.6)(800)\theta = 0$$

$$\ddot{\theta} + \frac{(288 \text{ N} \cdot \text{m})}{21.12 \text{ kg} \cdot \text{m}^2} \theta = 0 \quad \omega_n = \sqrt{\frac{288}{21.12}} = 3.693 \text{ s}^{-1}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.693} = 1.701 \text{ s}$$

$$(b) (v_B)_{\text{MAX}} = (1.2)(\dot{\theta})_{\text{MAX}} \quad \theta_m = \frac{y_B}{1.2} = \frac{0.025}{1.2} = 0.02083 \text{ rad}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

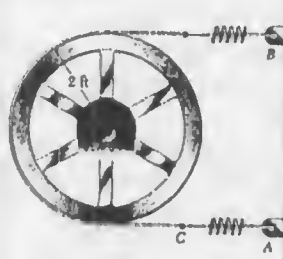
$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{\text{MAX}} = \theta_m \omega_n = (0.02083)(3.693) = 0.07694 \text{ rad/s}$$

$$(v_B)_{\text{MAX}} = (1.2)(\dot{\theta}_{\text{MAX}}) = (1.2)(0.07694) = 0.09233 \text{ m/s}$$

$$(v_B)_{\text{MAX}} = 92.3 \text{ mm/s}$$

19.43

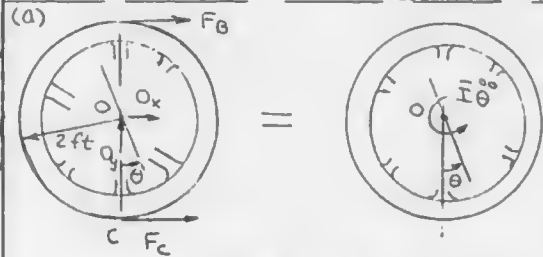


GIVEN:

600-lb FLYWHEEL OF  
RADIUS OF GYRATION = 20 in  
 $k = 75 \text{ lb/in}$  FOR EACH  
SPRING.  
POINT C IS PULLED  
TO THE RIGHT 1 in.  
AND RELEASED

FIND:

- (a) PERIOD OF VIBRATION  
(b) MAXIMUM ANGULAR  
VELOCITY OF THE FLYWHEEL



$$\sum M_B = (\sum M)_{\text{eff}}$$

$$F_C = k(\delta_{ST})_C - 2\theta \quad F_B = k(2\theta + (\delta_{ST})_B)$$

$$\uparrow + 2F_C - 2F_B = \bar{I} \ddot{\theta}$$

$$2k[(\delta_{ST})_C - 2\theta] - 2k[(\delta_{ST})_B + 2\theta] = \bar{I} \ddot{\theta} \quad (1)$$

AT EQUILIBRIUM ( $\theta = 0$ )  $F_B = k(\delta_{ST})_B$ ,  $F_C = k(\delta_{ST})_C$

$$\sum M_B = 0 = 2k(\delta_{ST})_C - 2k(\delta_{ST})_B \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\bar{I} \ddot{\theta} + 8k\theta = 0$$

$$\bar{I} = m \bar{k}^2 = \frac{(600 \text{ lb})(20/12 \text{ ft})^2}{(32.2 \text{ ft/s}^2)} = 51.76 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$k = (8 \text{ ft}) (75 \times 12 \text{ lb/ft}) = 7200 \text{ lb} \cdot \text{ft}$$

$$\omega_n^2 = \frac{8k}{\bar{I}} = \frac{7200 \text{ lb} \cdot \text{ft}}{51.76 \text{ lb} \cdot \text{ft} \cdot \text{s}^2} = 139.1 \text{ s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{139.1}} = 0.533 \text{ s}$$

$$(b) \theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{\text{MAX}} = \theta_m \omega_n$$

$$\omega_n = \sqrt{139.1} = 11.79 \text{ rad/s}$$

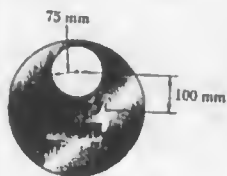
$$\theta_m = x_C / r = \frac{1/12}{2}$$

$$\theta_m = 0.04167 \text{ rad}$$

$$\dot{\theta}_{\text{MAX}} = (0.04167)(11.79) = 0.491 \text{ rad/s}$$

$$\omega = 0.491 \text{ rad/s}$$

19.44

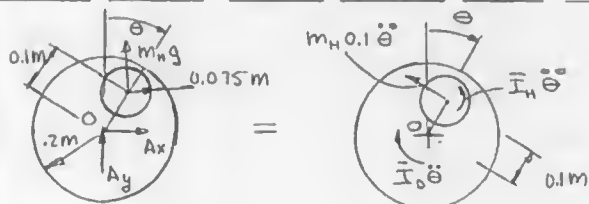


GIVEN:

DISK ATTACHED TO A  
FRICTIONLESS PIN AT ITS  
GEOMETRIC CENTER  
AS SHOWN

FIND:

- (a.) PERIOD OF SMALL  
OSCILLATIONS  
(b.) LENGTH OF A SIMPLE  
PENDULUM OF THE  
SAME PERIOD



$$\sum M_O = (\sum M_O)_{\text{eff}}$$

$$(+ \curvearrowleft) -m_h g (0.1) \sin \theta = \bar{I}_O \ddot{\theta} - \bar{I}_H \ddot{\theta} - (0.1)^2 m_h \ddot{\theta}$$

$$m_D = 8t\pi R^2 = (8t\pi)(.2)^2 = (0.04)\pi 8t$$

$$m_H = 8t\pi r^2 = (8t\pi)(0.075)^2 = (0.005625)\pi 8t$$

$$I_D = \frac{1}{2} m_D R^2 = \frac{1}{2} (0.04\pi 8t)(.2)^2 = 800 \times 10^{-6} \pi t$$

$$I_H = \frac{1}{2} m_H r^2 = \frac{1}{2} (0.005625\pi 8t)(0.075)^2 = 15.82 \times 10^{-6} \pi t$$

SMALL ANGLES  $\sin \theta \approx \theta$ 

$$[800 \times 10^{-6} \pi - 15.82 \times 10^{-6} \pi - (0.1)^2 (0.005625\pi)] t \ddot{\theta} + (0.005625\pi 8t)(9.81)(.1) \theta = 0$$

$$727.9 \times 10^{-6} \ddot{\theta} + 5.518 \times 10^{-3} \theta = 0$$

$$\omega_n^2 = \frac{5.518 \times 10^{-3}}{727.9 \times 10^{-6}} = 7.581$$

$$\omega_n = 2.753 \text{ RAD/S}$$

$$\tau_n = \frac{2\pi}{\omega_n} \quad \tau_n = \frac{2\pi}{2.753} = 2.28 \text{ s}$$

(b) PERIOD OF A SIMPLE PENDULUM

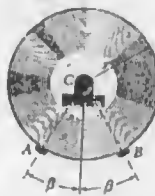
$$\tau_n = 2\pi \sqrt{l/g}$$

$$l = (\tau_n / 2\pi)^2 g$$

$$l = [(2.28) / 2\pi]^2 (9.81 \text{ m/s}^2)$$

$$l = 1.294 \text{ m}$$

19.45

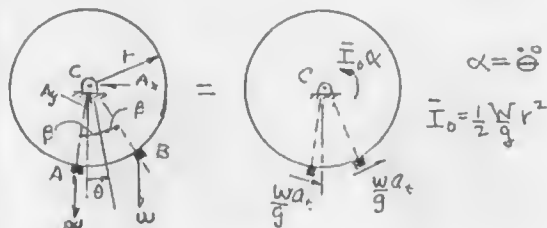


GIVEN:

WEIGHTS  $W$  AT  $A$  AND  $B$ , AND DISK  $W$   
FOR  $\beta = 0$ , PERIOD  $= \tau_0$

FIND:

ANGLE  $\beta$  FOR A PERIOD OF  $2\tau_0$



$$\sum M_C = (\sum M_C)_{\text{eff}}$$

$$(+ \curvearrowleft) W r \sin(\beta - \theta) - W r \sin(\beta + \theta) = \frac{2W}{g} r a_t + \bar{I} \alpha$$

$$r[\sin(\beta - \theta) - \sin(\beta + \theta)] = -2W r \sin \theta \cos \beta$$

$$\sin \theta \approx \theta \quad a_t = r \ddot{\theta}$$

$$\left( \frac{2W r^2}{g} + \frac{W}{2g} r^2 \right) \ddot{\theta} + (2W r \cos \beta) \theta = 0$$

$$\omega_n = \sqrt{\frac{2W g \cos \beta}{(2W + W/2) r}} = \sqrt{\frac{4g \cos \beta}{4 + W/W}} \quad (1)$$

$$\beta = 0 \quad \tau_0 = \frac{2\pi}{\omega_0} = 2\pi / \sqrt{\frac{4g}{4 + W/W}}$$

$$\tau_n = 2\pi / \sqrt{\frac{\cos \beta}{(4 + W/W) r}} = 2\tau_0 = 4\pi / \sqrt{\frac{4g}{4 + W/W}}$$

$$\cos \beta = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\beta = 75.5^\circ$$

19.46

REFER TO FIGURE IN PROB 19.45  
ABOVE

GIVEN:

$$w = 0.1 \text{ lb}, W = 3 \text{ lb}, r = 4 \text{ in.}, \beta = 60^\circ$$

FIND:

FREQUENCY OF SMALL OSCILLATIONS

FROM DERIVATION IN PROB 19.45 (EQ. 1)

$$\omega_n = \sqrt{\frac{24g \cos \beta}{(24 + W/w) r}}$$

$$\omega_n = \sqrt{\frac{(4)(32.2 \text{ ft/s}^2) \cos 60^\circ}{(4 + 3/0.1)(4/12)}} = 2.384 \text{ r/s}$$

$$f_n = \omega_n / 2\pi = 2.384 / 2\pi$$

$$f_n = 0.379 \text{ Hz}$$

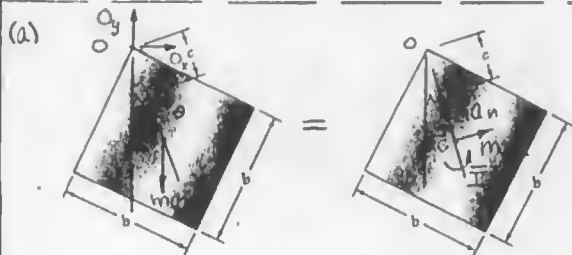
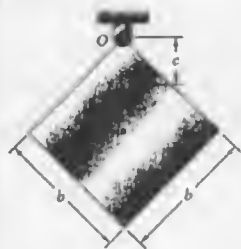
19.47

GIVEN:

SQUARE PLATE,  $b = 0.3\text{m}$ 

FIND:

- (a) PERIOD OF SMALL OSCILLATIONS ABOUT O  
(b) DISTANCE C FROM O TO A POINT A FROM WHICH THE PLATE SHOULD BE SUSPENDED TO MINIMIZE THE PERIOD



$$\Sigma M_O = (\Sigma M_O)_{\text{eff}} \quad \alpha = \ddot{\theta} \quad a_t = (OG)(\alpha)$$

$$\bar{I} = \frac{1}{6} m b^2$$

$$OG = b\sqrt{2}/2$$

$$a_t = (b\sqrt{2}/2) \ddot{\theta}$$

$$(b\sqrt{2}/2) m (b\sqrt{2}/2) \ddot{\theta} + \frac{1}{6} m b^2 \ddot{\theta} + (b\sqrt{2}/2) mg \theta = 0$$

$$(b\sqrt{2}/2) m (b\sqrt{2}/2) \ddot{\theta} + \frac{1}{6} m b^2 \ddot{\theta} + (b\sqrt{2}/2) mg \theta = 0$$

$$(b\sqrt{2}/2) m \ddot{\theta} + \frac{1}{6} m b^2 \ddot{\theta} + (b\sqrt{2}/2) mg \theta = 0$$

$$\ddot{\theta} + \frac{(\sqrt{2}/2) g}{(b/3)b} \theta = 0 \quad b = 0.3\text{m}$$

$$(T_n)_0 = \frac{2\pi}{(\omega_n)_0} = 2\pi \sqrt{\frac{(2/3)b}{(\sqrt{2}/2)g}} = 2\pi \sqrt{\frac{4(0.3\text{m})}{(3\sqrt{2})(9.81\text{m/s}^2)}}$$

$$(T_n)_0 = 1.067\text{s}$$

(b) SUSPENDED ABOUT A

$$\Sigma M_A = (\Sigma M_A)_{\text{eff}} \quad a_t = (OG-c)\alpha$$

$$(OG-c)(\sin\theta)(mg) = -(OG-c)m a_t - \bar{I} \alpha$$

$$((b\sqrt{2}/2 - c)^2 + \frac{1}{6} b^2) m \ddot{\theta} + (OG-c) mg \theta = 0$$

$$(T_n)_A^2 = \frac{(2\pi)^2}{\omega_n^2} = \frac{4\pi^2 [(b\sqrt{2}/2 - c)^2 + b^2/6]}{(b\sqrt{2}/2 - c)^2}$$

FOR MINIMUM PERIOD  $\frac{d(T_n)_A}{dc} = 0$ 

$$0 = 2(b\sqrt{2}/2 - c)(-1)(b\sqrt{2}/2 - c) - (-1)[(b\sqrt{2}/2 - c)^2 + b^2/6]$$

$$(b\sqrt{2}/2 - c)^2 + b^2/6 = 0 \quad b = 0.3\text{m}$$

$$b\sqrt{2}/2 - c = \frac{b}{\sqrt{6}} \quad c = 0.3 \left[ \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{6}} \right] = 0.08966\text{m}$$

$$c = 89.7\text{mm}$$

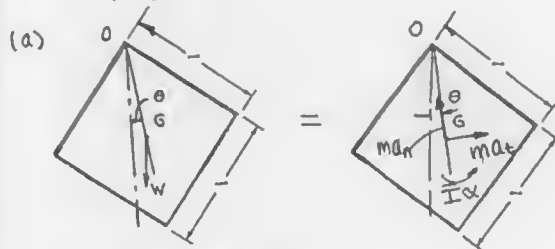
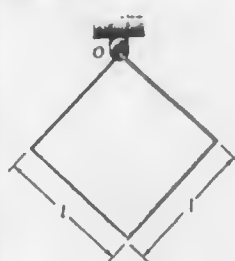
19.48

GIVEN:

THIN WIRE,  $\ell = 1.2\text{ft}$ 

FIND:

- (a) PERIOD ABOUT O  
(b) PERIOD ABOUT A POINT AT THE MIDPOINT OF ONE OF THE SIDES



M = MASS OF THE FRAME

$$\Sigma M_O = (\Sigma M_O)_{\text{eff}} \quad \alpha = \ddot{\theta} \quad a_t = (OG)(\alpha)$$

$$OG = \ell\sqrt{2}/2$$

$$a_t = (\ell\sqrt{2}/2) \ddot{\theta}$$

$$\ell \quad G \quad \ell \quad m/4$$

$$\text{FOR ONE LEG} \quad (I_G)_1 = I_G + (m/4)(\ell/2)^2$$

$$I_G = \frac{1}{12} m \ell^2$$

$$(I_G)_1 = \frac{m\ell^2}{4} \left[ \frac{1}{12} + \left(\frac{1}{2}\right)^2 \right] = \frac{m\ell^2}{4} \left( \frac{1}{3} \right)$$

FOR COMPLETE WIRE FRAME

$$\bar{I} = 4(I_G)_1 = (4) \frac{m\ell^2}{4} \left( \frac{1}{3} \right) = \frac{1}{3} m \ell^2$$

$$-mg \ell \frac{\sqrt{2}}{2} \sin\theta = \bar{I} \alpha + (m/4) \ell \frac{\sqrt{2}}{2} \sin\theta$$

$$\left( \frac{1}{3} + \frac{1}{4} \right) m \ell^2 \ddot{\theta} + mg \ell \frac{\sqrt{2}}{2} \theta = 0$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{(\sqrt{2}/2)g}{(5/12)\ell}}} = 2\pi \sqrt{\frac{5(1.2\text{ft})}{(3\sqrt{2})(32.2\text{ft/s}^2)}}$$

$$T_n = 1.317\text{s}$$

(b) FOR FRAME SUSPENDED FROM MIDPOINT



$$OG = \frac{\ell}{2}$$

$$\bar{I} = (4) \frac{m\ell^2}{4} \left[ \frac{1}{12} + \left(\frac{1}{2}\right)^2 \right] = \frac{1}{3} m \ell^2$$

$$-mg \frac{\ell}{2} \sin\theta = \bar{I} \ddot{\theta} + (m/4) \frac{\ell}{2} \ddot{\theta} = \left( \frac{1}{3} + \frac{1}{4} \right) m \ell^2 \ddot{\theta}$$

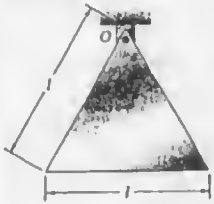
$$\frac{1}{12} m \ell^2 \ddot{\theta} + mg \frac{\ell}{2} \theta = 0$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{g}{\frac{1}{12}\ell}}} = 2\pi \sqrt{\frac{(\ell/6)(1.2\text{ft})}{(32.2\text{ft/s}^2)}} = 1.310\text{s}$$

$$T_n = 1.310\text{s}$$



19.49

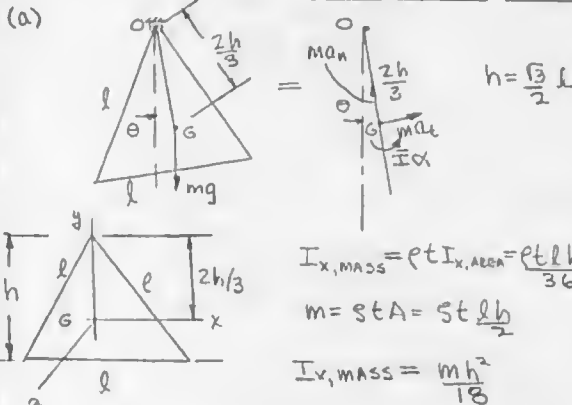


GIVEN:

UNIFORM EQUILATERAL  
TRIANGLE OF SIDE  $l = 0.3 \text{ m}$

FIND:

- (a) PERIOD IF PLATE IS  
SUSPENDED FROM ONE  
OF ITS VERTICES  
(b) PERIOD IF PLATE IS  
SUSPENDED FROM THE  
MIDPOINT OF ONE OF ITS  
SIDES



$$I_{x, \text{mass}} = \rho t I_{x, \text{area}} = \frac{\rho t l h^3}{36}$$

$$m = \rho t A = \rho t \frac{lh}{2}$$

$$I_{x, \text{mass}} = \frac{m h^2}{18}$$

$$I_{y, \text{mass}} = \rho t I_{y, \text{area}} \quad I_{y, \text{area}} = \frac{h l^3}{48}$$

$$I_{y, \text{mass}} = \frac{m l^2}{24}$$

$$I_g = \bar{I} = I_x + I_y = \frac{m h^2}{18} + \frac{m l^2}{24}$$

$$h = l \sqrt{3}/2 \quad \bar{I} = m l^2 \left[ \frac{3/4}{18} + \frac{1}{24} \right] = m l^2 / 12$$

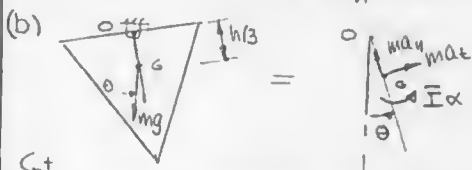
$$(a) \quad \alpha = \ddot{\theta} \quad a_t = \frac{\sqrt{3}}{3} l \ddot{\theta} \quad \sin \theta \approx \theta$$

$$\sum M_o = (\sum M_o)_{\text{eff}} \quad -m g \frac{\sqrt{3}}{3} l \sin \theta = \bar{I} \alpha + \frac{\sqrt{3}}{3} l m a_t$$

$$\left( \frac{1}{12} + \frac{1}{3} \right) m l^2 \ddot{\theta} + m g \frac{\sqrt{3}}{3} l \theta = 0$$

$$\omega_n^2 = \frac{\sqrt{3}/3 \cdot g}{5/12 \cdot l} = \frac{(\sqrt{3})(4)}{5} \left( \frac{9.81 \text{ m/s}^2}{0.3 \text{ m}} \right) = 45.31 \text{ s}^{-2}$$

$$\omega_n = 6.731 \text{ r/s} \quad \tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{6.731} = 0.933 \text{ s}$$



$$\sum M_o = (\sum M_o)_{\text{eff}} \quad -m g h/3 \sin \theta = \bar{I} \ddot{\theta} + m (h/3) \ddot{\theta}$$

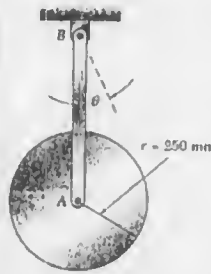
$$h = l \sqrt{3}/2 \quad \bar{I} = m l^2 / 12 \quad \left( \frac{1}{12} + \left( \frac{\sqrt{3}}{2} \right)^2 \right) m l^2 \ddot{\theta} + m g \frac{\sqrt{3}}{3} l \theta = 0$$

$$\frac{1}{6} \ddot{\theta} + \frac{\sqrt{3}}{6} \frac{g}{l} \theta = 0$$

$$\omega_n^2 = \frac{\sqrt{3} g}{l} = \frac{(\sqrt{3}) 9.81 \text{ m/s}^2}{0.3 \text{ m}} = 56.63 \text{ s}^{-2} \quad \omega_n = 7.5258 \text{ r/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{7.526} = 0.835 \text{ s}$$

19.50

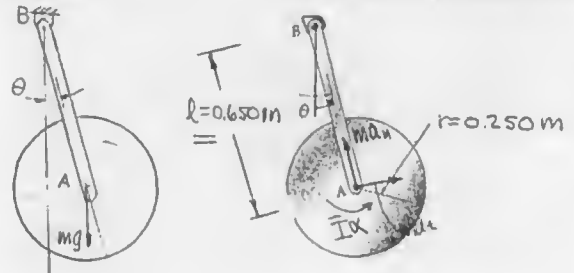


GIVEN:

ROD AB OF NEGLIGIBLE  
MASS ATTACHED TO A DISK  
OF MASS  $m$ .  $AB = l = 0.650 \text{ m}$   
 $r = 0.250 \text{ m}$

FIND:

- THE PERIOD OF SMALL  
OSCILLATIONS IF  
(a) THE DISK IS FREE TO  
ROTATE IN A BEARING AT A  
(b) THE DISK IS RIVETED AT A



$$I = \frac{1}{2} m r^2 = \frac{1}{2} (0.250)^2 m = \frac{m}{32}$$

$$l_t = l \alpha = 0.650 \alpha \quad \alpha = \ddot{\theta}$$

- (a) THE DISK IS FREE TO ROTATE AND IS IN  
CURVILINEAR TRANSLATION  
THUS  $\bar{I} \alpha = 0$

$$\sum M_o = (\sum M_o)_{\text{eff}}$$

$$\downarrow -m g l \sin \theta = l m a_t \quad \sin \theta \approx \theta$$

$$m l^2 \ddot{\theta} + m g l \theta = 0$$

$$\omega_n^2 = \frac{g}{l} = \frac{9.81 \text{ m/s}^2}{0.650 \text{ m}} = 15.092$$

$$\omega_n = 3.885 \quad \tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.885} = 1.617 \text{ s}$$

- (b) WHEN THE DISK IS RIVETED AT A, IT ROTATES  
AT AN ANGULAR ACCELERATION  $\alpha$

$$\sum M_o = (\sum M_o)_{\text{eff}}$$

$$\downarrow -m g l \sin \theta = \bar{I} \alpha + l m a_t \quad \bar{I} = \frac{1}{2} m r^2$$

$$\left( \frac{1}{2} m r^2 + m l^2 \right) \ddot{\theta} + m g l \theta = 0$$

$$\omega_n^2 = \frac{g l}{(r^2/2 + l^2)} = \frac{(9.81 \text{ m/s}^2)(0.650 \text{ m})}{[(0.250^2)/2 + (0.650^2)]} = 14.053 \text{ s}^{-2}$$

$$\omega_n = 3.749 \text{ r/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.749} = 1.676 \text{ s}$$

19.51

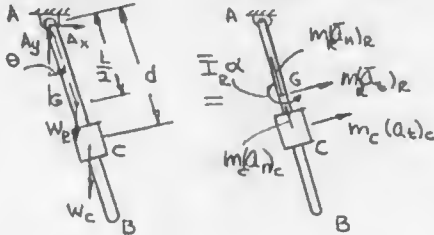
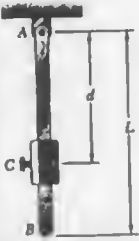
GIVEN:

COLLAR C WEIGHT,  $W_C = 2 \text{ lb}$   
 ROD AB WEIGHT,  $W_R = 6 \text{ lb}$ ,  $L = 3 \text{ ft}$

FIND:

PERIOD OF SMALL OSCILLATIONS  
 WHEN,

- (a)  $d = 3 \text{ ft}$   
 (b)  $d = 2 \text{ ft}$



$$\sum M_A = (\sum M_A)_{\text{eff}}$$

$$-W_R \frac{L}{2} \sin \theta - W_C d \sin \theta = \bar{I}_R \alpha + m_R \frac{L}{2} (\ddot{\theta})_R + m_C d (\ddot{\theta})_C$$

$$\sin \theta \approx \theta \quad \alpha = \ddot{\theta}, (\ddot{\theta})_R = \frac{L}{2} \alpha = \frac{L}{2} \ddot{\theta}, (\ddot{\theta})_C = d \ddot{\theta}$$

$$(\bar{I}_R + m_R (\frac{L}{2})^2 + m_C d^2) \ddot{\theta} + (m_R g \frac{L}{2} + m_C g d) \theta = 0$$

$$\bar{I}_R = \frac{1}{12} m_R L^2 \quad \bar{I}_R + m_R (\frac{L}{2})^2 = \frac{m_R L^2}{3}$$

$$(m_R L^2/3 + m_C d^2) \ddot{\theta} + (m_R g \frac{L}{2} + m_C g d) \theta = 0$$

$$\ddot{\theta} + \frac{(L/2 + \frac{m_C d}{m_R}) g}{(L^2/3 + \frac{m_C d^2}{m_R})} \theta = 0$$

$$\frac{m_C}{m_R} = \frac{W_C}{W_R} = \frac{2}{6} \quad L = 3 \text{ ft}$$

$$\ddot{\theta} + \frac{(\frac{3}{2} + \frac{1}{3} d) g}{(3 + \frac{1}{3} d^2)} \theta = 0$$

$$\tau_n = 2\pi / \omega_n = 2\pi \sqrt{\frac{(3 + d^2/3)}{(\frac{3}{2} + d/3)g}}$$

(a)  $d = 3 \text{ ft}$ 

$$\tau_n = 2\pi \sqrt{\frac{(3+3)}{(\frac{3}{2}+1)(32.2)}} = 1.715 \text{ s.}$$

(b)  $d = 2 \text{ ft}$ 

$$\tau_n = 2\pi \sqrt{\frac{(3+4/3)}{(3/2+2/3)(32.2)}} = 1.566 \text{ s}$$

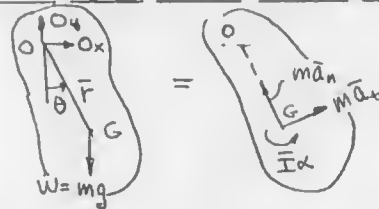
19.52

GIVEN:

COMPOUND PENDULUM WHICH  
 OSCILLATES ABOUT O  
 $\bar{k}$  = CENTROIDAL RADIUS OF GYRATION  
 $GA = \bar{k}^2 / F$

SHOW THAT:

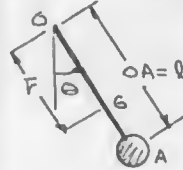
PERIOD EQUALS THE PERIOD  
 OF A SIMPLE PENDULUM  
 OF LENGTH OA.



$$\sum \tau_{M_O} = \sum (\tau_{M_O})_{\text{eff}}: -W \bar{r} \sin \theta = \bar{I} \alpha + m \bar{r}^2 \ddot{\theta}$$

$$-mg \bar{r} \sin \theta = m \bar{k}^2 \ddot{\theta} + m \bar{r}^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{g \bar{r}}{\bar{r}^2 + \bar{k}^2} \sin \theta = 0 \quad (1)$$

FOR A SIMPLE PENDULUM OF LENGTH  $OA = l$ 

$$\ddot{\theta} + \frac{g}{l} \theta = 0 \quad (2)$$

COMPARING EQUATIONS (1)  
AND (2)

$$l = \frac{\bar{r}^2 + \bar{k}^2}{\bar{r}}$$

$$GA = l - \bar{r} = \bar{k}^2 / \bar{r} \quad (\text{QED})$$

19.53

GIVEN:

COMPOUND PENDULUM AS IN  
 PROB. 19.52 SHOWN ABOVE

SHOW THAT:

SMALLEST PERIOD OF OSCILLATION OCCURS  
 WHEN  $F = \bar{k}$

SEE SOLUTION TO PROB. 19.52 FOR DERIVATION OF

$$\ddot{\theta} + \frac{g \bar{r}}{\bar{r}^2 + \bar{k}^2} \sin \theta = 0$$

FOR SMALL OSCILLATIONS  $\sin \theta \approx \theta$  AND

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g \bar{r}}} = \frac{2\pi}{\sqrt{g}} \sqrt{\bar{r} + \frac{\bar{k}^2}{\bar{r}}}$$

FOR SMALLEST  $\tau_n$  WE MUST HAVE  $\bar{r} + \frac{\bar{k}^2}{\bar{r}}$   
A MINIMUM

$$\frac{d(\bar{r} + \frac{\bar{k}^2}{\bar{r}})}{d\bar{r}} = 1 - \frac{\bar{k}^2}{\bar{r}^2} = 0$$

$$\bar{r}^2 = \bar{k}^2$$

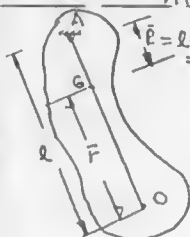
$$\bar{r} = \bar{k} \quad (\text{QED})$$

19.54

GIVEN:

COMPOUND PENDULUM OF PROB. 19.52  
SUSPENDED FROM A

SHOW THAT:

PERIOD IS THE SAME AS BEFORE AND  
THE NEW CENTER OF OSCILLATION IS  
AT O.

SAME DERIVATION AS IN  
PROB. 19.52 WITH  $\bar{F}$   
REPLACED BY  $\bar{R}$ . THUS,  
 $\bar{\Theta} + \frac{g\bar{R}}{\bar{R}^2 + \bar{k}} \Theta = 0$

LENGTH OF THE EQUIVALENT  
SIMPLE PENDULUM IS  
 $L = \frac{\bar{R}^2 + \bar{k}}{\bar{R}} = \bar{R} + \frac{\bar{k}}{\bar{R}}$   
 $L = (l - \bar{r}) + \frac{\bar{k}}{\bar{k}^2/\bar{F}} = l$

THUS THE LENGTH OF THE EQUIVALENT SIMPLE PENDULUM  
IS THE SAME AS IN PROB. 19.52. IT FOLLOWS THAT  
THE PERIOD IS THE SAME AND THAT THE NEW CENTER OF  
OSCILLATION IS AT O (Q.E.D.)

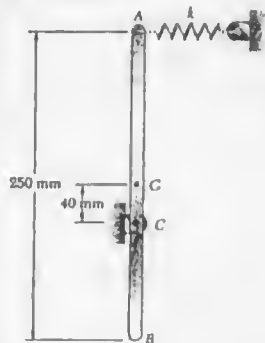
19.55

GIVEN:

8-kg BAR AB  
 $k = 500 \text{ N/m}$ 

FIND:

- (a) FREQUENCY OF  
SMALL OSCILLATIONS  
(b) SMALLEST  $k$  FOR  
WHICH OSCILLATIONS  
WILL OCCUR



$$F_s = (0.165\theta)k$$

$$\bar{I} = \frac{1}{12} m k^2 = \frac{1}{12} (8) (0.25)^2$$

$$\bar{I} = 0.04167 \text{ kg}\cdot\text{m}^2$$

$$\bar{I} \alpha = \bar{I} \ddot{\Theta}$$

$$\alpha = \ddot{\Theta}$$

$$\alpha_L = 0.04 \ddot{\Theta}$$

$$= 0.04 \ddot{\Theta}$$

$$\sin \Theta \approx \Theta$$

$$\Sigma M_C = \Sigma (M_C)_{\text{eff}}$$

$$-(0.165)^2 k \Theta + 0.04 mg \Theta = \bar{I} \ddot{\Theta} + (0.04)^2 m \ddot{\Theta}$$

$$(0.04167 + 0.01280) \ddot{\Theta} + (0.02722 k - 0.329) \Theta = 0 \quad (1)$$

(a)  $k = 500 \text{ N/m}$ 

$$0.05447 \ddot{\Theta} + 110.47 \Theta = 0$$

$$f_n = \frac{\omega_n}{2\pi} = (\sqrt{10.47/0.05447})/2\pi = 2.21 \text{ Hz}$$

(b) FOR  $\tau_n \rightarrow \infty$   $\omega_n \rightarrow 0$  OSCILLATIONS WILL  
NOT OCCUR

$$\text{FROM EQUATION (1), } \omega_n^2 = \frac{0.02722 k - 0.329}{0.05447} = 0$$

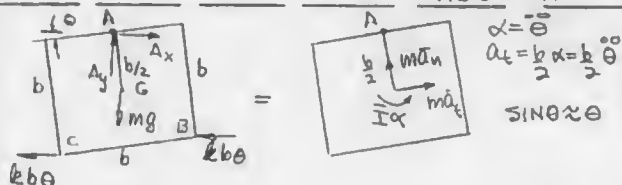
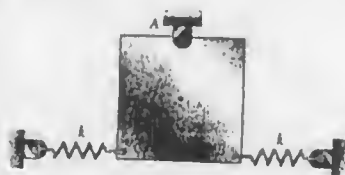
$$k = \frac{0.329}{0.02722} = \frac{(0.32)(9.81)}{(0.02722)} = 115.3 \text{ N/m}$$

19.56

GIVEN:

45-lb SQUARE  
PLATE WITH  
1.2 ft SIDES  
 $k = 8 \text{ lb/in. EACH}$ 

FIND:

FREQUENCY OF  
VIBRATION

$$\Sigma M_O = \Sigma (M_O)_{\text{eff}} \quad -mg \frac{b}{2} \Theta - 2kb^2 \Theta = \bar{I} \alpha + \left(\frac{b}{2}\right)^2 m \alpha$$

$$\bar{I} + m\left(\frac{b}{2}\right)^2 = \frac{1}{6} mb^2 + m\frac{b^2}{4} = \frac{5}{12} mb^2$$

$$\frac{5}{12} mb^2 \ddot{\Theta} + (mg \frac{b}{2} + 2kb^2) \Theta = 0$$

$$f_n = \omega_n / 2\pi = \sqrt{\frac{mg/2 + 2kb}{\frac{5}{12} mb}} / 2\pi \quad mg = 45 \text{ lb}$$

$$k = 8 \frac{\text{lb}}{\text{in}} = 96 \frac{\text{lb}}{\text{ft}} \quad b = 1.2 \text{ ft}$$

$$f_n = \sqrt{\frac{(45/2 + 2)(96)(1.2)}{[\frac{5}{12} \frac{45 \text{ lb}}{32.2 \text{ ft/s}^2} (1.2 \text{ ft})]}} / 2\pi$$

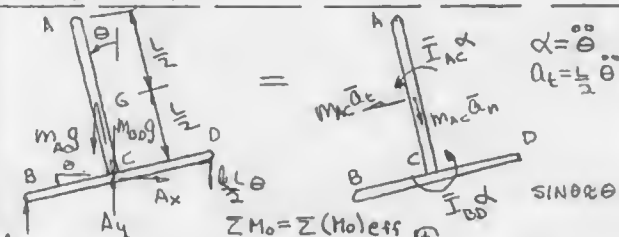
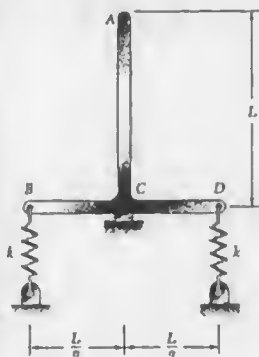
$$f_n = 3.03 \text{ Hz}$$

19.57

GIVEN:

m = 12 kg FOR AC  
m = 12 kg FOR BD  
L = 0.8 m  
 $k = 500 \text{ N/m}$ 

FIND:

FREQUENCY OF SMALL  
OSCILLATIONS

$$\Sigma M_O = \Sigma (M_O)_{\text{eff}}$$

$$[M_{AC} g \frac{L}{2} - 2k(\frac{L}{2})^2 \Theta] \Theta = (\bar{I}_{AC} + \bar{I}_{BD}) \alpha + M_{AC} (\frac{L}{2})^2 \alpha$$

$$m_{BD} = m_{AC} = m \quad \bar{I}_{BD} = \bar{I}_{AC} = \bar{I} = \frac{1}{12} mL^2$$

$$(\frac{1}{6} + \frac{1}{4}) mL^2 \ddot{\Theta} + [2k(\frac{L}{2})^2 - mg \frac{L}{2}] \Theta = 0$$

$$f_n = \omega_n / 2\pi = \sqrt{\frac{2(500)(0.4)^2 - (2)(9.81)(0.4)}{(\frac{5}{12})(12)(0.8)^2}} / 2\pi$$

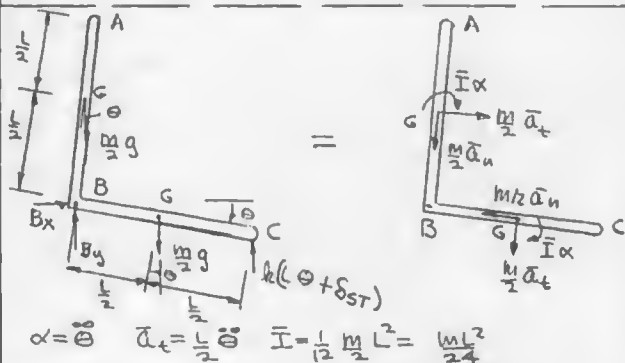
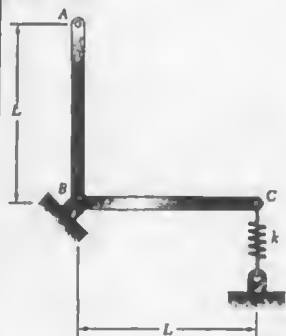
$$f_n = 0.945 \text{ Hz}$$

19.58

GIVEN:

ROD ABC OF TOTAL MASS  $M$ 

FIND:

FREQUENCY OF SMALL OSCILLATIONS IN TERMS OF  $M, L$  AND  $k$ .

$$\alpha = \ddot{\theta} \quad \bar{a}_t = \frac{L}{2} \ddot{\theta} \quad \bar{I} = \frac{1}{12} M L^2 = \frac{M L^2}{24}$$

$$\sum M_B = \sum (M_B)_{\text{eff}} \quad \sin \theta \approx \theta \quad \cos \theta \approx 1$$

$$\frac{Mg}{2} \frac{L}{2} \sin \theta + \frac{Mg}{2} \frac{L}{2} \cos \theta - k L (L \theta + \delta_{ST}) \cos \theta = \frac{1}{2} \bar{I} \ddot{\theta} + 2 \frac{M}{2} \bar{a}_t \frac{L}{2}$$

$$\frac{Mg L}{4} \theta + \frac{Mg L}{4} - k L^2 \theta - k L^2 \delta_{ST} = \frac{M L^2}{12} \ddot{\theta} + \frac{M L^2}{4} \ddot{\theta} \quad (1)$$

BUT FOR EQUILIBRIUM ( $\theta = 0$ )

$$\sum M_B = 0 = \frac{M}{2} g \frac{L}{2} - k L^2 \delta_{ST} \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\left( \frac{Mg L}{4} - k L^2 \right) \theta = \frac{M L^2}{8} \ddot{\theta}$$

$$\ddot{\theta} + \frac{(k L^2 - Mg L/4)}{M L^2/8} \theta = 0$$

$$\omega_n^2 = \frac{3k}{m} - \frac{3}{4} \frac{g}{L} \quad \omega_n = \sqrt{3} \sqrt{\frac{k}{m} - \frac{g}{4L}}$$

$$f_n = \omega_n / 2\pi$$

$$f_n = \frac{\sqrt{3}}{2\pi} \sqrt{\frac{k}{m} - \frac{g}{4L}}$$

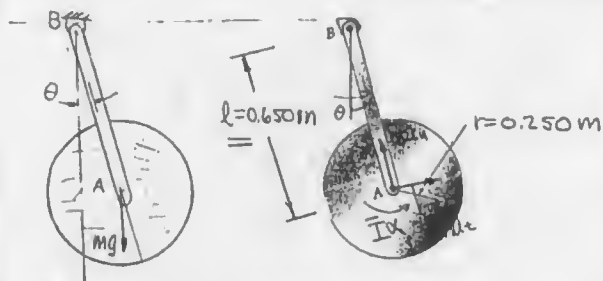
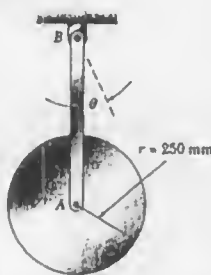
19.59

GIVEN:

ROD AB LENGTH  $l = 0.650 \text{ m}$   
 MASS OF AB IS  
 NEGUGIBLE  
 AB IS DISPLACED  $2^\circ$   
 FROM THE POSITION  
 SHOWN AND RELEASED

FIND:

MAXIMUM VELOCITY OF  
 A IF THE DISK IS,  
 (a) FREE TO ROTATE ABOUT A  
 (b) RIVETED TO AB AT A



$$\bar{I} = \frac{1}{2} M r^2 = \frac{1}{2} (0.250)^2 M = \frac{M}{32} \quad \alpha = \ddot{\theta}$$

$$\bar{a}_t = l \alpha = 0.650 \ddot{\theta}$$

(a) THE DISK IS FREE TO ROTATE AND IS IN  
 CURVILINEAR TRANSLATION. THUS  $\bar{I} \alpha = 0$   
 $\sum M_B = \sum (M_B)_{\text{eff}}$

$$\sum -Mg l \sin \theta = l m \bar{a}_t$$

$$M l^2 \ddot{\theta} + M g l \theta = 0 \quad \omega_n^2 = \frac{g}{l}$$

FROM 19.17, THE SOLUTION TO THIS EQUATION IS

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\text{At } t = 0, \theta = 2 \cdot \frac{\pi}{180} = \frac{\pi}{90} \text{ RAD}, \dot{\theta} = 0$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$t = 0, \quad 0 = \theta_m \omega_n \cos \phi \quad \phi = \frac{\pi}{2}$$

$$\frac{\pi}{90} = \theta_m \sin(0 + \frac{\pi}{2})$$

$$\theta_m = \frac{\pi}{90} \text{ RAD}$$

$$\text{THUS } \theta = \frac{\pi}{90} \sin(\omega_n t + \frac{\pi}{2})$$

$$(v_A)_{\text{MAX}} = l \dot{\theta}_{\text{MAX}} = l \theta_m \omega_n \quad l = 0.650 \text{ m} \quad \theta_m = \frac{\pi}{90} \quad \omega_n = \sqrt{\frac{g}{l}}$$

$$(v_A)_{\text{MAX}} = (0.650 \text{ m}) \left( \frac{\pi}{90} \right) \left( \sqrt{\frac{9.81 \text{ m/s}^2}{0.650 \text{ m}}} \right)$$

$$(v_A)_{\text{MAX}} = 0.08815 \text{ m/s} \quad (v_A)_{\text{MAX}} = 88.1 \frac{\text{mm}}{\text{s}}$$

(b) FOR DISK RIVETED AT A ( $\bar{I} \alpha$  INCLUDED)

$$\sum M_B = \sum (M_B)_{\text{eff}} \quad -Mg l \sin \theta = \bar{I} \alpha + l m \bar{a}_t$$

$$\left( \frac{1}{2} M r^2 + M l^2 \right) \ddot{\theta} + M g l \theta = 0$$

$$\omega_n^2 = \frac{g l}{r^2/2 + l^2}$$

$$\theta = \frac{\pi}{90} \sin(\omega_n t + \frac{\pi}{2}) \quad (\text{SEE (a)})$$

$$(v_A)_{\text{MAX}} = l \dot{\theta}_{\text{MAX}} = l \theta_m \omega_n$$

$$(v_A)_{\text{MAX}} = (0.650 \text{ m}) \sqrt{\frac{(9.81 \text{ m/s}^2)(0.650 \text{ m})}{(0.250^2/2 + 0.650^2) \text{ m}^2}} = 0.0851 \text{ m/s}$$

$$(v_A)_{\text{MAX}} = 85.1 \frac{\text{mm}}{\text{s}}$$

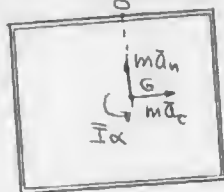
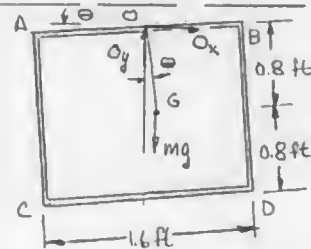
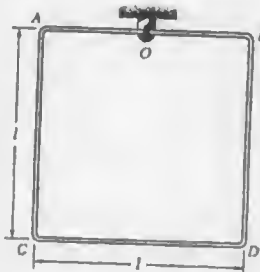
19.60

GIVEN:

THIN WIRE OF SIDE  
 $l = 1.2 \text{ ft}$ .  
 CORNER B PUSHED  
 DOWN 0.6 in. AND  
 RELEASED

FIND:

- (A) MAXIMUM VELOCITY  
 OF POINT B  
 (B) CORRESPONDING  
 MAGNITUDE OF THE  
 ACCELERATION


 $M = \text{TOTAL MASS}$ 

$$I = \frac{1}{12} M l^2 + M \left(\frac{l}{2}\right)^2 = \frac{M l^2}{3}$$

$$\alpha = \ddot{\theta} \quad \ddot{a}_t = \frac{l}{2} \alpha = \frac{l}{2} \ddot{\theta} \quad \theta \propto \sin \theta$$

$$\sum M_O = \sum (M_O)_{\text{eff}} \quad -mg \sin \theta \frac{l}{2} = \bar{I} \alpha + M \left(\frac{l}{2}\right)^2 \alpha$$

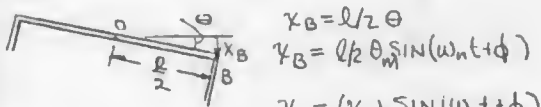
$$\left(\frac{M l^2}{3} + M \frac{l^2}{4}\right) \ddot{\theta} + mg \frac{l}{2} \theta = 0$$

$$\omega_n^2 = \frac{g l / 2}{\frac{1}{12} l^2} = \frac{6}{1} \frac{g}{l} = \frac{6}{1} \left( \frac{32.2 \text{ ft/s}^2}{1.6 \text{ ft}} \right) = 23.0 \text{ s}^{-2}$$

$$\omega_n = 4.796 \text{ RAD/S}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

(a)



$$\text{AT } t=0 \quad x_B = 0.6 \text{ in} = 0.05 \text{ ft} \quad \dot{x}_B = 0$$

$$t=0 \quad \dot{x}_B = 0 = (x_B)_m \omega_n \cos(0 + \phi) \quad \phi = \pi/2$$

$$x_B = 0.05 \text{ ft} = (x_B)_m \sin(0 + \frac{\pi}{2}), (x_B)_m = 0.05 \text{ ft}$$

$$(\dot{x}_B)_m = (x_B)_m \omega_n = (0.05 \text{ ft})(4.796 \text{ rad/s}) = 0.2398 \text{ ft/s}$$

$$(b) \quad x_B = (0.05 \text{ ft}) \sin(4.796 t + \pi/2) \quad (\dot{x}_B)_m = 0.2398 \text{ ft/s} = 2.88 \text{ in/s}$$

$$\dot{x}_B = (0.2398 \text{ ft/s}) \cos(4.796 t + \frac{\pi}{2})$$

$$\dot{x}_B = -(0.2398 \text{ ft/s})(4.796 \text{ rad/s}) \sin(4.796 t + \frac{\pi}{2})$$

$$\text{MAX VELOCITY WHEN } (4.796 t + \frac{\pi}{2}) = 0 \text{ OR } x_B = 0$$

$$\text{AND } \dot{x}_B = 0.2398 \text{ ft/s} \quad \text{AND } \dot{x}_B = 0$$

$$a_t = \ddot{x}_B = 0$$

$$a_n = (\dot{x}_B)^2 / r = (0.2398 \text{ ft/s})^2 / (0.8 \text{ ft}) = (0.07188 \text{ ft/s}^2)$$

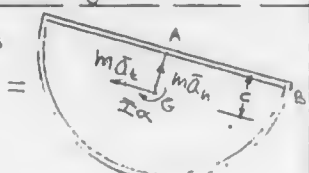
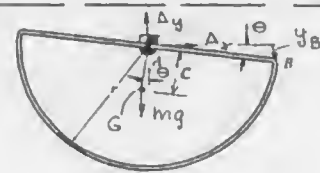
$$a_B = \sqrt{a_t^2 + a_n^2} = 0.07188 \text{ ft/s}^2 = 0.863 \text{ in/s}^2$$

19.61

GIVEN:

THIN WIRE OF RADIUS  
 $r = 0.220 \text{ m}$ .  
 POINT B IS PUSHED  
 DOWN 0.020 m AND  
 RELEASED

FIND:

 $v_B$  AT 8 S.


DETERMINE LOCATION OF THE CENTROID G.

LET  $\rho$  = MASS PER UNIT LENGTHTHEN TOTAL MASS  $M = \rho(2r + \pi r) = \rho r(2 + \pi)$ ABOUT C  $mgc = 0 + \pi r \rho \left(\frac{2r}{\pi}\right) g = 2r^2 \rho g$ 

$$\rho r(2 + \pi) c = 2r^2 \rho g$$

$$c = \frac{2r}{(2 + \pi)}$$

$$\bar{y} = \frac{2r}{\pi}$$

$$\sum M_O = \sum (M_O)_{\text{eff}} \quad \alpha = \ddot{\theta} \quad a_t = c \alpha = c \ddot{\theta}$$

$$-mgc \sin \theta = \bar{I} \alpha + M c a_n \quad \sin \theta \approx \theta$$

$$(\bar{I} + mc^2) \ddot{\theta} + mgc \theta = 0 \quad I_O \ddot{\theta} + mgc \theta = 0$$

$$\text{BUT } \bar{I} + mc^2 = I_O$$

$$I_O = (I_O)_{\text{SEMI CIRC}} + (I_O)_{\text{LINE}} = m_{\text{S-CIRC}} r^2 + m_{\text{LINE}} \left(\frac{2r}{3}\right)^2$$

$$m_{\text{S-CIRC}} = \rho \pi r \quad m_{\text{LINE}} = \rho 2r \quad \bar{y} = \frac{m}{r(2 + \pi)}$$

$$I_O = \rho \left[ \pi r \cdot r^2 + \frac{2r \cdot r^2}{3} \right] = \frac{\rho r^3}{(2 + \pi)} \left[ \pi + \frac{2}{3} \right]$$

$$\frac{\rho r^3}{(2 + \pi)} \left( \pi + \frac{2}{3} \right) \ddot{\theta} + mg \frac{2r}{(2 + \pi)} \theta = 0$$

$$\omega_n^2 = \frac{2g}{\left(\pi + \frac{2}{3}\right) r} = \frac{2(9.81 \text{ m/s}^2)}{\left(\pi + \frac{2}{3}\right)(0.220 \text{ m})}$$

$$\omega_n^2 = 23.42 \text{ s}^{-2} \quad \omega_n = 4.839 \text{ RAD/S}$$

$$\theta = \theta_m \sin(\omega_n t + \phi) \quad y_B = r \theta$$

$$y_B = r \theta_m \sin(\omega_n t + \phi) = (y_B)_m \sin(\omega_n t + \phi)$$

$$\text{AT } t=0 \quad y_B = 0.02 \text{ m} \quad \dot{y}_B = 0$$

$$(t=0) \quad \dot{y}_B = 0 = (y_B)_m \cos(0 + \phi) \quad \phi = \frac{\pi}{2}$$

$$y_B = 0.02 = (y_B)_m \sin(0 + \frac{\pi}{2}) \quad (y_B)_m = 0.02 \text{ m}$$

$$y_B = 0.02 \sin(\omega_n t + \frac{\pi}{2}) \quad \omega_n = 4.839 \text{ RAD}$$

$$\dot{y}_B = (0.02)(\omega_n) \cos(\omega_n t + \frac{\pi}{2}) = -(0.02) \omega_n \sin \omega_n t$$

$$\text{AT } t=8 \text{ s} \quad v_B = \dot{y}_B = -(0.02)(4.839) \sin[(4.839)(8)]$$

$$v_B = -0.0821 \text{ m/s} \quad v_B = 82.1 \text{ mm/s}$$

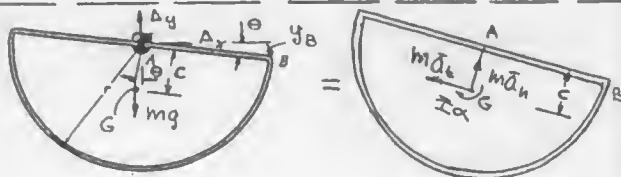
19.62



GIVEN:

THIN WIRE OF RADIUS  
 $r = 16$  in.  
 POINT B IS PUSHED  
 DOWN 1.5 in. AND  
 RELEASED

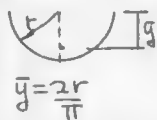
FIND:

 $a_B$  AT 10 s


DETERMINE LOCATION OF THE CENTROID G.

LET  $\rho$  = MASS PER UNIT LENGTH

THEN TOTAL MASS  $M = \rho(2r + \pi r) = \rho r(2 + \pi)$   
 ABOUT C  $mgc = 0 + \pi r e \left(\frac{2r}{\pi}\right) \rho g = 2r^2 \rho g$



$$\rho r(2 + \pi)c = 2r^2 \rho$$

$$c = \frac{2r}{(2 + \pi)}$$

$$\sum M_O = \sum (M_O)_{eff} \quad \ddot{\theta} = \ddot{\alpha} \quad a_c = c\ddot{\alpha} = c\ddot{\theta}$$

$$-mgc \sin \theta = \bar{I}\ddot{\alpha} + mca_n \quad \sin \theta \approx \theta$$

$$(\bar{I} + mc^2)\ddot{\theta} + mgc\theta = 0 \quad I_O \ddot{\theta} + mgc\theta = 0$$

$$\text{BUT } \bar{I} + mc^2 = I_O$$

$$I_O = (I_O)_{\text{semi circ}} + (I_O)_{\text{LINE}} = m_{\text{S-CIRC}} r^2 + m_{\text{LINE}} \left(\frac{2r}{2}\right)^2$$

$$m_{\text{S-CIRC}} = \rho \pi r \quad m_{\text{LINE}} = \rho 2r \quad c = rn/(2 + \pi)r$$

$$I_O = \rho \left[ \pi r^3/2 + 2r \cdot r^2/3 \right] = \frac{mr^2}{(2 + \pi)} \left[ \pi + \frac{4}{3} \right]$$

$$\frac{mr^2}{(2 + \pi)} \left[ \pi + \frac{4}{3} \right] \ddot{\theta} + mg \frac{2r}{(2 + \pi)} \theta = 0$$

$$\omega_n^2 = \frac{2g}{\left(\pi + \frac{4}{3}\right)r} = \frac{2(32.2 \text{ ft/s}^2)}{\left(\pi + \frac{4}{3}\right)(16/12 \text{ ft})}$$

$$\omega_n^2 = 12.683 \quad \omega_n = 3.561 \text{ r/s}$$

$$\theta = \theta_m \sin(\omega_n t + \phi) \quad y_B = r\theta$$

AT  $t = 0 \quad y_B = 1.5/12 = 0.125 \text{ ft} \quad \dot{y}_B = 0$   
 $y_B = 0 = (y_B)_m \cos(0 + \phi), \phi = \frac{\pi}{2}$   
 $y_B = 0.125 \text{ ft} = (y_B)_m \sin(\omega_n t + \frac{\pi}{2}), (y_B)_m = 0.125 \text{ ft}$

$$y_B = 0.125 \sin(\omega_n t + \frac{\pi}{2}) \quad \omega_n = 3.561 \text{ r/s}$$

$$\dot{y}_B = 0.125 \omega_n \cos(\omega_n t + \frac{\pi}{2}) = -0.125 \omega_n \sin \omega_n t$$

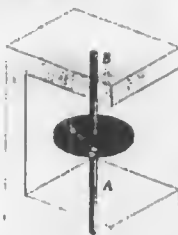
$$\dot{y}_B = -(0.125)(\omega_n^2) \sin(\omega_n t + \frac{\pi}{2}) = 0.125 \omega_n^2 \cos \omega_n t$$

AT  $t = 10 \text{ s} \quad (a_B)_t = \ddot{y}_B = (0.125)(3.561)^2 \cos[(3.561)(10)] = -0.7811$

$$(v_B)_t = \dot{y}_B = (0.125)(3.561) \sin[(3.561)(10)] = 0.3874 \text{ ft/s}$$

$$a_B = \left[ (a_B)_t^2 + \frac{v_B^2}{r} \right]^{1/2} = \left[ (-0.7811)^2 + \left( \frac{0.3874}{16/12} \right)^2 \right]^{1/2} = 0.789 \text{ ft/s}^2$$

19.63



GIVEN:

DISK OF RADIUS  $r = 120$  mm IS  
 WELDED TO ROD AB WHICH  
 IS FIXED AT A AND B.  
 DISK ROTATES  $8^\circ$  WHEN  
 A  $500 \text{ N}\cdot\text{m}$  IS APPLIED  
 PERIOD  $T_n = 1.3 \text{ s}$  WHEN THE  
 COUPLE IS REMOVED

FIND:

(a) THE MASS OF THE DISK

(b) PERIOD IF ONE ROD IS REMOVED

$$K = \frac{T}{\theta} = \frac{0.5 \text{ N}\cdot\text{m}}{(8)(\pi/180)}$$

$$K = 3.581 \text{ N}\cdot\text{m/rad}$$

$$\sum M_O = \sum (M_O)_{eff}$$

$$-K\theta = J\ddot{\theta} \quad \ddot{\theta} + \frac{K}{J}\theta = 0$$

$$T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{J/K} \quad J = \frac{T_n^2 K}{(2\pi)^2} = \frac{(1.3)^2 (3.581 \text{ N}\cdot\text{m/r})}{(2\pi)^2}$$

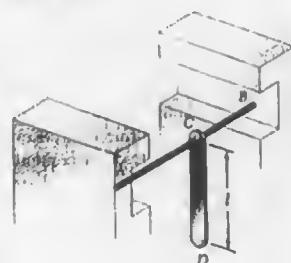
$$J = 0.1533 \text{ N}\cdot\text{m}\cdot\text{s}^2 = \frac{1}{2} m r^2 = \frac{1}{2} m (0.120 \text{ m})^2$$

$$m = \frac{(0.1533 \text{ N}\cdot\text{m}\cdot\text{s}^2)(2)}{(0.120 \text{ m})^2} = 21.3 \text{ kg}$$

(b) WITH ONE ROD REMOVED  $K' = K/2 = \frac{3.581}{2} = 1.791 \text{ N}\cdot\text{m/rad}$ 

$$T = 2\pi \sqrt{J/K'} = 2\pi \sqrt{\frac{0.1533 \text{ N}\cdot\text{m}\cdot\text{s}^2}{1.791 \text{ N}\cdot\text{m/rad}}} = 1.838 \text{ s}$$

19.64



GIVEN:

10-lb ROD CD OF  
 LENGTH  $l = 2.2$  ft  
 WELDED ROD FIXED  
 AT A AND B WITH  
 $K = 18 \text{ lb}\cdot\text{ft/rad}$

FIND:

PERIOD OF SMALL  
 OSCILLATIONS IF THE  
 EQUILIBRIUM POSITION  
 IS,

(a) VERTICAL AS SHOWN

(b) HORIZONTAL

(a)  $\sum M_C = \sum (M_C)_{eff}$   
 $-K\theta - mg \sin \theta = J\ddot{\theta} + m l \ddot{\alpha}$   
 $\alpha = \ddot{\theta} \quad a_c = \frac{l}{2} \ddot{\alpha} = \frac{l}{2} \ddot{\theta}$   
 $(\bar{J} + m(\frac{l}{2})^2) \ddot{\theta} + K\theta + mgl/2 = 0$   
 $\bar{J} + m(\frac{l}{2})^2 = J_C = \frac{1}{3} m l^2$

$$J_C = \frac{1}{3} (10 \text{ lb})(2.2 \text{ ft})^2 = 0.501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{J_C}{(K + mg l/2)}} \quad T_n = 2\pi \sqrt{\frac{0.501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2}{(18 + 10)(0.5)(2.2) \text{ lb}\cdot\text{ft}}}$$

$$T_n = 0.826 \text{ s}$$

(b)  $\sum M_C = \sum (M_C)_{eff}$   
 $-K(\theta + \theta_{st}) + mgl/2 = J\ddot{\theta} + m l \ddot{\alpha}$   
 $\alpha = \ddot{\theta} \quad a_c = l/2 \ddot{\theta}$

BUT IN EQUILIBRIUM ( $\theta = 0$ )  $\sum M_C = 0 = -K\theta_{st} + mgl/2$   
 THUS  $J_C \ddot{\theta} + K\theta = 0$   
 $T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{J_C}{K}} = 2\pi \sqrt{\frac{0.501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2}{18 \text{ lb}\cdot\text{ft/rad}}} = 1.048 \text{ s}$

19.65

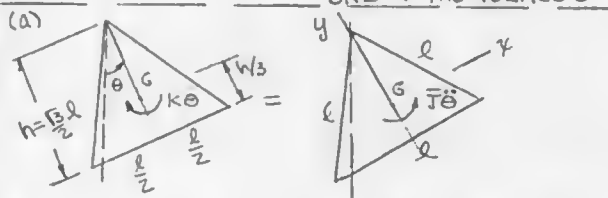
GIVEN:

1.8-kg PLATE IN THE SHAPE OF AN EQUILATERAL TRIANGLE SUSPENDED FROM A WIRE AT ITS CENTER OF GRAVITY FOR THE WIRE  $k = 35 \text{ N}\cdot\text{m}/\text{rad}$  PLATE IS ROTATED  $360^\circ$  AND RELEASED.



FIND:

- (a) PERIOD OF OSCILLATION  
(b) MAXIMUM VELOCITY OF ONE OF THE VERTICES



$$\sum M_G = \sum (M_G)_{\text{eff}} \quad -k\theta = J_G \ddot{\theta} \quad \alpha = \ddot{\theta}$$

$$J_G = (I_x)_G + (I_y)_G \quad A = \frac{1}{2}bh \quad m = \frac{\rho t bh}{2}$$

$$(I_x)_G = \frac{1}{36}bh^3 \quad (I_y)_G = \frac{1}{48}bh^3 \quad St = \frac{2m}{bh}$$

$$J_G = \frac{1}{36}bh^3 \rho t + \frac{1}{48}bh^3 \rho t = m \left[ \frac{h^2}{18} + \frac{b^2}{24} \right]$$

$$J_G = (1.8 \text{ kg}) \left[ \frac{(150/2)^2}{18} + \frac{1}{24} \right] (0.150)^2 = 3.375 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$J_G \ddot{\theta} + k\theta = 0 \quad 3.375 \times 10^{-3} \ddot{\theta} + 35 \times 10^{-3} \theta = 0$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{3.375 \times 10^{-3} \text{ kg}\cdot\text{m}^2}{35 \times 10^{-3} \text{ N}\cdot\text{m}}} \quad \tau_n = 1.951 \text{ s}$$

(b)

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{35 \times 10^{-3} \text{ N}\cdot\text{m}}{3.375 \text{ kg}\cdot\text{m}^2}} = 3.22 \text{ rad/s}^2$$

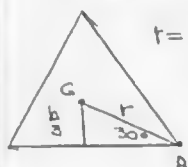
$$\text{AT } t=0 \quad \theta = 2\pi \text{ rad} \quad \dot{\theta} = 0$$

$$\ddot{\theta} = 0 = \theta_m \omega_n \cos(0 + \phi) \quad \phi = \pi/2$$

$$\theta = 2\pi = \theta_m \sin(0 + \pi/2) \quad \theta_m = 2\pi \text{ rad}$$

$$\theta = 2\pi \sin(3.22t + \pi/2) = 2\pi \cos(3.22t)$$

$$\dot{\theta}_{\text{max}} = \theta_m \omega_n = (2\pi)(3.22) = 20.23 \text{ rad/s}$$



$$r = \frac{h/3}{\sin 30^\circ} = 2h/3 = 15/3 \text{ mm}$$

$$(v_A)_{\text{max}} = r \dot{\theta}_{\text{max}}$$

$$(v_A)_{\text{max}} = (15/3)(0.150 \text{ m})(20.23 \text{ rad/s})$$

$$(v_A)_{\text{max}} = 1.752 \text{ m/s}$$

19.66

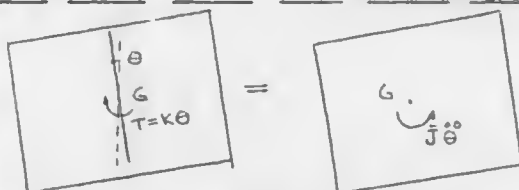
GIVEN:

ROTATION ABOUT A VERTICAL AXIS  
 $\tau = 2.2 \text{ s}$ , EMPTY PLATFORM  
 $\tau_n = 3.8 \text{ s}$ , WHEN OBJECT A IS ADDED  
 $k = 20 \text{ lb}\cdot\text{ft}/\text{rad}$  FOR WIRE



FIND:

CENTROIDAL MOMENT OF INERTIA FOR OBJECT A



$$\sum M_G = \sum (M_G)_{\text{eff}} \quad -k\theta = J_G \ddot{\theta} \quad J_G \ddot{\theta} + k\theta = 0$$

$$\text{EMPTY PLATFORM} \quad J_P = \text{CENTROIDAL } J \text{ OF PLATFORM}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{k/J_P}} \quad J_P = \frac{k\tau_n^2}{4\pi^2} = \frac{(20 \text{ lb}\cdot\text{ft}/\text{rad})(2.2 \text{ s})^2}{4\pi^2}$$

$$J_P = 2.452 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

PLATFORM WITH OBJECT A

J\_A = CENTROIDAL J OF A

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{k/(J_P + J_A)}} \quad J_P + J_A = \frac{k\tau_n^2}{4\pi^2}$$

$$J_A = \frac{(20 \text{ lb}\cdot\text{ft}/\text{rad})(3.8 \text{ s})^2}{4\pi^2} - 2.452 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$J_A = 7.315 - 2.452 = 4.863 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \quad J_A = 4.86 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

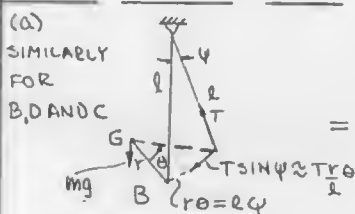
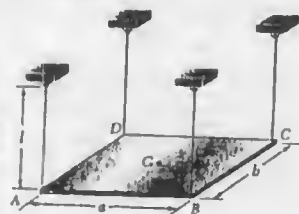
19.67

GIVEN:

THIN RECTANGULAR PLATE SUSPENDED AS SHOWN

FIND:

- PERIOD FOR A  
(a) ROTATION ABOUT A VERTICAL AXIS THROUGH G  
(b) HORIZONTAL DISPLACEMENT PERPENDICULAR TO AB  
(c) HORIZONTAL DISPLACEMENT PERPENDICULAR TO DC



$$\sum M_G = \sum (M_G)_{\text{eff}} \quad -4T\theta \cdot r = J_G \ddot{\theta} \quad T = mg/4$$

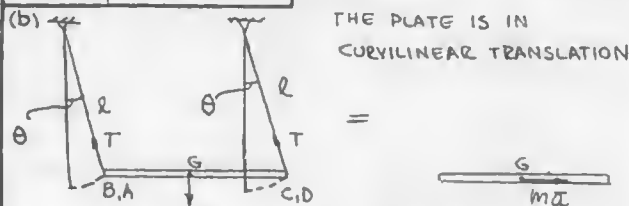
$$\ddot{\theta} + \frac{mgr^2}{J_G} \theta = 0 \quad J_G = \frac{1}{12}m(a^2 + b^2)$$

$$r^2 = \frac{1}{4}(a^2 + b^2)$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{J_G}{mgr^2}}$$

$$\tau_n = 2\pi \sqrt{\frac{\frac{1}{12}m(a^2 + b^2)}{mg \frac{1}{4}(a^2 + b^2)}} = 2\pi \sqrt{\frac{2}{3g}}$$

# 19.67 CONTINUED



THE PLATE IS IN CURVILINEAR TRANSLATION

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1$$

$$l \ddot{\theta} \cos \theta \approx l \ddot{\theta} = \ddot{a}$$

$$+\uparrow \sum F_y = 0 = 4(T \cos \theta) - mg = 0 \quad T = mg/4$$

$$+\rightarrow \sum F_x = \sum (F_x)_{\text{eff}} \quad -4T \sin \theta = m \ddot{a}$$

$$l \ddot{\theta} + g \theta = 0$$

$$\tau_n = 2\pi \sqrt{\frac{l}{g}}$$

(C) SINCE THE OSCILLATION ABOUT AXES PARALLEL TO AB (AND CD) IS INDEPENDENT OF THE LENGTH OF THE SIDES OF THE PLATE, THE PERIOD OF VIBRATION ABOUT AXES PARALLEL TO BC (AND AD) IS THE SAME

$$\tau_n = 2\pi \sqrt{\frac{l}{g}}$$

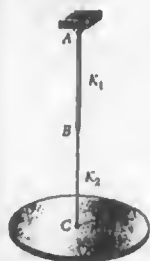
# 19.68

GIVEN:

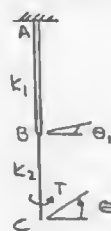
2.2-kg CIRCULAR DISK  
 $r = 0.8 \text{ m}$   
 WIRE AB,  $k_1 = 10 \text{ N}\cdot\text{m/rad}$   
 WIRE BC,  $k_2 = 5 \text{ N}\cdot\text{m/rad}$

FIND:

PERIOD OF OSCILLATION ABOUT AXIS AC



# EQUIVALENT TORSIONAL SPRING CONSTANT



$$T = k_e \theta, T = k_2(\theta - \theta_1), T = k_1 \theta_1$$

$$k_2 \theta = (k_1 + k_2) \theta_1$$

$$\theta_1 = \frac{k_2}{k_1 + k_2} \theta$$

$$T = k_e \theta = k_1 \theta_1$$

$$k_e \theta = k_1 \frac{k_2}{k_1 + k_2} \theta$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

NEWTON'S LAW



$$\bar{J} = \frac{1}{2} m r^2$$

$$\frac{1}{2} m r^2 \ddot{\theta} + k_e \theta = 0$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{2k_e}{mr^2}}} = 2\pi \sqrt{\frac{(2.2 \text{ kg})(0.8 \text{ m})^2}{2(10)(5)/(10+5) \text{ N}\cdot\text{m}}}$$

$$\tau_n = 2.89 \text{ s}$$

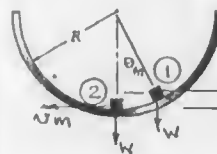
# 19.69

GIVEN:

PARTICLE WHICH MOVES WITHOUT FRICTION INSIDE A CURVED SURFACE

FIND:

PERIOD OF SMALL OSCILLATIONS



DATUM AT (2)

POSITION (1)

$$T_1 = 0$$

$$V_1 = WR(1 - \cos \theta_m)$$

SMALL OSCILLATIONS

$$(1 - \cos \theta_m) \approx 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_1 = WR \frac{\theta_m^2}{2}$$

POSITION (2)

$$U_m = R \dot{\theta}_m$$

$$T_2 = \frac{1}{2} m \dot{U}_m^2 = \frac{1}{2} m R^2 \dot{\theta}_m^2$$

$$V_2 = 0$$

CONSERVATION OF ENERGY  $T_1 + V_1 = T_2 + V_2$

$$0 + WR \frac{\theta_m^2}{2} = \frac{1}{2} m R^2 \dot{\theta}_m^2 + 0 \quad \dot{\theta}_m = \omega_n \theta_m$$

$$W = mg$$

$$mg R \frac{\theta_m^2}{2} = \frac{1}{2} m R^2 \omega_n^2 \theta_m^2$$

$$\omega_n = \sqrt{g/R}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{R}{g}}$$

# 19.70

GIVEN:

1402. SPHERE A  
 1002. SPHERE C  
 ROD AC OF NEGUGIBLE WEIGHT

FIND:

PERIOD OF SMALL OSCILLATIONS OF THE ROD



(1)  $\theta_m$

(2)  $\theta_m$

(3)  $\theta_m$

(4)  $\theta_m$

(5)  $\theta_m$

(6)  $\theta_m$

(7)  $\theta_m$

(8)  $\theta_m$

(9)  $\theta_m$

(10)  $\theta_m$

(11)  $\theta_m$

(12)  $\theta_m$

(13)  $\theta_m$

(14)  $\theta_m$

(15)  $\theta_m$

(16)  $\theta_m$

(17)  $\theta_m$

(18)  $\theta_m$

(19)  $\theta_m$

(20)  $\theta_m$

(21)  $\theta_m$

(22)  $\theta_m$

(23)  $\theta_m$

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(25)  $\theta_m$

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19.71

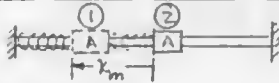


GIVEN:

1.8 kg COLLAR A  
ATTACHED TO A SPRING  
 $k = 800 \text{ N/m}$ , NO FRICTION  
COLLAR MOVED TO mm TO  
THE LEFT AND RELEASED

FIND:

THE MAXIMUM VELOCITY  
THE MAXIMUM ACCELERATION



DATUM AT ①  
POSITION ①

$$T_1 = 0 \quad V_1 = \frac{1}{2} k x_m^2$$

POSITION ②

$$T_2 = \frac{1}{2} m v_2^2 \quad v_2 = 0 \quad v_2 = \dot{x}_m$$

$$\dot{x}_m = \omega_n x_m$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{1}{2} k x_m^2 = \frac{1}{2} m \dot{x}_m^2 + 0$$

$$\frac{1}{2} k x_m^2 = \frac{1}{2} m \omega_n^2 x_m^2 \quad \omega_n = \frac{k}{m} = \frac{800 \text{ N/m}}{1.8 \text{ kg}}$$

$$\omega_n = 444.4 \text{ s}^{-2} \quad \omega_n = 21.08 \text{ rad/s}$$

$$\dot{x}_m = \omega_n x_m = (21.08 \text{ s}^{-1})(0.070 \text{ m}) = 1.476 \text{ m/s}$$

$$\ddot{x}_m = \omega_n^2 x_m = (21.08 \text{ s}^{-1})^2 (0.070 \text{ m}) = 31.1 \text{ m/s}^2$$

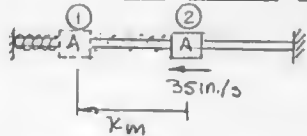
19.72

GIVEN:

3-lb COLLAR A, ATTACHED  
TO A SPRING OF CONSTANT  
 $k = 516 \text{ lb/in.}$   
COLLAR INITIAL VELOCITY  
 $= 35 \text{ in./s}$ , NO FRICTION

FIND:

THE AMPLITUDE  
THE MAXIMUM ACCELERATION



DATUM AT ①  
POSITION ①

$$35 \text{ in./s} = 2.917 \text{ ft/s}$$

$$k = 516 \text{ lb/in.} = 60 \text{ lb/ft}$$

$$T_1 = 0 \quad V_1 = \frac{1}{2} k x_m^2 = \frac{1}{2} (60 \text{ lb/ft}) x_m^2$$

POSITION ②

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} \left( \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (2.917 \text{ ft/s})^2 = 0.7925 \text{ ft}$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{60}{2} x_m^2 = \frac{0.7925}{2}$$

$$x_m = 0.1149 \text{ ft} = 1.379 \text{ in.}$$

$$\text{AT POSITION 2} \quad T_2 = \frac{1}{2} m \dot{x}_2^2 = \frac{1}{2} \left( \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (\omega_n^2) (x_m^2)$$

$$T_2 = \frac{0.09317}{2} \omega_n^2 x_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{60}{2} x_m^2 = \frac{0.09317}{2} \omega_n^2 x_m^2$$

$$\omega_n^2 = 644 \text{ s}^{-2} \quad \omega_n = 25.38 \text{ rad/s}$$

$$\dot{x}_m = \omega_n x_m = (25.38 \text{ s}^{-1})(0.1149 \text{ ft})$$

$$\ddot{x}_m = 74.0 \text{ ft/s}^2 = 888 \text{ in./s}^2$$

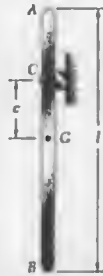
19.73

GIVEN:

ROTATION OF A UNIFORM ROD  
ABOUT A HORIZONTAL AXIS  
AT C. SMALL OSCILLATIONS

FIND:

VALUE OF THE DISTANCE C FOR  
WHICH THE FREQUENCY OF  
SMALL OSCILLATIONS WILL BE  
MAXIMUM

FIND  $\omega_n$  AS A FUNCTION OF C.

DATUM AT ②

POSITION ①

$$T_1 = 0 \quad V_1 = mgh$$

$$V_1 = mgc(1 - \cos \theta_m)$$

$$1 - \cos \theta_m \approx 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_1 = mgc \frac{\theta_m^2}{2}$$

POSITION ②

$$T_2 = \frac{1}{2} I_C \dot{\theta}_m^2$$

$$I_C = \bar{I} + mc^2 = \frac{1}{12} m l^2 + mc^2$$

$$T_2 = \frac{1}{2} m (l^2/12 + c^2) \dot{\theta}_m^2 \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + mgc \frac{\theta_m^2}{2} = m (l^2/12 + c^2) \frac{\dot{\theta}_m^2}{2} + 0$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$g c = m (l^2/12 + c^2) \omega_n^2$$

$$\omega_n^2 = \frac{g c}{(l^2/12 + c^2)}$$

MAXIMUM C, WHEN

$$\frac{d\omega_n^2}{dc} = 0 = g \frac{(l^2/12 + c^2) - 2c^2}{(l^2/12 + c^2)^2} = 0$$

$$l^2/12 - c^2 = 0 \quad c = l/\sqrt{12}$$

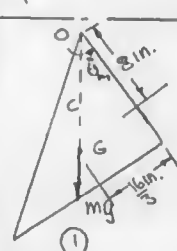
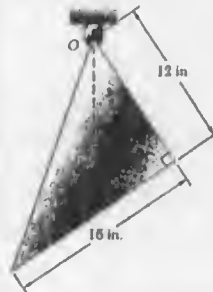
19.74

GIVEN:

THIN PLATE CUT INTO  
THE SHAPE OF A  
RIGHT TRIANGLE  
AND SUSPENDED FROM  
O IN A VERTICAL  
PLANE

FIND:

PERIOD FOR SMALL  
OSCILLATIONS



DATUM AT ①

POSITION ②

$$T_2 = 0$$

$$I h = c(1 - \cos \theta_m)$$

$$V_2 = mgc(1 - \cos \theta_m)$$

$$1 - \cos \theta_m \approx 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = mgc \frac{\theta_m^2}{2}$$

# 19.74 CONTINUED

POSITION ①

$$T_1 = \frac{1}{2} J_o \dot{\theta}_m^2 \quad V_1 = 0$$

CONSERVATION OF ENERGY  $T_1 + V_1 = T_2 + V_2$

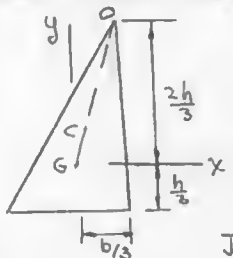
$$\frac{1}{2} J_o \dot{\theta}_m^2 + 0 = 0 + m g c \frac{\theta_m^2}{2}$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$J_o \omega_n^2 \theta_m^2 = m g c \theta_m^2$$

$$\omega_n^2 = \frac{m g c}{J_o}$$

DETERMINE C AND  $J_o$



$$c = \left[ \left( \frac{2b}{3} \right)^2 + \left( \frac{2h}{3} \right)^2 \right]^{1/2}$$

$$c = \frac{1}{3} [4b^2 + 4h^2]^{1/2}$$

$$h = 12 \text{ in.} = 1 \text{ ft}$$

$$b = 16 \text{ in.} = 4/3 \text{ ft}$$

$$c = \left[ 4 \left( 1^2 + \left( \frac{4}{3} \right)^2 \right) \right]^{1/2} = \frac{\sqrt{52}}{3}$$

$$J_o = \bar{J} + m c^2$$

$$\bar{J} = \int \bar{r}^2 dm = \int \left[ \frac{1}{36} b h^3 + \frac{1}{36} h b^3 \right]$$

$$m = \int \frac{1}{2} b h$$

$$\bar{J} = \frac{m}{18} [h^3 + b^3] = \frac{m}{18} [1 + \left( \frac{4}{3} \right)^3] = \frac{25}{162} m$$

$$J_o = \frac{25}{162} m + \frac{52}{81} m = \frac{129}{162} m \quad 1b \cdot ft \cdot s^2$$

$$\omega_n^2 = \frac{m g c}{J_o} = \frac{m (32.2) \left( \frac{\sqrt{52}}{3} \right)}{m (129/162)} = 32.4 \text{ s}^{-2} \quad \omega_n = 5.592 \text{ r/s}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.592} = 1.1045 \text{ s}$$

# 19.75

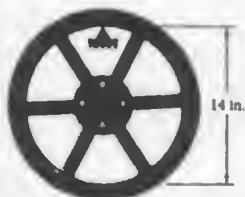
GIVEN:

85-lb FLYWHEEL

PERIOD = 1.26 s FOR SMALL OSCILLATIONS

FIND:

CENTROIDAL MOMENT OF INERTIA,  $\bar{J}$



DATUM AT ①

POSITION ①

$$T_1 = \frac{1}{2} J_o \dot{\theta}_m^2 \quad V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = m g h$$

$$h = r(1 - \cos \theta_m) = r 2 \sin^2 \theta_m / 2 \approx r \theta_m^2 / 2$$

$$V_2 = m g r \theta_m^2 / 2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2} J_o \dot{\theta}_m^2 + 0 = 0 + m g r \theta_m^2 / 2$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$J_o \omega_n^2 \theta_m^2 = m g r \theta_m^2$$

$$\omega_n^2 = \frac{m g r}{J_o}$$

$$T_n^2 = \frac{4\pi^2}{\omega_n^2} = \frac{4\pi^2 J_o}{m g r}$$

$$J_o = \bar{J} + m r^2$$

$$\bar{J} + m r^2 = \frac{T_n^2 (m g r)}{4\pi^2}$$

$$\bar{J} = \frac{T_n^2 (m g r)}{4\pi^2} - m r^2 = \frac{(1.26)^2 (85 \text{ lb}) \left( \frac{7}{12} \text{ ft} \right) \left( \frac{32.2 \text{ ft/s}^2}{12} \right)}{4\pi^2} - \left( \frac{85 \text{ lb}}{32.2 \text{ ft/s}^2} \right) \left( \frac{7}{12} \text{ ft} \right)^2$$

$$\bar{J} = 1.994 - 0.8983 = 1.096 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

# 19.76

GIVEN:

FOR SMALL OSCILLATIONS

PERIOD ABOUT A =  $T_A = 0.895 \text{ s}$

PERIOD ABOUT B =  $T_B = 0.805 \text{ s}$

$$r_a + r_b = 0.270 \text{ m}$$

FIND:

(a) LOCATION OF THE MASS CENTER G

(b) CENTROIDAL RADIUS OF GYRATION  $\bar{k}$ .



CONSIDER GENERAL PENDULUM OF CENTROIDAL RADIUS OF GYRATION  $\bar{k}$ .



DATUM AT ①

$$T_1 = \frac{1}{2} J_o \dot{\theta}_m^2 \quad V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = m g h$$

$$h = r(1 - \cos \theta_m) = r 2 \sin^2 \theta_m / 2 \approx r \theta_m^2 / 2$$

$$V_2 = m g r \theta_m^2 / 2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} J_o \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} m g r \theta_m^2$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$J_o \omega_n^2 \theta_m^2 = m g r \theta_m^2$$

$$\omega_n^2 = \frac{m g r}{J_o}$$

$$T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{J_o}{m g r}}$$

$$J_o = \bar{J} + m r^2 = m \bar{k}^2 + m r^2$$

$$(a) \quad T_n = 2\pi \sqrt{\frac{\bar{k}^2 + r^2}{g r}}$$

FOR THE ROD SUSPENDED AT A

$$T_n = 0.895 \text{ s} = 2\pi \sqrt{\frac{\bar{k}^2 + r_a^2}{g r_a}} \quad F = r_a \quad (1)$$

FOR THE ROD SUSPENDED AT B

$$T_n = 0.805 \text{ s} = 2\pi \sqrt{\frac{\bar{k}^2 + r_b^2}{g r_b}} \quad F = r_b \quad (2)$$

$$\text{BUT } r_a + r_b = 0.270 \text{ m} \quad (3)$$

$$\text{FROM (1)} \quad \bar{k}^2 + r_a^2 = g r_a \left( \frac{0.895}{2\pi} \right)^2 \quad (1')$$

$$\text{FROM (2)} \quad \bar{k}^2 + r_b^2 = g r_b \left( \frac{0.805}{2\pi} \right)^2 \quad (2')$$

SUBTRACTING (2') FROM (1')

$$r_a^2 - r_b^2 = (g/4\pi^2) (0.801 r_a - 0.648 r_b) \quad (4)$$

DIVIDING (4) BY (3) MEMBER BY MEMBER

$$r_a - r_b = \frac{1}{0.270} (g/4\pi^2) (0.801 r_b - 0.648 r_b)$$

$$r_a - r_b = \frac{9.81/4\pi^2}{0.270} (0.801 r_b - 0.648 r_b) = 0.7372 r_b - 0.5963 r_b$$

$$r_b = 0.6510 r_a \quad (5)$$

SUBSTITUTE FOR  $r_b$  FROM (5) INTO (3)

$$r_a + 0.6510 r_a = 0.270 \quad r_a = 0.1635 \text{ m}$$

$$r_b = 0.1065 \text{ m}$$

(b) FROM (1')

$$\bar{k}^2 = (9.81)(0.1635) \left( \frac{0.895}{2\pi} \right)^2 - (0.1635)^2$$

$$\bar{k}^2 = 0.03254 - 0.02673 = 0.05812 \text{ m}^2 \quad \bar{k} = 76.2 \text{ mm}$$

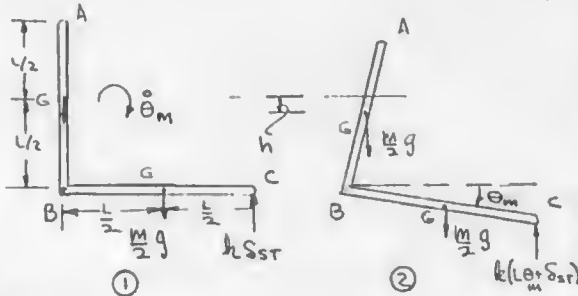
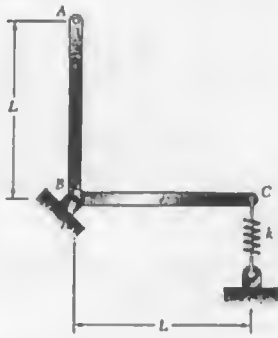
19.77

GIVEN:

ROD ABC OF MASS  $M$  IN A VERTICAL PLANE PINNED AT B AND SUPPORTED BY A SPRING AT C OF CONSTANT  $k$ .

FIND:

FREQUENCY OF SMALL OSCILLATIONS IN TERMS OF  $M, L$  AND  $k$ .



POSITION ①

$$T_1 = \frac{1}{2} I_B \dot{\theta}_m^2 \quad V_1 = \frac{1}{2} k (\delta_{ST})^2$$

POSITION ②

$$T_2 = 0 \quad V_2 = -Mgh - \frac{M}{2} g \frac{L}{2} \sin \theta_m + \frac{1}{2} k (L\theta_m + \delta_{ST})^2$$

$$h = \frac{L}{2} (1 - \cos \theta_m) = \frac{L}{2} \sin^2 \theta_m$$

$$h \approx \frac{L}{4} \theta_m^2 \quad V_2 = -\frac{M}{2} g \frac{L}{4} \theta_m^2 - \frac{M}{2} g \frac{L}{2} \theta_m + \frac{1}{2} k (L\theta_m + \delta_{ST})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} I_B \dot{\theta}_m^2 + \frac{1}{2} k \delta_{ST}^2 = 0 - \frac{M}{2} g \frac{L}{4} \theta_m^2 - \frac{M}{2} g \frac{L}{2} \theta_m + \frac{1}{2} k [L^2 \theta_m^2 + \delta_{ST}^2 + 2L\delta_{ST}\theta_m]$$

WHEN THE ROD IS IN EQUILIBRIUM,

$$\sum \mathcal{M}_B = 0 = \frac{M}{2} g \frac{L}{2} - k \delta_{ST} \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$I_B \ddot{\theta}_m = (kL^2 - \frac{MgL}{4}) \theta_m \quad \ddot{\theta}_m = \omega_n^2 \theta_m$$

$$I_B \omega_n^2 \theta_m = (kL^2 - \frac{MgL}{4}) \theta_m$$

$$\omega_n^2 = \frac{kL^2 - MgL/4}{I_B}$$

$$I_B = 2 \left( \frac{1}{3} \frac{M}{2} L^2 \right) = \frac{ML^2}{3}$$

$$\omega_n^2 = \frac{kL^2 - MgL/4}{ML^2/3} = \frac{3k}{M} - \frac{3g}{4L}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{3}}{2\pi} \sqrt{\frac{k}{M} - \frac{g}{4L}}$$

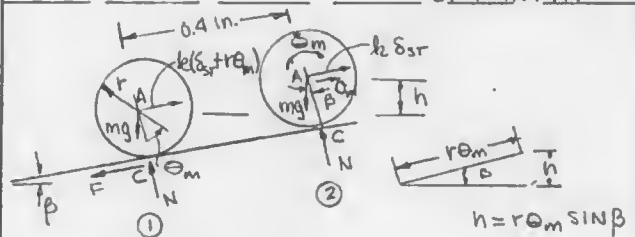
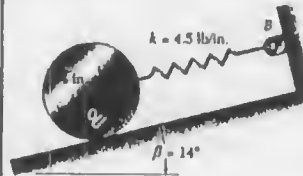
19.78

GIVEN:

15-lb DISK WHICH ROLLS WITHOUT SLIDING. POINT A IS MOVED DOWN 0.4 IN. AND RELEASED.

FIND:

(a) PERIOD  
(b) MAXIMUM VELOCITY OF POINT A.



(a) POSITION ①

$$T_1 = 0 \quad V_1 = \frac{1}{2} k (\delta_{ST} + r\theta_m)^2$$

POSITION ②

$$T_2 = \frac{1}{2} \bar{J} \dot{\theta}_m^2 + \frac{1}{2} m \bar{v}_m^2 \quad V_2 = mgh + \frac{1}{2} k (\delta_{ST})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2} k (\delta_{ST} + r\theta_m)^2 = \frac{1}{2} \bar{J} \dot{\theta}_m^2 + \frac{1}{2} m \bar{v}_m^2 + mgh + \frac{1}{2} k (\delta_{ST})^2$$

$$k \delta_{ST}^2 + 2k \delta_{ST} r \theta_m + \frac{1}{2} k r^2 \theta_m^2 = \frac{1}{2} \bar{J} \dot{\theta}_m^2 + \frac{1}{2} m \bar{v}_m^2 + 2mgh + k \delta_{ST}^2 \quad (1)$$

WHEN THE DISK IS IN EQUILIBRIUM

$$\sum \mathcal{M}_C = 0 = mgsin\beta r - k \delta_{ST} r$$

$$\text{ALSO } h = r \sin \beta \theta_m$$

THUS

$$mgh - k \delta_{ST} r = 0 \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\frac{1}{2} k r^2 \theta_m^2 = \frac{1}{2} \bar{J} \dot{\theta}_m^2 + \frac{1}{2} m \bar{v}_m^2$$

$$\bar{v}_m = \omega_n \theta_m \quad \bar{v}_m = r \dot{\theta}_m = r \omega_n \theta_m$$

$$\frac{1}{2} k r^2 \theta_m^2 = (\frac{1}{2} \bar{J} + \frac{1}{2} m r^2) \omega_n^2 \theta_m^2$$

$$\omega_n^2 = \frac{k r^2}{\bar{J} + m r^2} \quad \bar{J} = \frac{1}{2} m r^2$$

$$\omega_n^2 = \frac{k r^2}{\frac{1}{2} m r^2 + m r^2} = \frac{2}{3} \frac{k}{m}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{2}{3} \frac{4.5 \times 12 \text{ lb/in}}{32.2 \text{ ft/s}^2}}} = 0.715 \text{ s}$$

(b)

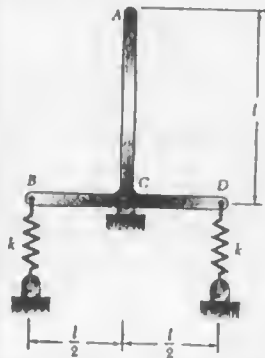
$$\bar{v}_m = r \dot{\theta}_m$$

$$\bar{v}_m = \theta_m \omega_n$$

$$\bar{v}_m = r \theta_m \omega_n \quad r \theta_m = 0.4 \text{ ft}$$

$$\bar{v}_m = \left( \frac{0.4 \text{ ft}}{12} \right) \left( \frac{2\pi}{0.715 \text{ s}} \right) = 0.293 \text{ ft/s}$$

19.79



GIVEN:

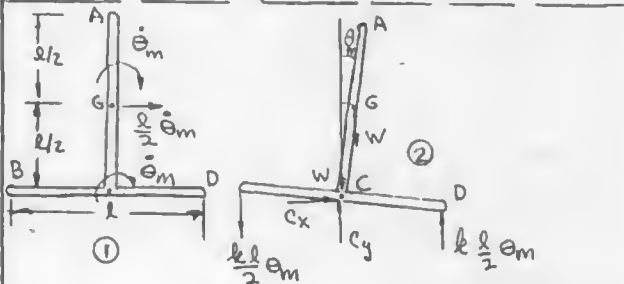
$$W_{AC} = W_{BD} = W = 1.2 \text{ lb}$$

$$l = 8 \text{ in.}$$

$$k = 0.6 \text{ lb/in}$$

FIND:

FREQUENCY OF SMALL OSCILLATIONS



POSITION ①

$$T_1 = 2\left(\frac{1}{2} \bar{I} \dot{\theta}_m^2\right) + \frac{1}{2} m \left(\frac{l}{2} \dot{\theta}_m\right)^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = -W \frac{l}{2} (1 - \cos \theta_m) + \frac{1}{2} k \left(\frac{l}{2} \theta_m\right)^2$$

$$V_2 = -\frac{Wl}{2} \frac{\theta_m^2}{2} + \frac{k l^2}{4} \theta_m^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} (2\bar{I}) \dot{\theta}_m^2 + \frac{1}{2} m \frac{l^2}{4} \dot{\theta}_m^2 + 0 = 0 - \frac{Wl}{2} \frac{\theta_m^2}{2} + \frac{k l^2}{4} \theta_m^2$$

$$\dot{\theta}_m = \omega_n \theta_m \quad \bar{I} = \frac{1}{12} W \frac{l^2}{g}$$

$$\left(\frac{W}{g} + \frac{W}{4g}\right) l^2 \omega_n^2 \theta_m^2 = \left(-\frac{Wl}{2} + \frac{k l^2}{2}\right) \theta_m^2$$

$$\omega_n^2 = \frac{-\frac{W}{2} + \frac{k l}{2}}{\frac{3}{12} (W/g) l} = \frac{6}{5} \left(-\frac{g}{2} + \frac{k l}{W/g}\right)$$

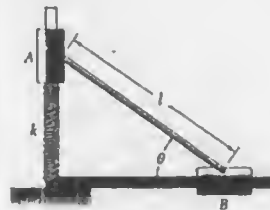
$$\omega_n^2 = \frac{6}{5} \left(-\frac{32.2 \text{ ft/s}^2}{2} + \frac{(0.6 \times 12 \text{ lb/ft})}{(1.2 \text{ lb}/32.2 \text{ ft/s}^2)}\right)$$

$$\omega_n^2 = \frac{6}{5} (-48.3 + 193.2) = 173.9 \text{ s}^{-2}$$

$$\omega_n = 13.19 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{13.19}{2\pi} = 2.10 \text{ Hz.}$$

19.80



GIVEN:

$$\text{Negl. ROD AB}$$

$$l = 0.6 \text{ m}$$

$$\text{COLLARS A AND B OF NEGLIGIBLE MASS}$$

$$k = 1.2 \text{ kN/m}$$

$$\theta = 40^\circ \text{ AT EQUILIBRIUM}$$

FIND:

PERIOD OF VIBRATION

VERTICAL ROD

$$y = l \sin \theta$$

$$\delta y = l \cos \theta \delta \theta$$

$$\delta \dot{y} = l \cos \theta \delta \dot{\theta}$$

$$\dot{x} = -l \sin \theta \dot{\theta}$$

$$\delta x = -l \sin \theta \delta \theta$$

$$\delta \dot{x} = -l \sin \theta \delta \dot{\theta}$$

$$\bar{y} = y/2 \quad \bar{x} = x/2$$

POSITION ① (MAXIMUM VELOCITY,  $\delta \dot{\theta}_m$ )

$$T_1 = \frac{1}{2} \bar{I} \delta \dot{\theta}_m^2 + \frac{1}{2} m (\delta \dot{x}_m^2 + \delta \dot{y}_m^2)$$

$$T_1 = \frac{1}{2} \left( \frac{1}{12} m l^2 \right) (\delta \dot{\theta}_m^2) + \frac{1}{2} m \left[ \left( \frac{l}{2} \sin \theta \right)^2 + \left( \frac{l}{2} \cos \theta \right)^2 \right] (\delta \dot{\theta}_m^2)$$

$$T_1 = \frac{1}{2} m l^2 \left[ \frac{1}{12} + \frac{1}{4} \right] (\delta \dot{\theta}_m^2) = \frac{1}{2} m l^2 \frac{5}{3} (\delta \dot{\theta}_m^2)$$

$$V_1 = \frac{1}{2} k (\delta s_T)^2 + mg \bar{y}$$

POSITION ② (ZERO VELOCITY, MAXIMUM  $\delta \theta_m$ )

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k (\delta y + \delta s_T)^2 + mg (\bar{y} - \delta y_m)$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m l^2 \frac{5}{3} (\delta \dot{\theta}_m^2) + \frac{1}{2} k \delta s_T^2 + mg \bar{y} = 0 + \frac{1}{2} k (\delta y + \delta s_T)^2 + mg (\bar{y} - \delta y_m)$$

$$m l^2 \frac{5}{3} (\delta \dot{\theta}_m^2) + k \delta s_T^2 + mg \bar{y} = k (\delta y^2 + 2 \delta y \delta s_T + \delta s_T^2) + mg (\bar{y} - \delta y_m)$$

BUT WHEN THE ROD IS IN EQUILIBRIUM,

$$\sum M_B = mg \frac{l}{2} - k \delta s_T l = 0 \quad mg = 2k \delta s_T \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$m l^2 \frac{5}{3} (\delta \dot{\theta}_m^2) = k \delta y_m^2 \quad \delta y_m = l \cos \theta \delta \theta_m$$

$$m l^2 \frac{5}{3} (\delta \dot{\theta}_m^2) = k l^2 \cos^2 \theta (\delta \theta_m)^2$$

FOR SIMPLE HARMONIC MOTION

$$\delta \theta = \delta \theta_m \sin(\omega_n t + \phi)$$

$$\delta \dot{\theta} = \delta \theta_m \omega_n$$

$$\frac{1}{3} m (\delta \theta_m)^2 \omega_n^2 = k l^2 \cos^2 \theta (\delta \theta_m)^2$$

$$\omega_n^2 = 3 \frac{k}{m} \cos^2 \theta = 3 \frac{(1200 \text{ N/m})}{8 \text{ kg}} \cos^2 40^\circ$$

$$\omega_n^2 = 264.07$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{264.07}} = 0.387 \text{ s}$$

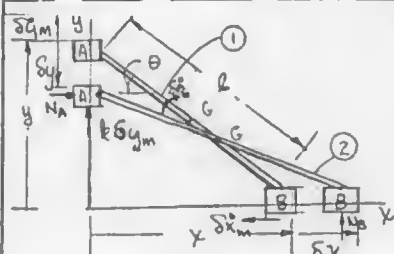
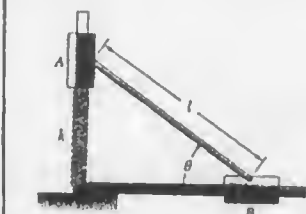
19.81

GIVEN:

$$\begin{aligned} l &= 0.6 \text{ m} \\ m_A &= m_B = m = 8 \text{ kg} \\ k &= 1.2 \text{ kN/m} \\ \theta &= 40^\circ \\ \text{ROD AB OF} \\ &\text{NEGLECTIBLE MASS} \end{aligned}$$

FIND:

PERIOD OF VIBRATION

POSITION ① (MAXIMUM VELOCITY,  $\delta \dot{\theta}_m$ )

$$T_1 = \frac{1}{2} m (\delta \dot{y}_m)^2 + \frac{1}{2} m (\delta \dot{x}_m)^2$$

$$T_1 = \frac{1}{2} m [(l \cos \theta)^2 + (l \sin \theta)^2] (\delta \dot{\theta}_m)^2$$

$$T_1 = \frac{1}{2} m l^2 (\delta \dot{\theta}_m)^2$$

$$V_1 = 0$$

POSITION ② (ZERO VELOCITY, MAXIMUM  $\delta \theta$ )

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k \delta y_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m l^2 (\delta \dot{\theta}_m)^2 + 0 = \frac{1}{2} k \delta y_m^2$$

$$\delta y_m = l \cos \theta \delta \theta_m$$

$$m l^2 (\delta \dot{\theta}_m)^2 = k l^2 \cos^2 \theta (\delta \theta_m)^2$$

SIMPLE HARMONIC MOTION

$$\delta \theta = \delta \theta_m \sin(\omega_n t + \phi)$$

$$\delta \ddot{\theta}_m = \delta \theta_m \omega_n^2$$

$$m l^2 (\delta \ddot{\theta}_m) \omega_n^2 = k l^2 \cos^2 \theta (\delta \theta_m)^2$$

$$\omega_n^2 = \frac{k}{m} \cos^2 \theta$$

$$\omega_n^2 = \frac{1200 \text{ N/m}}{8 \text{ kg}} \cos^2 40^\circ = 88.02 \text{ s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{88.02}} = 0.66915$$

$$\tau_n = 0.670 \text{ s}$$

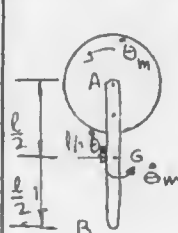
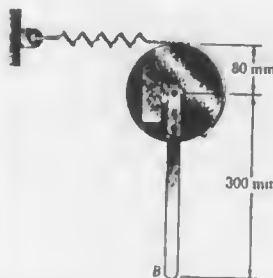
19.82

GIVEN:

$$\begin{aligned} m_{AB} &= 3 \text{ kg} \\ m_{\text{disk}} &= 5 \text{ kg} \\ \text{SPRING IS} \\ &\text{UNSTRETCHED IN} \\ &\text{THE POSITION} \\ &\text{SHOWN, } k = 280 \text{ N/m} \end{aligned}$$

FIND:

PERIOD OF SMALL OSCILLATIONS



①

POSITION ①

$$T_1 = \frac{1}{2} \bar{I}_{\text{disk}} \dot{\theta}_m^2 + \frac{1}{2} (I_A)_{\text{rod}} \dot{\theta}_m^2$$

$$V_1 = 0$$

$$\begin{aligned} \bar{I}_{\text{disk}} &= \frac{1}{2} m r^2 \\ (I_A)_{\text{rod}} &= \frac{1}{3} m_{AB} l^2 \end{aligned}$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k (r \theta_m)^2 + m_{AB} g \frac{l}{2} (1 - \cos \theta_m)$$

$$1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2} k r^2 \theta_m^2 + \frac{m_{AB} g l}{2} \theta_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} \left( \frac{1}{2} m r^2 + \frac{1}{3} m_{AB} l^2 \right) \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} k r^2 \theta_m^2 + \frac{1}{2} m_{AB} g l \theta_m^2$$

$$\ddot{\theta}_m = \omega_n^2 \theta_m$$

$$\left( \frac{1}{2} m r^2 + \frac{1}{3} m_{AB} l^2 \right) \omega_n^2 \theta_m^2 = \left( k r^2 + m_{AB} g l \right) \theta_m^2$$

$$\omega_n^2 = \frac{k r^2 + m_{AB} g l}{\frac{1}{2} m r^2 + \frac{1}{3} m_{AB} l^2}$$

$$\omega_n^2 = \frac{(280 \text{ N/m})(0.08 \text{ m})^2 + (3 \text{ kg})(9.81 \text{ m/s}^2)(0.3/2 \text{ m})}{\frac{1}{2} (5 \text{ kg})(0.08 \text{ m})^2 + \frac{1}{3} (3 \text{ kg})(0.300 \text{ m})^2}$$

$$\omega_n^2 = \frac{6.207}{0.106} = 58.55$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{58.55}} = 0.821 \text{ s}$$

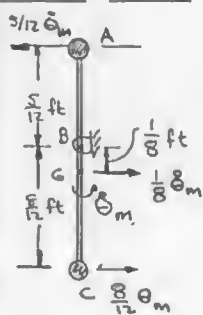
19.83

GIVEN:

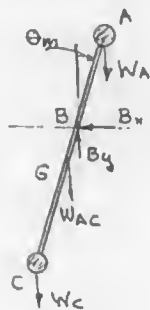
$W_A = 14 \text{ oz}$   $W_C = 10 \text{ oz}$   
 ROD AC WEIGHT = 20 oz  
 VERTICAL PLANE

FIND:

PERIOD OF SMALL OSCILLATIONS



①



②

POSITION ①

$$T_1 = \frac{1}{2} \frac{W_A}{g} \left( \frac{5}{12} \dot{\theta}_m \right)^2 + \frac{1}{2} \frac{W_C}{g} \left( \frac{13}{12} \dot{\theta}_m \right)^2 + \frac{1}{2} \frac{W_{AC}}{g} \left( \frac{13}{12} \dot{\theta}_m \right)^2 + \frac{1}{2} I_{AC} \dot{\theta}_m^2$$

$$I_{AC} = \frac{1}{12} \frac{W_{AC}}{g} \left( \frac{13}{12} \right)^2$$

$$T_1 = \frac{1}{2} g \left[ \frac{14}{16} \left( \frac{5}{12} \right)^2 + \frac{10}{16} \left( \frac{13}{12} \right)^2 + \frac{20}{16} \left( \frac{13}{12} \right)^2 + \frac{1}{2} \left( \frac{20}{16} \right) \left( \frac{13}{12} \right)^2 \right] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} (32.2 \text{ ft/s}^2) [0.1519 + 0.2778 + 0.0453 + 0.1223] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} \left( \frac{0.5715 \text{ lb} \cdot \text{ft}^2}{32.2 \text{ ft/s}^2} \right) \dot{\theta}_m^2 = \frac{1}{2} (0.01775) \dot{\theta}_m^2 (\text{lb} \cdot \text{ft})$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = -W_A \frac{5}{12} (1 - \cos \theta_m) + W_C \frac{13}{12} (1 - \cos \theta_m) + W_{AC} \frac{13}{12} (1 - \cos \theta_m)$$

$$1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \left[ -\left( \frac{14}{16} \right) \left( \frac{5}{12} \right) + \left( \frac{10}{16} \right) \left( \frac{13}{12} \right) + \left( \frac{20}{16} \right) \left( \frac{13}{12} \right) \right] \frac{\theta_m^2}{2} (\text{lb} \cdot \text{ft})$$

$$V_2 = [-0.3646 + 0.4167 + 0.1563] \frac{\theta_m^2}{2}$$

$$V_2 = 0.2084 \frac{\theta_m^2}{2}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} (0.01775) \dot{\theta}_m^2 + 0 = 0 + 0.2084 \frac{\theta_m^2}{2}$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$\omega_n^2 = \frac{0.2084}{0.01775} = 11.738$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{11.738}} = 1.8345$$

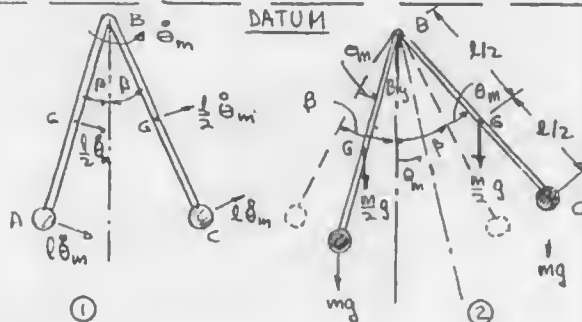
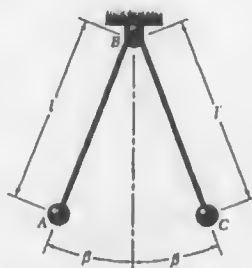
19.84

GIVEN:

SPHERES AND ROD ABC  
 ALL OF MASS M  
 $\beta = 40^\circ$   
 $l = 0.5 \text{ m}$

FIND:

FREQUENCY OF SMALL OSCILLATIONS



①

②

POSITION ①

$$T_1 = \frac{1}{2} m (\dot{x}_A)_m^2 + \frac{1}{2} m (\dot{x}_C)_m^2 + \frac{1}{2} (2I_A) (\dot{\theta}_m^2) + \frac{1}{2} (2 \frac{m}{2} l^2 \dot{\theta}_m^2)$$

$$I_C = \frac{1}{12} m l^2 \quad (x_A)_m = (x_C)_m = l \dot{\theta}_m$$

$$T_1 = m l^2 \dot{\theta}_m^2 + \left( \frac{m l^2}{24} + \frac{m l^2}{8} \right) \dot{\theta}_m^2 = \frac{7}{6} m l^2 \dot{\theta}_m^2$$

$$V_1 = -2 m g l \cos \beta - m g \frac{l}{2} \cos \beta = -\frac{5}{2} m g l \cos \beta$$

POSITION ②

$$T_2 = 0$$

$$V_2 = -m g l \cos (\beta - \theta_m) - \frac{m}{2} g \frac{l}{2} \cos (\beta - \theta_m) - m g l \cos (\beta + \theta_m) - \frac{m}{2} g \frac{l}{2} \cos (\beta + \theta_m)$$

$$V_2 = -\frac{5}{2} m g l [\cos \beta \cos \theta_m + \sin \beta \sin \theta_m + \cos \beta \cos \theta_m + \sin \beta \sin \theta_m]$$

$$V_2 = -\frac{5}{2} m g l \cos \beta \cos \theta_m$$

$$\cos \theta_m \approx 1 - \frac{\theta_m^2}{2} \quad (\text{SMALL ANGLES})$$

$$V_2 = -\frac{5}{2} m g l \cos \beta [1 - \frac{\theta_m^2}{2}]$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{7}{6} m l^2 \dot{\theta}_m^2 - \frac{5}{2} m g l \cos \beta = 0 - \frac{5}{2} m g l \cos \beta (1 - \frac{\theta_m^2}{2})$$

$$\dot{\theta}_m = \omega_n \theta_m$$

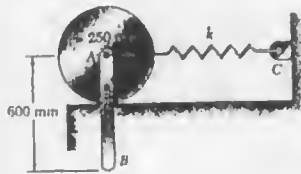
$$\frac{7}{6} l \omega_n^2 \theta_m^2 = \frac{5}{4} g \cos \beta \theta_m^2$$

$$\omega_n^2 = \frac{15}{14} g \cos \beta$$

$$\omega_n^2 = \frac{15}{14} \left( \frac{9.81 \text{ m/s}^2}{0.5 \text{ m}} \right) \cos 40^\circ = 16.10 \text{ s}^{-2}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{16.10}}{2\pi} = 0.639 \text{ Hz}$$

19.85

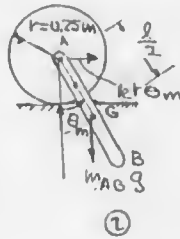
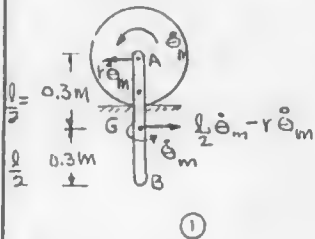


GIVEN:

0.8 kg ROD BOLTED TO  
1.2 kg DISK  
 $k = 12 \text{ N/m}$   
DISK ROLLS WITHOUT  
SLIDING.

FIND:

PERIOD OF SMALL  
OSCILLATIONS



POSITION ①

$$T_1 = \frac{1}{2} (\bar{I}_C) \dot{\theta}_m^2 + \frac{1}{2} m_{AB} \left( \frac{1}{2} - r \right)^2 \dot{\theta}_m^2 + \frac{1}{2} (\bar{I}_C) \dot{\theta}_m^2 + \frac{1}{2} m_{\text{DISK}} r^2 \dot{\theta}_m^2$$

$$(I_C)_{AB} = \frac{1}{2} m l^2 = \frac{1}{2} (0.8) (0.6)^2 = 0.024 \text{ kg} \cdot \text{m}^2$$

$$m_{AB} \left( \frac{1}{2} - r \right)^2 = (0.8) (0.3 - 0.25)^2 = 0.002 \text{ kg} \cdot \text{m}^2$$

$$(I_C)_{\text{DISK}} = \frac{1}{2} m_{\text{DISK}} r^2 = \frac{1}{2} (1.2) (0.25)^2 = 0.0375 \text{ kg} \cdot \text{m}^2$$

$$m r^2 = 1.2 (0.25)^2 = 0.0750 \text{ kg} \cdot \text{m}^2$$

$$T_1 = \frac{1}{2} [0.024 + 0.002 + 0.0375 + 0.0750] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} [0.1385] \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k (r \theta_m)^2 + m_{AB} g \frac{l}{2} (1 - \cos \theta_m)$$

$$1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2} \text{ (SMALL ANGLES)}$$

$$V_2 = \frac{1}{2} (12 \text{ N/m}) (0.25 \text{ m})^2 \theta_m^2 + (0.8 \text{ kg}) (9.81 \frac{\text{m}}{\text{s}^2}) (\frac{0.6 \text{ m}}{2}) \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2} [0.750 + 2.354] \theta_m^2 = \frac{1}{2} (3.104) \theta_m^2 \text{ J} \cdot \text{m}$$

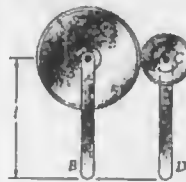
$$T_1 + V_1 = T_2 + V_2 \quad \dot{\theta}_m^2 = \omega_n^2 \theta_m^2$$

$$\frac{1}{2} (0.1385) \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} (3.104) \theta_m^2$$

$$\omega_n^2 = \frac{(3.104 \text{ J} \cdot \text{m})}{(0.1385 \text{ kg} \cdot \text{m}^2)} = 22.41 \text{ s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{22.41}} = 1.327 \text{ s}$$

19.86

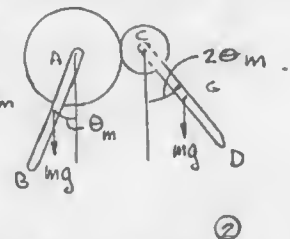
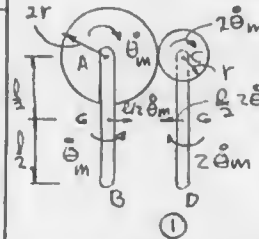


GIVEN:

RODS AB AND CD EACH OF  
MASS  $m$  AND LENGTH  $l$   
ATTACHED TO GEARS A AND C  
MASS OF GEAR A =  $4m$   
MASS OF GEAR C =  $m$

FIND:

PERIOD OF SMALL OSCILLATIONS



KINEMATICS

$$2r \theta_A = r \theta_C \quad 2\theta_A = \theta_C$$

$$\text{LET } \theta_A = \theta_m \quad 2\theta_m = (\theta_C)_m$$

$$2\dot{\theta}_m = (\dot{\theta}_C)_m$$

POSITION ①

$$T_1 = \frac{1}{2} \bar{I}_A \dot{\theta}_m^2 + \frac{1}{2} \bar{I}_C (2\dot{\theta}_m)^2 + \frac{1}{2} \bar{I}_{AB} \dot{\theta}_m^2 + \frac{1}{2} \bar{I}_{CD} (2\dot{\theta}_m)^2$$

$$+ \frac{1}{2} m_{AB} \left( \frac{l}{2} \dot{\theta}_m \right)^2 + \frac{1}{2} m_{CD} \left( \frac{l}{2} 2\dot{\theta}_m \right)^2$$

$$\bar{I}_A = \frac{1}{2} (4m) (2r)^2 = 8mr^2$$

$$\bar{I}_C = \frac{1}{2} (m) (r)^2 = \frac{1}{2} mr^2$$

$$\bar{I}_{AB} = \frac{1}{2} m l^2 \quad \bar{I}_{CD} = \frac{1}{2} m l^2$$

$$T_1 = \frac{1}{2} m [8r^2 + (r^2/2) 4 + l^2/2 + l^2/3 + l^2/4 + l^2]$$

$$T_1 = \frac{1}{2} m [10r^2 + \frac{5}{3} l^2] \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = m g \frac{l}{2} (1 - \cos \theta_m) + m g l (1 - \cos 4\theta_m)$$

$$\text{SMALL ANGLES} \quad 1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$1 - \cos 4\theta_m = 2 \sin^2 2\theta_m \approx 2 \theta_m^2$$

$$V_2 = \frac{1}{2} m g l (\frac{\theta_m^2}{2} + 2\theta_m^2) = \frac{1}{2} m g l \frac{5\theta_m^2}{2}$$

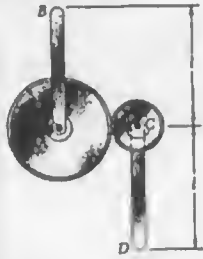
$$T_1 + V_1 = T_2 + V_2 \quad \dot{\theta}_m^2 = \omega_n^2 \theta_m^2$$

$$\frac{1}{2} m [10r^2 + \frac{5}{3} l^2] \omega_n^2 \theta_m^2 + 0 = 0 + \frac{1}{2} m g l \frac{5\theta_m^2}{2}$$

$$\omega_n^2 = \frac{\frac{5}{2} g l}{10r^2 + \frac{5}{3} l^2} = \frac{3 g l}{12r^2 + 2l^2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{12r^2 + 2l^2}{3 g l}}$$

19.87

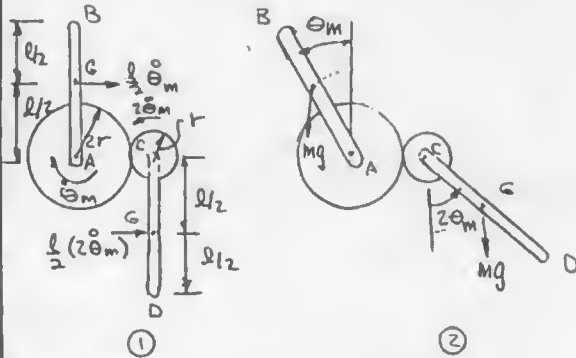


GIVEN:

RODS AB AND BC EACH  
OF MASS  $m$   
GEAR A OF MASS  $4m$   
GEAR C OF MASS  $m$

FIND:

PERIOD OF SMALL  
OSCILLATIONS

KINEMATICS  $2r\dot{\theta}_A = r\dot{\theta}_C$  $2\dot{\theta}_A = \dot{\theta}_C$  $2\dot{\theta}_A = \dot{\theta}_C$ LET  $\theta_A = \theta_m$  $2\dot{\theta}_m = (\dot{\theta}_C)_m$  $2\ddot{\theta}_m = (\ddot{\theta}_C)_m$ 

POSITION ①

$$T_1 = \frac{1}{2} \bar{I}_A \dot{\theta}_m^2 + \frac{1}{2} \bar{I}_C (\dot{\theta}_m)^2 + \frac{1}{2} \bar{I}_{AB} \dot{\theta}_m^2 + \frac{1}{2} \bar{I}_{BC} (2\dot{\theta}_m)^2 + \frac{1}{2} m_{AB} \left( \frac{l}{2} \dot{\theta}_m \right)^2 + \frac{1}{2} m_{BC} \left( \frac{l}{2} 2\dot{\theta}_m \right)^2$$

$$\bar{I}_A = \frac{1}{2} (4m) (2r)^2 = 8mr^2$$

$$\bar{I}_C = \frac{1}{2} (m) (r^2) = \frac{1}{2} mr^2$$

$$\bar{I}_{AB} = \frac{1}{12} ml^2 \quad \bar{I}_{BC} = \frac{1}{12} ml^2$$

$$T_1 = \frac{1}{2} m [8r^2 + (r^2/2) + l^2/12 + l^2/3 + l^2/4 + l^2] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} m [10r^2 + \frac{5}{3} l^2] \dot{\theta}_m^2 \quad V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = -mg \frac{l}{2} (1 - \cos \theta_m) + mg \frac{l}{2} (1 - \cos 2\theta_m)$$

SMALL ANGLES  $1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$ 

$$1 - \cos 2\theta_m = 2 \sin^2 \theta_m \approx 2\theta_m^2$$

$$V_2 = -mg \frac{l}{2} \frac{\theta_m^2}{2} + mg \frac{l}{2} 2\theta_m^2 = \frac{1}{2} mg \frac{3}{2} l \theta_m^2$$

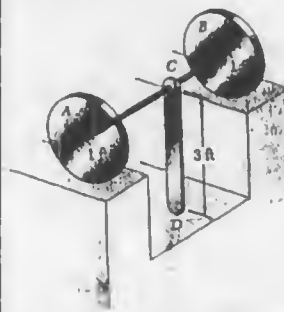
$$T_1 + V_1 = T_2 + V_2 \quad \ddot{\theta}_m = \omega_n^2 \theta_m$$

$$\frac{1}{2} m [10r^2 + \frac{5}{3} l^2] \theta_m^2 \omega_n^2 + 0 = 0 + \frac{1}{2} mg \frac{3}{2} l \theta_m^2$$

$$\omega_n^2 = \frac{3gl}{10r^2 + \frac{5}{3} l^2} = \frac{9gl}{60r^2 + 10l^2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{60r^2 + 10l^2}{9gl}}$$

19.88

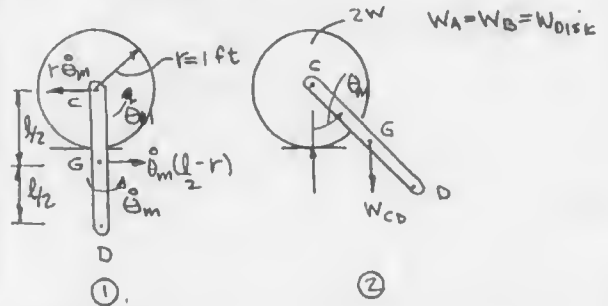


GIVEN:

10-lb ROD CD  
DISKS A AND B EACH  
WEIGH 20 lb  
AC OF NEGLIGIBLE  
WEIGHT  
NO SLIDING

FIND:

PERIOD OF SMALL  
OSCILLATIONS



POSITION ①

$$T_1 = \frac{1}{2} 2 \bar{I}_A \dot{\theta}_m^2 + \frac{1}{2} (2W_{disk}) \left( \frac{l}{2} \dot{\theta}_m \right)^2 + \frac{1}{2} \bar{I}_C \dot{\theta}_m^2 + \frac{1}{2} W_{CD} \left( \frac{l}{2} - r \right)^2 \dot{\theta}_m^2$$

$$\bar{I}_A = \frac{1}{2} \frac{W_{disk}}{g} r^2 = \frac{1}{2} \left( \frac{20}{g} \right) (1)^2 = \frac{10}{g}$$

$$\bar{I}_C = \frac{1}{12} \frac{W_{CD}}{g} l^2 = \frac{1}{12} \left( \frac{10}{g} \right) (3)^2 = \frac{15}{2g}$$

$$T_1 = \frac{1}{2} g [20 + 40 + \frac{15}{2} + \frac{5}{2}] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} g (70) \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = W_{CD} \frac{l}{2} (1 - \cos \theta_m)$$

SMALL ANGLES  $1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$ 

$$V_2 = \frac{1}{2} W_{CD} l \frac{\theta_m^2}{2} = \frac{1}{2} (10) (1.5) \theta_m^2 = \frac{1}{2} 15 \theta_m^2$$

$$T_1 + V_1 = T_2 + V_2 \quad \ddot{\theta}_m = \omega_n^2 \theta_m$$

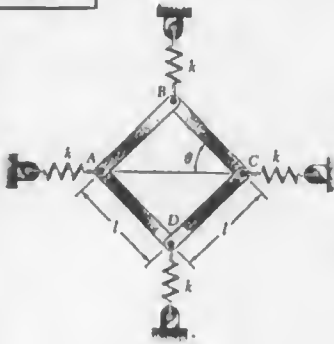
$$\frac{1}{2} g (70) \omega_n^2 \theta_m^2 + 0 = 0 + \frac{1}{2} 15 \theta_m^2$$

$$\omega_n^2 = \frac{15g}{70}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{70}{(15)(32.2)}} = 2.39 \text{ s}$$



19.89

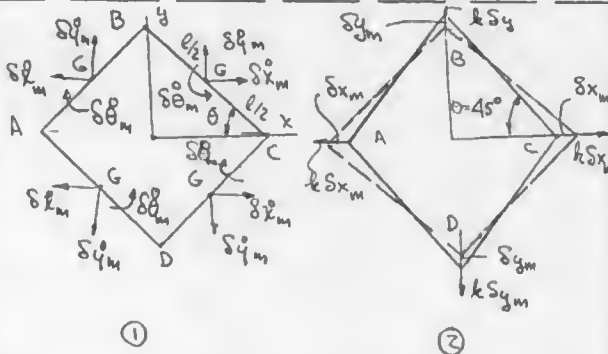


GIVEN:

FOUR BARS  
OF EQUAL  
MASS  $m$   
AND EQUAL  
LENGTH  $l$   
 $\theta = 45^\circ$   
HORIZONTAL  
PLANE

FIND:

PERIOD OF  
VIBRATION IF  
A AND C ARE  
GIVEN SMALL EQUAL  
DISPLACEMENTS  
AND RELEASED



KINEMATICS

$$\begin{aligned} BC \quad x_C &= l/2 \cos \theta & \delta x_C &= -l/2 \sin \theta \delta \theta \\ & & \delta \dot{x}_C &= -l/2 \sin \theta \delta \dot{\theta} \\ y_C &= l/2 \sin \theta & \delta y_C &= l/2 \cos \theta \delta \theta \\ & & \delta \dot{y}_C &= l/2 \cos \theta \delta \dot{\theta} \end{aligned}$$

$$\begin{aligned} x_C &= l \cos \theta & \delta x_C &= -l \sin \theta \delta \theta \\ & & \delta \dot{x}_C &= -l \sin \theta \delta \dot{\theta} \\ y_C &= l \sin \theta & \delta y_C &= l \cos \theta \delta \theta \\ & & \delta \dot{y}_C &= l \cos \theta \delta \dot{\theta} \end{aligned}$$

$$\begin{aligned} y_C &= 0 \\ y_B &= l \sin \theta & \delta y_B &= l \cos \theta \delta \theta \\ & & \delta \dot{y}_B &= l \cos \theta \delta \dot{\theta} \end{aligned}$$

THE KINETIC ENERGY IS THE SAME FOR ALL FOUR BARS.

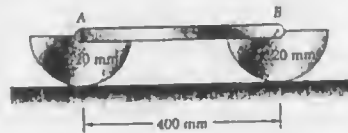
POSITION ①

$$\begin{aligned} T_1 &= 4 \left[ \frac{1}{2} \bar{I} (\delta \dot{\theta}_m)^2 + \frac{1}{2} m [(\delta \dot{x}_C)^2 + (\delta \dot{y}_C)^2] \right] \\ \bar{I} &= \frac{1}{12} m l^2 \\ T_1 &= 2 m l^2 \left[ \frac{1}{12} + \frac{1}{4} (\sin^2 \theta_m + \cos^2 \theta_m) \right] \delta \dot{\theta}_m^2 \\ T_1 &= \frac{2}{3} m l^2 \delta \dot{\theta}_m^2 \\ V_1 &= 0 \end{aligned}$$

POSITION ②

$$\begin{aligned} T_2 &= 0 \\ V_2 &= (2) \frac{1}{2} k (\delta x_m)^2 + (2) \frac{1}{2} k (\delta y_m)^2 \\ V_2 &= k (l^2 \sin^2 \theta + l^2 \cos^2 \theta) \delta \theta_m^2 = k l^2 \delta \theta_m^2 \\ T_1 + V_1 &= T_2 + V_2 \\ \frac{2}{3} m l^2 (\delta \dot{\theta}_m)^2 + 0 &= 0 + k l^2 (\delta \theta_m)^2 \\ \delta \dot{\theta}_m &= \omega_n \delta \theta_m \\ \omega_n^2 &= \frac{3}{2} k/m \\ \tau_n &= 2\pi \sqrt{\frac{2m}{3k}} \end{aligned}$$

19.90

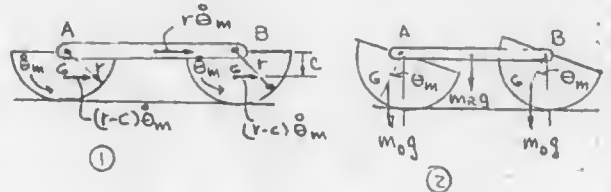


GIVEN:

DISKS OF MASS 3 kg  
EACH  
MASS OF ROD AB  
= 2 kg  
NO SLIDING

FIND:

PERIOD FOR  
SMALL OSCILLATIONS



POSITION ①

$$T_1 = 2 \left( \frac{1}{2} \right) \bar{I}_D \dot{\theta}_m^2 + 2 \left( \frac{1}{2} \right) m_D (r-c)^2 \dot{\theta}_m^2 + \frac{1}{2} m_r l^2 \dot{\theta}_m^2$$

FOR ONE DISK

$$\bar{I}_D = (\bar{I}_O)_A - m_D c^2 = \frac{1}{2} m_D r^2 - m_D \left( \frac{4r}{3\pi} \right)^2 = m_D \left[ \frac{r^2}{2} - \frac{16r^2}{9\pi^2} \right]$$

$$\bar{I}_D = 0.3199 m_D r^2$$

$$m_D (r-c)^2 = m_D r^2 \left( 1 - \frac{4}{3\pi} \right)^2 = 0.3315 m_D r^2$$

$$T_1 = [(0.3199 + 0.3315) m_D r^2 + 0.5 m_r l^2] \dot{\theta}_m^2$$

$$T_1 = [0.6512 m_D + 0.5 m_r] r^2 \dot{\theta}_m^2$$

POSITION ②

$$T_2 = 0$$

$$V_2 = 2 m_D g c (1 - \cos \theta_m)$$

$$c = \frac{4r}{3\pi} \quad 1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2} \quad (\text{SMALL ANGLES})$$

$$V_2 = 2 m_D g \frac{4r}{3\pi} \frac{\theta_m^2}{2}$$

$$V_2 = m_D r \frac{g(4)}{3\pi} = m_D r \frac{(9.81)(4)}{(3\pi)} = 4.164 m_D r$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.6512 m_D + 0.5 m_r) r^2 \dot{\theta}_m^2 = 0 + 4.164 m_D r$$

$$\dot{\theta}_m = \omega_n \theta_m$$

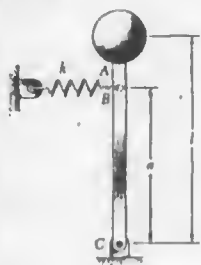
$$\omega_n^2 = \frac{4.164 m_D r}{(0.6512 m_D + 0.5 m_r) r^2} = \frac{(4.164)(3)}{(0.6512)(3) + 0.5(2)} 0.120$$

$$\omega_n^2 = \frac{12.490}{0.3544} = 35.24$$

$$\omega_n = 5.936$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.936} = 1.058 \text{ s}$$

19.91



GIVEN:

SPHERE OF WEIGHT  $W$   
BAR ABC OF NEGUGIBLE  
WEIGHT

FIND:

- (a) FREQUENCY OF SMALL  
OSCILLATIONS  
(b) SHALEST VALUE OF  
 $a$  FOR WHICH  
OSCILLATIONS WILL  
OCCUR.

19.92



GIVEN:

SPHERE OF WEIGHT  $W$   
 $f_n = 1.5 \text{ Hz}$  WHEN  $W = 2 \text{ lb}$   
 $f_n = 0.8 \text{ Hz}$  WHEN  $W = 4 \text{ lb}$

FIND:

FOR GIVEN  $k, a$  AND  $l$ , THE  
LARGEST VALUE OF  $W$   
FOR WHICH OSCILLATIONS  
WILL OCCUR

SEE SOLUTION TO PROB 19.91 FOR THE  
FREQUENCY IN TERMS OF  $W, k, a$  AND  $l$

$$f_n = \frac{1}{2\pi} \sqrt{g/l \left( \frac{ka^2}{Wl} - 1 \right)}$$

$$f_n = 1.5 \text{ Hz } W = 2 \text{ lb} \quad 1.5 = \frac{1}{2\pi} \sqrt{g/l \left( \frac{ka^2}{2l} - 1 \right)} \quad (1)$$

$$f_n = 0.8 \text{ Hz } W = 4 \text{ lb} \quad 0.8 = \frac{1}{2\pi} \sqrt{g/l \left( \frac{ka^2}{4l} - 1 \right)} \quad (2)$$

DIVIDE (1) BY (2)

$$\left( \frac{1.5}{0.8} \right)^2 = \left( \frac{ka^2}{2l} - 1 \right) / \left( \frac{ka^2}{4l} - 1 \right)$$

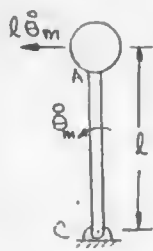
$$3.516 \frac{ka^2}{4l} - 3.516 = \frac{ka^2}{2l} - 1$$

$$\frac{ka^2}{l} \left[ \frac{3.516}{4} - \frac{1}{2} \right] = 2.516$$

$$\frac{ka^2}{l} = 6.640$$

$$f_n = \frac{1}{2\pi} \sqrt{g/l \left( \frac{6.640}{W} - 1 \right)}, \quad f_n = 0, \quad \frac{6.640}{W} - 1 = 0$$

$$W \leq 6.64 \text{ lb}$$



(a) ①

POSITION ①

$$T_1 = \frac{1}{2} m (l \dot{\theta}_m)^2 = \frac{1}{2} m l^2 \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = \frac{1}{2} k (a \sin \theta_m)^2 - W l (1 - \cos \theta_m)$$

SMALL ANGLES  $\sin \theta_m \approx \theta_m$

$$1 - \cos \theta_m \approx 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2} k a^2 \frac{\theta_m^2}{2} - W l \frac{\theta_m^2}{2} = \frac{1}{2} [ka^2 - Wl] \theta_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m l^2 \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} [ka^2 - Wl] \theta_m^2$$

$$\dot{\theta}_m = \omega_n \theta_m \quad \omega_n = W/g$$

$$\frac{W}{g} l^2 \omega_n^2 \theta_m^2 = ka^2 - Wl$$

$$\omega_n^2 = \frac{ka^2 - Wl}{(W/g) l^2} = g/l \left[ \frac{ka^2}{Wl} - 1 \right]$$

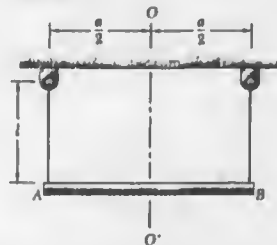
$$f_n = \frac{1}{2\pi} \omega_n = \frac{1}{2\pi} \sqrt{g/l \left( \frac{ka^2}{Wl} - 1 \right)}$$

$$(b) f_n = 0$$

$$\frac{ka^2}{Wl} - 1 > 0$$

$$a > \sqrt{Wl/k}$$

19.93



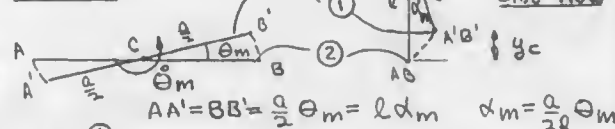
GIVEN:

PIPE SUSPENDED FROM  
TWO CABLES AT A  
AND B

FIND:

FREQUENCY  
VIBRATION FOR A  
SMALL ROTATION  
ABOUT OO'

TOP VIEW



POSITION ①

$$T_1 = 0 \quad V_1 = m g y_c = m g l (1 - \cos \alpha)$$

SMALL ANGLES  $1 - \cos \alpha \approx 2 \sin^2 \frac{\alpha}{2} \approx \frac{\alpha^2}{2}$

$$V_1 = m g l \left( \frac{\alpha^2}{8 l^2} \right) \theta_m^2$$

POSITION ②

$$T_2 = \frac{1}{2} I \dot{\theta}_m^2 = \frac{1}{2} \left( \frac{1}{2} m a^2 \right) \dot{\theta}_m^2 \quad V_2 = 0$$

$$\dot{\theta}_m = \omega_n \theta_m$$

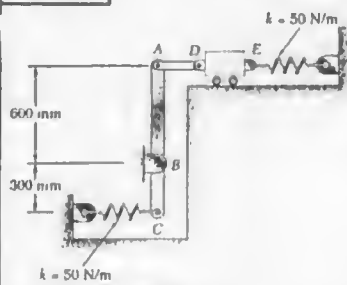
$$T_1 + V_1 = T_2 + V_2$$

$$m g l \left( \frac{\alpha^2}{8 l^2} \right) \theta_m^2 + 0 = \frac{1}{4} m a^2 \omega_n^2 \theta_m^2$$

$$\omega_n^2 = 3g/l$$

$$f_n = \frac{1}{2\pi} \sqrt{3g/l}$$

19.94

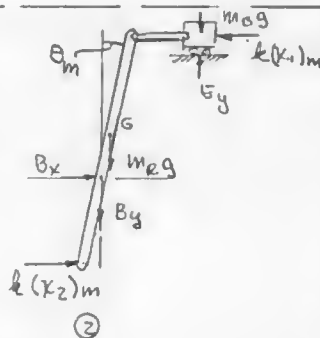
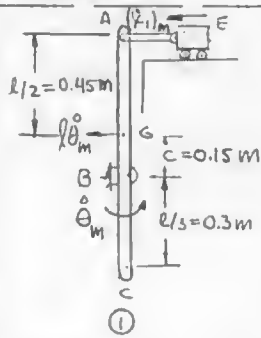


GIVEN:

2 kg ROD ABC  
2 kg BLOCK DE  
SPRINGS ACT  
IN TENSION OR  
COMPRESSION

FIND:

FREQUENCY OF  
SMALL VIBRATIONS



POSITION ①

$$T_1 = \frac{1}{2}(\bar{I}_R)\dot{\theta}_m^2 + \frac{1}{2}(m_R C^2)\dot{\theta}_m^2 + \frac{1}{2}m_E(\dot{x}_1)^2$$

$$\bar{I}_R = \frac{1}{12}m_R l^2 = \frac{1}{12}(2 \text{ kg})(0.9 \text{ m})^2 = 0.135 \text{ kg} \cdot \text{m}^2$$

$$m_E C^2 = 2 \text{ kg} (0.15 \text{ m})^2 = 0.0225 \text{ kg} \cdot \text{m}^2$$

$$x_1 = 0.6 \theta_m \quad m_E(\dot{x}_1)^2 = (2 \text{ kg})(0.6 \dot{\theta}_m \text{ m})^2$$

$$\dot{x}_1 = 0.6 \dot{\theta}_m$$

$$m_E(\dot{x}_1)^2 = 0.72 \dot{\theta}_m^2 \text{ m}^2$$

$$T_1 = \frac{1}{2} [0.135 + 0.0225 + 0.72] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} (0.9) \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k(x_1)^2 + \frac{1}{2} k(x_2)^2 - m_R g C (1 - \cos \theta_m)$$

$$(x_1)_m = 0.6 \theta_m \quad x_2 = 0.3 \theta_m$$

$$1 - \cos \theta_m = 2 \sin^2 \frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2} [(50 \text{ N/m})(0.6 \text{ m})^2 \theta_m^2 + (50 \text{ N/m})(0.3 \text{ m})^2 \theta_m^2 - (2 \text{ kg})(9.81 \text{ m/s}^2)(0.15) \theta_m^2]$$

$$V_2 = \frac{1}{2} [18 + 4.5 - 2.943] \theta_m^2$$

$$V_2 = \frac{1}{2} (19.55) \theta_m^2$$

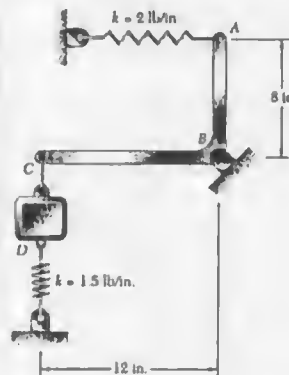
$$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2} (0.9) \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} (19.55) \theta_m^2$$

$$\dot{\theta}_m = \omega_n^2 \theta_m^2 \quad (0.9)(\omega_n^2) \theta_m^2 = (19.55) \theta_m^2$$

$$\omega_n^2 = \frac{19.55}{0.9} = 21.73 \text{ s}^{-2}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{21.73}}{2\pi} = 0.742 \text{ Hz}$$

19.95

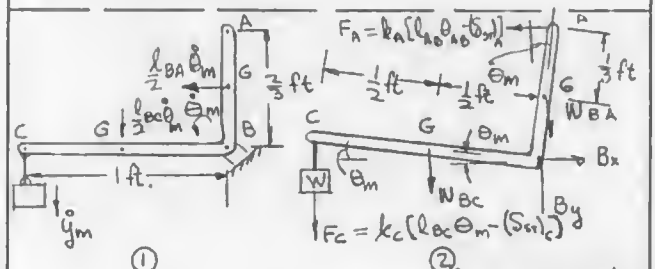


GIVEN:

$W_{ABC} = 1.4 \text{ lb}$   
 $W = 3 \text{ lb}$   
SPRINGS ACT IN  
TENSION OR  
COMPRESSION

FIND:

FREQUENCY OF  
SMALL OSCILLATIONS



POSITION ①

$$T_1 = \frac{1}{2} \bar{I}_{BC}(\dot{\theta}_m)^2 + \frac{1}{2} m_{BC}(\dot{x}_1)^2 + \frac{1}{2} \bar{I}_{BA}(\dot{\theta}_m)^2 + \frac{1}{2} m_{BA}(\dot{x}_2)^2 + \frac{1}{2} m \dot{y}_m^2$$

$$\bar{I}_{BC} + m_{BC}(\frac{l}{2})^2 = \frac{1}{12} m_{BC} l_{BC}^2 + \frac{1}{4} m_{BC} l_{BC}^2 = \frac{1}{3} m_{BC} l_{BC}^2$$

$$\bar{I}_{BA} + m_{BA}(\frac{l}{2})^2 = \frac{1}{12} m_{AB} l_{AB}^2 + \frac{1}{4} m_{AB} l_{AB}^2 = \frac{1}{3} m_{AB} l_{AB}^2$$

$$W_{BC} = \frac{12}{20} W_{ABC} = \frac{3}{5} (1.4 \text{ lb}) = 0.840 \text{ lb}$$

$$W_{BA} = \frac{8}{20} W_{ABC} = \frac{2}{5} (1.4 \text{ lb}) = 0.560 \text{ lb}$$

$$\frac{1}{3} m_{BC} l_{BC}^2 = \frac{1}{3} \left( \frac{0.840 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (1 \text{ ft})^2 = 0.008696 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\frac{1}{3} m_{BA} l_{BA}^2 = \frac{1}{3} \left( \frac{0.560 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (3 \text{ ft})^2 = 0.002577 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\dot{y}_m = l_{BC} \dot{\theta}_m$$

$$m \dot{y}_m^2 = m l_{BC}^2 \dot{\theta}_m^2 = \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} (1 \text{ ft})^2 \dot{\theta}_m^2 = 0.09317 \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} [0.008696 + 0.002577 + 0.09317] \dot{\theta}_m^2 = 0.1044 \frac{\dot{\theta}_m^2}{2}$$

$$V_1 = \frac{1}{2} k_c (S_{sr})_c^2 + \frac{1}{2} k_a (S_{sr})_a^2$$

POSITION ②  $T_2 = 0$ 

$$V_2 = W l_{BC} \theta_m + W_{BC} l_{BC} (2 \theta_m - W_{BA} l_{AB} / 2) (1 - \cos \theta_m) + \frac{1}{2} k_c (l_{BC} \theta_m + (S_{sr})_c)^2 + \frac{1}{2} k_a (l_{AB} \theta_m + (S_{sr})_a)^2$$

WHEN THE SYSTEM IS IN EQUILIBRIUM ( $\theta = 0$ )

$$\sum M_B = 0 = W l_{BC} + W_{BC} l_{BC} - k_c (S_{sr})_c l_{BC} - k_a (S_{sr})_a l_{AB} (1)$$

# 19.95 CONTINUED

$$1 - \cos \Theta_m = 2 \sin^2 \frac{\Theta_m}{2} \approx \frac{\Theta_m^2}{2}$$

$$V_2 = [W_{BC} + W_{BC}(\ell_{BC}/2)] \Theta_m - [W_{BA} \ell_{AB} / 2 (\Theta_m^2/2)] + \frac{1}{2} k_c \ell_{BC}^2 \Theta_m^2 - k_s \ell_{BC} \Theta_m + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\ell_{AB}^2 \Theta_m^2 - \ell_{AB} \delta s r \Theta_m + \frac{1}{2} k_A (\delta s r)^2)$$

TAKING EQUATION (1) INTO ACCOUNT

$$V_2 = -[W_{BA}(\ell_{AB}/2) \Theta_m^2/2 + \frac{1}{2} k_c \ell_{BC}^2 \Theta_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A \ell_{AB}^2 \Theta_m^2 + \frac{1}{2} k_A (\delta s r)^2]$$

$$V_2 = \frac{1}{2} [-0.560 (\frac{1}{3}) + 18 (1)^2 + 24 (\frac{2}{3})^2] \Theta_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\delta s r)^2$$

$$V_2 = \frac{1}{2} [-0.1867 + 18 + 10.67] \Theta_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\delta s r)^2$$

$$V_2 = \frac{1}{2} [28.48] \Theta_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\delta s r)^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} (0.1044) \dot{\Theta}_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\delta s r)^2 = 0 + \frac{1}{2} (28.48) \dot{\Theta}_m^2 + \frac{1}{2} k_c (\delta s r)^2 + \frac{1}{2} k_A (\delta s r)^2$$

$$\dot{\Theta}_m = \omega_n \Theta_m$$

$$0.1044 \omega_n^2 \Theta_m^2 = 28.48 \Theta_m^2$$

$$\omega_n^2 = \frac{28.48}{0.1044} = 272.8 \text{ s}^{-2} \quad f_n = \frac{\sqrt{272.8}}{2\pi} = 2.63 \text{ Hz}$$

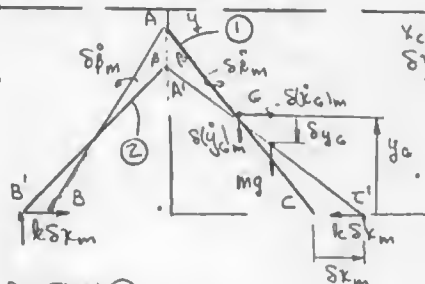
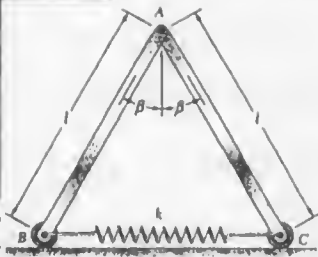
# 19.96

GIVEN:

RODS AB AND AC EACH OF MASS  $m$  AND LENGTH  $l$

FIND:

PERIOD WHEN A IS GIVEN A SMALL DOWN DEFLECTION AND RELEASED



$$\begin{aligned} x_c &= l \sin \beta \\ \delta x_c &= l \cos \beta \delta \beta \\ x_c &= \frac{l}{2} \cos \beta \\ \delta x_c &= -\frac{l}{2} \sin \beta \delta \beta \\ y_c &= \frac{l}{2} \sin \beta \\ \delta y_c &= \frac{l}{2} \cos \beta \delta \beta \\ \delta y_c &= \frac{1}{2} \cos \beta \delta \beta \end{aligned}$$

POSITION ①

$$T_1 = 2 \left[ \frac{1}{2} I \dot{\Theta}_m^2 + \frac{1}{2} m (\dot{s}_x^2 + \dot{s}_y^2) \right] = \left( \frac{1}{2} m l^2 + m l^2 (\sin^2 \beta + \cos^2 \beta) \right) \dot{\Theta}_m^2 = \frac{3}{2} m l^2 \dot{\Theta}_m^2$$

POSITION ②

$$T_1 = 0$$

$$V_2 = \frac{1}{2} k (2 \delta x_m)^2 = \frac{1}{2} (4 l^2 \cos^2 \beta) = 2 l^2 \cos^2 \beta$$

$$\dot{\Theta}_m = \omega_n \dot{\Theta}_m$$

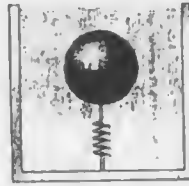
$$\frac{1}{2} m l^2 \omega_n^2 \dot{\Theta}_m^2 + 0 = 0 + 2 l^2 \cos^2 \beta \dot{\Theta}_m^2$$

$$\omega_n^2 = \frac{4 \cos^2 \beta}{m l} \quad T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\cos \beta} \sqrt{\frac{m l}{4}}$$

# 19.97

GIVEN:

SUCHARGE'S SPHERE  
V = VOLUME OF THE SPHERE  
KINETIC ENERGY =  $\frac{1}{2} \rho V U^2$   
WHERE  $\rho$  = MASS DENSITY  
AND  $U$  = VELOCITY OF THE SPHERE  
SPHERE MASS = 500 g  
 $k$  = 500 N/m, HOLLOW OF  
SPHERE RADIUS = 80 mm

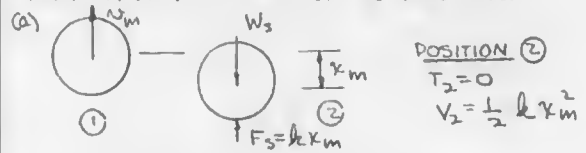


FIND:

(A) PERIOD WHEN DISPLACED VERTICALLY AND RELEASED

(B) PERIOD WHEN THE TANK IS ACCELERATED UPWARD AT  $0.5 \text{ m/s}^2$

THIS IS NOT A DAMPED VIBRATION. HOWEVER THE KINETIC ENERGY OF THE FLUID MUST BE INCLUDED



POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2} k x_m^2$$

$$\text{POSITION ①} \quad T_1 = T_{\text{SPHERE}} + T_{\text{FLUID}} = \frac{1}{2} m_s U_m^2 + \frac{1}{2} \rho V U_m^2$$

$$V_1 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2} m_s U_m^2 + \frac{1}{2} \rho V U_m^2 + 0 = 0 + \frac{1}{2} k x_m^2$$

$$\frac{1}{2} (m_s + \frac{1}{2} \rho V) U_m^2 = \frac{1}{2} k x_m^2 \quad U_m = x_m \omega_n \quad \omega_n^2 = \frac{k}{m_s + \frac{1}{2} \rho V}$$

$$\omega_n^2 = \frac{500 \text{ N/m}}{(0.5 \text{ kg} + (\frac{1}{2} \rho V))} \quad \frac{1}{2} \rho V = \frac{1}{2} (1000 \text{ kg/m}^3) (\frac{4}{3} \pi (0.08 \text{ m})^3) = 1.072 \text{ kg}$$

$$\omega_n^2 = 318 \text{ s}^{-2} \quad T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{318}} = 0.352 \text{ s}$$

(B) ACCELERATION DOES NOT CHANGE MASS,  $T_n = 0.352 \text{ s}$

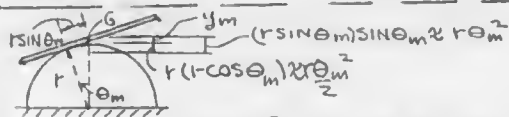
# 19.98

GIVEN:

PLATE ON A SEMI-CIRCULAR CYLINDER AS SHOWN

FIND:

PERIOD FOR SMALL OSCILLATIONS



POSITION ① MAX. DEFLECTION  $T_1 = 0$

$$V_1 = W y_m = m g r \Theta_m^2 / 2$$

POSITION ② ( $\Theta = 0$ )

$$T_2 = \frac{1}{2} I \dot{\Theta}_m^2 = \frac{1}{2} (\frac{1}{2} m l^2) \dot{\Theta}_m^2$$

$$\dot{\Theta}_m = \omega_n \dot{\Theta}_m$$

$$T_2 = \frac{1}{2} (\frac{1}{2} m l^2) \omega_n^2 \dot{\Theta}_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

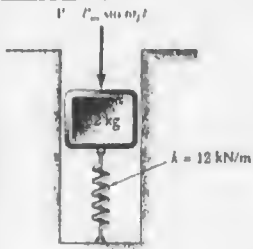
$$0 + \frac{1}{2} m g r \Theta_m^2 = \frac{1}{2} (\frac{1}{2} m l^2) \omega_n^2 \dot{\Theta}_m^2$$

$$\omega_n^2 = \frac{12 g r}{l^2}$$

$$T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{l^2}{12 g r}}$$

$$T_n = \frac{\pi l}{\sqrt{3 g r}}$$

19.99



GIVEN:

SYSTEM AS SHOWN  
WITH  $\omega_f = 10 \text{ rad/s}$   
NO FRICTION  
AMPLITUDE = 15 mm.

FIND:  
 $P_m$ 

EQ. (19.33)  $\gamma_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$   $P_m = \gamma_m (1 - (\omega_f/\omega_n)^2) k$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{12 \times 10^3 \text{ N/m}}{32 \text{ kg}}} = 19.365 \text{ rad/s}$

$P_m = (0.015 \text{ m})(12000 \text{ N/m})(1 - (10/19.365)^2)$

$$P_m = 132.0 \text{ N}$$

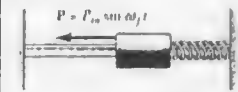
19.100

GIVEN:

9-lb COLLAR ATTACHED  
TO A SPRING,  $k = 2.5 \text{ lb/in.}$   
 $P_m = 3 \text{ lb.}$  NO FRICTION

FIND:

AMPLITUDE IF  
(a)  $\omega_f = 5 \text{ rad/s}$   
(b)  $\omega_f = 10 \text{ rad/s}$



EQ. (19.33)  $\gamma_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(2.5 \times 12 \text{ lb/ft})}{(9 \text{ lb} / 32.2 \text{ ft/s}^2)}} = 10.36 \text{ s}^{-1}$

$P_m/k = 3 \text{ lb} / (2.5 \times 12 \text{ lb/ft}) = 0.100 \text{ ft}$

$\gamma_m = \frac{0.100}{1 - (\omega_f/10.36)^2}$  (a)  $\omega_f = 5 \text{ s}^{-1}$

$\gamma = \frac{0.100 \text{ ft}}{1 - (5/10.36)^2} = 0.1304 \text{ ft}$  (IN PHASE)

(b)  $\omega_f = 10 \text{ rad/s}$   $\gamma = \frac{0.100 \text{ ft}}{1 - (10/10.36)^2} = 1.464 \text{ ft}$  (IN PHASE)

19.101

REFER TO FIGURE FOR PROB 19.100  
SHOWN ABOVE

GIVEN:

9-lb COLLAR ATTACHED TO A SPRING OF  
CONSTANT  $k$ .  $P_m = 2 \text{ lb}$ ,  $\omega_f = 5 \text{ rad/s}$   
AMPLITUDE OF MOTION  $\gamma_m = 6 \text{ in.}$

FIND:

$k$ , IF (a)  $\gamma_m$  IS IN PHASE WITH  $P$   
(b)  $\gamma_m$  IS OUT OF PHASE WITH  $P$

EQ. (19.33)  $\gamma_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$   $\omega_n^2 = k/m$

$\gamma_m = \frac{P_m}{k - m\omega_f^2}$

$k = \frac{P_m}{\gamma_m} + m\omega_f^2$

(a) IN PHASE  $k = \frac{2 \text{ lb}}{1/2 \text{ ft}} + (9 \text{ lb})(5 \text{ s}^{-1})^2 = 10.99 \text{ lb/ft}$

(b) OUT OF PHASE  $\gamma_m = -1/2 \text{ ft}$

$k = \frac{-2 \text{ lb}}{-1/2 \text{ ft}} + (9 \text{ lb})(5 \text{ s}^{-1})^2 = 2.99 \text{ lb/ft}$

19.102

GIVEN:

COLLAR OF MASS  $m$   
ATTACHED TO A SPRING  
OF CONSTANT  $k$

FIND:

RANGE OF  $\omega_f$  FOR WHICH  
AMPLITUDE EXCEEDS THREE  
TIMES THE STATIC DEFLECTION  
CAUSED BY  $P_m$

EQ. (19.33)

$\gamma_m = \frac{P_m/k}{1 - \omega_f^2/\omega_n^2}$

$P_m/k = 8 \text{ ST}$   $\omega_n^2 = \frac{k}{m}$

$\frac{8 \text{ ST}}{1 - \omega_f^2/\omega_n^2} \geq 3 \text{ ST}$

$1 - \omega_f^2/\omega_n^2 \leq 1/3$

$\frac{\omega_f^2}{\omega_n^2} > \frac{2}{3}$

ALSO  $\frac{8 \text{ ST}}{1 - \omega_f^2/\omega_n^2} < -3 \text{ ST}$

$1 - \omega_f^2/\omega_n^2 < -1/3$

$\frac{\omega_f^2}{\omega_n^2} < \frac{4}{3}$

$\frac{4}{3} > \frac{\omega_f^2}{\omega_n^2} > \frac{2}{3}$

$\sqrt{\frac{4k}{3m}} > \omega_f > \sqrt{\frac{2k}{3m}}$

19.103

GIVEN:

8-lb DISK  
RADIUS  $r = 200 \text{ mm}$   
WELDED TO A SHAFT  
FIXED AT B.  
DISK ROTATES  $3^\circ$   
WHEN A STATIC  
COUPLE  $50 \text{ N}\cdot\text{m}$  IS  
APPLIED.  $T_m = 60 \text{ N}\cdot\text{m}$



FIND:

RANGE OF VALUES OF  $\omega_f$   
FOR WHICH THE AMPLITUDE  
IS LESS THAN THE STATIC  
DEFLECTION CAUSED BY  $T_m$

ANALOGOUS TO EQ. (19.33)

$\Theta_m = \frac{T_m/k}{1 - \omega_f^2/\omega_n^2} = \frac{\Theta_{ST}}{1 - \omega_f^2/\omega_n^2}$

WHERE  $\omega_n^2 = \frac{k}{I}$   $k$  IS THE TORSIONAL SPRING  
CONSTANT AND  $I$  IS THE

CENTROIDAL MOMENT OF INERTIA  
OF THE DISK (SEE SAMPLE PROB. 13)

$k = \frac{50 \text{ N}\cdot\text{m}}{3^\circ (2\pi/360)} = \frac{3000 \text{ N}\cdot\text{m}}{\pi \text{ rad}}$

$I = \frac{1}{2} m r^2 = \frac{1}{2} (8 \text{ kg})(0.2 \text{ m})^2 = 0.160 \text{ kg}\cdot\text{m}^2$

$\omega_n^2 = \frac{k}{I} = \frac{3000 \text{ N}\cdot\text{m}}{(0.160 \text{ kg}\cdot\text{m}^2)} = 5968 \text{ s}^{-2}$



$\Theta_m = \frac{\Theta_{ST}}{1 - \omega_f^2/\omega_n^2} < -\Theta_{ST}$

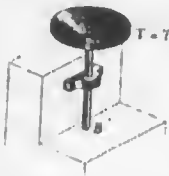
$1 < -1 + \omega_f^2/\omega_n^2$

$\omega_f^2 > 2\omega_n^2$

$\omega_f > \sqrt{2} \sqrt{5968 \text{ s}^{-2}} = 109.3 \text{ rad/s}$

NOTE:  $\omega_f$  IS INDEPENDENT OF  $T_m$

19.104



GIVEN:

8 kg DISK,  $r = 200 \text{ mm}$ ,  
WELDED TO A SHAFT FIXED  
AT B. DISK ROTATES  $3^\circ$  WHEN  
A STATIC COUPLE OF  $50 \text{ Nm}$   
IS APPLIED.  $T_m = 60 \text{ N}\cdot\text{m}$

FIND:

RANGE OF VALUES FOR WHICH  
THE AMPLITUDE OF VIBRATION  
IS LESS THAN  $3.5^\circ$

ANALOGOUS TO EQ (19.33)

$$\theta_m = \frac{T_m/k}{1 - \omega_f^2/\omega_n^2} = \frac{Q_m T}{1 - \omega_f^2/\omega_n^2}$$

$\omega_n^2 = k/I$  WHERE  $k$  IS THE TORSIONAL SPRING CONSTANT  
AND  $I$  IS THE CENTROIDAL MOMENT OF INERTIA  
OF THE DISK. (SEE SAMPLE PROBLEM 19.5)

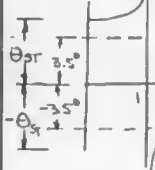
$$k = \frac{50 \text{ N}\cdot\text{m}}{(3^\circ)(\pi/360)} = \frac{3000}{\pi} \frac{\text{N}\cdot\text{m}}{\text{rad}}$$

$$I = \frac{1}{2} m r^2 = \frac{1}{2} (8 \text{ kg})(0.2 \text{ m})^2 = 0.160 \text{ kg}\cdot\text{m}^2$$

$$\omega_n^2 = k/I = (3000/\pi)/(0.160) = 59683.5 \text{ s}^{-2}$$

$$\theta_{st} = T_m/k = (60 \text{ N}\cdot\text{m})/(3000/\pi) = 0.06283 \text{ rad}$$

$$\theta_m = 3.5^\circ = 0.06107 \text{ rad}$$



$$\frac{\theta_{st}}{1 - \omega_f^2/\omega_n^2} < \theta_m$$

$$0.06283 > -0.06107$$

$$0.06283 < \left[ \left( \omega_f^2/59683.5 \right) - 1 \right] [0.06107]$$

$$1.02857 < \left( \omega_f^2/59683.5 \right) - 1$$

$$\omega_f^2 > 121075 \text{ s}^{-2} \quad \omega_f > 110.0 \text{ rad/s}$$

19.105

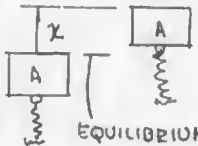


GIVEN:

4-lb BLOCK A  
SPRING  $k = 8 \text{ lb/ft}$   
 $\delta_m = 1 \text{ in.}$ ,  $\omega_f = 5 \text{ rad/s}$

FIND:

(a) AMPLITUDE OF MOTION  
OF THE BLOCK  
(b) AMPLITUDE OF  
FLUCTUATING FORCE OF  
THE SPRING ON BLOCK

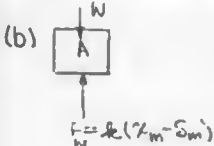
FROM EQ. (19.33')  $x_m = \delta_m / (1 - (\omega_f/\omega_n)^2)$ 

$$(a) \omega_n^2 = \frac{k}{m} = \frac{(2 \text{ lb/ft})}{(4 \text{ lb}/32.2 \text{ ft/s}^2)}$$

$$\omega_n^2 = 64.4 \text{ s}^{-2}$$

$$x_m = \frac{(4/12 \text{ ft})}{1 - (25/64.4)} = 0.5448 \text{ ft}$$

$$x_m = 0.545 \text{ ft}$$

SINCE  $\omega_f < \omega_n$ ,

$x$  AND  $\delta$  ARE IN PHASE  
AND NET SPRING DEFLECTION  
IS  $x - \delta$  AND  $F = k(x - \delta)$

$$F_m = (8 \text{ lb/ft})(0.5448 \text{ ft} - 0.333 \text{ ft})$$

$$F_m = 1.692 \text{ lb}$$

19.106



GIVEN:

8 kg BLOCK A  
SPRING  $k = 1.6 \text{ kN/m}$   
 $\delta_m = 150 \text{ mm}$

FIND:

VALUES OF  $\omega_f$  FOR WHICH  
THE FLUCTUATING FORCE  
OF THE SPRING ON THE  
BLOCK IS LESS THAN  
120 N.

FROM EQ. (19.33')  $x_m = \delta_m / (1 - \omega_f^2/\omega_n^2)$ 

$$\omega_n^2 = k/m = \frac{1.6 \times 10^3 \text{ N/m}}{8 \text{ kg}} = 200 \text{ s}^{-2}$$

IN PHASE

$$F_m = k(x_m - \delta_m) = k\delta_m / (1 - \omega_f^2/\omega_n^2) - k\delta_m < 120 \text{ N}$$

$$1 / (1 - \omega_f^2/\omega_n^2) - 1 < 120 / (1600)(150) = 1/2$$

$$F_m = 240 \text{ N}$$

$$2/3 < 1 - \omega_f^2/\omega_n^2$$

$$\omega_f^2 < \frac{1}{3} \omega_n^2$$

$$\omega_f < 200/3 \quad \omega_f < 8.16 \text{ rad/s}$$

OUT OF PHASE

$$F_m = k(x_m + \delta_m) = 1600(x_m + 0.150)$$

$$= 1600x_m + 240 \text{ N} > 120 \text{ N}$$

THERE IS NO VALUE FOR  $x_m$  WHICH WILL MAKE $F_m < 120 \text{ N}$  WHEN  $x$  AND  $\delta$  ARE OUT OF PHASE

19.107

GIVEN:

$(\delta_{st})_B = 2 \text{ in.}$   
 $\delta_m = 0.5 \text{ in.}$

FIND:

RANGE OF  $\omega_f$  FOR  
WHICH  $|x_m|_B < 1 \text{ in.}$



$$\delta = \delta_m \sin \omega_f t$$

$$k(x - \delta) = m\ddot{x}$$

$$m\ddot{x} = -k(x - \delta)$$

$$m\ddot{x} + kx = k\delta = k\delta_m \sin \omega_f t$$

THUS, FROM EQ (19.31) AND (19.33')

$$(x_m)_B = \frac{\delta_m}{1 - \omega_f^2/\omega_n^2}$$

$$\omega_n^2 = \frac{k}{m} = \frac{g}{\delta_{st}} = \frac{32.2 \text{ ft/s}^2}{2/12 \text{ ft}} = 193.2 \text{ s}^{-2}$$

$$(x_m)_B = \frac{0.5}{1 - \omega_f^2/193.2}$$

$$1 < \frac{1/2}{1 - \omega_f^2/193.2}$$

IN PHASE

$$1 - \omega_f^2/193.2 < \frac{1}{2}$$

$$\omega_f^2 < \frac{193.2}{2}$$

$$\omega_f < 9.83 \text{ rad/s}$$

OUT OF PHASE

$$1 - \omega_f^2/193.2 > \frac{1}{2}$$

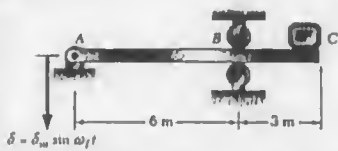
$$\omega_f < 9.83 \text{ rad/s}$$

$$\omega_f^2 > (3/2)(193.2)$$

$$\omega_f > 17.02 \text{ rad/s}$$

$$\omega_f = 17.02 \text{ rad/s}$$

19.108



GIVEN:

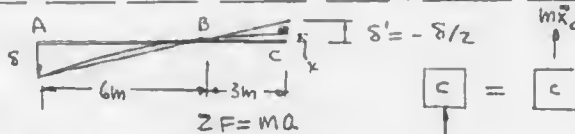
$$(\delta_{st})_C = 15 \text{ mm}$$

$$m_C = 120 \text{ kg}$$

$$\omega_f = 18 \text{ rad/s}$$

$$\delta_m = 10 \text{ mm}$$

FIND:

(a)  $m$ 

$$Z F = m A$$

$$m \ddot{x}_C = k(s - x) \quad s' = -s/2$$

$$\ddot{x}_C + \frac{k}{m} x_C = -k \frac{\delta_m}{2} \sin \omega_f t$$

$$\text{FROM EQ (19.31 AND 19.33)} \quad (x_C)_m = \frac{-\delta_m/2}{1 - (\omega_f/\omega_n)^2}$$

$$\text{THUS } (x_C)_m = \frac{-0.010/2}{1 - (18^2/654)}$$

$$(\ddot{s}_{st})_C = 0.015 \text{ m}$$

$$\omega_n^2 = 9 = 4.91 \text{ m/s}^2$$

$$\omega_n = 654 \text{ s}^{-2}$$

$$(x_C)_m = -0.009909 \text{ m}$$

$$x_C = (x_C)_m \sin \omega_f t$$

$$\ddot{x}_C = (a_C)_m = -(x_C)_m \omega_f^2$$

$$(a_C)_m = (0.009909 \text{ m})(18 \text{ s}^{-1})^2$$

$$(a_C)_m = 3.21 \text{ m/s}^2$$

19.109

GIVEN:

$$8\text{-kg Block A}$$

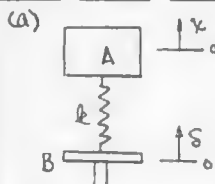
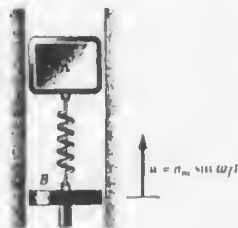
$$\text{Spring } k = 120 \text{ N/m}$$

$$a_m = 1.5 \text{ m/s}^2$$

$$\omega_f = 5 \text{ rad/s}$$

FIND:

- (a) MAXIMUM DISPLACEMENT OF A
- (b) AMPLITUDE OF THE FLUCTUATING FORCE EXERTED BY THE SPRING ON THE BLOCK



SUPPORT MOTION

$$a = \ddot{s} = a_m \sin \omega_f t$$

$$\ddot{s} = -(a_m/\omega_f^2) \sin \omega_f t$$

$$\ddot{s}_m = -a_m/\omega_f^2 = -1.5 \text{ m/s}^2$$

$$\ddot{s}_m = 0.060 \text{ m}$$

FROM EQ (19.31 AND 19.33')

$$x_m = \frac{\ddot{s}_m}{1 - (\omega_f/\omega_n)^2}$$

$$x_m = \frac{-0.060}{1 - (25/15)} = 0.090 \text{ m}$$

$$\omega_n^2 = k/m = 120 \text{ N/m} / 8 \text{ kg}$$

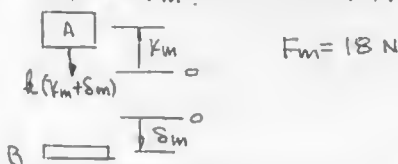
$$\omega_n^2 = 15 \text{ s}^{-2}$$

$$x_m = 0.090 \text{ m}$$

- (b)  $x$  IS OUT OF PHASE WITH  $s$  FOR  $\omega_f = 5 \text{ rad/s}$

THUS

$$F_m = k(x_m + s_m) = 120 \text{ N/m}(0.09 \text{ m} + 0.060 \text{ m})$$



19.110

GIVEN:

$$0.8\text{-lb BALL}$$

$$\text{ELASTIC CORD AB}$$

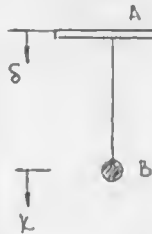
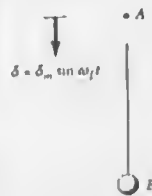
$$k = 516 \text{ lb/ft}$$

$$\delta_m = 8 \text{ in}$$

$$f_f = 0.5 \text{ Hz}$$

FIND:

AMPLITUDE OF THE MOTION OF B



$$Z F = m A$$

$$k(s-x) = m \ddot{x}$$

$$\ddot{x} + (k/m)x = \ddot{s}$$

FROM EQ. (19.31 AND 19.33')

$$\omega_f = 2\pi f_f = 2\pi(0.5) = \pi \text{ s}^{-1}$$

$$\omega_f^2 = \pi^2 \text{ s}^{-2}$$

$$\omega_n^2 = k/m = (516 \text{ lb/ft}) / (0.8 \text{ lb} / 32.2 \text{ ft/s}^2) = 20.25 \text{ s}^{-2}$$

$$\delta_m = (8)(1/12) = 2/3 \text{ ft}$$

$$x_m = \frac{\delta_m}{(1 - \omega_f^2/\omega_n^2)} = 0.7011 \text{ ft}$$

$$(1 - \pi^2/20.25) \quad (\text{IN PHASE})$$

CHECK TO SEE WHETHER CORD GOES SLACK

$$\text{STATIC DEFLECTION } \delta_{st} = W/k = 0.8 \text{ lb} / 516 \text{ lb/ft} = 0.16 \text{ ft}$$

$$\text{SINCE } x \text{ AND } s \text{ ARE IN PHASE THE MAXIMUM DEFLECTION OF THE CORD IS } x_m - \delta_m = 0.7011 - 0.6667$$

$$= 0.0344 \text{ ft}$$

$$\text{WHICH IS LESS THAN THE STATIC DEFLECTION OF } 0.16 \text{ ft}$$

$$x_m = 0.701 \text{ ft}$$

19.111

GIVEN:

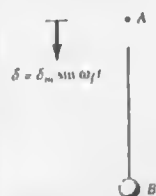
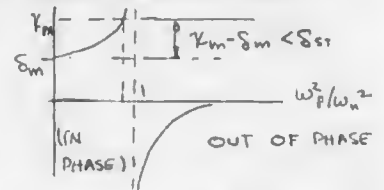
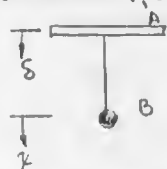
$$0.8\text{-lb BALL}$$

$$\text{ELASTIC CORD AB}$$

$$k = 516 \text{ lb/ft}$$

$$\delta_m = 8 \text{ in.}$$

FIND:

MAXIMUM  $\omega_f$  IF CORD IS NOT TO GO SLACKCORD BECOMES SLACK WHEN THE NET DEFLECTION OF THE CORD IS GREATER THAN THE STATIC DEFLECTION,  $\delta_{st}$ 

$$\delta_{st} = \frac{W}{k} = \frac{0.8 \text{ lb}}{516 \text{ lb/ft}} = 0.16 \text{ ft}$$

$$\omega_n^2 = \frac{k}{m} = \frac{516 \text{ lb/ft}}{0.8 \text{ lb} / 32.2 \text{ ft/s}^2}$$

FROM EQ (19.31 AND 19.33')

$$x_m - s_m < \delta_{st} \quad \delta_m \left[ \frac{1}{1 - \omega_f^2/\omega_n^2} - 1 \right] < \delta_{st}$$

$$\frac{1}{1 - (\omega_f^2/20.25)} < \frac{0.16 + 1}{2/3}$$

$$\frac{1}{1.24} < 1 - \frac{\omega_f^2}{20.25}$$

$$-\omega_f^2 > 20.25(0.806 - 1)$$

$$\omega_f^2 < 38.95 \text{ s}^{-2}$$

$$\omega_f < 6.24 \text{ rad/s}$$

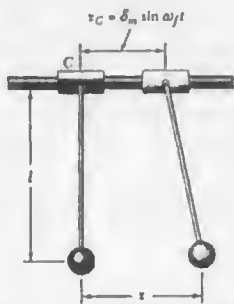
19.112

GIVEN:

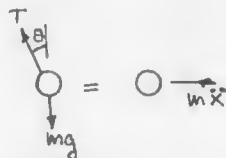
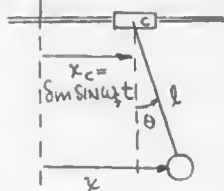
1.2 kg BOB  
 $l = 0.600 \text{ m}$   
 1.4 kg COLLAR C  
 $\delta_m = 0.010 \text{ m}$   
 $f_f = 0.5 \text{ Hz}$

FIND:

(a) AMPLITUDE OF MOTION OF THE BOB  
 (b) FORCE APPLIED TO THE COLLAR TO MAINTAIN THE MOTION



(a)



$$\Sigma F_x = m a_x$$

$$-T \sin \theta = m \ddot{x}$$

SMALL ANGLES  
 $\cos \theta \approx 1$  ACCELERATION  
 IN THE y DIRECTION IS  
 SECOND ORDER AND  
 IS NEGLECTED

$$\Sigma F_y = T \cos \theta - mg = 0$$

$$T = mg$$

$$m \ddot{x} = -mg \sin \theta$$

$$\sin \theta = \frac{x - x_c}{l}$$

$$m \ddot{x} + \frac{mg}{l} x = \frac{mg}{l} x_c = \frac{mg}{l} \delta_m \sin \omega_f t$$

$$\ddot{x} + \omega_n^2 x = \omega_n^2 \delta_m \sin \omega_f t$$

FROM EQ.(19.33')

$$x_m = \frac{\delta_m}{1 - \omega_f^2 / \omega_n^2}$$

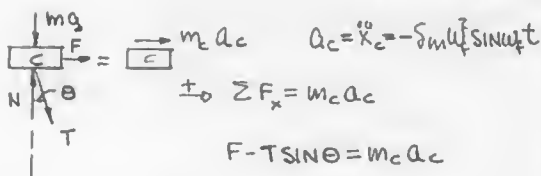
$$\omega_f^2 = (2\pi f_f)^2 = 4\pi^2 \frac{1}{0.6^2} = \pi^2 \text{ s}^{-2}$$

$$\omega_n^2 = g/l = \frac{9.81 \text{ m/s}^2}{0.6 \text{ m}} = 16.35 \text{ s}^{-2}$$

$$x_m = \frac{0.010 \text{ m}}{1 - \pi^2 / 16.35} = 0.02522 \text{ m}$$

$$x_m = 25.2 \text{ mm}$$

(b)



$$\Sigma F_x = m_c a_c$$

$$F - T \sin \theta = m_c a_c$$

$$\text{FROM PART (a)} \quad T = mg, \quad \sin \theta = \frac{x - x_c}{l}$$

THUS

$$F = -mg \left[ \frac{x - x_c}{l} \right] + m_c \ddot{x}_c$$

$$F = -m \omega_n^2 x + m \omega_n^2 x_c + m_c \ddot{x}_c$$

$$F = -m \omega_n^2 x_m \sin \omega_f t + m \omega_n^2 \delta_m \sin \omega_f t - m_c \omega_f^2 \sin \omega_f t$$

$$F = [-(1.2 \text{ kg})(16.35 \text{ s}^{-2})(0.02522 \text{ m}) + (1.2 \text{ kg})(16.35 \text{ s}^{-2})(0.01 \text{ m}) - (1.4 \text{ kg})(\pi^2 \text{ s}^{-2})(0.01 \text{ m})] \sin \pi t$$

$$F = -0.437 \sin \pi t \text{ (N)}$$

19.113

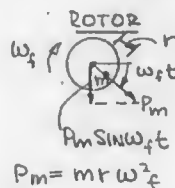
GIVEN:

MOTOR OF MASS M SUPPORTED BY SPRINGS WITH EQUIVALENT CONSTANT k. EQUIVALENT ROTOR MASS UNBALANCE m AT A DISTANCE r FROM THE AXIS OF ROTATION. ANGULAR VELOCITY OF MOTOR,  $\omega_f$

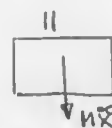
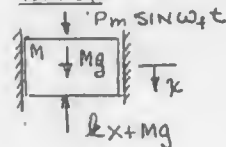
SHOW THAT:

AMPLITUDE OF THE MOTION OF THE MOTOR

$$x_m = \frac{r(m/m)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$



MOTOR



$$\Sigma F = ma \quad P_m \sin \omega_f t - kx = M \ddot{x}$$

$$M \ddot{x} + kx = P_m \sin \omega_f t$$

$$\ddot{x} + \omega_n^2 x = \frac{P_m}{M} \sin \omega_f t$$

FROM EQ.(19.33)

$$x_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$$

$$\text{BUT } P_m/k = m r \omega_f^2 / k \quad k = M \omega_n^2$$

$$P_m/k = r(m/m)(\omega_f/\omega_n)^2$$

THUS

$$x_m = \frac{r(m/m)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2} \quad \text{QED}$$

19.114

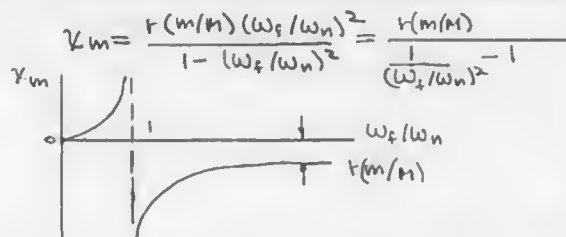
GIVEN:

100-kg MOTOR; UNBALANCED 15-kg ROTOR. SPEED INCREASED AND AT VERY HIGH SPEEDS THE AMPLITUDE NEARS 3.3 mm

FIND:

THE DISTANCE BETWEEN THE MASS CENTER OF THE ROTOR AND ITS AXIS OF ROTATION

USE THE EQUATION DERIVED IN PROB 19.113 (ABOVE)

FOR VERY HIGH SPEEDS  $\frac{1}{(\omega_f/\omega_n)^2} \rightarrow 0$  AND

$$x_m \rightarrow r(m/m), \text{ THUS } 3.3 \text{ mm} = r(15/100)$$

$$r = 22 \text{ mm}$$



### 19.115 GIVEN:

SPRING SUPPORTED MOTOR WHOSE SPEED IS INCREASED FROM 200 TO 300 RPM AMPLITUDE DUE TO UNBALANCE INCREASES CONTINUOUSLY FROM 2.5 TO 8 MM

FIND:

SPEED AT RESONANCE

FROM PROB 19.113

$$x_m = \frac{r(m/m)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$

$$2.5 = \frac{r(m/m)(200/\omega_n)^2}{1 - (200/\omega_n)^2}$$

$$8 = \frac{r(m/m)(300/\omega_n)^2}{1 - (300/\omega_n)^2}$$

$$\frac{2.5}{8} = \frac{1 - (300/\omega_n)^2}{1 - (200/\omega_n)^2} \left( \frac{200}{300} \right)^2$$

$$0.703 - 0.703(200/\omega_n)^2 = 1 - (300/\omega_n)^2$$

$$\frac{1}{\omega_n^2} [90 \times 10^3 - 28.125 \times 10^3] = 0.2969$$

$$\omega_n^2 = 208.4 \quad \omega_n = 457 \text{ rpm}$$

RESONANCE WHEN  $\omega_f = \omega_n \quad \omega_f = 457 \text{ rpm}$

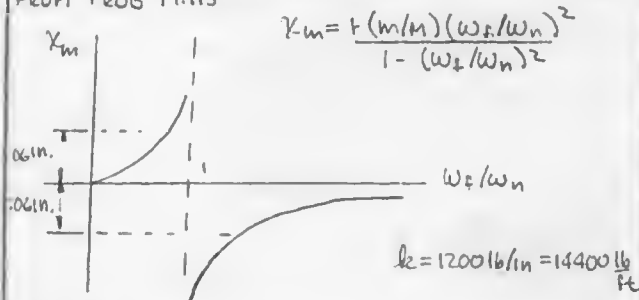
### 19.116 GIVEN:

400-16 MOTOR SUPPORTED BY SPRINGS WITH TOTAL  $k = 1200 \text{ lb/in}$  ROTOR UNBALANCE IS 102, 8 IN FROM THE AXIS OF ROTATION

FIND:

RANGE OF ALLOWABLE VALUES OF MOTOR SPEED IF THE AMPLITUDE OF VIBRATION IS NOT TO EXCEED 0.06 IN

FROM PROB 19.113



$$\omega_n^2 = k/m = \frac{(14400 \text{ lb/ft})}{(400 \text{ lb}/32.2 \text{ ft/s}^2)} = 1159.2 \text{ s}^{-2}$$

$$r m/m = (8/12 \text{ ft}) \left( \frac{1/16 \text{ lb}}{400 \text{ lb}} \right) = 104.17 \times 10^{-6} \text{ ft}$$

$$x_m = 0.06 \text{ in} \quad (0.06/12 \text{ ft}) \leq \frac{104.17 \times 10^{-6} \text{ ft} (\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$

$$47.998 - 47.998 (\omega_f/\omega_n)^2 \leq (\omega_f/\omega_n)^2$$

$$(\omega_f/\omega_n)^2 \leq \frac{47.998}{98.998}$$

$$\omega_f/\omega_n < 0.9897 \quad \omega_f \leq 0.9897 (1159.2)^{1/2}$$

$$\omega_f \leq (33.69 \text{ rad/s}) \left( \frac{1}{2\pi} \right) \left( \frac{1}{60} \right) = 322 \text{ RPM}$$

$$x_m = -0.06 \text{ in} \quad (-0.06/12 \text{ ft}) \geq \frac{104.17 \times 10^{-6} \text{ ft} (\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$

$$(\omega_f/\omega_n)^2 \geq \frac{47.998}{46.998}$$

$$\omega_f \geq (1.0106) (1159.2)^{1/2} \quad \omega_f \geq 39.40 \text{ rad/s} = 329 \text{ rpm}$$

### 19.117

GIVEN:

220-16 MOTOR  
UNBALANCE OF THE ROTOR = 202, 4 IN  
FROM THE AXIS OF ROTATION  
RESONANCE AT 400 RPM

FIND:

AMPLITUDE AT (a) 800 rpm, (b) 200 rpm, (c) 425 rpm

FROM PROB 19.113

$$x_m = \frac{r(m/m)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$

RESONANCE AT 400 RPM MEANS THAT  $\omega_n = 400 \text{ rpm}$

$$r(m/m) = (4 \text{ in.}) (2/16) / (220) = 2.2727 \times 10^{-3} \text{ in.}$$

$$(a) (\omega_f/\omega_n)^2 = (800/400)^2 = 4$$

$$x_m = \frac{2.2727 \times 10^{-3} \text{ in.} (4)}{1 - 4} = 0.00303 \text{ in.}$$

$$(b) (\omega_f/\omega_n)^2 = (200/400)^2 = 1/4$$

$$x_m = \frac{2.2727 \times 10^{-3} (1/4)}{1 - 1/4} = 0.000758 \text{ in.}$$

$$(c) (\omega_f/\omega_n)^2 = (425/400)^2 = 1.1289$$

$$x_m = \frac{2.2727 \times 10^{-3} (1.1289)}{1 - 1.1289} = -0.01990 \text{ in.}$$

### 19.118

GIVEN:

180-kg MOTOR  
UNBALANCE OF THE ROTOR = 28 g  
150 mm FROM AXIS OF ROTATION  
STATIC DEFECTION  $\delta_{st} = 12 \text{ mm}$

FIND:

MASS OF A PLATE ADDED TO THE BASE OF THE MOTOR SO THAT AMPLITUDE OF VIBRATION IS LESS THAN  $60 \times 10^{-6} \text{ m}$  FOR MOTOR SPEEDS ABOVE 300 rpm.

FROM PROB 19.113

$$x_m = \frac{r(m/m)(\omega_f/\omega_n)^2}{1 - \omega_f^2/\omega_n^2}$$

SINCE  $M\omega_n^2 = k$

$$x_m = (mr/k) \omega_f^2 / (1 - \omega_f^2/\omega_n^2)$$

BEFORE THE PLATE IS ADDED,  $\omega_n^2 = \frac{g}{\delta_{st}} = \frac{9.81 \text{ m/s}^2}{0.012 \text{ m}}$

$$k = M\omega_n^2 = (180 \text{ kg}) (817.55^2) \quad \omega_n^2 = 817.5 \text{ s}^{-2}$$

$$k = 147.15 \times 10^3 \text{ N/m}$$

$$mr/k = (28 \times 10^{-3} \text{ kg}) (0.150 \text{ m}) / (147.15 \times 10^3 \text{ N/m}) = 28.542 \times 10^{-9} \text{ m} \cdot \text{s}^2$$

AFTER THE PLATE IS ADDED THE NATURAL FREQUENCY OF THE SYSTEM CHANGES SINCE THE MASS CHANGES  $\omega_n'^2 = k/M'$

SINCE THE VIBRATION IS TO BE LESS THAN  $60 \times 10^{-6} \text{ m}$  FOR MOTOR SPEEDS ABOVE 300 rpm, WE HAVE

$$x_m = -60 \times 10^{-6} \text{ m} = \frac{(28.542 \times 10^{-9} \text{ m} \cdot \text{s}^2) (300 \cdot 2\pi/60)^2}{1 - (300 \cdot 2\pi/60)^2 / \omega_n'^2}$$

$$-2.1299 + 2.1299 \left( \frac{986.96}{\omega_n'^2} \right) = 1$$

$$\omega_n'^2 = \frac{2.1299 (986.96)}{2.1299} = 671.6 \text{ s}^{-2} = \frac{k}{M'}$$

$$M' = (147.15 \times 10^3 \text{ N/m}) / (671.6 \text{ s}^{-2}) = 219.1 \text{ kg}$$

$$\Delta M = M' - M = 219 - 180 = 39.1 \text{ kg}$$

19.119



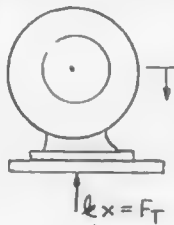
GIVEN:

400-lb MOTOR  
UNBALANCE OF 3 OZ.  
6 IN FROM AXIS OF  
ROTATION  
FORCE TRANSMITTED TO  
FOUNDATION LIMITED  
TO 0.2 lb WHEN  
MOTOR IS RUN AT  
100 RPM AND ABOVE

FIND:

- (a) MAXIMUM ALLOWABLE SPRING CONSTANT  $k$  OF A PAD PLACED BETWEEN THE MOTOR AND THE FOUNDATION  
(b) CORRESPONDING AMPLITUDE OF THE FLUCTUATING FORCE WHEN THE MOTOR IS RUN AT 200 RPM

(a) FROM PROB. (19.113)



$$\chi_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$

$$(F_T)_m = k \chi_m \quad \frac{k}{M} = \omega_n^2$$

$$(F_T)_m = \frac{rM \omega_f^2}{1 - \omega_f^2 M/k} \quad (1)$$

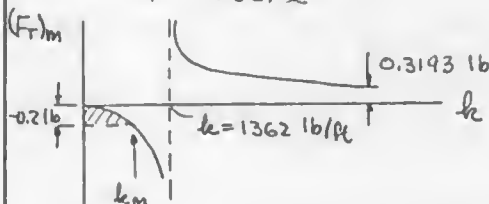
$$rM = (6/12 \text{ ft})(3/16 \text{ lb})/(32.2 \text{ ft/s}^2)$$

$$rM = 0.002912 \text{ lb} \cdot \text{s}^2$$

$$\text{AT } \omega_f = 100 \text{ rpm} = 100(2\pi/60) = 10.472 \text{ rad/s}$$

$$(F_T)_m = \frac{(0.002912 \text{ lb} \cdot \text{s}^2)(10.472 \text{ s}^{-1})^2}{1 - (10.472 \text{ s}^{-1})^2(400 \text{ lb}/32.2 \text{ ft/s}^2)}$$

$$(F_T)_m = \frac{0.31928}{1 - 1362/k}$$



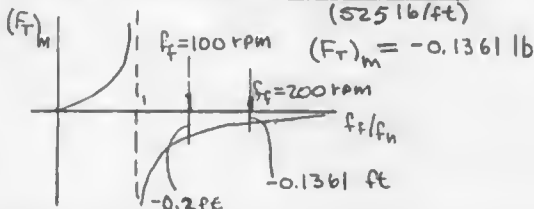
$$-0.2 = 0.319 / (1 - 1362/k)$$

$$-0.2 + 0.2(1362)/k_m = 0.31928$$

$$k_m = \frac{(0.2)(1362)}{0.51928} = 525 \text{ lb/ft}$$

(b) AT 200 RPM,  $\omega_f = (200)(2\pi/60) = 20.94 \text{ rad/s}$   
FROM (1), AND USING  $k$  FOUND IN PART (a)

$$(F_T)_m = \frac{(0.002912 \text{ lb} \cdot \text{s}^2)(20.94 \text{ s}^{-1})^2}{1 - (20.94 \text{ s}^{-1})^2(400 \text{ lb}/32.2 \text{ ft/s}^2)}$$



19.120

GIVEN:

180-kg MOTOR, SUPPORTED BY SPRINGS  
OF TOTAL CONSTANT  $k = 150 \text{ kN/m}$   
UNBALANCE OF THE ROTOR IS  
28-g AT 150 MM

FIND:

RANGE OF SPEEDS FOR WHICH THE FLUCTUATING  
FORCE  $(F_T)_m$  IS LESS THAN 20 N

FROM PROB. (19.113)

$$\chi_m = \frac{r(m/M)(\omega_f^2)}{1 - (\omega_f/\omega_n)^2}$$

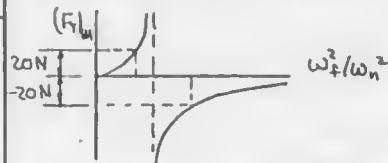
$$(F_T)_m = k \chi_m \quad k/M = \omega_n^2$$

$$F_T = rM \omega_f^2 / (1 - (\omega_f/\omega_n)^2)$$

$$rM = (0.150 \text{ m})(0.028 \text{ kg}) = 0.0042 \text{ m} \cdot \text{kg}$$

$$\omega_n^2 = k/M = (150 \times 10^3 \text{ N/m}) / (180 \text{ kg}) = 833.3 \text{ s}^{-2}$$

$$(F_T)_m = [(0.0042)(\omega_f^2)] / (1 - \omega_f^2/833.3)$$



$$(F_T)_m (1 - \omega_f^2/833.3) = 0.0042 \omega_f^2$$

$$F_T_m = [(F_T)_m / 833.3] \omega_f^2 + 0.0042 \omega_f^2$$

$$\omega_f^2 = (F_T)_m / [(F_T)_m / 833.3 + 0.0042]$$

$$\text{FOR } (F_T)_m = 20 \text{ N}$$

$$\omega_f^2 < \frac{20}{0.024 + 0.0042} = 709.2 \text{ s}^{-2}$$

$$\omega_f \leq 26.63 \text{ rad/s}$$

$$\omega < 26.63 \left( \frac{60}{2\pi} \right) = 254 \text{ rpm}$$

$$\text{FOR } (F_T)_m = -20 \text{ N}$$

$$\omega_f^2 > \frac{-20}{-0.024 + 0.0042} = 1010 \text{ s}^{-2}$$

$$\omega_f > 31.78 \text{ rad/s}$$

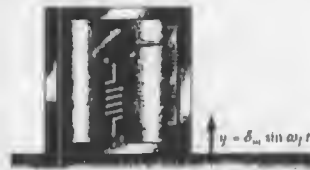
$$\omega > 31.78 \left( \frac{60}{2\pi} \right) = 303 \text{ rpm}$$

19.121

GIVEN:

$$f_n = 120 \text{ Hz}$$

$Z_m$  = AMPLITUDE  
RELATIVE TO THE  
BOX IS USED AS A  
MEASURE OF  $S_m$



FIND:

(a) % ERROR FOR  $f_f = 600 \text{ Hz}$ (b)  $f_f$  FOR ZERO ERROR

$$\chi = \left( \frac{S_m}{1 - \omega_f^2/\omega_n^2} \right) \sin \omega_f t$$

$$y = S_m \sin \omega_f t$$

 $z$  = RELATIVE MOTION

$$z = \chi - y = \left( \frac{S_m}{1 - \omega_f^2/\omega_n^2} - S_m \right) \sin \omega_f t$$

$$Z_m = S_m \left[ \frac{1}{1 - \omega_f^2/\omega_n^2} - 1 \right] = \frac{S_m \omega_f^2/\omega_n^2}{1 - \omega_f^2/\omega_n^2}$$

$$(a) \frac{Z_m}{S_m} = \frac{\omega_f^2/\omega_n^2}{1 - \omega_f^2/\omega_n^2} = \frac{(600/120)^2}{1 - (600/120)^2} = \frac{25}{24} = 1.0417$$

$$\text{ERROR} = 4.17\%$$

$$(b) \frac{Z_m}{S_m} = 1 = \frac{\omega_f^2/\omega_n^2}{1 - \omega_f^2/\omega_n^2}$$

$$1 = 2 \frac{\omega_f^2}{\omega_n^2} \quad f_f = \frac{\sqrt{2}}{2} f_n = \frac{\sqrt{2}}{2} (120) = 84.9 \text{ Hz}$$

19.122

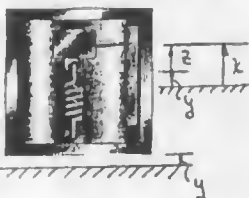


FIND:

% ERROR IN  $Q_m$  WHEN  $f_f = 600$  Hz

GIVEN:

$f_n = 2200$  Hz  
 $Z_m =$  AMPLITUDE  
 RELATIVE TO THE BOX  
 $Z_m \omega_n^2$  USED AS  
 A MEASURE OF  
 THE ACCELERATION  
 OF THE VIBRATING  
 SURFACE,  $Q_m = \frac{Z_m \omega_n^2}{\omega_f^2}$



ABSOLUTE MOTION OF  
 THE MASS

$$x = \frac{\delta_m}{1 - \omega_f^2/\omega_n^2} \sin \omega_f t$$

RELATIVE MOTION OF  
 THE MASS

$$z = x - y = \left[ \frac{\delta_m}{1 - \omega_f^2/\omega_n^2} - \delta_m \right] \sin \omega_f t$$

$$Z_m = \frac{\delta_m \omega_f^2}{1 - \omega_f^2/\omega_n^2}$$

$$Z_m \omega_n^2 = \frac{\delta_m \omega_f^2}{1 - \omega_f^2/\omega_n^2}$$

$$Q_m = \delta_m \omega_f^2 \quad Z_m \omega_n^2 = Q_m / (1 - \omega_f^2/\omega_n^2)$$

$$(\omega_f/\omega_n)^2 = \left( \frac{600}{2200} \right)^2 = 0.07438$$

$$\frac{Z_m \omega_n^2}{Q_m} = \frac{1}{1 - 0.07438} = 1.0804$$

$$\text{ERROR} = 8.04\%$$

19.123

$$P = P_m \sin \omega_f t$$



(1)



(2)

GIVEN:

TWO SYSTEMS  
 AS SHOWN

FIND: TRANSMISSIBILITY, I.E.

(1) THE RATIO OF  
 THE TRANSMITTED  
 FORCE TO THE  
 IMPRESSED FORCE  
 (2) RATIO OF THE  
 TRANSMITTED  
 DISPLACEMENT TO  
 THE IMPRESSED  
 DISPLACEMENT

SHOW THAT:

TO REDUCE TRANSMISSIBILITY,  $\omega_f/\omega_n > \sqrt{2}$

(1) FROM EQ. (19.33)  $X_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$

FORCE TRANSMITTED,  $(P_T)_m = k X_m = k \left[ \frac{P_m/k}{1 - (\omega_f/\omega_n)^2} \right]$

THUS TRANSMISSIBILITY =  $\frac{(P_T)_m}{P_m} = \frac{1}{1 - (\omega_f/\omega_n)^2}$

(2) FROM EQ. (19.33')

DISPLACEMENT TRANSMITTED  $X_m = \frac{\delta_m}{1 - (\omega_f/\omega_n)^2}$

TRANSMISSIBILITY =  $\frac{X_m}{\delta_m} = \frac{1}{1 - (\omega_f/\omega_n)^2}$

FOR  $\frac{(P_T)_m}{P_m}$  OR  $\frac{X_m}{\delta_m}$  TO BE LESS THAN 1

$$\frac{1}{1 - (\omega_f/\omega_n)^2} < 1$$

$$1 < 1 - (\omega_f/\omega_n)^2$$

$$(\omega_f/\omega_n)^2 > 2 \quad \omega_f/\omega_n > \sqrt{2} \quad \text{QED}$$

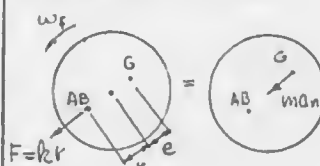
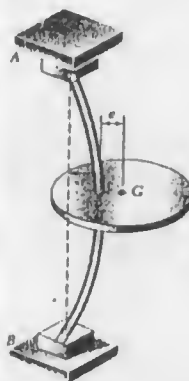
19.124

GIVEN:

60-lb DISK  
 $e = 0.006$  in.  
 $k = 40,000$  lb/ft FOR THE  
 SHAFT WHICH ROTATES  
 AT A CONSTANT ANGULAR  
 VELOCITY  $\omega_f$  ABOUT AB

FIND:

(a)  $\omega_f$  FOR RESONANCE  
 (b) DEFLECTION  $r$  WHEN  
 $\omega_f = 1200$  rpm



DEFLECTION OF  
 THE SHAFT IS.  
 THUS  $F = k r$

G DESCRIBES A  
 CIRCLE ABOUT THE  
 AXIS AB OF RADIUS  
 $r + e$ .  
 THUS  $a_n = (r + e) \omega_f^2$

$$F = m a_n \quad k r = m (r + e) \omega_f^2$$

$$\omega_n^2 = \frac{k}{m} \quad m = \frac{k}{\omega_n^2}$$

$$k r = \frac{k}{\omega_n^2} (r + e) \omega_f^2$$

$$r = \frac{e \omega_f^2}{\omega_n^2 - \omega_f^2}$$

(a) RESONANCE OCCURS WHEN  $\omega_f = \omega_n$ , I.E.  $r \rightarrow \infty$   
 $\omega_n = \sqrt{k/m} = \sqrt{\frac{40,000 \text{ lb/ft}}{60 \text{ lb}/32.2 \text{ ft/s}^2}} = 146.5 \text{ rad/s}$

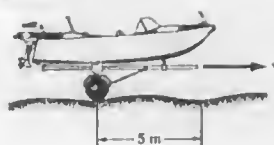
$$\omega_n = \omega_f = (146.5) \left( \frac{60}{2\pi} \right) = 1399.1$$

(b)

$$r = \frac{e \omega_f^2}{\omega_n^2 - \omega_f^2} = \frac{0.006 \text{ in.} (1200)^2}{(1399.1)^2 - (1200)^2} = 0.0126 \text{ in.}$$

19.125

GIVEN:



MASS OF TRAILER  
 AND LOAD = 250 kg  
 TRAILER SUPPORTED  
 BY TWO SPRINGS  
 EACH OF  $k = 10 \text{ kN/m}$   
 ROAD SURFACE  
 IS A SINE CURVE  
 WITH AMPLITUDE  
 OF 40 mm AND  
 WAVELENGTH 5 m

FIND:

(a) SPEED  $U$  AT WHICH  
 RESONANCE WILL OCCUR  
 (b) AMPLITUDE OF THE TRAILER

VIBRATION AT  $U = 50 \text{ km/h}$

TOTAL SPRING CONSTANT  $k_T = 2k = 20 \text{ kN/m}$   
 $\omega_n^2 = k_T/m = (20,000 \text{ N/m})/(250 \text{ kg}) = 80 \text{ s}^{-2}$

(a)  $y = \delta_m \sin \frac{2\pi}{\lambda} x \quad x = U t$   
 $y = \delta_m \sin \left( \frac{2\pi U}{\lambda} t \right) \quad \omega_f = \frac{2\pi U}{\lambda}$   
 $\omega_f = 2\pi/\lambda$ , THUS  
 $y = 0.04 \sin \omega_f t$   
 WHERE  $\omega_f = \frac{2\pi U}{\lambda} = \frac{2\pi (50)}{5}$



FROM EQ. (19.33')

$$X_m = \frac{\delta_m}{1 - \omega_f^2/\omega_n^2}$$

RESONANCE,  $\omega_f = \omega_n = \sqrt{80} \text{ s}^{-1}$   
 $U = 7.11 \text{ m/s} = 25.6 \text{ km/h}$

(b)  $\omega_f = \frac{2\pi (50)}{5} = 17.45 \text{ s}^{-1}$ ,  $X_m = \frac{0.04}{1 - \frac{17.45^2}{80}} = 0.0475 \text{ m}$

19.126

GIVEN:

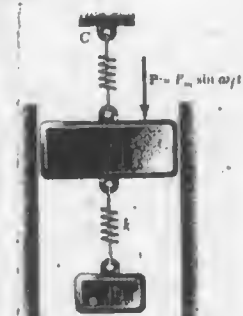
$$P_m = 20 \text{ N}$$

$$\omega_f = 2 \text{ rad/s}$$

$$m_B = 22 \text{ kg}$$

FIND:

- (a) VALUE OF  $k$  WHICH  
WILL PREVENT STEADY  
STATE VIBRATION OF  
A
- (b) CORRESPONDING  
AMPLITUDE OF  
BLOCK B



AT STEADY STATE BLOCK A DOES NOT MOVE AND IS THEREFORE REMAINS IN ITS ORIGINAL EQUILIBRIUM POSITION.

(a) BLOCK A

$$F_0 = m_A g$$

$$P = P_m \sin \omega_f t$$

$$\uparrow \Sigma F = 0$$

$$kx = -P_m \sin \omega_f t \quad (1)$$

BLOCK B

$$\uparrow \Sigma F = m_B \ddot{x}$$

$$m_B \ddot{x} + kx = 0$$

$$x = x_m \sin \omega_n t, \quad \omega_n = k/m_B$$

FROM (1)

$$k x_m \sin \omega_n t = -P_m \sin \omega_f t$$

$$\omega_n = \omega_f = 2 \text{ rad/s}, \quad k x_m = -P_m$$

$$\sqrt{\frac{k}{m_B}} = 2 \text{ s}^{-1}$$

$$k = (4 \text{ s}^{-2})(22 \text{ kg})$$

$$k = 88 \text{ N/m}$$

(b)  $k x_m = -P_m$   $x_m = \frac{-20 \text{ N}}{88 \text{ N/m}} = -0.227 \text{ m}$

19.127

GIVEN:

HEAVY DAMPING,  $C > C_c$ 

SHOW THAT:

A BODY NEVER PASSES THROUGH ITS EQUILIBRIUM POSITION 0 IF,

- (a) IT IS RELEASED FROM ANY POSITION WITH NO INITIAL VELOCITY
- (b) IT IS STARTED FROM 0 WITH AN ARBITRARY INITIAL VELOCITY

SINCE  $C > C_c$  WE USE EQ. (19.42), WHERE

$$\lambda_1 < 0, \quad \lambda_2 < 0$$

$$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (1)$$

$$v = \frac{dx}{dt} = C_1 \lambda_1 e^{\lambda_1 t} + C_2 \lambda_2 e^{\lambda_2 t} \quad (2)$$

(a)  $t=0, x=x_0, v=0$

FROM (1) AND (2)

$$x_0 = C_1 + C_2$$

$$0 = C_1 \lambda_1 + C_2 \lambda_2$$

SOLVING FOR  $C_1$  AND  $C_2$

$$C_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1} x_0 \quad C_2 = -\frac{\lambda_1}{\lambda_2 - \lambda_1} x_0$$

SUBSTITUTING FOR  $C_1$  AND  $C_2$  IN (1)

$$x = \frac{x_0}{\lambda_2 - \lambda_1} [\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}]$$

19.127, CONTINUED

FOR  $x=0$  WHEN  $t \neq \infty$ , WE MUST HAVE

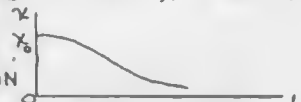
$$\lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t} = 0 \quad \frac{\lambda_2}{\lambda_1} = e^{(\lambda_2 - \lambda_1)t} \quad (3)$$

RECALL THAT

$\lambda_1 < 0, \lambda_2 < 0$ . CHOOSING  $\lambda_1$  AND  $\lambda_2$   
SO THAT  $\lambda_1 < \lambda_2 < 0$ , WE HAVE  
 $0 < \frac{\lambda_2}{\lambda_1} < 1$  AND  $\lambda_2 - \lambda_1 > 0$

THUS A POSITIVE SOLUTION FOR  $t > 0$  FOR EQ. (3) CANNOT EXIST SINCE IT WOULD REQUIRE THAT  $e$  RAISED TO A POSITIVE POWER BE LESS THAN 1, WHICH IS IMPOSSIBLE. THUS  $x$  IS NEVER 0.

THE  $x-t$  CURVE FOR THIS CASE IS SHOWN



(b)  $t=0, x=0, v=v_0$  EQ. (1) AND (2), YIELD

$$0 = C_1 + C_2 \quad v_0 = C_1 \lambda_1 + C_2 \lambda_2$$

SOLVING FOR  $C_1$  AND  $C_2$ ,  $C_1 = \frac{-v_0}{\lambda_2 - \lambda_1}, C_2 = \frac{v_0}{\lambda_2 - \lambda_1}$

SUBSTITUTING INTO (1)

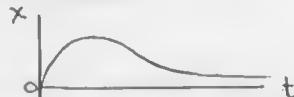
$$x = \frac{v_0}{\lambda_2 - \lambda_1} [e^{\lambda_2 t} - e^{\lambda_1 t}]$$

FOR  $x=0$ ,  $t \neq \infty$

$$e^{\lambda_2 t} = e^{\lambda_1 t}$$

FOR  $C > C_c, \lambda_1 \neq \lambda_2$ ; THUS NO SOLUTION CAN EXIST FOR  $t$  AND  $x$  IS NEVER 0

THE  $x-t$  CURVE FOR THIS MOTION IS AS SHOWN



19.128

GIVEN:

HEAVY DAMPING,  $C > C_c$ 

SHOW THAT:

A BODY RELEASED FROM AN ARBITRARY POSITION WITH AN ARBITRARY VELOCITY CANNOT PASS THROUGH ITS EQUILIBRIUM POSITION MORE THAN ONCE.

SUBSTITUTE THE INITIAL CONDITIONS,  $t=0, x=x_0, v=v_0$  IN EQS (1) AND (2) OF PROB. 19.127

$$x_0 = C_1 + C_2 \quad v_0 = C_1 \lambda_1 + C_2 \lambda_2$$

SOLVING FOR  $C_1$  AND  $C_2$ ,  $C_1 = \frac{(v_0 - \lambda_2 x_0)}{\lambda_2 - \lambda_1}, C_2 = \frac{(v_0 - \lambda_1 x_0)}{\lambda_2 - \lambda_1}$  AND SUBSTITUTING IN (1)

$$x = \frac{1}{\lambda_2 - \lambda_1} [(v_0 - \lambda_1 x_0) e^{\lambda_2 t} - (v_0 - \lambda_2 x_0) e^{\lambda_1 t}]$$

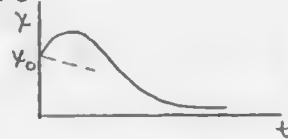
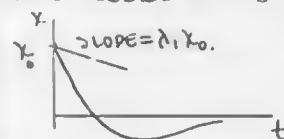
FOR  $x=0, t \neq \infty$   $(v_0 - \lambda_1 x_0) e^{\lambda_2 t} = (v_0 - \lambda_2 x_0) e^{\lambda_1 t}$

$$e^{(\lambda_2 - \lambda_1)t} = \frac{(v_0 - \lambda_2 x_0)}{(v_0 - \lambda_1 x_0)}$$

$$t = \frac{1}{(\lambda_2 - \lambda_1)} \ln \frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0}$$

THIS DEFINES ONE VALUE OF  $t$  ONLY FOR  $x=0$ , WHICH WILL EXIST IF THE NATURAL LOG IS POSITIVE I.E IF  $\frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0} > 1$ . ASSUMING  $\lambda_1 < \lambda_2 < 0$

THIS OCCURS IF  $v_0 < \lambda_1 x_0$



19.129

GIVEN:

LIGHT DAMPING,  $C < C_c$ 

SHOW THAT:

THE RATIO OF ANY TWO SUCCESSIVE MAXIMUM DISPLACEMENTS  $x_n$  AND  $x_{n+1}$  IN FIG. 19.11 IS A CONSTANT AND THAT THE NATURAL LOGARITHM OF THIS RATIO CALLED THE LOGARITHMIC DECREMENT IS,

$$\ln \frac{x_n}{x_{n+1}} = \frac{2\pi(C/C_c)}{\sqrt{1-(C/C_c)^2}}$$

FOR LIGHT DAMPING,  $C < C_c$ 

$$\text{EQ (19.46)} \quad x = x_0 e^{-(C/2m)t} \sin(\omega_0 t + \phi)$$

AT GIVEN MAX. DISPLACEMENT,  $t = t_n$ ,  $x = x_n$ 

$$\sin(\omega_0 t_n + \phi) = 1, \quad x_n = x_0 e^{-(C/2m)t_n}$$

AT NEXT MAX. DISPLACEMENT,  $t = t_{n+1}$ ,  $x = x_{n+1}$ 

$$\sin(\omega_0 t_{n+1} + \phi) = 1, \quad x_{n+1} = x_0 e^{-(C/2m)t_{n+1}}$$

$$\text{BUT } \omega_0 t_{n+1} - \omega_0 t_n = 2\pi$$

$$t_{n+1} - t_n = 2\pi/\omega_0$$

RATIO OF SUCCESSIVE DISPLACEMENTS:

$$\frac{x_n}{x_{n+1}} = \frac{x_0 e^{-(C/2m)t_n}}{x_0 e^{-(C/2m)t_{n+1}}} = e^{-(C/2m)(t_n - t_{n+1})} = e^{+(C/2m) \frac{2\pi}{\omega_0}}$$

$$\text{THUS } \ln \frac{x_n}{x_{n+1}} = \frac{C\pi}{m\omega_0} \quad (1)$$

$$\text{FROM EQS. (19.45)} \quad \omega_0 = \omega_n \sqrt{1 - (\frac{C}{C_c})^2}$$

$$\text{AND (19.41)} \quad \omega_0 = \frac{C_c}{2m} \sqrt{1 - (\frac{C}{C_c})^2}$$

THUS

$$\ln \frac{x_n}{x_{n+1}} = \frac{C\pi}{m} \frac{2m}{C_c} \frac{1}{\sqrt{1 - (\frac{C}{C_c})^2}}$$

$$\ln \frac{x_n}{x_{n+1}} = \frac{2\pi(C/C_c)}{\sqrt{1 - (C/C_c)^2}} \quad (\text{Q.E.D.})$$

19.130

GIVEN:

LIGHT DAMPING  $C/C_c < 1$ 

SHOW THAT:

SHOW THAT THE LOGARITHMIC DECREMENT CAN BE EXPRESSED AS  $1/k \ln(x_n/x_{n+k})$ , WHERE  $k$  IS THE NUMBER OF CYCLES BETWEEN READINGS OF THE MAXIMUM DISPLACEMENT

AS IN PROB. 19.129, FOR MAXIMUM DISPLACEMENTS  $x_n$  AND  $x_{n+k}$  AT  $t_n$  AND  $t_{n+k}$ ,  $\sin(\omega_0 t_n + \phi) = 1$  AND  $\sin(\omega_0 t_{n+k} + \phi) = 1$ .

$$x_n = x_0 e^{-(C/2m)t_n} \quad x_{n+k} = x_0 e^{-(C/2m)t_{n+k}}$$

RATIO OF MAXIMUM DISPLACEMENTS

$$\frac{x_n}{x_{n+k}} = \frac{x_0 e^{-(C/2m)t_n}}{x_0 e^{-(C/2m)t_{n+k}}} = e^{-(C/2m)(t_n - t_{n+k})}$$

$$\text{BUT } \omega_0 t_{n+k} - \omega_0 t_n = k(2\pi) \quad t_n - t_{n+k} = k \frac{2\pi}{\omega_0}$$

$$\text{THUS } \frac{x_n}{x_{n+k}} = e^{+(C/2m)(\frac{2k\pi}{\omega_0})}; \ln \frac{x_n}{x_{n+k}} = \frac{kC\pi}{m\omega_0} \quad (2)$$

BUT FROM PROB. 19.129 EQ.(1)

$$\text{LOG DECREMENT} = \ln \frac{x_n}{x_{n+1}} = \frac{C\pi}{m\omega_0}$$

COMPARING WITH EQ (2)

$$\text{LOG DECREMENT} = \frac{1}{k} \ln \frac{x_n}{x_{n+k}} \quad \text{Q.E.D.}$$

19.131

GIVEN:

LIGHT DAMPING,  $C < C_c$ 

$$T_0 = 2\pi/\omega_0$$

SHOW THAT:

- (a) TIME BETWEEN A MAXIMUM POSITIVE DISPLACEMENT AND THE FOLLOWING MAX NEGATIVE DISPLACEMENT IS  $T_0/2$   
 (b) TIME BETWEEN TWO SUCCESSIVE ZERO DISPLACEMENTS IS  $T_0/2$   
 (c) TIME BETWEEN A MAXIMUM POSITIVE DISPLACEMENT AND THE FOLLOWING ZERO DISPLACEMENT IS GREATER THAN  $T_0/4$

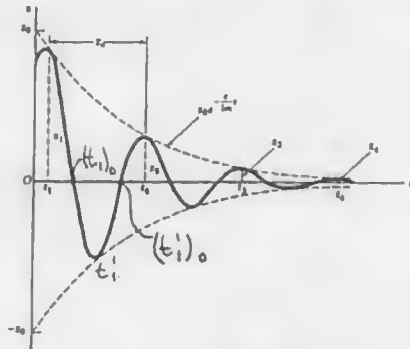


FIG. 19.11

EQ. (19.46)

$$x = x_0 e^{-(C/2m)t} \sin(\omega_0 t + \phi)$$

(a) MAXIMA (POSITIVE OR NEGATIVE) WHEN  $\dot{x} = 0$ 

$$\dot{x} = x_0 (-C/2m) e^{-(C/2m)t} \sin(\omega_0 t + \phi) + x_0 \omega_0 e^{-(C/2m)t} \cos(\omega_0 t + \phi)$$

THUS ZERO VELOCITIES OCCUR AT TIMES WHEN  $\dot{x} = 0$ , OR  $\tan(\omega_0 t + \phi) = 2m\omega_0/C$  (1)THE TIME TO THE FIRST ZERO VELOCITY,  $t_1$ , IS

$$t_1 = [\tan^{-1}(2m\omega_0/C) - \phi] / \omega_0 \quad (2)$$

THE TIME TO THE NEXT ZERO VELOCITY WHERE THE DISPLACEMENT IS NEGATIVE, IS

$$t_1' = [\tan^{-1}(2m\omega_0/C) - \phi + \pi] / \omega_0 \quad (3)$$

SUBTRACTING (2) FROM (3)

$$t_1' - t_1 = \pi/\omega_0 = \frac{\pi T_0}{2} = T_0/2 \quad \text{Q.E.D.}$$

(b) ZERO DISPLACEMENTS OCCUR WHEN

$$\sin(\omega_0 t + \phi) = 0 \quad \text{OR AT INTERVALS OF } \omega_0 t + \phi = \pi, 2\pi, \dots$$

THUS,  $(t_1)_0 = (\pi - \phi)/\omega_0$  AND  $(t_1')_0 = (2\pi - \phi)/\omega_0$ 

$$\text{TIME BETWEEN } 0's = (t_1')_0 - (t_1)_0 = \frac{2\pi - \pi}{\omega_0} = \frac{\pi T_0}{2} = T_0/2 \quad \text{Q.E.D.}$$

PLOT OF EQ. (1)



PLOT OF EQ. (1)

(c) THE FIRST MAXIMA OCCURS AT  $t_1'(\omega_0 t_1' + \phi)$ THE FIRST ZERO OCCURS AT  $(\omega_0(t_1)_0 + \phi) = \pi$ FROM THE ABOVE PLOT  $(\omega_0(t_1')_0 + \phi) - (\omega_0(t_1)_0 + \phi) > \pi$ 

$$\text{OR } (t_1')_0 - t_1 > \pi/2\omega_0 \quad (t_1')_0 - t_1 > T_0/4 \quad \text{Q.E.D.}$$

SIMILAR PROOFS CAN BE MADE FOR SUBSEQUENT MAX AND MIN

19.132

GIVEN:

BLOCK IN EQUILIBRIUM AS SHOWN IS DEPRESSED 1.2 IN. AND RELEASED AFTER 10 CYCLES THE MAXIMUM DISPLACEMENT OF THE BLOCK IS 0.5 IN.



FIND:

- (a) THE DAMPING FACTOR  $c/c_c$   
(b) THE VALUE OF THE COEFFICIENT OF VISCOUS DAMPING  $c$

FROM PROB 19.130 AND 19.129

$$(\gamma_k) \ln(\gamma_n/\gamma_{n+k}) = \frac{2\pi c/c_c}{\sqrt{1-(c/c_c)^2}}$$

WHERE  $k$  = NUMBER OF CYCLES = 10  
(a) FIRST MAXIMA IS,  $x_1 = 1.2$  IN.

THUS,  $n=1$   $\frac{\gamma_1}{\gamma_{1+10}} = \frac{1.2}{0.5} = 2.4$

$$\frac{1}{10} \ln 2.4 = 0.08755 = \frac{2\pi c/c_c}{\sqrt{1-(c/c_c)^2}}$$

$$1-(c/c_c)^2 = \left(\frac{2\pi}{0.08755}\right)^2 (c/c_c)^2$$

$$\left(\frac{c}{c_c}\right)^2 \left[\left(\frac{2\pi}{0.08755}\right)^2 + 1\right] = 1$$

$$\left(\frac{c}{c_c}\right)^2 = 1/(5150.1) = 0.0001941$$

$$c/c_c = 0.01393$$

(b)  $c_c = 2m\sqrt{k/m}$  (EQ. 19.41)

OR  $c_c = 2\sqrt{k/m}$

$$c_c = 2\sqrt{(8 \text{ lb/ft}) / (9 \text{ lb}/32.2 \text{ ft/s}^2)}$$

$$c_c = 2.991 \text{ lb}\cdot\text{s}/\text{ft}$$

FROM (a)  $\frac{c}{c_c} = 0.01393$   $c = (0.01393)(2.991)$

$$c = 0.0417 \text{ lb}\cdot\text{s}/\text{ft}$$

19.133

GIVEN:

SUCCESSIVE MAXIMUM DISPLACEMENTS OF A SPRING-MASS-DASHPOT SYSTEM ARE 25, 15, AND 9 MM  
 $m = 18 \text{ kg}$ ,  $k = 2100 \text{ N/m}$

FIND:

(a) THE DAMPING FACTOR  $c/c_c$ (b) THE COEFFICIENT OF VISCOUS DAMPING  $c$ .

(a) FROM PROB 19.29  $\ln \frac{\gamma_n}{\gamma_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$

FOR  $\gamma_n = 25 \text{ mm}$  AND  $\gamma_{n+1} = 15 \text{ mm}$

$$\ln \frac{25}{15} = 0.5108 = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\left(\frac{c}{c_c}\right)^2 \left[\left(\frac{2\pi}{0.5108}\right)^2 + 1\right] = 1$$

$$\left(\frac{c}{c_c}\right)^2 = \frac{1}{(151.3+1)} = 0.006566, \quad \frac{c}{c_c} = 0.0810$$

(b)  $c_c = 2m\sqrt{k/m}$  (EQ. 19.41)

$$c_c = 2\sqrt{k/m} = 2\sqrt{(2100 \text{ N/m})/(18 \text{ kg})} = 0.3888 \text{ N}\cdot\text{s}/\text{m}$$

FROM (a)  $\frac{c}{c_c} = 0.0810$

$$c = (0.0810)(0.3888) = 31.5 \text{ N}\cdot\text{s}/\text{m}$$

19.134

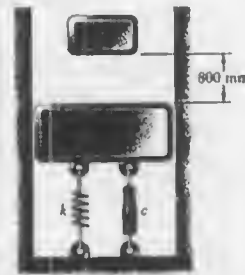
GIVEN:

4-kg BLOCK A

9-kg BLOCK B

 $k = 1500 \text{ N/m}$  $c = 230 \text{ N}\cdot\text{s}/\text{m}$ 

BLOCK A IS DROPPED FROM AN 800 MM HEIGHT ONTO B WHICH IS AT REST  
NO REBOUND



FIND:

MAXIMUM DISTANCE BLOCKS MOVE AFTER IMPACT

VELOCITY OF BLOCK A JUST BEFORE IMPACT

$$v_A = \sqrt{2gh} = \sqrt{2(9.81)(0.8)} = 3.962 \text{ m/s}$$

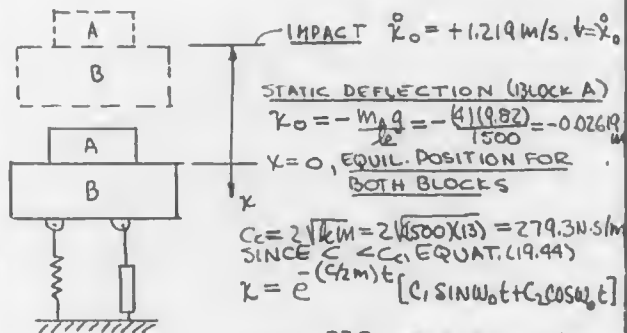
VELOCITY OF BLOCKS A AND B IMMEDIATELY AFTER IMPACT

CONSERVATION OF MOMENTUM

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

$$(4)(3.962) + 0 = (4+9) v'$$

$$v' = 1.219 \text{ m/s}$$

IMPACT  $\dot{x}_0 = +1.219 \text{ m/s}$ ,  $x_0 = 0$ 

STATIC DEFLECTION (BLOCK A)

$$x_0 = -\frac{m_A g}{k} = -\frac{4(9.81)}{1500} = -0.02619 \text{ m}$$

 $x = 0$ , EQUIL. POSITION FOR BOTH BLOCKS

$$c_c = 2\sqrt{k/m} = 2\sqrt{(1500)(13)} = 279.3 \text{ N}\cdot\text{s}/\text{m}$$

SINCE  $c < c_c$ , EQUAT. (19.44)

$$x = e^{-(c/2m)t} [C_1 \sin \omega_d t + C_2 \cos \omega_d t]$$

$$c/2m = \frac{230}{(2)(13)} = 8.846 \text{ s}^{-1}$$

FROM TOP OF PAGE 1221  $\omega_0^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$ 

$$\omega_0 = \sqrt{\frac{1500}{13} - (8.846)^2} = 6.094 \text{ rad/s}$$

$$x = e^{-8.846t} (C_1 \sin 6.094t + C_2 \cos 6.094t)$$

INITIAL CONDITIONS  $x_0 = -0.02619 \text{ m}$ 

(t=0)

$$\dot{x}_0 = +1.219 \text{ m/s}$$

$$x_0 = -0.02619 = e^0 [C_1(0) + C_2(1)]$$

$$C_2 = -0.02619$$

$$\dot{x}(0) = -8.846 e^0 [C_1(0) + (-0.02619)(1)]$$

$$+ e^{-8.846(0)} [6.094 C_1(1) + C_2(0)] = 1.219$$

$$1.219 = (-8.846)(-0.02619) + 6.094 C_1$$

$$C_1 = 0.16202$$

$$x = e^{-8.846t} (0.16202 \sin 6.094t - 0.02619 \cos 6.094t)$$

MAXIMUM DEFLECTION OCCURS WHEN  $\dot{x} = 0$ 

$$\dot{x} = 0 = -8.846 e^{-8.846t} (0.16202 \sin 6.094t - 0.02619 \cos 6.094t) + e^{-8.846t} [6.094] [0.16202 \cos 6.094t + 0.02619 \sin 6.094t]$$

$$0 = (-8.846)(0.16202) + (6.094)(0.02619) \sin 6.094t + 8.846(-0.02619) + (6.094)(0.16202) \cos 6.094t$$

### 19.134 CONTINUED

$$0 = -1.274 \sin 6.094t + 1.219 \cos 6.094t$$

$$\tan 6.094t = \frac{1.219}{1.274} = 0.957$$

$$\text{TIME AT MAX DEFLECTION} = t_m = \frac{\tan^{-1} 0.957}{6.094} = 0.1253 \text{ s}$$

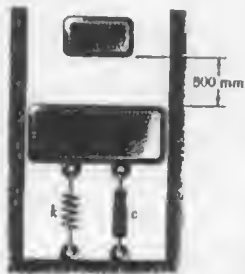
$$y_m = e^{-18.846(0.1253)} [0.1620 \sin(6.094)(0.1253) - 0.02619 \cos(6.094)(0.1253)]$$

$$y_m = (0.3301)(0.1120 - 0.0189) = 0.307 \text{ m}$$

BLOCKS MOVE, STATIC DEFLECTION +  $y_m$

$$\text{TOTAL DISTANCE} = 0.02619 + 0.307 = 0.333 \text{ m} = 333 \text{ mm}$$

### 19.135



GIVEN:

4 kg BLOCK A  
9 kg BLOCK B  
 $k = 1500 \text{ N/m}$   
 $c = 300 \text{ N·s/m}$   
BLOCK A IS DROPPED  
FROM AN 800 mm HEIGHT  
ONTO B WHICH IS AT  
REST  
NO REBOUND

FIND:

MAXIMUM DISTANCE BLOCKS  
MOVE AFTER IMPACT

VELOCITY OF BLOCK A JUST BEFORE IMPACT

$$v_A = \sqrt{2gh} = \sqrt{2(9.81)(0.8)} = 3.962 \text{ m/s}$$

VELOCITY OF BLOCKS A AND B IMMEDIATELY

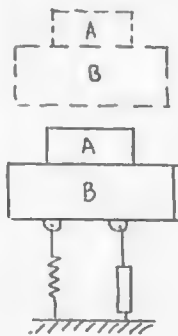
AFTER IMPACT

CONSERVATION OF MOMENTUM

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

$$(4)(3.962) + 0 = (4 + 9) v'$$

$$v' = 1.219 \text{ m/s} = \dot{x}_0$$



IMPACT  $\dot{x}_0 = +1.219 \text{ m/s} \downarrow$

STATIC DEFLECTION (BLOCK A)

$$x_0 = -\frac{m_A g}{k} = -\frac{4(9.81)}{1500} = -0.02619 \text{ m}$$

$x = 0$ , EQUIL. POSITION FOR BOTH BLOCKS

$$c_c = 2\sqrt{kM} = 2\sqrt{(1500)(13)}$$

$$c_c = 279.3 \text{ N·s/m}$$

SINCE  $c = 300 \text{ N·s/m} > c_c$   
SYSTEM IS HEAVILY DAMPED  
AND (EQ. (19.42))

$$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$\text{EQ. (19.40)} \quad \lambda = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\lambda = \frac{-300}{26} \pm \sqrt{\left(\frac{300}{26}\right)^2 - \frac{1500}{13}}$$

$$\lambda = -11.538 \pm 4.213$$

$$\lambda_1 = -15.751 \quad \lambda_2 = -7.325$$

$$x = C_1 e^{-15.751t} + C_2 e^{-7.325t}$$

### 19.135 CONTINUED

INITIAL CONDITIONS  $x_0 = -0.02619 \text{ m}$ ,  $\dot{x}_0 = 1.219 \text{ m/s}$

$$x(0) = x_0 = -0.02619 = C_1 e^0 + C_2 e^0$$

$$\dot{x}(0) = \dot{x}_0 = 1.219 = (-15.751)C_1 + (-7.325)C_2$$

SOLVING SIMULTANEOUSLY FOR  $C_1$  AND  $C_2$

$$C_1 = -0.1219, \quad C_2 = 0.09571$$

$$x(t) = -0.1219 e^{-15.75t} + 0.09571 e^{-7.325t}$$

MAXIMUM DEFLECTION WHEN  $\dot{x} = 0$

$$\dot{x} = 0 = (-1219)(-15.75) e^{-15.75t} + (0.09571)(-7.325) e^{-7.325t}$$

$$0 = 1.920 e^{-15.75t} - 0.701 e^{-7.325t}$$

$$\frac{1.920}{0.701} = e^{(-7.325 + 15.75)t}$$

$$2.739 = e^{8.425t}$$

$$\ln 2.739 = t_m$$

$$\frac{1}{8.425}$$

$$t_m = 0.1196 \text{ s}$$

$$x_m = (-0.1219) e^{-(15.75)(0.1196)} + (0.09571) e^{-(7.325)(0.1196)}$$

$$x_m = -0.01851 + 0.03986 = 0.02136 \text{ m}$$

TOTAL DEFLECTION = STATIC DEFLECTION +  $x_m$

$$\text{TOTAL DEFLECTION} = 0.02619 + 0.02136$$

$$= 0.0475 \text{ m} = 47.5 \text{ mm}$$

### 19.136 GIVEN:

GUN BARREL WEIGHT = 1500 lb  
RECUPERATOR CONSTANT  $c = 1100 \text{ lb·s/ft}$

FIND:

(a) CONSTANT  $k$  FOR RECUPERATOR TO RETURN THE BARREL TO ITS FIRING POSITION IN THE SHORTEST TIME WITHOUT OSCILLATION

(b) THE TIME NEEDED FOR THE BARREL TO MOVE TWO THIRDS OF THE WAY FROM ITS

MAXIMUM RECOIL POSITION TO ITS FIRING POSITION

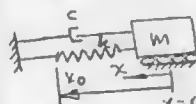
(a) A CRITICALLY DAMPED SYSTEM REGAINS ITS EQUILIBRIUM POSITION IN THE SHORTEST TIME  
THUS  $c = c_c = 1100 = 2m\sqrt{\frac{k}{m}} = 2\sqrt{kM}$  EQ. (19.41)

$$k = \frac{(c_c/2)^2}{m} = \frac{(1100/2 \text{ lb·s/ft})^2}{(1500 \text{ lb}/32.2 \text{ ft/s}^2)} = 6493.7$$

(b)

FOR A CRITICALLY DAMPED  
SYSTEM EQ. (19.43)

$$x = (C_1 + C_2 t) e^{-\omega_n t}$$



WE TAKE  $t = 0$  AT MAXIMUM  
DEFLECTION.  $x_0$

THUS  $\dot{x}(0) = 0$ ,  $x(0) = x_0$

INITIAL CONDITIONS

$$x(0) = x_0 = (C_1 + 0) e^0 \quad C_1 = x_0$$

$$x = (x_0 + C_2 t) e^{-\omega_n t}$$

$$\dot{x} = -\omega_n (x_0 + C_2 t) e^{-\omega_n t} + C_2 e^{-\omega_n t}$$

$$\dot{x}(0) = 0 = -\omega_n x_0 + C_2 \quad C_2 = \omega_n x_0$$

$$x = x_0 (1 + \omega_n t) e^{-\omega_n t}$$

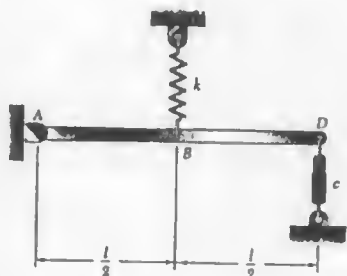
$$\text{FOR } x = \frac{x_0}{3} \quad \frac{1}{3} = (1 + \omega_n t) e^{-\omega_n t} \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$\text{BY TRIAL } \omega_n t = 2.289 \quad \omega_n = \sqrt{6490 \text{ lb/ft} / (1500 \text{ lb}/32.2 \text{ ft/s}^2)}$$

$$\omega_n = 11.806 \text{ s}^{-1} \quad t = 2.289 / 11.806 = 0.1939 \text{ s}$$



19.137

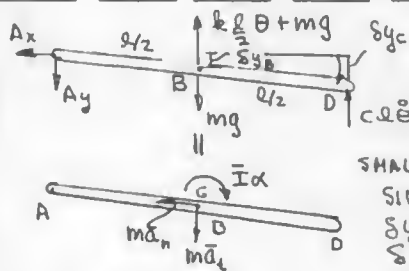


GIVEN:

ROD OF MASS  $m$   
PINNED AT A

FIND:

INTERMS OF  
 $m, k,$  AND  $c$   
(a) DIFFERENTIAL  
EQUATION OF  
MOTION  
(b) CRITICAL  
DAMPING  
COEFFICIENT  
 $c_c$



IN EQUILIBRIUM  
THE FORCE  
IN THE SPRING  
IS  $mg$   
SHALL ANGLES  
 $\sin \theta \approx \theta$   $\cos \theta \approx 1$   
 $\delta y_B = l/2 \theta$   
 $\delta y_C = l \theta$

(a) NEWTONS LAW  $\sum M_A = (\sum M_A)_{eff}$ 

$$\uparrow + mg(l/2) - (k(l/2\theta + mg)(l/2) - cl\dot{\theta}) = \bar{I}\ddot{\theta} + m\ddot{a}_G(l/2)$$

KINEMATICS  $\alpha = \ddot{\theta}$   $\bar{a}_G = l/2 \alpha = l/2 \ddot{\theta}$

$$[\bar{I} + m(l/2)^2]\ddot{\theta} + cl^2\dot{\theta} + k(l/2)^2\theta = 0$$

$$\bar{I} + m(l/2)^2 = \frac{1}{3}ml^2$$

$$\ddot{\theta} + (3c/m)\dot{\theta} + (3k/4m)\theta = 0$$

(b) SUBSTITUTING  $\theta = e^{\lambda t}$  INTO THE DIFFERENTIAL  
EQUATION OBTAINED IN (a), WE OBTAIN THE  
CHARACTERISTIC EQUATION,

$$\lambda^2 + (3c/m)\lambda + 3k/4m = 0$$

AND OBTAIN THE ROOTS

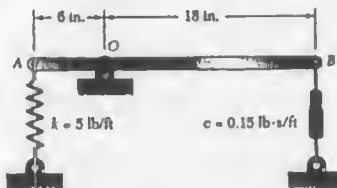
$$\lambda = \frac{-3c/m \pm \sqrt{(3c/m)^2 - (3k/m)}}{2}$$

THE CRITICAL DAMPING COEFFICIENT  $c_c$ , IS THE  
VALUE OF  $c$  IN THE RADICAL TO ZERO.  
THUS

$$(3c_c/m)^2 = 3k/m$$

$$c_c = \sqrt{km/3}$$

19.138



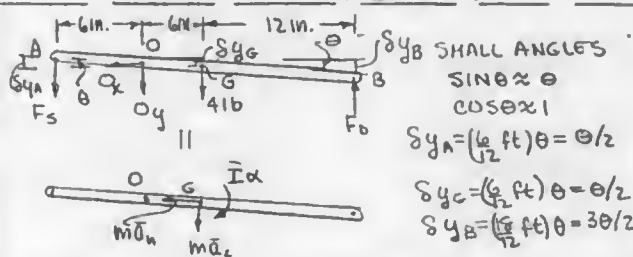
GIVEN:

4-lb ROD AB  
PINNED AT O.  
AND SUPPORTED  
BY A SPRING  
AT A. DIMENSIONS  
AND OTHER  
CONSTANTS  
AS SHOWN

FIND:

FOR SMALL OSCILLATIONS

(a) THE DIFFERENTIAL EQUATION OF MOTION  
(b) THE FORMED BY THE ROD WITH THE  
HORIZONTAL JS AFTER END B IS PUSHED  
DOWN 0.9 in. AND RELEASED

(a) NEWTONS LAW  $\sum M_O = (\sum M_O)_{eff}$ 

$$\uparrow + (-\frac{6}{12} \text{ ft}) F_s + (\frac{6}{12} \text{ ft})(4 \text{ lb}) - (\frac{18}{12} \text{ ft}) F_D = \bar{I}\ddot{\theta} + (\frac{6}{12} \text{ ft}) m\ddot{a}_G$$

(1)

$$F_s = k(\delta y_A + \delta y_{st})_A = k(\frac{6}{12} \theta + \delta y_{st})_A$$

$$F_D = c \delta \dot{y}_B = c \frac{3}{2} \dot{\theta}$$

$$\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12} m (24 \text{ in.})^2 = \frac{1}{3} m$$

KINEMATICS  $\alpha = \ddot{\theta}$   $\bar{a}_G = (\frac{6}{12} \text{ ft}) \alpha = \frac{\ddot{\theta}}{2}$ 

THUS FROM (1)

$$[\frac{m}{3} + \frac{m}{4}]\ddot{\theta} + (3/2)^2 c \dot{\theta} + (k/2)(\frac{6}{12} + \delta y_{st})_A - 2 = 0$$

(2)

BUT IN EQUILIBRIUM  $\sum M_O = 0$ 

$$\uparrow + k(\delta y_{st})_A (\frac{6}{12}) - (4 \text{ lb})(\frac{6}{12}) = 0, \quad \frac{1}{2}(\delta y_{st})_A = 2$$

EQ (2) BECOMES

$$(7/12)m\ddot{\theta} + (9/4)c\dot{\theta} + (k/4)\theta = 0$$

$$\frac{7}{12}m = (\frac{7}{12})(4/32.2) = 0.07246, \quad 9/4c = (9/4)(0.15) = 0.3375$$

$$k/4 = 5/4 = 1.25$$

$$0.07246\ddot{\theta} + 0.3375\dot{\theta} + 1.25\theta = 0$$

(b) SUBSTITUTING  $e^{\lambda t}$  INTO THE ABOVE DIFFERENTIAL EQUATION  
 $0.07246\lambda^2 + 0.3375\lambda + 1.25 = 0$

$$\lambda = \frac{-0.3375 \pm \sqrt{(0.3375)^2 - 4(0.07246)(1.25)}}{2(0.07246)}$$

$$\lambda = \frac{-0.3375 \pm \sqrt{-0.2484}}{2(0.07246)}$$

$$\lambda = -2.329 \pm 3.439i$$

SINCE THE ROOTS ARE COMPLEX AND CONJUGATE  
(LIGHT DAMPING), THE SOLUTION TO THE  
DIFFERENTIAL EQUATION IS, (EQ. 19.46),

$$\theta = \theta_0 e^{-2.329t} \sin(3.439t + \phi) \quad (3)$$

(CONTINUED)



### 19.138 CONTINUED

INITIAL CONDITIONS  $(y_0(0)) = 0.9 \text{ in.}$   
 $\theta(0) = (y_0)/18 \text{ in} = \frac{0.9}{18}$   
 $\dot{\theta}(0) = 0.05 \text{ rad}$   
 $\ddot{\theta}(0) = 0$

FROM (3)

$$\theta(0) = 0.05 = \theta_0 \sin \phi$$

$$\dot{\theta}(0) = 0 = -2.329 \theta_0 \sin \phi + 3.439 \theta_0 \cos \phi$$

$$\tan \phi = 3.439/2.329$$

$$\phi = 0.9755 \text{ rad}$$

$$\theta_0 = \frac{0.05}{\sin(0.9755)} = 0.06039 \text{ rad}$$

SUBSTITUTING INTO (3)

$$\theta = 0.06039 e^{-2.329t} \sin(3.439t + 0.9752)$$

AT  $t = 5 \text{ s}$

$$\theta(5) = 0.06039 e^{-2.329(5)} \sin[3.439(5) + 0.9752]$$

$$\theta(5) = -0.333 \times 10^{-6} \text{ rad}$$

$$\theta(5) = (0.01904 \times 10^{-6}) \text{ ABOVE HORIZONTAL}$$

### 19.139 GIVEN:

1100-lb MACHINE SUPPORTED BY TWO SPRINGS EACH WITH  $k = 3000 \text{ lb/ft}$   
 PERIODIC FORCE APPLIED OF 30-lb AT 2.8 Hz.  
 $C = 110 \text{ lb/ft}$

FIND:

AMPLITUDE OF STEADY STATE VIBRATION

EQ. (19.52)  $x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (C\omega_f)^2}}$

TOTAL SPRING CONSTANT  $k = (2)(3000 \text{ lb/ft}) = 6000 \text{ lb/ft}$

$$\omega_f = 2\pi f_f = 2\pi(2.8) = 5.6\pi \text{ rad/s}$$

$$m = W/g = 1100 \text{ lb}/(32.2 \text{ ft/s}^2) = 34.161 \text{ lb-s}^2/\text{ft}$$

$$x_m = \frac{30 \text{ lb}}{\sqrt{(6000 - (34.161)(5.6\pi)^2)^2 + (110(5.6\pi))^2} \left(\frac{\text{lb}}{\text{ft}}\right)^2}$$

$$x_m = \frac{30}{\sqrt{20.914 \times 10^6 + 3.745 \times 10^6}}$$

$$x_m = 0.00604 \text{ ft}$$

$$x_m = 0.0725 \text{ in.}$$

### 19.140 GIVEN:

1100-lb MACHINE SUPPORTED BY TWO SPRINGS  
 PERIODIC FORCE OF 30 lb APPLIED AT 2.8 Hz.  $C = 110 \text{ lb/ft}$   
 AMPLITUDE OF VIBRATION,  $x_m = 0.05 \text{ in}$

FIND:

SPRING CONSTANT OF EACH SPRING

EQ. (19.52)  $x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (C\omega_f)^2}}$

$$[(k - m\omega_f^2)^2 + (C\omega_f)^2] x_m^2 = P_m^2$$

$$k = \sqrt{(P_m/x_m)^2 - (C\omega_f)^2} + m\omega_f^2$$

$$\omega_f = 2\pi f_f = 2\pi(2.8) = 5.6\pi \quad m = \frac{W}{g} = \frac{1100 \text{ lb}}{32.2 \text{ ft/s}^2} = 34.161 \frac{\text{lb-s}^2}{\text{ft}}$$

$$k = \sqrt{\left(\frac{30 \text{ lb}}{0.05/12 \text{ ft}}\right)^2 - (110(5.6\pi))^2 + (34.161)(5.6\pi)^2}$$

$$k = \sqrt{51.84 \times 10^6 - 3.745 \times 10^6 +}$$

$$k = 6935 + 10573 = 17508 \text{ lb/ft}$$

$$k/2 = 8750 \text{ lb/ft}$$

### 19.141 GIVEN:

FORCED VIBRATING SYSTEM

FIND:

VALUES OF  $C/C_c$  FOR WHICH THE MAGNIFICATION FACTOR WILL DECREASE AS  $\omega_f/\omega_n$  INCREASES

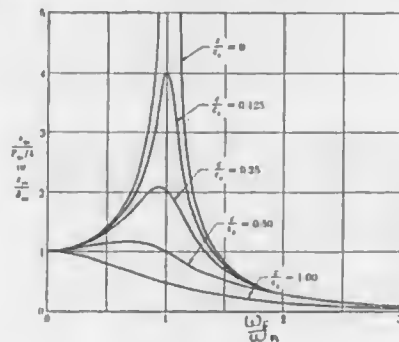


FIG. 19.12

EQ. (19.53)'

MAG. FACTOR  $\frac{x_m}{P_m/k} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(C/C_c)(\omega_f/\omega_n)]^2}}$

FIND VALUE OF  $C/C_c$  FOR WHICH THERE IS NO MAXIMUM FOR  $\frac{x_m}{P_m/k}$  AS  $\omega_f/\omega_n$  INCREASES

$$\frac{d\left(\frac{x_m}{P_m/k}\right)^2}{d(\omega_f/\omega_n)^2} = -\frac{[2(1 - (\omega_f/\omega_n)^2)(-1) + 4C^2/C_c^2]}{\{[1 - (\omega_f/\omega_n)^2]^2 + [2(C/C_c)(\omega_f/\omega_n)]^2\}^2} = 0$$

$$-2 + 2(\omega_f/\omega_n)^2 + 4C^2/C_c^2 = 0$$

$$(\omega_f/\omega_n)^2 = 1 - 2C^2/C_c^2$$

FOR  $C^2/C_c^2 \geq \frac{1}{2}$  THERE IS NO MAXIMUM FOR

$$\frac{x_m}{P_m/k}$$

AND THE MAGNIFICATION FACTOR WILL DECREASE AS  $\omega_f/\omega_n$  INCREASES

$$C/C_c \geq 1/\sqrt{2} \quad C/C_c \geq 0.707$$

19.142

GIVEN:

FORCED VIBRATING SYSTEM  
SHALL  $C/C_c$

SHOW THAT:

MAXIMUM AMPLITUDE OCCURS WHEN  
 $\omega_f \approx \omega_n$  AND THAT THE CORRESPONDING  
VALUE OF THE MAGNIFICATION  
FACTOR IS  $\frac{1}{2} C/C_c$ .

EQ. (19.53')

$$\text{MAG. FACTOR} = \frac{x_m}{P_m/k} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(C/C_c)(\omega_f/\omega_n)]^2}}$$

FIND VALUE OF  $\omega_f/\omega_n$  FOR WHICH  $\frac{x_m}{P_m/k}$   
IS A MAXIMUM

$$0 = \frac{d(\frac{x_m}{P_m/k})^2}{d(\omega_f/\omega_n)^2} = - \frac{[2(1 - (\omega_f/\omega_n)^2)(-1) + 4(C^2/C_c^2)]}{\{[1 - (\omega_f/\omega_n)^2]^2 + [2(C/C_c)(\omega_f/\omega_n)]^2\}^2}$$

$$-2 + 2(\omega_f/\omega_n)^2 + 4(C/C_c)^2 = 0$$

FOR SMALL  $C/C_c$   $\omega_f/\omega_n \approx 1$   $\omega_f \approx \omega_n$

$$\text{FOR } \omega_f/\omega_n = 1, \frac{x_m}{P_m/k} = \frac{1}{\sqrt{[1-1]^2 + [2(C/C_c)(1)]^2}}$$

$$\frac{x_m}{(P_m/k)} = \frac{1}{2} \frac{C}{C_c}$$

19.143

GIVEN:

15-kg MOTOR SUPPORTED BY FOUR  
SPRINGS EACH OF CONSTANT  
 $k = 45 \text{ kN/m}$   
MOTOR UNBALANCE IS EQUIVALENT  
TO MASS OF 20g AT 125mm  
FROM AXIS OF ROTATION

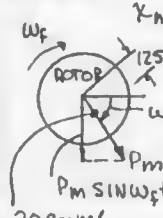
FIND:

AMPLITUDE OF STEADY STATE VIBRATION AT  
A SPEED OF 1500 RPM ASSUMING,

(a) NO DAMPING

(b) DAMPING FACTOR  $C/C_c = 1.3$ 

EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (C\omega_f)^2}}$$


$$\omega_f^2 = [1500(2\pi)/60]^2 = 24674 \text{ s}^{-2}$$

$$k = (4)(4500) = 180000 \text{ N/m}$$

$$P_m = m' r \omega_f^2 = (0.02 \text{ kg})(0.125 \text{ m})(24674 \text{ s}^{-2})$$

$$P_m = 61.685 \text{ N}$$

(a)  $C = 0$ 

$$x_m = \frac{61.685 \text{ N}}{[180000 - 15(24674)] \text{ (N/m)}}$$

$$x_m = -0.324 \times 10^{-3} \text{ m} = -0.324 \text{ mm}$$

(b) FOR  $C/C_c = 1.3$ 

$$\text{EQ. (19.41)} \quad C_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{m k} = 2\sqrt{15 \text{ kg}(180000 \text{ N/m})}$$

$$C_c = 3286 \text{ N.s/m} \quad C = (1.3)(3286) = 4272 \text{ N.s/m}$$

$$x_m = \frac{61.685 \text{ N}}{\sqrt{[180000 - 15(24674)]^2 + (4272)^2 (24674)}}$$

$$x_m = 0.0884 \times 10^{-3} \text{ m} = 0.0884 \text{ mm}$$

19.144

GIVEN:

18-kg MOTOR BOLTED TO  
A BEAM HAS A STATIC  
DEFLECTION  $\delta_{st} = 1.5 \text{ mm}$   
UNBALANCE IS EQUIVALENT  
TO A MASS OF 20g  
LOCATED 125mm FROM  
AXIS OF ROTATION



FIND:

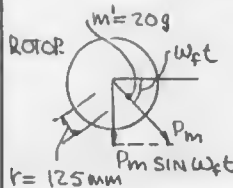
AMPLITUDE AT A MOTOR SPEED OF 900 RPM

(a) FOR NO DAMPING

(b) FOR  $C/C_c = 0.055$ 

EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (C\omega_f)^2}}$$



$$\omega_f^2 = [900(2\pi)/60]^2 = 8882.6 \text{ s}^{-2}$$

FIND SPRING CONSTANT  $k$   
FOR THE BEAM

$$k = \frac{mg}{\delta_{st}} = \frac{(18 \text{ kg})(9.81 \text{ m/s}^2)}{(1.5 \times 10^{-3} \text{ m})}$$

$$k = 117720 \text{ N/m}$$

$$P_m = m' r \omega_f^2 = (0.020 \text{ kg})(0.125 \text{ m})(8882.6 \text{ s}^{-2})$$

$$P_m = 22.20 \text{ N}$$

(a)  $C = 0$ 

$$x_m = \frac{22.20 \text{ N}}{[(117720 - (18)(8882.6 \text{ N/m}))^2]}$$

$$x_m = -0.527 \times 10^{-3} \text{ m} = -0.527 \text{ mm}$$

(b) FOR  $C/C_c = 0.055$ 

$$\text{EQ. (19.41)} \quad C_c = 2\sqrt{\frac{k}{m}} = 2\sqrt{117720(18)}$$

$$C_c = 2911 \text{ N.s/m}$$

$$C = 0.055 \quad C = (0.055)(2911) = 160.12 \text{ N.s/m}$$

$$x_m = \frac{22.21}{\sqrt{[117720 - (18)(8882.6)]^2 + (160.12)^2 (8882.6)}}$$

$$x_m = \frac{22.21}{\sqrt{(1.779 \times 10^9) + (0.2278 \times 10^9)}} = 0.000496 \text{ m}$$

$$x_m = 0.496 \text{ mm}$$

19.145

GIVEN:

100-lb MOTOR BOLTED TO BEAM WHICH HAS A STATIC DEFLECTION  $\delta_{st} = 0.25$  in. UNBALANCE IS 4 oz. AT 3 in. AMPLITUDE  $x_m = 0.010$  in AT 300 rpm

FIND:

(a) DAMPING FACTOR  $C/C_c$ (b) COEFFICIENT OF DAMPING  $C$ 

EQ. (19.53')

$$x_m = \frac{P_m/k}{\sqrt{(1 - (\omega_f/\omega_n)^2)^2 + (2(C/C_c)(\omega_f/\omega_n))^2}}$$

$\omega_f = 300 \text{ rpm} = 31.4 \text{ rad/s}$   
 $r = 3 \text{ in.}$   
 $\omega_n^2 = \frac{g}{\delta_{st}} = \frac{32.2 \text{ ft/s}^2}{0.25/12 \text{ ft}} = 1546 \text{ rad/s}^2$   
 $\omega_n = 39.3 \text{ rad/s}$   
 $\omega_f^2 = (300 \times \pi/30)^2 = 987.2 \text{ s}^{-2}$   
 $m' = 4 \text{ oz.}$   
 $P_m = m' r \omega_f^2 = (4/16 \text{ lb}) (3 \text{ ft}) (987.2 \text{ s}^{-2}) = 74.14 \text{ lb-ft}$   
 $C_c = \omega_n^2 m = (1546)(100/32.2) = 4801 \text{ lb-ft/s}$   
 $P_m/k = 74.14/4801 = 0.01544 \text{ ft}$   
 $\frac{0.01}{0.01544} = \frac{0.0003991}{\sqrt{(1 - 0.6387)^2 + (4)(0.6387)(C/C_c)^2}}$   
 $0.2293 = 0.1305 + 2.555 (C/C_c)^2$   
 $(C/C_c)^2 = \frac{0.0988}{2.555} = 0.03867$   
 $C/C_c = 0.1966$

(b) EQ. (19.41)  $C_c = 2m\omega_n$ 

$$C_c = 2 \left( \frac{100 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (39.3)^{1/2}$$

$$C_c = 244.2 \text{ lb-s/ft}$$

$$\frac{C}{C_c} = 0.1957$$

$$C = (244.2)(0.1957) = 48.0 \frac{\text{lb-s}}{\text{ft}}$$

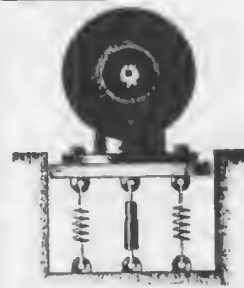
19.146

GIVEN:

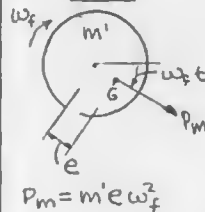
100-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH OF CONSTANT  $k = 90 \text{ kN/m}$  DASHPOT  $C = 6500 \text{ N-s/m}$  AMPLITUDE  $x_m = 2.1 \text{ mm}$  AT A SPEED OF 1200 RPM MASS OF THE ROTOR  $m' = 15 \text{ kg}$

FIND:

DISTANCE BETWEEN THE MASS CENTER OF THE ROTOR AND THE AXIS OF SHAFT



ROTOR



EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (C\omega_f)^2}}$$

$$\omega_f^2 = (1200 \times 2\pi/60)^2 = 15791 \text{ s}^{-2}$$

$$k = 4(90,000 \text{ N/m}) = 360,000 \text{ N/m}$$

$$P_m = m' e \omega_f^2$$

$$P_m = (15 \text{ kg})(e)(15791 \text{ s}^{-2})$$

$$P_m = 236870 e$$

$$0.0021 = \frac{236870 e}{\sqrt{(360,000 - (100)(15791))^2 + (6500)^2 (15791)^2}}$$

$$(1.4674 \times 10^6)(0.0021) = (236870) e$$

$$e = 0.1301 \text{ m}$$

$$e = 130.1 \text{ mm}$$

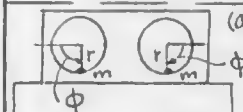
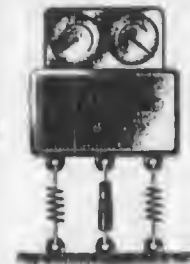
19.147

GIVEN:

TWO 400-g MASSES AT  $r = 150 \text{ mm}$  ROTATE AT THE SAME SPEED OF 1200 RPM IN OPPOSITE SENSES WHEN THE MASSES ARE EXACTLY BENEATH THEIR RESPECTIVE ROTATION AXES AMPLITUDE OF THE MOTION AT THIS SPEED EQUALS 15 mm TOTAL MASS = 140 kg

FIND:

(a) THE COMBINED SPRING CONSTANT  $k$   
 (b) THE DAMPING FACTOR  $C/C_c$



(a)  $\phi = \pi/2$ , AT 1200 RPM  
 EQ. (19.54)  $\tan \phi = \frac{2(C/C_c)(\omega_f/\omega_n)}{1 - (\omega_f/\omega_n)^2}$

$$\text{SINCE } \phi = \pi/2 \text{ TAN } \phi = \infty$$

$$\text{THUS } 1 - (\omega_f/\omega_n)^2 = 0$$

$$\omega_n = \omega_f = (1200 \times 2\pi/60) = 40\pi \text{ s}^{-1}$$

$$\omega_n^2 = \frac{k}{M} \quad k = M\omega_n^2 = (140 \text{ kg})(40\pi \text{ s}^{-1})^2 = 2210 \frac{\text{kN}}{\text{m}}$$

(b)

$$\text{EQ. (19.53), } \frac{x_m}{P_m/k} = \frac{1}{\sqrt{(1 - \omega_f^2/\omega_n^2)^2 + (2(C/C_c)(\omega_f/\omega_n))^2}}$$

$$\omega_f/\omega_n = 1 \quad P_m = 2mr\omega_f^2 = (2)(0.4 \text{ kg})(0.150 \text{ m})(40\pi \text{ s}^{-1})^2$$

$$P_m = 1895 \text{ N}, (0.015 \text{ m}) / [(1895 \text{ N}) / (2210 \times 10^3)] = 1 / [2(C/C_c)]$$

$$C/C_c = 0.0286 \frac{\text{N-s}}{\text{N}}$$

19.148

GIVEN:

MACHINE SUPPORTED BY SPRINGS AND CONNECTED TO A DASHPOT AS SHOWN

SHOW THAT:

THE AMPLITUDE OF THE FLUCTUATING FORCE TRANSMITTED TO THE FOUNDATION IS,

$$F_m = P_m \sqrt{\frac{1 + [z(c/c_c)(\omega_f/\omega_n)]^2}{[1 - (\omega_f/\omega_n)^2]^2 + [z(c/c_c)(\omega_f/\omega_n)]^2}}$$

FROM EQ. (19.48) THE MOTION OF THE MACHINE IS,

$$x = x_m \sin(\omega_f t - \phi)$$

THE TRANSMITTED TO THE FOUNDATION IS,

$$\text{SPRINGS } F_s = kx = kx_m \sin(\omega_f t - \phi)$$

$$\text{DASHPOT } F_d = c\dot{x} = cx_m \omega_f \cos(\omega_f t - \phi)$$

$$F_T = x_m [k \sin(\omega_f t - \phi) + c\omega_f \cos(\omega_f t - \phi)]$$

OR RECALLING THE IDENTITY,

$$A \sin y + B \cos y = \sqrt{A^2 + B^2} \sin(y + \psi)$$

$$\sin \psi = B / \sqrt{A^2 + B^2}$$

$$\cos \psi = A / \sqrt{A^2 + B^2}$$

$$F_T = [x_m \sqrt{k^2 + (c\omega_f)^2}] \sin(\omega_f t - \phi + \psi)$$

THUS THE AMPLITUDE OF  $F_T$  IS

$$F_m = x_m \sqrt{k^2 + (c\omega_f)^2} \quad (1)$$

FROM EQ. (19.53)  $x_m = \frac{P_m/k}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [z(c/c_c)(\omega_f/\omega_n)]^2}}$

SUBSTITUTING FOR  $x_m$  IN EQ (1)

$$F_m = \frac{P_m \sqrt{1 + (c\omega_f/k)^2}}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [z(c/c_c)(\omega_f/\omega_n)]^2}} \quad (2)$$

$$\omega_n^2 = k/m$$

AND EQ. (19.41)  $c_c = 2m\omega_n$   $m = c\omega_n/z$

$$c\omega_f/k = c\omega_f/m\omega_n^2 = z(c/c_c)(\omega_f/\omega_n)$$

SUBSTITUTING IN (2)

$$F_m = \frac{P_m \sqrt{1 + [z(c/c_c)(\omega_f/\omega_n)]^2}}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [z(c/c_c)(\omega_f/\omega_n)]^2}} \quad \text{Q.E.D.}$$

19.149

GIVEN:

SYSTEM AS SHOWN ABOVE IN PROB 19.148 WITH WEIGHT  $W = 200$  lb, FOUR SPRINGS, EACH WITH  $k = 12$  lb/ft, AND APPLIED PERIODIC FORCE WITH FREQUENCY  $f_f = \omega_f/2\pi = 0.8$  Hz AND AMPLITUDE  $P_m = 20$  lb

FIND:

AMPLITUDE OF FORCE  $F_m$  TRANSMITTED TO FOUNDATION

(a) IF  $c = 25$  lb-s/ft (b)  $c = 0$

REFER TO THE EQUATION DERIVED IN PROB 19.148

$$\omega_f = 2\pi f_f = 2\pi(0.8) = 1.6\pi \quad \omega_n^2 = k/m = 48/(200/32.2)$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 = \frac{(1.6\pi)^2}{(2.780)^2} = 3.269, \quad \frac{c}{c_c} = \frac{25}{2m\omega_n} = \frac{25}{(2)(200/32.2)(2.780)}$$

$$\frac{c}{c_c} = 0.7239, \quad \left(\frac{c}{c_c}\right)^2 = 0.52406$$

$$(a) F_m = \frac{20 \sqrt{1 + (4)(0.52406)(3.269)}}{\sqrt{(1 - 3.269)^2 + 4(0.52406)(3.269)}} = \frac{20 \sqrt{1 + 6.8526}}{\sqrt{5.148 + 6.8526}}$$

$$F_m = 16.18 \text{ lb}$$

$$(b) c = 0, F_m = (20)/\sqrt{5.148} = 8.81 \text{ lb}$$

19.150

GIVEN:

STEADY STATE VIBRATION UNDER A HARMONIC FORCE

SHOW THAT:

MECHANICAL ENERGY DISSIPATED PER CYCLE IS  $E = \pi c x_m^2 \omega_f$

ENERGY IS DISSIPATED BY THE DASHPOT

FROM EQ (19.48) THE DEFLECTION OF THE SYSTEM IS  $x = x_m \sin(\omega_f t - \phi)$

THE FORCE ON THE DASHPOT,  $F_d = c\dot{x}$

$$F_d = cx_m \omega_f \cos(\omega_f t - \phi)$$

THE WORK DONE IN A COMPLETE CYCLE WITH  $T_f = 2\pi/\omega_f$

$$E = \int_0^{2\pi/\omega_f} F_d dx \quad (\text{I.E. FORCE} \times \text{DISTANCE})$$

$$dx = x_m \omega_f \cos^2(\omega_f t - \phi) dt$$

$$E = \int_0^{2\pi/\omega_f} cx_m^2 \omega_f^2 \cos^2(\omega_f t - \phi) dt$$

$$\cos^2(\omega_f t - \phi) = (1 - 2 \cos(2\omega_f t - 2\phi))/2$$

$$E = cx_m^2 \omega_f^2 \int_0^{2\pi/\omega_f} \frac{1 - 2 \cos(2\omega_f t - 2\phi)}{2} dt$$

$$E = \frac{cx_m^2 \omega_f^2}{2} \left[ t - \frac{2 \sin(2\omega_f t - 2\phi)}{2\omega_f} \right]_0^{2\pi/\omega_f}$$

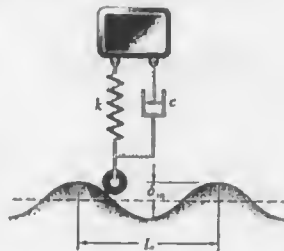
$$E = \frac{cx_m^2 \omega_f^2}{2} \left[ \frac{2\pi}{\omega_f} - \frac{2}{\omega_f} (\sin(2\pi - 2\phi) - \sin(-2\phi)) \right]$$

$$E = \pi c x_m^2 \omega_f \quad \text{Q.E.D.}$$

19.151

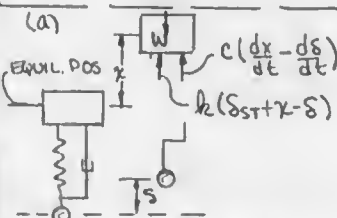
GIVEN:

SPRING-DASHPOT SYSTEM AS SHOWN WITH MASS  $M$  MOVING AT  $U$ , OVER A ROAD WITH A SINUSOIDAL CROSS SECTION OF AMPLITUDE  $S_m$  AND WAVELENGTH  $L$ .



FIND:

(a) DIFFERENTIAL EQUATION OF VERTICAL DISPLACEMENT OF MASS  $M$   
(b) EXPRESSION FOR THE AMPLITUDE OF  $M$



$$+\downarrow \Sigma F = ma: W - k(s + x) - c\left(\frac{dx}{dt} - \frac{ds}{dt}\right) = m \frac{d^2 x}{dt^2}$$

RECALLING THAT  $W = k s_{st}$ , WE WRITE

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = k s + c \frac{ds}{dt} \quad (1)$$

(CONTINUED)

### 19.151 CONTINUED

MOTION OF WHEEL IS A SINE CURVE,  $\delta = \delta_m \sin \omega_f t$   
THE INTERVAL OF TIME NEEDED TO TRAVEL A DISTANCE  $L$  AT A SPEED  $U$ , IS  $t = L/U$ , WHICH IS THE PERIOD OF THE ROAD SURFACE.

$$\text{THUS } \omega_f = 2\pi/T_f = \frac{2\pi}{L/U} = 2\pi U/L$$

$$\text{AND } \delta = \delta_m \sin \omega_f t \quad \frac{d\delta}{dt} = \frac{\delta_m 2\pi}{L/U} \cos \omega_f t$$

THUS EQ. (1) IS

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = (k \sin \omega_f t + c \omega_f \cos \omega_f t) \delta_m$$

(b) FROM THE IDENTITY

$$A \sin y + B \cos y = \sqrt{A^2 + B^2} \sin(y + \psi)$$

$$\sin \psi = B/\sqrt{A^2 + B^2}$$

$$\cos \psi = A/\sqrt{A^2 + B^2}$$

WE CAN WRITE THE DIFFERENTIAL EQUATION

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = \delta_m \sqrt{k^2 + (c\omega_f)^2} \sin(\omega_f t + \psi)$$

$$\psi = \tan^{-1} \frac{c\omega_f}{k}$$

THE SOLUTION TO THIS EQUATION

IS (ANALOGOUS TO EQ'S 19.47 AND 19.48, WITH  $P_m = \delta_m \sqrt{k^2 + (c\omega_f)^2}$ )

$x = x_m \sin(\omega_f t - \phi + \psi)$  WHERE ANALOGOUS TO EQ'S (19.52)

$$x_m = \frac{\delta \sqrt{k^2 + (c\omega_f)^2}}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$$

$$\tan \phi = \frac{c\omega_f}{k - m\omega_f^2}$$

$$\tan \psi = c\omega_f/k$$

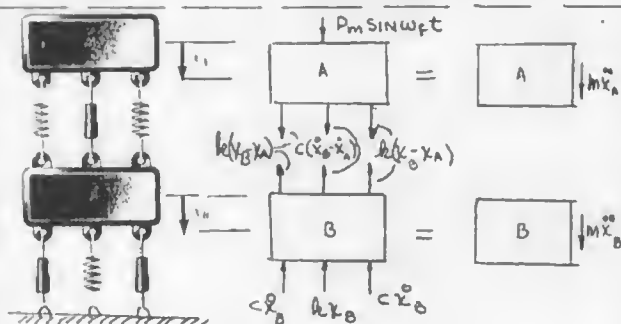
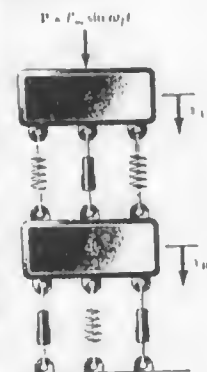
### 19.152

GIVEN:

BLOCKS A AND B HAVE MASS  $m$   
THREE SPRINGS, EACH HAVE CONSTANT  $k$   
THREE DAMPERS, EACH HAVE CONSTANT  $c$ .  
BLOCK A ACTED UPON BY A FORCE  $P = P_m \sin \omega_f t$

FIND:

DIFFERENTIAL EQUATIONS DEFINING THE DISPLACEMENTS  $x_A$  AND  $x_B$  OF THE BLOCKS FROM THEIR EQUILIBRIUM POSITION.



### 19.152 CONTINUED

SINCE THE ORIGINS OF COORDINATE ARE CHOSEN FROM THE EQUILIBRIUM POSITION, WE MAY OMIT THE INITIAL SPRING COMPRESSIONS AND THE EFFECT OF GRAVITY  
FOR LOAD A

$$+\downarrow \Sigma F = m a_A; P_m \sin \omega_f t + 2k(x_B - x_A) + c(\dot{x}_B - \dot{x}_A) = m \ddot{x}_A \quad (1)$$

FOR LOAD B

$$+\downarrow \Sigma F = m a_B; -2k(x_B - x_A) - c(\dot{x}_B - \dot{x}_A) - kx_B - c\dot{x}_B = m \ddot{x}_B$$

REARRANGING EQS (1) AND (2), WE FIND:

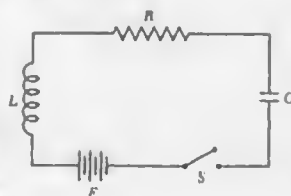
$$m \ddot{x}_A + c(\dot{x}_A - \dot{x}_B) + 2k(x_A - x_B) = P_m \sin \omega_f t$$

$$m \ddot{x}_B + 3c\dot{x}_B - c\dot{x}_A + 3kx_B - 2kx_A = 0$$

### 19.153

GIVEN:

R, L, C CIRCUIT AS SHOWN WITH SUDDENLY APPLIED VOLTAGE  $E$  WHEN THE SWITCH IS CLOSED



FIND:

VALUES OF  $R$  FOR WHICH OSCILLATIONS WILL TAKE PLACE WHEN THE SWITCH IS CLOSED

FOR A MECHANICAL SYSTEM OSCILLATIONS TAKE PLACE IF  $c < c_c$ . (LIGHTLY DAMPED) BUT FROM EQ. (19.41),

$$c_c = 2m \sqrt{k/m} = 2 \sqrt{k m}$$

THEREFORE

$$c < 2 \sqrt{k m} \quad (1)$$

FROM TABLE 19.2:

$$\begin{aligned} c &\rightarrow R \\ m &\rightarrow L \\ k &\rightarrow 1/C \end{aligned} \quad (2)$$

SUBSTITUTING IN (1) THE ANALOGOUS ELECTRICAL VALUES IN (2), WE FIND THAT OSCILLATIONS WILL TAKE PLACE IF,

$$R < 2 \sqrt{(1/C)(L)}$$

$$R < 2 \sqrt{L/C}$$

19.154

GIVEN:

R, L, C CIRCUIT OF FIG. PROB 19.153  
WITH CAPACITOR C REMOVED

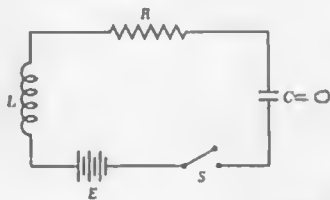
FIND:

IF SWITCH S IS CLOSED AT  $t=0$ 

(a) THE FINAL VALUE OF THE CURRENT IN THE CIRCUIT

(b) THE TIME  $t$  AT WHICH THE CURRENT WILL HAVE REACHED  $(1 - 1/e)$  TIMES ITS FINAL VALUE. (I.E. THE TIME CONSTANT)

ELECTRICAL SYSTEM



MECHANICAL SYSTEM

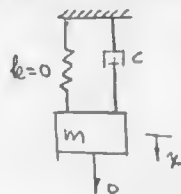


TABLE 19.2 FOR ANALOGUE  
CLOSING SWITCH S IS EQUIVALENT TO SUDDENLY  
APPLYING A CONSTANT FORCE OF MAGNITUDE P  
TO THE MASS

(a) FINAL VALUE OF THE CURRENT CORRESPONDS  
TO THE FINAL VELOCITY OF THE MASS. SINCE THE  
CAPACITANCE IS ZERO THE SPRING CONSTANT  
IS ALSO ZERO

$$P - c \frac{dx}{dt} = m \frac{d^2x}{dt^2} \quad (1)$$

FINAL VELOCITY OCCURS WHEN

$$\frac{d^2x}{dt^2} = 0$$

$$P - c \left. \frac{dx}{dt} \right|_{\text{FINAL}} = 0 \quad \left. \frac{dx}{dt} \right|_{\text{FINAL}} = \frac{P}{c}$$

$$V_{\text{FINAL}} = P/c$$

FROM TABLE 19.2:  $V \rightarrow I$ ,  $P \rightarrow E$ ,  $C \rightarrow R$   
THUS

$$I_{\text{FINAL}} = E/R$$

(b) REARRANGING EQ. (1), WE HAVE

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} = P$$

SUBSTITUTE  $\frac{dx}{dt} = Ae^{-\lambda t} + \frac{P}{c}$ ;  $\frac{d^2x}{dt^2} = -A\lambda e^{-\lambda t}$

$$m[-A\lambda e^{-\lambda t}] + c[Ae^{-\lambda t} + \frac{P}{c}] = P$$

$$-m\lambda + c = 0 \quad \lambda = c/m$$

THUS

$$\frac{dx}{dt} = Ae^{-(c/m)t} + \frac{P}{c}$$

AT  $t=0$   $\frac{dx}{dt} = 0$   $0 = A + P/c$   $A = -P/c$

$$V = \frac{dx}{dt} = \frac{P}{c} [1 - e^{-(c/m)t}]$$

FROM TABLE 19.2.  $V \rightarrow I$ ,  $P \rightarrow E$ ,  $C \rightarrow R$ ,  $m \rightarrow L$

$$I = \frac{E}{R} [1 - e^{-(R/L)t}]$$

FOR  $I = (E/R)(1 - 1/e)$ ,  $(R/L)t = 1$

$$t = \frac{L}{R}$$

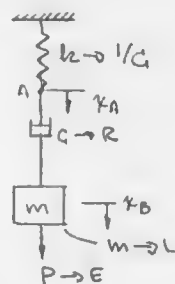
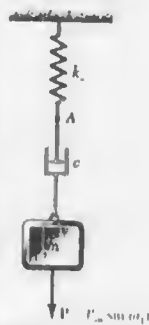
19.155

GIVEN:

MECHANICAL SYSTEM  
SHOWN

DRAW:

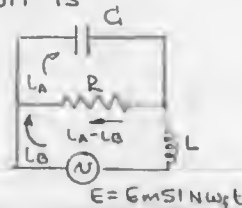
THE ELECTRICAL ANALOGUE  
CIRCUIT



WE NOTE THAT BOTH THE  
SPRING AND THE DASHPOT  
AFFECT THE MOTION OF  
POINT A. THUS ONE LOOP  
IN THE ELECTRICAL CIRCUIT  
SHOULD CONSIST OF A  
CAPACITOR ( $k \rightarrow 1/C$ ) AND A  
RESISTANCE ( $c \rightarrow R$ )

THE OTHER LOOP CONSISTS  
OF ( $m \sin \omega t \rightarrow E \sin \omega t$ ), AN  
INDUCTOR ( $m \rightarrow L$ ) AND THE  
RESISTOR ( $c \rightarrow R$ )

SINCE THE RESISTOR IS COMMON TO BOTH LOOPS,  
THE CIRCUIT IS



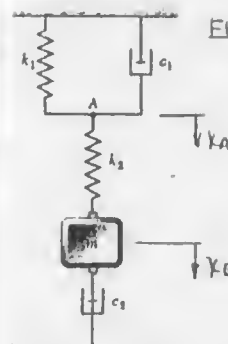
19.156

GIVEN:

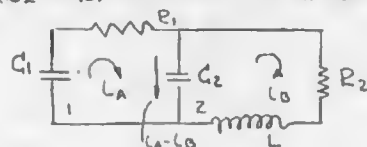
MECHANICAL SYSTEM SHOWN

FIND:

THE ELECTRICAL ANALOGUE  
CIRCUIT



LOOP 1 (POINT A)  $k_1 \rightarrow \frac{1}{C_1}$ ,  $k_2 \rightarrow \frac{1}{C_2}$ ,  $C_1 \rightarrow R_1$   
LOOP 2 (MASS m)  $k_2 \rightarrow \frac{1}{C_2}$ ,  $m \rightarrow L$ ,  $C_2 \rightarrow R_2$   
WITH ( $k_2 \rightarrow 1/C_2$ ) COMMON TO BOTH LOOPS,



19.157

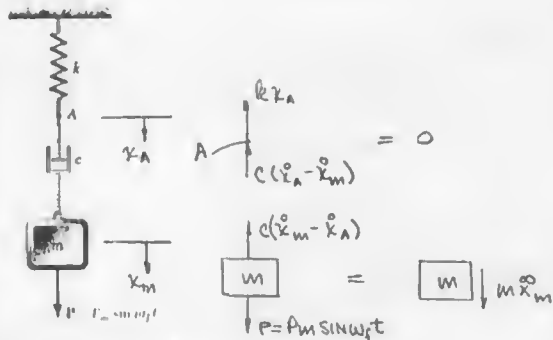
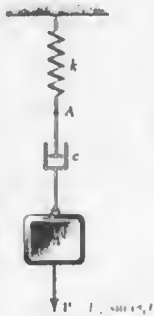
GIVEN:

MECHANICAL SYSTEM SHOWN

FIND:

THE DIFFERENTIAL EQUATIONS  
DEFINING

- (a) THE DISPLACEMENTS OF MASS  $M$   
AND OF THE POINT  $A$   
(b) THE CHARGES ON THE CAPACITORS  
OF THE ELECTRICAL ANALOGUE



(a) MECHANICAL SYSTEM

POINT A

$$\uparrow \sum F = 0 \quad kx_A + c \frac{d}{dt}(x_A - x_m) = 0$$

MASS  $M$ 

$$\uparrow \sum F = ma \quad c \frac{d}{dt}(x_m - x_A) - P_m \sin \omega_f t = -m \frac{d^2 x_m}{dt^2}$$

$$m \frac{d^2 x_m}{dt^2} + c \frac{d}{dt}(x_m - x_A) = P_m \sin \omega_f t$$

(b) ELECTRICAL ANALOGUE

FROM TABLE 19.2

$M \rightarrow L$   
 $C \rightarrow R$   
 $k \rightarrow 1/C$   
 $x \rightarrow q$   
 $P \rightarrow E$

SUBSTITUTING INTO THE RESULTS FROM PART (a), THE  
ANALOGOUS ELECTRICAL CHARACTERISTICS,

$$(1/C)q_A + R \frac{d}{dt}(q_A - q_m) = 0$$

$$L \frac{d^2 q_m}{dt^2} + R \frac{d}{dt}(q_m - q_A) = E_m \sin \omega_f t$$

NOTE:

THESE EQUATIONS CAN ALSO BE OBTAINED  
BY SUMMING THE VOLTAGE DROPS AROUND  
THE LOOPS IN THE CIRCUIT OF PROB 19.155

19.158

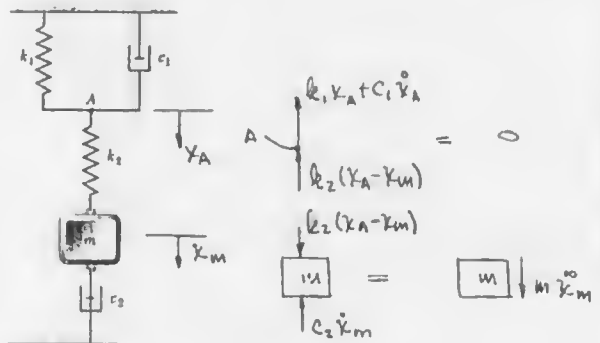
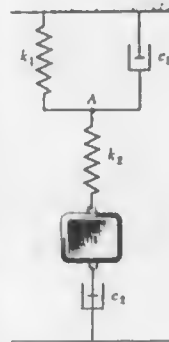
GIVEN:

MECHANICAL SYSTEM SHOWN

FIND:

THE DIFFERENTIAL EQUATIONS  
DEFINING

- (a) THE DISPLACEMENTS OF  
MASS  $M$  AND OF THE POINT  $A$   
(b) THE CHARGES ON THE  
CAPACITORS OF THE  
ELECTRICAL ANALOGUE



(a) MECHANICAL SYSTEM

POINT A

$$\uparrow \sum F = 0 \quad k_1 x_A + c_1 \frac{dx_A}{dt} + k_2(x_A - x_m) = 0$$

$$c_1 \frac{dx_A}{dt} + (k_1 + k_2)x_A - k_2 x_m = 0$$

MASS  $M$ 

$$\uparrow \sum F = ma \quad k_2(x_A - x_m) - c_2 \frac{dx_m}{dt} = m \frac{d^2 x_m}{dt^2}$$

$$m \frac{d^2 x_m}{dt^2} + c_2 \frac{dx_m}{dt} + k_2(x_m - x_A) = 0$$

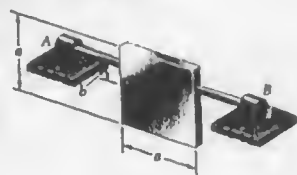
(b) ELECTRICAL ANALOGUE

SUBSTITUTING INTO THE RESULTS FROM PART (a)  
USING THE ANALOGOUS ELECTRICAL CHARACTERISTICS  
FROM TABLE 19.2 (SEE LEFT),

$$R_1 \frac{dq_A}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)q_A - \frac{1}{C_2}q_m = 0$$

$$L \frac{d^2 q_m}{dt^2} + R_2 \frac{dq_m}{dt} + \frac{1}{C_2}(q_m - q_A) = 0$$

19.159

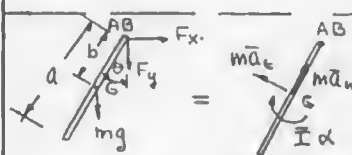


GIVEN:

THIN SQUARE PLATE  
OF SIDE  
OSCILLATIONS ABOUT  
AB AT A DISTANCE  
b FROM G

FIND:

- (a) PERIOD IF  $b = a/2$   
(b) A SECOND VALUE OF  
b WHICH GIVES THE  
SAME PERIOD AS IN (a)



NEWTON'S LAW  
 $\sum M_A = (\sum I A_B)_{eff}$

$$+ \quad mgb \sin \theta = -I \alpha - (m \bar{a}_t)(b)$$

$$\alpha = \ddot{\theta}$$

$$a_t = b \alpha = b \ddot{\theta}$$

$$\sin \theta \approx \theta$$

$$(\bar{I} + mb^2) \ddot{\theta} + mgb \theta = 0$$

$$\bar{I}_x = \frac{1}{12} ma^2$$

$$\ddot{\theta} + \frac{g b}{\frac{1}{12} a^2 + b^2} \theta = 0$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{a^2 + 12b^2}{12gb}} \quad (1)$$

(a) WHEN  $b = a/2$ 

$$\tau_n = 2\pi \sqrt{\frac{a^2 + 12(a^2/4)}{12g(a/2)}} = 2\pi \sqrt{\frac{2a}{3g}}$$

(b) EQUATING THE RESULT FROM PART (a)  
TO EQ. (1) AND SQUARING BOTH SIDES,

$$\frac{a^2 + 12b^2}{12gb} = \frac{2a}{3g}$$

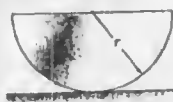
$$36b^2 - (24a)(b) + 3a^2 = 0$$

$$b^2 - \left(\frac{2}{3}a\right)b + \frac{a^2}{12} = 0$$

$$b = \frac{+2a \pm \sqrt{4a^2 - \frac{4}{3}a^2}}{2} = \frac{a}{2}, \frac{a}{6}$$

$$b = \frac{a}{6}$$

19.160



GIVEN:

HALF SECTION OF A SOLID  
CYLINDER IS ROTATED THROUGH  
A SMALL ANGLE AND RELEASED

FIND:

PERIOD OF OSCILLATION (USUALLY)

POSITION ①

$$V = 0 \quad T_1 = 0$$

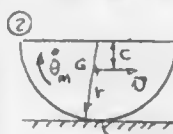
$$V_1 = m g c (1 - \cos \theta_m) \approx m g c \frac{\theta_m^2}{2}$$

POSITION ②

$$V_2 = 0$$

$$T_2 = \frac{1}{2} m \bar{V}^2 + \frac{1}{2} \bar{I} \dot{\theta}_m^2$$

$$\bar{V} = (r - c) \dot{\theta}$$

INSTANTANEOUS  
CENTER

$$I_0 = \bar{I} + mc^2 \quad \bar{I} = I_0 - mc^2 = \frac{1}{2} mr^2 - mc^2 = m[r^2/2 - c^2]$$

$$T_2 = \frac{1}{2} m [(r - c)^2 + (\frac{r^2}{2} - c^2)] \dot{\theta}_m^2$$

$$T_2 = \frac{1}{2} m [r^2 - 2cr + \frac{r^2}{2} - c^2] \dot{\theta}_m^2$$

$$c = \frac{4r}{3\pi} \quad T_2 = \frac{1}{2} m \left[ \frac{3}{2} r^2 - 2\left(\frac{4r}{3\pi}\right)r \right] \dot{\theta}_m^2$$

$$T_2 = \frac{1}{2} m r^2 \left[ \frac{3}{2} - \frac{8}{3\pi} \right] \dot{\theta}_m^2 = 0.3256 m r^2 \dot{\theta}_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + m g \left( \frac{4r}{3\pi} \right) \frac{\theta_m^2}{2} = 0.3256 m r^2 \dot{\theta}_m^2$$

FOR SMALL OSCILLATIONS

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_m = \theta_m \omega_n$$

$$g r \left( \frac{4}{3\pi} \right) \theta_m^2 = 0.3256 m r^2 \dot{\theta}_m^2 \omega_n^2$$

$$\omega_n^2 = \frac{0.2122}{0.3256} \frac{g}{r} = 0.6518 \frac{g}{r} \text{ s}^{-2}$$

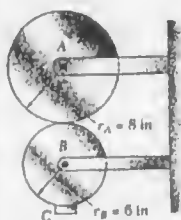
$$\omega_n = 0.8073 \sqrt{\frac{g}{r}}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{0.8073} \sqrt{\frac{r}{g}}$$

$$\tau_n = 7.78 \sqrt{\frac{r}{g}} \text{ s}$$



19.161

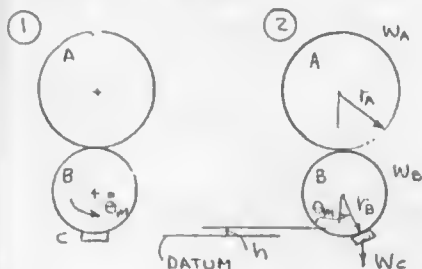


GIVEN:

$W_A = 30 \text{ lb}$ ,  $W_B = 12 \text{ lb}$   
 $W_C = 5 \text{ lb}$ , ATTACHED TO B  
 NO SLIPPING

FIND:

PERIOD OF SMALL OSCILLATIONS



SMALL OSCILLATIONS  $h = r_B(1 - \cos \theta_m) \approx r_B \theta_m^2/2$

POSITION ①  $r_B \dot{\theta}_B = r_A \dot{\theta}_A$ 

$$T_1 = \frac{1}{2} m_C (r_B \dot{\theta}_m)^2 + \frac{1}{2} \bar{I}_B \dot{\theta}_m^2 + \frac{1}{2} \bar{I}_A \left( \frac{r_B}{r_A} \dot{\theta}_m \right)^2$$

$$\bar{I}_B = \frac{W_B r_B^2}{2} \quad \bar{I}_A = \frac{W_A r_A^2}{2}$$

$$T_1 = \frac{1}{2} [m_C r_B^2 + m_B r_B^2/2 + (m_A r_A^2/2) (r_B/r_A)^2] \dot{\theta}_m^2$$

$$T_1 = \frac{1}{2} [(m_C + m_B/2 + m_A/2) r_B^2] \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = m_C g h = m_C g \theta_m^2/2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$\frac{1}{2} [(m_C + m_B/2 + m_A/2) r_B^2] \omega_n^2 \theta_m^2 + 0 =$$

$$0 + m_C g r_B \theta_m^2/2$$

$$\omega_n^2 = \frac{m_C}{m_C + (m_B + m_A)/2} \frac{g}{r_B}$$

$$\omega_n^2 = \frac{5}{5 + (12 + 30)/2} \frac{(32.2 \text{ ft/s}^2)}{(6/12) \text{ ft}}$$

$$\omega_n^2 = 12.39 \text{ s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{12.39}} = 1.785 \text{ s.}$$

19.162



GIVEN:

4.02 GYROSCOPE ROTOR;  
 $\tau_n = 6.00 \text{ s}$  WHEN ROTOR IS  
 SUSPENDED FROM A WIRE  
 AS SHOWN  
 WHEN 1.25 IN. DIAMETER  
 SPHERE IS SUSPENDED IN  
 THE SAME FASHION THE  
 PERIOD  $(\tau_n)_s = 3.80 \text{ s.}$

FIND:

RADIUS OF GYRATION  $\bar{k}$  OF THE ROTOR $k =$  SPRING CONSTANT OF THE WIRE

FOR SPHERE OR ROTOR



$$Z H_0 = (\Sigma H_0)_{\text{eff}}$$

$$k\theta = -\bar{I}\alpha \quad \alpha = \ddot{\theta}$$

$$\ddot{\theta} + \frac{k}{\bar{I}} \theta = 0$$

$$\omega_n^2 = \frac{k}{\bar{I}} \quad \tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\bar{I}/k} \quad (1)$$

ROTOR

$$\bar{I} = m_R \bar{k}^2 = \frac{(4/16 \text{ lb})}{(32.2 \text{ ft/s}^2)} \bar{k}^2 = 7.764 \times 10^{-3} \bar{k}^2$$

$$\text{FROM (1)} \quad 6 = 2\pi \sqrt{\bar{I}/k} \quad (2)$$

SPHERE

$$\bar{I}_s = \frac{2}{5} m r^2 \quad \text{SP WT} = 490 \text{ lb/ft}^3$$

$$m = \frac{W}{g} \quad W = (\text{VOL})(\text{SP WT}) = \frac{4}{3} \pi r^3 \rho$$

$$m = \frac{4}{3} \pi \left[ \frac{(1.25/2)}{(12)} \right]^3 [490 \text{ lb/ft}^3]$$

$$m = 9.006 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\bar{I}_s = \frac{2}{5} (9.006 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[ (1.25/2)/(12) \text{ ft} \right]^2$$

$$\bar{I}_s = 9.772 \times 10^{-6} \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

FROM (1)

$$3.80 = 2\pi \sqrt{\frac{9.772 \times 10^{-6}}{k}} \quad (3)$$

DIVIDE EQ. (2) BY EQ. (3) AND SQUARING,

$$\left( \frac{6}{3.80} \right)^2 = \frac{(7.764 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \bar{k}^2}{9.772 \times 10^{-6} \text{ lb} \cdot \text{s}^2 \cdot \text{ft}}$$

$$\bar{k}^2 = \frac{(9.772 \times 10^{-6})}{(7.764 \times 10^{-3})} \left( \frac{6}{3.80} \right)^2 = 3.138 \text{ ft}^2$$

$$\bar{k} = 0.0560 \text{ ft}$$

$$\bar{k} = 0.672 \text{ in.}$$

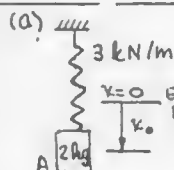
19.163

GIVEN:

1.5-kg BLOCK B CONNECTED BY A CORD TO A 2-kg BLOCK A SUSPENDED FROM A SPRING OF  $k = 3 \text{ kN/m}$  SYSTEM AT REST WHEN THE CORD IS CUT

FIND:

- FREQUENCY, AMPLITUDE AND MAXIMUM VELOCITY OF THE RESULTING MOTION
- MINIMUM TENSION IN THE SPRING DURING THE MOTION
- VELOCITY OF A, 0.3 S AFTER THE CORD IS CUT



POSITION IMMEDIATELY AFTER THE CORD IS CUT

$$x_0 = \frac{m_B g}{k} = \frac{(1.5 \text{ kg})(9.81 \text{ m/s}^2)}{3000 \text{ N/m}}$$

$$x_0 = 0.004905 \text{ m}$$

EQ. (19.10)

$$x = x_m \sin(\omega_n t + \phi) \quad \omega_n = \sqrt{k/m_A}$$

WHERE  $x$  IS MEASURED FROM THE EQUILIBRIUM POSITION

$$\omega_n = \sqrt{\frac{3000 \text{ N/m}}{2 \text{ kg}}} = 38.73 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{38.73}{2\pi} = 6.16 \text{ Hz}$$

INITIAL CONDITIONS ( $t=0$ )

$$x_0 = 0.004905 \text{ m} \quad \dot{x}(0) = 0$$

$$0.004905 = x_m \sin \phi$$

$$0 = x_m \omega_n \cos \phi \quad \phi = \pi/2$$

$$0.004905 = x_m(1)$$

$$x_m = 0.004905 \text{ m} = 4.91 \text{ mm}$$

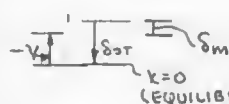
$$\text{MAXIMUM VELOCITY } v_m = \omega_n x_m = (38.73)(0.004905)$$

$$v_m = 0.1900 \text{ m/s}$$

- (b) MINIMUM TENSION IN THE SPRING OCCURS WHEN ITS DEFLECTION IS MINIMUM

$$\delta_{ST} = \frac{m_A g}{k} = \frac{(2 \text{ kg})(9.81 \text{ m/s}^2)}{3000 \text{ N/m}}$$

$$\delta_{ST} = 0.00654 \text{ m}$$



$$\delta_m = \delta_{ST} - x_m$$

$$\delta_m = 0.00654 - 0.004905$$

$$\delta_m = 0.001635 \text{ m}$$

$$F_m = k \delta_m = (3000 \text{ N/m})(0.001635 \text{ m})$$

$$F_m = 4.91 \text{ N}$$

(c) FROM (a)  $x = 0.004905 \sin(38.73t + \pi/2)$

$$\dot{x} = (0.004905)(38.73) \cos(38.73t + \pi/2)$$

At  $t = 0.3 \text{ s}$   $\dot{x} = 0.1900 \cos(38.73(0.3) + \pi/2)$

$$\dot{x}(0.3) = 0.1542 \text{ m/s} \downarrow$$

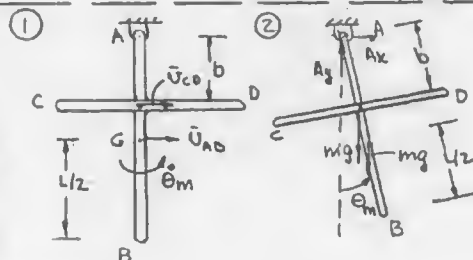
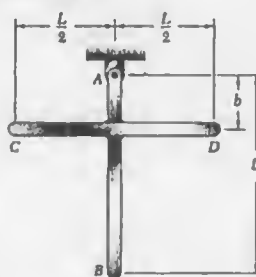
19.164

GIVEN:

TWO RODS EACH OF MASS  $m$  AND LENGTH  $L$ , WELDED TOGETHER TO FORM THE ASSEMBLY SHOWN

FIND:

- THE DISTANCE  $b$  FOR WHICH THE FREQUENCY OF SMALL OSCILLATIONS IS MAXIMUM
- THE CORRESPONDING MAXIMUM FREQUENCY



POSITION ①

$$v_1 = 0$$

$$T_1 = \frac{1}{2} [m \bar{v}_{CD}^2 + m \bar{v}_{AB}^2 + \bar{I}_{CD} \dot{\theta}_m^2 + \bar{I}_{AB} \dot{\theta}_m^2]$$

$$\bar{v}_{CD} = b \dot{\theta}_m \quad \bar{v}_{AB} = (L/2) \dot{\theta}_m$$

$$\bar{I}_{CD} = \bar{I}_{AB} = \frac{1}{12} m L^2$$

$$T_1 = \frac{1}{2} m [b^2 + (L/2)^2 + \frac{1}{12} L^2 + \frac{1}{12} L^2] \dot{\theta}_m^2 = \frac{m}{2} [b^2 + 5(L/2)^2] \dot{\theta}_m^2$$

POSITION ②

$$V_2 = mg b (1 - \cos \theta_m) + mg (L/2) (1 - \cos \theta_m)$$

SMALL ANGLES  $1 - \cos \theta_m \approx 2 \sin^2(\theta_m/2) \approx \theta_m^2/2$

$$V_2 = mg \frac{\theta_m^2}{2} (b + L/2)$$

$$T_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m [b^2 + \frac{5}{12} L^2] \dot{\theta}_m^2 + 0 = 0 + \frac{mg}{2} [b + L/2] \theta_m^2$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$\omega_n^2 = \frac{g(b + L/2)}{(b^2 + 5/12 L^2)} \quad (1)$$

MAX  $\omega_n^2$  WHEN  $d\omega_n^2/db = 0$ 

$$\frac{d\omega_n^2}{db} = \frac{(b + L/2)g - g(b + L/2)(2b)}{(b^2 + 5/12 L^2)^2} = 0$$

$$-b^2 - Lb + (5/12)L^2 = 0$$

$$b = \frac{-L \pm \sqrt{L^2 + (20/12)L^2}}{2} = 0.316L, 1.317L$$

$$b = 0.316L$$

(b) FROM EQ (1) AND THE ANSWER TO (a)

$$\omega_n^2 = \frac{g[0.316 + 0.5]}{[(0.316)^2 + 5/12]L} = 1.580 g/L$$

$$f_n = \frac{\sqrt{\omega_n^2}}{2\pi} = \frac{\sqrt{1.580}}{2\pi} \sqrt{g/L} = 0.200 \sqrt{g/L} \text{ Hz}$$

19.165

GIVEN:

SPRING SUPPORTED MOTOR SPEED  
INCREASED FROM 200 RPM TO  
500 RPM  
AMPLITUDE OF VIBRATION DECREASES  
CONTINUOUSLY FROM 8 mm TO 2.5 mm

FIND:

(a) RESONANT SPEED

(b) AMPLITUDE OF STEADY STATE VIBRATION AT 100 rpm

(1) FOR A MOTOR WITH A ROTOR UNBALANCE  
THE AMPLITUDE OF VIBRATION IS GIVEN  
BY (SEE SAMPLE PROB 19.5)

$$x_m = \frac{P_m / k}{1 - (\omega_f / \omega_n)^2}, \quad P_m = m r \omega_f^2$$

AT 200 rpm

$$-8 = \frac{m r (200)^2 / k}{(1 - (200 / f_n)^2)} \quad (1)$$

AT 500 rpm

$$-2.5 = \frac{m r (500)^2 / k}{(1 - (500 / f_n)^2)} \quad (2)$$

DIVIDING EQ (1) BY EQ. (2) TERM BY TERM,

$$\frac{8}{2.5} = \frac{1 - (500 / f_n)^2 (200)^2}{(1 - (200 / f_n)^2) (500)^2}$$

$$(3.2) (1 - (200 / f_n)^2) = 0.160 (1 - (500 / f_n)^2)$$

$$3.2 (f_n^2 - (200)^2) = 0.160 (f_n^2 - (500)^2)$$

$$(3.2 - 0.160) (f_n^2) = 3.2 (200)^2 - (0.160) (500)^2$$

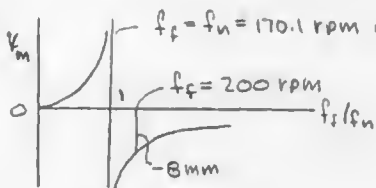
$$f_n^2 = 28947$$

$$f_n = 170.14 \text{ rpm}$$

$$f_n = 170.1 \text{ rpm}$$

RESONANCE WHEN  $f_f = f_n$ 

$$f_f = 170.1 \text{ rpm}$$

(b)  $f_f = f_n = 170.1 \text{ rpm}$  (RESONANCE)

$$x_m = \frac{m r}{k} \omega_f^2 \quad \text{AT 200 rpm} \quad \omega_f = \frac{2\pi(200)}{60}$$

$$\omega_f = \frac{20\pi}{3} \text{ rad/s}$$

$$-8 = \frac{m r (20\pi/3)^2}{1 - (200/170.14)^2} \quad (\text{Eq. 1})$$

$$\frac{m r}{k} = \frac{(-8) (-0.3818)}{(20\pi/3)^2} = 0.006963$$

AT 100 rpm

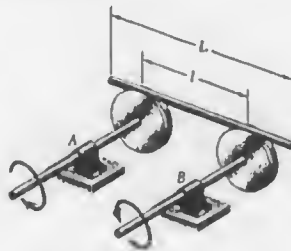
$$x_m = \frac{(0.006963) (10\pi/3)^2}{1 - (100/170.14)^2} = 1.1666 \text{ mm}$$

$$x_m = 1.167 \text{ mm}$$

19.166

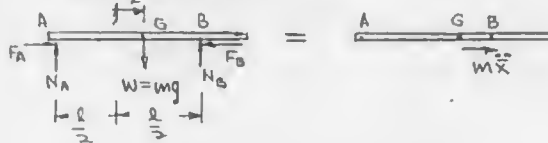
GIVEN:

ROD OF MASS  $m$  AND  
LENGTH  $L$  RESTS  
ON TWO PULLEYS  
WHICH ROTATE  
IN OPPOSITE  
DIRECTIONS AS  
SHOWN  
 $\mu_k$  = COEFFICIENT  
OF KINETIC FRICTION  
BETWEEN THE ROD  
AND THE PULLEYS



FIND:

FREQUENCY OF VIBRATION IF THE ROD IS  
GIVEN A SMALL DISPLACEMENT TO THE RIGHT  
AND RELEASED



$$+\circlearrowleft \sum M_A = \sum (M_A)_{\text{eff}}: L N_B - \left(\frac{L}{2} + x\right) mg = 0$$

$$N_B = \left(\frac{1}{2} + \frac{x}{L}\right) mg$$

$$+\uparrow \sum F_y = \sum (F_y)_{\text{eff}}: N_A + \left(\frac{1}{2} + \frac{x}{L}\right) mg - mg = 0$$

$$N_A = \left(\frac{1}{2} - \frac{x}{L}\right) mg$$

THUS

$$F_A = \mu_k N_A = \mu_k \left(\frac{1}{2} - \frac{x}{L}\right) mg$$

$$F_B = \mu_k N_B = \mu_k \left(\frac{1}{2} + \frac{x}{L}\right) mg$$

 $\rightarrow$ 

$$\sum F = \sum (F_x)_{\text{eff}}$$

$$F_A - F_B = m \ddot{x}$$

$$\mu_k \left(\frac{1}{2} - \frac{x}{L}\right) mg - \mu_k \left(\frac{1}{2} + \frac{x}{L}\right) mg = m \ddot{x}$$

$$m \ddot{x} + \frac{2\mu_k g}{L} x = 0$$

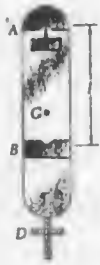
$$\omega_n^2 = \frac{2\mu_k g}{L}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2\mu_k g}{L}}$$

19.167

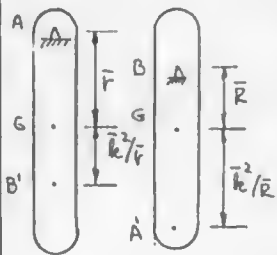
GIVEN:

COMPOUND PENDULUM WITH KNIFE EDGES AT A AND B A DISTANCE  $l$  APART COUNTERWEIGHT D IS ADJUSTED SO THAT THE PERIOD IS THE SAME WHEN EITHER KNIFE EDGE IS USED



SHOW THAT:

THE PERIOD IS THE SAME AS A SIMPLE PENDULUM OF LENGTH  $l$  (I.E.  $T_n = 2\pi\sqrt{l/g}$ ) AND THAT  $g = 4\pi^2 l / T^2$



FROM PROB 19.52 THE LENGTH OF AN EQUIVALENT SIMPLE PENDULUM IS:

$$l_A = F + l^2/F$$

AND

$$l_B = R + l^2/R$$

$$\text{BUT } T_A = T_B$$

$$2\pi\sqrt{\frac{l_A}{g}} = 2\pi\sqrt{\frac{l_B}{g}}$$

THUS

$$l_A = l_B$$

$$\text{FOR } l_A = l_B$$

$$F + \frac{l^2}{F} = R + \frac{l^2}{R}$$

$$F^2 R + l^2 R = F R^2 + l^2 F$$

$$FR(F - R) = l^2(F - R)$$

$$(F - R) \neq 0$$

$$\text{THUS } FR = l^2$$

$$\text{OR } F = \frac{l^2}{R}, \quad R = \frac{l^2}{F}$$

$$\text{THUS } AG = GA' \quad \text{AND} \quad BG = GB'$$

$$\text{THAT IS, } A = A' \quad \text{AND} \quad B = B'$$

$$\text{NOTING THAT } l_A = l_B = l$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{OR } g = \frac{4\pi^2 l}{T^2}$$

19.168

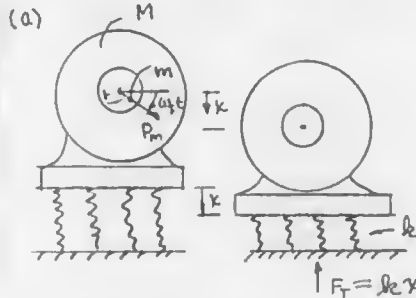
GIVEN:

400-kg MOTOR SUPPORTED BY FOUR SPRINGS. EACH SPRING HAS A CONSTANT OF 150 kN/m UNBALANCE IS 23 g AT 100 mm FROM THE AXIS OF ROTATION

FIND:

FOR A SPEED OF 800 RPM

- (a) THE AMPLITUDE OF THE FLUCTUATING FORCE TRANSMITTED TO THE FOUNDATION  
(b) THE AMPLITUDE OF THE VERTICAL MOTION OF THE MOTOR



FROM EQ (19.33)

$$x_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2} \quad (1)$$

$$\text{THUS } F_T = k x_m = \frac{P_m}{1 - \omega_f^2/\omega_n^2} \quad (2)$$

$$k = (4)(150,000 \text{ N/m}) = 600,000 \text{ N/m}$$

$$\omega_n^2 = k/m = \frac{600,000}{400} = 1500 \text{ s}^{-2}$$

$$\omega_f^2 = (2\pi f_f)^2 = [(2\pi)(800/60)]^2 = 7018 \text{ s}^{-2}$$

$$P_m = m r \omega_f^2 = (0.023 \text{ kg})(0.100 \text{ m})(7018 \text{ s}^{-2})$$

$$P_m = 16.14 \text{ N}$$

SUBSTITUTING THE ABOVE VALUES INTO EQ. 2

$$F_T = \frac{16.14}{1 - (7018/1500)} = -4.388 \text{ N}$$

$$F_T = 4.39 \text{ N}$$

(b)

$$x_m = F_T/k = \frac{(4.388 \text{ N})}{(600,000 \text{ N/m})}$$

$$x_m = 0.00731 \times 10^{-3} \text{ m}$$

$$x_m = 0.00731 \text{ mm}$$

19.169

GIVEN:

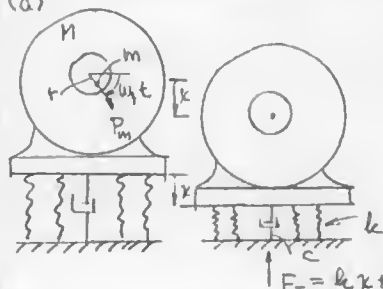
400 kg MOTOR SUPPORTED BY  
FOUR SPRINGS EACH WITH  
 $k = 150 \text{ kN/m}$ ,  
AND A DASHPOT WITH  
 $C = 6500 \text{ N·s/m}$   
UNBALANCE IS 23 g AT 100 mm  
FROM THE AXIS OF ROTATION

FIND:

FOR A SPEED OF 800 rpm

- (a) AMPLITUDE OF THE FLUCTUATING FORCE  
TRANSMITTED TO THE FOUNDATION  
(b) AMPLITUDE OF THE VERTICAL MOTION  
OF THE MOTOR

(a)



$$x = x_m \sin(\omega_f t + \phi)$$

$$\dot{x} = x_m \omega_f \cos(\omega_f t + \phi)$$

$$F_T = kx + C\dot{x} = kx_m \sin(\omega_f t + \phi) + Cx_m \omega_f \cos \omega_f t$$

AMPLITUDE,  $(F_T)_m = x_m \sqrt{k^2 + C^2 \omega_f^2}$  (1)

FROM EQ. (19.52)  $x_m = \frac{P_m}{\sqrt{(k - M\omega_f^2)^2 + (C\omega_f)^2}}$  (2)

$$k = 4(150,000 \text{ N/m}) = 600,000 \text{ N/m}$$

$$\omega_n^2 = k/M = 600,000/400 = 1500 \text{ s}^{-2}$$

$$\omega_f^2 = (2\pi f_f)^2 = [2\pi(800)/60]^2 = 7018 \text{ s}^{-2}$$

$$P_m = m r \omega_f^2 = (0.023 \text{ kg})(0.100 \text{ m})(7018) = 16.14 \text{ N}$$

FROM (2)  $x_m = \frac{16.14}{\sqrt{(600,000 - 400 \times 7018)^2 + (6500)^2 (7018)}}$

$$x_m = 7.10 \times 10^{-6} \text{ m} \quad (3)$$

FROM (1)

$$(F_T)_m = 7.10 \times 10^{-6} \text{ m} \sqrt{(600,000)^2 + (6500)^2 (7018)}$$

$$(F_T)_m = 5.75 \text{ N}$$

(b) FROM (3)

$$x_m = 0.00710 \text{ mm}$$

NOTE: COMPARING RESULTS WITH PROB. 19.168

IN WHICH THERE IS NO DASHPOT, THE  
AMPLITUDE OF THE FORCE HAS INCREASED  
WHILE THE AMPLITUDE OF VERTICAL MOTION  
DECREASES.

19.170



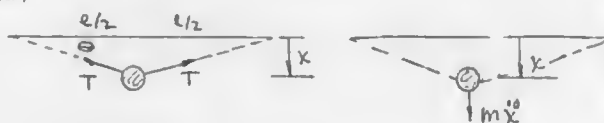
GIVEN:

SHALL MASS  $m$  ATTACHED TO AN ELASTIC  
CORD OF LENGTH  $l$ , IN A HORIZONTAL  
PLANE  
TENSION IN THE CORD REMAINS  
CONSTANT AS THE BALL IS GIVEN  
A SMALL DISPLACEMENT PERPENDICULAR  
TO THE CORD AND RELEASED

FIND:

- (a) DIFFERENTIAL EQUATION OF MOTION OF  
THE BALL  
(b) THE PERIOD OF VIBRATION

(a)



$$+ \uparrow \Sigma F = m\ddot{x}$$

$$2T \sin \theta = m\ddot{x}$$

FOR SMALL  $x$ ,  $\sin \theta \approx \tan \theta = x/(l/2)$

$$m\ddot{x} + \frac{2T}{(l/2)} x = 0$$

$$m\ddot{x} + \frac{4T}{l} x = 0$$

(b)  $\omega_n^2 = \frac{4T}{ml}$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{4T/ml}} = \pi \sqrt{\frac{ml}{T}}$$

# 19.C1 GIVEN:

PERIOD OF A SIMPLE PENDULUM OF LENGTH  $l$  IS,

$$T_n = 2\pi \sqrt{\frac{l}{g}} \left[ 1 + \left(\frac{1}{2}\right)^2 C^2 + \left(\frac{1 \times 3}{2 \times 4}\right)^2 C^4 + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 C^6 + \dots \right]$$

WHERE  $C = \sin \frac{1}{2} \theta_m$  AND  $\theta_m$  IS THE AMPLITUDE

## FIND:

THE SUM OF THE SERIES IN BRACKETS USING SUCCESSIVELY 1, 2, 4, 8 AND 16 TERMS FOR VALUES OF  $\theta_m$  FROM  $30^\circ$  TO  $120^\circ$  USING  $30^\circ$  INCREMENTS. EXPRESS RESULTS WITH FIVE SIGNIFICANT FIGURES

REWRITE GIVEN SERIES IN TERMS OF  $n = 1, 2, 3, \dots$

$$\text{AMPLITUDE} = \theta_m$$

$$C = \frac{1}{2} \sin \theta_m$$

$$\text{LET } T = 2\pi \sqrt{\frac{l}{g}} [B] \text{ WHERE } B = \left[ 1 + \left(\frac{1}{2}\right)^2 C^2 + \left(\frac{1 \times 3}{2 \times 4}\right)^2 C^4 + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 C^6 + \dots \right]$$

WE MAY COMPUTE  $B$  AS FOLLOWS:

$$n=1: B = \left[ 1 + \left(\frac{2n-1}{2n}\right)^2 C^2 \right]$$

$$n=2: B = \left[ 1 + \left(\frac{2n-1}{2n}\right)^2 C^2 + \left(\frac{2n-1}{2n}\right)^2 C^2 \right]$$

$$n=3: B = \left[ 1 + \left(\frac{2n-1}{2n}\right)^2 C^2 + \left(\frac{2n-1}{2n}\right)^2 C^2 + \left(\frac{2n-1}{2n}\right)^2 C^2 \right]$$

AT EACH STEP THE QUANTITY ABOVE THE  $\frac{2n-1}{2n}$  IS THE CHANGE IN  $B$  AND IS DENOTED BY  $\Delta B$

AND THE QUANTITY  $\left(\frac{2n-1}{2n}\right)^2 C^2$  IS DENOTED BY FACTOR =  $\frac{2n-1}{2n} C$

## OUTLINE OF PROGRAM

CALCULATE  $C = \frac{1}{2} \sin \theta_m$  FOR  $\theta_m = 30^\circ$   
 CALCULATE  $B$ , USING THE ALGORITHM ABOVE  
 FOR  $n = 1, 2, 4, 8, 16$   
 PRINT  $B$  FOR  $\theta_m$  AND  $n$   
 REPEAT FOR  $\theta_m = 60^\circ, 90^\circ$  AND  $120^\circ$

## PROGRAM OUTPUT

Amplitude = 30 degree

N	Bracket
1	1.01675
2	1.01738
4	1.01741
8	1.01741
16	1.01741

Amplitude = 60 degree

N	Bracket
1	1.06250
2	1.07129
4	1.07311
8	1.07310
16	1.07310

Amplitude = 90 degree

N	Bracket
1	1.12500
2	1.16016
4	1.17704
8	1.18022
16	1.18034

Amplitude = 120 degree

N	Bracket
1	1.18750
2	1.26660
4	1.33146
8	1.36460
16	1.37240

## 19.C2

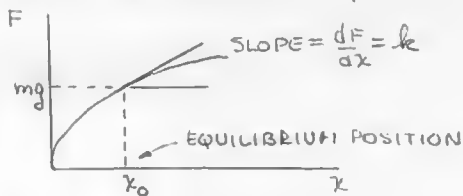
GIVEN:

FORCE DEFLECTION EQUATION FOR A CLASS OF SPRING IS  $F = 5x^{1/n}$  WHERE  $F$  IS IN NEWTONS AND  $x$  IS THE DEFLECTION IN METERS

FIND:

FOR A BLOCK OF MASS  $m$  SUSPENDED FROM THE SPRING AND IS GIVEN A SMALL DOWNWARD DISPLACEMENT FROM ITS EQUILIBRIUM POSITION, THE FREQUENCY OF VIBRATION OF THE BLOCK FOR  $m = 0.2, 0.6$  AND  $1.0$  kg AND FOR VALUES OF  $n$  FROM 1 TO 2 USING 0.2 INCREMENTS

ANALYSIS

FORCE-DEFLECTION CURVE  $F = 5x^{1/n}$ 

$$k = \frac{dF}{dx} = \frac{5}{n} x^{\frac{1}{n}-1} = \frac{5}{n} x_0^{\frac{1-n}{n}}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{n}} x_0^{\frac{1-n}{2n}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{5}{n}} x_0^{\frac{1-n}{2n}} \quad (1)$$

FOR ANY  $mg$ , THE EQUILIBRIUM POINT IS  $F = mg = 5x_0^{1/n}$

$$x_0 = \left(\frac{mg}{5}\right)^n \quad (2)$$

OUTLINE OF PROGRAM

1. CALCULATE  $x_0$  FROM EQ (2) FOR  $m = 0.2$  kg AND  $n = 2$
2. SUBSTITUTE  $x_0$  FROM (2) INTO (1),
3. CALCULATE  $f_n$  AND PRINT  $f_n$ ,  $m$  AND  $n$
4. REPEAT STEPS 1-3 FOR  $n = 1.8, 1.6, 1.4, 1.2$  AND  $1.0$
5. REPEAT STEPS 1-4 FOR  $m = 0.6$  AND  $1.0$  kg

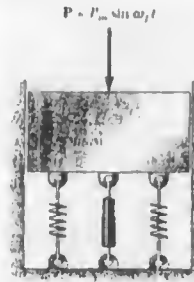
PROGRAM OUTPUT

n	m (kg)	f (Hz)
2.0	0.20	0.898
1.8	0.20	0.862
1.6	0.20	0.833
1.4	0.20	0.811
1.2	0.20	0.798
1.0	0.20	0.796
2.0	0.60	0.299
1.8	0.60	0.321
1.6	0.60	0.346
1.4	0.60	0.376
1.2	0.60	0.413
1.0	0.60	0.459
2.0	1.00	0.180
1.8	1.00	0.203
1.6	1.00	0.230
1.4	1.00	0.263
1.2	1.00	0.304
1.0	1.00	0.356

## 19.C3

GIVEN:

MACHINE ELEMENT SUPPORTED BY SPRINGS AND CONNECTED TO A DASHPOT IS SUBJECTED TO A PERIODIC FORCE AS SHOWN



FIND:

FOR FREQUENCY RATIOS  $\omega_f/\omega_n$  EQUAL TO 0.8, 1.4 AND 2.0 AND FOR DAMPING FACTORS  $c/c_c$  EQUAL TO 0, 1, AND 2, THE TRANSMISSIBILITY  $T_m = F_m/P_m$  WHERE  $F_m$  IS THE MAXIMUM FORCE TRANSMITTED TO THE FOUNDATION TO THE MAXIMUM VALUE  $P_m$

ANALYSIS

FROM PROB. 19.148,

$$T_m = \frac{P_m}{F_m} = \frac{1 + [2(c/c_c)(\omega_f/\omega_n)]^2}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2}}$$

OUTLINE OF PROGRAM (USING THE ABOVE PROGRAM)

1. INPUT  $c/c_c = 0$
2. INPUT  $\omega_f/\omega_n = 0.8$
3. CALCULATE  $T_m$  AND PRINT FOR  $c/c_c$  AND  $\omega_f/\omega_n$  THE VALUE OF  $T_m$
4. REPEAT STEPS 2 AND 3 FOR  $\omega_f/\omega_n = 1.4$  AND THEN FOR  $\omega_f/\omega_n = 2.0$
5. REPEAT STEPS 1 THROUGH 4 FOR  $c/c_c = 1.0$  AND THEN FOR  $c/c_c = 2.0$

PROGRAM OUTPUT

$\omega_f/\omega_n$	$c/c_c$	$T_m$
FREQ. RATIO	DAMPING FACTOR	TRAN. RATIO
0.80	0.0	2.778
1.40	0.0	1.042
2.00	0.0	0.333
0.80	1.0	1.150
1.40	1.0	1.004
2.00	1.0	0.825
0.80	2.0	1.041
1.40	2.0	1.001
2.00	2.0	0.944

19.C4

GIVEN:

15-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH OF CONSTANT 60 kN/m. UNBALANCE EQUALS 20 g AT 125 mm FROM AXIS OF ROTATION.

FIND:

AMPLITUDE AND ACCELERATION FOR MOTOR SPEEDS OF 1000 TO 2500 rpm USING 100 rpm INCREMENTS

ANALYSIS

$$\text{FROM EQ. (19.33)} \quad \gamma_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2} \quad (1)$$

WHERE  $P_m = m r \omega_f^2$  (SAMPLE PROB. 19.5)

$$k = 4 \times 60,000 \text{ N/m} = 240,000 \text{ N/m}$$

$$P_m = (0.020)(0.125) \omega_f^2 = 2500 \times 10^{-6} \omega_f^2$$

$$\omega_n^2 = \frac{k}{M} = \frac{240,000 \text{ N/m}}{15 \text{ kg}} = 16000 \text{ s}^{-2}$$

SUBSTITUTE THE ABOVE VALUES INTO (1)

$$\gamma_m = \frac{(2500 \times 10^{-6} \omega_f^2) / (240,000)}{1 - \omega_f^2 / 16000} \text{ m} \quad (2)$$

$$a_m = \omega_f^2 \gamma_m \text{ m/s}^2 \quad (3)$$

$$\omega_f = (\text{RPM})(2\pi)/60 \quad (4)$$

OUTLINE OF PROGRAM

1. USING EQ. (2) AND NOTING EQ. (4) INPUT AN INITIAL VALUE OF MOTOR SPEED OF 1000 rpm
2. CALCULATE  $\gamma_m$
3. CALCULATE FROM EQ. (3),  $a_m$
4. PRINT rpm,  $\gamma_m$  AND  $a_m$
5. REPEAT STEPS 1 THROUGH 4 FOR MOTOR SPEEDS OF 1100 TO 2500 rpm IN STEPS OF 100 rpm

PROGRAM OUTPUT

TO OBTAIN THE UNITS CORRESPONDING TO THE ANSWERS, BELOW, MULTIPLY EQ. (2) BY 1000, AND IF THE RESULT (IN MM) IS USED IN EQ. (3), DIVIDE IT BY 1000.

SPEED (RPM)	AMP. (mm)	ACCEL. (m/s**2)
1000	0.363	3.98
1100	0.810	10.75
1200	12.615	199.21
1300	-1.219	22.60
1400	-0.652	14.02
1500	-0.474	11.70
1600	-0.388	10.88
1700	-0.337	10.67
1800	-0.303	10.77
1900	-0.280	11.07
2000	-0.262	11.51
2100	-0.249	12.05
2200	-0.239	12.66
2300	-0.230	13.35
2400	-0.223	14.10
2500	-0.217	14.90

19.C5

GIVEN:

SAME AS 19.C4 AT LEFT WITH A DASHPOT HAVING A COEFFICIENT OF DAMPING  $c = 2.5 \text{ kN/s}$  IS CONNECTED TO THE MOTOR BASE AND THE GROUND

FIND:

AMPLITUDE AND ACCELERATION FOR MOTOR SPEEDS OF 1000 TO 2500 rpm USING 100 rpm INCREMENTS

ANALYSIS

FROM EQ. (19.52)

$$\gamma_m = \frac{P_m}{\sqrt{(k - M\omega_f^2)^2 + (c\omega_f)^2}} \quad (1)$$

$$k = 4 \times 60,000 \text{ N/m} = 240,000 \text{ N/m}$$

$$P_m = m r \omega_f^2 = (0.020)(0.125) \omega_f^2 = 2500 \times 10^{-6} \omega_f^2$$

SUBSTITUTE INTO (1)

$$\gamma_m = \frac{2500 \times 10^{-6} \omega_f^2}{\sqrt{(240,000 - 15(\omega_f^2))^2 + (2500)^2 \omega_f^2}} \text{ m} \quad (2)$$

$$a_m = \omega_f^2 \gamma_m \quad (3)$$

$$\omega_f = (\text{RPM})(2\pi)/60 \quad (4)$$

OUTLINE OF PROGRAM

1. USING EQ. (2) AND NOTING EQ. (4), INPUT AN INITIAL VALUE OF MOTOR SPEED OF 1000 rpm
2. CALCULATE  $\gamma_m$  (IN METERS)
3. CALCULATE FROM EQ. (3) THE ACCELERATION  $a_m$
4. PRINT rpm,  $\gamma_m$ ,  $a_m$
5. REPEAT STEPS 1 THROUGH 4 FOR MOTOR SPEEDS OF 1100 TO 2500 rpm IN INCREMENTS OF 100 rpm

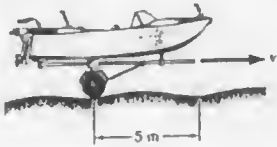
PROGRAM OUTPUT

SEE NOTE AT LEFT

SPEED (RPM)	AMP. (mm)	ACCEL. (m/s**2)
1000	0.1006	1.103
1100	0.1140	1.513
1200	0.1257	1.984
1300	0.1353	2.507
1400	0.1430	3.074
1500	0.1491	3.679
1600	0.1538	4.318
1700	0.1574	4.987
1800	0.1601	5.688
1900	0.1621	6.419
2000	0.1637	7.180
2100	0.1649	7.972
2200	0.1657	8.796
2300	0.1664	9.653
2400	0.1669	10.542
2500	0.1673	11.464



## 19.C6



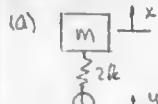
GIVEN:

TRAILER AND LOAD MASS  
 $= 250 \text{ kg}$   
 SUPPORTED BY TWO  
 SPRINGS EACH WITH  
 $k = 10 \text{ kN/m}$   
 ROAD IS A SINE  
 CURVE WITH AN  
 AMPLITUDE OF  $40 \text{ mm}$   
 AND WAVE LENGTH OF  
 $5 \text{ m}$ .

FIND:

- (a) AMPLITUDE OF VIBRATION AND MAXIMUM VERTICAL  
 ACCELERATION OF THE TRAILER FOR SPEEDS OF  
 10 TO  $80 \text{ km/h}$  USING  $5 \text{ km/h}$  INCREMENTS  
 (b) USING APPROPRIATE SMALLER INCREMENTS  
 DETERMINE THE RANGE OF VALUES OF THE SPEED  
 FOR WHICH THE TRAILER WILL LEAVE THE GROUND.

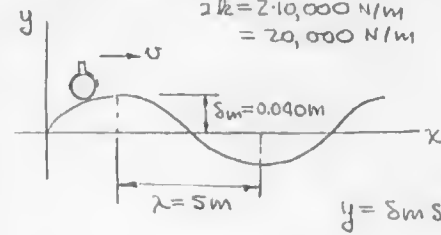
ANALYSIS



FROM EQ (19.33')

$$y = \delta_m \sin \omega_f t \quad x_m = \frac{\delta_m}{1 - (\omega_f / \omega_n)^2}$$

$$2k = 2 \cdot 10,000 \text{ N/m} = 20,000 \text{ N/m}$$



$$y = \delta_m \sin \frac{2\pi x}{\lambda}$$

$$\omega_n^2 = \frac{2k}{m} = \frac{20,000 \text{ N/m}}{250 \text{ kg}}$$

$$\omega_n^2 = 80 \text{ s}^{-2}$$

$$x = vt$$

$$y = \delta_m \sin(\omega_f / \lambda) t$$

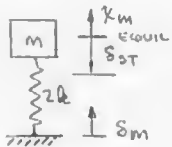
$$c = \lambda / v \quad \omega_f = 2\pi / c$$

$$\omega_f = \frac{2\pi v}{\lambda} = \frac{2\pi v}{5}$$

$$\text{THUS } x_m = \frac{40 \text{ mm}}{1 - \left( \frac{2\pi v (1000)}{3600} \right)^2 / 80} \text{ mm} \quad (1)$$

v IN km/h

- (b) WHEN  $y$  AND  $x$  ARE IN PHASE THEY  
 HAVE THE SAME SIGN (LE  $x_m +$ )



THE TRAILER LEAVES THE  
 GROUND WHEN THE FORCE  
 IN THE SPRING IS ZERO.  
 THIS OCCURS WHEN

$$x_m > \delta_m + \delta_{ST} \text{ WHERE}$$

$$\delta_{ST} = \frac{mg}{2k} = \frac{250 \times 9.81}{20,000}$$

$$\delta_{ST} = 0.1226 \text{ m} = 122.6 \text{ mm}$$

THUS WHEN  $x_m > 122.6 + 40 = 162.6 \text{ mm}$  THE  
 TRAILER WILL LEAVE THE GROUND

WHEN  $y$  AND  $x$  ARE OUT OF PHASE ( $x_m -$ ) THE  
 TRAILER WILL LEAVE THE GROUND WHEN

$$x_m < -122.6 + 40 = -82.6 \text{ mm}$$

## 19.C6 CONTINUED

## OUTLINE OF PROGRAM

- (a) INPUT TO EQ. 1 VALUES OF VELOCITY FROM  
 10 TO  $80 \text{ km/h}$  IN  $5 \text{ km/h}$  INTERVALS  
 AND PRINT THE RESULTS  
PROGRAM OUTPUT

SPEED (km/h)	AMPLITUDE (mm)
10.0	47.19
15.0	60.85
20.0	102.36
25.0	832.11
30.0	-107.88
35.0	-46.20
40.0	-27.84
45.0	-19.19
50.0	-14.25
55.0	-11.09
60.0	-8.92
65.0	-7.36
70.0	-6.19
75.0	-5.29
80.0	-4.57

- (b) FROM PART (b) OF THE ANALYSIS WE NOTE  
 THAT IF  $x_m > 162.6 \text{ mm}$  OR  $x_m < -82.6 \text{ mm}$   
 THE TRAILER WILL LEAVE THE GROUND. FROM  
 THE RESULTS OF PART (a) WE NOTE THAT  
 THIS OCCURS BETWEEN THE VELOCITIES  
 OF  $20 \text{ km/h}$  AND  $35 \text{ km/h}$   
 RERUN EQ. (1) FOR VELOCITIES OF  $20 \text{ km/h}$   
 TO  $35 \text{ km/h}$  AT INTERVALS OF  $0.1 \text{ km/h}$  AND  
 PRINT THE RESULTS

SPEED (km/h)	AMPLITUDE (mm)	SPEED (km/h)	AMPLITUDE (mm)
22.2	160.41	26.8	-425.78
22.3	164.89	26.9	-391.68
22.4	169.65	27.0	-362.54
22.5	174.72	27.1	-337.35
22.6	180.13	27.2	-315.35
22.7	185.90	27.3	-295.98
22.8	192.09	27.4	-278.79
22.9	198.73	27.5	-263.44
23.0	205.88	27.6	-249.64
23.1	213.60	27.7	-237.17
23.2	221.96	27.8	-225.85
23.3	231.04	27.9	-215.53
23.4	240.94	28.0	-206.08
23.5	251.77	28.1	-197.39
23.6	263.68	28.2	-189.37
23.7	276.82	28.3	-181.96
23.8	291.41	28.4	-175.08
23.9	307.70	28.5	-168.68
24.0	326.00	28.6	-162.72
24.1	346.70	28.7	-157.14
24.2	370.31	28.8	-151.91
24.3	397.49	28.9	-147.00
24.4	429.12	29.0	-142.39
24.5	466.39	29.1	-138.04
24.6	510.94	29.2	-133.94
24.7	565.14	29.3	-130.06
24.8	632.52	29.4	-126.38
24.9	718.53	29.5	-122.90
25.0	832.14	29.6	-119.59
25.1	989.16	29.7	-116.45
25.2	1220.36	29.8	-113.45
25.3	1594.54	29.9	-110.60
25.4	2303.69	30.0	-107.88
25.5	4161.94	30.1	-105.28
25.6	*****	30.2	-102.80
25.7	*****	30.3	-100.42
25.8	*****	30.4	-96.14
25.9	*****	30.5	-95.96
26.0	*****	30.6	-93.86
26.1	*****	30.7	-91.85
26.2	-878.93	30.8	-89.91
26.3	-747.58	30.9	-88.05
26.4	-650.06	31.0	-86.26
26.5	-574.79	31.1	-84.54
26.6	-514.95	31.2	-82.88
26.7	-466.22	31.3	-81.27